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A mathematical one-man one-vote rationale for Madisonian presidential voting based on maximum individual voting power

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Abstract. The One-man One-vote (OMOV) criterion is defined formally to demand (1) Maximum and equal voting power over the final outcome for each individual voter, and (2) Equal power-per-vote for all groups of voters. We show that if it allotted Representative votes on a popular-vote-cast (rather than on the present census) basis, the Madisonian Electoral College system would assure individual voters the largest possible total power available to a "simple" system. This popular-vote basis ("MVP") modification would relieve the present electoral impotence of voters in poorly-contested states without disturbing the voting power now enjoyed in closely-contested states.

> "The worth of a state, in the long run, is the worth of the individuals composing it." John Stuart Mill
> "And be these juggling fiends no more believed/That palter with us in a double sense,/That keep the word of promise to our ear,/And break it to our hope." Shakespeare, *Macbeth*, Act V Scene 8.

1. Introduction

Individual votes have maximum fair impact on the final outcome only when an election is closely contested, and close contests virtually never occur in large electorates. If, however, the electorate is districted, the voter can affect the outcome by affecting the outcome of his own district, provided it is closely contested. If not, he can still affect the national outcome (by withholding his vote from an unacceptable presumed winner) if districts are awarded national weight on the the basis of popular-vote-cast.

The above sketches the argument to follow: Voting systems for large electorates that aspire to assure individual votes maximum and equal power over the outcome should be divided into the largest possible closely-contested districts, and those given national weight in proportion to the popular vote they cast. The key underlying consideration is that of "close contest," the obstacles to it are "bloc" effects (e.g., that of electorate size), and these are countered by simple design tactics (e.g., by districting the electorate.) The undefined

terms are given mathematical definitions, and play key roles in a central theorem.

We will analyze the family of simple voting systems of which the present Madisonian system is the focal member and argue that it alone possesses, apart from the popular-vote basis, all the simple refinements available. That is, the argument provides a rationale for the present Madisonian system on the basis of the One-Man One-Vote (OMOV) criterion.

The formalized OMOV criterion demands that a voting system assure maximum and equal voting power for individuals, and equal power-per-vote for all groups of individuals. The (specific) power of an individual vote is defined as the probability that the outcome will turn on it, under its system, assuming nothing else changes.

Voting power measures the bargaining power of the individual vote with the leading candidate, the sensitivity of the outcome to individual consent, and indirectly, the voter's incentive to acquiesce in an unfavorable outcome. Because an electorate's voting-power sum varies from system to system, OMOV requires the largest possible equal voting power for its individual voters. It excludes, for example, a system that would give zero power, equally, to all its voters. We focus on the simple design tactics, and argue that there have been historical errors in appreciating them.

2. The implied model

We assume an independent-voter model (Natapoff, 1995 = "I") in an electorate of an odd number (2r + 1) of voters each of whom has independent probability (p, q) = e.g., (.60, .40) of voting for candidate (A, B). As seen by any individual voter, the other 2r votes are likely to produce a margin of 2r (p-q) with variance u = 4pq (= .96) per vote cast.

The index u measures the quality of contest for individual votes and ranges from u = 0 (p = 1, q = 0, uncontested) to u = 1 (p = q = ½, perfectly contested.) The quality of contest for the full electorate is measured (inversely) by $z^2 = [2r\ (p\text{-}q)]^2/2ru = 2r\ (1-u)/u$, where z = the expected margin, in units of its own standard error. When z > 1 for a group, we call the group a "bloc". Thus, any sufficiently large (2r > u/(1 − u)) electorate is a bloc, unless each individual vote in it is perfectly (u = 1) contested.

Our illustrative electorate, (u = .96, 1 − u = .04), is a bloc if 2r > .96/.04 = 24, i.e., if it has 25 or more independent voters in it, because in that case its expected margin will be greater than one standard error.

