Trade-in Remanufacturing, Strategic Customer Behavior, and Government Subsidies

Fuqiang Zhang, Renyu Zhang

Olin Business School, Washington University in St. Louis, St. Louis, MO 63130

{fzhang22@wustl.edu, renyu.zhang@wustl.edu}

September 14, 2015

Abstract

Remanufacturing has been increasingly used in industry. To facilitate the collection of cores for remanufacturing, many firms offer rebates that allow repeat customers to trade in used products for upgraded ones at a discounted price. This paper studies the impact of strategic customer behavior on the economic and environmental values of such trade-in remanufacturing practice. There are several major findings. First, under trade-in remanufacturing, a firm may earn a higher profit with strategic customers than with myopic customers, which differs from the common belief that firms dislike forward-looking customer behavior due to its detrimental effect on profit. This is because strategic customers can anticipate the future price discount brought by the trade-in option, so when the revenue-generating effect of remanufacturing is strong enough, they might be willing to pay a higher first-period price than the myopic customers. Second, we show that strategic customer behavior may create a tension between profitability and sustainability: On one hand, by exploiting the forward-looking customer behavior, trade-in remanufacturing is more valuable to the firm with strategic customers than with myopic customers; on the other hand, with strategic customers, trade-in remanufacturing may have a negative impact on the environment and also on social welfare, since it may give rise to a significantly higher production quantity without improving customer surplus. Therefore, our research demonstrates that it is important to understand the interaction between trade-in remanufacturing and strategic customer behavior. Finally, to resolve the above tension, we study how a social planner (e.g., the government) should design a public policy to maximize social welfare. It has been shown that subsidizing remanufactured products alone may lead to undesired outcomes; however, the social optimum can be achieved by using a simple linear subsidy and tax scheme for all product versions.

Key words: remanufacturing; trade-in rebates; strategic customer behavior; environment; subsidization

1 Introduction

It is a common practice for firms to offer trade-in rebates to recycle used products. For example, Apple offers both in-store and online trade-in programs, which allow customers to exchange their used iPhones, iPads, and Macs for credits to purchase new ones (Apple Online Store, 2015). Analogously, Amazon allows Kindle owners to trade in their old products for newer versions at a discount price (Copy, 2011). More examples on the adoption of trade-in rebates to collect cores for remanufacturing have been reported in industries like furniture, carpets, and power tools, etc. (see Ray et al., 2005).

Recycling used products through trade-in rebates has been lauded for its various benefits. From the economic perspective, the return product flow from trade-in rebates serves as an important source for generating revenue and reducing costs. With the recycled products, firms can recover the residual values by either remanufacturing them into new ones or reusing their components and materials. Following the literature (e.g., Ray et al., 2005), throughout the paper we use the term *remanufacturing* to represent the general revenue-generating process through recycling and recovering used products. In practice, the revenue-generating/cost-saving effect of trade-in based remanufacturing could be significant. Xerox, which partly bases its remanufacturing on trade-in returns, has saved several hundred million dollars each year, which accounts for 40%-65% of the company's manufacturing costs (Savaskan et al., 2004). From the strategic perspective, trade-in rebates may improve firm profitability by elevating customer switching costs (Klemperer, 1987), discouraging second-hand markets (Levinthal and Purohit, 1989), increasing purchase frequency (Van Ackere and Reyniers, 1995), and reducing inefficiencies arising from the lemon problem (Rao et al., 2009). From the environmental perspective, trade-in rebates encourage customers to return used products, thus generating less waste and disposals. In particular, the electronics market is featured with frequent product introductions and generates more than one million tons of so-called e-wastes each year (Plambeck and Wang, 2009). Using trade-in rebates, Apple collected more than 40,000 tons of e-wastes in 2014, which account for more than 75 percent of the products they sold seven years earlier (see Apple Environmental Responsibility, 2015).

It has been empirically verified that customers exhibit forward-looking behaviors in the electronics market due to frequent product introductions (Plambeck and Wang, 2009). In particular, when the firm offers trade-in rebates, strategic customer behavior naturally arises, because customers can anticipate a possible price discount in the *future* if making a purchase *now* (Van Ackere and Reyniers, 1995; Fudenberg and Tirole, 1998; Rao et al., 2009). Moreover, advances in information technology enable customers to easily obtain product and price information. For example, Kayak launched the price forecast service to help customers decide when to book a flight (Empson, 2013). As a consequence, strategic customer behavior has become more prevalent in today's business world. Although strategic customer behavior has been widely acknowledged in the literature, it is not clear how such behavior would affect the economic and environmental benefits of trade-in remanufacturing.

Governments around the world have made tremendous efforts to promote recycling and remanufacturing used products. One commonly used strategy is to provide subsidies for remanufacturing. For instance, in January 2015, the Chinese government released a policy to subsidize the use of remanufactured vehicle engines and transmissions (Chen, 2015). Analogously, the Chinese government established a special fund in 2011 to provide subsidies to companies engaged in the recycling and recovering of waste electrical and electronic products (e.g., Xie and Bai, 2010). As another example, a recent report backed by the Scottish government and Zero Waste Scotland (ZWS) concluded that Scotland was in a unique position to develop a circular economy and called for government subsidies to help boost closed loop recycling, reuse, biorefining, and remanufacturing (The Recycler, 2015). In the literature, the effects of government subsidies for remanufacturing/trade-in remanufacturing have been studied in settings without explicitly modeling customer behaviors (e.g., Mitra and Webster, 2008; Ma et al., 2013). Despite its importance, the question of how the government should design the subsidization policy under strategic customer behavior to induce the social optimum has not been thoroughly explored.

The primary goal of this paper is to analyze how strategic customer behavior influences the value of trade-in remanufacturing from the perspectives of the firm, the environment, and the government. For this purpose, we develop a two-period model in which a profit-maximizing firm sells two generations of a product in an *ex-ante* uncertain market. To highlight the impact of strategic customer behavior, we consider two scenarios, one with strategic customers and the other with myopic customers. Strategic customers make their purchasing decisions based on both *current* and anticipated *future* utilities, whereas myopic customers make decisions based on *current* utilities only. In the first period, the firm sells the first-generation product in the market. In the second period, the firm sells the second-generation product to new customers (who have not purchased in the first period); meanwhile the firm offers trade-in rebates through which repeat customers (who have purchased in the first period) exchange used products for new second-generation ones at a discounted price. The firm generates revenue by remanufacturing the recycled products. This remanufacturing process also reduces the (negative) environmental impact of the business, because it decreases energy and raw material consumption, as well as waste disposal. We model the government as a policy-maker whose subsidy/tax policy may affect the firm's pricing and production strategy as well as the customers' purchasing decisions. The objective of the government is to maximize the social welfare, i.e., the sum of firm profit and customer surplus less environmental impact.

We find that strategic customer behavior has important implications on the practice of trade-in remanufacturing. First, under trade-in remanufacturing, the firm can earn a higher profit with strategic customers than with myopic customers if the revenue-generating effect of remanufacturing is sufficiently strong. In other words, strategic customer behavior may improve firm profit, which is in contrast with the commonly believed notion that strategic customer behavior hurts firm profit. When the firm employs trade-in remanufacturing, strategic customers will anticipate the *future* trade-in rebate (i.e., price discount) in the second period, which depends on the additional value generated by remanufacturing. Note that a deeper discount in the second period will induce a higher willingness-to-pay in the first period. Thus, strategic customers may be willing to pay a higher first-period price than myopic customers if the revenue-generating effect of remanufacturing is strong enough, which allows the firm to extract a higher profit. This implies that when early purchases (of strategic customers) can be induced by the additional benefits (i.e., the trade-in option and the deep discount brought by remanufacturing), a firm may benefit from strategic customer behavior. Without trade-in remanufacturing, however, strategic customer behavior always hurts the firm's profit, as reported in the literature.

Second, with strategic customers, the adoption of trade-in remanufacturing may create a tension between firm profitability and environmental sustainability. Trade-in rebate essentially offers an early purchase reward and thus can deliver additional value by exploiting the forward-looking behavior of strategic customers. As a result, trade-in remanufacturing is more valuable to the firm with strategic customers than with myopic customers. However, the early-purchase inducing effect of trade-in remanufacturing also prompts the firm to increase production quantities significantly under strategic customer behavior. The increased production quantities may outweigh the environmental advantage of remanufacturing unless the unit environmental benefit of remanufacturing is very high. Hence, trade-in remanufacturing generally hurts the environment with strategic customers. Moreover, we find that trade-in remanufacturing decreases customer surplus, and consequently, the social welfare may decrease as well. Therefore, our results call for caution on the adoption of trade-in remanufacturing under strategic customer behavior, because it is likely to be severely detrimental to the environment and the society.

With myopic customers, however, trade-in manufacturing generally benefits the environment. The price discrimination effect of trade-in rebates increases the expected unit profit from new customers in the second period. This effect drives the firm to decrease the first-period production quantity and thus increase the potential second-period market size of new customers. As long as the unit environmental benefit of remanufacturing is not too low, trade-in remanufacturing induces lower production quantities and, thus, benefits the environment. Therefore, for the scenario with myopic customers, the tension between firm and environment does not exist in general.

The tension between firm profitability and environmental sustainability under strategic customer behavior motivates us to study how government intervention can achieve the socially optimal outcome. Specifically, we focus on the subsidization policy the government can use to promote the activities of used products recycling (e.g., trade-in rebates, remanufacturing, and take-backs; see Mitra and Webster, 2008; Wang et al., 2014; The Recycler, 2015). An intuitive policy observed in practice is to subsidize the firm/customers for selling/purchasing remanufactured products. However, we find that subsidizing remanufactured products alone actually hurts the environment and is not sufficient to achieve the social optimum. This cautions the policy-makers about how to promote remanufacturing through subsidization. With either strategic or myopic customers, in order to induce the social optimum, it suffices for the government to use a simple linear subsidy/tax scheme for the sales of both product generations and remanufacturing. In addition, if the total unit economic and environmental value of remanufacturing is low, the government should provide more subsidies to the firm with strategic customers than with myopic customers, and vice versa.

The rest of the paper is organized as follows. In Section 2, we position this paper in the related literature. The base model and the equilibrium analysis are presented in Section 3. In Section 4, we analyze the impact of trade-in remanufacturing upon the firm and the environment. In Section 5, we characterize the government policy that can induce the social optimum. This paper concludes with Section 6. All proofs are given in the Appendix.

2 Literature Review

This paper builds upon two streams of research in the literature: (1) remanufacturing and closed-loop supply chain management, and (2) strategic customer behavior.

There is a rapidly growing stream of literature on remanufacturing and closed-loop supply chain management. Comprehensive reviews of this literature are given by Guide and Van Wassenhove (2009) and Souza (2013). Several papers study the optimal inventory policy with return flows of used products; see, e.g., Van der Laan et al. (1999); Toktay et al. (2000), and Gong and Chao (2013). These papers focus on characterizing the cost-minimizing inventory policy in a system with exogenously given demand rate, price, and remanufacturability. More recently, researchers start to explicitly model some strategic issues related to remanufacturing, such as used product acquisition, demand segmentation, product cannibalization, and competition. Savaskan et al. (2004) study the optimal reverse channel structure for the collection of used products from customers. Ferguson and Toktay (2005) analyze the competition between new and remanufactured products (i.e., the cannibalization effect) and characterize the optimal recovery strategy. When remanufacturability is an endogenous decision, Debo et al. (2005) investigate a joint pricing and production technology selection problem of a manufacturer who sells a remanufacturable product to heterogeneous customers. Under the cannibalization effect of remanufactured products, Ferrer and Swaminathan (2006) study the competition between an original equipment manufacturer (OEM) and an independent operator who only sells remanufactured products. Atasu et al. (2008) show that remanufacturing could serve as a marketing strategy to target the customers in the green segment and, hence, enhance the profitability of the OEM. Oraiopoulos et al. (2012) characterize the optimal relicensing strategy of an OEM to mitigate the cannibalization effect in the secondary market. Galbreth et al. (2013) study how the rate of product innovation affects the firm's reuse and remanufacturing decisions. Gu et al.

(2015) investigate the quality design and environmental consequences of green consumerism with remanufacturing. There are papers that address behaviorial issues related to remanufacturing such as how the remanufactured products affect the customer valuation of new products (Agrawal et al., 2015). Government regulations on remanufacturing have also been studied in the literature; see, e.g., Ma et al. (2013). Cohen et al. (2015) study the impact of demand uncertainty on government subsidies for green technology adoption. The impact of trade-in rebates has also received some attention in the remanufacturing literature. For example, Ray et al. (2005) examine the value of price discrimination for new and repeat customers with differentiated ages (and qualities) of the returned products.

The impact of strategic customer behavior has received an increasing amount of attention in the operations management literature. Shen and Su (2007) provide a comprehensive review on customer behavior models in revenue management and auctions. Bensako and Winston (1990) show that rational customers drive a monopolist firm to charge a lower price for any given state in each period. Su (2007) characterizes the optimal pricing strategy with a heterogenous group of strategic customers. When customers are forward-looking, Aviv and Pazgal (2008) study the optimal single mark-down timing with finite inventories. In a newsvendor model where customers anticipate the likelihood of stockout before deciding whether to make a purchase, Dana and Petruzzi (2001) and Su and Zhang (2008, 2009) study the impact of strategic customer behavior on newsvendor profit, supply chain performance, and the role of product availability in inducing demand, respectively. Liu and Van Ryzin (2008) propose the effective capacity rationing strategy to induce early purchases with strategic customers. Cachon and Swinney (2009, 2011) and Swinney (2011) demonstrate how quick response can be employed to mitigate strategic customer behavior. Jerath et al. (2010) study opaque selling and last-minute selling with strategic customers in a revenue management framework. In a cheap talk framework, Allon et al. (2011) show that, though nonverifiable, the availability information improves the profit of a service firm and the expected utility of its customers. Allon and Bassamboo (2011) further demonstrate that a single retailer providing availability information on its own cannot create any credibility with homogeneous customers. Chu and Zhang (2011) investigate the integrated information and pricing strategy with strategic customers and the customer preorders before product release. Parlakturk (2012) demonstrates how vertical product differentiability helps mitigate strategic customer behavior. Recently, there are papers addressing the optimal strategy with multiple product introductions and strategic customer behavior. For example, in the presence of strategic consumers, Liang et al. (2014) characterize the optimal product rollover strategies, whereas Lobel et al. (2015) study the new product launch strategy.

There are a few papers that investigate trade-in rebates with forward-looking customers. Van Ackere and Reyniers (1995) show that, under strategic customer behavior, trade-ins can serve as a mechanism to achieve price commitment. Fudenberg and Tirole (1998) study the monopoly pricing of overlapping generations of a durable good with and without a second-hand market. In an infinite-horizon model setting, Rao et al. (2009) demonstrate that trade-in rebates can alleviate the inefficiencies arising from the lemon problem.

Our paper contributes to the aforementioned streams of research by studying the interaction between trade-in remanufacturing and strategic customer behavior, and how such interaction affects the economic and environmental values of trade-in remanufacturing. We demonstrate that strategic customer behavior may benefit the firm, but give rise to a tension between firm profitability and environmental sustainability under trade-in remanufacturing. In addition, we characterize how the government can achieve the social optimum, using a simple linear subsidy/tax scheme with either strategic or myopic customers.

3 Model and Equilibrium Analysis

3.1 Model Setup

We consider a monopoly firm (he) in the market who sells a product to customers (she) in a two-period sales horizon. In the first period, the firm produces the first-generation product at a unit production cost c_1. The potential market size X, which is the total number of potential customers, is *ex-ante* uncertain. The customers are infinitesimal, each requesting at most one unit of the product in any period. Demand uncertainty is a common feature with new product introduction, but the firm can obtain more accurate demand information as the market matures. Hence, in the second period, the market uncertainty is resolved so the realized market size X becomes known to the firm. Without loss of generality, we assume that $X > 0$, with a distribution function $F(\cdot)$ and density function $f(\cdot) = F'(\cdot)$.

A customer's valuation V for the first-generation product is independently drawn from a continuous distribution with a distribution function $G(\cdot)$ supported on $[\underline{v}, \bar{v}]$ ($0 \leq \underline{v} < \bar{v}$). We call the customer with product valuation V the type-V customer. At the beginning of the sales horizon, each customer only knows the distribution of her own valuation $G(\cdot)$, but not the realization V. This assumption captures the customers' uncertainties about the quality, and fits the situation where the product is brand new. In the second period, all customers observe their own type V. For the customers who purchased the product in period 1, they learn their type V by consuming the product. For the customers who did not purchase the product in period 1, they learn its quality and fit (thus, their type V) through social learning platforms (e.g., Facebook and Amazon customer review systems). Hence, the customers are homogeneous *ex ante* (i.e., at the beginning of period 1), but heterogeneous *ex post* (i.e., at the beginning of period 2). This is a common setting in the models concerning strategic customer behavior (see, e.g., Xie and Shugan, 2001; Su, 2009). We assume that the valuation distribution $G(\cdot)$ has an increasing failure rate, i.e., $g(v)/\bar{G}(v)$ is increasing in v, where $g(\cdot) = G'(\cdot)$ is the density function and $\bar{G}(\cdot) = 1 - G(\cdot)$. This is a mild assumption and can be satisfied by most commonly used distributions. Let $\mu := \mathbb{E}(V) > c_1$, i.e., in expectation, a customer's valuation exceeds the production cost.

