Inventory Commitment and Monetary Compensation under Competition

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Inventory commitment and monetary compensation have been widely recognized as effective marketing strategies in monopoly settings when customers concern about stock-outs. To attract more customer traffic, a firm reveals its inventory availability information to customers before the sales season, or offers monetary compensation to placate customers if the product is out of stock. In this study, we seek to integrate the newsvendor and Hotelling models to investigate these two strategies when retailers compete on both price and inventory availability. We develop a game-theoretic framework to analyze the strategic interactions among the retailers and customers and draw the following insights. First, both inventory commitment and monetary compensation would lead to a prisoner's dilemma. Although these strategies are preferred regardless of the competitor's price and inventory decisions, the equilibrium profit of each retailer could be lower in the presence of inventory commitment or monetary compensation, they both would intensify the competition between the retailers. Second, contrary to the common wisdom, we find that market competition may hurt social welfare by reducing the product availability under equilibrium. The inventory commitment and monetary compensation strategies further intensify the competition between the retailers, therefore causing an even lower social welfare. Our study implies that, although inventory commitment and monetary compensation improve the retailer profit and social welfare under monopoly, these strategies should be used with caution under competition.

Key words: Inventory availability, retail competition, inventory commitment, monetary compensation

1. Introduction

Inventory commitment and monetary compensation are widely-adopted marketing strategies for the firms when consumers are worried about potential stock-outs. For example, the retailer credibly reveals the inventory stocking quantity to its customers (see Su and Zhang 2009). Many e-commerce firms, including BestBuy.com and Neweggs.com, offer real-time availability information in the store and online. Target and Walmart also allow consumers to check the inventory availability of a particular product at local stores using the zip code and the DPCI number (Morgan 2015). Moreover, consumers have access to technologies and applications that help them track product availability information. For example, TrackAToy offers availability information for many retailers if consumers enter the product's name on the web page (see `http://www.trackatoy.com/`). An alternative strategy to placate consumers is to offer a monetary compensation if the product is out of stock when the consumers are visiting the store. Sloot et al. (2005) find that monetary compensation such as discount coupons, rain checks, and additional services are effective in placating consumers upon stock-outs. Bhargava et al. (2006) report that MAP LINK (the U.S. largest map distributor), VERGE (a U.S. media network publisher), and IntelliHome (a U.S. smart home technology company) offer discounts of 2%, 5%, and 10%, respectively, for all backlogged items. Many car dealers provide price reductions if automobile consumers choose is out of stock, whereas restaurants offer free dishes if consumers' original choices are sold out. In the context of online retailing, sellers usually waive delivery fees if items are backlogged.

The strategies of inventory commitment and monetary compensation improve firms' profits in a monopoly market environment (see, e.g., Su and Zhang 2009). In the first place, the two strategies attract demand as they signal an assurance of high inventory quantity in-stock. Consumers would not make necessary up-front costs (i.e., time) to patronize a firm if the product they are looking for is out-of-stock. Moreover, the monetary compensation reimburses consumers when the product is out-of-stock, which encourages the consumers to visit the firms even if there is a certain probability that the product is unavailable. These two strategies become even more important in a competitive marketplace as product availability is a key leverage to capture market demand, especially in the era of e-commerce and online shopping. For example, in December 2011, BestBuy.com canceled some online orders due to the overwhelming demand for hot product offerings. Soon after the cancellation, many customers moved to Amazon.com with a click of button, as reported by TradeGecko (Tao 2014).

Being aware of the importance of inventory availability, firms may compete aggressively to attract market demand by committing to high inventory quantity in-stock and/or offering high compensation upon stock-outs. Although the strategies of inventory commitment and monetary compensation have been acknowledged to benefit the firms in monopoly settings, there has been little research studying their effectiveness in a competitive marketplace. On one hand, these strategies provide incentives for customers to visit the retailers and enhance their competitive edge. On the other hand, it is also possible that firms will battle to over-commit inventory quantities and/or provide higher compensations in order to win a larger market share under competition. This phenomenon is analogous to the "price war" in many industries which has economically devastated many small businesses. For example, as major airlines went toe-to-toe in matching and exceeding one another's reduced fares, the whole industry records a higher volume of air travel as well as an alarming record of profit losses (see `https://hbr.org/2000/03/how-to-fight-a-price-war`). Similarly, the two marketing strategies, if adopted by competing firms, may result in an escalated competition on product availability, and eventually lead to excess inventory in-stock throughout the whole industry.

In view of the potential alarming results due to over-competition on product availability, this paper examines the inventory commitment and monetary compensation strategies under market competition. We model two competing retailers as newsvendors located at the endpoints of a Hotelling line market. Customers are uniformly distributed on the Hotelling line. As in the standard Hotelling model, customers incur a travel cost to patronize a retailer. The closer a customer is located to a retailer, the less travel cost she will incur. Before demand is realized, each retailer sets its price and inventory order quantity to maximize the expected profit. The prices are observable to customers and the other retailer, whereas the inventory order quantity is each retailer's private information. Individual customers choose which retailer to patronize based on product price, travel cost, and belief about inventory availability. Under the inventory commitment strategy, a retailer credibly reveals its inventory order information to the public, whereas under the monetary compensation strategy, a retailer compensates the customers who cannot get the product due to stock-out.

The primary goal of this paper is to evaluate the commonly used inventory commitment and monetary compensation strategies under competition. To this end, we adopt the perfect Bayesian equilibrium (PBE) framework to study the strategic interactions between the retailers and the customers under competition on inventory availability and price. The retailers are competing on both price and inventory availability. In particular, the retailers' trade-off is between decreasing price (which implies low inventory availability) and increasing inventory availability (which requires a high price). Depending on the competition intensity, we characterize the market equilibrium and deliver the following set of insights.

First, inventory commitment and monetary compensation may decrease retailers' profit under market competition. In the monopoly setting, it has been shown that the inventory commitment and monetary compensation strategies benefit the retailer in the presence of strategic customers, because these strategies help reassure customers in the presence of stock-out risks (Su and Zhang 2009). Specifically, under market competition, if the inventory commitment option is allowed for each competing retailer, a prisoner's dilemma may arise. Specifically, although both retailers have incentives to commit to an inventory order quantity, the equilibrium profits of both retailers may decrease if inventory commitment is adopted. Revealing inventory information to customers intensifies market competition and results in inventory over-stocking (with an order quantity higher than the optimal newsvendor inventory), thus rendering lower profits for both retailers. Likewise, the monetary compensation may prompt the retailer to overcompensate customers so as to signal high product availability, thus backfiring on the retailers and hurting their profits.

Second, contrary to the common wisdom, we find that market competition may hurt social welfare under equilibrium. The inventory commitment and monetary compensation strategies further intensify the competition between the retailers, therefore reducing the social welfare in a competitive market. It has been well documented in the economics literature that competition increases social welfare (see, e.g., Stiglitz 1981). In our setting, one may intuit that competition will enhance the social welfare as well, because competition lowers the equilibrium prices. Our results, however, indicate that market competition leads to lower product availability and, therefore, lowers the social welfare. Inventory commitment and monetary compensation strategies, while improving social welfare in a monopoly market (e.g., Su and Zhang 2009), induce the retailers to compete more aggressively on inventory availability, which turn out to further decrease the social welfare under market competition.

The remainder of this paper is organized as follows. Section 2 positions this paper in the relevant literature. The model is introduced in Section 3. Sections 4 and 5, respectively, study the value of the inventory commitment and monetary compensation strategies under inventory availability competition. The analysis of customer surplus and social welfare is given in Section 6. Finally, Section 7 concludes this paper. All the proofs are presented in the Appendix.

2. Literature Review

The impact of inventory availability has been extensively studied in the operations management literature. Dana and Petruzzi (2001) consider a newsvendor model where consumers are concerned about inventory availability and can choose whether to visit the firm. Su and Zhang (2008) introduce the strategic waiting behavior of customers into the newsvendor setting and investigate the impact of such behavior on a firm's pricing and stocking decisions. Liu and Van Ryzin (2008) demonstrate that one can mitigate the strategic waiting behavior by limiting inventory availability over repeated selling horizons. Since then there have been a growing number of operations studies that involve strategic customer behavior and availability considerations in various settings. For example, Su and Zhang (2009) and Cachon and Feldman (2014) further include a search cost for the stock-out-conscious customers. Cachon and Swinney (2009, 2011) focus on the value of quick response under strategic customer behavior. Prasad et al. (2014), Li and Zhang (2013), and Wei and Zhang (2017a) investigate the advance selling strategy where product availability may affect customers' optimal timing of purchase. Allon and Bassamboo (2011) use a cheap talk framework to quantify the value of providing inventory availability information to customers; Liang et al. (2014) examine a firm's product rollover strategies under consumers' forward-looking behavior. With variable assortment depth, Bernstein and Martínez-de Albéniz (2016) study the optimal dynamic product rotation strategy in the presence of strategic customers. Tereyagoglu and Veeraraghavan (2012) study a retailer's problem when selling to conspicuous consumers whose consumption utility depends on the availability of the product. Finally, Gao and Su (2016) study the role of inventory availability in the context of omni-channel retailing. Wei and Zhang (2017b) provide a recent review of this line of research. Despite the fast growth of this topic, the majority of studies in the literature focus on single-firm settings; our paper, instead, contributes to the above literature by studying the impact of product availability in a competitive setting.

In a competitive marketplace, if a stock-out occurs at one firm, unsatisfied demand may switch to the other firms. Such stock-out-based substitution has also received significant attention in the operations management literature. Lippman and McCardle (1997) propose several ways to model demand allocation between competing newsvendors and show that competition leads to overstocking relative to the centralized solution. Netessine and Rudi (2003) develop a tractable model to compare inventory management under centralized vs. decentralized control. Several studies extend the static substitution model to dynamic ones; see, for example, Bassok et al. (1999), Shumsky and Zhang (2009), and Yu et al. (2015). This line of research does not explicitly model individual customer behavior, which is a key focus of our work. Therefore, our paper differs from this research in terms of both the model setting and insights.

Another stream of papers study the competition on product availability in the economics literature. Carlton (1978) is among the first to formally consider the issue of product availability in a competitive market and argues that only equilibrium outcome with zero firm profit will arise. As a follow-up to Carlton's work, Deneckere and Peck (1995) consider a game where firms can decide on both price and capacity and demonstrate that a pure-strategy equilibrium exists if and only if the number of firms is sufficiently large. Lei (2015) studies a similar integrated newsvendor and Hotelling model but with asymmetric unit costs. He finds that firms with the lowest unit cost may survive in the long run. Along this line of research, Daughety and Reinganum (1991) and Dana (2001) are closely related to our paper. Specifically, Daughety and Reinganum (1991) consider a setting where consumers have imperfect information on both price and stocking levels at firms. An important finding is that in equilibrium, the duopoly price is lower than the monopoly price if consumers' search cost is low, while the duopoly price is the same as the monopoly price if consumers' search cost is high. In contrast, we find that retailers may charge a strictly *higher* price to signal high product availability and thus attract more demand in the presence of market competition. Dana (2001) adopts a newsvendor setup to model retailers competing on product availability. It has been shown that the retailers can enjoy a positive profit (i.e., they can charge a price higher than marginal cost) even though the products are perfectly substitutable because the retailers can signal a high probability of product availability using a high price. Our paper also uses a similar newsvendor paradigm, but with several important differences. First, we use the Hotelling setup to incorporate heterogeneous travel costs of customers, which which lead to different insights. Second, we examine the impact of availability competition on customer surplus, while Dana (2001) focuses on the equilibrium outcome from the firms' perspectives. Finally, we also study the effectiveness of operational strategies such as stock-out compensations and inventory commitment, which are absent from the above economics literature.

The economics literature has also studied the impact of competition on customer surplus. For example, Brynjolfsson et al. (2003) summarize two channels of how market competition of product variety improves consumer surplus. In their study of the on-line bookstores market, the increased product variety competition induces around three million dollars more consumer welfare in 2000. In the food industry, Hausman and Leibtag (2007) empirically verify that the entry of new business and the expansion of existing business improve average consumer surplus by approximately 25%. Goolsbee and Petrin (2004) show that the competition between broadcast satellites (DBS) and cable leads to a consumer welfare gain of \$2.5 billion for satellite buyers and \$3 billion for cable subscribers. Our contribution to this literature is that we demonstrate the adverse effect of market competition on social welfare if customers are concerned about inventory availability.

3. Model

To study the inventory commitment and monetary compensation strategies in the presence of inventory availability competition, we build our model upon the classical newsvendor and Hotelling frameworks. The newsvendor setup captures the key features of demand uncertainty and perishable inventory, which are common for a retail setting where the inventory availability concern is most relevant. The Hotelling model highlights the competition between the retailers, as well as the heterogeneous tastes/preferences of the customers. These salient features are often ignored in the literature studying inventory availability. We now introduce our model from the perspectives of retailers (he) and customers (she) separately.

We model the market as a Hotelling line with a unit length, denoted by $\mathcal{M} = [0,1]$. Two retailers, R_i $(i = 1,2)$, are located at the two endpoints of the Hotelling market $\mathcal{M}$. Without loss of generality, we assume R_1 is at 0 whereas R_2 is at 1. Each retailer sells a substitutable product that has the same procurement cost c. Each retailer R_i stocks q_i units of inventory and charges a price p_i to maximize his own expected profit.

In the market $\mathcal{M}$, customers are uniformly distributed over the interval $[0,1]$. Each customer has infinitesimal mass in $\mathcal{M}$, and purchases at most one unit of the product. The valuation of the product to all customers is homogeneous and denoted by v. Such a modeling setup helps single out the effect of competition. The aggregate market demand D (i.e., the total mass of the Hotelling line) is uncertain and follows a known distribution $F(\cdot)$. We assume that the demand distribution has an increasing failure rate, which can be satisfied by most commonly used distributions. For conciseness, we define $\mathbb{E}[\cdot]$ as the expectation operation, and $x \wedge y := \min(x,y)$ as the minimum operation. To visit a retailer, each customer incurs a travel cost that increases linearly with her travel distance. More specifically, the travel cost of a customer located at $x \in \mathcal{M}$ to visit R_1 (resp. R_2) is sx (resp. $s(1-x)$), where s is the unit distance travel cost. It is worth noting that the travel cost can also be interpreted as the search cost for customers. The longer the distance between a customer and her focal retailer, the more costly for her to patronize. The customer aims to maximize her expected payoff by choosing to visit (or not to visit) a retailer.

