

# Bush 631-600: Quantitative Methods

Lecture 10 (11.08.2022): Uncertainty vol. I

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## What is today's plan?

- ▶ Calculating uncertainty: detecting 'real' findings.
- ▶ From r.v.s. to estimators.
- ▶ Types of estimators: data, surveys, experiments.
- ▶ Simulations.
- ▶ Confidence intervals
- ▶ R work: `table()`, loops, simulations, plots.

## Final project

- ▶ Research proposal lessons:
  - ▶ Clear objective / research question.
  - ▶ Literature - what do we know?
  - ▶ Focus on concepts, not measures.

# Final project

Data report:

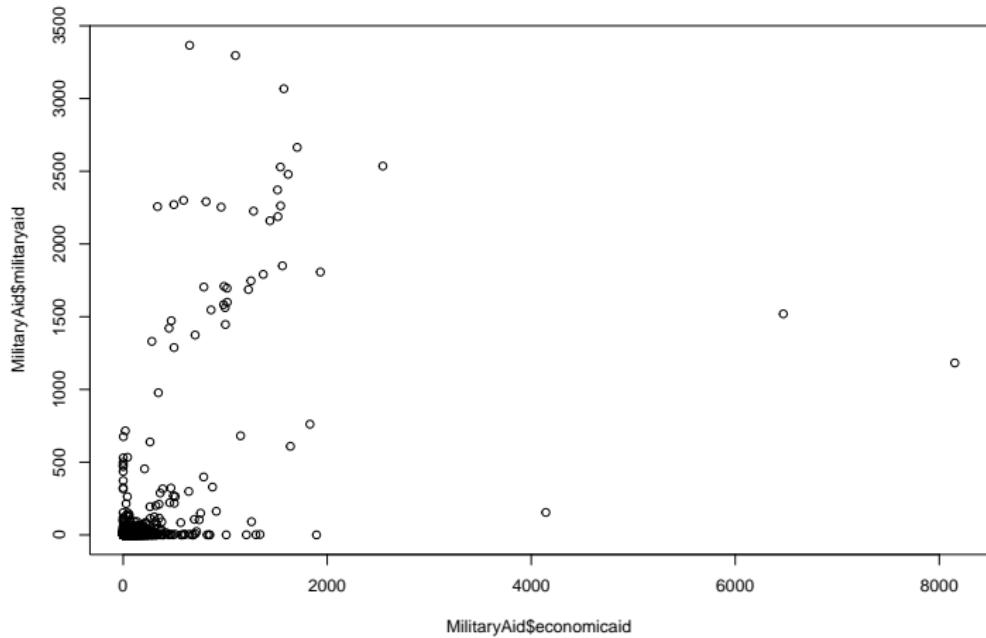
- ▶ Succinct description of topic and importance.
- ▶ What are my central arguments?
- ▶ Clear variable names.
- ▶ Variable values.

Visuals?

- ▶ Labels (axis, ticks).
- ▶ Title.
- ▶ Attention grabbing - use colors and add relevant text.

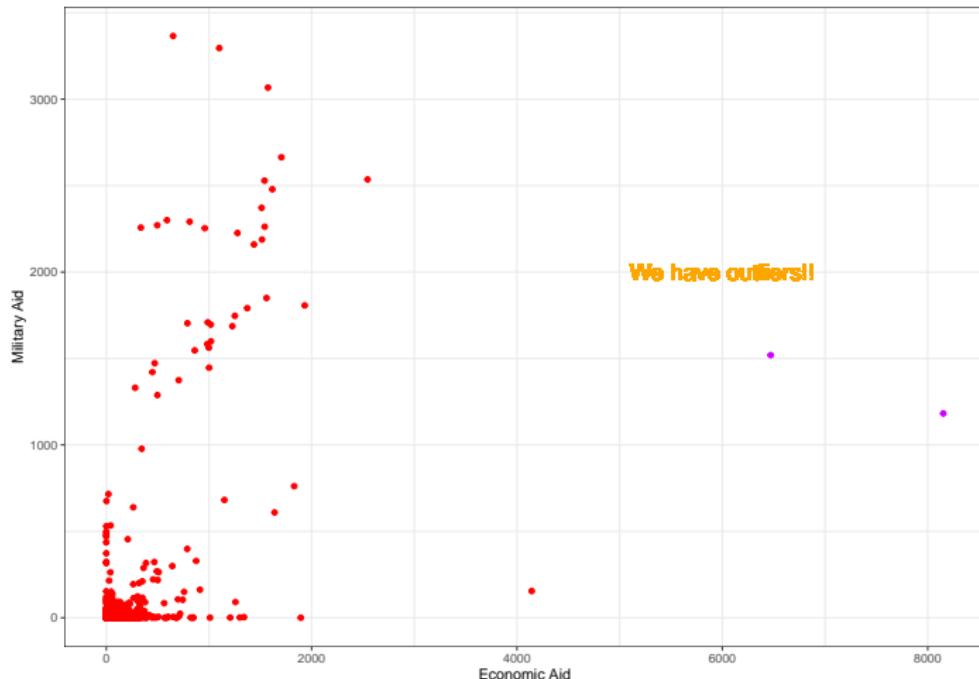
# Useful visuals

What looks better?



# Useful visuals

Or this...



# We have findings!!!

- ▶ Data patterns are systematic? Or noise?
- ▶ Our estimates → real relationship or random?
- ▶ Using probability calculations.

## Events to numbers

- ▶ Random variables: map outcomes to numbers.
- ▶ Assess quantities in population → we cannot.
- ▶ Use sample: r.v.s and the values of concepts.
- ▶ Define a random variable  $X$ :
  - ▶  $X=1$  if 'random' person supports president, 0 otherwise.
  - ▶  $\bar{X} = E[\bar{X}] = \mu ??$
  - ▶ Yes!!
  - ▶ Large samples to the rescue.

# Uncertainty on the diamond

## 2022 MLB Predictions

Updated after every game.

Standings Games Pitchers Teams

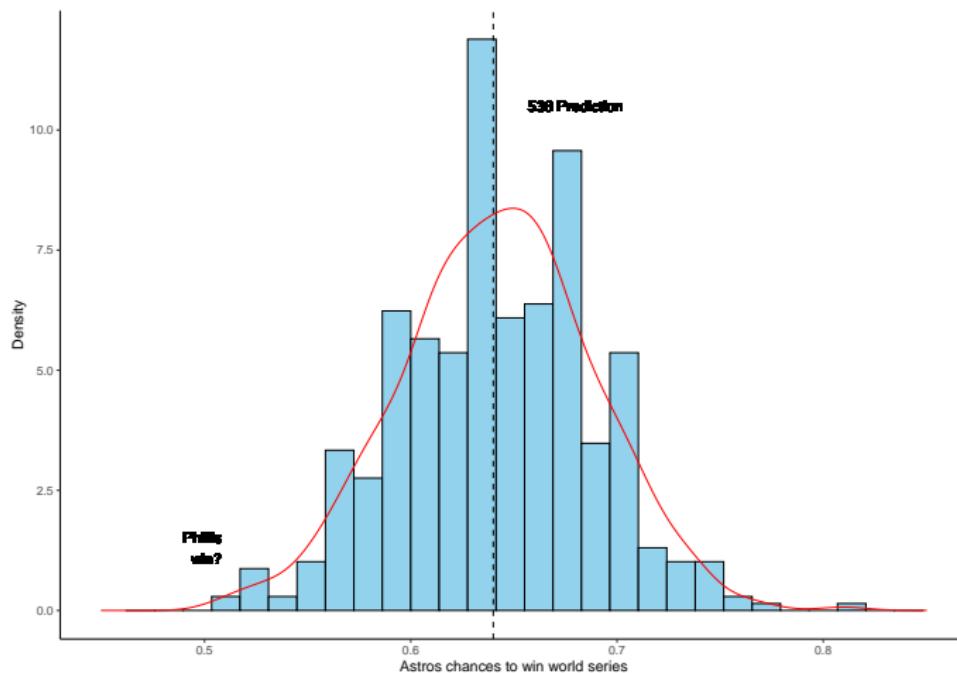
TEAM	DIVISION	TEAM RATING	1-WEEK CHANGE	CHANCE OF MAKING ...			WIN WORLD SERIES
				DIV. SERIES	LEAGUE CHAMP.	WORLD SERIES	
 <b>Astros</b> 3-0, 3-0	AL West	1588	+7	✓	✓	95%	64%
 <b>Phillies</b> 3-1, 4-1	NL East	1547	+6	✓	✓	✓	33%

- ▶ No shot for the Phils?
- ▶ What does 64% mean?

