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BRNO UNIVERSITY OF TECHNOLOGY

FACULTY OF MECHANICAL ENGINEERING

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Topic 6: Ziegler-Nichols method 1 and 2, Root-Locus

Course : Control Theory I (VA1-A 18/19Z)
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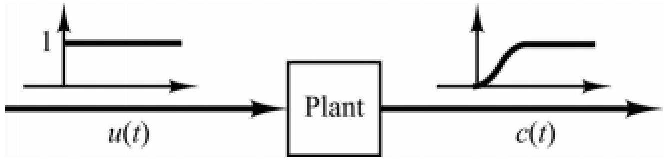
Location : A1/0636

Ziegler-Nichols Rules

Ziegler and Nichols came up with two methods for setting the parameters of PID controllers. These are **rules of thumb** and there is no guarantee that the resulting system behaves optimally. Ziegler-Nichols provides only a starting point for further tuning.

Method 1: Applies if the system's response to a unit-step is S-shaped, indicating that the plant involves no pure integration and the system response is not dominated by a pair of complex-conjugate poles: Notice that this method is applied on the plant itself, without feedback.

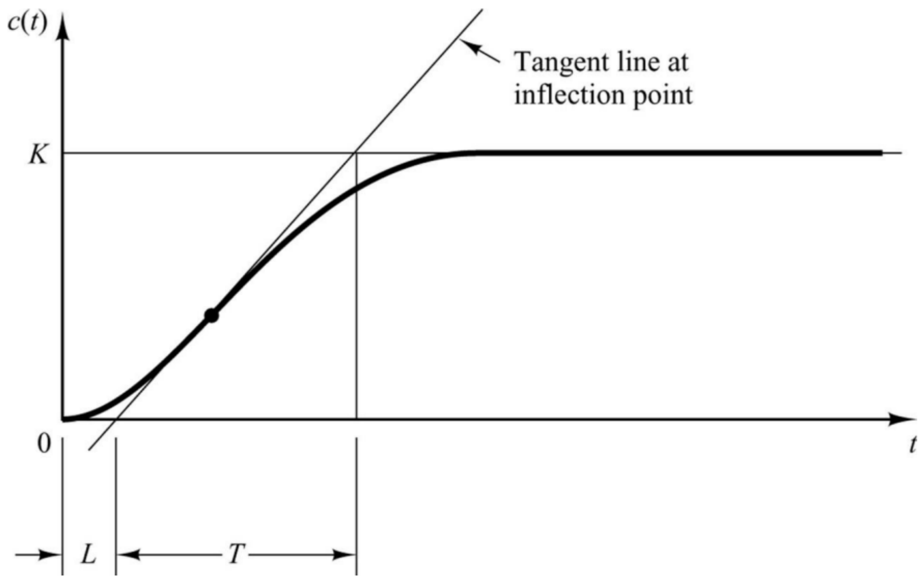


Method 2: The system appears to involve some pure integration and/or dominant complex-conjugate poles (i.e. the response is similar to an underdamped 2nd order response). This method is applied on the closed-loop system, with feedback.

Ziegler-Nichols Method 1

Ziegler–Nichols Tuning 1st Method S-shaped Step Input Response Curve

The S-shaped reaction curve can be characterized by two constants, delay time L and time constant T. These parameters can be obtained by drawing a tangent line at the inflection point of the curve:



L is the intersection of the tangent line with the time axis. L + T is the time at which the tangent line intersects the steady-state value.

Ziegler-Nichols Method 1

Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Using the parameters K, L and T, we can set the values of KP, KI and KD according to the formula shown in the table below.