The sequence of events unfolds as follows. At the beginning of the sales season, each retailer R_i simultaneously decides his inventory stocking quantity q_i and announces the retail price p_i. Both the inventory level q_i and the price p_i cannot be adjusted throughout the sales horizon. Customers observe the prices (p_1, p_2), but not the inventory levels (q_1, q_2), and decide which retailer to visit (or not to visit any of them). The demand D_i for retailer R_i is realized as a result of customers' cumulative purchasing decisions. If $D_i \leq q_i$, all customers requesting a product can get one. Otherwise, $D_i > q_i$, stock-out occurs and customers not receiving the product leave the market. Upon stock-out, the customers may switch to the other retailer for a substitute. However, due to the symmetry of the retailers, rational customers will not search for a substitute under equilibrium, as the other retailer will also run out of inventory if the focal retailer faces stock-out. The equilibrium analysis for asymmetric retailers is more challenging, and we leave it as a direction for future research. Finally, the transactions occur and the retailers collect the revenues.

3.1. Equilibrium Characterization

Next, we analyze the equilibrium of the base model. To this end, we adopt the Perfect Bayesian Equilibrium (PBE) concept. Under the PBE, customers, upon observing the prices (p_1, p_2), form beliefs about inventory availability and make purchasing decisions to maximize their own expected utilities, whereas retailers (at the beginning of the sales horizon) base their pricing and inventory decisions on anticipations about customers' purchasing behaviors to maximize profits. Furthermore, under equilibrium, both the customers' beliefs about inventory availability and the retailers' anticipations should be consistent with the actual outcomes according to the Bayes rule. Following Dana (2001), we restrict our attention to equilibria where the retailers play symmetric pure strategies.

Customers' Problem. We first analyze the customers' problem. Consider a customer located at $x \in \mathcal{M}$. Her surplus to visit R_1 is $v - p_1 - sx$ (resp. $-sx$) if the product is in stock (resp. out of stock). Similar analysis can be applied if she visits R_2. The customer gains zero surplus if she does not visit any retailer. Since customers cannot observe retailers' inventory status, they form a belief about it (see Dana 2001). To facilitate the analysis, we assume customers form beliefs about the (unobservable) inventory availability probability instead of order quantity, because the influence of inventory stocking quantity on the expected utility (and thus the purchasing behavior) of a customer boils down to the availability probability it induces. Specifically, let $\theta_i(p_1, p_2) \in [0,1]$ be the in-stock probability of R_i given the price. Thus, the expected utility of a customer located at x to visit R_1 (resp. R_2) is $\mathcal{U}_1(x) := (v - p_1)\theta_1(p_1, p_2) - sx$ (resp. $\mathcal{U}_2(x) := (v - p_2)\theta_2(p_1, p_2) - s(1-x)$).

Customers base their purchasing decisions on their beliefs about product availability. More specifically, a customer chooses to visit the retailer from which she can earn the highest non-negative expected payoff (otherwise, she will not visit anyone). Since a customer is infinitesimal, without loss of generality, a customer located at x will patronize R_i if $\mathcal{U}_i(x) \geq \max\{0, \mathcal{U}_{-i}(x)\}$ and will not visit any retailer if $\mathcal{U}_i(x) < 0$ $(i = 1, 2)$. Therefore, there exist two thresholds $\underline{x}(p_1, p_2)$ and $\bar{x}(p_1, p_2)$ such that a customer located at x will patronize R_1 if $x \leq \underline{x}(p_1, p_2)$, will patronize R_2 if $x \geq \bar{x}(p_1, p_2)$, and will not visit any retailer if $x \in (\underline{x}(p_1, p_2), \bar{x}(p_1, p_2))$. Algebraic manipulation yields that

$$\begin{cases} \underline{x}(p_1, p_2) = \mathcal{P}_{[0,1]}\left(\min\left\{\frac{(v-p_1)\theta_1(p_1,p_2)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta_1(p_1,p_2)-(v-p_2)\theta_2(p_1,p_2)}{2s}\right\}\right), \\ \bar{x}(p_1, p_2) = \mathcal{P}_{[0,1]}\left(\max\left\{1 - \frac{(v-p_2)\theta_2(p_1,p_2)}{s}, \frac{1}{2} + \frac{(v-p_2)\theta_2(p_1,p_2)-(v-p_1)\theta_1(p_1,p_2)}{2s}\right\}\right), \end{cases}$$

where $\mathcal{P}_{[0,1]}(x) := \min\{\max\{x, 0\}, 1\}$ is the projection operator onto the interval $[0,1]$.

Retailer's Problem. Next, we analyze the retailer's pricing and inventory problem. Each retailer strategizes his price and inventory decisions in anticipation of customers' purchasing decisions (thus his market share). Specifically, the demand for R_1 (resp. R_2) is $\underline{x}(p_1, p_2)D$ (resp. $(1 - \bar{x}(p_1, p_2))D$). Given the competitor's price, the retailer R_i's profit maximization problem is:

$$\max_{(p_i, q_i)} \{p_i \mathbb{E}(\alpha_i(p_1, p_2)D \wedge q_i) - cq_i\},$$

where $\alpha_1(p_1, p_2) = \underline{x}(p_1, p_2)$ and $\alpha_2(p_1, p_2) = 1 - \bar{x}(p_1, p_2)$ represent the respective market share. Therefore, given a price p_i, the retailer R_i's optimal inventory order strategy is the newsvendor solution: $q_i = \alpha_i(p_1, p_2) F^{-1}\left(\frac{p_i - c}{p_i}\right)$.

To characterize the PBE, we need to specify the off-equilibrium customer belief on inventory availability (see, e.g., Dana 2001). Moreover, we refine the off-equilibrium belief to rule out implausible equilibria. Consistent with the equilibrium refinement strategy of Dana (2001), customers rationally believe that the retailers are stocking the optimal amount of inventory given any observed price. Specifically, given the price (p_1, p_2), the customers believe that the inventory order quantity of the retailer R_i is $q_i = \alpha_i(p_1,p_2)F^{-1}\left(\frac{p_i-c}{p_i}\right)$. Conditioned on the existence of a customer, her belief about the total demand for the retailer R_i is a random variable with probability density function $g_i(y|p_1,p_2) := \frac{y}{\alpha_i(p_1,p_2)\mu} f\left(\frac{y}{\alpha_i(p_1,p_2)}\right)$, where $\mu := \mathbb{E}[D]$ (see, e.g., Dana 2001, Su and Zhang 2009). As shown by Dana (2001), without horizontal heterogeneity (i.e., all customers have the same distance to the competing retailers), each customer holds an identical belief of the inventory availability for R_i $\int_{y=0}^{\infty} \frac{\min(q_i,\alpha_i(p_1,p_2)y)f(y)}{\alpha_i(p_1,p_2)\mu}\mathrm{d}y$. In our model, however, customers have different distances to the retailers and will arrive at different time, which requires us to carefully model how inventory is rationed to customers upon stock-out.

Efficient Rationing. Although customers make retailer patronage decisions simultaneously, a customer closer to the focal retailer will arrive earlier due to the shorter travel distance and will be served with a higher priority. As shown in the behavioral economics literature, the stock-out information lowers customers satisfaction and reduces purchase motivation (Kim and Lennon 2011), and thus can be viewed as an exogenous imposed tax (Sicular 1988) to the customers. Hence, customers with higher surpluses are more likely to visit the retailer earlier and would be served earlier. This phenomenon motivates us to apply an inventory allocation rule that accounts for the effect of travel distance. Specifically, we adopt the efficient rationing rule which prioritizes customers closer to the retailer upon stockout. We remark that if the retailers adopt the uniform/random rationing rule, i.e., inventory will be allocated to each customer who visits a retailer with equal probability, our managerial conclusions will not be affected. We refer interested readers to a comprehensive discussion of efficient rationing by Tasnádi (1999).

We first specify the inventory availability probability for a customer located at the purchasing threshold. By the efficient rationing rule, the customer located at $\underline{x}(p_1,p_2)$ (resp. $\bar{x}(p_1,p_2)$) observes the lowest probability of product availability from R_1 (resp. R_2) among all customers located in the interval $[0,\underline{x}(p_1,p_2)]$ (resp. $[\bar{x}(p_1,p_2),1]$). That is, the customers at the purchasing thresholds can receive the product if and only if the total demand for R_i does not exceed its inventory stocking quantity. Therefore, the belief of the customers (at the purchasing threshold) about R_i's inventory availability probability is: $\theta_i(p_1,p_2) = \mathbb{P}\Bigg(\alpha_i(p_1,p_2)D \le q_i\Bigg) = \frac{1}{\mu}\int_{y=0}^{F^{-1}\left(\frac{p_i-c}{p_i}\right)} yf(y)\mathrm{d}y$. Note that the belief of product availability only depends on the price of the focal retailer. We remark that this is driven by our equilibrium refinement rule that customers believe the retailers will stock the optimal newsvendor inventory, which induces a service level that depends on the price of the focal retailer only. For the subsequent analysis, we shall use $\theta^*(p_1) = \frac{1}{\mu}\int_{y=0}^{F^{-1}\left(\frac{p_i-c}{p_i}\right)} yf(y)\mathrm{d}y$ to denote customers' belief of R_i's inventory availability for those at the purchasing thresholds.

For customers who are between the purchasing thresholds i.e., $x \in (\underline{x}(p_1,p_2),\bar{x}(p_1,p_2))$, their beliefs of R_i's product availability are lower than $\theta^*(p_i)$ for $i = 1,2$; whereas for customers who are closer to R_1 (resp. R_2) than its associated purchasing threshold i.e., $x < \underline{x}(p_1, p_2)$ (resp. $x > \bar{x}(p_1, p_2)$, their beliefs of R_1's (resp. R_2's) product availability are higher than $\theta^*(p_1)$ (resp. $\theta^*(p_2)$). Hence, specifying the equilibrium belief of customers at the purchasing thresholds is sufficient to characterize the purchasing behaviors of *all* customers. In other words, given that customers hold identical beliefs of inventory availability probability as those at the purchasing thresholds, they will exhibit exactly the same purchasing behavior as they hold the "true" equilibrium beliefs which depend on their specific locations.

Equilibrium. We are now ready to characterize the symmetric equilibrium price and inventory decisions of the retailers. Under the symmetric PBE, both retailers charge the same equilibrium price p^*. Customers hold the same belief of equilibrium product availability, and we have $\theta^*(p^*) = \frac{1}{\mu}\int_{y=0}^{F^{-1}\left(\frac{p^*-c}{p^*}\right)} yf(y)\mathrm{d}y$ and $\alpha_i(p_i, p^*) = \min\{\frac{(v-p_i)\theta^*(p_i)}{s}, \frac{1}{2}\}$ for $i = 1, 2$. The travel cost s can also be viewed as a measure of competition intensity. In particular, if s is large enough, customers located at the middle of the Hotelling line $\mathcal{M}$ will visit neither of the retailers. In this scenario, the market is fully separated and, hence, there is no competition between the retailers. On the other hand, if s is small, the surplus difference of customers to purchase from different retailers is small, so the two retailers compete on price and inventory availability.

If s is large, the market is not fully covered and, therefore, there is no competition between the two retailers. Then, the equilibrium price $p^* = \arg\max_{0\le p\le v} \Pi_m(p)$ is the optimal monopoly price solved for the following problem:

$$\begin{aligned} \max_{0\le p\le v} \quad & \Pi_m(p) = p\mathbb{E}\left[\alpha(p)D \wedge q(p)\right] - cq(p) \\ \text{s.t.} \quad & q(p) = \alpha(p)F^{-1}\left(\frac{p-c}{p}\right), \\ & \alpha(p) = \frac{(v-p)\theta^*(p)}{s}, \end{aligned} \tag{1}$$

where $\theta^*(p) = \frac{1}{\mu}\int_{y=0}^{F^{-1}\left(\frac{p-c}{p}\right)} yf(y)\mathrm{d}y$. The first constraint represents that the inventory order quantity is prescribed by the newsvendor model. The second constraint follows from a retailer's market share equation. Since there is no competition between two retailers, under equilibrium, we must have $\alpha(p^*) < 1/2$.

If s is small, the market is fully covered and the two retailers compete with each other. In this case, the equilibrium price $p^* = \arg\max_{0\le p\le v} \Pi(p, p^*)$ is solved by the following:

$$\begin{aligned} \max_{0\le p\le v} \quad & \Pi(p, p^*) = p\mathbb{E}\left[\alpha(p, p^*)D \wedge q(p, p^*)\right] - cq(p, p^*) \\ \text{s.t.} \quad & q(p, p^*) = \alpha(p, p^*)F^{-1}\left(\frac{p-c}{p}\right), \\ & \alpha(p, p^*) = \frac{1}{2} + \frac{(v-p)\theta^*(p) - (v-p^*)\theta^*(p^*)}{2s}. \end{aligned} \tag{2}$$

Note that the above two cases do not necessarily cover all the possible scenarios. It is also possible that although the two retailers cover the entire market, they do not compete with each other. This case occurs when the travel cost s is in the moderate range. More specifically, in this case, each retailer covers 50% of the market and the customer located at the middle of the market $\mathcal{M}$ (i.e., $x = 0.5$) is indifferent between visiting either retailer or not visiting anyone, i.e., the equilibrium price p^* satisfies that $\alpha(p^*) = \frac{(v-p^*)}{s}\theta^*(p^*) = \frac{1}{2}$. By Su and Zhang (2009), p^* should be the larger root of the equation (of variable p) $\frac{(v-p)}{s}\theta^*(p) = \frac{1}{2}$. Finally, we note that, in this case, p^* is also the solution to the stochastic program (1). Furthermore, under equilibrium, we impose the constraint $\alpha(p^*) = \frac{1}{2}$ in this case with full market coverage without competition.

We are now ready to characterize the symmetric PBE $(p^*, q^*, \theta^*(\cdot))$ in the base model.

Proposition 1. *(a) There exists a unique symmetric PBE $(p^*, q^*, \theta^*(\cdot))$ in the base model.*

(b) There exists a threshold $\underline{s}$ such that if $s \le \underline{s}$, we have $p^* = \arg\max_{0\le p\le v}\Pi(p, p^*)$ (see Eq. (2)), $q^* = \frac{1}{2}F^{-1}\left(\frac{p^*-c}{p^*}\right)$, and $\theta^*(p^*) = \frac{1}{\mu}\int_{y=0}^{2q^*} yf(y)\mathrm{d}y$. In this case, the two retailers compete with each other and each covers half of the entire market.

(c) There exists another threshold $\bar{s} > \underline{s}$, such that if $\underline{s} < s \le \bar{s}$, we have $p^* = v - \frac{s}{2\theta^*(p^*)}$, $q^* = \frac{1}{2}F^{-1}\left(\frac{p^*-c}{p^*}\right)$, and $\theta^*(p^*) = \frac{1}{\mu}\int_{y=0}^{2q^*} yf(y)\mathrm{d}y$. In this case, each retailer covers half of the entire market without competing with each other.