# Uncertainty on the diamond

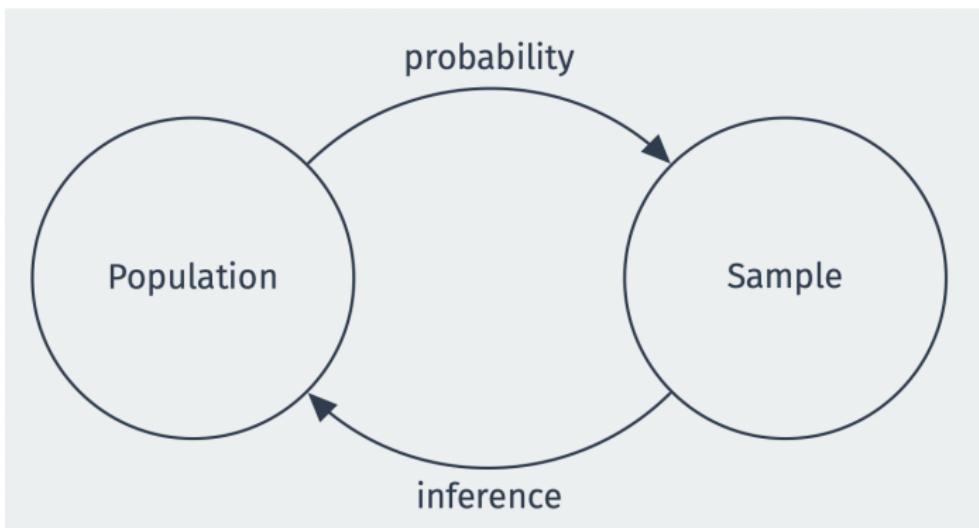
```
# Simulate the World series
sim <- 500
draws <- rbinom(sim, size = 100, prob = 0.64) / 100
mean(draws)

## [1] 0.64176
```



## Our data - our research interests

- ▶ Making inferences from data to population



# Uncertainty

- ▶ Research questions:
  1. President's gender and FP actions?
  2. Regime type and frequency of terrorism?
  3. Regional trade zone and countries trade balance?
- ▶ Treatment / Factor has an effect:
  - ▶ Women are more aggressive in defense spending and public threats.
  - ▶ Democratic regimes experience more terror incidents.
  - ▶ Regional trade zone increased the trade balance with neighbors.
- ▶ Are these effects real or just noise?

# Uncertainty in data: US and WW II

## Pearl Harbor (December 7, 1941)

“Signals to noise ratio”  
(Wohlstetter 1962)

Diplomatic .vs. military intelligence  
(Kahn, 1991)



*David Kahn*  
THE INTELLIGENCE FAILURE  
OF PEARL HARBOR

**O**n a low-sunrise morning in 1940, Frank B. Roskin, a 37-year-old civilian employee of the U.S. Army, climbed into his Ford sedan in Arlington, Virginia, and drove to his job in Washington, D.C. Though his work was all but classified and required him to remain silent about it, Roskin was disciplined and kept his mind on the traffic. He parked in a lot behind the Munitions Building on Constitution Avenue, arriving at 7 a.m., an hour early, as was his custom. He walked down one of the wings that stretched out from the building's main entrance, past a guardhouse, and an armed guard locked the entrance to Rooms 3416 and 3418. They were among the most secure in the entire structure, and Roskin was one of them among the most secret in the U.S. government.

In the days before codebreakers, he had been one of many trying

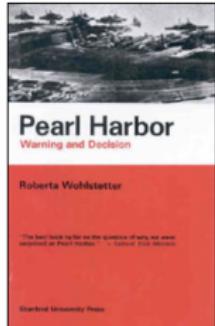
to crack the most difficult cipher of the Empire of Japan,

a machine that American cryptanalysts called PURPLE,

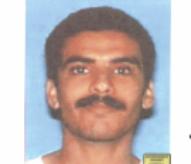
and within hours on that day, Friday, September 29, he would

be celebrating one of the greatest moments in American

cryptology.



# Uncertainty in data: 9/11 Intelligence failure

MEMORANDUM FOR:	
FROM:	
OFFICE:	
SUBJECT:	Re: Khalid Al-Mihdhar
REFERENCE:	
Original Text of Original Text of	[Redacted]
TO:	
FROM:	
OFFICE:	
DATE:	08/21/2001 04:05:02 PM
SUBJECT:	Re: Khalid Al-Mihdhar
<p>WHAT??: Same passport number? How interesting. I know his fellow travelers made one or two trips to the US in the same January time frame, yes? Probably would be useful to memorialize the US visits of the party in a cable.....</p> <p>[Redacted]</p> <p>[Redacted] as I was reviewing all the cables on Khalid Al-Mihdhar, I noticed he had a U.S. Visa in his passport. I asked INS to check and they just came back and said he entered the U.S. on 15 January 2000 and listed the Los Angeles [Redacted] as his destination. He departed the U.S. on 10 June 2000. I looked through traffic and could not find anything else.</p> <p>I'll be sending [Redacted] to FBI to pass what we know of Khalid Al-Mihdhar and that he entered the U.S. on 15 January. Maybe there is something they can do — perhaps run his name by Ressam? I will be here in the morning, and will then be meeting with [Redacted] in the early afternoon to talk about the U.S.S. Cole and will give her a head's up. Let me know if you need me to do anything.</p>	

# Estimation

- ▶ *Quantity of interest* in population.
- ▶ *Point estimation* → a ‘best guess’.
- ▶ Many possible point estimators:
  - ▶ Population mean ( $\mu$ ): elections turnout.
  - ▶ ‘Special population’ mean ( $\mu$ ): likelihood of joining international treaty.
  - ▶ Variance of a r.v. ( $\sigma^2$ ): variation in support for sanctioning China/Russia.
  - ▶ Population ATE ( $\mu_1 - \mu_0$ ): difference b-w treatment and control groups.

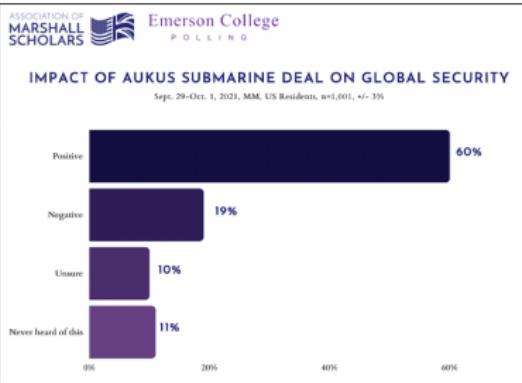
# Estimation

## Estimator $\theta$



**AUKUS PARTNERSHIP**

The UK, the United States and Australia have agreed a landmark defence and security partnership that will defend our shared interests around the world



# How to estimate public opinion?

CNNB MARKETS BUSINESS INVESTING TECH POLITICS CNBC TV WATCHLIST CRAMER PRO

POLITICS

## Iran will return to nuclear talks after months of stalled negotiations, State Department says

PUBLISHED WED, NOV 3 2021 4:29 PM EDT

Amanda Macias @AMANDA\_M\_MACIAS

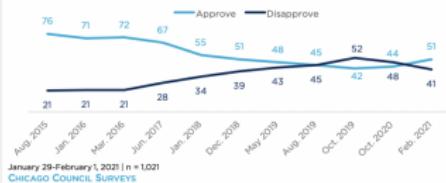
SHARE f t in e

The screenshot shows a news article from CNN Politics. The headline reads "Iran will return to nuclear talks after months of stalled negotiations, State Department says". The article was published on Wednesday, November 3, 2021, at 4:29 PM EDT. The author is Amanda Macias (@AMANDA\_M\_MACIAS). There are social sharing options for Facebook, Twitter, LinkedIn, and Email.



