Forecasting Apple Sales Revenue with Holt-Winters Model

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Marketing Analytics I

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Year Qtr	Revs	Trend
19794	19.54	1
19801	23.55	2
19802	32.569	3
19803	41.467	4
19804	67.621	5
19811	78.765	6
19812	90.719	7
19813	97.678	8
19814	133.553	9
19821	131.019	10
19822	142.681	11
19823	175.808	12
19824	214.293	13
19831	227.982	14
19832	267.284	15
19833	273.21	16
19834	316.228	17
19841	300.102	18
19842	422.143	19

Year Qtr	Year and Quarter (1–4)
Revs	Sales Revenue
Trend	Trend multiplier

Raw Data

The raw data is Apple sales revenue from 1979 to 2005.

In the following analysis, I will use multiple methods to forecast revenue and test model performance with visualization.

alpha	1

SSE	11573760.3

Year Qtr	Revs	Trend	2 Period MA forecast	Smoothed Forecast	Error	Error^2
19794	19.54	1				
19801	23.55	2				
19802	32.569	3	21.54499996	21.54499996	11.024	121.528575
19803	41.467	4	28.05949992	21.54499996	19.922	396.886081
19804	67.621	5	37.01799989	28.05949992	39.5615	1565.11227
19811	78.765	6	54.54399985	37.01799989	41.747	1742.81201
19812	90.719	7	73.19299984	54.54399985	36.175	1308.63063
19813	97.678	8	84.74199987	73.19299984	24.485	599.515231
19814	133.553	9	94.19849992	84.74199987	48.811	2382.51373
19821	131.019	10	115.6155	94.19849992	36.8205	1355.7492
19822	142.681	11	132.2859998	115.6155	27.0655	732.54128
19823	175.808	12	136.8499997	132.2859998	43.522	1894.16447
19824	214.293	13	159.2444997	136.8499997	77.443	5997.41825
19831	227.982	14	195.0504997	159.2444997	68.7375	4724.84393
19832	267.284	15	221.1374998	195.0504997	72.2335	5217.67849
19833	273.21	16	247.6329997	221.1374998	52.0725	2711.54518
19834	316.228	17	270.2469993	247.6329997	68.595	4705.27403
19841	300.102	18	294.7189994	270.2469993	29.855	891.321028
19842	422.143	19	308.1649995	294.7189994	127.424	16236.8759
19843	477.399	20	361.1224995	308.1649995	169.234	28640.1467
19844	698.296	21	449.7709995	361.1224995	337.1735	113685.969
19851	435.344	22	587.8474994	449.7709995	-14.427	208.138316
19852	374.929	23	566.8199997	587.8474994	-212.918	45334.2874
19853	409.709	24	405.1364999	566.8199997	-157.111	24683.8664
19854	533.89	25	392.3189998	405.1364999	128.7535	16577.4636
19861	408.943	26	471.7994995	392.3189998	16.624	276.357378
19862	448.279	27	471.4164996	471.7994995	-23.5205	553.21393
19863	510.786	28	428.6109996	471.4164996	39.3695	1549.95751
19864	662.253	29	479.5324993	428.6109996	233.642	54588.5836