(d) If $s > \bar{s}$, we have $p^* = \arg\max_{0\le p\le v}\Pi_m(p)$ (see Eq. (1)), $q^* = \frac{(v-p^*)\theta^*(p^*)}{s}F^{-1}\left(\frac{p^*-c}{p^*}\right)$, and $\theta^*(p^*) = \frac{1}{\mu}\int_{y=0}^{F^{-1}\left(\frac{p^*-c}{p^*}\right)} yf(y)\mathrm{d}y$. In this case, each retailer covers $\frac{(v-p^*)\theta^*(p^*)}{s} < \frac{1}{2}$ of the entire market, so the market is partially covered by the retailers without competition.

Proposition 1 implies that a unique symmetric PBE exists. Furthermore, we find that if the travel cost is small ($s \le \underline{s}$), the competition is intensive and the two retailers will cover the entire market competitively, each with a 50% market share. If the travel cost is moderate ($\underline{s} < s \le \bar{s}$), although the two retailers remain covering the entire market (each still with a 50% market share), there is no competition in the market. Finally, if the travel cost is high ($s > \bar{s}$), the market is completely segmented and some customers will opt not to visit any retailer, so the market share of each retailer is smaller than 50%. Such a market outcome structure is consistent with that in the standard Hotelling model without demand uncertainty (see Lemma 2 in the Appendix B). Proposition 1 also indicates that once a consumer meets stockout in her focal retailer, the other retailer also runs out of inventory. This result partially justifies our modeling assumption that, upon the stockout of one retailer, customers will not visit the other one (under equilibrium). For the rest of this paper, we focus on studying how the widely adopted inventory commitment and monetary compensation strategies would impact the retailers and the social welfare in a competitive market.

4. Inventory Commitment

Inventory commitment is a commonly-used ex ante strategy (i.e., it is used before demand realization) to enhance a retailer's competitive edge in the presence of availability-concerned customers (see, e.g., Su and Zhang 2009). Under this strategy, the retailer should credibly announce its order quantity to both the customers and the other retailer. For example, Amazon.com recently provided a lightning deal platform to allow retailers to promote their products. A salient feature of "lightning deals" is that sellers have to announce the amount of inventory to customers. In particular, a customer can see a real-time status bar on the web page of the seller indicating the current price, inventory, and percentage of units that have already been claimed by other customers. In some other circumstances, a retailer has to publicize its inventory information to customers, even if he is not willing to do so by himself. For instance, the affiliated stores of Great Clips (a hair salon franchise in the United States and Canada) post their real-time information of available slots online. Customers can check the anticipated waiting times of all stores around and add their names to the waitlist before actually visiting the salon. In this case, the competing franchised stores are forced to reveal their available inventory information.

It has been shown in the literature that the inventory commitment strategy benefits the monopoly retailer (e.g., Su and Zhang 2009). In this section, we strive to analyze this strategy in a competitive market. Our results imply that the inventory commitment strategy may lead to an undesirable prisoner's dilemma: Although both retailers will voluntarily reveal their inventory information under equilibrium, the equilibrium profit of each retailer will be lower than in the base model where the retailers cannot credibly announce the order quantity information. Therefore, the inventory commitment strategy may not serve as an effective tool for retailers in a competitive market where customers are concerned about the inventory stock-out risk.

We now formally model the inventory commitment strategy in our duopoly market. We use subscript v to represent the model with inventory commitment. At the beginning of the sales horizon, the competing retailers first decide whether to reveal the inventory information to the public (i.e., whether to adopt the inventory commitment strategy). Then, the retailers will announce price and order inventory accordingly. If a retailer commits to publicizing its inventory information, he will truthfully announce its order quantity to the whole market. Finally, the customers observe the prices of the retailers and the amount of inventory ordered by the retailer who adopts the inventory commitment strategy, and decide which retailer to visit. As in the base model, we adopt the PBE framework to analyze the equilibrium market outcome. There are three cases to consider: (i) both retailers do not reveal the inventory order quantities, which is reduced to the base model; (ii) both retailers adopt the inventory commitment strategy; (iii) one retailer adopts the inventory commitment strategy whereas the other one does not reveal its inventory. Section 3 presents a detailed analysis for case (i). We now analyze cases (ii) and (iii).

Both Retailers Adopt Inventory Commitment Strategy. In the presence of inventory commitment, individual customers do not need to form beliefs about inventory availability, but directly optimize their purchasing decisions after observing both prices and inventory stocking quantities. Specifically, after observing retailer R_i's price p_i and stocking quantity q_i, where $i \in \{1,2\}$, customers estimate the in-stock probability of each retailer conditioned on her existence. Similar to the base model, there exists a threshold $\underline{x}_v$ (resp. $\bar{x}_v$) such that customers with locations $x \le \underline{x}_v$ (resp. $x > \bar{x}_v$) will visit retailer R_1 (resp. retailer R_2). Algebraic manipulation yields that (θ_i is open information, where $i \in \{1,2\}$)

$$\begin{cases} \underline{x}(p_1,p_2) = \mathcal{P}_{[0,1]}\left(\min\left\{\frac{(v-p_1)\theta_1}{s}, \frac{1}{2} + \frac{(v-p_1)\theta_1-(v-p_2)\theta_2}{2s}\right\}\right), \\ \bar{x}(p_1,p_2) = \mathcal{P}_{[0,1]}\left(\max\left\{1-\frac{(v-p_2)\theta_2}{s}, \frac{1}{2} + \frac{(v-p_2)\theta_2-(v-p_1)\theta_1}{2s}\right\}\right), \end{cases}$$

where $\mathcal{P}_{[0,1]}(x) := \min\{\max\{x,0\},1\}$ is the projection operator onto the interval $[0,1]$. It suffices to characterize the perceived inventory availability probabilities at the purchasing thresholds $\underline{x}_v$ and $\bar{x}_v$. We define θ_1^v as the perceived inventory availability probability for a customer located at $\underline{x}_v$ to visit retailer R_1, and θ_2^v as the perceived inventory availability probability for a customer located at $\bar{x}_v$ to visit retailer R_2. Then, we have $\theta_i^v = \frac{1}{\mu}\int_{y=0}^{q_i/\alpha_i^v} yf(y)\mathrm{d}y$, where $\alpha_1^v := \underline{x}_v$ is the market share of retailer R_1 and $\alpha_2^v := 1-\bar{x}_v$ is the market share of retailer R_2. Denote retailer R_i's profit as $\Pi_v(p_i,q_i) = p_i\mathbb{E}[\alpha_i(p_i,q_i)D \wedge q_i] - cq_i$. As in the base model, we focus on the case of symmetric equilibrium $(p_v^*, q_v^*, \alpha_v^*)$, where p_v^* is the equilibrium price, q_v^* is the equilibrium order quantity, and α_v^* is the equilibrium market share.

Similar to the base model, if s is large, the market is segmented and $\alpha_v^* = \frac{(v-p_v^*)\theta_v^*}{s}$. If s is small, the market is fully covered and the retailers compete on price and order quantity to win market share, so the market share of retailer R_i, α_i^v, is the unique solution to the following equation (of α): $\frac{v-p_i}{\mu}\int_{y=0}^{q_i/\alpha} yf(y)\mathrm{d}y - s\alpha = \frac{v-p_v^*}{\mu}\int_{y=0}^{q_v^*/(1-\alpha)} yf(y)\mathrm{d}y - s(1-\alpha)$. If s is moderate, the market is fully covered but the retailers do not compete with each other, so $\alpha_v^* = \frac{1}{2}$. Summarizing the above three cases, the following proposition characterizes the equilibrium outcome if both retailers commit to revealing their inventory information to the market.

Proposition 2. *If both retailers adopt the inventory commitment strategy, the following statements hold:*

(a) There exists a unique symmetric PBE $(p_v^*, q_v^*, \alpha_v^*)$.

(b) There exists a threshold $\underline{s}_v$, *such that if* $s \le \underline{s}_v$, *we have* $(p_v^*, q_v^*) = \arg\max_{0\le p\le v, q\ge 0} \Pi_v(p,q)$ *subject to the constraint* $\frac{v-p}{\mu}\int_{y=0}^{q_i/\alpha(p,q)} yf(y)\mathrm{d}y - s\alpha(p,q) = \frac{v-p_v^*}{\mu}\int_{y=0}^{q_v^*/(1-\alpha(p,q))} yf(y)\mathrm{d}y - s(1-\alpha(p,q))$. *In this case, each retailer covers half of the entire market and competes with each other.*

(c) There exists another threshold $\bar{s}_v > \underline{s}_v$, *such that if* $\underline{s}_v < s \le \bar{s}_v$, *we have* $(p_v^*, q_v^*) = \arg\max_{0\le p\le v, q\ge 0} \Pi_v(p,q)$ *subject to the constraints* $\frac{v-p}{s\mu}\int_{y=0}^{2q} yf(y)\mathrm{d}y = \frac{1}{2}$ *and* $\alpha(p,q) = \frac{1}{2}$. *In this case, each retailer covers half of the entire market without competition.*

(c) If $s > \bar{s}_v$, we have $(p_v^*, q_v^*) = \arg\max_{0 \le p \le v, q \ge 0} \Pi_v(p,q)$ where $\alpha(p,q) = \frac{v-p}{s\mu}\int_{y=0}^{q/\alpha(p,q)} yf(y)\mathrm{d}y$. In this case, each retailer covers $\alpha_v^* = \frac{v-p_v^*}{s\mu}\int_{y=0}^{q_v^*/\alpha_v^*} yf(y)\mathrm{d}y < \frac{1}{2}$ of the entire market, so the market is partially covered and there is no competition between the retailers.

Proposition 2 implies that the equilibrium outcome of the scenario where both retailers adopt the inventory commitment strategy shares the same structure as the base model. Specifically, Proposition 2(b) refers to the case with a low travel cost s, which results in full market coverage with competition, with each retailer serving half of the market and competing with each other. Proposition 2(c) refers to the case with complete market coverage without competition if the travel cost is moderate. Finally, the last part of Proposition 2 refers to the case with partial market coverage when the travel cost is high. Clearly, there is no competition in this case.

Incentive for Inventory Commitment In this subsection, we demonstrate that, regardless of whether the competitor adopts the inventory commitment strategy and the competitor's price and order quantity, the focal retailer will earn a higher profit by credibly revealing its inventory information to customers. This implies that inventory commitment is a dominating strategy for the retailers. As a consequence, the equilibrium outcome of the market with the inventory commitment option will be that both retailers voluntarily publicize their inventory order quantity, charge the price p_v^*, and order q_v^* units of inventory as prescribed in Proposition 2.

Let retailer R_1 as the focal retailer, we shall consider both the case where retailer R_2 credibly announces q_2 and the case where retailer R_2 does not reveal his order quantity. Given retailer R_2's price and inventory decision (p_2, q_2), we use $\Pi_{i,j}$ $(i,j \in \{d,v\})$ to denote the maximum profit of retailer R_1 if he adopts strategy i and retailer R_2 adopts strategy j, where subscript d refers to no inventory commitment and subscript v refers to inventory commitment. For example, $\Pi_{d,v}$ refers to the maximum profit of retailer R_1 if he does not adopt the inventory commitment strategy and retailer R_2 adopts this strategy. The derivations of $\Pi_{i,j}$ $(i,j \in \{v,d\})$ are given in the Appendix.

PROPOSITION 3. *For any (p_2, q_2) set by retailer R_2, we have that $\Pi_{v,d} > \Pi_{d,d}$ and $\Pi_{v,v} > \Pi_{d,v}$.*

An important implication of Proposition 3 is that, if the competing retailers can credibly reveal their inventory information to the market, adopting the inventory commitment strategy would be a *dominating* strategy for each of the retailers, regardless of the price and inventory decisions of the competitor. Therefore, the equilibrium outcome of the market under the inventory commitment *option* is that both retailers voluntarily reveal their inventory order quantity. Our next result examines the profit implication of the inventory commitment strategy under market competition. We use Π_v^* (resp. Π^*) to denote the equilibrium profit of a retailer with (resp. without) the inventory commitment strategy.

Proposition 4. *If the retailers have the option to credibly announce their inventory information, the following statements hold:*

(a) Under equilibrium, both retailer R_1 and retailer R_2 adopt the inventory commitment strategy.

(b) There exist a threshold $\bar{s}_{vd}$ for the unit travel cost and a threshold $\bar{c}_{vd}$ for the unit production cost. If $s < \bar{s}_{vd}$ and $c < \bar{c}_{vd}$, then we have $\Pi_v^* < \Pi^*$. Otherwise, $s \geq \bar{s}_{vd}$ or $c \geq \bar{c}_{vd}$, we have $\Pi_v^* \geq \Pi^*$.

As shown in Proposition 4, if both the unit travel cost s and the ordering cost c are low (i.e., $s < \bar{s}_{vd}$ and $c < \bar{c}_{vd}$), and if the inventory commitment strategy is adopted, the inventory stocking quantity can directly influence the purchasing behaviors of the customers. Therefore, the competition between retailers may be intensified by this strategy. The retailers may over-commit to inventory in a competitive market, thus reducing the profit of each retailer. Recall that in our base model, the stocking quantity is not observable by customers but signaled by price, so the only competitive leverage of a retailer is the prevailing price he charges. However, if the retailers can commit to their pre-announced inventory order quantities, they have more flexibility to influence demand. Furthermore, the signaling power of price is diluted if the inventory information is directly available to customers. In particular, when the unit cost c is high, the inventory commitment strategy helps the retailers increase the willingness-to-pay of the customers, thus attracting higher demand. On the other hand, if the unit cost is low, this strategy may backfire by triggering an over-commitment of stocking quantity. If, in addition, the market competition is intensive (i.e., $s < \bar{s}_{vd}$), each retailer will aggressively order a large amount of inventory to attract customers, which in turn exacerbates market competition and decreases the profits of both retailers ($\Pi_v^* < \Pi^*$ when $c < \bar{c}_{vd}$ and $s < \bar{s}_{vd}$). Therefore, whenever the cost c and customer travel cost s are both low, the retailers are actually worse off in the presence of the inventory commitment option due to the induced inventory over-commitment and intensified market competition. Proposition 3 and Proposition 4 together deliver a new and interesting insight that inventory commitment strategy may give rise to a prisoner's dilemma under market competition. Although this strategy is preferred by either retailer regardless of the competitor's inventory and price decisions, the retailers would be worse off if both of them adopt the inventory commitment strategy.