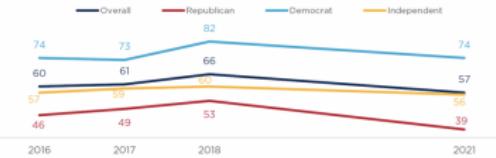
### Iranians More Likely to Approve of JCPOA

As you may know, in July 2015, Iran and the P5+1 countries reached a comprehensive agreement in regard to Iran's nuclear program, which is also known as the JCPOA. In general and based on what you know about the JCPOA, to what degree do you approve or disapprove of this agreement? Do you: (%)



### Democrats and Republicans Diverge on Iran Agreement

Based on what you know, do you think the United States should or should not participate in the following international agreements? (% participate) The agreement that lifts some international economic sanctions against Iran in exchange for strict limits on its nuclear weapons



- ▶ Random sample of respondents.

## Estimating with public samples

- ▶ Assume:  $X_1 \dots X_n$  iid *Bernoulli distributed* random variables.
- ▶ Proportion of support for deal  $\rightarrow p$ .
- ▶ An estimate: one realization of estimator (random variables right??)
- ▶ Estimate:

$$\hat{\theta} = \bar{X}_n \rightarrow \text{population } p$$

## Still, an estimation...

- ▶ Is our estimate good?
- ▶ **Estimation error:** difference with 'true value'.
- ▶ Error =  $\bar{X}_n - p$
- ▶ p is unknown, now what?
- ▶ Calculate average magnitude of estimation error.
- ▶ Hypothetical repetition of sampling:
  - ▶ Multiple estimate values ( $\hat{\theta}$ )
  - ▶ Multiple estimation error values.

# Estimation

- ▶ Repetition → sampling distribution of  $\hat{\theta}$
- ▶ Estimation error / bias using *expectations*
- ▶  $bias = E(\text{est.Error}) = E(\text{Estimate} - \text{truth}) = E(\bar{X}_n) - p = p - p = 0$
- ▶ **Unbiasedness:** Sample proportion is on average equal to the population proportion.
- ▶ Accuracy over multiple samples (not a single-shot survey)
- ▶ Estimator is *unbiased*

## Estimators in experiments

- ▶ Treatment(s) and control groups.
- ▶ *Estimator* → diff-in-means.
- ▶ Sample Average Treatment Effect (SATE):

$$SATE = \frac{1}{n} * \sum_{i=1}^n [Y_i(1) - Y_i(0)]$$

## Diff-in-means estimator

- ▶ Random sampling of population.
- ▶ Random assignment into treatment(s).
- ▶ Population Average Treatment Effect (PATE)
- ▶  $PATE = E[Y(1) - Y(0)]$
- ▶ Diff-in-means estimator is *unbiased*

# Unbiased estimator

## ► Monte-Carlo simulations

```
# Create Sample, Control and treatment groups (means and SDs)
n <- 500
mu0 <- 0
sd0 <- 1
mu1 <- 1
sd1 <- 1

# Create sampling distributions
y0 <- rnorm(n, mean = mu0, sd = sd0)
head(y0)

## [1] -0.99274116 -1.32054615 -2.79817571  0.48616276 -0.03684791  1.41178990
y1 <- rnorm(n, mean = mu1, sd = sd1)

# calculate diff-in-means (SATE)
tau <- y1 - y0
head(tau)

## [1]  2.0929783  4.1806412  2.9382569  0.6697544  1.5358116 -0.2688237
SATE <- mean(tau)
SATE

## [1] 1.014856
```

# Increasing the sample

- ▶ Simulate & randomly assign treatment

```
# Repeat
sims <- 5000
diff.means <- rep(NA,sims)

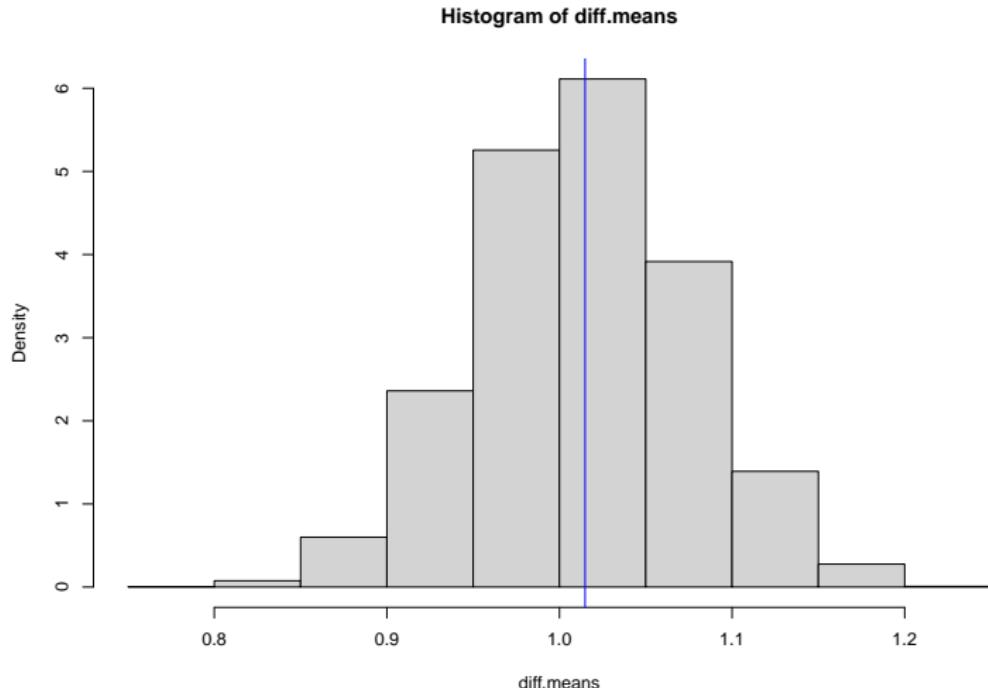
for (i in 1:sims){
  treat <- sample(c(rep(1, n/2), rep(0, n/2)), size = n, replace = FALSE)
  diff.means[i] <- mean(y1[treat == 1] - mean(y0[treat == 0]))
}

est.error <- diff.means - SATE
summary(est.error)

##          Min.    1st Qu.     Median      Mean    3rd Qu.      Max.
## -0.2459536 -0.0418603 -0.0015130 -0.0008485  0.0408982  0.2020634
```

## SATE estimator (large sample simulation)

```
hist(diff.means, freq = FALSE)  
abline(v=SATE, col = "blue")
```



# Estimator distribution

- ▶ Calculate variation with SD (estimator)

```
# SD of estimator  
sd(diff.means)  
  
## [1] 0.06236986  
sqrt(mean((diff.means - SATE)^2))  
  
## [1] 0.0623694
```

- ▶ Calculate SD - only with a simulation.
- ▶ Reality → one sample, SD is unknown.

# SD of sample

- ▶ **Standard error:** estimated degree of deviation from expected value
- ▶ Variability of our (single!) sample
- ▶  $\sqrt{\hat{V}(\hat{Y})} = \sqrt{\frac{\bar{Y}*(1-\bar{Y})}{n}}$

```
# Simulate and add SE calculate
sims2 <- 5000
diff.means2 <- rep(NA,sims)
diff.se <- rep(NA, sims)

for (i in 1:sims){
  Y0 <- rnorm(n, mean = mu0, sd = sd0)
  Y1 <- rnorm(n, mean = mu1, sd = sd1)
  treat <- sample(c(rep(1, n/2), rep(0, n/2)), size = n, replace = FALSE)
  diff.means2[i] <- mean(Y1[treat == 1] - mean(Y0[treat == 0]))
  diff.se[i] <- sqrt(var(Y1[treat == 1])/(n/2) + var(Y0[treat == 0])/(n/2))
}

sd(diff.means2)

## [1] 0.08911718
mean(diff.se)

## [1] 0.08936297
```

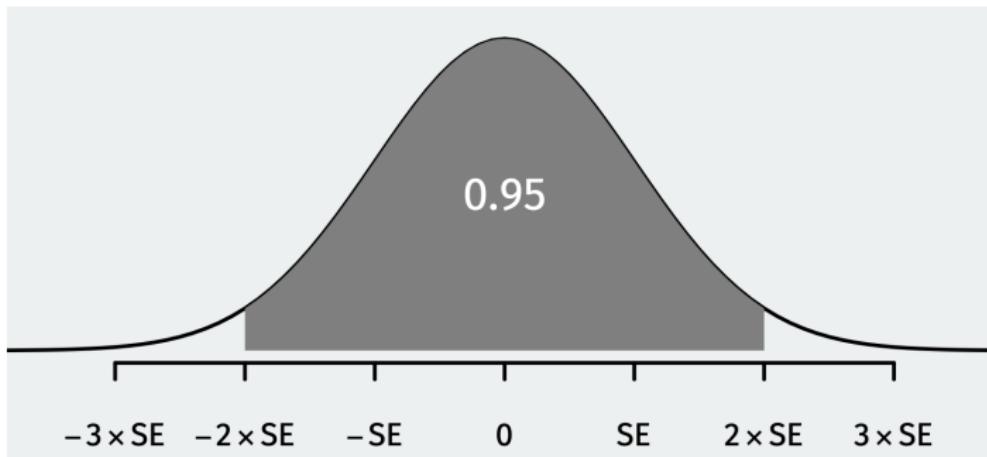
## Broader approach to estimator distribution

- ▶ Quantities beyond means and SD.