The firm offers an upgraded version of the product in period 2. This is a customary practice for product categories like consumer electronics, home appliances, and furniture. A type-V customer has a valuation of $(1+\alpha)V$ for the upgraded second-generation product, where $\alpha \geq 0$ is exogenously given and captures the innovation level (e.g., the improved features) of the upgraded product. Accordingly, let the production cost of the second-generation product be c_2. To model the product depreciation, we take the approach of Van Ackere and Reyniers (1995): If a type-V customer has already bought the product in period 1, her valuation of consuming the used product in period 2 is $(1-k)V$, where $k \in [0,1]$ refers to the depreciation factor. Specifically, if $k = 0$, the product is completely durable; if $k = 1$, the product is completely useless after the first period (either the product is worn out or the technology is obsolete). Therefore, the willingness-to-pay of a type-V customer in period 2 is $(1+\alpha)V$ if she *did not* purchase the product in period 1 (i.e., a new customer), and is $(1+\alpha)V - (1-k)V = (k+\alpha)V$ if she *purchased* the product in period 1 (i.e., a repeat customer).

As widely recognized in the literature, the firm can generate revenue from remanufacturing by reusing the materials and components of the recycled products (see, e.g., Savaskan et al., 2004; Ray et al., 2005). We now model the revenue-generating effect of remanufacturing. There are two types of remanufacturing in our model. First, the firm recycles the unsold first-generation products at the end of period 1. The recycled leftover inventory in the first period is remanufactured and can generate a net per-unit revenue r_1 ($r_1 < c_1$) for the firm. That is, in the base model we assume no excess inventory is carried over to the second period. This assumption applies when the inventory holding cost is sufficiently high or the firm does not want to dilute the sales of the newer generation product, which is usually the case in the electronics market. Moreover, this assumption facilitates the technical tractability of our model. In the Appendix, we present an extension where the firm may hold leftover inventory and offer both product generations in the second period. The second type of remanufacturing is by using the returned products in period 2, i.e., customers who bought the product in period 1 can trade the old product for a second-generation one at a discounted price in period 2. The net revenue of remanufacturing from a used product in period 2 is r_2 ($r_2 < c_2$). Following Savaskan et al. (2004), we assume that all remanufactured products are upgraded to the quality standards of new ones, so that consumers cannot distinguish them from newly made products. Relaxing this assumption will not affect our qualitative results.

The environmental impact of the product is the aggregate (negative) lifetime impact of the product on the environment. The total environmental impact is the production quantity of the product multiplied by the per-unit impact (see, e.g., Thomas, 2011; Agrawal et al., 2012). Let $\kappa_1 > 0$ denote the unit environmental impact of the first-generation product. Analogously, we denote $\kappa_2 > 0$ as the unit environmental impact of the second-generation product. Such impact may refer to the use of natural resources, emission of harmful gases, and generation of solid wastes. Moreover, κ_1 and κ_2 can be estimated by the conventional life-cycle analysis (see, e.g., Agrawal et al., 2012). To model the environmental benefit of remanufacturing, let ι_1 ($\iota_1 < \kappa_1$) be the unit environmental benefit of recycling the first-period leftover inventory, and ι_2 ($\iota_2 < \kappa_2$) be that of recycling the used products through trade-in rebates. Here, ι_1 and ι_2 refer to the reductions in both the production environmental impact in period 2 and the end-of-use and end-of-life product disposal, by recycling and reusing the materials and components of the first-generation products. To capture the environmental advantage of the second-generation product, we assume that $\kappa_1 - \iota_t \geq \kappa_2$ ($t = 1, 2$), i.e., the total environmental impact of the first-generation product dominates that of the second-generation product even if the end-of-use/end-of-life first-generation products are recycled and remanufactured.

The sequence of events unfolds as follows. At the beginning of period 1, the firm announces the price p_1 and decides the production quantity Q_1. Each customer observes p_1, but not Q_1, and makes her decision whether to order the product or to wait until period 2. The first-period demand $X_1 \leq X$ is then realized, the firm collects his first-period revenue, and all customers stay in the market. Note that X_1 is determined by the collective effect of all customers' purchasing behaviors. If $X_1 \leq Q_1$, any customer who requests a product can get one in period 1. Otherwise, $X_1 > Q_1$, then the Q_1 products are randomly allocated to the demand, and $X_1 - Q_1$ customers have to wait due to the limited availability. At the end of period 1, the firm recycles and remanufactures the leftover inventory. At the beginning of period 2, the firm learns the realized total market size X, and each individual customer learns her type V. The firm then announces the price p_2^n for new customers as well as the trade-in price $p_2^r \leq p_2^n$ ($p_2^n - p_2^r$ is the trade-in rebate); all new customers decide whether to purchase the second-generation product, whereas all repeat customers decide whether to trade their used products in for new second-generation ones. Finally, the firm produces the second-generation products, recycles and remanufactures the used products from repeat customers, and collects the second-period revenue.

For notational convenience, we will use $\mathbb{E}[\cdot]$ to denote the expectation operation, $x \wedge y$ to denote the minimum of two numbers x and y, and $\epsilon_1 \overset{d}{=} \epsilon_2$ to denote that two random variables ϵ_1 and ϵ_2 follow the same distribution. The scenario with myopic customers will be denoted with "$\tilde{}$".

3.2 Equilibrium Analysis

We consider two scenarios, one with strategic customers and the other with myopic customers. Strategic customers maximize their total expected surplus over the two-period horizon, whereas myopic customers maximize their expected current-period surplus in each period. In both scenarios, the firm seeks to maximize his total expected profit over the entire horizon. For expositional convenience, we assume there is a common discount factor for the firm and the customers in period 2, denoted by $\delta \in (0, 1]$. To highlight the impact of strategic customer behavior upon the economic and environmental values of trade-in remanufacturing, we assume that the customers are either purely strategic or completely myopic. In reality, the actual customer behavior may take a form between these two extremes. Our model can be easily adapted to capture this situation by fixing the discount factor of the firm at δ, and allowing the discount factor of the customers δ_c to vary in the interval $[0, \delta]$. The higher the δ_c, the greater the customers' concern about future utilities, and thus the more strategic they are. In particular, $\delta_c = \delta$ ($\delta_c = 0$) corresponds to the scenario with purely strategic (myopic) customers.

To characterize the game outcome, we adopt the rational expectation (RE) equilibrium concept. The RE equilibrium was proposed by Muth (1961) and has been widely used in the operations management literature (e.g., Su and Zhang, 2008, 2009; Cachon and Swinney, 2009, 2011). Using backward induction, we start with the decisions of the two parties in period 2. There are $X_2^n = X - (X_1 \wedge Q_1)$ new customers and $X_2^r = X_1 \wedge Q_1$ repeat customers in the market. Note that, since period 2 is the final period, strategic and myopic customers exhibit the same purchasing behavior therein. Hence, regardless of customer behavior, the firm should adopt the same pricing strategy in period 2 as well. Given (X_2^n, X_2^r), let $p_2^n(X_2^n, X_2^r)$ and $Q_2^n(X_2^n, X_2^r)$ be the equilibrium price and production quantity for new customers in period 2. Analogously, $p_2^r(X_2^n, X_2^r)$ and $Q_2^r(X_2^n, X_2^r)$ are the equilibrium trade-in price and production quantity for repeat customers, respectively. Correspondingly, we denote $\pi_2(X_2^n, X_2^r)$ as the equilibrium second-period profit of the firm.

LEMMA **1** *(a) For any (X_2^n, X_2^r), $p_2^n(X_2^n, X_2^r) \equiv p_2^{n*}$ and $p_2^r(X_2^n, X_2^r) \equiv p_2^{r*}$, where*

$$p_2^{n*} = \mathrm{argmax}_{p_2^n \geq 0}(p_2^n - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right) \text{ and } p_2^{r*} = \mathrm{argmax}_{p_2^r \geq 0}(p_2^r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right).$$

Moreover, $p_2^{r*} < p_2^{n*}$ if and only if $k < 1$ or $r_2 > 0$.

(b) For any (X_2^n, X_2^r), $Q_2^n(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) X_2^n$, and $Q_2^r(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) X_2^r$.

(c) There exist two positive constants β_n^* and β_r^*, such that $\pi_2(X_2^n, X_2^r) = \beta_n^* X_2^n + \beta_r^* X_2^r$ for all (X_2^n, X_2^r), where

$$\beta_n^* = \max_{p_2^n \geq 0}(p_2^n - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right) \text{ and } \beta_r^* = \max_{p_2^r \geq 0}(p_2^r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right).$$

Lemma 1 characterizes the equilibrium pricing and production strategy of the firm in period 2. Specifically, both the equilibrium price for new customers and the equilibrium trade-in price are independent of the realized market size (X_2^n, X_2^r). Hence, the equilibrium production quantity for new (repeat) customers is a fixed fraction of the corresponding market size X_2^n (X_2^r) in period 2. As long as the used product is not completely useless to customers in period 2 (i.e., $k < 1$) or remanufacturing generates a positive revenue (i.e., $r_2 > 0$), the firm offers positive trade-in rebates to repeat customers. Moreover, the equilibrium profit of the firm in period 2, $\pi_2(X_2^n, X_2^r)$, is linearly separable in X_2^n and X_2^r.

We now analyze the firm's and the customers' decisions in period 1. We begin with the customers' purchasing behavior. A strategic customer forms beliefs about the first-period product availability probability $\mathfrak{a}$, the second-period price for new customers $\mathfrak{p}_2^n$, and the second-period trade-in price $\mathfrak{p}_2^r$, where $\mathfrak{a}$, $\mathfrak{p}_2^n$, and $\mathfrak{p}_2^r$ are all nonnegative random variables. Based on the belief vector $(\mathfrak{a}, \mathfrak{p}_2^n, \mathfrak{p}_2^r)$ and the observed first-period price p_1, she computes the expected utility of making an immediate purchase, $\mathcal{U}_p := \mathfrak{a}(\mathbb{E}[V]+\delta\mathbb{E}[(k+\alpha)V-\mathfrak{p}_2^r]^+-p_1)+(1-\mathfrak{a})\delta\mathbb{E}[(1+\alpha)V-\mathfrak{p}_2^n]^+$, and the expected utility of waiting, $\mathcal{U}_w := \delta\mathbb{E}[(1+\alpha)V-\mathfrak{p}_2^n]^+$. Hence, the first-period reservation price of a strategic customer, ξ_r, is given by $\xi_r := \max\{p_1 : \mathcal{U}_p \geq \mathcal{U}_w\}$, and she will make a purchase in period 1 if and only if $p_1 \leq \xi_r$. The decision-making process of a myopic customer is simpler because she does not form beliefs about the first-period availability and second-period prices, but bases her purchasing decision on the current utility only. Hence, the first-period reservation price for a myopic customer equals her expected valuation of the product, i.e., $\tilde{\xi}_r = \mathbb{E}[V] = \mu$. Following the standard approach in the marketing (Xie and Shugan, 2001) and the strategic customer behavior (Su and Zhang, 2008; Cachon and Swinney, 2011) literature with homogeneous customers, we assume that all customers will make a purchase in period 1 if p_1 equals their reservation prices (ξ_r for strategic customers and $\tilde{\xi}_r$ for myopic customers). Thus, with strategic (myopic) customers, the first-period demand, X_1, is given by $X_1 = X \cdot \mathbf{1}_{\{p_1 \leq \xi_r\}}$ ($X_1 = X \cdot \mathbf{1}_{\{p_1 \leq \tilde{\xi}_r\}}$).

Next, we consider the firm's problem in period 1. The firm does not know the exact reservation price of strategic (myopic) customers ξ_r ($\tilde{\xi}_r$), but forms a belief $\mathfrak{r}_1$ ($\tilde{\mathfrak{r}}_1$) about it. To maximize his expected profit, the firm sets the first-period price p_1 ($\tilde{p}_1$) equal to the expected reservation price $\mathfrak{r}_1$ ($\tilde{\mathfrak{r}}_1$), which is the highest price (the firm believes) strategic (myopic) customers are willing to pay in the first period. Thus, the firm believes that the first-period demand $X_1 = X$. Thus, the second-period market size of new customers is $X_2^n = (X - Q_1)^+$, and that of repeat customers is $X_2^r = X \wedge Q_1$. Moreover, the firm sets the first-period production quantity Q_1 to maximize the total expected profit with strategic (myopic) customers $\Pi_f(Q_1)$ ($\tilde{\Pi}_f(Q_1)$), where $\Pi_f(Q_1) = \mathbb{E}\{p_1(X \wedge Q_1) - c_1Q_1 + r_1(Q_1 - X)^+ + \delta\pi_2(X_2^n, X_2^r)\}$ and $\tilde{\Pi}_f(Q_1) = \mathbb{E}\{\tilde{p}_1(X \wedge Q_1) - c_1Q_1 + r_1(Q_1 - X)^+ + \delta\pi_2(X_2^n, X_2^r)\}$, with $p_1 = \mathfrak{r}_1$, $\tilde{p}_1 = \tilde{\mathfrak{r}}_1$, $X_2^n = (X - Q_1)^+$, and $X_2^r = X \wedge Q_1$. Finally, under the RE equilibrium, all beliefs are rationally formulated and thus consistent with the actual outcomes.

Let $(p_1^*, Q_1^*, \xi_r^*, \mathfrak{r}^*, \mathfrak{a}^*, \mathfrak{p}_2^{n*}, \mathfrak{p}_2^{r*})$ and $(\tilde{p}_1^*, \tilde{Q}_1^*, \tilde{\xi}_r^*, \tilde{\mathfrak{r}}^*)$ be the RE equilibria in the market with strategic and myopic customers, respectively. For concision, the formal definitions of the RE equilibria in both scenarios are given in the Appendix. To characterize the RE equilibrium, we define two auxiliary variables $m_1^* := \mu + \delta[\beta_r^* - \beta_n^* + \mathbb{E}((k+\alpha)V - p_r^*)^+ - \mathbb{E}((1+\alpha)V - p_n^*)^+]$ and $\tilde{m}_1^* := \mu + \delta(\beta_r^* - \beta_n^*)$. As will be clear in our subsequent analysis, m_1^* ($\tilde{m}_1^*$) is the first-period effective marginal revenue with strategic (myopic) customers, which summarizes the impact of the second-period decisions on the first-period firm profit. Based on Lemma 1, we can characterize the RE equilibrium market outcome in the scenario with either strategic or myopic customers.

Theorem 1 *(a) With strategic customers, an RE equilibrium* $(p_1^*, Q_1^*, \xi_r^*, \mathfrak{r}^*, \mathfrak{p}_2^{n*}, \mathfrak{p}_2^{r*}, \mathfrak{a}^*)$

exists with (i) $p_1^* = \mu + \delta[\mathbb{E}((k+\alpha)V - p_2^{r*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+]$*; and (ii)* $Q_1^* = \bar{F}^{-1}(\frac{c_1 - r_1}{m_1^* - r_1})$*. Moreover, all RE equilibria give rise to the identical expected total profit of the firm,* $\Pi_f^* = (m_1^* - r_1)\mathbb{E}(X \wedge Q_1^*) - (c_1 - r_1)Q_1^* + \delta\beta_n^*\mathbb{E}(X)$.

(b) With myopic customers, an RE equilibrium $(\tilde{p}_1^*, \tilde{Q}_1^*, \tilde{\xi}_r^*, \tilde{\mathfrak{r}}^*)$ *exists with (i)* $\tilde{p}_1^* = \mu$*; and (ii)* $\tilde{Q}_1^* = \bar{F}^{-1}(\frac{c_1 - r_1}{\tilde{m}_1^* - r_1})$*. Moreover, all RE equilibria give rise to the identical expected total profit of the firm,* $\tilde{\Pi}_f^* = (\tilde{m}_1^* - r_1)\mathbb{E}(X \wedge \tilde{Q}_1^*) - (c_1 - r_1)\tilde{Q}_1^* + \delta\beta_n^*\mathbb{E}(X)$.

Theorem 1(a) and (b) characterize the RE equilibrium market outcomes in the scenarios with strategic and myopic customers, respectively. In each scenario, the first-period price equals the corresponding expected reservation price of the customers, and the first-period production quantity can be determined by the solution of a corresponding newsvendor problem. In equilibrium, the total environmental impact should be the difference between the total environmental impact of production/disposal and the total environmental benefit of remanufacturing. Hence, the equilibrium environmental impact with strategic customers is $I_e^* = \mathbb{E}\{\kappa_1 Q_1^* + \delta\kappa_2(Q_2^n(X_2^{n*}, X_2^{r*}) + Q_2^r(X_2^{n*}, X_2^{r*})) - \iota_1(Q_1^* - X)^+ - \delta\iota_2 Q_2^r(X_2^{n*}, X_2^{r*})\}$, where $X_2^{n*} = (X - Q_1^*)^+$ and $X_2^{r*} = X \wedge Q_1^*$; whereas that with myopic customers is $\tilde{I}_e^* = \mathbb{E}\{\kappa_1\tilde{Q}_1^* + \delta\kappa_2(Q_2^n(\tilde{X}_2^{n*}, \tilde{X}_2^{r*}) + Q_2^r(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})) - \iota_1(\tilde{Q}_1^* - X)^+ - \delta\iota_2 Q_2^r(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})\}$, where $\tilde{X}_2^{n*} = (X - \tilde{Q}_1^*)^+$ and $\tilde{X}_2^{r*} = X \wedge \tilde{Q}_1^*$.