Our analysis demonstrates that the inventory commitment strategy does not always benefit the retailers under competition, which is in sharp contrast to the monopoly setting. There is a large body of research in the literature focusing on the inventory commitment strategy. A central message in this literature is that the inventory commitment strategy is beneficial for retailers. For example, Cachon and Swinney (2011) and Liu and Van Ryzin (2008) propose two-stage models to explore how to use availability information to manipulate customers' expectations and thus induce them to buy early. In a competitive market setting, revealing inventory information to customers

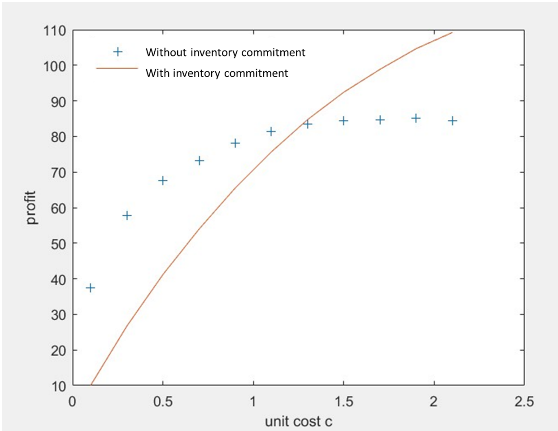

Figure 1 **Retailer profits in equilibrium.**

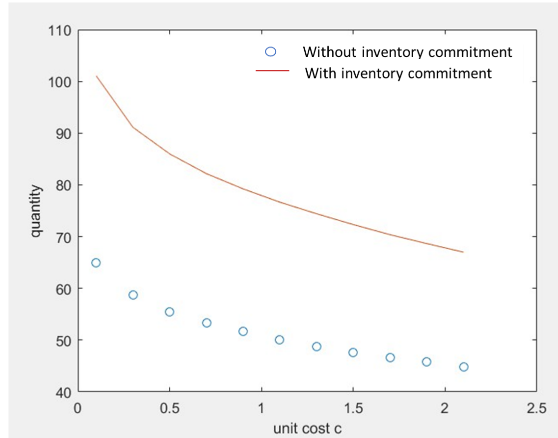

Figure 2 **Retailer order quantities in equilibrium.**

may lead to a higher equilibrium price, and, as a result, improves the firms' profits (see Dana 2001, Carlton 1978, and Dana and Petruzzi 2001). In a supply chain setting, Su and Zhang (2008) demonstrate that the firm's profit can be improved by promising either that the available inventory will be limited (quantity commitment) or that the price will be kept high (price commitment). In a monopoly setting, Su and Zhang (2009) further shows that the inventory commitment strategy offers customers information to more accurately assess their chances of securing the product. Thus, the inventory commitment strategy increases customers' willingness-to-pay and improves the profit of a monopoly firm. In a Hotelling competition setting, however, our results demonstrate that the inventory commitment strategy may give rise to a prisoner's dilemma and hurt the retailers.

We complement our theoretical analysis with numerical experiments to further illustrate the effect of inventory commitment. We compare the equilibrium profits and stocking quantities in the base model and the model with inventory commitment. In our numerical experiments, we set $s = 0.1$, $v = 10$, and the market demand D to follow a Gamma distribution with mean 90 and standard deviation 30. Figures 1 and 2 plot the equilibrium profits and order quantities, respectively, for the base model and the model with inventory commitment. Figure 1 clearly shows that the equilibrium profit of a retailer will be lower in the presence of inventory commitment whenever the ordering cost c is low. Figure 2 further demonstrates that, with inventory commitment, the retailers will order much more than they would have without revealing the inventory information to the market.

To conclude this section, we remark that, implementing the inventory commitment strategy relies heavily on the retailers' credibility in the market. The retailers should be able to credibly reveal their order quantity information to their competitors and their customers in the market. Otherwise, if the retailers fail to credibly convince the market, the effect of inventory commitment will be diluted. In the next section, we analyze another *ex post* monetary compensation strategy which is applicable without such commitment power.

5. Monetary Compensation

Next, we proceed to analyze the widely used monetary compensation strategy, which is an ex post strategy. After customers visit a retailer and find that the product is out of stock, the retailer will compensate them for such inconvenience. This strategy could reassure the customers in the presence of potential stock-outs, thus motivating customers to visit the retailer. In practice, the compensation is offered in the form of coupons, gift cards, price discounts for future orders, and free shipping opportunities. For example, FoodLand offers consumers a rain check for the out-of-stock items (see `https://www.foodland.com/if-i-have-coupon-product-out-stock-may-i-receive-rain-check-product`). The simplest and most direct compensation strategy is to placate customers for stock-outs with cash, which we refer to as the monetary compensation strategy. In this section, we focus on studying the effect of monetary compensation under competition.

The monetary compensation strategy has proven beneficial to a monopoly retailer (see Su and Zhang 2009), because it incentivizes strategic customers to visit him. In a competitive market, however, the story is different. Our analysis in this section shows that, when monetary compensation is an option, competing retailers will (voluntarily) overcompensate customers to attract higher demand, which in turn decreases their profits compared with the baseline setting where monetary compensation is not allowed.

To model the monetary compensation strategy, we assume that each retailer offers a compensation $m_i \geq 0$ ($i \in \{1,2\}$) to customers who face stock-out. The special case where $m_i = 0$ refers to that R_i does not offer monetary compensation. So both retailers have the flexibility to decide whether to offer monetary compensation upon stock-outs and the amount of compensation. As in the base model, customers observe the retailers' prices and monetary compensation terms, but not their inventory stocking quantities. The retailers set the price and inventory stocking quantity to maximize their profits, whereas customers decide where to make a purchase to maximize their expected surplus. Following the same equilibrium analysis paradigm as in the base model, we consider the symmetric PBE in the model with monetary compensation. We use the subscript c to denote the model with monetary compensation.

For any customer located at x, she will visit the retailer that yields the highest non-negative expected payoff and receive monetary compensation upon stockout. Hence, the customer's expected payoff is $(v-p_1)\theta_1 + m_1(1-\theta_1) - sx$ (resp. $(v-p_2)\theta_2 + m_2(1-\theta_2) - s(1-x)$), where θ_1 (resp. θ_2) is the customer's belief about R_1's (resp. R_2's) inventory availability probability. Indeed, a rigorous definition of the inventory availability probability is $\theta_i(p_1, p_2, m_1, m_2)$, where $i \in \{1,2\}$, which is a function of price and monetary compensation. For the simplicity of notations, we use θ_i to represent retailer R_i's inventory availability probability, where $i \in \{1,2\}$, in the analysis.

The retailer R_i's, where $i \in \{1,2\}$, decision problem can be formalized as:

$$\max_{(p_i,m_i,q_i)} \Pi_i^c(p_i,m_i,q_i) = p_i \mathbb{E}(\alpha_i^c D \wedge q_i) - m_i \mathbb{E}(\alpha_i^c D - q_i)^+ - cq_i,$$

where $\alpha_1^c = \underline{x}_c$ and $\alpha_2^c = 1 - \bar{x}_c$. Algebraic manipulation yields that

$$\begin{cases} \underline{x}(p_1,p_2) = \mathcal{P}_{[0,1]}\left(\min\left\{\frac{(v-p_1)\theta_1+m_1(1-\theta_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta_1-(v-p_2)\theta_2+m_1(1-\theta_1)-m_2(1-\theta_2)}{2s}\right\}\right), \\ \bar{x}(p_1,p_2) = \mathcal{P}_{[0,1]}\left(\max\left\{1 - \frac{(v-p_2)\theta_2+m_2(1-\theta_2)}{s}, \frac{1}{2} + \frac{(v-p_2)\theta_2-(v-p_1)\theta_1+m_2(1-\theta_2)-m_1(1-\theta_1)}{2s}\right\}\right), \end{cases}$$

where $\mathcal{P}_{[0,1]}(x) := \min\{\max\{x,0\},1\}$ is the projection operator onto the interval $[0,1]$. Therefore, the retailer R_i orders the newsvendor quantity $q_i^c = \alpha_i^c F^{-1}\left(\frac{p_i+m_i-c}{p_i+m_i}\right)$. Recall that the retailers adopt efficient rationing, the in-stock probability of a customer at the purchasing threshold is $\theta_i^c = \theta^*(p_i+m_i) = \frac{1}{\mu}\int_{y=0}^{F^{-1}(\frac{p_i+m_i-c}{p_i+m_i})} yf(y)\mathrm{d}y$. Here, the customer belief on inventory availability depends on (p_i,m_i) through the effective margin p_i+m_i.

Again we focus on the symmetric PBE $(p_c^*, m_c^*, q_c^*, \theta^*(\cdot))$, where p_c^* is the equilibrium price, m_c^* is the equilibrium compensation, and q_c^* is the equilibrium order quantity. Similar to the base model, the equilibrium market share of retailer R_i, α_i^c takes one of the following three forms: (a) if s is large, the market is completely segmented without competition; (b) if s is small, the market is fully covered and the retailers compete with each other; and (c) if s is moderate, the market is fully covered but the retailers do not compete with each other. The following proposition characterizes the PBE in the presence of monetary compensation.

PROPOSITION 5. *For the model with the monetary compensation option, the following statements hold:*

(a) There exists a unique symmetric PBE $(p_c^*, m_c^*, q_c^*, \theta^*(\cdot))$.

(b) There exists a threshold $\underline{s}_c$, such that if $s \le \underline{s}_c$, we have $(p_c^*, m_c^*) = \arg\max_{0\le p\le v, m\ge 0} \Pi^c(p,m,q)$, subject to $q = \alpha(p,m)F^{-1}\left(\frac{p+m-c}{p+m}\right)$ and $\alpha(p,m) = \frac{1}{2} + \frac{(v-p-m)\theta^*(p+m)+m-(v-p_c^*-m_c^*)\theta^*(p_c^*+m_c^*)-m_c^*}{2s}$. In equilibrium, $q_c^* = \frac{1}{2}F^{-1}\left(\frac{p_c^*+m_c^*-c}{p_c^*+m_c^*}\right)$, $\theta_c^* = \frac{1}{\mu}\int_{y=0}^{2q_c^*} yf(y)\mathrm{d}y$, and each retailer covers half of the entire market with competition.

(c) If $\underline{s}_c < s \le \bar{s}_c$ ($\underline{s}_c < \bar{s}_c$), we have $(p_c^*, m_c^*) = \arg\max_{0\le p\le v, m\ge 0} \Pi^c(p,m,q)$, subject to $q = \frac{1}{2}F^{-1}\left(\frac{p+m-c}{p+m}\right)$, $\alpha = \frac{1}{2}$, and $p+m = v - \frac{s+2m}{2\theta^*(p+m)}$. In this case, each retailer covers half of the entire market without competition.

(d) If $s > \bar{s}_c$, we have we have $(p_c^*, m_c^*) = \arg\max_{0\le p\le v, m\ge 0} \Pi^c(p,m,q)$, subject to $q = \alpha(p,m)F^{-1}\left(\frac{p+m-c}{p+m}\right)$ and $\alpha(p,m) = \frac{(v-p-m)\theta^*(p+m)+m}{s}$. In equilibrium, $q_c^* = \alpha(p_c^* + m_c^*)F^{-1}\left(\frac{p_c^*+m_c^*-c}{p_c^*+m_c^*}\right)$, $\theta_c^* = \frac{1}{\mu}\int_{y=0}^{q_c^*/\alpha(p_c^*+m_c^*)} yf(y)\mathrm{d}y$, and each retailer covers less than half of the entire market without competition.

Similar to Proposition 1, Proposition 5 shows the existence and uniqueness of PBE in the model with monetary compensation. Depending on the value of s, the retailers may cover the entire market or just a portion of it. If s is small ($s \le \underline{s}_c$), the retailers compete with each other and achieve full market coverage. Otherwise, the market may not be fully covered and there is essentially no market competition.

To examine the impact of the monetary compensation on the retailers' profit, we denote the equilibrium profit of a retailer in the model with monetary compensation as Π_c^* (the equilibrium profit of the retailers in the base model is Π^*). The following proposition shows that monetary compensation may hurt the retailers if market competition is intense.

PROPOSITION 6. *For the model with the monetary compensation option, the following statements hold:*

(a) There exists a critical threshold $\bar{s}_{cd} < \bar{s}_c$, such that if $s < \bar{s}_{cd}$, we have $\Pi_c^* < \Pi^*$.

(b) Otherwise, $s \ge \bar{s}_{cd}$, we have $\Pi_c^* \ge \Pi^*$.

Proposition 6 delivers an interesting message that, if the retailers have the option to offer monetary compensations upon stock-outs, they will earn a lower profit as long as the competition is sufficiently intense ($s \le \bar{s}_{cd}$). This is in contrast with the effect of monetary compensation in the monopoly setting, which always benefits the retailer (see Su and Zhang 2009). By offering a monetary compensation, the retailer, on one hand, is equipped with another lever in the competitive landscape; but, on the other hand, he competes more aggressively through direct subsidies to customers upon stock-outs. If s is large, the former effect dominates, which results in a higher profit in the presence of monetary compensation. If s is small, however, the latter effect dominates and monetary compensation leads to severe competition, which will in turn diminish the profit of each retailer. As a consequence, if the market competition is already fierce (i.e., s is small), the monetary compensation option will further intensify the competition and hurt the retailers. In a similar spirit to the classical Hotelling model (see Lemma 2 in the Appendix C), the intensified competition induced by stock-out compensations drags the equilibrium effective margin $p_c^* + m_c^*$ down to the marginal cost c as s approaches zero. Hence, if the unit travel cost s is sufficiently small (i.e., the model proposed by Dana 2001), both retailers may earn zero profit in the presence of the monetary compensation option.

In the existing literature, many studies have demonstrated that retailers can extract more profit by offering monetary compensation in a monopoly market. To convince customers of inventory availability, retailers adopt monetary compensation as a self-punishment mechanism upon stock-outs. With such a mechanism, customers will anticipate a high service level and increase their willingness-to-pay, which in turn can boost the firm's profit. For example, Su and Zhang (2009) show that monetary compensation can increase the retailer's product availability in a monopoly model. Besides such a short-run effect, it is widely believed that the monetary compensation also has a long-run effect to expand a firm's market share. Compensating customers upon stock-outs has a positive effect on customers' shopping experience, and thus cultivates customer loyalty. In other words, by purposefully providing compensations for stock-outs, retailers have the potential to increase their demand in the long run (see, e.g., Bhargava et al. 2006). Albeit with all these benefits, our results (i.e., Proposition 6), nevertheless, deliver a new insight to our understanding of the monetary compensation strategy by demonstrating that this strategy may backfire and lead to profit losses for the retailers.