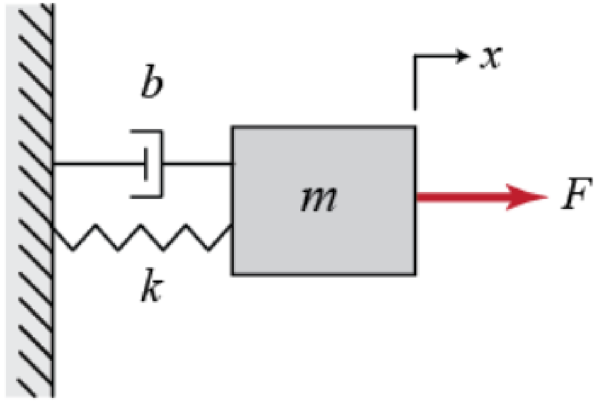
Controller type	K_p	T_i	T_d
P	$\frac{1}{K}\frac{T}{L}$		
PI	$0.9\frac{1}{K}\frac{T}{L}$	$3.5L$	
PD	$1.2\frac{1}{K}\frac{T}{L}$		$0.25L$
PID	$1.2\frac{1}{K}\frac{T}{L}$	$2L$	$0.5L$

Example: MSD Ziegler-Nichols Type 1

Controller type	K_p	T_i	T_d
P	$\frac{1}{K}\frac{T}{L}$		
PI	$0.9\frac{1}{K}\frac{T}{L}$	$3.5L$	
PD	$1.2\frac{1}{K}\frac{T}{L}$		$0.25L$
PID	$1.2\frac{1}{K}\frac{T}{L}$	$2L$	$0.5L$

Parameters of the system:

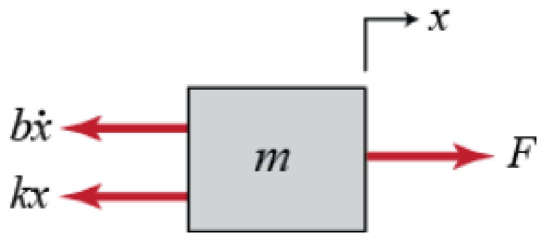
```
m  = 1;   % mass               [kg]
k  = 20;  % spring constant    [N/m]
b  = 10;  % damping constant   [Ns/m]
F  = 1;   % input force        [N]
```



$K = ?$

$L, T = ?$

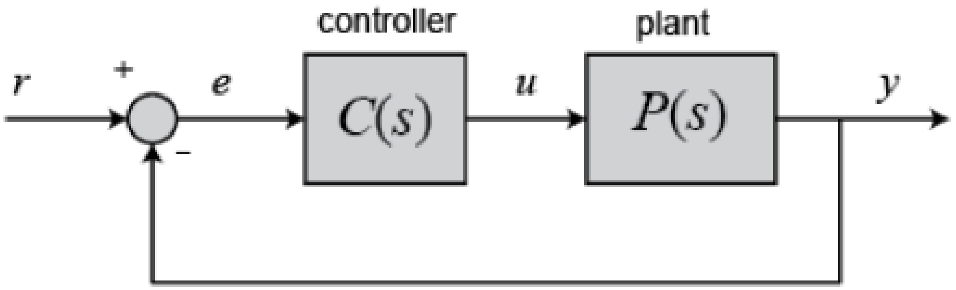
ZN Type 1: PID = ?