1

Method 1 Simple Average

2-period (2 quarter) moving average (MA) forecast

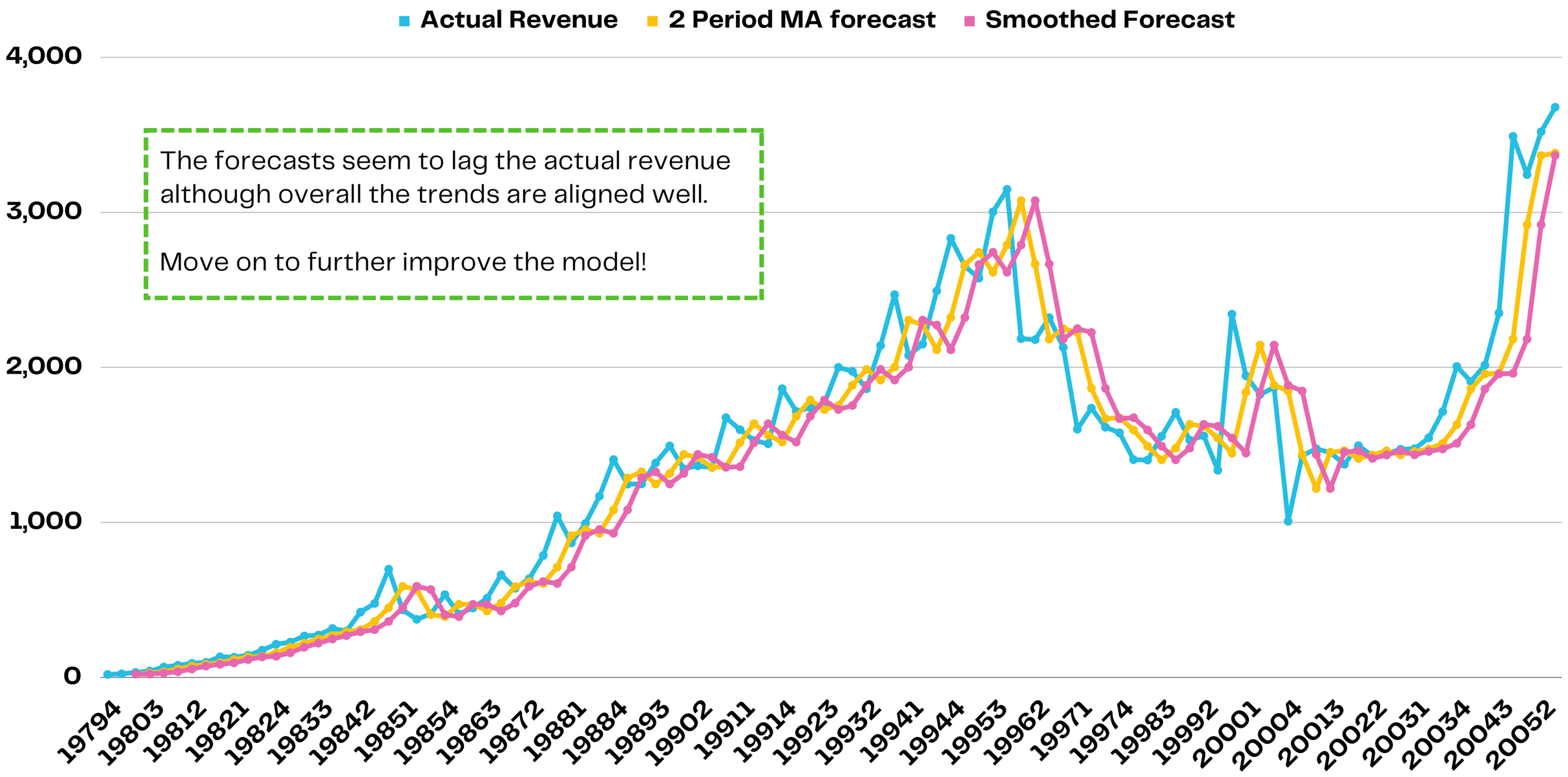
2

Method 2

exponentially smoothed forecast

$$\hat{x}_{t+1} = \alpha \times (x_t) + (1-\alpha) \times (\hat{x}_t)$$

- Set alpha to a random number to start **(alpha is a smoothing parameter between 0 and 1)**
- Calculate Error=Actual-Forecast
- Sum Error^2
- Minimize SSE to get optimal alpha
- Alpha=1 in this case


Actual Revenue
2 Period MA forecast
Smoothed Forecast
4,000
3,000
2,000
1,000
0
The forecasts seem to lag the actual revenue although overall the trends are aligned well.
Move on to further improve the model!
19794
19803
19812
19821
19824
19833
19842
19851
19854
19863
19872
19881
19884
19893
19902
19911
19914
19923
19932
19941
19944
19953
19962
19971
19974
19983
19992
20001
20004
20013
20022
20031
20034
20043
20052

3 Method 3 Holt-Winters model

Break data down into dynamic component pieces, use exponential smoothing to dynamically update the level, trend, and seasonal components of the forecast, and minimize SSE.

