Motion of Electron in Uniform Electric Field

Let's consider two parallel plates separated by a distance **d** with upper plate maintained at higher(positive) potential and the lower plate is earthed so that a uniform Electric field of intensity**(E)** is produced between these plates. Let **V** be the potential difference between two plates as shown in the figure. Let an electron with an initial velocity **u** enters into the electric field, perpendicular to the direction of the field.

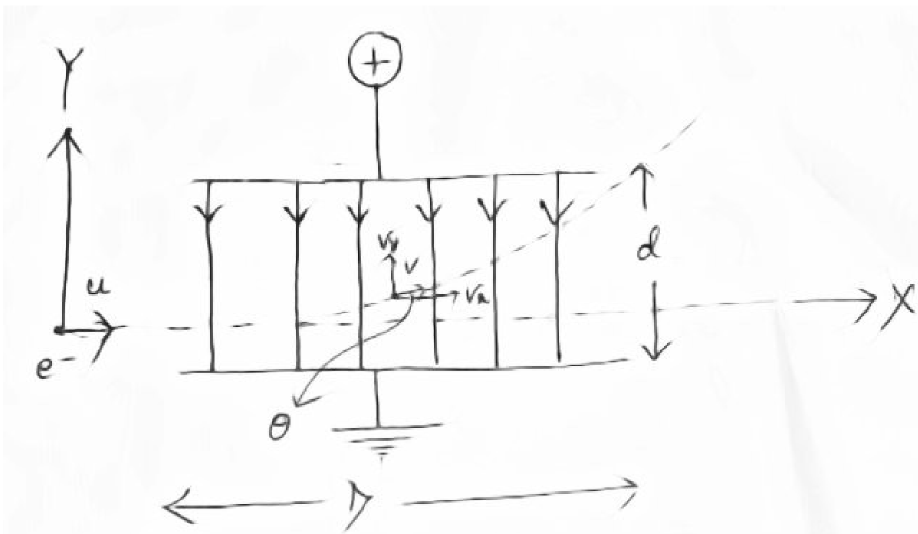

Figure 1: Deflection of the electron in an electric field

Here, the velocity of the electron along the horizontal direction(X-direction) is uniform, so $a_x = 0$. The horizontal distance travelled by the the electron is given by,

$x = u_x t + \frac{1}{2} a_x t^2$

$x = ut + \frac{1}{2}.0.t^2$

$x = ut$

Now,

$t = \frac{x}{u}$..(i)

Also, the vertical distance of electron along the y-direction is given by,

$y = u_y t + \frac{1}{2} a_y t^2$

$y = 0.t + \frac{1}{2} a_y t^2$

$y = \frac{1}{2} a_y t^2$ ....(ii)

The electric force exerted by the electric field on the electron which deflects the electron is given by,

$F_E = ma_y$

$eE = ma_y$

$e\frac{V}{d} = ma_y$

$a_y = \frac{eV}{md}$ ....(iii)

Substituting eqns(i) and (iii) into (ii),

$y = \frac{1}{2}\frac{eV}{md}\left(\frac{x}{u}\right)^2$

$y = \left(\frac{eV}{2mdu^2}\right)x^2$ ....(iv)

Equation(iv) is in the form of the equation of parabola. So, we can conclude that the motion of electron in an electric field is parabolic in nature.

Angle of deflection of electron in the electric field

Here,

$v_x = u$

$v_y = u_y + a_y t$

$= 0 + a_y t$

$= \frac{eV}{md}\frac{x}{u}$

$\therefore v_y = \frac{eVx}{mdu}$

Now,

$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{eVx}{mdu}}{u} = \frac{eVx}{mdu^2}$

So, the angle of deflection is,

$\theta = tan^{-1}\left(\frac{eVx}{mdu^2}\right)$ ....(v)

The velocity of the electron at any point in the electric field is,

$v = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{u^2 + \left(\frac{eVx}{mdu}\right)^2}$ .... (vi)

After crossing the electric field, the electron continues in its motion in a straight line path tangent to the parabola, because at that point, the applied electric field terminates, so no force acts on the electron to change its velocity. Thus, the gain in kinetic energy is given by,

$\Delta K.E. = (K.E)_f - (K.E.)_i$

$= \frac{1}{2}m\left[u^2 + \left(\frac{eED}{mu}\right)^2\right] - \frac{1}{2}mu^2$ (Here, $E = \frac{V}{d}$ and D = length of the plate)

$$= \frac{1}{2}m\left(\frac{eED}{mu}\right)^2$$
$$= \frac{e^2E^2D^2}{2mu^2}$$