Quantization of Energy

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Postulates of Bohr's Atomic model

In 1913, a new atomic model was proposed by Neil Bohr, incorporating the Planck's quantum theory of radiation. The basic postulates are as follows:

1. Electrons revolve round the nucleus in certain, fixed circular, concentric orbits, which are known as stationary states or orbits. As long as the electron remains in these stationary orbits, it will not radiate energy. In other words, the energy of the electron remains constant (stationary, i.e. non-changing) as long as it remains in the same orbit. The stationary states are also called Energy Levels because each orbit is associated with a definite energy. These stationary orbits are non-radiating or closed orbits.

2. The stationary orbits are only those in which the angular momentum of the electron in that orbit is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant. If 'm' is the mass of an electron, 'v' is its velocity and 'r' is the radius of the electron orbit, then the angular momentum is 'mvr'. So, condition for a stationary orbit is,
$$mvr = \frac{nh}{2\pi}$$
This is known as Bohr's quantization condition.

3. Energy is absorbed or emitted only when an electron jumps from one energy state to another energy state. When the electron transits from higher energy state to lower energy state, it radiates or emits energy in the form of photon and absorbs the energy to jump from the lower energy state to the higher energy state. The energy of the emitted or absorbed photon is equal to the difference in energy between the two energy states i.e.
If the electron jumps from the higher energy state E_i to the lower energy state E_f, then the energy of the emitted photon is
$$hf = E_i - E_f$$
where, f is the frequency of emitted radiation
Similarly, if the electron jumps from the lower energy state E_i to the higher energy state E_f by absorbing the photon, then the energy of the absorbed photon is
$$hf = E_f - E_i$$
where, f is the frequency of absorbed radiation

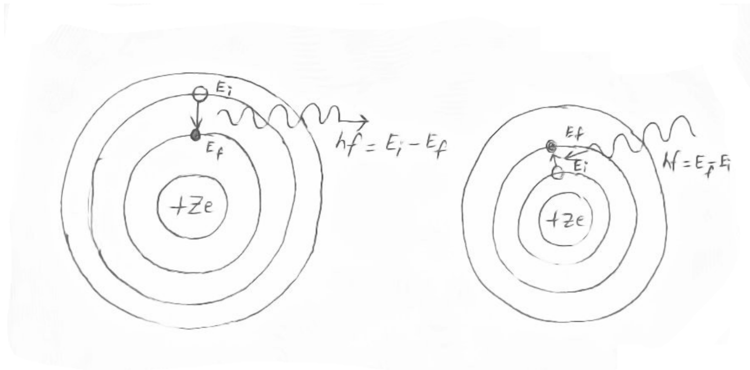

Figure 1: Transitions of electron in an atom

Bohr's theory of hydrogen atom

Let Ze be the charge of nucleus and e be the charge of an electron that revolves around the nucleus through an orbit of radius r.

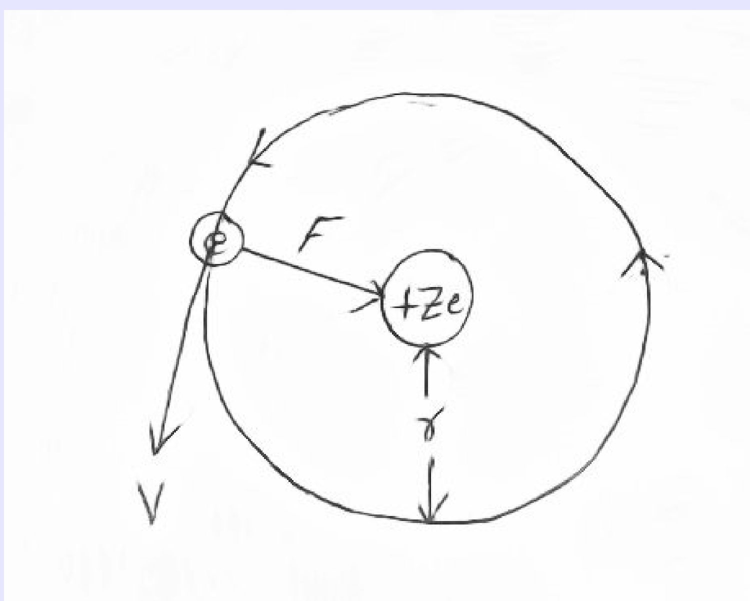

Figure 2: Transitions of electron in an atom

The electrostatic force of attraction between the nucleus and electron is,

$$F_e = \frac{kZe.e}{r^2} = \frac{kZe^2}{r^2} \tag{1}$$

where, $k = \frac{1}{4\pi\epsilon_0}$, $Z =$ atomic number of an atom, $\epsilon_0 =$ permittivity of free space