The central technical result (I) is that districting a homogeneous bloc increases the voting power of each individual voter, E, in it. An undistricted (raw) electorate of 9 votes, for example, has the probability

Table 1. Specific individual voting powers: raw (135 × 1) vs. districted (45 × 3) voting, at several levels, p, of voter preference for the favored candidate

The u-index, (=4pq) ranges from u = 0 [representing an uncontested, (p, q) = (1, 0) electorate] to u = 1 [representing a perfect, (p, q) = (½, ½) contest.]

The z-parameter = the expected margin, in units of its own standard error: $z^* \approx 1$.

Voter preference for leading candidate, A.			Individual voting powers, L, under		
p(%)	u	z	Raw	Districted	
			L (135×1)	L (45×3)	L_{Raw}/L_{Dist}
50.0	1.000	0.000	.0688	.0598	1.051
54.5*	.992*	1.040	.0401	.0399	1.005
61.1	.951	2.628	.00238	.00475	.500
64.8	.912	3.596	.000144	.000581	.247
65.7	.901	3.837	.000064	.000316	.202

Notes. The specific voting power, L, of an individual vote = the probability of a deadlock among the remainder of the electorate = the fraction of the power (=1) it would enjoy in a one-voter electorate – where it would always be decisive.

Notice that in contests that are (more poorly, as well, better) contested than the critical level (*), that is in those for which p (>, =, <) p* = .545, or u (<, =, >) u* = .992, power under the districted system is (greater, equal to, less) than under the raw system. The last column shows that districted voting gives (1, 2, 4, 5) times as much power as raw voting when u = (.992, .951, .912, .901). It gives 100 or 1000 times as much for lower values of u, i.e. for sufficiently poor contests.

$$L\,(9\times1) = (35/128)\,(4pq)^4 = (35/128)\,u^4 \tag{1}$$

of hinging on E's vote, whereas that probability is

$$L\,(3\times3) = (1/16)\,u^3(u+3) \tag{2}$$

under 3 × 3 districting. The first, L(9 × 1), is the probability that the eight votes besides E's will split evenly; the second, L(3 × 3), that E's two district-mates will divide evenly, and that the two other districts will also divide evenly so that E's vote will decide his district, and his district the election. We can calculate that

$$L\,(9\times1)\,(<,\,=,\,>)\,L\,(3\times3) \text{ when } u\,(<,\,=,\,>)\,u^*, \tag{3}$$

for $u^* = 8/9$, the critical u-value. The critical expected margin is $z^* = \sqrt{(2r}$ $(1-u)/u) = \sqrt{(8\,(1/9)/(8/9))} = 1$ standard deviation, which corresponds pre-

Table 2. The simple electoral devices for presidential elections and the 5 (of a total of 18 possible) combinations of them that characterize simple REFERENCE ELECTORAL SYSTEMS prominent in the legislative debate on electoral reform. The present MADISONIAN system, for example, is based on State districts, Senatorial Votes conferred on a winner-take-all basis, and a census (as opposed to popular-vote)-based formula for determining a state's size

District	Basis for conferring a state's senatorial electoral votes	Size basis for assigning national electoral weight to a district	
		Popular vote	Census population
State	Winner-take-all	MVP	MADISONIAN
	Proportional		
	No senatorial votes		
Congressional District	Winner-take-all		DISTRICT
	Proportional		
	No senatorial votes		
Individual	Winner-take-all		
	Proportional	PROPORTIONAL	
	No senatorial votes	RAW POPULAR	

cisely to our definition, $z = 1$, of a bloc. The connection between ($u = u^*$) and ($z = 1$), precise in this example, is coarse in general.

The theorem (I) establishes that there is always a unique critical value, u^*, for every districtable electorate, X, and each simple districting, X′, of it. When u ($<$, $=$, $>$) u^*, undistricted L(X) ($<$, $=$, $>$) districted L(X′). Districting, therefore, always improves individual voting power in blocs, but lowers it in well-contested voting units.

Table 1 shows the voting powers (L) in a 135-vote electorate for different u-values, and illustrates the increase – from $L(135 \times 1)$ to $L(45 \times 3)$ – that comes from districting the electorate when ($u < u^* = .992$) it is a bloc.

