Design and Implementation of the Controller for the selected systems

VA1 - project

1. Differential equation
2. Laplace transform -> transfer function $G_s(s)$
3. System analysis (time response overview, frequency response overview, stability, etc.)
4. Controller design (use 2-3 methods of design, comparison of methods)
5. Result: algorithm (script, function, etc) + (1-2) xA4 paper from each example

Software: MATLAB/Simulink, Python, C/C++

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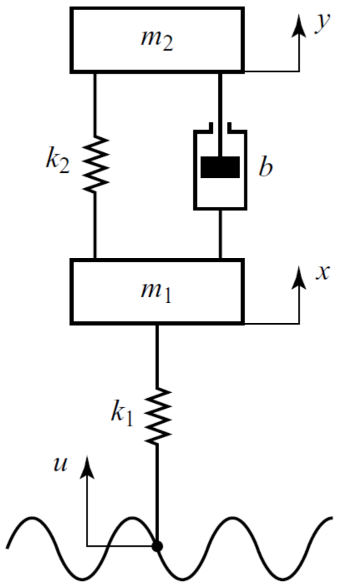
Example 1: Mass-Spring-Damper

Obtain the transfer function

$$\frac{Y(s)}{U(s)}$$

of the system shown in the Figure below. The input u is a displacement input.



m_1	1.35	mass [kg]
m_2	1.45	mass [kg]
k_1	0.95	Spring constant [N/m]
k_2	1.05	Spring constant [N/m]
b	0.25	Damping constant [Ns/m]

Solution:

Assume that displacements x and y are measured from respective steady-state positions in the absence of the input u. Applying the Newton's second law to this system, we obtain

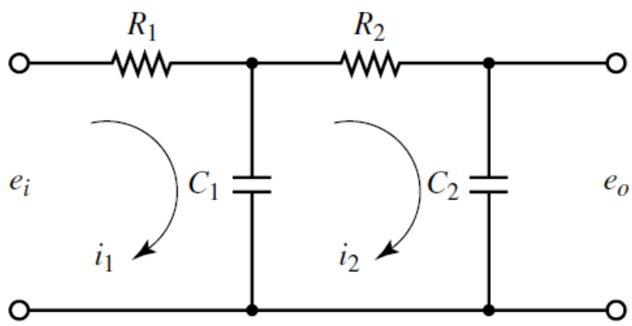
$$m_1\ddot{x} = k_2(y - x) + b(\dot{y} - \dot{x}) + k_1(u - x)$$
$$m_2\ddot{y} = -k_2(y - x) + b(\dot{y} - \dot{x})$$

Example 2: RC (resistor, capacitor) circuit

Obtain the transfer function

$$\frac{E_0(s)}{E_i(s)}$$

of the system shown in the Figure below. Assume that e_i is the input and e_o is the output. The capacitances C_1 and C_2 are not charged initially.



C_1	1.10^{-6}	capacitor [F]
C_2	$1.15.10^{-6}$	capacitor [F]
R_1	5.10^5	resistor [Ω]
R_2	$0.5.10^4$	resistor [Ω]

Solution:

It will be shown that the second stage of the circuit (R_2C_2 portion) produces a loading effect on the first stage (R_1C_1 portion). The equations for this system are

$$\frac{1}{C_1}\int (i_1 - i_2)dt + R_1 i_1 = e_i$$

and

$$\frac{1}{C_1}\int (i_2 - i_1)dt + R_2 i_2 + \frac{1}{C_2}\int i_2 dt = 0$$

$$\frac{1}{C_2}\int i_2 dt = e_0$$

Example 3: DC Motor Position/Speed

Obtain the transfer function

a) *position*

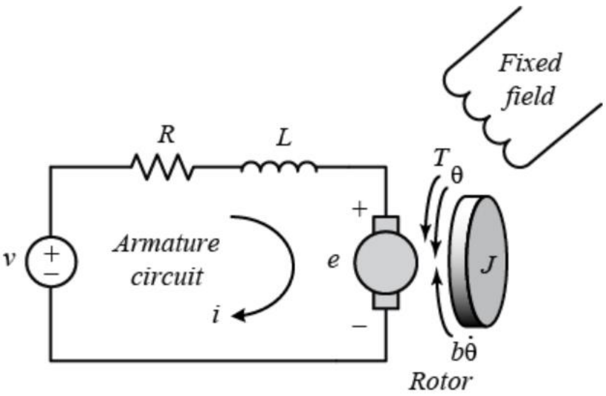
$$\frac{\theta(s)}{V(s)}$$

b) *speed*

$$\frac{\dot{\theta}(s)}{V(s)}$$

of the system shown in the Figure below. Assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the position of the shaft (θ). The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure.



J	$3.2284.10^{-6}$	moment of inertia of the rotor $[kg.m^2]$
b	$3.5077.10^{-6}$	motor viscous friction constant $[N.m.s]$
K_b	0.0274	electromotive force constant $[V/rad/sec]$
K_t	0.0274	motor torque constant $[N.m/A]$
R	4	electric resistance [Ω]
L	$2.75.10^{-6}$	electric inductance $[H]$

Solution:

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current $\boldsymbol{i}$ by a constant factor $\boldsymbol{K_t}$ as shown in the equation below. This is referred to as an armature-controlled motor.

$$T = K_t i$$

The back emf, $\boldsymbol{e}$, is proportional to the angular velocity of the shaft by a constant factor $\boldsymbol{K_b}$.

$$e = K_b \dot{\theta}$$

In SI units, the motor torque and back emf constants are equal, that is, $\boldsymbol{K_t} = \boldsymbol{K_b}$ therefore, we will use $\boldsymbol{K}$ to represent both the motor torque constant and the back emf constant.

From the figure above, we can derive the following governing equations based on Newton's 2nd law and Kirchhoff's voltage law.

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$