Numerical Problems on Photoelectric effect

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Q.No. 1

Sodium has a work function of 2 eV. Calculate the maximum energy and speed of the emitted electrons when sodium is illuminated by a radiation of 150 nm. What is the threshold frequency of radiation for which electrons are emitted from sodium surface.

Work function $(\phi) = 2 \text{ eV} = 2 \times 1.6 \times 10^{-19} \text{ J} = 3.2 \times 10^{-19} \text{ J}$

Wavelength of radiation $(\lambda) = 150 \text{ nm} = 150 \times 10^{-9} \text{ m}$

Maximum kinetic energy of emitted electrons $(K.E_{\max}) = ?$

Maximum speed of emitted electrons $(v_{\max}) = ?$

Threshold frequency $(f_0) = ?$

We have,

$$hf = \phi + K.E_{\max}$$

$$\text{or, } K.E_{\max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{150 \times 10^{-9}} - 3.2 \times 10^{-19}$$

$$= \mathbf{1.006 \times 10^{-18} J}$$

Now,

$$\frac{1}{2} m v_{\max}^2 = 1.006 \times 10^{-18} J$$

$$\text{or, } v_{\max} = \sqrt{\frac{2 \times 1.006 \times 10^{-18}}{m}}$$

$$\text{or, } v_{\max} = \sqrt{\frac{2 \times 1.006 \times 10^{-18}}{9.1 \times 10^{-31}}}$$

$$\therefore \boldsymbol{v_{\max} = 1.487 \times 10^{6} m/s}$$

Again,

$$\phi = hf_0$$

$$\therefore f_0 = \frac{\phi}{h}$$

$$= \frac{3.2 \times 10^{-19}}{6.63 \times 10^{-34}} = \mathbf{4.83 \times 10^{14}} Hz$$

Q.No. 2

The maximum kinetic energy of the electrons emitted from a metallic surface is 1.6×10^{-19} J when the frequency of the radiation is 7.5×10^{14} Hz. Calculate the minimum frequency of the radiation for which electrons will be emitted.

Here,

Maximum kinetic energy ($K.E_{\max}$) = 1.6×10^{-19} J

Frequency of radiation (f) = 7.5×10^{14} Hz

Minimum(threshold) frequency (f_0) = ?

We have,

$$\begin{aligned} hf &= \phi + K.E_{\max} \\ hf &= hf_0 + K.E_{\max} \\ hf_0 &= hf - K.E_{\max} \\ f_0 &= \frac{hf - K.E_{\max}}{h} \\ &= \frac{6.63 \times 10^{-34} \times 7.5 \times 10^{14} - 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \\ \therefore f_0 &= 5.09 \times 10^{14} Hz \end{aligned}$$

Q.No. 3

The maximum energy of photoelectrons emitted from a metal plate is 1.2 eV. If the threshold wavelength is 2.48×10^{-9} m, calculate the wavelength of incident light.

Here,

Maximum energy of photoelectrons ($K.E_{\max}$) = 1.2 eV = $1.2 \times 1.6 \times 10^{-19}$ J

Threshold wavelength (λ_0) = 2.48×10^{-9} m

Wavelength of incident light (λ) = ?

We have,

$$hf = \phi + K.E_{\max}$$

$$hf = hf_0 + K.E_{\max}$$

$$
\begin{aligned}
K.E_{\max} &= h(f - f_0) \\
K.E_{\max} &= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \\
1.2 \times 1.6 \times 10^{-19} &= 6.63 \times 10^{-34} \times 3 \times 10^{8}\left(\frac{1}{\lambda} - \frac{1}{2.48 \times 10^{-9}}\right) \\
\frac{1.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} &= \left(\frac{1}{\lambda} - \frac{1}{2.48 \times 10^{-9}}\right) \\
9.65 \times 10^{5} + \frac{1}{2.48 \times 10^{-9}} &= \frac{1}{\lambda} \\
\frac{1}{\lambda} &= 4.04 \times 10^{8} \\
\lambda &= \frac{1}{4.04 \times 10^{8}} \\
\therefore \lambda &= 2.48 \times 10^{-9} m
\end{aligned}
$$

Q.No. 4

When ultraviolet light with a wavelength of 400 nm falls on a certain metal surface, the maximum kinetic energy of the emitted electrons is 1.10 eV. What is the maximum kinetic energy of the photoelectrons when light of wavelength 300 nm falls on the same surface?

Here,

Wavelength of UV light (λ_1) = 400 nm

Maximum kinetic energy for λ_1 ($K.E_1$) = 1.10 eV

Wavelength of UV light (λ_2) = 300 nm

Maximum kinetic energy for λ_2 ($K.E_2$) = ?

We have,

$$hf = \phi + K.E_{\max}$$

So,

$$\frac{hc}{\lambda_1} = \phi + K.E_1 \tag{1}$$

$$\frac{hc}{\lambda_2} = \phi + K.E_2 \tag{2}$$

Subtracting equation (1) from (2) we have,

$$
\begin{aligned}
K.E_2 - K.E_1 &= hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) \\
or, K.E_2 &= K.E_1 + hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) \\
&= 1.10 \times 1.6 \times 10^{-19} + 6.63 \times 10^{-34}\left(\frac{1}{300 \times 10^{-9}} - \frac{1}{400 \times 10^{-9}}\right) \\
&= 3.42 \times 10^{-19} J \\
&= \frac{3.42 \times 10^{-19}}{1.6 \times 10^{-19}} \\
&\boldsymbol{= 2.13eV}
\end{aligned}
$$

Millikan's experiment for the verification of Einstein's photoelectric equation

Einstein's photoelectric equation was experimentally verified by R.A. Millikan. The experimental arrangement used by Millikan is shown in figure 1. He measured stopping electrons (maximum K.E.) of photoelectrons emitted by a number of alkali metals over a wide range of frequencies of incident radiation.