The "simple" voting systems (Table 2) – are the 18 possible configurations of the few electoral devices that have earned support in a Congressional vote. These include the four reference systems (Madisonian, Raw Popular, Proportional, and District) that Congress has voted on, and the popular-vote-based version of the present Madisonian system (the "Maximum Voting Power", MVP) design which is made from the same devices (FitzGerald, 1970). In comparing these 18 across the three key dimensions (district, size-basis, and Senatorial votes) that span all simple systems, we will call "best" the system that inhibits the largest number of accessible bloc effects.

The conclusions are consequences of the OMOV criterion, not an argument for adopting a popular-vote basis. Such an argument would have to treat such issues as deadlock resolution, uniformity of voting qualifications, and electoral fraud (Natapoff, 1969; 1977: 368).The present narrowly-defined analysis treats none of those .

The formal definition of voting power grows from the power indices initiated by Luther Martin during pre-Constitutional debates in 1787 (Riker, 1986), and invented again, in modern influential form by Shapley and Shubik (1954), Mann and Shapley (1960), Shubik (1964), and Banzhaf (1968). These pioneer indices measure relative voting power between voters within the electorate (not the overall level of that power) and, when they assume that each configuration of voters is as likely as every other, assume the singular case of ($u = 1$) perfectly-contested voters. That assumption explicitly keeps indices from reflecting the effects that changes in the degree of contest (u) for individual votes could have on their voting power.

By contrast, the definition of voting power (L) above permits us to analyze – if only in principle – the actual probability of affecting the outcome, the overall level of individual voting power, and the effects of changes in degree of contest (u). In particular, it can indicate when voters have (equally) no power at all – which happens, in theory and practice, whenever bloc effects are poorly controlled.

The key result for us is that the sum of actual voting powers, like the capacity to tip a boat, is sensitive to design: Some boats and electoral designs (e.g., canoes and districted systems) are intrinsically more tippable than others (e.g., rafts and undistricted systems). Each passenger in a canoe has equal and large, but on a raft equal and small capacity to tip it. Moreover, the difference in sensitivity between one nautical (or electoral) design and another provide (OMOV) reasons for choosing one design over another in the extent to which they compel or urgently invite an aspiring president to earn support over the whole electorate.

If we assume that Congress will consider in the near future only configurations of familiar, well-understood devices that it has voted on at least once before, our discussion of simple systems corresponds to the horizon of practical debate over presidential electoral reform that has dominated most of this century. We will conclude that no simple system meets the OMOV criterion, that our present Madisonian system meets it better than any other system ever voted on by the Congress, and that its popular-vote-based version is the best simple system under the OMOV criterion.

The Madisonian system distributes 538 electoral votes. Of these, 436 ($\approx$ 17/21) are "Representative" votes, allotted among states in proportion to (the power 1 of) census size, and 102 ($\approx$ 4/21) are "Senatorial" votes, allotted equally among the states (i.e., according to the 0th power of size). The former allotment favors large, the latter small states.

We will give the direct argument, reconcile it with historically prominent arguments, and introduce some intuitive analogies helpful to a synthesis.

3. Direct argument: *Elements of the design problem*

The historical evidence suggests that the current Madisonian system has improved individual voting power by creating *de facto* state-districts. We argued above that this would be true whenever national raw popular elections are not close, and only the 11 shown below of 42 elections, 1828–1992, have had raw margins even as small as 5%. (1844, 1876, 1880, 1884, 1888, 1892, 1896, 1916, 1948, 1960, 1968), (1.5, −3.0, .08, .64, −.87, 3.4, 4.2, 3.3, 4.6, .17, 0.7)%.

Candidate Truman's substantial 4.6% (over 2 million vote) popular margin in 1948, for example, contrasts with his narrow electoral vote margin, about half of which came from a victory by only 18 thousand votes in California. State districting made the national outcome vulnerable to smaller portions of the electorate than could plausibly have threatened it under raw popular voting.

This historic strength of state-districted voting has been interpreted in reverse by proponents of raw popular voting who regard the possible defeat of the raw popular vote winner ("raw-vote anomaly") as a disabling defect. In fact, although raw-vote anomaly should actually occur in about half of close elections, it has occurred only once (1888) in 41 cleanly-decided elections. This is evidence that closely-contested raw elections are rare, and that the large size of our electorate requires that it be divided into districts.