$$\sum F_x = F(t) - b\dot{x} - kx = m\ddot{x}$$

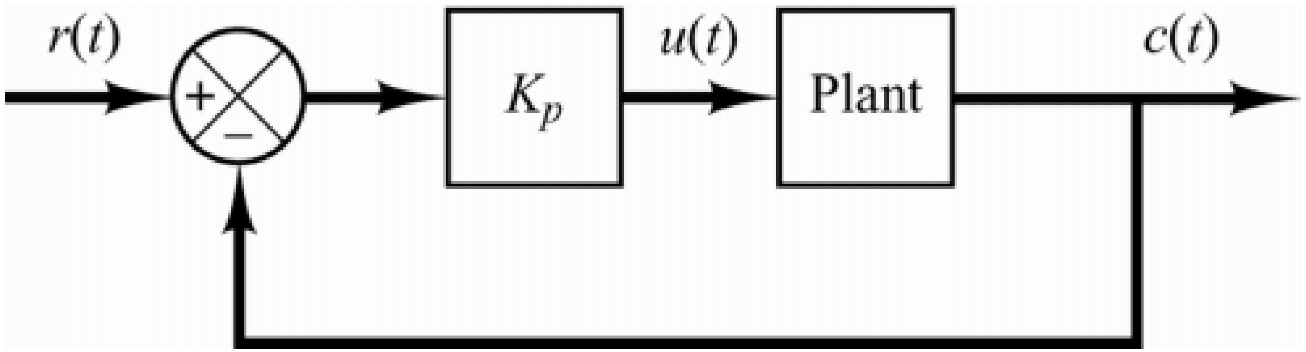
$$ms^2X(s) + bsX(s) + kX(s) = F(S)$$

$$\mathrm{P}(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+bs+k}$$



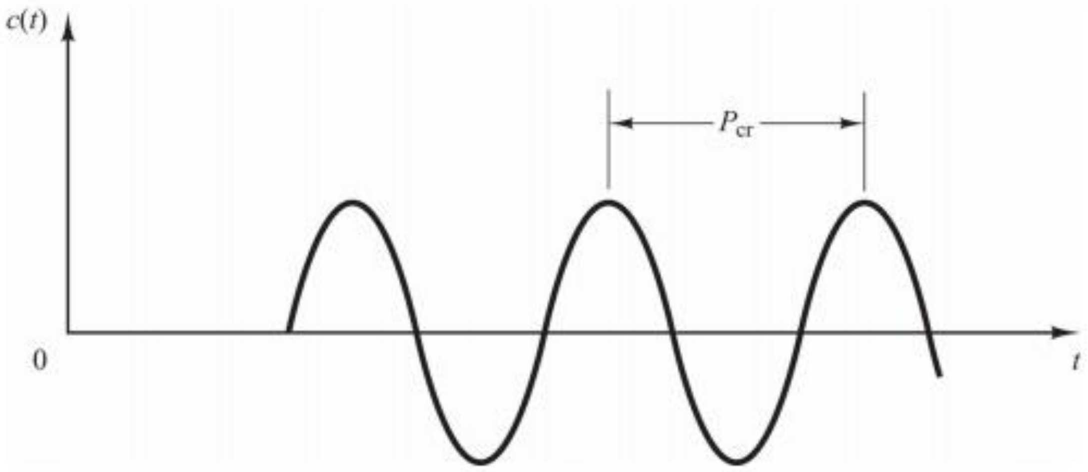
Ziegler-Nichols Method 2

To apply the second method we do a test on the system that varies Kp while keeping Kd = 0 and Ki = 0. The system being tested is as follows:



Kp is increased from 0 until it reaches a critical value Kcr at which the output exhibits sustained oscillations.

Ziegler-Nichols Method 1



At $K_p = K_{cr}$ the system's output will oscillate with period P_{cr}. These two values are used to determine the PID gains:

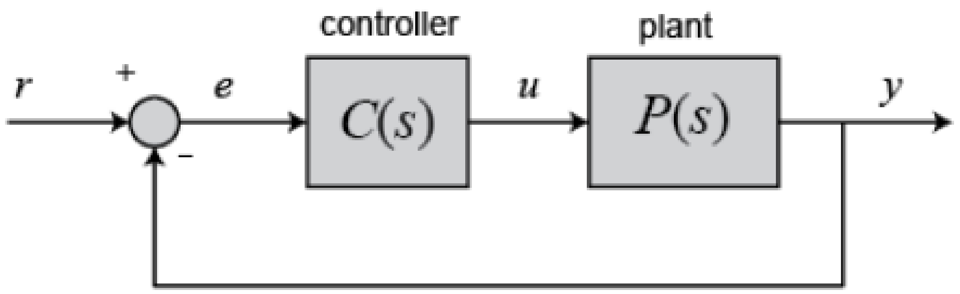
Controller type	K_p	T_i	T_d
P	$0.5K_{cr}$		
PI	$0.45K_{cr}$	$0.83P_{cr}$	
PD	$0.4K_{cr}$		$0.05P_{cr}$
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.12P_{cr}$

Example: MSD Ziegler-Nichols Type 2

Controller type	K_p	T_i	T_d
P	$0.5K_{cr}$		
PI	$0.45K_{cr}$	$0.83P_{cr}$	
PD	$0.4K_{cr}$		$0.05P_{cr}$
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.12P_{cr}$

Transfer function:

$$P(s) = \frac{1}{s(s+1)(s+2)}$$



$$K_{cr} = ?$$
$$P_{cr} = ?$$

ZN Type 2: PID = ?