4 Impact of Trade-in Remanufacturing

In this section, we analyze the impact of trade-in remanufacturing on the firm and the environment under different customer behaviors (i.e., strategic or myopic customers). Our focus is on how strategic customer behavior influences the economic and environmental values of trade-in remanufacturing.

To facilitate our comparison, we first introduce a benchmark model where the firm does not offer trade-in rebates to customers. As a consequence, the firm cannot recycle used products for remanufacturing in period 2. We call this the No Trade-in Remanufacturing (NTR) model, which is denoted by the superscript "u" hereafter. We use $p_2^u(X_2^n, X_2^r)$ to denote the equilibrium second-period pricing strategy of the firm in the NTR model, which does not depend on customer behavior. As in the base model, the firm forms a belief about the customers' expected willingness-to-pay in the first period, and bases his price and production decisions on this belief. The customers, on the other hand, form beliefs about the product availability and the second-period price, and time their purchases. Again, the formal definitions of the RE equilibrium in the NTR model are given in the Appendix. By the same argument in the proof of Theorem 1, we can show that an RE equilibrium exists with either strategic or myopic customers in the NTR model. Let (p_1^{u*}, Q_1^{u*}) denote the equilibrium first-period price and production decisions of the firm with strategic customers, and $(\tilde{p}_1^{u*}, \tilde{Q}_1^{u*})$ denote those with myopic customers in the NTR model. Accordingly, the associated equilibrium expected profit of the firm (environmental impact) is denoted by Π_f^{u*} (I_e^{u*}) in the scenario with strategic customers, and by $\tilde{\Pi}_f^{u*}$ ($\tilde{I}_e^{u*}$) in the scenario with myopic customers.

Let $\Pi_f^u(Q_1)$ ($\tilde{\Pi}_f^u(Q_1)$) be the expected profit of the firm with strategic (myopic) customers to produce Q_1 products in the period 1 in the NTR model. We characterize the objective functions $\Pi_f(\cdot)$, $\tilde{\Pi}_f(\cdot)$, $\Pi_f^u(\cdot)$, and $\tilde{\Pi}_f^u(\cdot)$ in the following lemma.

LEMMA **2** *The objective functions are given by* $\Pi_f(Q_1) = (m_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_2^{n*}\mathbb{E}(X)$, $\tilde{\Pi}_f(Q_1) = (\tilde{m}_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_2^{n*}\mathbb{E}(X)$,

$$\Pi_f^u(Q_1) = (m_1^u(Q_1) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}\{(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)X\},$$

and $\tilde{\Pi}_f^u(Q_1) = (\tilde{m}_1^u(Q_1) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}\{(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)X\}$, *where* $X_2^n = (X - Q_1)^+$, $X_2^r = X \wedge Q_1$. *The expressions of* $m_1^u(\cdot)$ *and* $\tilde{m}_1^u(\cdot)$ *are given in the Appendix.*

Lemma 2 implies that, in the NTR model, the effective first-period marginal revenue is production-quantity-dependent, and given by $m_1^u(\cdot)$ in the scenario with strategic customers and by $\tilde{m}_1^u(\cdot)$ with myopic customers. The economic interpretation of $m_1^u(Q_1)$ ($\tilde{m}_1^u(Q_1)$) is that, when the first-period production quantity is Q_1, it measures the additional expected marginal revenue to sell the product in period 1 over that in period 2 with strategic (myopic) customers. Hence, the higher the $m_1^u(Q_1)$ and $\tilde{m}_1^u(Q_1)$, the more profitable it is for the firm to sell the first-generation product in the NTR model with strategic and myopic customers, respectively. In other words, $m_1^u(\cdot)$ and $\tilde{m}_1^u(\cdot)$ capture the willingness-to-produce of the firm in period 1. Without loss of generality, we focus on the case where $m_1^u(\cdot) > 0$ and $\tilde{m}_1^u(\cdot) > 0$ for all $Q_1 \geq 0$, i.e., the firm can gain a positive revenue to sell the first-generation product. Otherwise, the firm will not produce or sell anything in period 1.

4.1 Impact on Firm Profit

This subsection investigates the value of trade-in remanufacturing to the firm. To begin with, we characterize the role of strategic customer behavior, depending on whether the firm adopts trade-in remanufacturing or not.

THEOREM **2** *(a) Under trade-in remanufacturing, let* $e^* := \mathbb{E}((k+\alpha)V - p_2^{r*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+$. *Then, we have (i)* $p_1^* > \tilde{p}_1^*$ *if and only if* $e^* > 0$, *(ii)* $Q_1^* > \tilde{Q}_1^*$ *if and only if* $e^* > 0$, *and (iii)* $\Pi_f^* > \tilde{\Pi}_f^*$ *if and only if* $e^* > 0$. *Moreover, there exists a threshold* $\bar{r} \geq \frac{1-k}{1+\alpha}c_2$, *such that* $e^* > 0$ *if and only if* $r_2 > \bar{r}$.

(b) Under no trade-in remanufacturing, we have (i) $p_1^{u*} \leq \tilde{p}_1^{u*}$, *where the inequality is strict if* $k < 1$, *(ii)* $Q_1^{u*} \leq \tilde{Q}_1^{u*}$, *and (iii)* $\Pi_f^{u*} \leq \tilde{\Pi}_f^{u*}$, *where the inequality is strict if* $k < 1$ *and* $\tilde{Q}_1^{u*} > 0$.

Under trade-in remanufacturing, Theorem 2(a) compares the equilibrium outcomes with different customer behaviors. We find that the key to this comparison is the difference between the expected surplus of a repeat customer and that of a new customer in period 2 (i.e., e^*). With strategic customers, the firm charges a higher first-period price, sets a higher first-period production level, and earns a higher total expected profit, if and only if the expected second-period surplus of a repeat customer is higher than that of a new customer (i.e, $e^* > 0$). In particular, the presence of strategic customer behavior benefits the firm if the revenue-generating effect of remanufacturing is strong enough (i.e., $r_2 > \bar{r}$). This result is in sharp contrast with the well-established notion in the literature that strategic customer behavior hurts a firm's profit (e.g., Aviv and Pazgal, 2008; Su and Zhang, 2008). Trade-in remanufacturing leads to a price discount for repeat customers in period 2, which can be perceived by strategic customers when deciding whether to make a purchase in period 1. This discount outweighs the benefit of strategic waiting if the revenue generated from remanufacturing is sufficiently high (i.e., $r_2 > \bar{r}$). In this case, the presence of forward-looking behavior will enable the firm to charge a higher price, produce more, and thus earn a higher profit. We emphasize that both the trade-in option and the revenue-generating effect of remanufacturing are essential for the firm to benefit from strategic customer behavior: The former induces strategic customers to anticipate the price discount for repeat customers, whereas the latter brings in the additional benefit that guarantees a deep discount so that strategic customers are willing to pay an even higher first-period price than myopic customers. In contrast, Theorem 2(b) shows that, without trade-in remanufacturing, the firm always suffers from strategic customer behavior, as reported in the existing literature.

Theorem 2 suggests that the presence of strategic customer behavior will make trade-in remanufacturing more attractive to the firm. Next, we study how trade-in remanufacturing influences the profit and the pricing strategy of the firm under different customer behaviors. The following theorem compares the equilibrium prices and profits in the NTR model and those in the base model with either strategic or myopic customers.

THEOREM **3** *(a) In period 2, $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r. Moreover, for any (X_2^n, X_2^r), $p_2^{r*} \le p_2^u(X_2^n, X_2^r) \le p_2^{n*}$, where the inequalities are strict if $k < 1$ and $X_2^n, X_2^r > 0$.*

(b) With strategic customers, we have (i) $p_1^{u*} \le p_1^*$, where the inequality is strict if $p_2^{r*} < p_2^{n*}$; and (ii) $\Pi_f^{u*} \le \Pi_f^*$, where the inequality is strict if $p_2^{r*} < p_2^{n*}$ and $Q_1^* > 0$.

(c) With myopic customers, we have (i) $\tilde{p}_1^{u*} = \tilde{p}_1^*$; and (ii) $\tilde{\Pi}_f^{u*} \le \tilde{\Pi}_f^*$, where the inequality is strict if $p_2^{r*} < p_2^{n*}$ and $\tilde{Q}_1^{u*} > 0$.

Theorem 3 shows that the equilibrium second-period price without trade-in remanufacturing, $p_2^u(\cdot, \cdot)$, is bounded from below by the equilibrium second-period trade-in price p_2^{r*}, and from above by the equilibrium second-period price for new customers p_2^{n*}. Hence, under trade-in remanufacturing, the expected utility of strategic customers to make a purchase in the first period increases (i.e., $\delta\mathbb{E}[(k+\alpha)V - p_2^{r*}]^+ \ge \delta\mathbb{E}[(k+\alpha)V - p_2^u(X_2^n, X_2^r)]^+$), whereas the benefit of waiting decreases decreases (i.e., $\delta\mathbb{E}[(1+\alpha)V-p_2^{n*}]^+ \leq \delta\mathbb{E}[(1+\alpha)V-p_2^u(X_2^n, X_2^r)]^+$. This implies that trade-in remanufacturing makes strategic customers more willing to purchase immediately than to wait until period 2. Therefore, trade-in remanufacturing enables the firm to exploit the forward-looking behavior of strategic customers and thus induces early purchases from them. With myopic customers, however, trade-in remanufacturing does not have an early-purchase inducing effect because myopic customers do not care about their future surplus. This result is also consistent with the finding in the durable product literature that the secondary market gives rise to greater resale value of a durable product and thus can increase the sales of the new product upfront (see, e.g., Hendel and Lizzeri, 1999; Waldman, 2003).

From Theorem 3, we can see there are three beneficial effects of trade-in remanufacturing that may improve firm profit: (a) the revenue-generating effect of remanufacturing, (b) the price discrimination effect of trade-in rebates, i.e., the differentiated prices for new and repeat customers helps the firm exploit the customer segmentation in period 2, and (c) the early-purchase inducing effect of trade-in rebates, i.e., the price discount to repeat customers enables the firm to exploit the forward-looking behavior of strategic customers by offering early-purchase rewards. The first two effects benefit the firm with either strategic or myopic customers, whereas the third effect improves the firm's profit with strategic customers only. In the following, we conduct extensive numerical experiments to quantify the third effect, and deliver insights on how strategic customer behavior influences the value of trade-in remanufacturing to the firm.

The design of the numerical study is as follows. Let the customer valuation V follow a uniform distribution on $[0, 1]$ ($\mu = \mathbb{E}(V) = 0.5$). The discount factor is $\delta = 0.95$, the unit environmental impact of the first-generation product is $\kappa_1 = 1$, and the unit environmental impact of the second-generation product is $\kappa_2 = 0.75$. To focus on the impact of customer behaviors, we set $r_1 = r_2 = 0$ (i.e., there is no revenue-generating effect associated with remanufacturing), and the unit environmental benefits of recycling/remanufacturing to be $\iota_1 = 0$ and $\iota_2 = 0.3$ (these two values will be useful when studying the environmental impact in Section 4.2). The unit production cost of the first-generation product is $c_1 \in \{0.05, 0.1, 0.15, 0.2, 0.25\}$. The innovation level of the second-generation product is $\alpha \in \{0, 0.05, 0.1, 0.15, 0.2\}$, and the unit production cost of the second-generation product is $c_2 = 0.25(1+\alpha) \in \{0.25, 0.2625, 0.275, 0.2875, 0.3\}$. We consider the depreciation factor $k \in \{0.3, 0.4, 0.5, 0.6, 0.7\}$. The demand X follows a gamma distribution with mean 100 and coefficient of variation $CV(X)$ taking values from the set $\{0.5, 0.6, 0.7, 0.8, 0.9\}$. Thus, we have a total of 625 parameter combinations that cover a wide range of reasonable problem scenarios. The above problem scenarios form a subset of the extensive experiments we have conducted. Our numerical findings are very robust. For concision, we will only present the results for the parameter combinations listed above.

We calculate the expected profit for each scenario with either strategic or myopic customers both in the base model, (Π_f^*, $\tilde{\Pi}_f^*$) and in the NTR model, (Π_f^{u*}, $\tilde{\Pi}_f^{u*}$). The two metrics of interest are: $\gamma_s := (\Pi_f^* - \Pi_f^{u*})/\Pi_f^{u*} \times 100\%$, and $\gamma_m := (\tilde{\Pi}_f^* - \tilde{\Pi}_f^{u*})/\tilde{\Pi}_f^{u*} \times 100\%$, i.e., γ_s (γ_m)

refers to the relative profit improvement of trade-in remanufacturing with strategic (myopic) customers. We evaluate γ_s and γ_m under the 625 parameter combinations and report that, under each combination, γ_s is significantly higher than γ_m. More specifically, γ_s is at least 5.8% and can be as high as 61.6%, with an average of 30.2%; whereas γ_m ranges from 0.008% to 11.7%, with an average of 3.1%. We give the summary statistics of γ_s and γ_m in Table 1.

	Min	5th percentile	Median	95th percentile	Max	Mean	Standard deviation
γ_s	5.8	11.3	28.3	55.8	61.6	30.2	13.1
γ_m	0.008	0.22	2.5	8.1	11.7	3.1	2.5

Table 1: **Summary Statistics: Firm Profit** (%)

Our numerical results deliver an important message on the economic value of trade-in remanufacturing: Trade-in remanufacturing delivers a much higher value to the firm with strategic customers than with myopic customers (γ_s is significantly higher than γ_m for each problem instance). Recall that, with myopic customers, trade-in remanufacturing only has the benefits of revenue-generating and price discrimination, whereas, with strategic customers, this strategy has the additional value of inducing early purchases. Therefore, these results indicate that the value of trade-in remanufacturing to the firm mainly comes from the early-purchase inducing effect of trade-in rebates to exploit strategic customer behavior, rather than from the revenue-generating effect of remanufacturing or the price discrimination effect of trade-in rebates to exploit customer segmentation.

4.2 Impact on Environment and Customer Surplus

Our next goal is to examine the environmental value of trade-in remanufacturing under different customer behaviors. We first characterize how trade-in remanufacturing influences the effective first-period marginal revenue and production quantities of the firm.

Theorem **4** *Assume $k < 1$.*

(a) With strategic customers, we have (i) $m_1^u(Q_1)$ is decreasing in Q_1; (ii) $m_1^u(Q_1) < m_1^*$ for all Q_1; (iii) $Q_1^{u*} \le Q_1^*$, where the inequality is strict if $Q_1^* > 0$.

(b) With myopic customers, we have (i) $\tilde{m}_1^u(Q_1)$ is increasing in Q_1; (ii) for each $r_2 < \bar{r}_2$[1], there exists a threshold $\bar{Q}(r_2)$ increasing in r_2, such that $\tilde{m}_1^u(Q_1) \le \tilde{m}_1^*$ for all $Q_1 \le \bar{Q}(r_2)$, and $\tilde{m}_1^u(Q_1) > \tilde{m}_1^*$ for all $Q_1 > \bar{Q}(r_2)$; (iii) for each $r_2 < \bar{r}_2$, there exists a threshold $\bar{c}_1(r_2) > 0$, such that $Q_1^{u*} > Q_1^*$ if $c_1 \le \bar{c}_1(r_2)$.

Theorem 4 provides an interesting comparison between the scenarios of strategic and myopic customers: With strategic customers, trade-in remanufacturing always increases the first-period production quantity of the firm, whereas it may prompt the firm to produce less with myopic

[1] The expression of $\bar{r}_2$ is given in the Appendix.

customers. More specifically, Theorem 4(a) shows that, under strategic customer behavior, the effective marginal revenue with trade-in remanufacturing always dominates that without (i.e., $m_1^u(\cdot) < m_1^*$). As a result, the firm produces more in period 1 under trade-in remanufacturing. Theorem 4(b), however, suggests that, with myopic customers, trade-in remanufacturing may give rise to a lower first-period effective marginal revenue if the production quantity is large (i.e., $\tilde{m}_1^u(Q_1) > \tilde{m}_1^*$ if $Q_1 > \bar{Q}(r_2)$), thus driving the firm to lower the first-period production quantity if the first-period unit production cost is low (i.e., $c_1 \leq \bar{c}_1(r_2)$). Recall from Theorem 3 that trade-in remanufacturing increases the first-period willingness-to-pay of strategic customers, which, in turn, drives the firm to produce more in period 1. Such early-purchase and, thus, early-production inducing effects of trade-in remanufacturing, however, are absent with myopic customers. In the scenario of myopic customers, on the other hand, the price discrimination effect of trade-in remanufacturing improves the unit profit generated from the new customers in period 2, thus leading to a lower effective first-period marginal revenue if the revenue-generating effect of remanufacturing is not too strong (i.e., $r_2 < \bar{r}_2$). As a consequence, the firm decreases the first-period production quantity to increase the second-period market size of new customers.

Theorem 4 demonstrates the contrasting effects of trade-in remanufacturing on production quantities under different customer behaviors. How does trade-in remanufacturing affect the environment? The answer is given in the next theorem.