Observed value = Baseline level at time t + Trend at time t + Seasonality at time t

Baseline	Trend	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Sum
-4.7273	13.6035	1.4706	-1.6464	-7.3215	7.4972	0

Initial estimates of Level, Trend, Seasonality
setting initial values to start with sum=0

Year Qtr	Revs	Trend	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Forecast	Error	Error ^2	SSE
19794	19.5400	1	0	0	0	1	16.3735	3.1665	10.0270	28.9677
19801	23.5500	2	1	0	0	0	23.9504	-0.4004	0.1603	
19802	32.5690	3	0	1	0	0	34.4369	-1.8679	3.4891	
19803	41.4670	4	0	0	1	0	42.3654	-0.8984	0.8071	
19804	67.6210	5	0	0	0	1	70.7876	-3.1666	10.0273	
19811	78.7650	6	1	0	0	0	78.3646	0.4004	0.1603	
19812	90.7190	7	0	1	0	0	88.8511	1.8679	3.4892	
19813	97.6780	8	0	0	1	0	96.7795	0.8985	0.8072	

Minimize

Initialize the model with 2 seasons of data

Trend multiplier

To capture seasonality use a dummy for each quarter and impose the constraint that the four dummies sum to 0

Baseline + Trend multiplier * Trend coeff + Quarter dummy * Quarter coeff

Model Initialization

Baseline	Trend	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Sum
-4.7273	13.6035	1.4706	-1.6464	-7.3215	7.4972	0

alpha	beta	gamma
0.6834	0.1581	0.2774

Smoothing Parameters

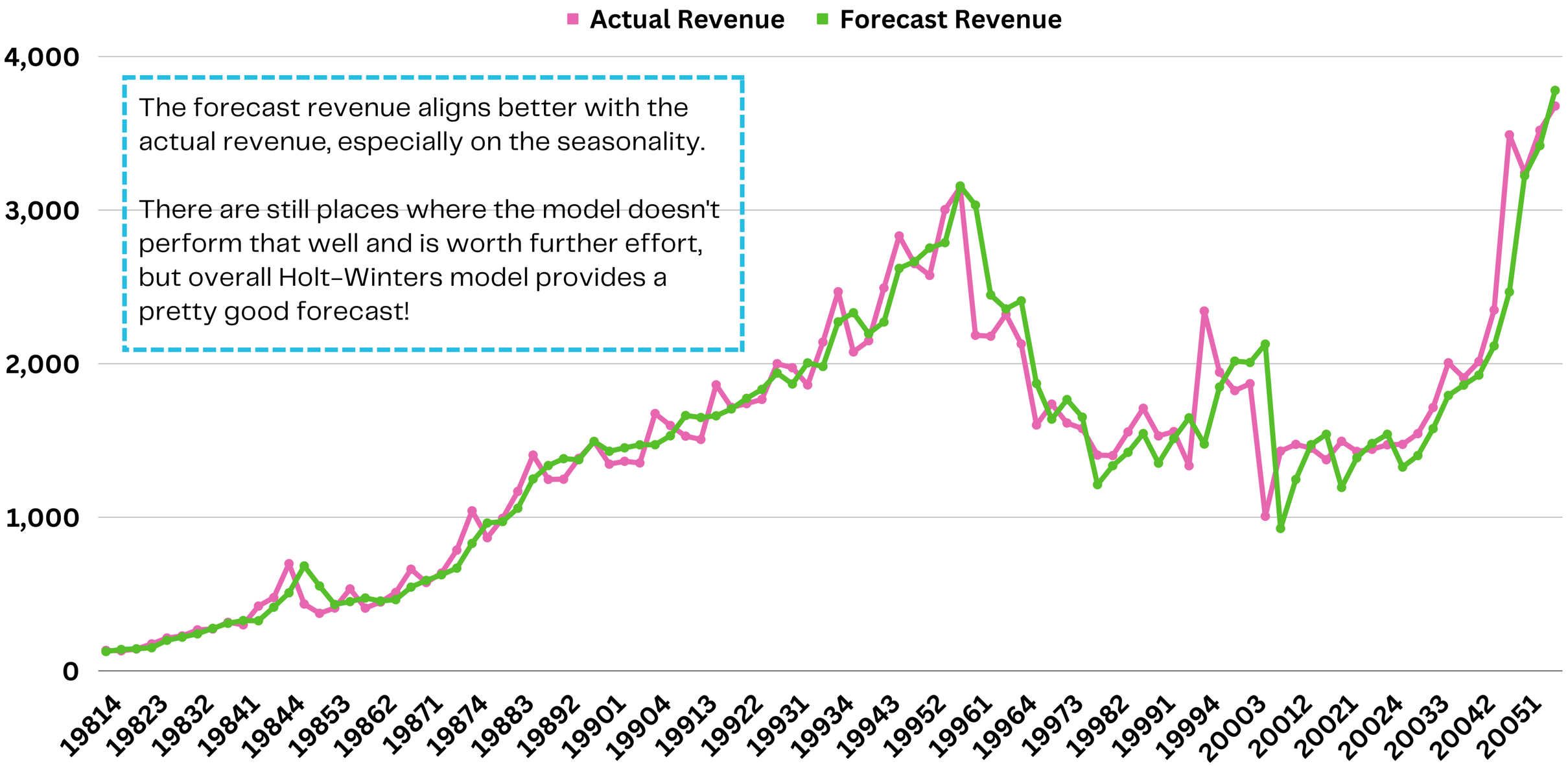
$$L_t : \alpha \times (x_t - S_{t-12}) + (1-\alpha) \times (L_{t-1} + T_{t-1})$$
$$T_t : \beta \times (L_t - L_{t-1}) + (1-\beta) \times (T_{t-1})$$
$$S_t : \gamma \times (x_t - L_t) + (1-\gamma) \times (S_{t-12})$$

