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INSTITUTE OF AUTOMATION AND COMPUTER SCIENCE

Topic 8: Digital Control System

Course : Control Theory I (VA1-A 18/19Z)

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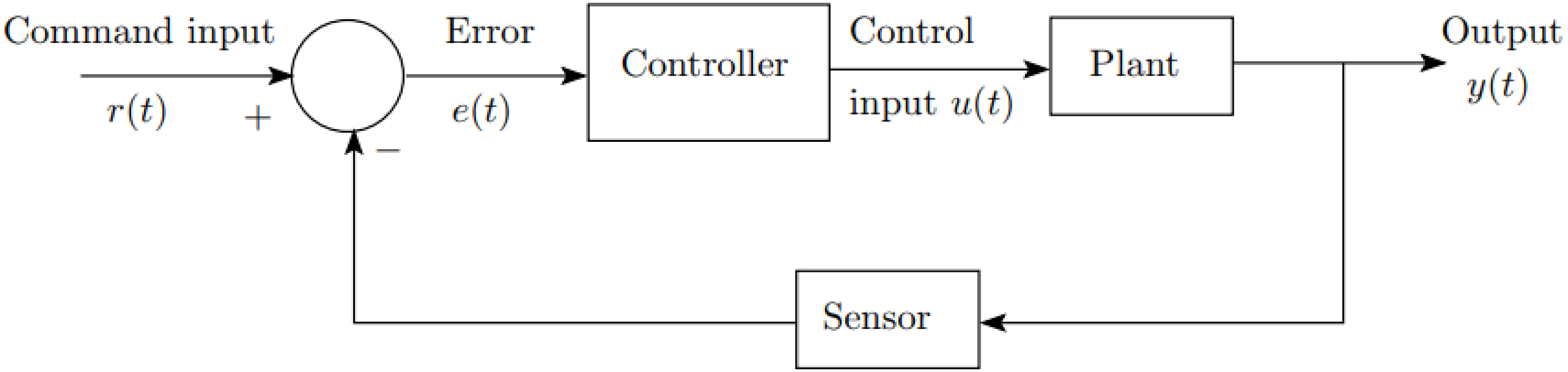
Location : A1/0636

Introduction

A digital control system model can be viewed from different perspectives including control algorithm, computer program, conversion between continuous and digital domains, system performance etc. One of the most important aspects is the sampling process level.

In continuous time control systems, all system variables are continuous signals. Whether the system is linear or nonlinear, all variables are continuously present and therefore known at all times. A typical continuous time control system is shown in Figure below.



Introduction

In a digital control system, the control algorithm is implemented in a digital computer. The error signal is discretized and fed to the computer by using an A/D (continuous to digital) converter. The controller output is again a discrete signal which is applied to the plant after using a D/A (digital to continuous) converter. General block diagram of a digital control system is shown in Figure below.

e(t) is sampled at intervals of T. In the context of control and communication, sampling is a process by which a continuous time signal is converted into a sequence of numbers at discrete time intervals. It is a fundamental property of digital control systems because of the discrete nature of operation of digital computer.