THEOREM **5** *(a) With strategic customers, there exists a threshold* $\bar{\iota}_2^u > 0$*, such that* $I_e^* \geq I_e^{u*}$ *if* $\iota_2 \leq \bar{\iota}_2^u$*.*

(b) Assume that $r_2 < \bar{r}_2$ *and* $c_1 \leq \bar{c}_1(r_2)$*. With myopic customers, there exists a threshold* $\tilde{\iota}_2^u < \kappa_2$*, such that* $\tilde{I}_e^{u*} \geq \tilde{I}_e^*$ *if* $\iota_2 \geq \tilde{\iota}_2^u$*.*

When customers are strategic, trade-in rebates encourage them to recycle the used first-generation products more frequently, so they also purchase the product more frequently. In this scenario, trade-in remanufacturing leads to a worsened outcome for the environment if the unit environmental benefit of remanufacturing is not high enough to justify the early-production inducing effect (i.e., $\iota_2 \leq \bar{\iota}_2^u$ in Theorem 5(a)). When the customers are myopic and the unit production cost is sufficiently low, trade-in remanufacturing motivates the firm to produce less in period 1 (see Theorem 4(b)). Hence, trade-in remanufacturing helps improve the environment as long as the unit environmental benefit of remanufacturing is not too low (i.e., $\iota_2 \geq \tilde{\iota}_2^u$ in Theorem 5(b)). Theorem 5 reveals the significant impact of customer behavior on the environmental value of trade-in remanufacturing. With strategic customers, the adoption of trade-in remanufacturing is likely to be detrimental to the environment, whereas, with myopic customers, adopting trade-in remanufacturing may benefit both the firm and the environment. Some papers in the literature (e.g., Debo et al., 2005; Galbreth et al., 2013; Gu et al., 2015) have also established that remanufacturing may increase the production quantity and thus environmental impact. Our paper, however, demonstrates that the environmental impact of trade-in remanufacturing depends critically on customer behavior.

We now numerically illustrate the environmental value of trade-in remanufacturing. We employ the same numerical setup as Section 4.1. Recall that I_e^* ($\tilde{I}_e^*$) is the expected environmental impact for the scenario with strategic (myopic) customers in the base model, and I_e^{u*} ($\tilde{I}_e^{u*}$) is that in the NTR model. We are interested in the following two metrics: $\eta_s := (I_e^* - I_e^{u*})/I_e^{u*} \times 100\%$, and $\eta_m := (\tilde{I}_e^* - \tilde{I}_e^{u*})/\tilde{I}_e^{u*} \times 100\%$, i.e., η_s (η_m) refers to the relative change of the environmental impact when adopting trade-in remanufacturing with strategic (myopic) customers.

We evaluate η_s and η_m under the 625 parameter combinations and obtain the following numerical findings: (i) Under each parameter combination, η_s is significantly higher than η_m; and (ii) For most of the parameter combinations, $\eta_s > 0$ but $\eta_m < 0$. Specifically, η_s takes values from -1.2% to 171.9%, with an average of 49.2%; whereas η_m ranges from -10.2% to 4.5%, with an average of -5.0%. Moreover, $\eta_s < 0$ (i.e., trade-in remanufacturing benefits the environment with strategic customers) for 10 out of the 625 (i.e., 1.6%) problem instances we examine, whereas $\eta_m < 0$ (i.e., trade-in remanufacturing benefits the environment with myopic customers) for 585 out of the 625 (i.e., 93.6%) problem instances. Table 2 summarizes the statistics of η_s and η_m.

	Min	5th percentile	Median	95th percentile	Max	Mean	Standard deviation
η_s	-1.2	2.0	37.8	117.8	171.9	49.2	41.4
η_m	-10.2	-8.5	-5.5	0.51	4.5	-5.0	2.7

Table 2: **Summary Statistics: Environmental Impact** (%)

Table 2 confirms that trade-in remanufacturing generally leads to much higher environmental impact with strategic customers than with myopic customers (η_s is significantly higher than η_m). Though beneficial to the firm (see Table 1), the early-purchase inducing effect of trade-in remanufacturing gives rise to much higher production quantities under strategic customer behavior, and thus leads to a much worse outcome from the environmental perspective. Therefore, strategic customer behavior has opposing effects on the value of trade-in remanufacturing to the firm and the environment: It makes this strategy more attractive to the firm, but less desirable to the environment.

The above results suggest that trade-in remanufacturing may create a tension between firm profitability and environmental sustainability with strategic customers, but benefits both the firm and the environment with myopic customers. Since η_s is significantly larger than zero for most of the numerical cases we examine, trade-in remanufacturing is detrimental to the environment for a large set of reasonable problem instances under strategic customer behavior. Hence, in general, the early-purchase inducing effect dominates the environmental benefit of remanufacturing with strategic customers. Under strategic customer behavior, the firm significantly benefits from trade-in remanufacturing, but the environment significantly suffers from this strategy (i.e., $\gamma_s > 0$ and, in general, $\eta_s > 0$). With myopic customers, however, both the firm and the environment would benefit from the adoption of trade-in remanufacturing (i.e., $\gamma_m > 0$ and, in general, $\eta_m < 0$).

Although an increased production quantity means more pressure on the environment, it also increases the consumption level of the product. To conclude this section, we explore how trade-in remanufacturing impacts the total customer surplus under different customer behaviors. We use S_c^* ($\tilde{S}_c^*$) and S_c^{u*} ($\tilde{S}_c^{u*}$) to denote the equilibrium total customer surplus for the scenarios with strategic (myopic) customers in the base model and the NTR model, respectively.

THEOREM **6** *(a) In the base model, we have* $S_c^* = \delta\mathbb{E}[((1+\alpha)V - p_2^{n*})^+ X]$ *and* $\tilde{S}_c^* = \delta\mathbb{E}[((1+\alpha)V - p_2^{n*})^+(X - \tilde{Q}_1^*)^+] + \delta\mathbb{E}[((k+\alpha)V - p_2^{r*})^+(X \wedge \tilde{Q}_1^*)]$.

(b) In the NTR model, we have $S_c^{u*} = \delta\mathbb{E}[((1+\alpha)V - \mathfrak{p}_2^{u*})^+ X]$ *and* $\tilde{S}_c^{u*} = \delta\mathbb{E}[((1+\alpha)V - \mathfrak{p}_2^{u*})^+(X - \tilde{Q}_1^{u*})^+] + \delta\mathbb{E}[((k+\alpha)V - \mathfrak{p}_2^{u*})^+(X \wedge \tilde{Q}_1^{u*})]$.

(c) We have the following relationship on the customer surpluses of strategic customers: $S_c^* \leq S_c^{u*}$, *where the inequality is strict if* $k < 1$ *and* $Q_1^{u*} > 0$.

Theorem 6(a) and (b) compute the total customer surpluses in the base model and the NTR model. Moreover, in Theorem 6(c), we demonstrate that, with strategic customers, the total customer surplus always decreases with the adoption of trade-in remanufacturing. This is because, with strategic customers, the total customer surplus only depends on the (perceived) price for new customers in period 2, which is higher under the adoption of trade-in remanufacturing (see Theorem 3(a)). By Theorem 4(a), one may argue that, under strategic customer behavior, trade-in remanufacturing increases production quantities and thus increases the customer surplus. Theorem 6(c), however, shows that the total customer surplus actually decreases in this scenario. Hence, under strategic customer behavior, trade-in remanufacturing gives rise to higher production quantities without improving the customer surplus. Further, the social welfare (i.e., firm profit plus customer surplus less environmental impact) is likely to decrease under trade-in remanufacturing as well. This has been confirmed in the numerical study we explored in this section.

To summarize, customer behavior plays an important role in the economic and environmental values of trade-in remanufacturing. With myopic customers, trade-in remanufacturing benefits both the firm and the environment. With strategic customers, trade-in remanufacturing would be even more beneficial to the firm; however, meanwhile it may hurt the environment, decrease customer surplus, and possibly lower social welfare. Therefore, it is important for firms and policy-makers to understand customer behavior when making decisions related to trade-in remanufacturing.

5 Social Optimum and Government Intervention

As shown in Section 4, adopting trade-in remanufacturing may create a tension between firm profitability and environmental sustainability under strategic customer behavior. In this section, we analyze how a policy-maker (e.g., the government) can design the public policy to resolve this tension and maximize the social welfare under different customer behaviors.

To characterize the socially optimal outcome, we assume that the government can set the prices and production levels, with an objective to maximize the social welfare. Let W_s denote the social welfare, which is defined by the expected profit of the firm Π_f, plus the expected customer surplus S_c, net the expected environmental impact I_e, i.e.,

$$W_s = \Pi_f + S_c - I_e.$$

By backward induction, we start with the second-period pricing and production problem. As in the base model, strategic and myopic customers exhibit the same purchasing behavior in period 2. For any given realized market size in period 2 (X_2^n, X_2^r), we use $(p_{s,2}^n(X_2^n, X_2^r), p_{s,2}^r(X_2^n, X_2^r))$ to denote the equilibrium pricing strategy, and $(Q_{s,2}^n(X_2^n, X_2^r), Q_{s,2}^r(X_2^n, X_2^r))$ to denote the equilibrium production strategy. Correspondingly, we denote $w_2(X_2^n, X_2^r)$ as the equilibrium second-period social welfare.

LEMMA **3** *(a) For any (X_2^n, X_2^r), $p_{s,2}^n(X_2^n, X_2^r) \equiv p_{s,2}^{n*}$ and $p_{s,2}^r(X_2^n, X_2^r) \equiv p_{s,2}^{r*}$, where $p_{s,2}^{n*} = c_2 + \kappa_2$ and $p_{s,2}^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$. Hence, $p_{s,2}^{n*} > p_{s,2}^{r*}$ if and only if $r_2 > 0$ or $\iota_2 > 0$.*

(b) For any (X_2^n, X_2^r), $Q_{s,2}^n(X_2^n, X_2^r) = \bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right) X_2^n$, and $Q_{s,2}^r(X_2^n, X_2^r) = \bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right) X_2^r$.

(c) There exist two positive constants $\beta_{s,n}^*$ and $\beta_{s,r}^*$, such that $w_2(X_2^n, X_2^r) = \beta_{s,n}^* X_2^n + \beta_{s,r}^* X_2^r$ for all (X_2^n, X_2^r), where $\beta_{s,n}^* = \mathbb{E}[(1+\alpha)V - p_{s,2}^{n*}]^+$ and $\beta_{s,r}^* = \mathbb{E}[(k+\alpha)V - p_{s,2}^{r*}]^+$.

Lemma 3 implies that, with either strategic or myopic customers, the socially optimal second-period pricing strategy takes the form that the prices for new and repeat customers are equal to the respective net unit production cost plus the net unit environmental impact (i.e., $p_{s,2}^{n*} = c_2 + \kappa_2$ and $p_{s,2}^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$). Moreover, the equilibrium social welfare is linear in the realized market size (X_2^n, X_2^r).

In period 1, strategic customers base their purchasing decisions on their rational expectations, whereas myopic customers decide whether to make a purchase by comparing the current price and the expected valuation. Let $(p_{s,1}^*, Q_{s,1}^*)$ denote the equilibrium first-period price and production quantity with strategic customers, and $(\tilde{p}_{s,1}^*, \tilde{Q}_{s,1}^*)$ denote those with myopic customers. As in the base model and the NTR model, we introduce the first-period effective marginal welfare with either strategic or myopic customers, $m_{s,1}^* = \tilde{m}_{s,1}^* := \mu + \delta[\beta_{s,r}^* - \beta_{s,n}^*]$. The following lemma characterizes the social welfare maximizing equilibrium outcomes.

LEMMA 4 *(a) With strategic customers, we have (i)* $p^*_{s,1} = m^*_{s,1}$; *(ii)* $Q^*_{s,1} = \bar{F}^{-1}(\frac{c_1+\kappa_1-r_1-\iota_1}{m^*_{s,1}-r_1-\iota_1})$; *and (iii) the equilibrium expected social welfare is* $W^*_s = (m^*_{s,1} - r_1 - \iota_1)\mathbb{E}(X \wedge Q^*_{s,1}) - (c_1 + \kappa_1 - r_1 - \iota_1)Q^*_{s,1} + \delta\beta^*_{s,n}\mathbb{E}[X]$.

(b) With myopic customers, we have (i) $\tilde{p}^*_{s,1} = \mu$; *(ii)* $\tilde{Q}^*_1 = \bar{F}^{-1}(\frac{c_1+\kappa_1-r_1-\iota_1}{\tilde{m}^*_{s,1}-r_1-\iota_1})$; *and (iii) the equilibrium expected social welfare is* $\tilde{W}^*_s = (\tilde{m}^*_{s,1} - r_1 - \iota_1)\mathbb{E}(X \wedge \tilde{Q}^*_{s,1}) - (c_1 + \kappa_1 - r_1 - \iota_1)\tilde{Q}^*_{s,1} + \delta\beta^*_{s,n}\mathbb{E}[X]$.

(c) Let $e^*_s := \beta^*_{s,r} - \beta^*_{s,n}$. *Then, we have (i)* $p^*_{s,1} \geq \tilde{p}^*_{s,1}$ *if and only if* $e^*_s \geq 0$; *(ii)* $Q^*_{s,1} = \tilde{Q}^*_{s,1}$; *and (iii)* $W^*_s = \tilde{W}^*_s$.

Since the social planner needs to balance firm profit, customer surplus, and environmental impact, whereas the firm maximizes its own profit only, the social-welfare-maximizing equilibrium outcome may be quite different from the profit-maximizing one, as shown by comparing Lemma 4 with Theorem 1. In particular, if the unit environmental impacts, κ_1 and κ_2, are sufficiently large, the social planner will set lower production quantities than the firm will do to limit the total environmental impacts. Lemma 4(c) characterizes how different customer behaviors influence the social-welfare-maximizing RE equilibrium outcome. Specifically, we show that the expected optimal social welfare with strategic customers is the same as that with myopic customers, and so is the optimal first-period production quantity. The equilibrium first-period price, however, depends on customer behavior. If the expected surplus of a repeat customer dominates that of a new customer (i.e., $e^*_s \geq 0$), the equilibrium first-period price is higher with strategic customers. Otherwise, $e^*_s < 0$, the equilibrium first-period price is higher with myopic customers. We notice that e^*_s is the counterpart of e^* (see Theorem 2), both of which characterize the additional expected utility of a repeat customer over a new one in period 2.

We now analyze how the government, whose objective is to maximize the expected social welfare W_s, could induce the firm, whose objective is to maximize his expected profit Π_f, to set the socially optimal prices and production quantities under different customer behaviors. A commonly-observed government subsidization policy is to subsidize the firm or customers for the remanufactured products (see, e.g., Mitra and Webster, 2008; Chen, 2015). To model this subsidization policy, we assume that the government offers the firm a per-unit subsidy s_r for remanufacturing leftover inventory and used products. The per-unit subsidy to the firm is without loss of generality, because all results and qualitative insights in this section continue to hold with the per-unit subsidy to customers, and the proportional subsidy[1] to either the firm or the customers. For expositional ease, we take the approach of per-unit subsidy to the firm.

We first study how the government subsidization policy for remanufactured products would influence the equilibrium outcome in the following theorem.

[1]The proportional subsidy refers to the government subsidization scheme under which the unit subsidy is proportional to (e.g., 10% of) the sales price/trade-in price.

Theorem **7** *(a) For any* (X_2^n, X_2^r)*, we have (i)* p_2^{r*} *is decreasing in* s_r*; and (ii)* $Q_2^r(X_2^n, X_2^r)$ *is increasing in* s_r*.*

(b) With strategic customers, we have (i) p_1^* *is increasing in* s_r*; (ii)* Q_1^* *is increasing in* s_r*; (iii)* Π_f^* *is increasing in* s_r*; and (iv)* I_e^* *is increasing in* s_r*.*

(c) With myopic customers, we have (i) $\tilde{p}_1^*$ *is independent of* s_r*; (ii)* $\tilde{Q}_1^*$ *is increasing in* s_r*; (iii)* $\tilde{\Pi}_f^*$ *is increasing in* s_r*; and (iv)* $\tilde{I}_e^*$ *is increasing in* s_r*.*

One of the main goals for the government to subsidize remanufacturing is to improve the environment (see Chen, 2015; The Recycler, 2015). Theorem 7(b,c), however, suggests that if the government only subsidizes for remanufacturing (i.e., $s_r > 0$), the environment actually suffers from this subsidization policy with either strategic or myopic customers (i.e., I_e^* and $\tilde{I}_e^*$ are increasing in s_r). This result follows from the rationale that subsidizing remanufactured products not only promotes the adoption of remanufacturing, but also increases the production levels of the first-generation product, which is the least environmentally friendly product version. The environment thus suffers from the increased production levels under the subsidization for remanufacturing alone. Therefore, the government should be careful about designing the subsidization policy, because haphazard subsidization for remanufacturing may result in an undesired outcome.