### CONFIDENCE INTERVALS

- ▶ Range of true values of estimator.
- ▶ Range of plausible values.
- ▶ Rest on assuming repeated sampling.

## Chance errors intervals



- ▶ Normal distribution empirical rule.
- ▶ 95% of values within 2 SD, in sample  $\rightarrow$  2 SEs.
- ▶ Range of possible values  $\rightsquigarrow \pm 1.96$  SEs

## BYO CIs

- ▶ Constructing confidence intervals.
- ▶ (1) What *confidence level*?
- ▶ Conventional: 95%.
- ▶ Defined using  $\alpha(0 - 1) = ?$
- ▶ (2) CI:  $100 * (1 - \alpha)\% = \bar{Y} \pm z_{\alpha/2} * SE$
- ▶  $\alpha = 0.05 \rightarrow 95\% \text{ CI.}$

# Confidence Intervals

- ▶ Formal CI:

$$CI(\alpha) = (\bar{X}_n - z_{\alpha/2} * SE, \bar{X}_n + z_{\alpha/2} * SE)$$

- ▶ *Critical value* =  $(1 - \alpha/2)$

$\alpha$	<i>Confidence level</i>	<i>Critical value</i> $z_{\alpha/2}$	<i>R expression</i>
0.01	99%	2.58	<code>qnorm(0.995)</code>
0.05	95%	1.96	<code>qnorm(0.975)</code>
0.1	90%	1.64	<code>qnorm(0.95)</code>

# Confidence intervals

- ▶ Finding the critical values
- ▶ `qnorm()` function: define `lower.tail = FALSE`

```
# find critical values
qnorm(0.05, lower.tail = FALSE)
```

```
## [1] 1.644854
qnorm(0.025, lower.tail = FALSE)
```

```
## [1] 1.959964
qnorm(0.005, lower.tail = FALSE)
```

```
## [1] 2.575829
```

# ClIs in R

- ▶ ClIs for our JCPOA survey

```
# Sample, Mean support and SE
n <- 2000
x.bar <- 0.6
Iran.se <- sqrt(x.bar * (1-x.bar)/n)
Iran.se

## [1] 0.01095445

# CIs
c(x.bar - qnorm(0.995) * Iran.se, x.bar + qnorm(0.995) * Iran.se) #99%
## [1] 0.5717832 0.6282168
c(x.bar - qnorm(0.975) * Iran.se, x.bar + qnorm(0.975) * Iran.se) #95%
## [1] 0.5785297 0.6214703
c(x.bar - qnorm(0.95) * Iran.se, x.bar + qnorm(0.95) * Iran.se) #90%
## [1] 0.5819815 0.6180185
```

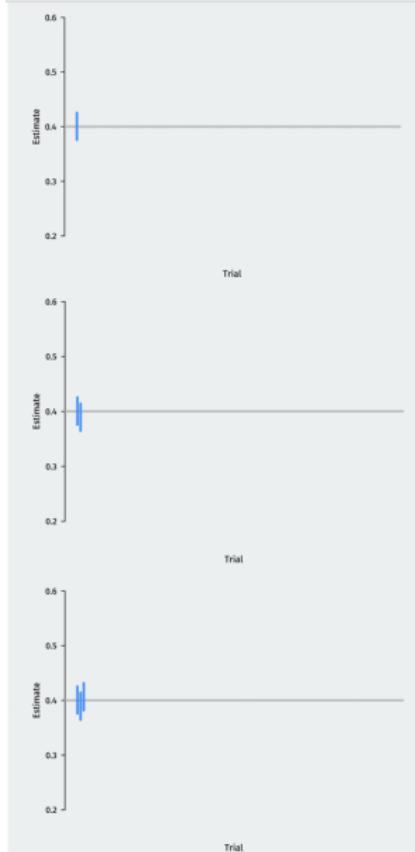
# Interpretation

- ▶ How to interpret CIs?
- ▶ NO → 95% chance true value is within the interval.
- ▶ Why? Estimator is unknown (value is 0/1).
- ▶ YES → Interval contains true value 95% of the times in repeated random samples.
- ▶ Not the **Wait What?** pic again right???
- ▶ **One more time:**

Interval contains the true value 95% of the times in repeated random samples

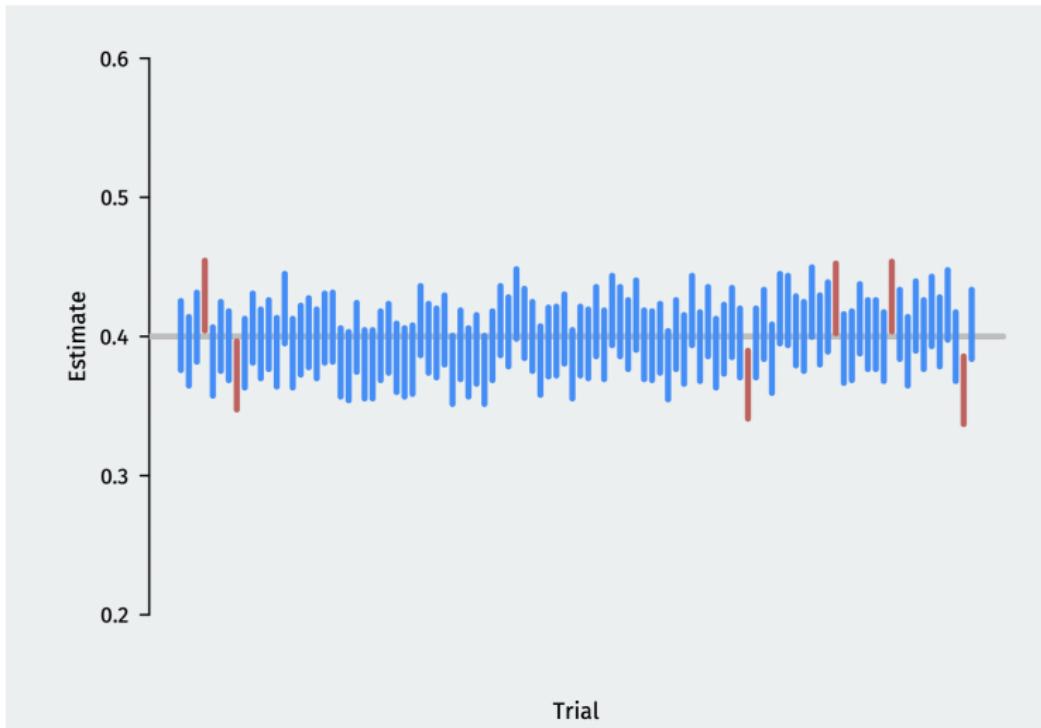
# Simulate CIs

- ▶ Policy: Global  $\text{CO}_2$  emissions reduction
- ▶ Sample = 1500 respondents
- ▶  $p = 0.4$  (assumed support)
- ▶ Calculate 95% CIs in multiple samples



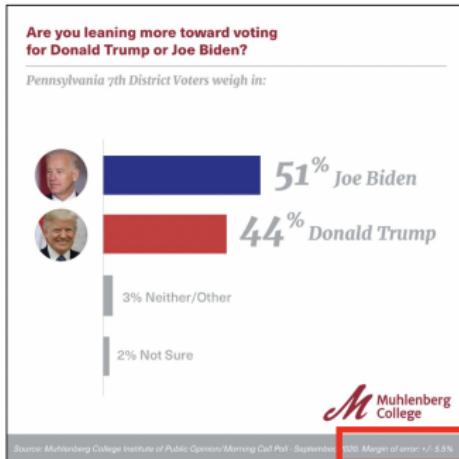
## Simulate CIs

- ▶ How many overlap with 'true' support?

