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INSTITUTE OF AUTOMATION
AND COMPUTER SCIENCE

BRNO UNIVERSITY OF TECHNOLOGY

FACULTY OF MECHANICAL ENGINEERING

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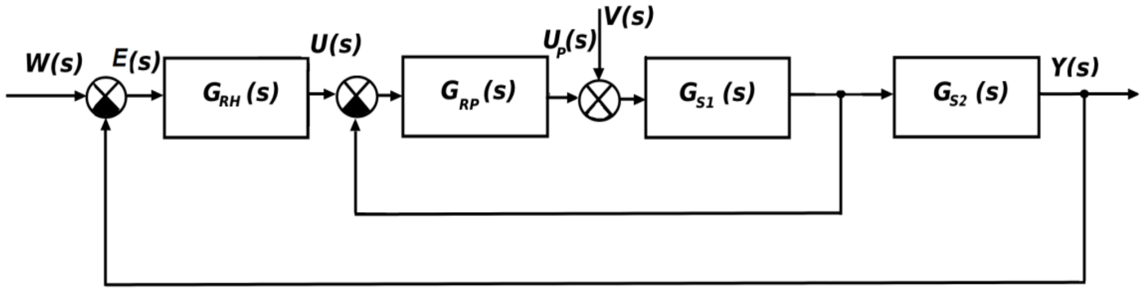
Topic 9: Branched control circuits

Course : Control Theory I (VA1-A 18/19Z)
Teacher : Ing. Roman Parák (Roman.Parak@vutbr.cz), A1/0642
Location : A1/0636

Circuit with auxiliary controlled variable



W(s): desired setpoint/ command variable
Y(s): controlled variable
E(s): error value
U(s): manipulated variable
V(s): disturbance variable

Control transfer function:

$$G_w(s) = \frac{Y(s)}{W(s)} = \frac{G_{RH}(s)\frac{G_{S1}(s)G_{RP}(s)}{1+G_{S1}(s)G_{RP}(s)}G_{S2}(s)}{1+G_{RH}(s)\frac{G_{S1}(s)G_{RP}(s)}{1+G_{S1}(s)G_{RP}(s)}G_{S2}(s)} =$$

$$= \frac{G_{RH}(s)G_{RP}(s)G_S(s)}{1+G_{S1}(s)G_{RP}(s)+G_{RH}(s)G_{RP}(s)G_S(s)}$$

Disturbance transfer function:

$$G_v(s) = \frac{Y(s)}{W(s)} = \frac{\left[1-\frac{G_{S1}(s)G_{RP}(s)}{1+G_{S1}(s)G_{RP}(s)}\right]G_{S1}(s)G_{S2}(s)}{1+G_{S2}(s)G_{RH}(s)\frac{G_{S1}(s)G_{RP}(s)}{1+G_{S1}(s)G_{RP}(s)}} =$$

$$= \frac{G_{S1}(s)G_{S2}(s)}{1+G_{S1}(s)G_{RP}(s)+G_{RH}(s)G_{S1}(s)G_{S2}(s)}$$

Characteristic equation:

$$1+G_{S1}(s)G_{RP}(s)+G_{RH}(s)G_{S1}(s)G_{S2}(s) = 0$$

Dynamics inversion design

System	Controller: Analog (T=0), Digital (T>0)				
	Type	r_0		T_i	T_D
		$T_d = 0$	$T_d > 0$		
$\frac{k_1}{s}.e^{-T_d}$	P	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	-
$\frac{k_1}{(T_1 s + 1)}.e^{-T_d}$	PI(PS)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	T_1	-
$\frac{k_1}{s(T_1 s + 1)}.e^{-T_d}$	PD	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	T_1
$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}.e^{-T_d}$ $T_1 \geq T_2$	PID(PSD)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	$T_1 + T_2$	$\frac{T_1.T_2}{T_1 + T_2}$

k	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
β	2.718	1.944	1.720	1.561	1.437	1.337	1.248	1.172	1.104

$$a = \frac{1}{\beta T_d}$$

Example: auxiliary controlled variable

1. Simple control circuit:

System transfer function:

$$G_S(s) = \frac{4}{(2s+1)(0.5s+1)} e^{-4s}$$

$$k_1 = 4, T_1 = 2, T_2 = 0.5, T_D = 4$$

Calculation (PID -> G_{RH}):

$$T_i = T_1 + T_2 = 2.5$$

$$T_d = \frac{T_1 \cdot T_2}{T_1 + T_2} = 0.4$$

$$a = \frac{1}{\beta T_D} = \frac{1}{2.718 \cdot 4} = 0.0919$$

$$r_0 = \frac{a T_i}{k_1} = \frac{0.0919 \cdot 2.5}{4} = 0.0574$$

2. Control circuit with auxiliary controlled variable:

System transfer function:

$$G_{S1}(s) = \frac{4}{(0.5s+1)} \longrightarrow k_1 = 4,\ T_i = T_1 = 0.5$$

$$G_{S2}(s) = \frac{1}{(2s+1)} e^{-4s}$$

Calculation (PI -> G_{RP}, PID -> G_{RH}):