We also remark that offering monetary compensation may cause a free-rider issue. Specifically, customers who are not interested in purchasing the product may still visit the retailer with the hope of being compensated, as long as the travel cost is not too high. These customers are referred to as free-riders. The free-riding behavior creates a moral hazard issue, so that retailers can hardly recognize their true customers. Fortunately, many marketing strategies and new technology tools can be used to alleviate, or even eliminate, such free-riding issue. For example, retailers may ask customers to claim their desired product in order to be eligible for compensation upon stock-out. If the claimed product is out of stock and no substitute can match the customer's need, then a monetary compensation is offered. Otherwise, the customers cannot receive the monetary compensation. Another mechanism the retailers can use is to solicit more information from customers through cheap talk. Once the retailer verifies a customer's true motivation for purchasing the product, a monetary compensation can be awarded. Therefore, throughout our analysis, we assume that the free-riding behavior is negligible. This is consistent with the business practice in various industries where retailers effectively compensates customers stock-outs to induce a higher demand (see, e.g., Bhargava et al. 2006, Su and Zhang 2009).

To summarize, although inventory commitment and monetary compensation have been widely recognized to benefit a monopolist retailer facing customers with inventory availability concerns, our results demonstrate that these strategies may hurt retailers in a competitive market if the competition is intensive. In addition to the well-known mechanism of inventory commitment and monetary compensation to increase customers' willingness-to-pay and, thus, attract more demand, these strategies may backfire by intensifying market competition. Our results deliver an important actionable insight that retailers should carefully evaluate their position in a competitive market before adopting these strategies to attract customers with availability concerns. Haphazardly employing monetary compensation or inventory commitment strategies may result in undesirable outcomes with lower profits for retailers under competition.

6. Social Welfare

In this section, we study two important questions regarding the social welfare of a competitive market. First, how will market competition impact the social welfare? Second, what are the social welfare implications of inventory commitment and monetary compensation under competition? A well-acknowledged insight in the economics literature is that competition will benefit social welfare (i.e., social welfare will increase when firms compete; see e.g., Stiglitz 1981). Our results in this section, however, deliver an insight in stark contrast with this common wisdom: When customers are concerned about inventory availability, competition may hurt the social welfare. It has been shown in the OM literature that inventory commitment and monetary compensation strategies improve social welfare in a monopoly market (e.g., Su and Zhang 2009). However, we demonstrate that these strategies induce the retailers to compete more aggressively on inventory availability, which turn out to further decrease the social welfare under market competition.

We begin our analysis by quantifying the average customer surplus and social welfare in different models, starting with the base model. Recall that, in the base model, the equilibrium outcome can be either the whole market coverage or the partial market coverage. We will only consider the whole market coverage case (i.e., $s \leq \bar{s}$) in this section, since the retailers do not compete with each other in the partial market coverage case. Under equilibrium, we evaluate the *average* customer surplus and social welfare in the base model as $CS^* = (v - p^*)\theta(p^*) - \frac{s}{4}$ and $SW^* = \mu\left(v\theta^*(p) - \frac{s}{4}\right)$, respectively, where $\theta(p^*) = \frac{1}{\mu}\int_0^{F^{-1}\left(\frac{p^*-c}{p^*}\right)} yf(y)dy$ and p^* is the equilibrium price characterized by Proposition 1. Note that the equilibrium price p^* plays a key role in determining the average customer surplus and social welfare. It *explicitly* influences the average customer surplus, and *implicitly* influences the social welfare through product availability.

To explore the impact of inventory availability competition, we introduce a benchmark model where retailers at the two endpoints of the Hotelling line belong to a single firm and are managed in a centralized fashion. The firm optimizes price and inventory decisions of the two retailers to maximize their total profit. It is clear that each of the two retailers will cover half of the whole market in this benchmark model without competition. In the subsequent analysis, we will use subscription b to denote the benchmark model. Analogous to our base model, the average customer surplus in the benchmark model is $CS_b^* = (v - p_b^*)\theta(p_b^*) - \frac{s}{4}$ and the social welfare is $SW_b^* = \mu\left(v\theta^*(p_b) - \frac{s}{4}\right)$, where $\theta(p_b^*) = \frac{1}{\mu}\int_0^{F^{-1}\left(\frac{p_b^*-c}{p_b^*}\right)} yf(y)dy$ and $p_b^* = v - \frac{s}{2\theta(p_b^*)}$. It is worth noting that the customer surplus and social welfare share the same structure for the cases with and without competition, but with different equilibrium prices. Therefore, the key to understanding the impact of competition boils down to analyzing how it affects the equilibrium prices. The following lemma characterizes the impact of equilibrium price on customer surplus and social welfare:

Lemma 1. *The following statements hold:*

(a) The average customer surplus function, $(v-p)\theta^*(p)-\frac{s}{4}$, is concave in price p. In particular, the equilibrium price in the base model (with competition), p^*, satisfies condition $p^* \in [\hat{p}, v)$, where $\hat{p} = \arg\max_p (v-p)\theta^*(p)$. Therefore, the average customer surplus in the base model, $CS^* = (v - p^*)\theta(p^*) - \frac{s}{4}$, is decreasing in p^*.

(b) The social welfare, $\mu(v\theta^*(p) - \frac{s}{4})$, is increasing in price p.

As shown by Lemma 1(a), the expected customer surplus is concave in price. Moreover, the equilibrium price in the base model is lower bounded by $\hat{p}$, which is the price that maximizes the expected customer surplus. As a result, the customer's expected surplus is concavely decreasing in price under equilibrium. Lemma 1(b) shows that the social welfare function is increasing in price. A high price signals a high product availability, which improves social welfare as well.

The impact of competition on customer surplus and social welfare is well-studied in the economics literature. A general insight in this literature is that competition between firms will improve customer surplus. For example, Brynjolfsson et al. (2003) summarize two mechanisms that drive market competition in product variety to improve consumer surplus. Increased market competition lowers market prices and expands product lines, both of which leads to increased customer surplus. Moreover, the economics literature does not have a conclusive answer to how competition affects social welfare. Although many researchers have shown that competition may potentially reduce social welfare (e.g., Stiglitz 1981), a widespread belief is that competition between firms will increase social welfare because the benefits from customer surplus dominates the losses from firm profits. Our model incorporates the competition on both price and inventory availability. Recall that a high price can signal high product availability under equilibrium. Therefore, it is unclear apriori whether competition will drive the retailers to lower the prices to directly attract customers or to increase the prices to indirectly signal high product availability. The following proposition addresses this question and characterizes the conditions under which either effect dominates.

Proposition 7. *The following statements hold:*

(a) If $s \le \underline{s}$, we have $CS^* \ge CS_b^*$ and $SW^* \le SW_b^*$;

(b) Otherwise, $s > \underline{s}$, we have $CS^* = CS_b^*$ and and $SW^* = SW_b^*$.

Proposition 7 shows that, if the travel cost is small (i.e., $s \le \underline{s}$), market competition benefits customers but hurts the social welfare. However, if the travel cost is large (i.e., $s > \underline{s}$), the retailers are not competing on market share (see parts (c) and (d) of Proposition 1), so the models with and without competition are equivalent to each other. This is in sharp contrast to the common wisdom in the economics literature that competition will increase the social welfare (Stiglitz 1981). To understand the rationale of Proposition 7, we identify two opposing effects. The first effect is referred to as the pricing effect, under which competition drives the retailers to charge lower prices as a promotion to attract customers. The second effect is called the product availability effect, under which competition drives the retailers to signal high inventory availability by increasing the prices. Specifically, as shown in Proposition 7(a), the retailers compete on capturing more market share by offering higher customer surplus and, thus the market competition is beneficial to the customers. Since the average customer surplus is decreasing in equilibrium price (see Lemma 1(a)), the retailers compete on offering lower price in the market competition (the pricing effect dominates). In contrast, the social welfare is increasing in equilibrium price (see Lemma 1(b)). As a consequence, when retailers are competing on offering lower price to attract more market share, the product availability decreases and, thus the social welfare decreases. In other words, although market competition improves average customer surplus, the loss from retailers dominates the benefit from customers so the social welfare declines. However, when retailers are not competing on market share, as shown in Proposition 7(b), the base and benchmark models provide the same average customer surplus and social welfare under equilibrium (note that both models have the same price p_b^*).

Another question we wish to address in this paper is how inventory commitment and monetary compensation strategies impact social welfare under competition. We now explore whether these two strategies can be used to improve the *average* consumer surplus and social welfare under market competition. The equilibrium *average* consumer surplus and social welfare functions under the inventory commitment strategy are given by $CS_v^* = (v - p_v^*)\theta(p_v^*) - \frac{s}{4}$ and $SW_v^* = \mu(v\theta(p_v^*) - \frac{s}{4})$, respectively, where v represents the case of inventory commitment strategy. Similarly, the equilibrium *average* consumer surplus and social welfare functions under the monetary compensation strategy are given by $CS_c^* = (v - p_v^* - m_c^*)\theta(p_c^* + m_c^*) - \frac{s}{4} + m_c^*$ and $SW_c^* = \mu(v\theta(p_c^* + m_c^*) - \frac{s}{4})$, respectively, where c represents the case of monetary compensation strategy. Note that the compensation term m_c^* will not directly affect the social welfare as it is a cash transfer between the retailers and customers. However, the compensation m_c^* does impact the equilibrium average consumer surplus because customers facing stockout will be compensated.

Proposition 8. *Under the inventory commitment or monetary compensation strategies, we have that (a) $CS_v^* \geq CS^*$, and (b) $CS_c^* \geq CS^*$.*

Proposition 8 shows that, although inventory commitment and monetary compensation do not necessarily benefit the retailers under competition, these strategies are always beneficial to the customers. Both strategies provide incentives to attract customers to patronize the retailers and, as a consequence, benefit the customers once adopted by the retailers.

Proposition 9. *The following statements hold:*

(a) Under the inventory commitment strategy, there exists a threshold s_{vw} such that $SW_v^* < SW^*$ for $s < s_{vw}$.

(b) Under the monetary compensation strategy, there exists a threshold s_{cw} such that $SW_c^* < SW^*$ for $s < s_{cw}$.

Different from Proposition 8, Proposition 9 shows that inventory commitment and monetary compensation strategies may hurt social welfare under intensive competition. Recall from Propositions 4 and 6 that, under intensive competition, both strategies will backfire and decrease the profit and inventory availability probability of the retailers. Similar rationale applies to Proposition 9 as well. Since the inventory commitment and monetary compensation strategies provide an alternative channel in which the retailers could compete for market share, the equilibrium price and product availability may decline when market competition is intensive. As a result, the social welfare will decrease as well. Combining Propositions 4, 6, 8 and 9, we find that inventory commitment and monetary compensation strategies will always make customers better off but retailers worse off under intensive market competition, with the former dominating the latter, so the social welfare will decrease under these strategies in this case.

To summarize, our analysis in this section shows that, contrary to the widely-held belief, competition may actually decrease the social welfare when customers have inventory availability concerns. The inventory commitment and monetary compensation strategies further intensify the competition between the retailers, therefore causing an even lower social welfare.

7. Conclusion

Inventory commitment and monetary compensation have been proposed in the literature to mitigate strategic customer behavior and enhance firm profit in a monopoly setting. This paper examines these strategies in a competitive setting when retailers compete on price and inventory availability. Combining the newsvendor and Hotelling frameworks, we characterize the strategic interactions among the retailers and the customers. Depending on competition intensity, we provide market equilibrium price and inventory availability and quantify the impact of these strategies on firms' profitability, average consumer surplus, and social welfare. There are two main results from this research.

First, we find that both strategies lead to a prisoner's dilemma: Although a retailer would benefit from either strategy regardless of the competitor's price and inventory decisions, both inventory commitment and monetary compensation will intensify market competition and hurt the retailers in a competitive market. This is in stark contrast to the common wisdom that these strategies improve the retailer profit under monopoly. Specifically, the inventory commitment strategy may dilute the signaling power of price, thus leading to over-stock of inventory for the competing retailers, while the monetary compensation strategy tends to overcompensate customers. Therefore, both strategies will intensify the market competition, thus, reducing the profit of both retailers on the market.

Second, we find that with customers' product availability concerns, competition decreases equilibrium retail prices, which decreases product availability and the social welfare. This contrasts the widely held belief that competition normally improves the social welfare. Furthermore, inventory commitment and monetary compensation may further intensify the competition between the retailers and, as a consequence, decrease the product availability and hurt the social welfare.

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Appendices to "Inventory Commitment and Monetary Compensation under Competition"

Appendix A: Summary of Notations

Table 1 Summary of Notations

R_i :	Retailer i ($i=1,2$)
p_i :	Price of Retailer i
q_i :	Stocking quantity of Retailer i
α_i :	Market share of Retailer i
Π_i :	Profit of Retailer i
D :	Market aggregate demand
s :	Unit travel cost
c :	Procurement cost
v :	Product valuation
$F(\cdot)$:	Distribution function of demand ($\bar{F}(x) := 1-F(x)$)
$f(\cdot)$:	Density function demand distribution
x :	Consumer's location on the Hotelling line, $x \in \mathcal{M}$ and $\mathcal{M}=[0,1]$
$\mathbb{E}[\cdot]$:	Taking expectation
$x \wedge y$:	Taking the minimum
θ_i :	Customers' (rational) expectation of R_i's inventory availability probability ($i=1,2$)

Appendix B: Deterministic Hotelling Model Benchmark

In this section, we introduce the classic Hotelling competition model with deterministic demand as the benchmark. The comparison between our focal model and the deterministic benchmark could help us crystallize the impact of demand uncertainty and customers' availability concern.

We consider the same Hotelling line setup as the base model presented in Section 3 but with deterministic total market size. Specifically, we assume the aggregate market demand D is deterministic and known to everyone in the market. Without loss of generality, we normalize $D=1$. In the absence of demand uncertainty, the retailers will order exactly the amount of their respective market share, so every customer will be able to obtain her requested product. The two retailers R_1 and R_2 determine their respective prices p_1 and p_2 to maximize their own profits, whereas each retailer choose whether and where to visit. As in the base model, we focus on the symmetric equilibrium $(\tilde{p}_d^*, \tilde{q}_d^*)$, where $\tilde{p}_d^*$ is the equilibrium price and $\tilde{q}_d^*$ is the equilibrium order quantity of each retailer. Here, we use "$\tilde{\ }$" to represent the deterministic benchmark model. It is then straightforward to observe that if s is small, R_1 and R_2 can serve the entire market, each covering 50% of the customers. If, otherwise, s is large, there is essentially no competition between the two retailers and the market is not completed covered. Formally, we characterize the equilibrium prices of the deterministic benchmark in the following lemma.

Lemma 2. *Assume that $D=1$ with certainty. The following statements hold:*

(a) If $s < \frac{2(v-c)}{3}$, $\tilde{p}_d^* = s+c$ and $q_d^* = \frac{1}{2}$. Hence, R_1 and R_2 each cover 50% of the market.