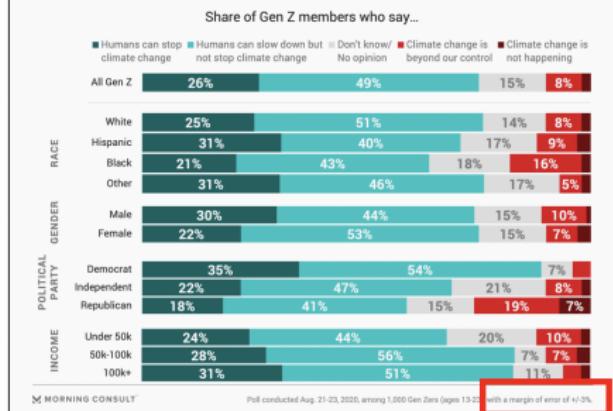


# Polls: the ‘fine print’

## Margin of error



### Most Gen Zers Say Climate Change's Progression Is Inevitable



## Margin of error

- ▶ MOE: half-width of a 95% CI.
- ▶ JCPOA sample proportion of support = 0.6
- ▶ JCPOA sample MOE =  $\pm 3\%$
- ▶ JCPOA 95% CI: [57%,63%]

## Margin of error

$$MOE = \pm z_{0.025} * SE \approx \pm 1.96 * \sqrt{\frac{\bar{X}_n * (1 - \bar{X}_n)}{n}}$$

- ▶ What is the minimum sample size?
- ▶ Popular stage in research design.
- ▶ Conduct **before** fielding the survey.

## MOE and Sample size

- ▶ Calculate multiple proportions of support.
- ▶ Define your MOE: 1% , 2%, 3%, 5%
- ▶ Possible sample sizes

```
# Define MOEs
moe <- c(0.01, 0.02, 0.03, 0.05)

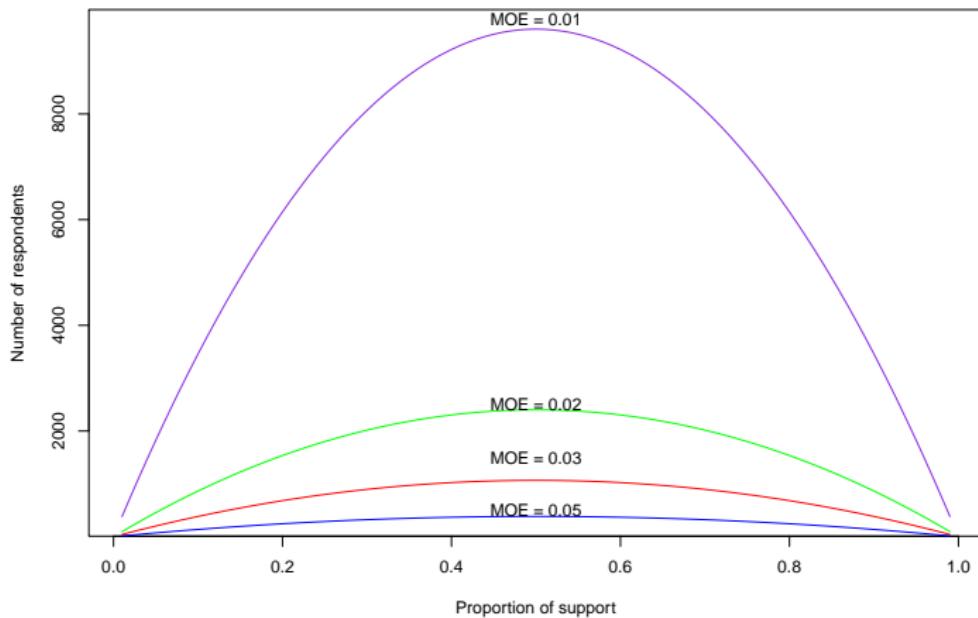
# Define vector of proportion of support (0-100 by 1%)
prop <- seq(from = 0.01, to = 0.99, by = 0.01)

# Using MOE and proportion for possible sample sizes
num <- 1.96^2 * prop * (1-prop) / moe[1]^2
head(num, n=10)

## [1] 380.3184 752.9536 1117.9056 1475.1744 1824.7600 2166.6624 2500.8816
## [8] 2827.4176 3146.2704 3457.4400
```

# MOE and Sample size

- ▶ Plotting our analysis
- ▶ CLT, SE and sample size...



## ClIs & Experiments

- ▶ Quantify uncertainty for causal effect analysis.
- ▶ JCPOA support among Americans → good!
- ▶ Variations of JCPOA support among groups → even better!
  - ▶ Men  $\longleftrightarrow$  Women.
  - ▶ Young  $\longleftrightarrow$  Old.
  - ▶ Vets (military)  $\longleftrightarrow$  no military background.
- ▶ Estimator: *population ATE*
- ▶  $\mu_T - \mu_C$

## Let's reduce plastics

- ▶ Environmental policy: 'fighting-back' against plastic bags.
- ▶ Policy, main aspects - financial incentives:
  1. Financial incentives: cash back.
  2. Financial incentives: fee for plastic bags.
- ▶ Define outcome:  $X_i = 1$  if support policy, 0 otherwise.
- ▶ Sample mean (treatment: cash back),  $\bar{X}_T = 0.43$
- ▶ Sample mean (control: plastics fee),  $\bar{X}_C = 0.32$

$$\hat{ATE} = \bar{X}_T - \bar{X}_C = 0.11$$

## Simulating policy support

- ▶ Sample diff-in-means on average equal to population diff-in-means
- ▶ Still, some variation

```
# Simulate our experiment in population
xt.sims <- rbinom(1000, size = 1000, prob = 0.43) / 1000
head(xt.sims)

## [1] 0.423 0.438 0.414 0.459 0.428 0.429

xc.sims <- rbinom(1000, size = 1000, prob = 0.32) / 1000
head(xc.sims)

## [1] 0.326 0.321 0.314 0.304 0.309 0.324

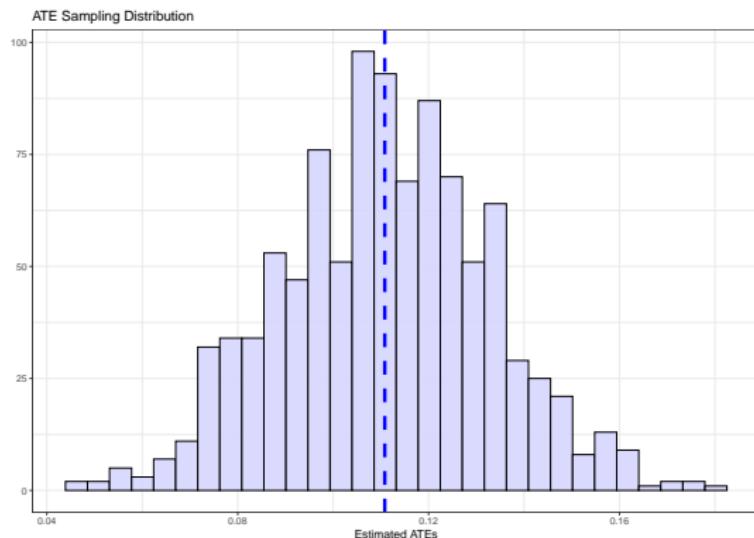
# Mean
mean(xt.sims-xc.sims)

## [1] 0.110806
```

# ATE distribution

- ▶ How our  $\hat{ATE} \approx 0.11$  looks like?

```
# Plot with tidyverse
hp <- data.frame(mn = (xt.sims-xc.sims))
ggplot(hp, aes(mn)) +
  geom_histogram(fill="#D6D7FF", color="black", alpha=0.9) +
  geom_vline(xintercept = mean(hp$mn), color = "blue", linetype = "dashed", size = 1.5) +
  xlab("Estimated ATEs") + ylab("") + ggtitle("ATE Sampling Distribution") +
  theme_bw()
```



