

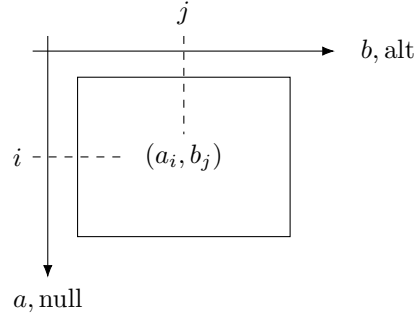
## aftermath::sequential

**parallel\_stopping\_time motivation.** Consider a rule  $(\tau, d)$  that decides between two hypotheses  $\mathcal{H}_0$  and  $\mathcal{H}_1$ , where  $\tau = \min \{t_0, t_1\}$ ,

$$t_0 = \inf \{n \geq 1 : R_n \geq a\}, \quad t_1 = \inf \{n \geq 1 : S_n \geq b\},$$

and  $d = j$  if  $\tau = t_j$ ,  $j = 0, 1$ . When both thresholds are crossed at once, i.e.,  $\tau = t_0 = t_1$ , we consider the decision erroneous.

For efficiency reasons, rather than perform simulations for a given pair of thresholds  $(a, b)$ , we construct a grid of thresholds  $(a_i, b_j)$  with  $a_1 < a_2 < \dots < a_m$  and  $b_1 < b_2 < \dots < b_n$ . We will call the thresholds  $a$ ,  $b$ , null and alternative, respectively.



The stopping time  $\tau$  will run as long as at least one of the threshold pairs in the matrix has not been crossed, or equivalently as long as  $(a_m, b_n)$  has not been crossed. For each pair of thresholds, we record the observed value of  $\tau$ , and the decision made (1 if  $d = 0$ ; 2 if  $d = 1$ ; 3 if  $\tau = t_0 = t_1$ ).

When the first observation is collected, the  $\Gamma$ -shaped region (dotted on the figure below) in the corresponding matrices will be filled. With each next step the decision will have been made in a  $\Gamma$ -shaped region. Let  $i_k$  and  $i_{k+1}$  denote the indices of the first uncrossed null threshold at steps  $k$  and  $k+1$ , respectively. Let  $j_k$  and  $j_{k+1}$  denote the corresponding indices of the first uncrossed alternative thresholds. Then the decisions associated with  $\tau = k+1$  will have the following form.