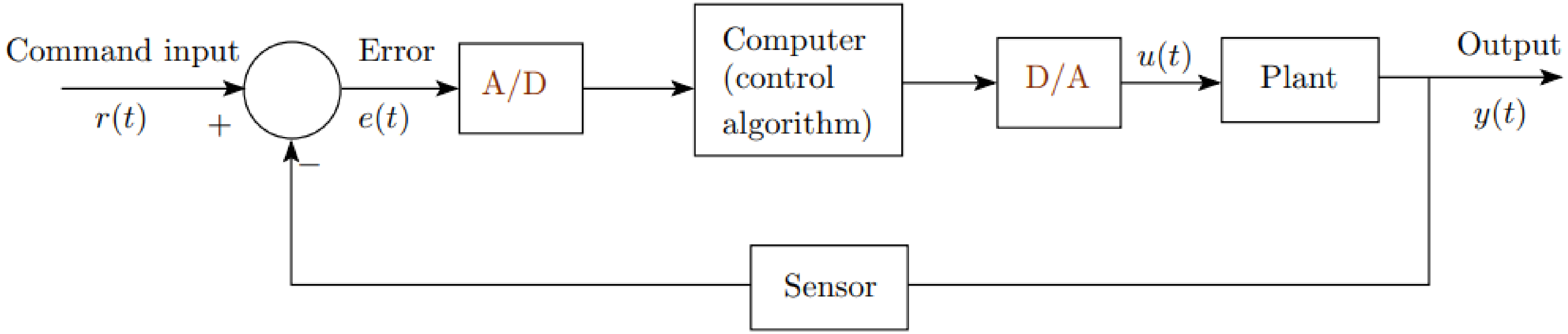
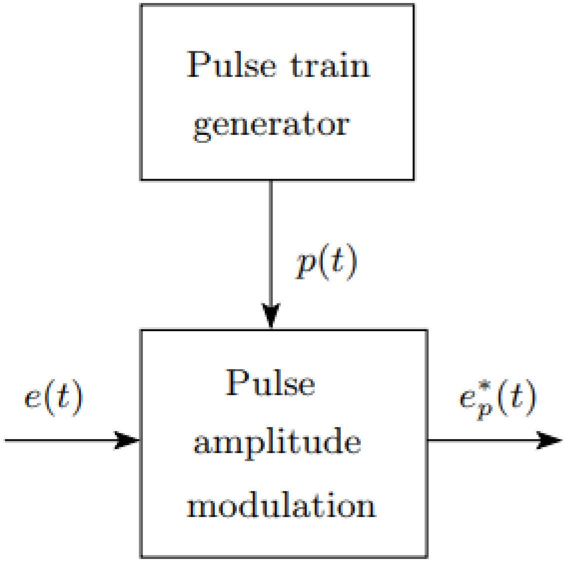
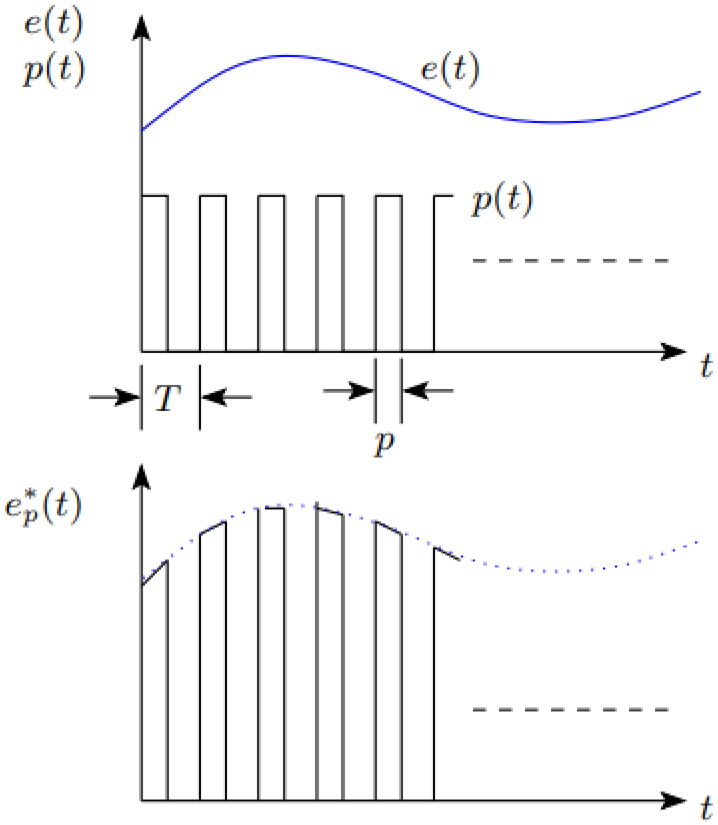


Figure below shows the structure and operation of a finite pulse width sampler, where left picture represents the basic block diagram and right picture illustrates the function of the same. T is the sampling period and p is the sample duration.





Z-transform

Difference equations:

a) negative shift $$y(k)+a_1y(k-1)+\ldots+a_ny(k-n)=b_0u(k)+b_1u(k-1)+\ldots+b_mu(k-m)$$

b) positive shift $$a_ny(k+n)+a_{n-1}y(k+n-1)+\ldots+a_0y(k)=b_mu(k+m)+\ldots+b_0u(k)$$

Z-transform:

$$G(z)=\frac{Z\{y(kT)\}}{Z\{u(kT)\}}=\frac{Y(z)}{U(z)}$$

a) negative shift

$$G(z)=\frac{Y(z)}{U(z)}=\frac{b_0+b_1z^{-1}+\ldots+b_mz^{-m}}{1+a_1z^{-1}+\ldots+a_nz^{-n}}$$

b) positive shift

$$G(z)=\frac{Y(z)}{U(z)}=\frac{b_mz^m+\ldots+b_1z+b_0}{a_nz^n+\ldots+a_1z+a_0}$$

Example: Z-transform

Difference equation:

$$y(k)-5y(k-1)+1{,}2y(k-2)=3{,}5u(k)+2u(k-1)-4u(k-2)$$

Z-transform:

a) positive shift

$$G(z)=\frac{3{,}5z^2+2z-4}{z^2-5z+1{,}2}$$

b) negative shift

$$G(z)=\frac{3{,}5+2z^{-1}-4z^{-2}}{1-5z^{-1}+1{,}2z^{-2}}$$

PSD Controller

PID (Proportional Integral Derivational) controller:

$$u(t) = r_0\left[e(t) + \frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt}\right]$$