## Simulating policy support

- ▶  $\hat{ATE} \approx 0.11 \rightarrow$  makes a difference?
- ▶ Use SEs to learn of variation of estimator

```
# Calculate SE
x.se <- sqrt((0.43*0.57)/1000 + (0.32*0.68)/1100)
x.se

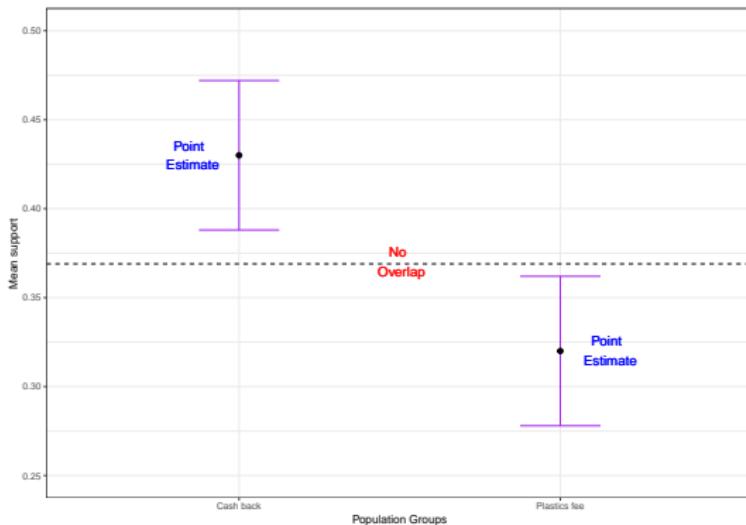
## [1] 0.02104562
# 95% CIs for meaningful results
c(0.43 - qnorm(0.975) * x.se, 0.43 + qnorm(0.975) * x.se)

## [1] 0.3887513 0.4712487
c(0.32 - qnorm(0.975) * x.se, 0.32 + qnorm(0.975) * x.se)

## [1] 0.2787513 0.3612487
```

# Plot and check effect

```
# plot with tidyverse
ggplot(se_plot, aes(x,y)) +
  geom_errorbar(aes(ymin = y-2*se, ymax = y+2*se), width = 0.25, color = "purple") +
  geom_point(size = 2) + ylim(0.25,0.5) +
  geom_hline(yintercept = 0.369, linetype = "dashed") +
  geom_text(x=1.5,y=0.37,label = "No \n Overlap", color = "red", size = 4.5) +
  geom_text(x=0.85,y=0.43,label = "Point \n Estimate", color = "blue", size = 4.5) +
  geom_text(x=2.15,y=0.32,label = "Point \n Estimate", color = "blue", size = 4.5) +
  ylab("Mean support") + xlab("Population Groups") +
  theme_bw()
```



## More simulations and data

- ▶ Create our own experimental data
- ▶ `library(fabricatr)`: Random data generator
- ▶ Steps:
  1. Create treatments (assign sample size and probabilities).
  2. Create binary outcome variables.
  3. Create continuous outcome variables.
- ▶ Join all variables into one large data set.
- ▶ Focus on treatment 1 and cont. outcome variable:
  - ▶ Regime of aid recipient (democracy or not).
  - ▶ Extent of aid provided.

# Create random data

- ▶ Code for treatments and all variables

```
## Create data
# Set seed for randomizer
set.seed(12345)

# Create treatments (sample size of 1000)
exp.dat <- fabricate(
  N = 1000,
  trt1 = draw_binary(N = 1000, prob = 0.5),
  trt2 = draw_binary(N = 1000, prob = 0.5))

# Create Binary & Continuous outcome variables
random_vars <- fabricate(
  N = 1000,
  dv_cor1 = correlate(given = exp.dat$trt1, rho = 0.8,
                      draw_binary, N = 1000, prob = 0.65),
  dv_cor2 = correlate(given = exp.dat$trt2, rho = 0.65,
                      draw_binary, N = 1000, prob = 0.35),
  cont_cor1 = correlate(given = exp.dat$trt1, rho = 0.55,
                        rnorm, mean = 1500, sd = 30),
  cont_cor2 = correlate(given = exp.dat$trt2, rho = 0.75,
                        rnorm, mean = 1450, sd = 45))
```

# Create random data

- ▶ Join variables and final data output

```
# Tidyverse approach to join columns
exp.dat <- left_join(exp.dat, random_vars, by = "ID")
```

```
# Our random experimental data
head(exp.dat, n=8)
```

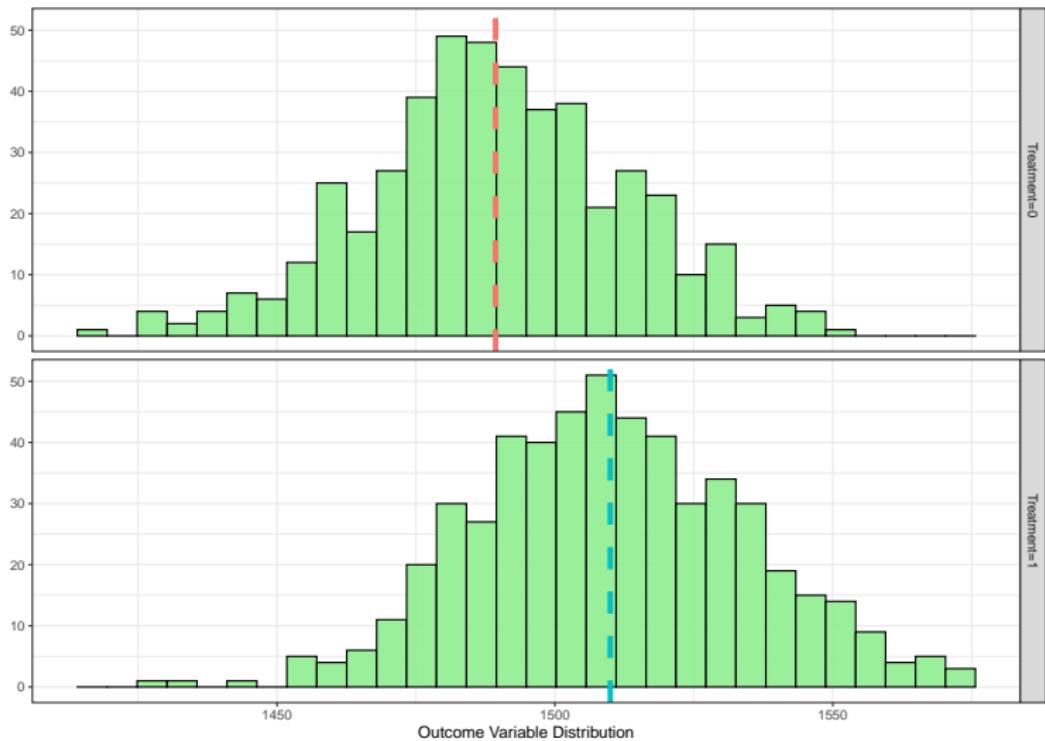
```
##      ID trt1 trt2 dv_cor1 dv_cor2 cont_cor1 cont_cor2
## 1 0001    1    0      1      0  1523.100  1395.533
## 2 0002    1    1      1      1  1492.402  1466.578
## 3 0003    1    0      1      0  1500.165  1431.904
## 4 0004    1    0      1      0  1510.011  1406.666
## 5 0005    0    1      0      1  1515.649  1442.158
## 6 0006    0    0      0      0  1512.053  1430.640
## 7 0007    0    0      0      1  1474.265  1451.380
## 8 0008    1    1      1      0  1498.759  1443.719
```

# Exploring the experimental data

- ▶ Random assignment of ‘respondents’?
- ▶ Calculate mean outcome for treatment 1 and ATE.

```
# How many 'respondents' assigned per treatment?  
n.zero <- sum(exp.dat$trt1 == 0)  
n.zero  
  
## [1] 469  
n.one <- sum(exp.dat$trt1 == 1)  
n.one  
  
## [1] 531  
# Mean outcome variable by treatment 1  
est.zero <- mean(exp.dat$cont_cor1[exp.dat$trt1 == 0])  
est.zero  
  
## [1] 1489.333  
est.one <- mean(exp.dat$cont_cor1[exp.dat$trt1 == 1])  
est.one  
  
## [1] 1509.973  
# calculate ATE ( $Y(1) - Y(0)$ )  
est.one - est.zero  
  
## [1] 20.6396
```