Motivated by the discrepancy between the intention and outcome of a commonly used government subsidization policy for remanufacturing, we consider an alternative more general government policy that subsidizes for/taxes on the production of both generation products and remanufacturing. Some other comprehensive government subsidization policies on production, recycling, remanufacturing, and trade-in rebates are discussed in, e.g., Webster and Mitra (2007); Ma et al. (2013), and Wang et al. (2014). The goal of such government subsidization programs is to promote the development of remanufacturing, curb pollution, and stimulate consumption. We assume that government subsidies (taxes) are provided (charged) for the sales of both generation products, and recycling/remanufacturing the leftover inventory and used products. Specifically, let $s_g := (s_1, s_2, s_r)$ denote the subsidy/tax scheme the government adopts. The government offers the firm a per-unit subsidy s_1 for sales of the first-generation product, a per-unit subsidy s_2 for sales of the second-generation product, and a per-unit subsidy s_r for remanufacturing. If $s_i < 0$ $(i = 1, 2, r)$, the firm taxes on the sales of the product or remanufacturing leftover inventory and used products. In particular, we remark that the aforementioned most common government subsidization policy for remanufacturing alone is a special case of this general subsidy/tax scheme with $s_1 = 0$, $s_2 = 0$, and $s_r > 0$.

We now analyze how the government should design the linear subsidy/tax scheme to induce the socially optimal outcome under different customer behaviors.

Theorem **8** *(a) With strategic customers, there exists a unique linear subsidy/tax scheme* $s_g^* = (s_1^*, s_2^*, s_r^*)$*, under which the socially optimal RE equilibrium outcome is achieved.*

Moreover, we have (i) s_2^* is the unique solution to $p_{s,2}^{n*} = \mathrm{argmax}_{p_2^n \geq 0}\{(p_2^n + s_2 - c_2)\bar{G}(\frac{p_2^n}{1+\alpha})\}$; (ii) s_r^* is the unique solution to $p_{s,2}^{r*} = \mathrm{argmax}_{p_2^r \geq 0}\{(p_2^r + s_r + s_2^* - c_2 + r_2)\bar{G}(\frac{p_2^r}{k+\alpha})\}$; (iii) s_1^* is the unique solution to $\frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*} = \frac{c_1-r_1}{m_1^s(s_1)-r_1}$, where $m_1^s(s_1) := s_1 + m_{s,1}^* + \delta[(\kappa_2 + s_2^* + s_r^* - \iota_2)\bar{G}(\frac{p_{s,2}^{r*}}{k+\alpha}) - (\kappa_2 + s_2^*)\bar{G}(\frac{p_{s,2}^{n*}}{1+\alpha})]$; (iv) s_1^* is decreasing in κ_1, s_2^* is decreasing in κ_2, and s_r^* is increasing in ι_2; and (v) there exists a threshold vector $(\bar{\kappa}_1^s, \bar{\kappa}_2^s, \bar{\iota}_2^s)$, such that $s_1^* \geq 0$ if and only if $\kappa_1 \leq \bar{\kappa}_1^s$, $s_2^* \geq 0$ if and only if $\kappa_2 \leq \bar{\kappa}_2^s$, and $s_r^* \geq 0$ if and only if $\iota_2 \geq \bar{\iota}_2^s$.

(b) *With myopic customers, there exists a unique linear subsidy/tax scheme $\tilde{s}_g^* = (\tilde{s}_1^*, \tilde{s}_2^*, \tilde{s}_r^*)$, under which the socially optimal RE equilibrium outcome is achieved. Moreover, we have (i) $\tilde{s}_2^* = s_2^*$; (ii) $\tilde{s}_r^* = s_r^*$; (iii) $\tilde{s}_1^*$ is the unique solution to $\frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{\tilde{m}_{s,1}^*-r_1-s_r^*} = \frac{c_1-r_1}{\tilde{m}_1^s(s_1)-r_1}$, where $\tilde{m}_1^s(s_1) := s_1 + \mu + \delta[(\kappa_2 + s_2^* + s_r^* - \iota_2)\bar{G}(\frac{p_{s,2}^{r*}}{k+\alpha}) - (\kappa_2 + s_2^*)\bar{G}(\frac{p_{s,2}^{n*}}{1+\alpha})]$; (iv) $\tilde{s}_1^*$ is decreasing in κ_1; and (v) there exists a threshold $\tilde{\bar{\kappa}}_1^s$, such that $\tilde{s}_1^* \geq 0$ if and only if $\kappa_1 \leq \tilde{\bar{\kappa}}_1^s$.*

(c) *We have (i) $s_1^* \geq \tilde{s}_1^*$ if and only if $e_s^* \leq 0$; and (ii) $\bar{\kappa}_1^s \geq \tilde{\bar{\kappa}}_1^s$ if and only if $e_s^* \leq 0$, where e_s^* is defined in Lemma 4(c).*

Theorem 8 demonstrates that the government can use a simple linear subsidy/tax scheme to induce the socially optimal outcome in the scenarios with either strategic or myopic customers. The linear subsidy/tax policy s_g helps control the margin of the firm and the willingness-to-pay of the customers. Hence, the government can use this incentive scheme to regulate the market and ensure the firm sets the socially optimal prices and production quantities with either strategic or myopic customers. More specifically, in both scenarios, the government should provide a combined subsidy/tax scheme for the sales of both product generations and the recycle of leftover inventory and used products. Since some components in s_g^* and $\tilde{s}_g^*$ may be negative, it is possible that the government taxes the firm on some product versions to discourage their sales. This phenomenon results from the government's goal of balancing the tradeoff between firm profit, customer surplus, and environmental impact. In particular, with either strategic or myopic customers, the government subsidizes more for (taxes less on) the sales of one product version if its unit environmental impact increases. Analogously, more subsidies (less taxes) should be provided for (charged on) remanufacturing if its unit environmental benefit is higher.

Comparing the scenarios with strategic and myopic customers (i.e., Theorem 8(c)) sheds light on how different customer behaviors influence the optimal government subsidy policy. We find that the optimal subsidy/tax rates for the second-generation product and remanufacturing are independent of whether the customers are strategic or myopic (i.e., $\tilde{s}_2^* = s_2^*$ and $\tilde{s}_r^* = s_r^*$). The optimal subsidy/tax rate for the first-generation product, however, is sensitive to customer behavior. The government should provide a higher subsidy/lower tax for sales of the first-generation product with strategic customers than with myopic customers (i.e., $s_1^* \geq \tilde{s}_1^*$) if and only if, in period 2, the expected surplus of a new customer dominates that of a repeat customer (i.e., $e_s^* \leq 0$). If $e_s^* \leq 0$, strategic customers are reluctant to make an immediate purchase, so, to regulate the market with strategic customers, the government should provide more subsidies for the sales of the first-generation product to induce early purchases. On the other hand, if $e_s^* > 0$, a repeat customer has higher expected surplus in period 2, and thus strategic customers are more willing to purchase the product immediately in period 1. In this case, to discourage strategic customers from overconsumption in period 1, the government offers less subsidies for the sales of the first-generation product with strategic customers than it does with myopic customers. The rationale behind the dichotomy in Theorem 8(c) is that, with the adoption of trade-in remanufacturing, strategic customers anticipate both the purchasing option as a new customer and the trade-in option as a repeat customer. Depending on which option has a higher expected utility, a strategic customer may have a higher or lower willingness-to-pay than a myopic customer does. Hence, the government may provide higher or lower incentives in period 1 to encourage or discourage the early purchases of strategic customers accordingly.

Based on Theorem 8, we now compare the total government costs of the optimal subsidy/tax scheme under different customer behaviors. For any subsidy/tax scheme s_g, we denote $C_g(s_g)$ ($\tilde{C}_g(s_g)$) as the associated expected total government cost under RE equilibrium with strategic (myopic) customers. Define $C_g^* := C_g(s_g^*)$ and $\tilde{C}_g^* := \tilde{C}_g(\tilde{s}_g^*)$ as the social-welfare-maximizing government costs with strategic and myopic customers, respectively.

Theorem **9** *(a)* $C_g^* - \tilde{C}_g^* = (s_1^* - \tilde{s}_1^*)\mathbb{E}(X \wedge Q_{s,1}^*)$.

(b) $C_g^* \geq \tilde{C}_g^*$ *if and only if* $e_s^* \leq 0$. *Moreover, there exists a threshold* $\bar{\mathcal{V}}_2 > 0$, *such that* $e_s^* \leq 0$, *if and only if* $r_2 + \iota_2 \leq \bar{\mathcal{V}}_2$.

Theorem 9 compares the social-welfare-maximizing government costs in scenarios with strategic and myopic customers. Specifically, we show that the total cost to regulate a market with strategic customers is higher than with myopic customers whenever the socially optimal subsidy for the first-generation product with strategic customers dominates that with myopic customers (i.e., $s_1^* \geq \tilde{s}_1^*$). Equivalently, according to Theorem 9(b), it costs the government more to regulate a market with strategic customers if the expected surplus of a new customer dominates that of a repeat customer in period 2 (i.e., $e_s^* \leq 0$). In this case, more subsidies should be provided to incentivise the more reluctant strategic customers to make an early purchase in period 1. Another implication of Theorem 9(b) is that if the total unit economic and environmental value of remanufacturing, $r_2 + \iota_2$, is sufficiently low (i.e., below the threshold $\bar{\mathcal{V}}_2$), the total government cost is lower with strategic customers. Therefore, our analysis delivers the new insight to the literature that strategic customer behavior has a negative (positive) impact upon the government if the total economic and environmental value of remanufacturing is low (high).

6 Conclusion

In this paper, we develop an analytical model to study how different customer behaviors influence the economic and environmental values of trade-in remanufacturing. From the firm's perspective, we show that trade-in remanufacturing is generally much more valuable with strategic customers than with myopic customers. This is because a trade-in rebate essentially offers an early purchase reward and thus can deliver additional value by exploiting the forward-looking behavior of strategic customers. In particular, with the adoption of trade-in remanufacturing, strategic customer behavior may help increase the firm's profit, which contrasts the common belief in the literature that strategic customer behavior hurts firm profit. In the trade-in remanufacturing setting, the price discount in the second period increases with the revenue generated from remanufacturing; thus when the revenue-generating effect is strong enough, the willingness-to-pay of strategic customers in the first period could be even higher than that of myopic customers, which allows the firm to extract more profit with strategic customers.

From the environmental perspective, trade-in remanufacturing decreases the unit environmental impact, but increases the production quantities through the early-purchase inducing effect with strategic customers. Moreover, under strategic customer behavior, adopting trade-in remanufacturing may decrease the customer surplus and social welfare. Hence, with strategic customers, caution is needed on the adoption of trade-in remanufacturing, because it could be detrimental to the environment and the society. With myopic customers, however, trade-in remanufacturing leads to a lower first-period production quantity in general. Our results indicate that customer behavior plays an important role in the value of trade-in remanufacturing. Specifically, with strategic customers, trade-in remanufacturing may create a tension between firm profitability and environmental sustainability; but, with myopic customers, it generally benefits both the firm and the environment.

To resolve the above tension caused by trade-in remanufacturing, we also study how the government should design a regulatory policy to balance firm profit, customer surplus, and environmental impact. A commonly observed policy is to subsidize the remanufactured products. However, we find that despite its intention to protect the environment, such a policy fails to achieve the social optimum and is actually harmful to the environment. To achieve the socially optimal outcome, we show that it suffices for the government to employ a simple linear incentive scheme. This scheme imposes either subsidy or tax on the sales of both product generations as well as the remanufactured products: A subsidy (tax) should be applied if the environmental impact of the product is sufficiently low (high).

Our research can be extended in several directions. First, this paper considers a product with only two generations. A natural extension is to study a multi-period model in which the product can have three or more generations. Second, the market may consist of multiple competing firms that offer partially substitutable products. It would be interesting to examine how competition affects the adoption of trade-in remanufacturing. Finally, one may extend the current model to consider supply chain settings. How trade-in remanufacturing affects the supply chain performance is also a promising direction for future research.

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Appendix A: Equilibrium Definitions

We now give the definitions of the RE equilibria in the four scenarios considered in this paper: (a) the base model with strategic customers, (b) the base model with myopic customers, (c) the NTR model with strategic customers, and (d) the NTR model with myopic customers. Let $A(Q_1) := \mathbb{E}[X \wedge Q_1]/\mathbb{E}[X]$ ($Q_1 \geq 0$) be the availability function given the first-period production quantity Q_1 (see Su and Zhang, 2009).

Definition 1 (Base model with strategic customers.) An RE equilibrium in the base model with strategic customers consists of $(p_1^*, Q_1^*, \xi_r^*, \mathfrak{r}^*, \mathfrak{a}^*, \mathfrak{p}_2^{n*}, \mathfrak{p}_2^{r*})$ satisfying

(a) $p_1^* = \mathfrak{r}^*$; $Q_1^* = \text{argmax}_{Q_1 \geq 0} \Pi_f(Q_1)$ where $\Pi_f(\cdot)$ is given in Lemma 2;

(b) $\xi_r^* = \mu + \delta \mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{r*}]^+ - \delta \mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+$;

(c) $\mathfrak{r}^* = \xi_r^*$;

(d) $\mathfrak{a}^* = A(Q_1^*)$; $(\mathfrak{p}_2^{n*}, \mathfrak{p}_2^{r*}) \stackrel{d}{=} (p_2^{n*}, p_2^{r*})$.

Definition 2 (Base model with myopic customers.) An RE equilibrium in the base model with myopic customers consists of $(\tilde{p}_1^*, \tilde{Q}_1^*, \tilde{\xi}_r^*, \tilde{\mathfrak{r}}^*)$ satisfying

(a) $\tilde{p}_1^* = \tilde{\mathfrak{r}}^*$; $\tilde{Q}_1^* = \text{argmax}_{Q_1 \geq 0} \tilde{\Pi}_f(Q_1)$ where $\tilde{\Pi}_f(\cdot)$ is given in Lemma 2;

(b) $\tilde{\xi}_r^* = \mu$;

(c) $\tilde{\mathfrak{r}}^* = \tilde{\xi}_r^*$.

Definition 3 (NTR model with strategic customers.) An RE equilibrium in the NTR model with strategic customers consists of $(p_1^{u*}, Q_1^{u*}, \xi_r^{u*}, \mathfrak{r}^{u*}, \mathfrak{a}^{u*}, \mathfrak{p}_2^{u*})$ satisfying

(a) $p_1^{u*} = \mathfrak{r}^{u*}$; $Q_1^{u*} = \text{argmax}_{Q_1 \geq 0} \Pi_f^u(Q_1)$, where $\Pi_f^u(\cdot)$ is given in Lemma 2;

(b) $\xi_r^{u*} = \mu + \delta \mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{u*}]^+ - \delta \mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{u*}]^+$;

(c) $\mathfrak{r}^{u*} = \xi_r^{u*}$;

(d) $\mathfrak{a}^{u*} = A(Q_1^{u*})$; $\mathfrak{p}_2^{u*} \stackrel{d}{=} p_2^u((X - Q_1^{u*})^+, X \wedge Q_1^{u*})$, where $p_2^u(\cdot,\cdot)$ is characterized in Theorem 3(a).

Definition 4 (NTR model with myopic customers.) An RE equilibrium in the NTR model with myopic customers consists of $(\tilde{p}_1^{u*}, \tilde{Q}_1^{u*}, \tilde{\xi}_r^{u*}, \tilde{\mathfrak{r}}^{u*})$ satisfying

(a) $\tilde{p}_1^{u*} = \tilde{\mathfrak{r}}^{u*}$; $\tilde{Q}_1^{u*} = \text{argmax}_{Q_1 \geq 0} \tilde{\Pi}_f^u(Q_1)$ where $\tilde{\Pi}_f^u(\cdot)$ is given in Lemma 2;

(b) $\tilde{\xi}_r^{u*} = \mu$;

(c) $\tilde{\mathfrak{r}}^{u*} = \tilde{\xi}_r^{u*}$.

In Definitions 1-4, conditions (a) and (b) follow from that the decisions are optimal given the rational beliefs, and conditions (c) and (d) follow from that the rational beliefs are consistent with actual outcomes.

Appendix B: Proofs of Statements

We use $h_1'(\cdot)$ to denote the derivative operator of a single variable function $h_1(\cdot)$, $\partial_x h_2(\cdot)$ to denote the partial derivative operator of a multi-variable function, $h_2(\cdot)$, with respect to variable x, and $1_{\{\cdot\}}$ to denote the indicator function. For any multivariate continuously differentiable function $h_2(x_1, x_2, \cdots, x_n)$ and $x' := (x_1', x_2', \cdots, x_n')$ in $h_2(\cdot)$'s domain, $\forall i$, we use $\partial_{x_i} h_2(x_1', x_2', \cdots, x_n')$ to denote $\partial_{x_i} h_2(x_1, x_2, \cdots, x_n)|_{x=x'}$.