(b) If $\frac{2(v-c)}{3} \le s \le v-c$, $p_d^* = v - \frac{1}{2}s$ and $q_d^* = \frac{1}{2}$. Hence, R_1 and R_2 each cover 50% of the market.

(c) If $s > v-c$, $\tilde{p}_d^* = \frac{1}{2}(v+c)$ and $q_d^* = \frac{1}{2s}(v-c) < \frac{1}{2}$. Hence, R_1 and R_2 each cover less than 50% of the market.

As shown in Lemma 2, when the competition is intensive ($s \leq \frac{2(v-c)}{3}$), the equilibrium price is increasing in s. If the competition is moderate ($\frac{2(v-c)}{3} \leq s \leq v-c$), the equilibrium price is decreasing in s. Finally, if the competition is mild ($s > v-c$), the equilibrium price is independent of s.

Proof of Lemma 2. In a deterministic Hotelling model, two retailers compete on market share by charging prices. Demand is determined and is an open knowledge to all players in the market, so there is no issue of product availability. Without loss of generality, we shall use retailer R_1 as an example in the analysis.

Case I. Full Market Coverage with competition. When the travel cost s is small, the two retailers cover the entire market. Given price p_1 and p_2, consumers located at $x \in [0,1]$ visit the retailer R_1 if $v - p_1 - sx \geq v - p_2 - s(1-x) > 0$. Thus, the retailer R_1 earns market share $\frac{-p_1+p_2+s}{2s}$, and accordingly profit $\pi_1(p_1) = (p_1 - c)\frac{-p_1+p_2+s}{2s}$. Taking first derivative of the profit function yields $p_1^* = \frac{p_2+s+c}{2}$. Since the two retailers are symmetric, retailer R_2 asks the same optimal price $p_2^* = \frac{p_1+s+c}{2}$ to maximize his own profit. In equilibrium, the two retailers have the same optimal solutions: $p_d^* = s + c$, $q_d^* = \frac{1}{2}$, and each covers half market share. Finally, to guarantee $v - p_d^* - \frac{s}{2} > 0$, we obtain $s < \frac{2(v-c)}{3}$.

Case II. Partial Market Coverage without competition. When the travel cost s is large, each retailer covers less than half market share. Given price p_1, consumers visit the retailer $R-1$ if $v - p_1 - sx > 0$. In other words, the retailer earns market share $\frac{v-p_1}{s}$ and profit $\pi_1(p_1) = (p_1 - c)\frac{v-p_1}{s}$. Taking first derivative of the profit function yields $p_1^* = \frac{v+c}{2}$. Similarly, by symmetry of the game, we obtain $p_2^* = \frac{v+c}{2}$. Thus, we have $p_d^* = \frac{v+c}{2}$, $q_d^* = \frac{v-c}{2s}$, and each retailer covers market share $\frac{v-c}{2s}$. Finally, to guarantee that equilibrium market share is less than half, we obtain $s > v - c$.

Case III. Full Market Coverage without competition. To make our analysis complete, we still need to analyze the case when $v - c \leq s \leq \frac{2(v-c)}{3}$. In this case, each retailer charges a price to cover just half market share (consumers in the middle of the Hotelling line earn zero surplus from either retailer, which ensures zero market competition), so we have $v - p_1 - \frac{s}{2} = 0$. Thus, we obtain $p_d^* = v - \frac{s}{2}$ and $q_d^* = \frac{1}{2}$. □

Appendix C: Proof of Statements

Proof of Proposition 1. Given the equilibrium retailer decisions (p^*, q^*), a customer located at x has an expected payoff of $(v - p^*)\theta^*(p^*) - sx$, where $x \in [0,1]$. Note that, if the unit travel cost s is small, retailers compete on both price and inventory availability and the market $\mathcal{M}$ is fully covered under equilibrium. If the unit travel cost s is large, $\mathcal{M}$ is not fully covered in equilibrium and, thus, the retailers do not directly compete with each other. In this case, the equilibrium outcome satisfies $(v - p^*)\theta^*(p^*) - s\alpha^* = 0$, where α^* is the equilibrium market share of a retailer. Hence, the expected payoff of the customers located at $x = \alpha^*$ and $x = 1 - \alpha^*$ should be 0. Finally, when the unit travel cost s is in a medium range, $\mathcal{M}$ is fully covered but the two retailers do not compete with each other. In this case, each retailer covers half of the market share under equilibrium. Thus, we have that $(v - p^*)\theta^*(p^*) - \frac{1}{2}s = 0$. For the rest of our proof, we use R_1 as the focal retailer.

Case I. Full Market Coverage with competition. Let p_i be the price charged by retailer R_i and α_i be the market share of R_i. Since the two retailers cover the entire market, a customer at the intersection of their respective market segments should be indifferent between visiting either retailer, i.e., $(v - p_1)\theta^*(p_1) - s\alpha_1 = (v - p_2)\theta^*(p_2) - s(1 - \alpha_1) \geq 0$. Under equilibrium, R_2 charges equilibrium price p^*, and we next analyze

R_1's best response given R_2's price p^*, which is denoted as $p_1(p^*)$. We write R_1's profit as $\Pi(p_1,p^*) := p_1\mathbb{E}(\alpha_1(p_1,p^*)D\wedge q_{1o}) - cq_{1o}$, where $q_{1o} = \alpha_1(p_1,p^*)F^{-1}(\frac{p_1-c}{p_1})$, and its market share $\alpha_1(p_1,p^*)$ satisfies the following equilibrium condition (the expected payoff to visit R_1 is the same as that to visit R_2): $(v - p_1)\theta^*(p_1) - s\alpha_1(p_1,p^*) = (v-p^*)\theta^*(p^*) - s(1-\alpha_1(p_1,p^*))$. For simplicity, we rewrite the equilibrium condition as $U(p_1) - s\alpha_1(p_1,p^*) = U(p^*) - s(1-\alpha_1(p_1,p^*))$, where $U(p) = (v-p)\theta^*(p)$. According to Lemma 1, we know that $U(p)$ is a decreasing concave function when $p\in[\hat{p},v)$. Under equilibrium, by symmetry, the market share satisfies the condition $\alpha_1^* = \frac{1}{2}$ and the price satisfies $p_1(p^*) = p^*$, i.e.,

$$p^* = p_1(p^*) := \underset{0\le p\le v}{\arg\max}\left(\frac{1}{2} + \frac{(v-p)\theta^*(p) - (v-p^*)\theta^*(p^*)}{2s}\right)\left\{p\mathbb{E}\left[D\wedge \bar{F}^{-1}\left(\frac{c}{p}\right)\right] - c\bar{F}\left(\frac{c}{p}\right)\right\}$$

To find R_1's best response $p_1(p^*)$, we take derivative of the profit function $\Pi(p_1,p^*)$ with respect to price p_1, which yields

$$\frac{\partial\Pi(p_1,p^*)}{\partial p_1} = \frac{1}{2s}U'(p_1)\pi(p_1) + \left(\frac{1}{2} + \frac{U(p_1)-U(p^*)}{2s}\right)\pi'(p_1),$$

where $\pi(p_1) = p_1\mathbb{E}\left(D\wedge F^{-1}\left(\frac{p_1-c}{p_1}\right)\right) - cF^{-1}\left(\frac{p_1-c}{p_1}\right)$.

According to Lemma 1, we known that $U(p)$ is a decreasing and concave function in p for $p\in[\hat{p},v)$. Moreover, we know that $U'(p) = 0$ at $p = \hat{p}$; $U'(p) < 0$ and $U(p) = 0$ at $p = v$. Since $\pi(p)$ is an increasing function, we have $\Pi'(p) > 0$ at $p = \hat{p}$ and $\Pi'(p) < 0$ at $p = v$. Hence, the first-order condition, $\Pi'(p) = 0$, results in a unique optimal price $p^*\in[\hat{p},v)$ and we have $U'(p^*)\pi(p^*) + s\pi'(p^*) = 0$ in equilibrium.

Next, we prove the existence and uniqueness of the equilibrium. The implicit function theorem and the envelope theorem together yield that $\frac{d^2p_1(p^*)}{d(p^*)^2} = \{-\frac{\partial}{\partial p^*}\frac{\partial^2\Pi(p_1)}{\partial p_1^2}\cdot\frac{\partial^2\Pi(p_1)}{\partial p_1\partial p^*} + \frac{\partial^2\Pi(p_1)}{\partial p_1^2}\cdot\frac{\partial}{\partial p^*}\frac{\partial^2\Pi(p_1)}{\partial p_1\partial p^*}\}/(\frac{\partial^2\Pi(p_1)}{\partial p_1\partial p^*})^2$. Thus, it can be easily verified that $\frac{\mathrm{d}p_1(p^*)}{\mathrm{d}p^*} > 0$ and $\frac{\mathrm{d}^2p_1(p^*)}{\mathrm{d}(p^*)^2} < 0$, i.e., $p_1(p^*)$ is concavely increasing in p^*. In addition, observe that $\lim_{p^*\to v}p_1(p^*) < v$ and $\lim_{p^*\to\hat{p}}p_1(p^*) \ge \hat{p}$. Thus, the function $p_1(p) - p$ has a unique root on $[\hat{p},v)$, which implies that the best-response function $p_1(\cdot)$ has a unique fixed point on the interval $[\hat{p},v)$. This proves the existence and uniqueness of the equilibrium. By the symmetry of the equilibrium outcome, we have $\alpha^* = \frac{1}{2}$ and $q^* = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p^*}\right)$ under equilibrium. This finishes the proof of part (b).

Case II. Partial Market Coverage. In this case, the two retailers have no direct competition and the market is partially covered. It is straightforward to observe that the equilibrium outcome of each retailer is identical to that of a monopoly retailer. This completes the proof of part (d).

Case III. Full Market Coverage without Competition. In this case, the two retailers have no direct competition, but each covers half of the market share. Hence, we have $U(p^*) = \frac{1}{2}s$, i,e, $p^* = v - \frac{s}{2\theta^*(p^*)}$. Since the equilibrium market share for each retailer is $\frac{1}{2}$, the equilibrium order quantity is given by $q^* = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p^*}\right)$. This completes the proof of part (c).

Finally, to complete the proof, we show the existence of the two critical thresholds $\underline{s}$ and $\bar{s}$. Define the threshold $\underline{s}$ as the unit travel cost satisfying $U(p^*) = \frac{1}{2}\underline{s}$, where p^* is the equilibrium price characterized in Case I. Thus, the threshold $\underline{s}$ represents the situation under which the two retailers are barely competing with each other. Similarly, we define the threshold $\bar{s}$ as the unit travel cost satisfying $U(p^*) = \frac{1}{2}\bar{s}$, where p^* is the equilibrium price characterized in Case II, i.e., the threshold $\bar{s}$ represents the situation under which the two retailers barely cover the entire market. Since the function $U(p)$ is concavely decreasing in p for $p\in[\hat{p},v)$, we have that $\underline{s} < \bar{s}$. This completes the proof of Proposition 1. $\square$

Proof of Proposition 2. Similar to the previous proof, we set R_1 as the focal retailer. As in the proof of Proposition 1, we consider three cases: (I) s is small so that the two retailers cover the entire market and compete, (II) s is moderate so that the two retailers cover the entire market without competition, and (III) s is big so that the two retailers partially cover the market. We start our analysis with case I, the case of full market coverage, i.e., s is small.

Assume that retailer R_2 charges the equilibrium price p^* and stocks the equilibrium inventory quantity q^*. Retailer R_1 maximizes its profit $\Pi(p,q) := p\mathbb{E}(\alpha(p,q)D \wedge q) - cq$. Note that R_1's market share $\alpha(p,q)$ satisfies the equilibrium condition $(v-p)\theta(\frac{q}{\alpha(p,q)}) - s\alpha(p,q) = (v-p^*)\theta(\frac{q^*}{1-\alpha(p,q)}) - s(1-\alpha(p,q)) \geq 0$, where $\theta(\frac{q}{\alpha}) = \frac{1}{\mu}\int_{y=0}^{\frac{q}{\alpha}} yf(y)\mathrm{d}y$. Taking derivative for both sides regards to quantity, we obtain that the market share $\alpha(p,q)$ is concavely increasing in the stocking quantity q. Then, we take the derivative of the profit function with respect to q, the first-order condition then implies that

$$q^*(p) = \alpha(p,q)F^{-1}\left(\frac{p-c}{p} + \frac{d\alpha(p,q)}{dq}\int_0^{\frac{q}{\alpha(p,q)}} xdF(x)\right).$$

Under equilibrium, we have $p = p^*$, $q = q^*(p^*) = \frac{1}{2}F^{-1}\left(\frac{p^*-c}{p^*} + \frac{d\alpha(p^*,q^*)}{dq^*}\int_0^{2q^*} xdF(x)\right)$, where $\frac{d\alpha(p,q)}{dq} > 0$.

Comparing the equilibrium order quantity in the base model, $q_o(p) = \frac{1}{2}F^{-1}(\frac{p-c}{p})$, and the equilibrium order quantity in the model with inventory commitment, $q^*(p) = \frac{1}{2}F^{-1}\{\frac{p-c}{p} + \frac{d\alpha(p,q)}{dq}\int_0^{2q} xdF(x)\} := g(p)$, we find that $g(p)$ shares the same functional properties as $F^{-1}(\frac{p-c}{p})$, which is concavely increasing in p. Moreover, given the same price as in the base model, the retailer in the inventory commitment model has a tendency to increase inventory stock.

Next, we examine how R_1 would determine the price given the optimal quantity decision $q^*(p)$. Note that the expected profit of R_1 is

$$p_1\mathbb{E}[(\alpha_1(p_1)D) \wedge q^*(p_1)] - cq^*(p_1),$$

where

$$q^*(p) = \alpha(p)F^{-1}\left(\frac{p-c}{p} + \frac{d\alpha(q)}{dq}\int_0^{\frac{q}{\alpha(q)}} ydF(y)\right)$$

and

$$\alpha(p) = \frac{1}{2} + \frac{1}{2s}\left\{(v-p)\int_0^{\frac{q^*(p)}{\alpha(p)}} ydF(y) - (v-p^*)\int_0^{\frac{q^*}{1-\alpha(p)}} ydF(y)\right\}.$$

Following the same argument in the proof of Proposition 1, we know the best-response function for price $p(p^*)$ is concavely increasing in R_2's price decision p^*. As in the proof of Proposition 1, this implies that a unique price equilibrium exists and we denote the equilibrium price as p_v^*. Putting everything together, we have that a unique symmetric equilibrium (p_v^*, q_v^*) exists for the case where the retailers cover the entire market.