Example: Zeros-Poles

Transfer function:

$$G(S) = \frac{s(3s+1)}{(2s+1)(s^2+2s+5)}$$

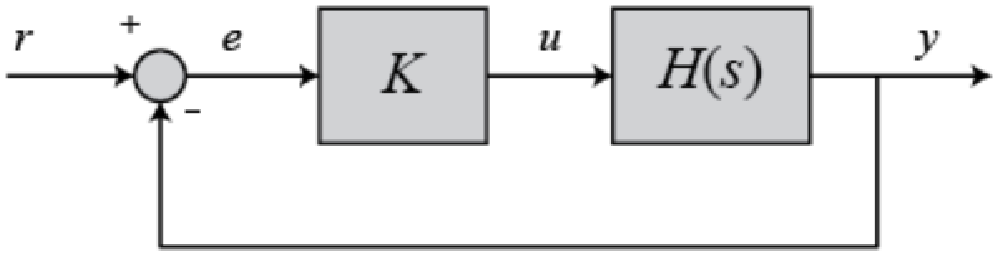
Zeros = ?

Poles = ?

Graph = ?

Root-Locus method

The root locus of an (open-loop) transfer function H(s) is a plot of the locations (locus) of all possible closed-loop poles with some parameter, often a proportional gain K, varied between 0 and infinity. The figure below shows a unity-feedback architecture, but the procedure is identical for any open-loop transfer function H(s), even if some elements of the open-loop transfer function are in the feedback path.



The closed-loop transfer function in this case is:

$$\frac{Y(s)}{R(s)} = \frac{KH(s)}{1 + KH(s)}$$

and thus the poles of the closed-loop system are values of s such that $1 + KH(s) = 0$

Root-Locus method

If we write $\mathrm{H}(s) = \frac{b(s)}{a(s)}$, then this equation can be rewritten as:

$$\Rightarrow a(s) + Kb(s) = 0$$

$$\Rightarrow \frac{a(s)}{K} + b(s) = 0$$

Let n be the order of a(s) and m be the order of b(s) (the order of the polynomial corresponds to the highest power of s).

We will consider all positive values of K. In the limit as K ->0, the poles of the closed-loop system are solutions of a(s) = 0 (poles of H(s)). In the limit as K ->infinity, the poles of the closed-loop system are solutions of b(s) = 0 (zeros of H(s)).

Root-Locus method

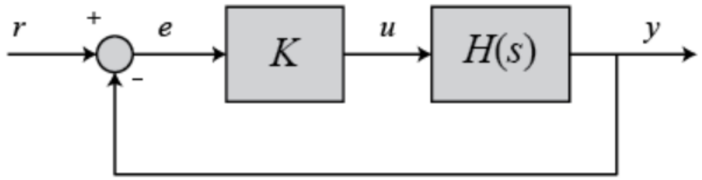
No matter our choice of K, the closed-loop system has n poles, where n is the number of poles of the open-loop transfer function $H(s)$. The root locus then has n branches, each branch starts at a pole of $H(s)$ and approaches a zero of $H(s)$. If $H(s)$ has more poles than zeros (as is often the case), $m < n$ and we say that $H(s)$ has zeros at infinity. In this case, the limit of $H(s)$ as $s \to$ infinity is zero. The number of zeros at infinity is $n-m$, the number of open-loop poles minus the number of open-loop zeros, and is the number of branches of the root locus that go to "infinity" (asymptotes).

Since the root locus consists of the locations of all possible closed-loop poles, the root locus helps us choose the value of the gain K to achieve the type of performance we desire. If any of the selected poles are on the right-half complex plane, the closed-loop system will be unstable. The poles that are closest to the imaginary axis have the greatest influence on the closed-loop response, so even if a system has three or four poles, it may still behave similar to a second- or a first-order system, depending on the location(s) of the dominant pole(s)

Example: Root-Locus

Transfer function:

$$\mathrm{H}(s) = \frac{s+7}{s(s+5)(s+15)(s+20)}$$



Let's assume our design criteria are 5% overshoot and 1 second rise time.

K = ? // proportional gain

ζ = ? // damping ration

ω_n = ? // natural frequency

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