The anomalous losing candidates (Jackson, Tilden, Cleveland) served an average of one term apiece as president after their defeats. Thus, raw-vote anomaly has been both rare and generally self-correcting – not a danger or a sign that Madisonian voting is intrinsically undemocratic, as its critics – proponents of raw voting – have suggested.

Those critics (e.g., Peirce, 1968) cite the many historical near-anomalies as disasters narrowly averted. In those elections, small numbers of votes in key close states could have turned the national outcome without turning the national raw vote margin. This suggests directly that Madisonian voting increases the susceptibility of outcomes to individual votes (as compared with raw voting) which is what OMOV demands.

A large homogeneous electorate is almost sure to be poorly-contested on a raw basis because any small systematic advantage enjoyed by one candidate with each voter, like any small advantage enjoyed by a casino on each turn of the wheel, creates the near-assurance of winning, on the average, in the long run. The many Madisonian near-anomalies in raw-vote-safe elections argue that state-districts have inhibited that statistical Size effect, and increased the chance that individual votes (in closely-contested states) could turn the outcome. The protection of voting power in poorly-contested states requires a design tactic beyond state districting.

3.1. Size, lion, canceling-voter, and wasted-voter bloc effects

Any effect that delivers a large expected margin (z-value) is a bloc effect. We saw earlier that any electorate that is large enough – even if it is perfectly homogeneous – is automatically a (Size) bloc unless its individual voters are perfectly (u = 1) contested.

When individual votes are uniformly poorly contested, electorates are also poorly contested, and small numbers of voters have no impact on the outcome (Wasted-Voter effect). The same is true of any closely-contested sub-group within a bloc: Its votes on either side cancel one other, and produce less net margin (and power) per vote than do the poorly-contested votes outside it (Canceling effect).

Under raw voting closely-contested states, inefficient at producing net national margin, would suffer from this canceling. They would have less national power per vote than would poorly-contested states, which violates the OMOV criterion. The present districting-by-state protects them.

In the same spirit, any system (e.g., Proportional Voting) that permitted an individual or district to distribute votes among candidates (and therefore, cancel a part of its own net effect on the national margin) would lower the relative national impact of those states that did so as against those who chose not to. Aversion to that potential Canceling effect may explain why all the states (except, at present, Maine) have chosen to vote as a unit.

Similarly, the Canceling effect would punish any well-contested state that distributed its Senatorial electoral votes on a proportional basis. (Only a fraction of its Senatorial votes would be transmitted to the final margin.)

On the other hand, a poorly-contested (bloc) part of an electorate contributes more net margin per vote, and like Aesop's Lion, takes much more than its proportional share – sometimes all – of the voting power. Under raw popular voting, for example, a dominant bloc would take all the voting power: Even a 49% minority would have no power per vote as a group, and *a fortiori* no individual voter (even within the majority) would have any.

In summary, large electorates are vulnerable to the Size effect, and districting is the reference device that counters it. Congressional districts, smaller than states, are often homogeneous (rather than closely-contested), create a Canceling effect in close states, and put individual voting power at risk every ten years when their boundaries are redrawn. They are, therefore, inferior to states which form the largest simple close districts and best district choice. (By extension, the full electorate itself would be the best district choice if it were closely-contested.)

The two remaining design dimensions (Senatorial electoral votes, and size basis) must minimize the two accessible vices of state districts, a potential large-state Lion effect if the states are of different sizes, and a Wasted-Voter effect within any poorly-contested state.

Large states have intrinsically greater power per vote than small (I), assuming that each state's votes are statistically independent of every other's. Therefore, Senatorial electoral votes are needed to counter a potential large-state Lion effect, and those should be given undivided to avoid Canceling effect, as noted. More, a popular-vote basis for measuring state size averts a Wasted Voter effect caused by a census basis, and is, therefore, superior.