Next, we consider case III: partial market coverage. The case II will be discussed later. In this case, $(v-p)\theta(p,q) - s\alpha(p,q) = 0$, where $\theta(p,q) = \frac{1}{\mu}\int_0^{\frac{q}{\alpha(p,q)}} yf(y)dy$. In other words, we have $p = v - \frac{s\alpha(p,q)}{\theta(p,q)}$. Hence, the optimization problem of R_1 is given by

$$\max_{(p,q)}\{p\mathbb{E}(\alpha(p,q)D \wedge q) - cq\},$$

subject to the constraint $p = v - \frac{s\alpha(p,q)}{\theta(p,q)}$. We use (p_v^{**}, q_v^{**}) to denote the equilibrium outcome with partial market coverage.

For case II, the two retailers have no competition but cover the entire market. Thus, we have $p = v - \frac{s}{2\theta(p,q)}$ by full market coverage. Each retailer maximizes profit

$$\max_{(p,q)} \left\{ p\mathbb{E}\left(\frac{1}{2}D \wedge q\right) - cq \right\},$$

subject to the constraint $p = v - \frac{s}{2\theta(p,q)}$. We use (p_v^{***}, q_v^{***}) to denote the equilibrium outcome of this case.

To conclude our proof, we still need to show that there exists two threshold $\underline{s}_v$ and $\bar{s}_v$, such that an equilibrium with partial market coverage exists if $s > \bar{s}_v$ and an competitive equilibrium with full market coverage exists if $s \le \underline{s}_v$. Define $\bar{s}_v$ as $(v - p_v^{**})\theta(p_v^{**}, q^*(p_v^{**})) = \frac{1}{2}\bar{s}_v$ and $(v - p_v^*)\theta(p_v^*, q^*(p_v^*)) = \frac{1}{2}\underline{s}_v$. Similar to the results in the base model, we have $p_v^* > p_v^{**}$ in equilibrium, which implies $\underline{s}_v \le \bar{s}_v$. Therefore, if $s \le \underline{s}_v$, each retailer covers half market and competes; if $\underline{s}_v < s \le \bar{s}_v$, each retailer covers half market but has no competition; otherwise, if $s > \bar{s}_v$, each retailer covers partial market and behaves as a local monopolist. This completes the proof of Proposition 2. □

Proof of Proposition 3. To compare the profit of R_1 under different strategies, we first calculate its profit in different circumstances.

We first examine the case where R_2 does not reveal its inventory information. In this case, a customer at the purchasing threshold forms the belief $\theta_2 = \theta^*(p_2) = \frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_2})} yf(y)\mathrm{d}y$ the inventory availability probability of R_2. A customer will visit R_1 if and only if her utility of visiting R_1 dominates that of visiting R_2 and that of visiting no one (i.e., 0). Therefore, if R_1 also does not reveal its inventory information to the market, its market share α_1 is given by

$$\alpha_1 = \min\left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1) - (v-p_2)\theta^*(p_2)}{2s}. \right\} \tag{3}$$

Thus, the maximum profit of R_1 if he does not adopt the inventory commitment strategy is

$$\Pi_{d,d} := \max_{0 \le p_1 \le v} \left\{ \min\left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1) - (v-p_2)\theta^*(p_2)}{2s} \right\} \cdot \left\{ p_1\mathbb{E}\left[D \wedge \bar{F}^{-1}\left(\frac{c}{p_1}\right)\right] - c\bar{F}^{-1}\left(\frac{c}{p_1}\right)\right]\right\}$$

Similarly, if R_1 adopts the inventory commitment strategy, its market share α_1 satisfies the following equation:

$$\alpha_1 = \min\left\{ \frac{v-p_1}{s\mu}\int_{y=0}^{q_1/\alpha_1} yf(y)\mathrm{d}y, \frac{1}{2} + \frac{v-p_1}{2s\mu}\int_{y=0}^{q_1/\alpha_1} yf(y)\mathrm{d}y - \frac{(v-p_2)\theta^*(p_2)}{2s} \right\}. \tag{4}$$

Therefore, the maximum profit of R_1 if he adopts the inventory commitment strategy is

$$\Pi_{v,d} := \max_{0 \le p_1 \le v, q_1 \ge 0} \left\{ p_1\mathbb{E}[\alpha_1 D \wedge q_1] - cq_1 \right\},$$

where α_1 satisfies equation (4).

We now turn our attention to the case where R_2 adopts the inventory commitment strategy. If R_1 does not reveal its inventory information to the market, customers at the purchasing threshold for R_1 form belief $\theta_1 = \theta^*(p_1) = \frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_1})} yf(y)\mathrm{d}y$ about the inventory availability probability of the retailer. Therefore, the market share α_1 is the solution to the following equation

$$\alpha_1 = \min\left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1)}{2s} - \frac{v-p_2}{2s\mu}\int_{y=0}^{q_2/(1-\alpha_1)} yf(y)\mathrm{d}y \right\}. \tag{5}$$

The maximum profit of R_1 if he does not reveal its inventory information is

$$\Pi_{d,v} := \max_{0\le p_1\le v, q_1\ge 0} \{p_1\mathbb{E}[\alpha_1 D \wedge q_1] - cq_1\},$$

where α_1 satisfies equation (5).

If R_1 adopts the inventory commitment strategy, its market share α_1 satisfies the following equation:

$$\alpha_1 = \min\left\{\frac{v-p_1}{s\mu}\int_{y=0}^{q_1/\alpha_1} yf(y)\mathrm{d}y, \frac{1}{2} + \frac{v-p_1}{2s\mu}\int_{y=0}^{q_1/\alpha_1} yf(y)\mathrm{d}y - \frac{v-p_2}{2s\mu}\int_{y=0}^{q_2/(1-\alpha_1)} yf(y)\mathrm{d}y\right\}. \tag{6}$$

The maximum profit of R_1 if he adopts the inventory commitment strategy is

$$\Pi_{v,v} := \max_{0\le p_1\le v, q_1\ge 0} \{p_1\mathbb{E}[\alpha_1 D \wedge q_1] - cq_1\},$$

where α_1 satisfies equation (6).

By comparing the profit function of R_1 under different strategy profiles, it is straightforward to observe that the equilibrium market share of R_1 is larger if he commits to an inventory order quantity, regardless of whether R_2 reveals his inventory order. Hence, the profit of R_1 will be higher under the inventory commitment strategy if the retailer commits to ordering an inventory level that leads to the same in-stock probability. Therefore, regardless of the price and inventory order quantity decisions for R_2 and regardless of whether R_2 adopts the inventory commitment strategy, the profit of R_1 is higher if he adopts the inventory commitment strategy, i.e., $\Pi_{v,d} > \Pi_{d,d}$ and $\Pi_{v,v} > \Pi_{d,v}$. This completes the proof of Proposition 3. □

Proof of Proposition 4. First, we prove $\Pi_v^* \ge \Pi^*$ when the market has no competition (i.e., s is sufficiently large). In the monopoly model, a retailer's profit function is

$$\Pi(p) = p\mathbb{E}(\alpha(p)D \wedge q_o(p)) - cq_o(p),$$

where $q_o(p) = \alpha(p)\bar{F}^{-1}(\frac{c}{p})$, $\alpha(p) = \frac{v-p}{s}\theta^*(p)$ and $\theta^*(p) = \frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}(\frac{c}{p})} yf(y)\mathrm{d}y$. In the model with inventory commitment, a retailer's profit function is

$$\Pi_v(p,q) = p\mathbb{E}(\alpha(p,q)D \wedge q) - cq,$$

where $\alpha(p,q) = \frac{v-p}{s}\theta(p,q)$ and $\theta(p,q) = \frac{1}{\mu}\int_{y=0}^{\frac{q}{\alpha(p,q)}} yf(y)\mathrm{d}y$. It is clear from the formulation of the profit functions that, the two models have the same profit functions but the model without inventory commitment has an additional constraint $q = \alpha(p)\bar{F}^{-1}(\frac{c}{p})$. Hence, $\Pi_v^* = \max_{(p,q)} \Pi_v(p,q) \ge \max_v \Pi(p, q(p)) = \max \Pi(p) = \Pi^*$. Therefore, if s is large such that the market is partially covered by the two retailers, we have $\Pi_v^* \ge \Pi^*$.

Now we turn to the case of full market coverage. To begin with, we analyze the equilibrium pricing policies of both models, starting with the base model. First observe that as $s = 0$, an infinitely small increase of expected payoff could attract all customers to visit. As a result, under equilibrium, both retailers will set the price at the customer-surplus maximizing one: $\hat{p}$. Hence, the retailer's profit in the base model is

$$\Pi^* = p^*\mathbb{E}\left(\frac{1}{2}D \wedge q(p^*)\right) - cq(p^*),$$

where $q(p^*) = \frac{1}{2}\bar{F}(\frac{p^*-c}{p^*})$ and $p^* = \hat{p}$. It is clear that $\Pi^* > 0$.

We now consider the model with inventory commitment. When $s=0$, by the first-order condition with respect to q, we have that the equilibrium order quantity q_v^* satisfies the equation $q_v^* = \frac{1}{2}F\left(\frac{p_v^*-c}{p_v^*}+\frac{\int_0^{2q_v^*}xf(x)dx}{4q_v^*}\right)$. Also note that if $c\to 0$, we have $\frac{\partial\alpha_v^*}{\partial p_v^*}=\frac{1}{2\mu}\frac{\theta(p_v^*,q_v^*)}{s+2(v-p_v^*)(2q_v^*)^2f(2q_v^*)}$ in the model of inventory commitment, which is smaller than $\frac{\partial\alpha^*}{\partial p_1}|_{p_1=p_v^*}$ in the base model. Thus, we have that $p_v^*<p^*$. Therefore,

$$\begin{aligned}\Pi^* &= p^*\mathbb{E}\left(\frac{1}{2}D\wedge q^*\right)-cq^* = \frac{1}{2}\left\{\hat{p}\mathbb{E}\left(D\wedge\bar{F}^{-1}(\frac{c}{\hat{p}})\right)-c\bar{F}^{-1}(\frac{c}{\hat{p}})\right\}\\ &> \frac{1}{2}\left\{p_v^*\mathbb{E}\left(D\wedge\bar{F}^{-1}(\frac{c}{p_v^*})\right)-c\bar{F}^{-1}(\frac{c}{p_v^*})\right\}\geq p_v^*\mathbb{E}\left(\frac{1}{2}D\wedge q_v^*\right)-cq_v^*=\Pi_v^*,\end{aligned}$$

where the first inequality follows from $p^*=\hat{p}>p_v^*$, and the second follows from $q_v^*\neq\bar{F}^{-1}(\frac{c}{p_v^*})$ (where the quantity $q=\bar{F}^{-1}(\frac{c}{p_v^*})$ is the optimal quantity that maximizes the profit function given the price p_v^*). Therefore, the inventory commitment strategy results in a lower profit if $s=0$ and $c\to 0$.

Finally, the two equilibrium profits, Π^* and Π_v^*, are both continuous in s and c. Moreover, we have just shown that $\lim\limits_{s,c\to 0}\Pi^*>\lim\limits_{s,c\to 0}\Pi_v^*$. Thus, there exist two thresholds $\bar{s}_{vd}$ (for s) and $\bar{c}_{vd}$ (for c), such that if $s<\bar{s}_{vd}$ and $c<\bar{c}_{vd}$, $\Pi_v^*<\Pi^*$; otherwise $\Pi_v^*\geq\Pi^*$. This concludes the proof of Proposition 4. □

Proof of Proposition 5. We continue to use retailer R_1 as the focal retailer. Analogous to the proof of Proposition 1, we need to consider three cases: (I) when the unit travel cost s is small, the two retailers cover the entire market and compete to each other under equilibrium; (II) when the unit travel cost s is medium, the two retailers cover the entire market, but have no market competition; and (III) when the unit travel cost s is large, the market is not fully covered under equilibrium and, thus, the retailers have no direct competition. In case (III), the problem can be decomposed into two separate monopoly problems.

We first focus on the case (I): complete market coverage. Given equilibrium price p^* and equilibrium compensation m^*, the expected profit of retailer R_1 is

$$(p_1+m_1)\mathbb{E}(\alpha_1D\wedge q_o)-cq_o-m_1\mathbb{E}(\alpha_1D),$$

where $q_o=\alpha_1F^{-1}(\frac{p_1+m_1-c}{p_1+m_1})$. Since the two retailers cover the entire market, we have the equilibrium condition $U(p_1+m_1)-s\alpha_1+m_1=U(p^*+m^*)-s(1-\alpha_1)+m^*$, where $U(p+m)=(v-p-m)\theta^*(p+m)$ and $\theta^*(p+m)=\frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p+m}\right)}yf(y)\mathrm{d}y$. In other words, the consumers at the intersection of their respective market coverages are indifferent between visiting either retailer. For ease of exposition, we define $t_1=p_1+m_1$, which refers to the effective marginal revenue of the product. Hence, the equilibrium condition on the customer's choice behavior at the intersection of the retailers' respective market coverage (i.e., $x=\alpha_1$) can be rewritten as

$$U(t_1)-s\alpha_1+m_1=U(t^*)-s(1-\alpha_1)+m^*.$$

Therefore, the profit function of R_1 can be rewritten as

$$t_1\mathbb{E}(\alpha_1D\wedge q_o)-cq_o-(A^*+2s\alpha_1-U(t_1))\,\mathbb{E}(\alpha_1D),$$

where $A^*=U(t^*)+m^*-s$. Following the same argument in the proof of Proposition 1, it can be shown that the expected profit of R_1 is concave in $t_1\in[\hat{p},v)$, which further implies that there exists a unique best response for R_1, $t_1(t^*)$. Given $t_1(t^*)$, we have best compensation response $m_1(m^*) = A^* + 2s\alpha_1 - (v - t_1(t^*))\theta^*(t_1(t^*))$ and best price response $p_1(p^*) = t_1(t^*) - m_1(m^*)$. Therefore, a unique equilibrium (p_c^*, m_c^*) exists. Exchanging the roles of R_1 and R_2, we have that, given R_1's decisions (p^*, m^*), the expected profit of R_2 is concave in t_2. The concavity of the expected profit function also implies that an equilibrium exists. This concludes the proof of the case with full market coverage.

We then consider the case (III): partial market coverage, i.e., $\alpha_1 + \alpha_2 < 1$. In this case, R_1 makes the price and monetary compensation decisions to maximize its profit $\Pi(p_1, m_1) = (t_1)\mathbb{E}(\alpha_1 D \wedge q_{1o}) - cq_{1o} - m_1\mathbb{E}(\alpha_1 D)$, where $q_{1o} = \alpha_1 F^{-1}(\frac{t_1 - c}{t_1})$, and $\alpha_1 = \frac{U(t_1)+m_1}{s}$. Thus, the expected profit can be rewritten as

$$t_1\mathbb{E}(\alpha_1 D \wedge q_o) - cq_0 - \{s\alpha_1 - U(t_1)\}\,\mathbb{E}(\alpha_1 D).$$

Following the same argument in the proof of Proposition 1, we know this expected profit function is concave in t_1. Hence, the equilibrium (p_c^{**}, m_c^{**}) exists if s is sufficiently large.