# How does it look?



# Regime treatment matters?

- ▶ Calculate margin of error → SEs
- ▶ Calculate CIs (define  $\alpha = 0.05$ )

```
# SEs for treatment 1 results
se.zero <- sd(exp.dat$cont_cor1[exp.dat$trt1 == 0]) / sqrt(n.zero)
se.zero

## [1] 1.068058

se.one <- sd(exp.dat$cont_cor1[exp.dat$trt1 == 1]) / sqrt(n.one)
se.one

## [1] 1.06291

# Define alpha
alpha <- 0.05

# CIs
ci.zero <- c(est.zero - qnorm(1-alpha / 2) *
               se.zero, est.zero + qnorm(1-alpha / 2) * se.zero)
ci.zero

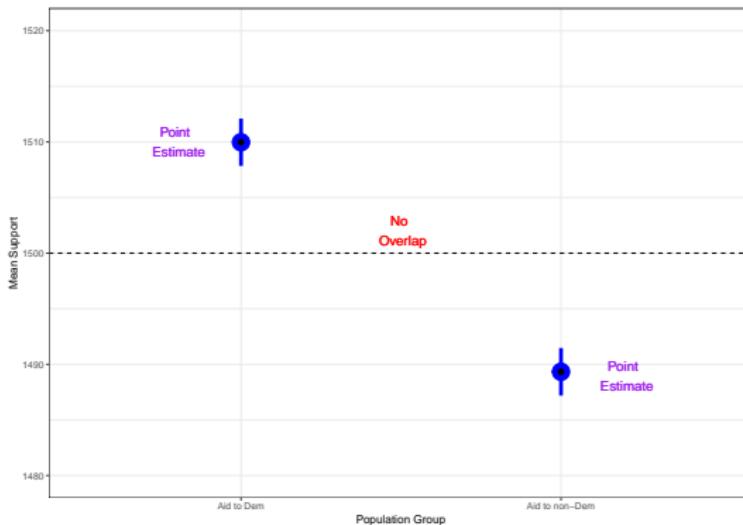
## [1] 1487.240 1491.427

ci.one <- c(est.one - qnorm(1-alpha / 2) *
               se.one, est.one + qnorm(1-alpha / 2) * se.one)
ci.one

## [1] 1507.890 1512.056
```

# How does our effect looks? matters?

```
# plot with tidyverse
ggplot(se_plot2, aes(x,y)) +
  geom_pointrange(aes(ymin = y-2*se, ymax = y+2*se), color = "blue", size = 1.75) +
  geom_point(size = 2) + ylim(1480,1520) +
  geom_hline(yintercept = 1500, linetype = "dashed") +
  geom_text(x=1.5,y=1502,label = "No \n Overlap", color = "red", size = 4.5) +
  geom_text(x=0.8,y=1510,label = "Point \n Estimate", color = "purple", size = 4.5) +
  geom_text(x=2.2,y=1489,label = "Point \n Estimate", color = "purple", size = 4.5) +
  ylab("Mean Support") + xlab("Population Group") +
  theme_bw()
```



## Clarifying objectives

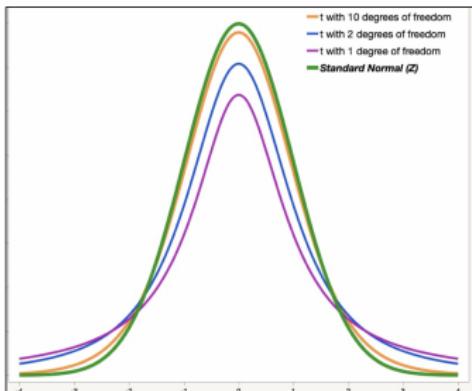
- ▶ What's with all the simulations?
- ▶ Real world: 1 sample, 1 mean...
- ▶ Research supported by simulations: public policy
  - ▶ Support for government policy: expand anecdotal findings.
  - ▶ Lobbying in the senate: women representatives example.
- ▶ Research supported by simulations: business world
  - ▶ Product design and development: expand A/B testing.

## Estimation approaches

- ▶ Estimation thus far → CLT
- ▶ ATE & CIs are based on CLT assumption
- ▶ Alternative: outcome variable  $\sim N(\mu, \sigma^2)$
- ▶ Use *student's t-distribution*:
  - ▶ Also describes DOF (degrees of freedom).
  - ▶ normal z-score == student's t-statistic.
  - ▶ Distribution has 'heavier tails'.

# student's t-distribution

- ▶  $DOF = (n - k)$ , ( $n$ = observations;  $k$ =model parameters).
- ▶ Critical value: t-statistic



