Set

A set is an unorded collection of object, where these objects are called elements or members of the set. We can write $a \in A$ to denote that a is an element of set A & $a \notin A$ to denote a is not an element of the set A.

eg: $V = \{a, e, i, o, u\}$
where V is the set of all vowels in English Alphabet

$O = \{1, 3, 5, 7, 9\}$; set of odd numbers less than 10.

$I = \{1, 2, 3 \ldots 99\}$, set of positive integers less than 100.

Note: A set may have unrelated elements.
$R = \{a, Sunday, 1, 8, Monday, 2\}$

Representation of set:
Two Method 1) Roster Method 2) Set Builder notation

1) Roster Method: Use of brackets to enclose elements
$O = \{1, 3, 5, 7, 9\}$

2) Set Builder Notation: Characterize all the elements in the set by stating the property that all the elements have.

eg $O = \{x \mid x \text{ is an odd and } x < 10\}$

$I = \{x \in Z^+ \mid x \text{ is odd and } x < 100\}$

Note: $N = \{1, 2, 3, \ldots\}$, set of natural numbers

$Z = \{\ldots, -3, -2, -1, 0, 1, 2, \ldots\}$, set of integers

$Z^+ = \{1, 2, 3, \ldots\}$ set of positive integers

$Q = \{p/q \mid p \in Z, q \in Z \ \& \ q \neq 0\}$, set of rational numbers

R is set of real number

C is set of complex number

Equal Set: Two sets are equal if and only if they have the same elements. Therefore, if A & B are sets, then A and B are equal if and only if $\forall x \, (x \in A \leftrightarrow x \in B)$. We can write it as $(A = B)$

Note: $A = \{2, 3, 4\}$, $B = \{2, 2, 3, 4, 3\}$

$C = \{4, 3, 2\}$, $D = \{2, 4, 4, 3, 3\}$

All the sets above are equal. $(A = B = C = D)$

Empty Set (ϕ): This is a special type of set that has no element. It is also called null set (ϕ)

$\phi = \{\ \}$

Singleton Set: This is a set which has only one element.

eg $A = \{1\}$

$B = \{\phi\}$

Note: The single element on B is a null set. So B is not an empty set.

Subset : The set A is a subset of B if and only if every element of A is also an element of B. The notation used is $A \subseteq B$.

$$\forall x\ (x \in A \rightarrow x \in B)$$

If A is not a subset of B it is represented as
$A \not\subseteq B$

eg if $A = \{1, 3, 5\}$

$B = \{x \mid x$ is an odd number and $x < 10\}$

$A \subseteq B$

$C = \{2, 3, 5\}$

$C \not\subseteq B$

Note: for any set S $\phi \subseteq S$ & $S \subseteq S$.

Proper Subset: If A is a subset of B and $A \neq B$ then the set A is a proper subset of B, denoted as $A \subset B$

eg $A = \{1, 3, 5\}$ $C = \{1, 3, 5\}$
$B = \{1, 3, 5, 7\}$

$A \subset B$. & $C \not\subset B$

Note: To show that two set A & B are equal, we can show that $A \subseteq B$ & $B \subseteq A$.

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Size of a Set (Cardinality) : If S is a finite set then if n is the 'n' is the cardinality of S the set S.
number of unique element of is denoted as $|S|$.
The cardinality of set S

eg $A = \{1, 1, 3, 5\}$ $B = \{\phi\}$
$|A| = 3$ $|B| = 1$

Note: $|\phi| = 0$

Cardinality of infinite set can't be determined
eg set of positive integers.

Power of a Set (Power Set) : Given a Set S, the power set of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$.

eg if $A = \{0, 1, 2\}$.
$P(A) = \{\phi, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\},$
$\{1,2,3\}\}$

if $B = \{\phi\}$
$P(B) = \{\phi, \{\phi\}\}$
$P(\phi) = \{\phi\}$

Note the cardanility of a set S in 'n' then the Cardinality of it power set is 2^n.
$|P(S)| = 2^n$

Venn Diagram: They are the Graphical representation of a set.