Proof of Lemma 1: Part (a). Given (p_2^n, p_2^r) ($p_2^r \le p_2^n$), a new customer will make a purchase if and only if $(1+\alpha)V \ge p_2^n$, whereas a repeat customer will make a purchase if and only if $(k+\alpha)V \ge p_2^r$. Thus, the *ex ante* probability that a new customer will purchase the second-generation product is $\bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$, whereas the probability that a repeat customer will join the trade-in program is $\bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$. Therefore, conditioned on the realized market size (X_2^n, X_2^r), the expected profit of the firm in period 2 is given by:

$$\begin{aligned}\Pi_2(p_2^n, p_2^r | X_2^n, X_2^r) :=& X_2^n(p_2^n - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right) + X_2^r(p_2^r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right) \\ =& X_2^n v_2^n(p_2^n) + X_2^r v_2^r(p_2^r),\end{aligned} \tag{1}$$

where $v_2^n(p_2^n) := (p_2^n - c_2)\bar{G}(\frac{p_2^n}{1+\alpha})$ and $v_2^r(p_2^r) := (p_2^r - c_2 + r_2)\bar{G}(\frac{p_2^r}{k+\alpha})$. We now show that $v_2^n(\cdot)$ is quasiconcave in p_2^n, and $v_2^r(\cdot)$ is quasiconcave in p_2^r. Note that

$$\partial_{p_2^n} v_2^n(p_2^n) = -\left(\frac{p_2^n - c_2}{1+\alpha}\right) g\left(\frac{p_2^n}{1+\alpha}\right) + \bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$$

and

$$\partial_{p_2^r} v_2^r(p_2^r) = -\left(\frac{p_2^r - c_2 + r_2}{k+\alpha}\right) g\left(\frac{p_2^r}{k+\alpha}\right) + \bar{G}\left(\frac{p_2^r}{k+\alpha}\right).$$

Because $g(v)/\bar{G}(v)$ is continuously increasing in v, $g(\frac{p_2^n}{1+\alpha})/\bar{G}(\frac{p_2^n}{1+\alpha})$ is continuously increasing in p_2^n and $g(\frac{p_2^r}{k+\alpha})/\bar{G}(\frac{p_2^r}{k+\alpha})$ is continuously increasing in p_2^r. Hence, $\partial_{p_2^n} v_2^n(p_2^n) = 0$ has a unique solution p_2^{n*} and $\partial_{p_2^r} v_2^r(p_2^r) = 0$ has a unique solution p_2^{r*}, where $v_2^n(\cdot)$ $[v_2^r(\cdot)]$ is strictly increasing on $[0, p_2^{n*})$ $[[0, p_2^{r*})]$ and strictly decreasing on $(p_2^{n*}, +\infty)$ $[(p_2^{r*}, +\infty)]$. Therefore, for any realized (X_2^n, X_2^r), $X_2^n v_2^n(\cdot)$ is quasiconcave in p_2^n, and $X_2^r v_2^r(\cdot)$ is quasiconcave in p_2^r. Thus, for any realized (X_2^n, X_2^r), $(p_2^n(X_2^n, X_2^r), p_2^r(X_2^n, X_2^r)) = (p_2^{n*}, p_2^{r*})$ maximizes $\Pi_2(\cdot, \cdot | X_2^n, X_2^r)$.

It remains to show that $p_2^{n*} > p_2^{r*}$ if and only if $k < 1$ or $r_2 > 0$. Note that p_2^{n*} satisfies

$$\left(\frac{p_2^{n*} - c_2}{1+\alpha}\right) \frac{g\left(\frac{p_2^{n*}}{1+\alpha}\right)}{\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)} = 1, \tag{2}$$

and p_2^{r*} satisfies

$$\left(\frac{p_2^{r*} - c_2 + r_2}{k+\alpha}\right) \frac{g\left(\frac{p_2^{r*}}{k+\alpha}\right)}{\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)} = 1. \tag{3}$$

If $k < 1$ or $r_2 > 0$, $\frac{p_2^{n*} - c_2 + r_2}{k+\alpha} > \frac{p_2^{n*} - c_2}{1+\alpha}$, and the increasing failure rate condition implies that $g\left(\frac{p_2^{n*}}{k+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right) \ge g\left(\frac{p_2^{n*}}{1+\alpha}\right) / \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$. Thus,

$$\left(\frac{p_2^{n*} - c_2 + r_2}{k+\alpha}\right) \frac{g\left(\frac{p_2^{n*}}{k+\alpha}\right)}{\bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right)} > \left(\frac{p_2^{n*} - c_2}{1+\alpha}\right) \frac{g\left(\frac{p_2^{n*}}{1+\alpha}\right)}{\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)} = 1,$$

and, hence, $\partial_{p_2^r} v_2^r(p_2^{n*}) < 0$. Since $v_2^r(\cdot)$ is quasiconcave, $p_2^{r*} < p_2^{n*}$. On the other hand, if $k = 1$ and $r_2 = 0$, $v_2^n(\cdot) \equiv v_2^r(\cdot)$ and thus $p_2^{n*} = p_2^{r*}$. This completes the proof of **Part (a)**.

Part (b). Because all new customers with willingness-to-pay $(1+\alpha)V$ greater than $p_2^n(X_2^n, X_2^r) \equiv p_2^{n*}$ would make a purchase. Hence,

$$Q_2^n(X_2^n, X_2^r) = \mathbb{E}[X_2^n 1_{\{(1+\alpha)V \geq p_2^{n*}\}} | X_2^n] = \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) X_2^n.$$

Analogously, all repeat customers with willingness-to-pay $(k+\alpha)V$ greater than $p_2^r(X_2^n, X_2^r) \equiv p_2^{r*}$ would make a purchase. Hence,

$$Q_2^r(X_2^n, X_2^r) = \mathbb{E}[X_2^r 1_{\{(k+\alpha)V \geq p_2^{r*}\}} | X_2^r] = \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) X_2^r.$$

This proves **Part (b)**.

Part (c). Since $\pi_2(X_2^n, X_2^r) := \max\{\Pi_2(p_2^n, p_2^r | X_2^n, X_2^r) : 0 \leq p_2^r \leq p_2^n\}$, it follows immediately that

$$\pi_2(X_2^n, X_2^r) = [\max v_2^n(p_2^n)] X_2^n + [\max v_2^r(p_2^r)] X_2^r.$$

To complete the proof, it remains to show that $\beta_n^* = [\max v_2^n(p_2^n)] > 0$ and $\beta_r^* = [\max v_2^r(p_2^r)] > 0$. By equations (2) and (3), we have $p_2^{n*} - c_2 > 0$, $\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) > 0$, $p_2^{r*} - c_2 + r_2 > 0$, and $\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) > 0$. Hence, $\beta_n^* = (p_2^{n*} - c_2)\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) > 0$ and $\beta_r^* = (p_2^{r*} - c_2 + r_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) > 0$. This completes the proof of **Part (c)**. $\square$

Proof of Theorem 1: Part (a). Since ξ_r^* satisfies that $\mathcal{U}_p = \mathcal{U}_w$, we have

$$\mathfrak{a}^*(\mathbb{E}[V] + \delta\mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{r*}]^+ - \xi_r^*) + (1-\mathfrak{a}^*)\delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+ = \delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+.$$

Direct algebraic manipulation yields that $\xi_r^* = \mu + \delta\mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+$. Hence, by Definition 1 and Lemma 1(a),

$$\begin{aligned} p_1^* = \mathfrak{r}_1^* = \xi_r^* &= \mu + \delta\mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+ \\ &= \mu + \delta\mathbb{E}[(k+\alpha)V - p_2^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - p_2^{n*}]^+. \end{aligned}$$

Hence,

$$\begin{aligned} \Pi_f(Q_1) &= p_1^*\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + r_1\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{\pi_2(X - (X \wedge Q_1), X \wedge Q_1)\} \\ &= (p_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[\beta_n^*(X - (X \wedge Q_1)) + \beta_r^*(X \wedge Q_1)] \\ &= (p_1^* + \delta(\beta_r^* - \beta_n^*) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_n^*\mathbb{E}(X) \\ &= (m_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_n^*\mathbb{E}(X), \end{aligned}$$

where the second equality follows from $(Q_1 - X)^+ = Q_1 - (X \wedge Q_1)$, and the last from the identity $m_1^* = \mu + \delta\mathbb{E}[(k+\alpha)V - \mathfrak{p}_2^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_2^{n*}]^+ + \delta(\beta_r^* - \beta_n^*)$. Therefore, Q_1^* is the solution to a newsvendor problem with marginal revenue $m_1^* - r_1$, marginal cost $c_1 - r_1$, and demand distribution $F(\cdot)$. Hence, $Q_1^* = \bar{F}^{-1}(\frac{c_1 - r_1}{m_1^* - r_1})$ and $\Pi_f^* = \Pi_f(Q_1^*) = (m_1^* - r_1)\mathbb{E}(X \wedge Q_1^*) - (c_1 - r_1)Q_1^* + \delta\beta_n^*\mathbb{E}(X)$. This proves **Part (a)**.

Part (b). Since myopic customers will make a purchase if and only if $p_1 \leq \mu$, $\tilde{p}_1^* = \tilde{\xi}_1^* = \mu$. Hence,

$$\begin{aligned} \tilde{\Pi}_f(Q_1) &= \tilde{p}_1^*\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + r_1\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{\pi_2(X - (X \wedge Q_1), X \wedge Q_1)\} \\ &= (\tilde{p}_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[\beta_n^*(X - (X \wedge Q_1)) + \beta_r^*(X \wedge Q_1)] \\ &= (\tilde{p}_1^* + \delta(\beta_r^* - \beta_n^*) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_n^*\mathbb{E}(X) \\ &= (\tilde{m}_1^* - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\beta_n^*\mathbb{E}(X), \end{aligned}$$

where the second equality follows from $(Q_1 - X)^+ = Q_1 - (X \wedge Q_1)$, and the last from the identity $\tilde{m}_1^* = \mu + \delta(\beta_r^* - \beta_n^*)$. Therefore, $\tilde{Q}_1^*$ is the solution to a newsvendor problem with marginal revenue $\tilde{m}_1^* - r_1$, marginal cost $c_1 - r_1$, and demand distribution $F(\cdot)$. Hence, $\tilde{Q}_1^* = \bar{F}^{-1}(\frac{c_1 - r_1}{m_1^* - r_1})$ and $\tilde{\Pi}_f^* = \tilde{\Pi}_f(\tilde{Q}_1^*) = (m_1^* - r_1)\mathbb{E}(X \wedge \tilde{Q}_1^*) - (c_1 - r_1)\tilde{Q}_1^* + \delta\beta_n^*\mathbb{E}(X)$. This proves **Part (b)**. $\square$

Proof of Lemma 2: The expressions for $\Pi_f(\cdot)$ and $\tilde{\Pi}_f(\cdot)$ have already been given in the proof of Theorem 1(a) and Theorem 1(b), respectively. We now compute $\Pi_f^u(Q_1)$. Following the same argument as the proof of Theorem 1(a), given the first-period production quantity Q_1, the first-period equilibrium price is

$$\begin{aligned} p_1^u(Q_1) =& \mathbb{E}[V] + \delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+] \\ =& \mu + \delta[\mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+ - \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+], \end{aligned}$$

where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. Let $\pi_2^u(X_2^n, X_2^r) := \max_{p_2^u} \Pi_2^u(p_2^u | X_2^n, X_2^r)$. Hence,

$$\begin{aligned} \pi_2^u(X_2^n, X_2^r) &= \max_{p_2^u \geq 0}\{X_2^n(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{1+\alpha}\right) + X_2^r(p_2^u - c_2)\bar{G}(\frac{p_2^u}{k+\alpha})\} \\ &= X_2^n(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) + X_2^r(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right), \end{aligned}$$

where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. Therefore,

$$\begin{aligned} \Pi_f^u(Q_1) &= p_1^u(Q_1)\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + r_1\mathbb{E}(X - Q_1)^+ + \delta\mathbb{E}[\pi_2^u(X_2^n, X_2^r)] \\ &= (p_1^u(Q_1) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[(X - Q_1)^+(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) \\ &\quad +(X \wedge Q_1)(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)] \\ &= (p_1^u(Q_1) + \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right) - (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)] \\ &\quad -r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)X] \\ &= (m_1^u(Q_1) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)X], \end{aligned}$$

where

$$\begin{aligned} m_1^u(Q_1) : &= \mu + \delta\{\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)] + \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+ \\ &\quad -\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)] - \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+\}, \end{aligned}$$

with $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$.

Analogously, since $\tilde{p}_1^{u*} = \mathbb{E}[V] = \mu$,

$$\begin{aligned}
\tilde{\Pi}_f^u(Q_1) &= \tilde{p}_1^{u*}\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + r_1 \mathbb{E}(X - Q_1)^+ + \delta \mathbb{E}[\pi_2^u(X_2^n, X_2^r)] \\
&= (\mu - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta \mathbb{E}[(X - Q_1)^+ (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) \\
&\quad +(X \wedge Q_1)(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)] \\
&= (\mu + \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right) - (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)] \\
&\quad - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) X] \\
&= (\tilde{m}_1^u(Q_1) - r_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1)Q_1 + \delta \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) X],
\end{aligned}$$

where

$$\tilde{m}_1^u(Q_1) := \mu + \delta\{\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)] - \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)]\},$$

with $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. $\square$

Before giving the proof of Theorem 2, we first prove Theorem 3.

Proof of Theorem 3: Part (a). If the firm charges a single price p_2^u in period 2, all new (repeat) customers with willingness-to-pay $(1+\alpha)V$ ($(k+\alpha)V$) greater than p_2^u will make a purchase (join the trade-in program). Hence, the second-period profit function of the firm $\Pi_2^u(p_2^u|X_2^n, X_2^r)$ is given by

$$\begin{aligned}
\Pi_2^u(p_2^u|X_2^n, X_2^r) &= X_2^n(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{1+\alpha}\right) + X_2^r(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{k+\alpha}\right) \\
&= X_2^n v_2^n(p_2^u) + X_2^r \hat{v}_2^r(p_2^u),
\end{aligned}$$

where $\hat{v}_2^r(p_2) := (p_2 - c_2)\bar{G}\left(\frac{p_2}{k+\alpha}\right)$. Clearly, $\hat{v}_2^r(\cdot)$ has a unique maximizer $\hat{p}_2^{r*}$, where $\hat{p}_2^{r*} \geq p_2^{r*}$ with the inequality being strict if $r_2 > 0$. Moreover, $\Pi_2^u(p_2^u|X_2^n, X_2^r) = \hat{\Pi}_2(p_2^u, p_2^u|X_2^n, X_2^r)$, where, by the proof of Lemma 1(a), $\hat{\Pi}_2(p_2^n, p_2^r|X_2^n, X_2^r) := X_2^n v_2^n(p_2^n) + X_2^r \hat{v}_2^r(p_2^r)$ is quasiconcave function of (p_2^n, p_2^r). Thus, the equilibrium second-period pricing strategy $p_2^u(X_2^n, X_2^r)$ is the maximizer of the second-period profit function, i.e., $p_2^u(X_2^n, X_2^r) = \mathrm{argmax}_{p_2^u \geq 0}\Pi_2^u(p_2^u|X_2^n, X_2^r)$. Note that since $\hat{\Pi}_2(\cdot, \cdot|X_2^n, X_2^r)$ is quasiconcave in (p_2^n, p_2^r), $\Pi_2^u(p_2^u|X_2^n, X_2^r) = \hat{\Pi}_2(p_2^u, p_2^u|X_2^n, X_2^r)$ is also quasiconcave in p_2^u.

Observe that

$$\partial_{p_2^u}\Pi_2^u(p_2^u|X_2^n, X_2^r) = X_2^n\left[\bar{G}\left(\frac{p_2^u}{1+\alpha}\right) - \left(\frac{p_2^u - c_2}{1+\alpha}\right) g\left(\frac{p_2^u}{1+\alpha}\right)\right] + X_2^r\left[\bar{G}\left(\frac{p_2^u}{k+\alpha}\right) - \left(\frac{p_2^u - c_2}{k+\alpha}\right) g\left(\frac{p_2^u}{k+\alpha}\right)\right].$$

Since $g(v)/\bar{G}(v)$ is increasing in v, $\partial_{p_2^u}\Pi_2^u(p_2^u|X_2^n, X_2^r) < 0$ if $p_2^u > p_2^{n*}$, and $\partial_{p_2^u}\Pi_2^u(p_2^u|X_2^n, X_2^r) > 0$ if $p_2^u < \hat{p}_2^{r*}$. Thus,

$$p_2^u(X_2^n, X_2^r) \in [\hat{p}_2^{r*}, p_2^{n*}] \subset [p_2^{r*}, p_2^{n*}] = [p_2^r(X_2^n, X_2^r), p_2^n(X_2^n, X_2^r)].$$

If $k < 1$, by the proof of Lemma 1(a), $\hat{p}_2^{r*} < p_2^{n*}$. Since $X_2^n, X_2^r > 0$, $\partial_{p_2^u}\Pi_2^u(\hat{p}_2^{r*}|X_2^n, X_2^r) = X_2^n\left[\bar{G}\left(\frac{\hat{p}_2^{r*}}{1+\alpha}\right) - \left(\frac{\hat{p}_2^{r*} - c_2}{1+\alpha}\right) g\left(\frac{\hat{p}_2^{r*}}{1+\alpha}\right)\right.$ 0 and $\partial_{p_2^u}\Pi_2^u(p_2^{n*}|X_2^n, X_2^r) = X_2^r\left[\bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right) - \left(\frac{p_2^{n*} - c_2}{k+\alpha}\right) g\left(\frac{p_2^{n*}}{k+\alpha}\right)\right] < 0$. Therefore, $p_2^{r*} \leq \hat{p}_2^{r*} < p_2^u(X_2^n, X_2^r) < p_2^{n*}$ for all $X_2^n, X_2^r > 0$.