4. Coarse historical data: Coherence of the Madisonian design

Senatorial votes seem to have inhibited large-state Lion effect. If, however, the largest states grew substantially faster than the average, or formed a large-state bloc, the present fraction (4/21) of Senatorial votes would be too small. The warning sign would be a pattern of winners who carried significantly (i.e., at least $\sqrt{51} \approx 7$) fewer states than their defeated rival.

Hughes (in 1916) nearly became the only such small-state-anomalous winner by carrying 12 fewer states (*18*) than Wilson (*30*). Wilson's margin 277 ($=217 + 30(2)$) over Hughes's 254 ($=218 + 18(2)$) was the narrowest in history, before or since, and was provided entirely by Senatorial votes. Although Senatorial votes are often criticized as undemocratic, they saved the popular-vote winner's victory in 1916, the only election that has ever turned on them.

The election of 1916, the closest electoral vote contest in history is also the only one in which a popular-vote-base would have changed the winner, and that because of a historically unique difference among the states in voter qualifications. In 1916, the last election before the 19th amendment (1920) gave women in all states the right to vote, Hughes's victory in Illinois (in which women voted) would have had, under a popular-vote base, twice as much national weight per current electoral vote as did the larger states of New York and Pennsylvania where women did not yet vote. That additional margin would have elected Hughes under a popular-vote base.

Wilson lost the three states (Illinois, New York, and Pennsylvania) casting the largest popular vote, but no other winner has ever lost even the two largest. (In 1948, however, Truman did lose the two largest in *electoral* vote.)

In historical elections, then, the winner has carried a non-negative plurality in the popular vote (except in 1888); of the states (except in 1960 and 1976, where the plurality was -3, and statistically insignificant); and, of the two largest states (except in 1916 and 1948.) (Note that two-thirds of the states are smaller than average.) This rarity of and balance in opposing anomalies suggest that Madisonian devices have protected both small- and large-state voters as the Madisonian design intended.

5. Elements of a solution: Popular-vote allotment basis

An example of the potential of a popular-vote basis to remedy the Wasted-Voter effect in a poorly-contested state is illustrated in a 15-vote electorate of 10 X's (X^{10}) and 5 Y's (Y^5) when districted as ($X^4Y^5 \mid X^6$): Each district's winner commands all and only the non-blank votes cast in it (popular-vote basis), not the maximum potential vote of the district (census basis). Blank votes do not count.

Raw voting would constitute an X-bloc gerrymander anticipating (X^{10} Y^5) $\rightarrow$ X^{15}; with a census basis, the design above would constitute a Y-bloc gerrymander, anticipating (X^4 $Y^5 \mid X^6$) $\rightarrow$ ($Y^9 \mid X^6$) $\rightarrow$ Y^{15}. A popular vote basis, however, would assure all individual voters power over the outcome anticipating (X^0 $Y^5 \mid X^6$) $\rightarrow$ ($Y^5 \mid X^6$) $\rightarrow$ X^{11} if the X-voters in district I cast defensive blank ballots. District I's X-voters cannot change their district's outcome, but their blank ballots could so deflate its national weight as to reverse the national outcome.

This illustrates the potential of the popular-vote-basis: It rewards a moderate candidate for the consent tacitly and paradoxically expressed (in votes cast for her opponent) by her loyal opposition, and reduces the incentive for pure bloc strategies. More, it protects the primary tactic of districting large electorates from the danger that the districts it draws will someday be poorly contested.

5.1. The popular-vote basis

At present, a (poorly-contested) state's winner, candidate A, receives national credit for many presumed (e.g., ineligible, indifferent, or involuntary) votes whose support she did not earn. In close states, census errors deny the winner credit for votes she earned fairly. The ineligible include felons, dead persons, and minors; the involuntary, those counted for A in the national results who find her indifferent or inimical to their interests, but had no voting power with which to oppose her. By diminishing the power of individual voters to affect the outcome, these distortions violate the OMOV criterion.

Under a popular-vote basis the popular vote equivalent of the present Senatorial electoral votes would be (2/436) (National Popular Vote) = (2/436) (104.4 million) = (480,000 in 1992), per state, when rounded to the nearest 10,000. That calculation preserves Senatorial votes at their present fraction, 102/538, of the total. Thus, each state commands its own actual total popular vote (say, 2,001,776) plus its Senatorial votes (480,000), or 2,481,776 in all.