Next, we turn to case (II): when the entire market is fully covered, but without competition. Obviously, each retailer in this case charges a price such that half market share is just covered, so we have constraint $\frac{U(t_1)+m_1}{s} = \frac{1}{2}$. Given this constraint, the retailer balances t_1 and m_1 to maximize profit $t_1\mathbb{E}(\frac{1}{2}D \wedge q_o) - cq_o - \frac{1}{2}m_1\mathbb{E}(D)$, where $q_o = \frac{1}{2}F^{-1}(\frac{t_1 - c}{t_1})$. Similar as analysis above, the profit function is concave in $t_1 \in [\hat{p}, v)$, so we have a unique solution t^*. By the constraint function, we have $m^* = \frac{1}{2}s - U(t^*)$. Therefore, we obtain a unique equilibrium solution (p_c^{***}, m_c^{***}).

Finally, we find thresholds $\underline{s}_c$ and $\bar{s}_c$ to separate the above three cases. When $s = \underline{s}_c$, the two retailers just cover the whole market with competition. When $s = \bar{s}_c$, the two retailers are monopolists and just cover the whole market. Similar as results in the Proposition 1, we have $\underline{s}_c \le \bar{s}_c$. Therefore, the equilibrium outcome will be (p_c^*, m_c^*) when $s \le \underline{s}_c$; be (p_c^{**}, m_c^{**}) when $s > \bar{s}_c$; and be (p_c^{***}, m_c^{***}) when $\underline{s}_c < s \le \bar{s}_c$. This concludes the proof of Proposition 5. □

Proof of Proposition 6. We start our proof by considering two extreme cases.

Case I. Zero searching cost (i.e., $s = 0$). In this case, the two retailers compete on offering higher consumer expected payoff, because $\alpha'(p, m) \to -\infty$. In the model of monetary compensation, the expected payoff function is $U(p + m) + m - sx$ for consumers located at $x \in [0, 1]$. The first term, $U(p + m)$, is concave with its maximum value, $U(\hat{p})$, at $p + m = \hat{p}$. The second term is linearly increasing in m. In other words, given $p + m = \hat{p}$, retailers can always capture the entire market by continuously increasing compensation m. However, each retailer's profit function is strictly decreasing in compensation m, so the retailers have to stop raising compensation at zero profit. Therefore, each retailer obtains zero profit under equilibrium when $s = 0$. In contrast, each retailer charges price $p = \hat{p}$ in the base model, because $\hat{p}$ implies the highest expected payoff. Since we always have $\hat{p} > c$, each retailer must have a positive profit in the base model. Therefore, we have $\Pi^* > \Pi_c^*$ when $s = 0$.

Case II. Infinitely large search cost (i.e., $s \to \infty$). In this case, the two retailers have no direct competition (i.e., partial market coverage). In the model of monetary compensation, each retailer maximizes its profit $\alpha(p+m)\left\{(p+m)\mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p+m})) - c\bar{F}^{-1}(\frac{c}{p+m}) - m\mathbb{E}(D)\right\}$, where $\alpha(p+m) = \frac{U(p+m)}{s}$. In the base model, each retailer maximizes its profit $\alpha(p)\left\{(p)\mathbb{E}(D\wedge \bar{F}^{-1}(\frac{c}{p}))-c\bar{F}^{-1}(\frac{c}{p})\right\}$, where $\alpha(p)=\frac{U(p)}{s}$. The profit function in the model of monetary compensation restores to the profit function in the base model when $m=0$. Since m is a free variable, the base model is a special case of the monetary compensation model when $s=0$. In other words,

$$\Pi_c^* = \max_{(p,m)} \Pi_c(p,m) \geq \max_p \Pi_c(p,0) = \max_p \Pi(p) = \Pi^*.$$

Therefore, we have $\Pi_c^* \geq \Pi^*$ when $s\to\infty$.

Finally, recall that Π^* and Π_c^* are quasi-concave in s. By proofs above, we have already obtained $\Pi^* > \Pi_c^*$ when $s=0$; and $\Pi_c^* \geq \Pi^*$ when $s\to\infty$. Therefore, there exists a threshold $\bar{s}_{cd}$ such that $\Pi_c^* < \Pi^*$ if $s<\bar{s}_{cd}$; $\Pi_c^* \geq \Pi^*$ if $s \geq \bar{s}_{cd}$. This completes the proof of Proposition 6. □

Proof of Lemma 1 We first show that the customer's expected payoff function is concave in price p. Suppose a customer located at x, her expected payoff is $(v-p_i)\theta_i^*(p)-sx$ if she chooses to buy from retailer R_i. Under the PBE equilibrium, we have $\theta_i^*(p_i)=\frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i}\right)} yf(y)\mathrm{d}y$. Hence, a customer located at x has an expected payoff of $U(p_i):=(v-p_i)\frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i}\right)} yf(y)\mathrm{d}y-sx$. We have

$$U'(p_i) = -\frac{1}{\mu}\int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i}\right)} yf(y)\mathrm{d}y + \frac{c(v-p_i)\bar{F}^{-1}\left(\frac{c}{p_i}\right)}{\mu p_i^2 f\left(\frac{c}{p_i}\right)},$$

which is strictly decreasing in p_i given that D follows a distribution with increasing failure rate. By setting $U'(p)=0$, the expected payoff $U(p)$ is maximized at $p=\hat{p}$, where $\hat{p}=\arg\max_p(v-p)\theta^*(p)$.

Before proving that the expected payoff function is concave, we first show that the equilibrium price falls into the interval $[\hat{p},v)$. First, we prove that $\Pi(\hat{p})>\Pi(p')$ for any $p'<\hat{p}$. Without loss of generality, we use retailer R_1 for illustration. Suppose retailer R_1 decreases its price from $\hat{p}$ to p', then some customers will not visit the retailer, because the expected payoff of the customers is maximized at the price $p=\hat{p}$. As a result, by decreasing price from $\hat{p}$ to p', the retailer will induce a lower demand and a strictly lower profit margin. This implies that $\Pi(\hat{p})>\Pi(p')$. Thus, the retailer must charge a price higher than $p\geq\hat{p}$. Next, we show that the equilibrium price cannot exceed v. If the price is greater than or equal to the product valuation, i.e., $p\geq v$, no customer can afford the product, which further implies that the demand is zero and the retailer earns zero profit. Therefore, the retailer's optimal price must be within the range of $[\hat{p},v)$.

Note that the average customer surplus function is $(v-p)\theta^*(p)-\frac{s}{4}=U(p)+sx-\frac{s}{4}$ and $U(p)$ is concave in price p, thus $(v-p)\theta^*(p)-\frac{s}{4}$ is concave in p. Moreover, since the equilibrium price p^* in the base model satisfies condition $p^*\in[\hat{p},v)$, the term $(v-p^*)\theta^*(p^*)$ is decreasing in price p^* and, thus the average customer surplus is also decreasing in price p^*.

Finally, we show that the social welfare function, $SW^*=\mu(v\theta^*(p)-\frac{s}{4})$ is increasing in price p. Clearly, the social welfare function is increasing in product availability $\theta^*(p)$. Recall that a high price signals a high product availability $\theta^*(p)$, so the social welfare function is increasing in price p. □

Proof of Proposition 7. We first show that p^* is quasi-concave in s in the base model. More specifically, we shall prove that the equilibrium price p^* increases in s when s is small and decreases in s when s is moderate.

First, if $s \le \underline{s}$, the equilibrium price p^* satisfies the first-order condition

$$U'(p^*)\pi(p^*) + s\pi'(p^*) = 0,$$

where $U(p) = (v-p)\theta^*(p)$ and $\pi(p) = p\mathrm{E}\left[D \wedge \bar{F}^{-1}\left(\frac{c}{p}\right)\right] - c\bar{F}^{-1}\left(\frac{c}{p}\right)$. Now, we consider the equilibrium price as a function of s: $p^*(s)$. Taking the derivative of the above first order equation with respect to s, we obtain

$$\frac{\mathrm{d}p^*(s)}{ds} = -\pi'(p^*(s))/\frac{\mathrm{d}}{\mathrm{d}p}(U'(p^*(s))\pi(p^*(s)) + s\pi'(p^*(s))).$$

Since $\pi'(p) > 0$ for all p, the numerator of the right side is negative. Since the profit function is concave, we have $\frac{d(U'(p)\pi(p)+s\pi'(p))}{dp} < 0$. Hence, we obtain $\frac{\mathrm{d}p^*(s)}{\mathrm{d}s} > 0$. Therefore, the equilibrium price $p^*(s)$ is increasing in s for $s \le \underline{s}$.

If $\underline{s} < s \le \bar{s}$, we have $p^*(s) = v - \frac{s}{2\theta^*(p^*)}$. Since $\theta^*(p)$ decreases in p for the range $p \in [\hat{p}, v)$, we have that $p^*(s)$ is decreasing in s for $s \in [\underline{s}, \bar{s}]$. It is worth noting that the equilibrium price in the base model equals the price in the benchmark model, $p^* = p_b^* = v - \frac{s}{2\theta^*(p_b^*)}$.

To summarize, we have $p^* \le p_b^*$ if $s \le \underline{s}$ and $p^* > p_b^*$ if $s > \underline{s}$. Note that when $s > \bar{s}$, the two retailers are not competing in the base model, so this case is trivial (the base model restores the benchmark model).

Recall that the average consumer surplus function, $CS = (v-p)\theta(p) - \frac{s}{4}$, is decreasing in price p; the social welfare function, $SW = \mu(v\theta(p) - \frac{s}{4})$, is increasing in price p. Therefore, we have (1) if $s \le \underline{s}$, $CS^* \ge CS_b^*$ and $SW^* \le SW_b^*$; (2) otherwise, $CS^* = CS_b^*$ and $SW^* = SW_b^*$. □

Proof of Proposition 8. To begin with, we show $SC_c^* \ge SC^*$. Suppose the market follows the equilibrium path of base competition model and achieves equilibrium solutions (p^*, q^*). In this case, the monetary compensation $m^* = 0$. Now, we allow the retailers to pay compensation to consumers. Accordingly, the equilibrium compensation switches from $m^* = 0$ to $m_c^* \ge 0$. A higher compensation rate increases consumer surplus and thus helps retailers earn more market share (but decreases its marginal revenue). If $m_c^* = 0$, the retailers have no incentive to compete more in market share, so the two models result in the same consumer surplus. If $m_c^* > 0$, the two retailers have incentives to compete more in market share, so a positive compensation rate raises consumer surplus. In short, since we always have $m_c^* \ge m^* = 0$, offering non-negative monetary compensation to customers upon stock can always increases the equilibrium average customer surplus, i.e., $SC_c^* \ge SC^*$.

Next, we show $SC_v^* \ge SC^*$. Similarly, suppose the market follows the equilibrium path of base model and achieves equilibrium solution (p^*, q^*). Now, we allow the retailers to announce quantity information to the market. As a result, the market switches to a new equilibrium path (p_v^*, q_v^*) under inventory commitment. In the case of inventory commitment, the retailers are motivated to increase quantity and decrease price.

First, the retailers have incentives to increase quantity. Assume the equilibrium price p^* is unchanged. Once the retailers commit inventory to the market, the inventory stock must not decrease. The argument is as follows. On one side, decreasing quantity decreases consumer surplus and thus decreases market share.

One the other side, deviating from the critical fractile quantity ($q = \frac{1}{2}F^{-1}(\frac{p-c}{p})$) decreases marginal revenue. As a result, by decreasing inventory quantity, the retailers must earn less profit, so the inventory quantity must not be decreased. However, the retailers may increase quantity. Although increasing stock quantity also deviates from the critical fractile quantity and thus decreases marginal revenue, it raises market share by offering higher product availability. Thus, the retailers may earn higher profit by increasing stock quantity. Therefore, given a equilibrium price, the retailers may choose to increase quantity.

Second, the retailers have incentives to decrease price. Similarly, assume the equilibrium quantity q^* is unchanged, the retailers have no incentive to increase retail price. The argument is as follows. On one side, increasing price decreases consumer surplus and thus decreases market share. One the other side, deviating from the critical fractile price ($p = c/\bar{F}(2q)$) decreases marginal revenue. As a result, by increasing price, the retailers must earn less profit, so the retail price must not be increased. However, retailers may decrease price. Although decreasing price also deviates from the critical fractile price and thus shrinks marginal revenue, it raises market share. In other words, the retailers may earn higher profit by decreasing price. Therefore, given a equilibrium quantity, the retailers may choose to decrease price.

In sum, once retailers apply inventory commitment, we have $q_v^* \geq q^*$ and(or) $p_v^* \leq p^*$. Since both increasing quantity and decreasing price are beneficial to consumers' surplus, we have $SC_v^* \geq SC^*$. □

Proof of Proposition 9. We focus on analyzing two extreme cases.

Case I. A small searching cost (i.e., $s \downarrow 0$). In this case, the entire market is fully covered. More specifically, we have $p^* \to \hat{p}$ ($\hat{p}$ is the price that maximizes the customer's expected utility) and profit $\Pi^*(\hat{p})$ in the base model. However, the retailer's profit under strategies of inventory commitment and monetary compensation are close to zero as $s \downarrow 0$, so the retailer will stock zero quantity and thus provides zero product availability.

Case II. A moderate searching cost (i.e., $\underline{s} \leq s < \bar{s}$). In this case, the entire market is fully covered without competition in the base model. Moreover, each retailer can be viewed as a monopoly retailer that serves half of the market. Su and Zhang (2008) have proved that the strategies of inventory commitment and monetary compensation provide a higher order quantity in the monopoly model setting, so we have $\theta^*(p_v^*) > \theta^*(p^*)$ and $\theta^*(p_c^*) > \theta^*(p^*)$.

Finally, recall that the function of product availability, $\theta^*(p^*)$, is continuous in equilibrium price p^* and the equilibrium price p^* is continuous in s, we conclude that: (1) there exists a threshold s_{vw} and we have $\theta^*(p_v^*) < \theta^*(p^*)$ if $s < s_{vw}$; and (2) there exists a threshold s_{cw} and we have $\theta^*(p_c^*) < \theta^*(p^*)$ if $s < s_{cw}$. Since the social welfare function is increasing in product availability, we have: (1) $SW_v^* < SW^*$ if $s < s_{vw}$; and (2) $SW_c^* < SW^*$ if $s < s_{cw}$. □