When $p_2^u \in [\hat{p}_2^{r*}, p_2^{n*}]$, $\bar{G}(\frac{p_2^u}{1+\alpha}) - (\frac{p_2^u - c_2}{1+\alpha})g(\frac{p_2^u}{1+\alpha}) \geq 0$ and $\bar{G}(\frac{p_2^u}{k+\alpha}) - (\frac{p_2^u - c_2}{k+\alpha})g(\frac{p_2^u}{k+\alpha}) \leq 0$. Thus, $\Pi_2^u(p_2^u|X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r if $p_2^u \in [\hat{p}_2^{r*}, p_2^{n*}]$, i.e., $\Pi_2^u(p_2^u|X_2^n, X_2^r)$ is supermodular in (p_2^u, X_2^n) on the lattice $[\hat{p}_2^{r*}, p_2^{n*}] \times [0, +\infty)$, and submodular in (p_2^u, X_2^r) on the lattice $[\hat{p}_2^{r*}, p_2^{n*}] \times [0, +\infty)$. Therefore, $p_2^u(X_2^n, X_2^r)$ is continuously increasing in X_2^n and continuously decreasing in X_2^r. This proves **Part (a)**.

Part (b). Note that $p_1^{u*} = \mu + \delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+]$, where $\mathfrak{p}_2^{u*} \stackrel{d}{=} p_2^u((X - Q_1^{u*})^+, X \wedge Q_1^{u*}) \in [p_2^{r*}, p_2^{n*}]$. Therefore, $\delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((k+\alpha)V - p_2^{r*})^+] \leq 0$, $\delta[\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+] \geq 0$, and thus

$$p_1^{u*} - p_1^* = \delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((k+\alpha)V - p_2^{r*})^+] - \delta[\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+] \leq 0.$$

If $p_2^{r*} < p_2^{n*}$, at least one of the following two inequalities are strict: $\delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((k+\alpha)V - p_2^{r*})^+] \leq 0$ and $\delta[\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+] \geq 0$. Hence, $p_1^{u*} < p_1^*$ if $p_2^{r*} < p_2^{n*}$.

It's straightforward to compute that, for any $Q_1 \geq 0$,

$$\begin{aligned}\Pi_f(Q_1) - \Pi_f^u(Q_1) =& (p_1^* - p_1^u(Q_1))\mathbb{E}(X \wedge Q_1) + \delta\mathbb{E}[(\beta_n^* - v_2^n(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X - Q_1)^+ \\ &+ (\beta_r^* - \hat{v}_2^r(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X \wedge Q_1)],\end{aligned}$$

where $p_1^u(Q_1) = \mu + \delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+] \leq p_1^{u*}$ with $\mathfrak{p}_2^{u*} \stackrel{d}{=} p_2^u((X - Q_1)^+, X \wedge Q_1)$. Since $\beta_n^* \geq v_2^n(p_2)$ and $\beta_r^* \geq v_2^r(p_2) \geq \hat{v}_2^r(p_2)$ for any $p_2 \geq 0$, $\Pi_f(Q_1) \geq \Pi_f^u(Q_1)$ for all $Q_1 \geq 0$, and thus $\Pi_f^* = \max_{Q_1} \Pi_f(Q_1) \geq \max_{Q_1} \Pi_f^u(Q_1) = \Pi_f^{u*}$. If $p_2^{r*} < p_2^{n*}$, by the proof of part (a), $p_1^* > p_1^u(Q_1)$ and, hence, $\Pi_f(Q_1) > \Pi_f^u(Q_1)$ for all $Q_1 > 0$. Therefore, $\Pi_f^* = \Pi_f(Q_1^*) \geq \Pi_f(Q_1^{u*}) > \Pi_f^u(Q_1^{u*}) = \Pi_f^{u*}$. This proves **part (b)**.

Part (c). $\tilde{p}_1^{u*} = \tilde{p}_1^* = \mu$ follows immediately from that μ is the willingness-to-pay of myopic customers. Moreover, direct computation yields that, for any $Q_1 \geq 0$,

$$\tilde{\Pi}_f(Q_1) - \tilde{\Pi}_f^u(Q_1) = \delta\mathbb{E}[(\beta_n^* - v_2^n(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X - Q_1)^+ + (\beta_r^* - \hat{v}_2^n(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X \wedge Q_1)] \geq 0$$

where the inequality follows from the proof of part (b). If $p_2^{r*} < p_2^{n*}$, at least one of $\mathbb{E}[(\beta_n^* - v_2^n(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X - Q_1)^+]$ and $\mathbb{E}[(\beta_r^* - \hat{v}_2^n(p_2^u((X - Q_1)^+, X \wedge Q_1)))(X \wedge Q_1)]$ is positive for $Q_1 > 0$. Hence, the same argument as the proof of part (b) yields that $\tilde{\Pi}_f^* > \tilde{\Pi}_f^{u*}$ if $\tilde{Q}_1^{u*} > 0$. This proves **part (c)**. □

Proof of Theorem 2: Part (a). Since $p_1^* - \tilde{p}_1^* = m_1^* - \tilde{m}_1^* = e^*$, it follows immediately that $p_1^* > \tilde{p}_1^*$ and $Q_1^* = \bar{F}^{-1}(\frac{c_1 - r_1}{m_1^* - r_1}) > \bar{F}^{-1}(\frac{c_1 - r_1}{\tilde{m}_1^* - r_1}) = \tilde{Q}_1^*$ if and only if $e^* > 0$. Moreover, for any Q_1, $\Pi_f(Q_1) - \tilde{\Pi}_f(Q_1) = e^*\mathbb{E}(X \wedge Q_1) > 0$ if and only if $e^* > 0$. Therefore, $\Pi_f^* = \max \Pi_f(Q_1) > \max \tilde{\Pi}_f(Q_1) = \tilde{\Pi}_f^*$ if and only if $e^* > 0$ and $Q_1^* > 0$.

Next, we show that $e^* > 0$ if and only if $r_2 > \bar{r}$. Observe that $v_2^r(p_2^r)$ is submodular in (p_2^r, r_2), so p_2^{r*} is decreasing in r_2. Moreover, e^* is decreasing in p_2^{r*}. Hence, e^* is increasing in r_2 and $e^* > 0$ if and only if $r_2 > \bar{r}$ for some $\bar{r}$. We now show that $\bar{r} \geq \frac{1-k}{1+\alpha}c_2$. It suffices to show that if $r_2 = \frac{1-k}{1+\alpha}c_2$, $e^* \leq 0$. If $r_2 = \frac{1-k}{1+\alpha}c_2$, $v_2^r(p_2^r) = (p_2^r - \frac{k+\alpha}{1+\alpha}c_2)\bar{G}(\frac{p_2^r}{k+\alpha})$. It's straightforward to check that $p_2^{r*} = \frac{k+\alpha}{1+\alpha}p_2^{n*}$. Hence,

$$\begin{aligned}e^* =& \mathbb{E}[(k+\alpha)V - p_2^{r*}]^+ - \mathbb{E}[(1+\alpha)V - p_2^{n*}]^+ \\ =& \mathbb{E}[(k+\alpha)V - \frac{k+\alpha}{1+\alpha}p_2^{n*}]^+ - \mathbb{E}[(1+\alpha)V - p_2^{n*}]^+ \\ =& -\frac{1-k}{1+\alpha}\mathbb{E}[(1+\alpha)V - p_2^{n*}]^+ \leq 0.\end{aligned}$$

This proves **Part (a)**.

Part (b). Observe that,

$$p_1^{u*} - \tilde{p}_1^{u*} = \delta[\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ - \mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+],$$

where $\mathfrak{p}_2^{u*} \overset{d}{=} p_2^u((X-Q_1^{u*})^+, X \wedge Q_1^{u*})$. Since $k \le 1$, $p_1^{u*} \le \tilde{p}_1^{u*}$ and the inequality is strict if $k<1$. This establishes **part (b-i)**.

We now show part (b-ii). Direct computation yields that

$$\tilde{m}_1^u(Q_1) - m_1^u(Q_1) = \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+ - \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+,$$

where $X_2^n = (X-Q_1)^+$ and $X_2^r = X \wedge Q_1$. Since $k<1$, we have $\mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+ - \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+ > 0$.

Let $\Pi(Q_1, 1) = \tilde{\Pi}_f^u(Q_1)$ and $\Pi(Q_1, 0) = \Pi_f^u(Q_1)$,

$$\begin{aligned}\Pi(Q_1,1) - \Pi(Q_1,0) &= \tilde{\Pi}_f^u(Q_1) - \Pi_f^u(Q_1) = (\mu - p_1^u(Q_1))\mathbb{E}(X\wedge Q_1)\\ &= [\mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+ - \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+]\mathbb{E}(X\wedge Q_1).\end{aligned}$$

Since $[\mathbb{E}((1+\alpha)V-p)^+ - \mathbb{E}((k+\alpha)V-p)^+]' = -\mathbb{P}(\frac{p}{1+\alpha} \le V \le \frac{p}{k+\alpha}) \le 0$ and $p_2^u(X_2^n, X_2^r)$ is decreasing in Q_1, $\Pi(Q_1,1) - \Pi(Q_1,0) = (\mu - p_1^u(Q_1))(X \wedge Q_1)$ is increasing in Q_1, and, hence, $\Pi(\cdot,\cdot)$ is a supermodular function on the lattice $[0,+\infty)\times\{0,1\}$. Thus, $\tilde{Q}_1^{u*} = \text{argmax}_{Q_1\ge 0}\Pi(Q_1,1) \ge \text{argmax}_{Q_1\ge 0}\Pi(Q_1,0) = Q_1^{u*}$. This proves **part (b-ii)**.

Finally, since $\tilde{\Pi}_f^u(Q_1) - \Pi_f^u(Q_1) = (\mu - p_1^u(Q_1))(X\wedge Q_1) \ge 0$ where the inequality is strict if $p_2^{r*} < p_2^{n*}$. Hence, $\tilde{\Pi}_f^{u*} = \max_{Q_1\ge 0}\tilde{\Pi}_f^u(Q_1) \ge \max_{Q_1 \ge 0}\Pi_f^u(Q_1) = \Pi_f^{u*}$. Moreover, the same argument as the proof of Theorem 3 (b-ii) implies that $\tilde{\Pi}_f^{u*} > \Pi_f^{u*}$ if $p_2^{r*} < p_2^{n*}$. This establishes **part (b)**. □

Proof of Theorem 4: Part (a). We first show that $m_1^u(Q_1)$ is decreasing in Q_1. Observe that $m_1^u(Q_1) = \mu + \delta[U_r(Q_1) - U_n(Q_1)]$, where

$$U_r(Q_1) := \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)] + \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+,$$

and

$$U_n(Q_1) := \mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)] + \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+.$$

Let $u_r(p) := (p-c_2)\bar{G}(\frac{p}{k+\alpha}) + \mathbb{E}((k+\alpha)V - p)^+ = \mathbb{E}[(k+\alpha)V - c_2]1_{\{(k+\alpha)V\ge p\}}$ and $u_n(p) := (p - c_2)\bar{G}(\frac{p}{1+\alpha}) + \mathbb{E}((1+\alpha)V-p)^+ = \mathbb{E}[(1+\alpha)V - c_2]1_{\{(1+\alpha)V \ge p\}}$. It's clear that $u_r(\cdot)$ and $u_p(\cdot)$ are continuously decreasing in p. Moreover, $U_r(Q_1) = \mathbb{E}[u_r(p_2^u(X_2^n, X_2^r))]$ and $U_n(Q_1) = \mathbb{E}[u_n(p_2^u(X_2^n, X_2^r))]$, where $X_2^n = (X-Q_1)^+$ and $X_2^r = X\wedge Q_1$. Since $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r, it is stochastically decreasing in Q_1. Hence, it suffices to show that $u_r(p) - u_n(p)$ is increasing in p. Observe that

$$\begin{aligned}u_r(p) - u_n(p) &= -[\int_{p/(1+\alpha)}^{p/(k+\alpha)}((1+\alpha)V - p)g(V)\,\mathrm{d}V + \int_{p/(k+\alpha)}^{\bar{v}}(1-k)Vg(V)\,\mathrm{d}V]\\ &= -[\int_{p/(1+\alpha)}^{\bar{v}}((1+\alpha)V - \max(p,(k+\alpha)V))g(V)\,\mathrm{d}V],\end{aligned}$$

which is continuously increasing in p. This establishes **part (a-i)**.

We now show that $m_1^u(Q_1) < m_1^*$ for all Q_1. Observe that

$$m_1^u(Q_1) - m_1^* = \mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] - \mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})].$$

Because $p_2^{r*} \le p_2^u(X_2^n, X_2^r) \le p_2^{n*}$, $\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$. Hence, $m_1^u(Q_1) \le m_1^*$. If $k < 1$, $p_2^{r*} < p_2^{n*}$, one of the inequalities $\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$ must be strict. Therefore, $m_1^u(Q_1) < m_1^*$ for all $Q_1 \ge 0$. This proves **part (a-ii)**.

Next, we show that $Q_1^{u*} \le Q_1^*$. Observe that

$$\Pi_f^u(Q_1) - \Pi_f(Q_1) = (m_1^u(Q_1) - m_1^*)(X \wedge Q_1) + \delta\mathbb{E}[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)] - \delta\beta_n^*\mathbb{E}(X).$$

Let $\Pi(Q_1, 1) = \Pi_f(Q_1)$ and $\Pi(Q_1, 0) = \Pi_f^u(Q_1)$. Then,

$$\Pi(Q_1, 1) - \Pi(Q_1, 0) \quad = \quad (m_1^* - m_1^u(Q_1))(X \wedge Q_1) + \delta\mathbb{E}X[\beta_n^* - (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)]$$

Note that for any realization of X, $p_2^u(X_2^n, X_2^r)$ and thus $(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha})$ is decreasing in Q_1. Therefore, by part (a-ii), $\Pi(Q_1, 1) - \Pi(Q_1, 0)$ is increasing in Q_1. Hence, $\Pi(\cdot, \cdot)$ is supermodular on the lattice $[0, +\infty) \times \{0, 1\}$. Hence, $Q_1^{u*} = \text{argmax}_{Q_1 \ge 0}\Pi_f^u(Q_1) \le \text{argmax}_{Q_1 \ge 0}\Pi_f(Q_1) = Q_1^*$. If $Q_1^{u*} > 0$, since $m_1^* > m_1^u(Q_1^{u*})$ $\Pi_f'(Q_1^{u*}) > \partial_{Q_1}\Pi_f^u(Q_1^{u*}) = 0$. Since $\Pi_f(\cdot)$ is concave in Q_1, $Q_1^* > Q_1^{u*}$. This proves **part (a-iii)**.

Part (b). We first show that $\tilde{m}_1^u(Q_1)$ is increasing in Q_1. Note that $\tilde{m}_1^u(Q_1) = \mu + \delta\mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r)) - v_2^n(p_2^u(X_2^n, X_2^r))]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. Because $\hat{p}_2^{r*} \le p_2^u(X_2^n, X_2^r) \le p_2^{n*}$ and $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r. Thus, $\hat{v}_2^r(p_2^u(X_2^n, X_2^r))$ is stochastically increasing in Q_1 and $v_2^n(p_2^u(X_2^n, X_2^r))$ is stochastically decreasing in Q_1. Therefore, $\tilde{m}_1^u(Q_1) = \mu + \delta\mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r)) - v_2^n(p_2^u(X_2^n, X_2^r))]$ is increasing in Q_1. This proves **part (b-i)**.

We now show **part (b-ii)**. Let $\hat{\beta}_r^* = \max_{p \ge 0} \hat{v}_2^r(p)$. It's clear that $\beta_r^* - \hat{\beta}_r^*$ is increasing in r_2, with $\beta_r^* = \hat{\beta}_r^*$ if $r_2 = 0$. Moreover, since $k < 1$, $\hat{\beta}_n^* := v_2^n(\hat{p}_2^{r*}) < \beta_n^*$. Therefore, let $\bar{r}_2 > 0$ be the threshold such that $\beta_r^* - \hat{\beta}_r^* = \beta_n^* - \hat{\beta}_n^*$. Hence, for all $r_2 < \bar{r}_2$, $\beta_r^* - \hat{\beta}_r^* < \beta_n^* - \hat{\beta}_n^*$. Moreover, by the monotone convergence theorem,

$$\lim_{Q_1 \to +\infty} \tilde{m}_1^u(Q_1) = \mu + \delta[v_2^r(\hat{p}_2^{r*}) - v_2^n(\hat{p}_2^{r*})] = \mu + \delta[\hat{\beta}_r^* - \hat{\beta}_n^*] < \mu + \delta[\beta_r^* - \beta_n^*] = \tilde{m}_1^*.$$

Part (b-i) shows that $\tilde{m}_1^u(Q_1)$ is increasing in Q_1. Hence, there exists a threshold $\bar{Q}(r_2)$ such that $\tilde{m}_1^u(Q_1) \ge \tilde{m}_1^*$ if and only if $Q_1 \ge \bar{Q}(r_2)$. To show that $\bar{Q}(r_2)$ is increasing in r_2, we observe that $\tilde{m}_1^*$ is increasing in r_2. Hence, $\bar{Q}(r_2) := \min\{Q_1 : \tilde{m}_1^u(Q_1) \ge \tilde{m}_1^*\}$ is increasing in r_2. This proves **part (b-ii)**.