There are no involuntary votes under a popular-vote basis. Invulnerable-A's supporters increase her national total vote-for-vote which is precisely the impact that raw popular voting would provide. On the other side, a voter who

finds A unacceptable can, by casting a blank ballot, deflate her national total by one vote. Every voter in a poorly-contested state can make a one-vote difference in the final national totals, whereas under a census basis, none could.

The popular-vote-basis rules are analogous to the rules of poker: The voter always votes (bets and plays out) a strong hand, but must be induced to vote a weak one by concessions, and the absence of signs (political hostility) that he would lose by playing. That incentive for candidates to sue for opposition acquiescence in the states with poorest contests would promote the present two-party system where it is now weakest, and thereby nourish individual voting power.

The popular-vote basis rewards locally dominant candidate A as much, vote-for-vote, for inducing the opposition to vote against her (and for protecting their right to do so) as for inducing her partisans to vote for her. This substantiates the political fiction that the winner represents both those voters who did, and those who did not vote for her – since all had the power to advance or inhibit her election. (Raw voting, which also uses a popular-vote-basis, gives dominant local candidates an incentive – in increased net margin transmitted to the national totals – to suppress opposition votes.)

6. Reconciliation with earlier popular views

Suppose an electorate, divided equally between Northern X's and Southern Y's, must draw two Congressional districts. If these are drawn North and South, both will be poorly-contested, no individual voter will have any power, and each bloc will win one seat. If the districts are drawn East and West, both will be closely-contested, and all *individual* voters (both X and Y) will have the largest possible voting power. In half of such elections, however, one of the major *blocs* will lose both seats, and be unrepresented. This electoral insult to dominant blocs is an unavoidable cost of maximizing the power of individual voters.

Proportional representation has the reverse effect, analogous to the first case. When voters are forced to select a single list of parliamentary candidates, they have been forced to vote in blocs, which destroyed their voting power. The prime minister elected by a coalition then reflects, by inference, a compromise of major-bloc, not of individual interests. This is tested when large blocs are deadlocked, and critical small blocs can exact concessions that individual dominant-bloc members would oppose.

Senator Birch Bayh led the most successful effort to abolish the Madisonian voting system since the innovation of popular voting for president in the Jacksonian period. He, who has arguably contributed more than any other legislator or legal scholar to the clarity of the political fundamentals of the debate

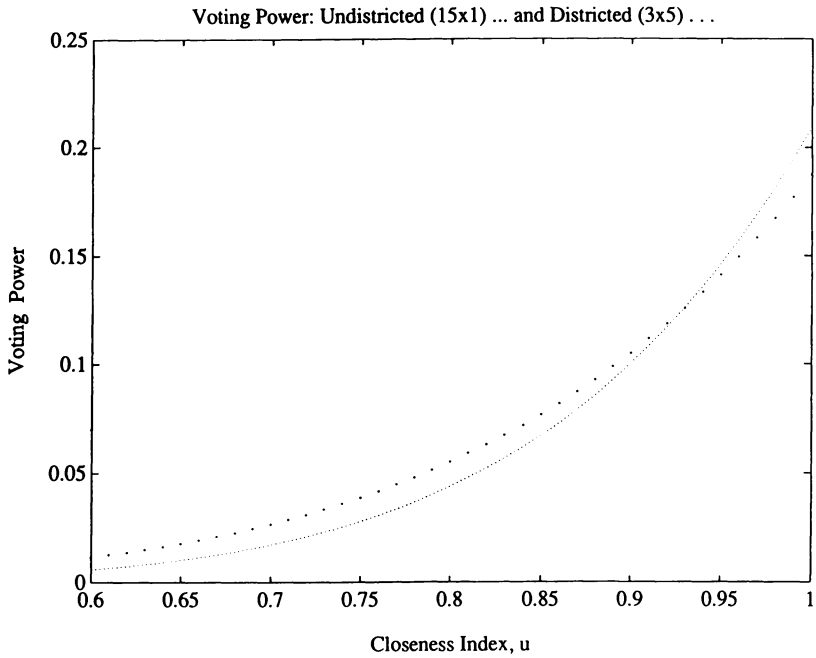

Figure 1.