Let m be the mass of the electron moving with a velocity v in a circular orbit of radius r, then the centripetal force is given by,

$$F_c = \frac{mv^2}{r} \tag{2}$$

Since the centripetal force is provided by the electrostatic force,

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

$$mv^2 = \frac{kZe^2}{r} \tag{3}$$

Radius of n^{th} orbit

From the Bohr's second postulate (Quantization of angular momentum),

$$mvr = \frac{nh}{2\pi}$$

$$v = \frac{nh}{2\pi mr} \tag{4}$$

Substituting (4) into (3), we have,

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{kZe^2}{r}$$

$$m\left(\frac{n^2h^2}{4\pi^2m^2r^2}\right) = \frac{kZe^2}{r}$$

$$\frac{n^2h^2}{4\pi^2mr} = kZe^2$$

$$r = \frac{n^2h^2}{4\pi^2kmZe^2}$$

$$r = \frac{n^2h^2}{4\pi^2 \times \frac{1}{4\pi\epsilon_0} \times mZe^2}$$

$$r = \frac{\epsilon_0 n^2h^2}{\pi mZe^2} \tag{5}$$

In general, for n^{th} orbit, we have,

$$r_n = \frac{\epsilon_0 n^2h^2}{\pi mZe^2} \tag{6}$$

This equation gives the radii of permitted orbits. The radii of permitted orbits are proportional to the square of principal quantum number (i.e $r_n \propto n^2$) since $\epsilon_0, h, \pi, m, Z, e$ are constant.

Thus, the stationary orbits are not equally spaced. Orbits nearer the nucleus are closer than the farther orbits.

The radius of hydrogen atom is

$$r_n = \frac{\epsilon_0 n^2h^2}{\pi me^2} \tag{7}$$

For first orbit, n = 1,

$$r_1 = \frac{\epsilon_0 h^2}{\pi m e^2}$$

So, equation(7) becomes,

$$r_n = n^2 r_1 \tag{8}$$

Now,

$$r_1 = \frac{8.85 \times 10^{-12} \times (6.63 \times 10^{-34})^2}{\pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = 0.53 \times 10^{-10} = 0.53\mathring{A}$$

This is the radius of the lowest orbit of hydrogen atom known as Bohr radius. From equation(8), we have,

$$r_n = n^2 0.53\mathring{A}$$

Velocity of electron in n^{th} orbit

From equation(4), we have,

$$v = \frac{nh}{2\pi m r} \tag{9}$$

In general, the speed of electron in n^{th} orbit is,

$$v_n = \frac{nh}{2\pi m r_n} \tag{10}$$

Also, from equation(6), the radius of the n^{th} orbit is,

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2} \tag{11}$$

Substituting the value of r_n from equation(11) into (10), we have,

$$v_n = \frac{nh}{2\pi m \times \frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}}$$

$$v_n = \frac{Ze^2}{2\epsilon_0 n h} \tag{12}$$

This equation gives the velocity of electron in n^{th} orbit of an atom. Providing other parameters are constant,

$$v_n \propto \frac{1}{n}$$

So, the speed of electron decreases as it goes to higher energy states and the speed is maximum for the innermost orbit. The speed of electron in hydrogen atom is,

$$v_n = \frac{e^2}{2\epsilon_0 n h} \tag{13}$$

For n = 1,

$$v_1 = \frac{e^2}{2\epsilon_0 h}$$

So, equation(13) becomes,

$$v_n = \frac{v_1}{n} \tag{14}$$

Now,

$$v_1 = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34}} = 2.19 \times 10^6 = \frac{1}{137}c$$

where, c = speed of light in vacuum. So, from equation(14), we have,

$$v_n = \left(\frac{1}{137}c\right)\frac{1}{n}$$

Total energy of electron in n^{th} orbit

The energy of electron revolving in a stationary orbit is the sum of the kinetic energy (due to the speed of the electron) and the potential energy (due to the position of the electron).