Cartesian Product
Let A and B be sets. The cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$, Hence
$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}$$

if $A = \{1, 2\}$ and $B = \{a, b, c\}$
$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

Find $B \times A$ and check if $(A \times B) = (B \times A)$

Note: $A \times B = B \times A$ if $A = B$ or $A = \phi$ or $B = \phi$

The Cartesian product of the set $A_1, A_2, \ldots, A_n$, denoted by $A_1 \times A_2 \cdots \times A_n$ is the set of ordered n tuples $(a_1, a_2, \ldots, a_n)$, where a_i belongs to A_i for $i = 1, 2, 3, \ldots n$.
$$A_1 \times A_2 \times A_3 \times \cdots \times A_n = \{(a_1, a_2, a_3, \ldots, a_n) \mid a_i \in A_i \text{ for } i = 1, 2, 3, \ldots, n\}$$

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if $A=\{0,1\}$, $B=\{1,2\}$ & $C=\{0,1,2\}$

$A\times B\times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0),$
$(0,2,2), \cancel{(0,2} (1,1,0), (1,1,1), (1,1,2),$
$(1,2,0), (1,2,1), (1,2,2)\}$

find $B\times A\times C$

if $A=\{1,2\}$ find $A^3 = A\times A\times A$

Set Operations

1) Union: Let A and B be sets. The union of the sets A and B is denoted by $A\cup B$, which is the set that contains those element that are either in A or B, or in both

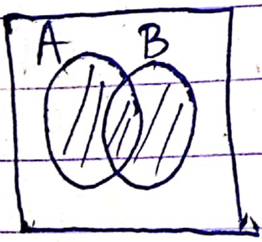
$A\cup B = \{x \mid x\in A \vee x\in B\}$

if $A=\{1,3,5\}$ & $B=\{1,2,3\}$

$A\cup B = \{1,2,3,5\}$

2) Intersection: Let A & B be sets. The intersection of the sets A & B is denoted by $A\cap B$, which is the set of element both in A and B.

$A\cap B = \{x \mid x\in A \wedge x\in B\}$



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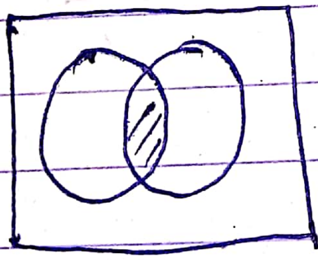


$A\cap B$

if $A = \{1, 2, 3\}$ & $B = \{3, 4, 5\}$
$A \cap B = \{3\}$

Note: if two set are disjoint then their intersection is the empty set.

3) Set Difference: The difference of A & B, denoted by $A - B$ is the set containing those elements that are in A but not in B. Also denoted as A/B

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

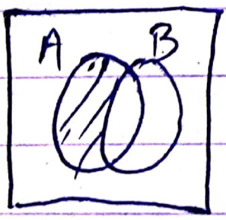
if $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$
$A - B = \{1, 2\}$

Note complement of a set A ($\bar{A}$) is the difference of U & A where U is the universal set

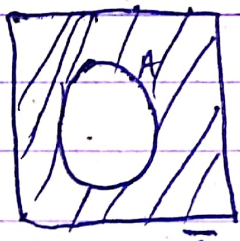
$$\bar{A} = \{x \in U \mid x \notin A\}$$

eg if $U = \{1, 2, 3, 4, 5\}$ $A = \{2, 5\}$

$$\bar{A} = \{1, 3, 4\}$$



$A - B$



$\bar{A} = U - A$

Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$ using set Builder notation

$\overline{A \cap B} = \{x \mid x \notin A \cap B\}$ [Complement]

$= \{x \mid \neg(x \in A \cap B)\}$ [using negation]

$= \{x \mid \neg(x \in A \wedge x \in B)\}$ [∵ intersection]

$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$ [first law of de morgan]

$= \{x \mid (x \notin A) \vee (x \notin B)\}$ [using negation]

$= \{x \mid x \in \bar{A} \vee x \in \bar{B}\}$ [using definition of complement]

$= \{x \mid x \in \bar{A} \cup \bar{B}\}$ [using definition of union]

$= \bar{A} \cup \bar{B}$.