on Constitutional electoral reform, wrote: "...voting power... is the ability of a voter to actually affect the outcome of an election. A democratic electoral system, it seems to me, would equalize voting power, insuring each voter an equal opportunity to influence the final choice. *The only electoral system that can meet this test, of course, is direct popular election.*" (Bayh, 1967: 934; 1977.) The informal opinion in the last sentence is widely held, illuminating, and as we have seen earlier, fundamentally in error. Many systems can give equal power, but OMOV also demands large power that raw voting cannot provide.

Consider, for example, voting systems (i)–(v) below for an electorate of $2r+1 = 15$ voters in which each voter has, independently, the probability (p, q) = (.65, .35) of voting for candidate (A, B). The electorate is a bloc: A's expected margin = 4.2 votes, its standard error = 3.57 votes, and z = (4.2/3.57) = 1.18 > 1. Thus, the rule-of-thumb predicts (correctly, as Table 3 and the graph in Fig. 1 confirm) that (either 3×5 or 5×3) districting would increase voting power.

All five systems are "fair": their individual voters are indistinguishable, have the same power, but different common powers under different systems. There is equality among voters *within* systems, but inequality *between* systems.

Table 3. Voting power, L (= Probability that a single vote among 15 will be decisive when the probability that any other voter will vote for 'A' is p = .65.)

(d × v) districting → d districts, each of v voters.

Voting system	d	v	Power L, of a vote	
(i) Tyranny				0.0
(ii) Lot			1/15 =	.0667
(iii) Raw: (15 × 1)	1	15		.1082
(iv) (3 × 5)	3	5		.1117
(v) (5 × 3)	5	3		.1118

6.1. Madison's warning and the Madisonian design

In Federalist 10, Madison (1961) warned of "..the violence.." and "..the mischiefs of faction," and "..the superior force of an interested and overbearing majority." He hoped "..to secure the public good and private rights against the danger of such a [majority-dominated] faction, and at the same time to preserve the spirit and form of popular [majority-ruled] government.." Despite its success in promoting these values, many see the Madisonian system as defective: It has elected the raw vote winner only 97%, not 100%, of the time and could defeat her again. That view rejects Madison's warning that under raw voting, individual rights are vulnerable to factions, violence and tyranny.

Madison suggested (in another connection): "..make it less probable that a majority of the whole will have a common motive to invade the rights of other citizens; or if such a common motive exists, ... more difficult for all who feel it to discover their own strength, and to act in unison with each other." Our current electoral rules assimilate that tactic: Individuals do not vote in a (bloc) national electorate, but (preferably) in a close state contest where the "majority .. [is] rendered by .. local situation, unable to concert and carry into effect schemes of oppression."

6.2. The American Bar Association's critique

The American Bar Association (1967) called the present Madisonian system "..archaic, undemocratic, complex, ambiguous, indirect, and dangerous," and advocated replacing it with national raw popular voting. Such criticisms are either incorrect or inconsequential.

The basic Madisonian system is archaic and indirect only in having live Electors. Its strategic design is nearly faultless, even by present OMOV standards, and the popular-vote-basis would mend its few accessible faults.

The claim that the Madisonian system is undemocratic, complex and dangerous presumably refers to potential electoral anomaly. We have seen, however, that anomaly-tolerance is the price of defending a large electorate against the Size effect. Since raw voting is defenseless against Size effect, its advocates must also be assuming that there is no danger, and never could be any danger from large-state Lion effect, even if the Madisonian inhibitions provided by state-districts and Senatorial electoral votes were removed. These approximations are far too severe, and therefore untenable in any practical debate.

The report says that under raw voting "..the vote of every individual in the constituency.. would be of equal weight, as it now is in elections for the United States Senate and House of Representatives and for statewide, municipal, county, town and village offices throughout the United States" (American Bar Association, 1967: 177). This explicitly ignores (or denies) Size effect, and focuses not on voting power (the capacity to change the outcome), but on voting weight (the capacity to change the numerical score) which plays only an incidental role in the OMOV criterion.