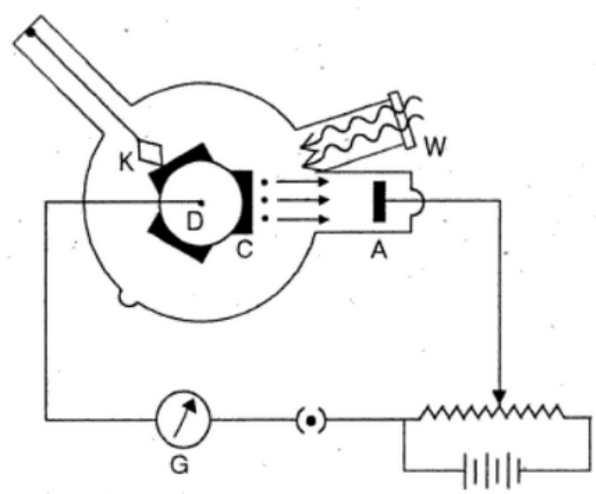

Figure 1: Millikan's apparatus for photoelectric effect

The apparatus consists of a drum D suitably mounted in an evacuated glass chamber, and capable of rotation from outside. Three different alkali metallic surfaces are mounted on the drum. Any surface can be scraped clean by a sharp knife edge K and then brought before a quartz window W through which monochromatic light of known frequency falls on the cleaned surface C. The emitted photoelectrons are collected by anode A by means of potential difference applied between C and A. The galvanometer G measures the photoelectric current.

When the light of known frequency is allowed to fall upon the metal at C, the photoelectrons are emitted from the cathode(emitter). These electrons are collected by the anode A (collector), so galvanometer shows deflection. Now, the polarity of the emitter and collector is reversed such that the collector is at negative potential, so the emitted photoelectrons have to do work against this negative potential at the expense of its own kinetic energy in order to reach the collector. The negative potential at collector repels the electrons reaching to it from emitter. If the negative potential at collector is increased with the help of rheostat, a stage will be finally reached when even the fastest electron will not be able to reach the collector. At this stage, photoelectric current becomes zero. *This negative potential applied at the anode A (collector) for which photoelectric current reduces to zero is called stopping potential* (V_s). At stopping potential we have,

$$eV_s = K.E_{\max} = \frac{1}{2}mv_{\max}^2 \tag{3}$$

Now, according to Einstein's photoelectric equation,

$$\begin{aligned} hf &= \phi + K.E_{\max} \\ K.E_{\max} &= hf - \phi \end{aligned}$$

From equation (3),

$$eV_s = hf - \phi$$

$$V_s = \frac{h}{e}f - \frac{\phi}{e} \tag{4}$$

This equation (4) represents a straight line between stopping potential V_s and frequency of incident radiation f in the form of

$$y = mx + c \tag{5}$$

where, m = slope and c = y-intercept,

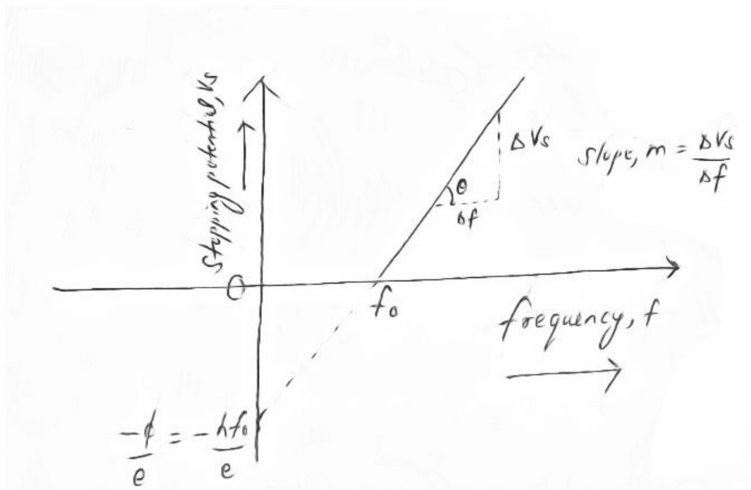

Figure 2: Plot between stopping potential and frequency of incident light

Using light of different frequencies, the corresponding stopping potentials are measured. Millikan plotted a graph between the stopping potential V_s and frequency of light f. A straight line is obtained as shown in the figure 2. This straight line verifies Einstein's photoelectric equation. Comparing equation (4) with (5) we have,

$m = \frac{h}{e}$ and $c = \frac{-\phi}{e}$

Since $m = \frac{h}{e}$,

$$h = m \times e$$

$$h = \frac{\Delta V_s}{\Delta f} \times e$$

Thus, by knowing the slope of the line AB, the value of h can be determined experimentally and its value is found to be $6.63 \times 10^{-34} Js$ which is a constant.

Similarly,

$$c = \frac{-\phi}{e}$$

$$\phi = -c \times e$$

$$\text{or, } \phi = hf_0$$

$$\text{or, } \phi = \frac{h}{e} \times e \times f_0$$

$$\therefore \phi = \frac{\Delta V_s}{\Delta f} \times e \times f_0$$

In this way, the value of work function of the material can also be determined experimentally.