Numbers in each row of the table are values on a <i>t</i> -distribution with <i>df</i> degrees of freedom for selected right-tail (greater-than) probabilities ( <i>p</i> ).								
<i>df</i>	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324803	1.000000	3.077804	6.313752	12.70620	31.00052	63.05624	68.1192
2	0.288675	0.819457	1.80518	2.919952	4.30353	6.96456	9.32484	31.5891
3	0.279671	0.748952	1.637144	2.553563	3.16045	4.54071	5.94091	12.9240
4	0.270722	0.649657	1.533026	2.31942	2.77445	3.74695	4.40491	6.6163
5	0.267811	0.726687	1.475884	2.01948	2.57053	3.28493	4.03214	6.8888
6	0.264825	0.717559	1.420956	1.94316	2.44681	3.14267	3.70743	5.9588
7	0.263617	0.711142	1.414824	1.898578	2.36462	2.9975	3.49949	5.4079
8	0.261821	0.70587	1.396115	1.869548	2.3060	2.89646	3.39539	5.0413
9	0.26095	0.702722	1.380229	1.83113	2.26276	2.82144	3.24984	4.7899
10	0.260185	0.699812	1.372194	1.802481	2.22814	2.76377	3.18927	4.5889
11	0.259594	0.697445	1.363400	1.79588	2.20899	2.71908	3.10581	4.4370
12	0.25903	0.695465	1.362117	1.792236	2.17881	2.68100	3.05454	4.3176
13	0.25891	0.693802	1.360171	1.79033	2.16027	2.65231	3.01228	4.2298
14	0.25821	0.692417	1.349230	1.781318	2.14407	2.62494	2.9784	4.1485
15	0.25785	0.691157	1.340606	1.75350	2.13145	2.59247	2.94671	4.0728
16	0.25769	0.690132	1.336757	1.74595	2.11981	2.56349	2.9079	4.0150
17	0.257347	0.689132	1.333379	1.73962	2.10862	2.53803	2.88023	3.9851
18	0.257122	0.688384	1.330391	1.73408	2.09052	2.52522	2.87644	3.9719
19	0.25692	0.687621	1.327728	1.729133	2.09302	2.51948	2.86093	3.8834
20	0.25674	0.686954	1.325451	1.724718	2.08586	2.52796	2.84534	3.8495
21	0.25658	0.686352	1.323188	1.720743	2.07965	2.51765	2.83116	3.8183
22	0.25642	0.685836	1.321237	1.717144	2.07387	2.50832	2.81676	3.7821
23	0.25627	0.685356	1.319460	1.713872	2.06868	2.49897	2.8074	3.7659
24	0.25612	0.684893	1.317836	1.710982	2.0639	2.49216	2.79694	3.7454
25	0.25599	0.684423	1.31645	1.708141	2.05954	2.48511	2.78144	3.7251
26	0.25585	0.684043	1.314872	1.70561	2.05553	2.47963	2.77071	3.7068
27	0.25574	0.683665	1.313703	1.70326	2.05183	2.47266	2.76968	3.6899
28	0.25564	0.683353	1.312627	1.701131	2.04841	2.46714	2.76339	3.6779
29	0.25554	0.683046	1.311434	1.699127	2.04523	2.46232	2.75923	3.6658
30	0.25565	0.682759	1.310415	1.697261	2.04227	2.45726	2.75609	3.6460
31	0.25554	0.682493	1.309452	1.695454	2.03925	2.45255	2.75283	3.6295
32	0.25544	0.682248	1.308482	1.69371	2.03725	2.44801	2.74963	3.6165
33	0.25534	0.682016	1.307512	1.69208	2.03525	2.44357	2.74643	3.6035
34	0.25524	0.681806	1.306542	1.69045	2.03335	2.43914	2.74323	3.5905
35	0.25514	0.681607	1.305572	1.68881	2.03145	2.43471	2.73999	3.5775
36	0.25504	0.681411	1.304602	1.68717	2.02955	2.42998	2.73669	3.5645
37	0.25494	0.681216	1.303632	1.68553	2.02765	2.42465	2.73339	3.5515
38	0.25484	0.681022	1.302662	1.68391	2.02575	2.41892	2.73009	3.5385
39	0.25474	0.680829	1.301692	1.68229	2.02385	2.41259	2.72679	3.5255
40	0.25464	0.680636	1.300722	1.68067	2.02195	2.40596	2.72349	3.5125
41	0.25454	0.680443	1.299752	1.67905	2.01995	2.39813	2.71999	3.4995
42	0.25444	0.680251	1.298782	1.67743	2.01795	2.38999	2.71669	3.4865
43	0.25434	0.680059	1.297812	1.67581	2.01595	2.38166	2.71339	3.4735
44	0.25424	0.679867	1.296842	1.67419	2.01395	2.37293	2.70999	3.4605
45	0.25414	0.679675	1.295872	1.67257	2.01195	2.36399	2.70669	3.4475
46	0.25404	0.679483	1.294902	1.67095	2.00995	2.35465	2.70339	3.4345
47	0.25394	0.679291	1.293932	1.66933	2.00795	2.34482	2.69999	3.4215
48	0.25384	0.679101	1.292962	1.66771	2.00595	2.33469	2.69669	3.4085
49	0.25374	0.678909	1.291992	1.66609	2.00395	2.32396	2.69339	3.3955
50	0.25364	0.678717	1.291022	1.66447	2.00195	2.31263	2.68999	3.3825
51	0.25354	0.678525	1.290052	1.66285	1.99995	2.30086	2.68669	3.3695
52	0.25344	0.678333	1.289082	1.66123	1.99795	2.28803	2.68339	3.3565
53	0.25334	0.678141	1.288112	1.65961	1.99595	2.27496	2.67999	3.3435
54	0.25324	0.677949	1.287142	1.65799	1.99395	2.26183	2.67669	3.3305
55	0.25314	0.677757	1.286172	1.65637	1.99195	2.24856	2.67339	3.3175
56	0.25304	0.677565	1.285202	1.65475	1.98995	2.23499	2.66999	3.3045
57	0.25294	0.677373	1.284232	1.65313	1.98795	2.22116	2.66669	3.2915
58	0.25284	0.677181	1.283262	1.65151	1.98595	2.20693	2.66339	3.2785
59	0.25274	0.676989	1.282292	1.64989	1.98395	2.19236	2.65999	3.2655
60	0.25264	0.676797	1.281322	1.64827	1.98195	2.17733	2.65669	3.2525
61	0.25254	0.676605	1.280352	1.64665	1.97995	2.16176	2.65339	3.2395
62	0.25244	0.676413	1.279382	1.64503	1.97795	2.14563	2.64999	3.2265
63	0.25234	0.676221	1.278412	1.64341	1.97595	2.12876	2.64669	3.2135
64	0.25224	0.676029	1.277442	1.64179	1.97395	2.11103	2.64339	3.1995
65	0.25214	0.675837	1.276472	1.63917	1.97195	2.09296	2.63999	3.1865
66	0.25204	0.675645	1.275502	1.63655	1.96995	2.07423	2.63669	3.1735
67	0.25194	0.675453	1.274532	1.63393	1.96795	2.05496	2.63339	3.1605
68	0.25184	0.675261	1.273562	1.63131	1.96595	2.03483	2.62999	3.1475
69	0.25174	0.675069	1.272592	1.62869	1.96395	2.01376	2.62669	3.1345
70	0.25164	0.674877	1.271622	1.62607	1.96195	1.99153	2.62339	3.1215
71	0.25154	0.674685	1.270652	1.62345	1.95995	1.96806	2.61999	3.1085
72	0.25144	0.674493	1.269682	1.62083	1.95795	1.94349	2.61669	3.0955
73	0.25134	0.674301	1.268712	1.61821	1.95595	1.91876	2.61339	3.0825
74	0.25124	0.674109	1.267742	1.61559	1.95395	1.89383	2.60999	3.0695
75	0.25114	0.673917	1.266772	1.61297	1.95195	1.86866	2.60669	3.0565
76	0.25104	0.673725	1.265802	1.61035	1.94995	1.84329	2.60339	3.0435
77	0.25094	0.673533	1.264832	1.60773	1.94795	1.81776	2.59999	3.0305
78	0.25084	0.673341	1.263862	1.60511	1.94595	1.79193	2.59669	3.0175
79	0.25074	0.673149	1.262892	1.60249	1.94395	1.76576	2.59339	3.0045
80	0.25064	0.672957	1.261922	1.59987	1.94195	1.73919	2.58999	2.9915
81	0.25054	0.672765	1.260952	1.59725	1.93995	1.71236	2.58669	2.9785
82	0.25044	0.672573	1.259982	1.59463	1.93795	1.68533	2.58339	2.9655
83	0.25034	0.672381	1.258912	1.59191	1.93595	1.65796	2.57999	2.9525
84	0.25024	0.672189	1.257942	1.58929	1.93395	1.63029	2.57669	2.9395
85	0.25014	0.671997	1.256972	1.58667	1.93195	1.59999	2.57339	2.9265
86	0.25004	0.671805	1.255992	1.58405	1.92995	1.56836	2.56999	2.9135
87	0.24994	0.671613	1.255022	1.58143	1.92795	1.53649	2.56669	2.9005
88	0.24984	0.671421	1.254052	1.57881	1.92595	1.50446	2.56339	2.8875
89	0.24974	0.671229	1.253082	1.57619	1.92395	1.47229	2.55999	2.8745
90	0.24964	0.671037	1.252112	1.57357	1.92195	1.43986	2.55669	2.8615
91	0.24954	0.670845	1.251142	1.57095	1.91995	1.40729	2.55339	2.8485
92	0.24944	0.670653	1.250172	1.56833	1.91795	1.37452	2.54999	2.8355
93	0.24934	0.670461	1.249202	1.56571	1.91595	1.34155	2.54669	2.8225
94	0.24924	0.670269	1.248232	1.56309	1.91395	1.30858	2.54339	2.8095
95	0.24914	0.670077	1.247262	1.56047	1.91195	1.27551	2.53999	2.7965
96	0.24904	0.669885	1.246292	1.55785	1.90995	1.24244	2.53669	2.7835
97	0.24894	0.669693	1.245322	1.55523	1.90795	1.20927	2.53339	2.7705
98	0.24884	0.669501	1.244352	1.55261	1.90595	1.17609	2.52999	2.7575
99	0.24874	0.669309	1.243382	1.54999	1.90395	1.14282	2.52669	2.7445
100	0.24864	0.669117	1.242412	1.54737	1.90195	1.10955	2.52339	2.7315
101	0.24854	0.668925	1.241442	1.54475	1.89995	1.07628	2.51999	2.7185
102	0.24844	0.668733	1.240472	1.54213	1.89795	1.04301	2.51669	2.7055
103	0.24834	0.668541	1.239502	1.53951	1.89595	1.01074	2.51339	2.6925
104	0.24824	0.668349	1.238532	1.53689	1.89395	0.97747	2.50999	2.6795
105	0.24814	0.668157	1.237562	1.53427	1.89195	0.94420	2.50669	2.6665
106	0.24804	0.667965	1.236592	1.53165	1.88995	0.91093	2.50339	2.6535
107	0.24794	0.667773	1.235622	1.52903	1.88795			

# t-distribution in R

- ▶ CIs are wider, more conservative
- ▶ Use qt() function

```
# CI: CLT vs. t-distribution
# Treatment = 0
ci.zero
```

```
## [1] 1487.240 1491.427
ci.zeroT <- c(est.zero - qt(0.975,df = n.zero - 1) * se.zero,
               est.zero + qt(0.975,df = n.zero - 1) * se.zero)
```

```
ci.zeroT
```

```
## [1] 1487.234 1491.432
# Treatment = 1
ci.one
```

```
## [1] 1507.890 1512.056
ci.oneT <- c(est.one - qt(0.975,df = n.one - 1) * se.one,
               est.one + qt(0.975,df = n.one - 1) * se.one)
ci.oneT
```

```
## [1] 1507.885 1512.061
```

# Wrapping up Week 10

- ▶ Summary:
  - ▶ The challenge of uncertainty: Separating signals and noise.
  - ▶ Estimation using sample mean or diff-in-means.
  - ▶ Simulations and estimators probability distributions.
  - ▶ SD, SEs and margin of errors.
  - ▶ Constructing CI - how to interpret 95% CI?
  - ▶ Estimators are uncertain, but meaningful?
  - ▶ Estimating with the t-distribution.

**Data report next week!**