The report implies that the design of village elections will serve presidential electorates, thousands of times larger. In fact, large electorates require a more sophisticated (districted) architecture to meet the same OMOV criterion, just as the elephant's large scale demands an architecture altered from that of its tiny mammalian cousin shrew, though it is made of similar tissue.

The raw popular voting plan the report advocated would be defenseless against Size and large-state Lion effects, and reduce individual voting power. Since it permits many parties, it could elect a president who had small national support, carried few or no states, and provided no evidence of support or acceptance by broad segments of the national electorate.

Despite its technical design shortcomings, the ABA's proposal was endorsed by its own convention and by the League of Women Voters. Polls of the 1970s reported overwhelming popular support. The House of Representatives passed it, and the Senate came within a handful of votes of doing the same.

6.3. Intuitive analogies: Inverse game theory and sporting rules

Electoral design is a problem in inverse game theory (i.e., game design). Where game theory takes a set of rules and pursues a winning strategy for it, inverse game theory takes a given strategy and designs rules under which it would be optimal – as a means of inducing players to adopt it. Electoral laws, anti-trust laws and sporting rules are game designs that enforce artful, energetic competition.

Since small electorates cannot serve as models for large because of Size effect, presidential electoral analysis must rely directly on a few dozen historical

presidential elections, little resembling one another, and spread over two centuries as its experimental data base. Thereby, large-statistics sport, with a history of millions of documented contests may offer indirect, but illuminating analogies.

No major professional large-statistics team sport, for example, confers its championship on the basis of highest raw total score, or of most games won over the season, or best gross average performance. Those are insensitive to small bits of performance, and insufficient to compel intense competition over all phases of all its contests (or states). They would not compel and reward victory in many crucial (e.g., playoff, or large-state) contests. Major sporting championships can and do turn on individual errors or flashes of brilliance at a critical moment of a critical game. These dramatic crises are regarded as memorable treasures, and (implicitly) as validation of the sport's rules and tradition. By analogy, a few key votes in a close state can turn a whole presidential outcome. That lesson to candidates is the moral of this sport.

A basketball team routinely suffers anomalous defeat (more field goals, but fewer points) when its defense lacks the refinement demanded of a champion.

Cleveland (1888), Wilson (1916), and Truman (1948) had larger raw scores (popular votes), but lost or nearly did because they lost too many key contests. Truman lost two key states, and Wilson three, but Cleveland lost the the five largest states including his own, New York – which was too many – and suffered anomalous defeat.

The sporting equivalent of electoral-anomaly-1888 has occurred in more than a dozen World Series since 1920. In each, the champion had a lower collective raw score than the loser. Although this occurred four times in five years (1971–5), and five times out of eight (1957–64), raw-score anomaly is so well understood as a symptom of close contest in high-statistics sports that it was remarked on (if at all) as a piece of sporting trivia, never as a pretext for the reform of the rules.

7. Conclusion

The theory and practice of the American constitutional electoral example have not been perfected within the one-man, one-vote ideal. They are, nevertheless, widely consulted as a model for new constitutions that aspire to protect individual rights and a coherent union against the threats of violence and schism posed by centrifugal diversity. The present Madisonian design, based on popular consent, gives large voting power to individuals, and protects it from key bloc effects, notably those of pure Size, and state size.

We treated the category of simple systems. In them, each vote is cast undivided for a single candidate, the full presidential term is served, undivided, by the

winner, and such demographic issues as Age (XXVI), Sex (XIX), Race (XV), and Income (XXIV) are resolved by Constitutional amendment (number indicated).

The questions left open are how (or if) the national electorate is to be districted, Senatorial electoral votes distributed, and state size measured. The practical problem, when so constrained, becomes an abstract puzzle accessible to mathematical analysis. We conclude that the present system would be the best simple one for American presidential elections by the OMOV criterion if it were modified to measure a state's size by its actual popular vote. The change would benefit all voters in poorly-contested states.

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