Kinetic energy of the electron in the n^{th} orbit is given by,

$$K.E_n = \frac{1}{2}mv_n^2 \tag{15}$$

Substituting the value of v_n from (12) into (15), we have,

$$K.E_n = \frac{1}{2}m\left(\frac{Ze^2}{2\epsilon_0 nh}\right)^2$$

$$K.E_n = \frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2} \tag{16}$$

Potential energy of the electron in the n^{th} orbit due to the electric field of the nucleus is given by,

$$P.E_n = -e\left(\frac{kZe}{r_n}\right)$$

Substituting the value of r_n from equation (11), we have,

$$P.E_n = \frac{-kZe^2}{\left(\frac{\epsilon_0 n^2 h^2}{\pi m Z e^2}\right)}$$

$$P.E_n = -\frac{1}{4\pi\epsilon_0}\frac{Ze^2 \pi m Z e^2}{\epsilon_0 n^2 h^2}$$

$$P.E_n = \frac{-mZ^2e^4}{4\epsilon_0^2 n^2 h^2} \tag{17}$$

Hence, the total energy of the electron in the n^{th} orbit is,

$$\begin{aligned} E_n &= P.E_n + K.E_n \\ &= \frac{-mZ^2e^4}{4\epsilon_0^2 n^2 h^2} + \frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2} \end{aligned}$$

$$\therefore E_n = -\frac{mZ^2e^4}{8\epsilon_0^2 n^2 h^2} \tag{18}$$

The total energy of electron is negative which shows that the electron is bound to the nucleus and some work should be done to separate it from the nucleus.

So, total energy of electron in the n^{th} orbit of hydrogen atom (Z=1) is,

$$E_n = -\frac{me^4}{8\epsilon_0^2 n^2 h^2} \tag{19}$$

Here, for Hydrogen atom,

$$E_n = -\frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times n^2 \times (6.63 \times 10^{-34})^2}$$

$$E_n = -\frac{13.6}{n^2} eV$$

$$i.e. E_n \propto -\frac{1}{n^2}$$

Here, as n increases E_n becomes less negative and hence increases i.e. the electron has greater energy at higher energy states. Also, when $n = \infty$, $E_\infty = 0$, which shows that the maximum energy of electron is zero. It means electron gets free from the nucleus.

Number of orbit or principal quantum number (n)	**Total energy,** $E_n = -\frac{13.6}{n^2} eV$
1	-13.6 eV
2	-3.40 eV
3	-1.51 eV
4	-0.85 eV
5	-0.54 eV
...	...
∞	0

Table 1: Total energy of various energy states of Hydrogen atom

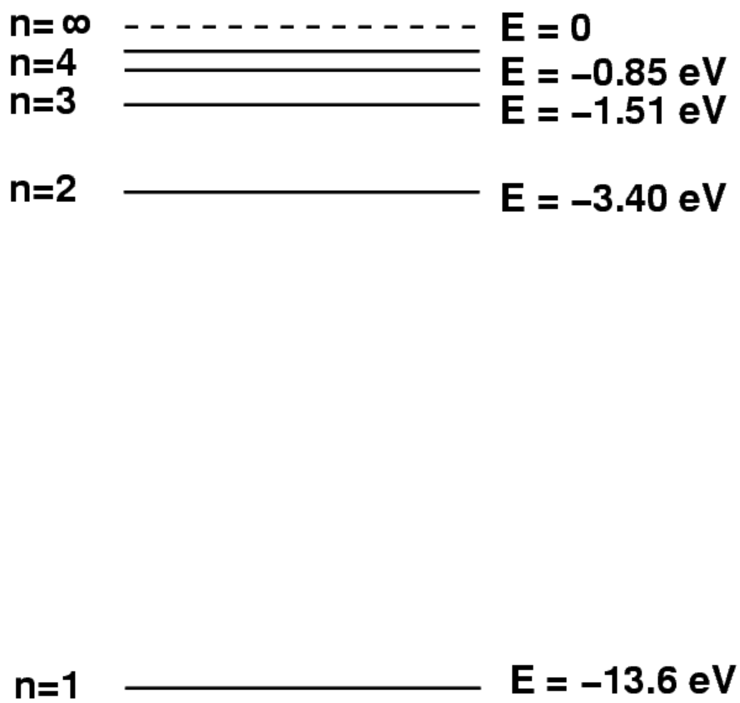

Figure 3: Energy level diagram

The energy of various energy states of hydrogen atom can be seen in the energy level diagram shown in Figure 3. The lowermost energy state corresponding to n = 1, is called ground state of a hydrogen atom. All other states above the ground state are called excited states, such as energy state of n = 2 is called first excited state, n = 3 is called second excited state and so on. Larger spacing between the energy states implies that greater energy is required for the transition of electron between these states.