Prove that a) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ using set Builder notation

b) $\overline{A \cup (B \cap C} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

Inclusion - exclusion principle:

It is a counting technique which generalizes the familiar method of obtaining the number of elements in the union of two or more finite set.

for 2 set $|A \cup B| = |A| + |B| - |A \cap B|$

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for 3 set $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Computer Representation of Sets.

Assume U is the universal set which is finite, in our case U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and ordering of element of U is done in increasing order. If A is the set of even number less than 11 i.e A = {2, 4, 6, 8, 10}.
We can represent set a as string of binary digit (bit) as follow
0101010101
Now to find $\bar{A}$ we can simply perform bitwise not to the string of bit of A which is
1010101010
We can convert this ~~bit~~ string of bits by comparing it to universal set and locating the ~~the~~ ones of the string
$\bar{A}$ = {1, 3, 5, 7, 9}.

Again if C = {1, 2, 3, 4, 5} and D = {1, 3, 5, 7, 9}
To find the union & intersection of the set we find the bit of string which are 1111100000 & 1010101010 for C & D respectively.

To find $C \cup D$ we perform bit wise or operation to the bit of string
1111100000 $\vee$ 1010101010 = 1111101010
which corresponds to set {1, 2, 3, 4, 5, 7, 9}

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Similarly to find $C \cap D$ we perform bitwise and operation.

$$1111100000 \wedge 1010101010 = 1010100000$$

which corresponds to the set $\{1, 3, 5\}$

If $A = \{1, 2, 4\}$ $B = \{2, 3, 5\}$

find $\bar{A}$, $A \cup B$ & $A \cap B$ using computer representation of sets technique

Function

Let A and B be non empty sets. A function f from A to B is an assignment of exactly one element of B to each element of A. We write $f(a) = b$ if b is a unique element of B assigned by the function f to the elements a of A. If f is a function from A to B, we write $f: A \to B$.

Also A is the domain of f and B is the codomain of f. If $f(a) = b$ b is the image of a & a is the preimage of b.

eg of function

$$f(x) = x+1$$

for $f(1) = 2$

If f_1 and f_2 are functions from A to R. Then $f_1 + f_2$ & $f_1 f_2$ are also function from A to R defined for all $x \in A$ by

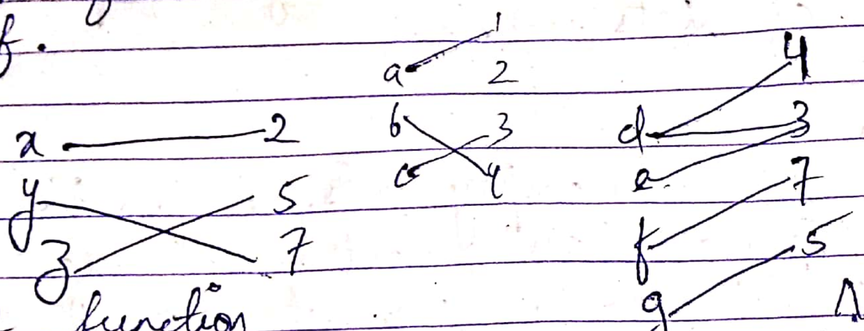
$$(f_1 + f_2)(x) = f_1(x) + f_2(x)$$

$$(f_1 f_2)(x) = f_1(x) f_2(x)$$

eg. If $f_1(x) = x^2$ & $f_2(x) = x - x^2$ find $f_1 + f_2$ & $f_1 f_2$

One to One (Injective) function

A function f is said to be one to one (Injective) if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.



One to One function

Not one to one & Not function

Determine whether function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$ & $f(d) = 3$ is one to one?

Determine if function $f(x) = x^2$ is one to one for Z & Z^+.

Determine if function $f(x) = x + 1$ is one to or not.