Calculates L,T, S with these

Year Qtr	Revs	Trend	Quarter 1	Quarter 2	Quarter 3	Quarter 4	Level	Trend	Seasonal	Forecast	Error	Error ^2	SSE
19794	19.5400	1	0	0	0	1							
19801	23.5500	2	1	0	0	0							
19802	32.5690	3	0	1	0	0							
19803	41.4670	4	0	0	1	0							
19804	67.6210	5	0	0	0	1			7.4972				
19811	78.7650	6	1	0	0	0			1.4706				
19812	90.7190								-1.6464				
19813	97.6780						104.9995	13.6035	-7.3215				
19814	133.5530						123.6965	14.4088	8.1518	126.1002	7.4528	55.5444	5740486.37
19821	131.0190	10	1	0	0	0	132.2572	13.4842	0.7191	139.5759	-8.5569	73.2209	
19822	142.6810	11	0	1	0	0	144.7750	13.3315	-1.7706	144.0950	-1.4140	1.9994	
19823	175.8080	12	0	0	1	0	175.2081	16.0350	-5.1238	150.7850	25.0230	626.1485	
19824	214.2930	13	0	0	0	1	201.4251	17.6447	9.4602	199.3949	14.8981	221.9533	
19831	227.9820	14	1	0	0	0	224.6692	18.5299	1.4387	219.7889	8.1931	67.1274	
19832	267.2840	15	0	1	0	0	260.8697	21.3234	0.5002	241.4286	25.8554	668.5041	
19833	273.2100	16	0	0	1	0	279.5555	20.9065	-5.4627	277.0694	-3.8594	14.8946	
19834	316.2280	17	0	0	0	1	304.7716	21.5878	10.0140	309.9222	6.3058	39.7636	
19841	300.1020	18	1	0	0	0	307.4309	18.5954	-0.9937	327.7981	-27.6961	767.0716	
19842	422.1430	19	0	1	0	0	391.3741	28.9262	8.8978	326.5264	95.6166	9142.5287	
19843	477.3990	20	0	0	1	0	463.0571	35.6855	0.0318	414.8375	62.5615	3913.9379	
19844	698.2960	21	0	0	0	1	628.2808	56.1641	26.6605	508.7566	189.5394	35925.1767	
19851	435.3440	22	1	0	0	0	514.8794	29.3577	-22.7841	683.4511	-248.1071	61557.1449	
19852	374.9290	23	0	1	0	0	422.4446	10.1036	-6.7533	553.1348	-178.2058	31757.3232	
19853	409.7090	24	0	0	1	0	416.9173	7.6325	-1.9769	432.5800	-22.8710	523.0828	

Initial Level = the observed value in that period adjusted by the appropriate seasonality factor

Minimize SSE


Actual Revenue
Forecast Revenue
4,000
3,000
2,000
1,000
0
The forecast revenue aligns better with the actual revenue, especially on the seasonality.
There are still places where the model doesn't perform that well and is worth further effort, but overall Holt-Winters model provides a pretty good forecast!
19814
19823
19832
19841
19844
19853
19862
19871
19874
19883
19892
19901
19904
19913
19922
19931
19934
19943
19952
19961
19964
19973
19982
19991
19994
20003
20012
20021
20024
20033
20042
20051

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Welcome to contact
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Thank you :)