$$r_0 = \frac{2T_i}{2kT_w} = \frac{2 \cdot 0.5}{2 \cdot 4 \cdot 0.5} = 0.25$$

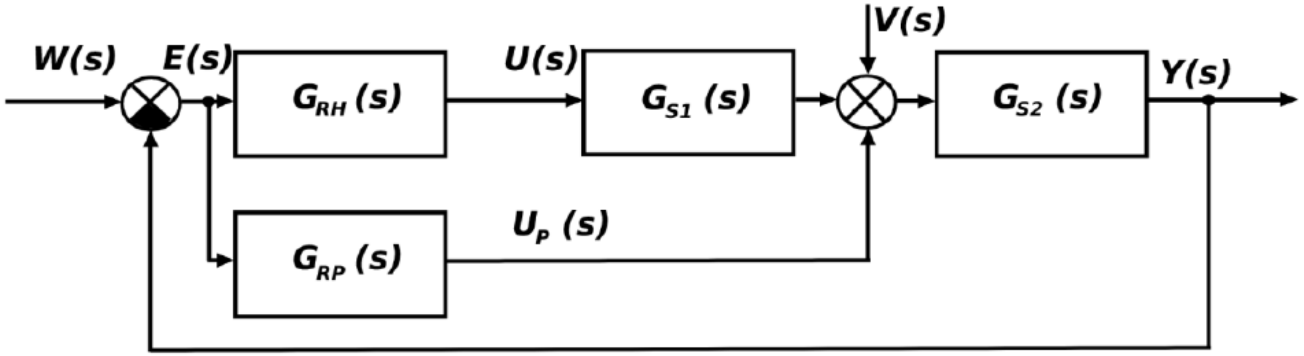
$$G_R(s) = r_0\left(1 + \frac{1}{T_i s}\right) = 0.25\left(1 + \frac{1}{0.5s}\right) = \frac{(0.125s + 0.25)}{0.5s}$$

$$G_{S1}^*(s) = \frac{G_{RP}(s) G_{S1}(s)}{1 + G_{RP}(s) G_{S1}(s)} = \frac{\frac{(0.125s+0.25)}{0.5s}\frac{4}{(0.5s+1)}}{1 + \frac{(0.125s+0.25)}{0.5s}\frac{4}{(0.5s+1)}} =$$

$$= \frac{0.5s + 1}{0.25(s^2 + 4s + 4)} = \frac{1}{0.5s + 1}$$

$$G_S^*(s) = \frac{1}{(0.5s+1)(2s+1)} e^{-4s} \longrightarrow k_1 = 1, T_1 = 2, T_2 = 0.5, T_D = 4$$

Circuit with auxiliary manipulated variable



W(s): desired setpoint/ command variable
Y(s): controlled variable
E(s): error value
U(s): manipulated variable
V(s): disturbance variable

Control transfer function:

$$G_w(s) = \frac{Y(s)}{W(s)} = \frac{\left(G_{RH}(s)G_{S1}(s) + G_{RP}(s)\right)G_{S2}(s)}{1 + \left(G_{RH}(s)G_{S1}(s) + G_{RP}(s)\right)G_{S2}(s)} =$$

$$= \frac{G_{RH}(s)G_S(s) + G_{RP}(s)G_{S2}(s)}{1 + G_{RH}(s)G_S(s) + G_{RP}(s)G_{S2}(s)}$$

Disturbance transfer function:

$$G_v(s) = \frac{Y(s)}{V(s)} = \frac{G_{S2}(s)}{1 + G_{S2}(s)\left(G_{RH}(s)G_{S1}(s) + G_{RP}(s)\right)} =$$

$$= \frac{G_{S2}(s)}{1 + G_{RH}(s)G_S(s) + G_{RP}(s)G_{S2}(s)}$$

Characteristic equation:

$$1 + G_{RH}(s)G_S(s) + G_{RP}(s)G_{S2}(s) = 0$$

Dynamics inversion design

System	Controller: Analog (T=0), Digital (T>0)				
	Type	r_0		T_i	T_D
		$T_d = 0$	$T_d > 0$		
$\frac{k_1}{s}.e^{-T_d}$	P	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	-
$\frac{k_1}{(T_1 s + 1)}.e^{-T_d}$	PI(PS)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	T_1	-
$\frac{k_1}{s(T_1 s + 1)}.e^{-T_d}$	PD	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	T_1
$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}.e^{-T_d}$ $T_1 \geq T_2$	PID(PSD)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	$T_1 + T_2$	$\frac{T_1.T_2}{T_1 + T_2}$

k	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
β	2.718	1.944	1.720	1.561	1.437	1.337	1.248	1.172	1.104

$$a = \frac{1}{\beta T_d}$$

Example: auxiliary manipulated variable

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1. Simple control circuit:

System transfer function:

$$G_S(s) = \frac{4}{(4s+1)(0.5s+1)}e^{-2s}$$

$$k_1 = 4, T_1 = 4, T_2 = 0.5, T_D = 2.$$

Calculation (PID -> G_{RH}):

$$T_i = T_1 + T_2 =$$

$$T_d = \frac{T_1 \cdot T_2}{T_1 + T_2} =$$

$$a = \frac{1}{\beta \mathrm{T}_D} =$$

$$r_0 = \frac{aT_i}{k_1} =$$

2. Control circuit with auxiliary controlled variable:

System transfer function:

$$G_{S1}(s) = \frac{2}{(4s+1)}e^{-2s}$$

$$G_{S2}(s) = \frac{2}{(0.5s+1)} \longrightarrow k_1 = 2\,,\; T_i = T_1 = 0.5$$

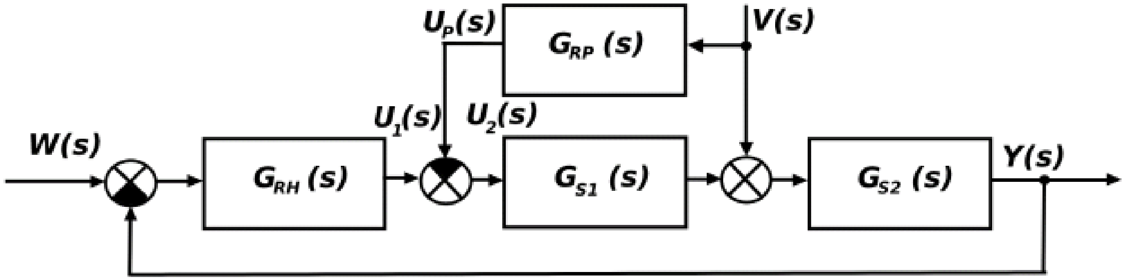
Calculation (PI -> G_{RP}):

$$T_i = T_1$$

$$a = \frac{1}{\beta \mathrm{T}_D} =$$

$$r_0 = \frac{aT_i}{k_1} =$$

Circuit with auxiliary measurement of disturbance variable



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V(s): disturbance variable

Control transfer function:

$$G_w(s) = \frac{Y(s)}{W(s)} = \frac{G_{RH}(s)G_S(s)}{1 + G_{RH}(s)G_S(s)}$$

Disturbance transfer function:

$$G_v(s) = \frac{Y(s)}{V(s)} = \frac{[1 - G_{RP}(s)G_{S1}(s)]G_{S2}(s)}{1 + G_{RH}(s)G_{S1}(s)G_{S2}(s)} = \frac{G_{S2}(s) - G_{RP}(s)G_S(s)}{1 + G_{RH}(s)G_S(s)}$$

Compensator transfer function (ideally setting of the compensator to achieve full invariance to the disturbance variable):

$$G_v(s) = 0, \qquad G_{S2}(s) - G_{RP}(s)G_S(s) = 0 \quad \Longrightarrow \quad G_{RP}(s) = \frac{G_{S2}(s)}{G_S(s)} = \frac{G_{S2}(s)}{G_{S1}(s)G_{S2}(s)} = \frac{1}{G_{S1}(s)}$$

Dynamics inversion design

System	Controller: Analog (T=0), Digital (T>0)				
	Type	r_0		T_i	T_D
		$T_d = 0$	$T_d > 0$		
$\frac{k_1}{s}.e^{-T_d}$	P	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	-
$\frac{k_1}{(T_1 s+1)}.e^{-T_d}$	PI(PS)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	T_1	-
$\frac{k_1}{s(T_1 s+1)}.e^{-T_d}$	PD	$\frac{2}{k_1(2.T_w)}$	$\frac{a}{k_1}$	-	T_1
$\frac{k_1}{(T_1 s+1)(T_2 s+1)}.e^{-T_d}$ $T_1 \geq T_2$	PID(PSD)	$\frac{2.T_i}{k_1(2.T_w)}$	$\frac{a.T_i}{k_1}$	$T_1 + T_2$	$\frac{T_1.T_2}{T_1+T_2}$

k	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40
β	2.718	1.944	1.720	1.561	1.437	1.337	1.248	1.172	1.104

$$a = \frac{1}{\beta T_d}$$

Example: auxiliary measurement of disturbance variable

1. Simple control circuit:

System transfer function:

$$G_S(s) = \frac{4}{(8s+1)(0.2s+1)} e^{-6s}$$

$$k_1 = 4, T_1 = 8, T_2 = 0.2, T_D = 6$$

Calculation (PID -> G_{RH}):

$$T_i = T_1 + T_2 =$$

$$T_d = \frac{T_1 \cdot T_2}{T_1 + T_2} =$$

$$a = \frac{1}{\beta T_D} =$$

$$r_0 = \frac{aT_i}{k_1} =$$

2. Control circuit with auxiliary controlled variable:

System transfer function:

$$G_{S1}(s) = \frac{2}{(0.2s+1)}$$

$$G_{S2}(s) = \frac{2}{(8s+1)} e^{-6s}$$

Calculation (G_{RP}):

$$G_v(s) = 0, \qquad G_{S2}(s) - G_{RP}(s) G_S(s) = 0$$

$$\Downarrow$$

$$G_{RP}(s) = \frac{G_{S2}(s)}{G_S(s)} = \frac{G_{S2}(s)}{G_{S1}(s) G_{S2}(s)} = \frac{1}{G_{S1}(s)}$$

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