PSD (Proportional – Summation – Differentiation) controller:

$$\int_0^{kT} e(t)dt \cong T\sum_{i=1}^{k} e(i) \longrightarrow \frac{de}{dt} \cong \frac{e(k)-e(k-1)}{T} \longrightarrow u(k) = r_0\left\{e(k) + \frac{T}{T_i}\sum_{i=1}^{k} e(i) + \frac{T_d}{T}\left[e(k)-e(k-1)\right]\right\}$$

$$\Delta u(k) = u(k) - u(k-1) \longrightarrow u(k-1) = r_0\left\{e(k-1) + \frac{T}{T_i}\sum_{i=1}^{k-1} e(i) + \frac{T_d}{T}\left[e(k-1)-e(k-2)\right]\right\}$$

PSD controller:

$$u(k)-u(k-1)=\underbrace{r_0\left(1+\frac{T_d}{T}+\frac{T}{T_i}\right)}_{q_0}e(k)-\underbrace{r_0\left(1+2\frac{T_d}{T}\right)}_{q_1}e(k-1)+\underbrace{r_0\frac{T_d}{T}}_{q_2}e(k-2)$$

$$u(k)-u(k-1)=q_0e(k)+q_1e(k-1)+q_2e(k-2)$$

$$\downarrow$$

$$q_0=r_0\left(1+\frac{T_d}{T}+\frac{T}{T_i}\right) \qquad q_1=-r_0\left(1+2\frac{T_d}{T}\right) \qquad q_2=r_0\frac{T_d}{T}$$

$$\downarrow$$

$$G_R(z)=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{1-z^{-1}}$$

Example: PSD Controller

PID -> PSD:

$$G_R(s) = 0{,}4\left(1 + \frac{1}{0{,}5s} + 0{,}1s\right)$$

PSD + Difference equation:

$$q_0 = r_0\left(1 + \frac{T_d}{T} + \frac{T}{T_i}\right) = 0{,}4\left(1 + \frac{0{,}1}{0{,}1} + \frac{0{,}1}{0{,}5}\right) = 0{,}88$$

$$q_1 = -r_0\left(1 + 2\frac{T_d}{T}\right) = -0{,}4\left(1 + 2\frac{0{,}1}{0{,}1}\right) = -1{,}2$$

$$q_2 = r_0\frac{T_d}{T} = 0{,}4\frac{0{,}1}{0{,}1} = 0{,}4$$

$$u(k) - u(k-1) = 0{,}88\,e(k) - 1{,}2\,e(k-1) + 0{,}4\,e(k-2)$$

$$G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} = \frac{0{,}88 - 1{,}2z^{-1} + 0{,}4z^{-2}}{1 - z^{-1}}$$

Dynamics inversion design

System	Controller: Analog (T=0), Digital (T>0)				
	Type	r_0		T_i	T_d
		$T_d = 0$	$T_d > 0$		
$\frac{k_1}{s}.e^{-T_d}$	P	$\frac{2}{k_1(2.T_w + T)}$	$\frac{a}{k_1}$	-	-
$\frac{k_1}{(T_1 s + 1)}.e^{-T_d}$	PI(PS)	$\frac{2.T_i}{k_1(2.T_w + T)}$	$\frac{a.T_i}{k_1}$	$T_1 - \frac{T}{2}$	-
$\frac{k_1}{s(T_1 s + 1)}.e^{-T_d}$	PD	$\frac{2}{k_1(2.T_w + T)}$	$\frac{a}{k_1}$	-	$T_1 - \frac{T}{2}$
$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}.e^{-T_d}$ $T_1 \geq T_2$	PID(PSD)	$\frac{2.T_i}{k_1(2.T_w + T)}$	$\frac{a.T_i}{k_1}$	$T_1 + T_2 - T$	$\frac{T_1.T_2}{T_1 + T_2} - \frac{T}{4}$

T – sampling period

$T < 0{,}32 T_d$

$T < 0{,}30 T_w$

$$a_{analog} = \frac{1}{\beta T_d}$$

$$a_{digital} = \frac{1}{\alpha T + \beta T_d}$$

k	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
α	1.282	0.984	0.884	0.832	0.763	0.697	0.669	0.640	0.618
β	2.718	1.944	1.720	1.561	1.437	1.337	1.248	1.172	1.104

Method of optimal module

System	Controller: Analog (T=0), Digital (T>0)			
	Type	r_0	T_i	T_d
$\frac{k_1}{(T_1 s + 1)}$	P	$\frac{1}{2k_1 + T_2}$	-	-
$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}$ $T_1 \geq T_2$	PI(PS)	$\frac{T_i}{2k_1 + T_2}$	$T_1 - 0.5T$	-
$\frac{k_1}{s(T_1 s + 1)(T_2 s + 1)}$ $T_1 \geq T_2$	PD	$\frac{1}{2k_1(T_2+0.5T)}$	-	$T_1 - 0.5T$
$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}$ $T_1 \geq T_2 \geq T_3$	PID(PSD)	$\frac{1}{2k_1(T_3+0.5T)}$	$T_1 + T_2 - T$	$\frac{T_1 . T_2}{T_1 + T_2} - \frac{T}{4}$

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