Part (b-iii). Without loss of generality, assume that $Q_1^{u*} > 0$. Otherwise, the result holds trivially. It's clear that $Q_1^{u*} \uparrow \bar{X}$ and $Q_1^* \uparrow \bar{X}$ as $c_1 \downarrow 0$, where $\bar{X}$ is the upper bound of the support of X. Hence, there exists a threshold $\tilde{c}(r_2) > 0$, dependent on r_2, such that if $c_1 < \tilde{c}(r_2)$, $Q_1^{u*}, Q_1^* > \bar{Q}(r_2)$. Let $\hat{\pi}_2(Q_1) := \delta\mathbb{E}[v_2^n(p_2^u(X_2^n, X_2^r))X]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. It's clear that $\hat{\pi}_2(\cdot)$ is differentiable and, by the chain rule

$$\hat{\pi}_2'(Q_1) = \delta\mathbb{E}[\partial_p v_2^n(p_2^u(X_2^n, X_2^r))(\partial_{X_2^n} p_2^u(X_2^n, X_2^r) + \partial_{X_2^r} p_2^u(X_2^n, X_2^r))1_{\{X \ge Q_1\}}X].$$

As $Q_1 \to \bar{X}$, for any realization of $X \le \bar{X}$, $\partial_{X_2^n} p_2^u(X_2^n, X_2^r)$ and $\partial_{X_2^r} p_2^u(X_2^n, X_2^r)$ converges to 0. Hence, by the dominated convergence theorem, there exits a threshold $\hat{Q} \in [\bar{Q}(r_2), \bar{X})$, such that $\hat{\pi}_2'(Q_1) \in [-\epsilon\mathbb{P}(X \ge Q_1), 0]$ for all $Q_1 \ge \hat{Q}$, where $\epsilon := (\tilde{m}_1^u(\hat{Q}) - \tilde{m}_1^*)/2 > 0$. Let $\bar{c}_1(r_2) \in (0, \tilde{c}(r_2)]$ be the threshold such that, if $c_1 < \bar{c}_1(r_1)$, we have $Q_1^{u*}, Q_1^* > \hat{Q} \geq \bar{Q}(r_2)$. Therefore,

$$\begin{aligned}
\tilde{\Pi}'_f(Q_1^{u*}) &= (\tilde{m}_1^* - r_1)\mathbb{P}(X \geq Q_1^{u*}) - (c_1 - r_1) \\
&< (\tilde{m}_1^u(Q_1^{u*}) - r_1)\mathbb{P}(X \geq Q_1^{u*}) - \epsilon\mathbb{P}(X \geq Q_1^{u*}) - (c_1 - r_1) \\
&\leq (\tilde{m}_1^u(Q_1^{u*}) - r_1)\mathbb{P}(X \geq Q_1^{u*}) + \hat{\pi}'_2(Q_1^{u*}) - (c_1 - r_1) \\
&\leq \partial_{Q_1}\tilde{\Pi}_f^u(Q_1^{u*}) \\
&= 0,
\end{aligned}$$

where the first inequality follows from $\tilde{m}_1^u(Q_1^{u*}) - \tilde{m}_1^* \geq (\tilde{m}_1^u(\hat{Q}) - \tilde{m}_1^*) = 2\epsilon > \epsilon$, the second from $\hat{\pi}'_2(Q_1^{u*}) \in [-\epsilon\mathbb{P}(X \geq Q_1^{u*}), 0]$, and the last from the monotonicity that $\tilde{m}_1^u(\cdot)$ is increasing in Q_1. Because $\tilde{\Pi}_f(\cdot)$ is concave in Q_1, $\tilde{Q}_1^* = \text{argmax}_{Q_1}\tilde{\Pi}_f(Q_1) < \tilde{Q}_1^{u*}$ follows immediately. This establishes **part (b-iii)** and thus **Theorem 4**. $\square$

Before presenting the proof Theorem 5, we give the following lemma that computes the equilibrium environmental impacts I_e^* and $\tilde{I}_e^*$.

LEMMA **5** *(a) With strategic customers, the total expected environmental impact of the RE equilibrium is $I_e^* = I_e(Q_1^*)$, where*

$$I_e(Q_1) := (\kappa_1 - \iota_1)Q_1 + (\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right))\mathbb{E}(Q_1 \wedge X) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - Q_1)^+.$$

(b) With myopic customers, the total expected environmental impact of the RE equilibrium is $\tilde{I}_e^* = I_e(\tilde{Q}_1^*)$.

(c) The function $I_e(\cdot)$ is strictly increasing in Q_1. Hence, $I_e^* \geq \tilde{I}_e^*$ if and only if $Q_1^* \geq \tilde{Q}_1^*$.

Proof of Lemma 5: Parts (a) and (b). Direct computation yields that

$$\begin{aligned}
I_e^* =&\mathbb{E}\{\kappa_1 Q_1^* + \delta\kappa_2(Q_2^n(X_2^{n*}, X_2^{r*}) + Q_2^r(X_2^{n*}, X_2^{r*})) - \iota_1(Q_1^* - X)^+ - \delta\iota_2 Q_2^r(X_2^{n*}, X_2^{r*})\} \\
=&\mathbb{E}\{\kappa_1 Q_1^* + \delta\kappa_2((X - Q_1^*)^+\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) + (X \wedge Q_1^*)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)) - \iota_1(Q_1^* - X)^+ - \delta\iota_2(X \wedge Q_1^*)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\} \\
=&(\kappa_1 - \iota_1)Q_1^* + (\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right))\mathbb{E}(X \wedge Q_1^*) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - Q_1^*)^+ \\
=&I_e(Q_1^*),
\end{aligned}$$

where the second inequality follows from $X_2^{n*} = (X - Q_1^*)^+$ and $X_2^{r*} = X \wedge Q_1^*$, the third from $(Q_1^* - X)^+ = Q_1^* - (X \wedge Q_1^*)$, and the last from the definition of the function $I_e(\cdot)$. Analogously,

$$\begin{aligned}
\tilde{I}_e^* =&\mathbb{E}\{\kappa_1 \tilde{Q}_1^* + \delta\kappa_2(Q_2^n(\tilde{X}_2^{n*}, \tilde{X}_2^{r*}) + Q_2^r(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})) - \iota_1(\tilde{Q}_1^* - X)^+ - \delta\iota_2 Q_2^r(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})\} \\
=&\mathbb{E}\{\kappa_1 \tilde{Q}_1^* + \delta\kappa_2((X - \tilde{Q}_1^*)^+\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) + (X \wedge \tilde{Q}_1^*)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)) - \iota_1(\tilde{Q}_1^* - X)^+ - \delta\iota_2(X \wedge \tilde{Q}_1^*)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\} \\
=&(\kappa_1 - \iota_1)\tilde{Q}_1^* + (\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right))\mathbb{E}(X \wedge \tilde{Q}_1^*) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - \tilde{Q}_1^*)^+ \\
=&I_e(\tilde{Q}_1^*).
\end{aligned}$$

This completes the proof of **Parts (a) and (b)**.

Part (c). To establish the monotonicity of $I_e(\cdot)$, observe that

$$I_e'(Q_1) = \kappa_1 - \iota_1 + (\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right))\mathbb{P}(X > Q_1) - \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{P}(X > Q_1) > \kappa_1 - \iota_1 - \delta\kappa_2 > 0,$$

where the first inequality follows from $\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) \leq 1$ and $\mathbb{P}(X > Q_1) \leq 1$, and the second from the assumption that $\kappa_1 > \iota_1 + \kappa_2$. Hence, $I_e(\cdot)$ is strictly increasing in Q_1. This proves **Part (c)**. $\square$

Proof of Theorem 5: Part (a). First, we compute I_e^{u*}. Given the market sizes (X_2^n, X_2^r), the equilibrium total second-period production quantity, $Q_2^u(X_2^n, X_2^r)$, is given by

$$Q_2^u(X_2^n, X_2^r) = X_2^n\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) + X_2^r\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right).$$

Therefore,

$$\begin{aligned} I_e^{u*} &= \mathbb{E}\{\kappa_1 Q_1^{u*} - \iota_1(Q_1^{u*} - X)^+ + \delta\kappa_2 Q_2^u(X_2^{n*}, X_2^{r*})\} \\ &= (\kappa_1 - \iota_1)Q_1^{u*} + \mathbb{E}\{[\iota_1 + \delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right)](Q_1^{u*} \wedge X)\} + \delta\kappa_2\mathbb{E}[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)(X - Q_1^{u*})^+], \end{aligned}$$

where $X_2^{n*} = (X - Q_1^{u*})^+$ and $X_2^{r*} = X \wedge Q_1^{u*}$. If $Q_1^* = 0$, $Q_1^{u*} = 0$ as well by Theorem 4(a). Hence, $I_e^* = I_e^{u*}$ regardless of the value of ι_2. In this case, part (a) trivially holds. On the other hand, if $Q_1^* > 0$, I_e^* is strictly linearly decreasing in ι_2. Thus, let $\bar{\iota}_2^u := \max\{\iota_2 : I_e^* \geq I_e^{u*}\}$. We have $I_e^* \geq I_2^{u*}$ if and only if $\iota_2 \leq \bar{\iota}_2^u$. In particular, if $\iota_2 = 0$, $Q_1^* > Q_1^{u*}$ implies that

$$(\kappa_1 - \iota_1)Q_1^* + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - Q_1^*)^+ > (\kappa_1 - \iota_1)Q_1^{u*} + \delta\kappa_2\mathbb{E}[\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{1+\alpha}\right)(X - Q_1^{u*})^+],$$

and

$$(\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right))\mathbb{E}(X \wedge Q_1^*) \geq \mathbb{E}[(\iota_1 + \delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*}, X_2^{r*})}{k+\alpha}\right))(X - Q_1^{u*})].$$

Thus, $\bar{\iota}_2^u > 0$. This establishes **part (a)**.

Part (b). As in the proof of part (a), we first compute $\tilde{I}_e^{u*}$:

$$\begin{aligned} \tilde{I}_e^{u*} &= \mathbb{E}\{\kappa_1\tilde{Q}_1^{u*} - \iota_1(\tilde{Q}_1^{u*} - X)^+ + \delta\kappa_2 Q_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})\} \\ &= (\kappa_1 - \iota_1)\tilde{Q}_1^{u*} + \mathbb{E}\{[\iota_1 + \delta\kappa_2\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{k+\alpha}\right)](\tilde{Q}_1^{u*} \wedge X)\} + \delta\kappa_2\mathbb{E}[\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{1+\alpha}\right)(X - \tilde{Q}_1^{u*})^+], \end{aligned}$$

where $\tilde{X}_2^{n*} = (X - \tilde{Q}_1^{u*})^+$ and $\tilde{X}_2^{r*} = X \wedge \tilde{Q}_1^{u*}$. By Theorem 5(b), $\tilde{Q}_1^{u*} \geq \tilde{Q}_1^*$. Hence,

$$(\kappa_1 - \iota_1)\tilde{Q}_1^* + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - \tilde{Q}_1^*)^+ \leq (\kappa_1 - \iota_1)\tilde{Q}_1^{u*} + \delta\kappa_2\mathbb{E}[\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{1+\alpha}\right)(X - \tilde{Q}_1^{u*})^+].$$

Let $\tilde{\iota}_2^u := (\bar{G}(\frac{p_2^{r*}}{k+\alpha}) - \bar{G}(\frac{p_2^{n*}}{k+\alpha}))\kappa_2/\bar{G}(\frac{p_2^{r*}}{k+\alpha}) < \kappa_2$. If $\iota_2 \geq \tilde{\iota}_2^u$, since $\tilde{Q}_1^{u*} \geq \tilde{Q}_1^*$,

$$\begin{aligned} \mathbb{E}\{[\iota_1 + \delta\kappa_2\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{k+\alpha}\right)](\tilde{Q}_1^{u*} \wedge X)\} &\geq \mathbb{E}\{[\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right)](\tilde{Q}_1^{u*} \wedge X)\} \\ &\geq \mathbb{E}\{[\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right)](Q_1^{u*} \wedge X)\}. \end{aligned}$$

Therefore, if $\iota_2 \geq \tilde{\iota}_2^u$,

$$\begin{aligned}
\tilde{I}_e^{u*} &= (\kappa_1 - \iota_1)\tilde{Q}_1^{u*} + \mathbb{E}\{[\iota_1 + \delta\kappa_2\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{k+\alpha}\right)](\tilde{Q}_1^{u*} \wedge X)\} + \delta\kappa_2\mathbb{E}[\bar{G}\left(\frac{p_2^u(\tilde{X}_2^{n*}, \tilde{X}_2^{r*})}{1+\alpha}\right)(X - \tilde{Q}_1^{u*})^+] \\
&\geq (\kappa_1 - \iota_1)\tilde{Q}_1^* + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}(X - \tilde{Q}_1^*)^+ + \mathbb{E}\{[\iota_1 + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{n*}}{k+\alpha}\right)](Q_1^{u*} \wedge X)\} \\
&= \tilde{I}_e^*,
\end{aligned}$$

which proves **part (b)**. $\square$

Proof of Theorem 6: Part (a). We first compute the equilibrium total customer surplus in the scenario of strategic customers, S_c^*. If a customer is a new customer in period 2, her expected total surplus is $\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+$ (since, by Lemma 1, $p_2^n(X_2^n, X_2^r) = p_2^{n*}$). Hence, the expected surplus of a strategic customer in the base model is given by:

$$\begin{aligned}
&\mathfrak{a}^*(\mu - p_1^* + \delta\mathbb{E}((k+\alpha)V - p_2^{r*})^+) + (1-\mathfrak{a}^*)\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+ \\
=&\mathfrak{a}^*(\mu - \mu + \delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+ - \delta\mathbb{E}((k+\alpha)V - p_2^{r*})^+ + \delta\mathbb{E}((k+\alpha)V - p_2^{r*})^+) + (1-\mathfrak{a}^*)\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+ \\
=&\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+.
\end{aligned}$$

Therefore, the equilibrium total customer surplus is given by $S_c^* = \mathbb{E}[\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+ X] = \delta\mathbb{E}[((1+\alpha)V - p_2^{n*})^+ X]$.

We now compute the equilibrium total customer surplus in the scenario of myopic customers, $\tilde{S}_c^*$. Since the customers are myopic, they get zero utility in period 1. Hence, in period 2, the expected surplus of a new customer is $\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+$, whereas that of a repeat customer is $\delta\mathbb{E}((k+\alpha)V - p_2^{r*})^+$. Therefore, the total customer surplus is given by

$$\begin{aligned}
\tilde{S}_c^* =&\mathbb{E}[\delta\mathbb{E}((1+\alpha)V - p_2^{n*})^+(X - \tilde{Q}_1^*)^+] + \mathbb{E}[\delta\mathbb{E}((k+\alpha)V - p_2^{r*})^+(X \wedge \tilde{Q}_1^*)] \\
=&\delta\mathbb{E}[((1+\alpha)V - p_2^{n*})^+(X - \tilde{Q}_1^*)^+] + \delta\mathbb{E}[((k+\alpha)V - p_2^{r*})^+(X \wedge \tilde{Q}_1^*)].
\end{aligned}$$

This proves **part (a)**.

Part (b). We first compute S_c^{u*}. If a customer is a new customer in period 2, her expected total surplus is $\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+$. Hence, the expected surplus of a strategic customer in the NTR model is given by:

$$\begin{aligned}
&\mathfrak{a}^{u*}(\mu - p_1^{u*} + \delta\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+) + (1-\mathfrak{a}^{u*})\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ \\
=&\mathfrak{a}^{u*}(\mu - \mu + \delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ - \delta\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+ + \delta\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+) + (1-\mathfrak{a}^{u*})\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ \\
=&\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+.
\end{aligned}$$

Therefore, the equilibrium total customer surplus is given by $S_c^{u*} = \mathbb{E}[\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+ X] = \delta\mathbb{E}[((1+\alpha)V - \mathfrak{p}_2^{u*})^+ X]$.

We now compute $\tilde{S}_c^{u*}$. Since the customers are myopic, they get zero utility in period 1. Hence, in period 2, the expected surplus of a new customer is $\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+$, whereas that of a repeat customer is $\delta\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+$. Therefore, the total customer surplus is given by

$$\begin{aligned}
\tilde{S}_c^{u*} =&\mathbb{E}[\delta\mathbb{E}((1+\alpha)V - \mathfrak{p}_2^{u*})^+(X - \tilde{Q}_1^{u*})^+] + \mathbb{E}[\delta\mathbb{E}((k+\alpha)V - \mathfrak{p}_2^{u*})^+(X \wedge \tilde{Q}_1^{u*})] \\
=&\delta\mathbb{E}[((1+\alpha)V - \mathfrak{p}_2^{u*})^+(X - \tilde{Q}_1^{u*})^+] + \delta\mathbb{E}[((k+\alpha)V - \mathfrak{p}_2^{u*})^+(X \wedge \tilde{Q}_1^{u*})].
\end{aligned}$$

This proves **part (b)**.

Part (c). Note that, by Theorem 3(a), $p_2^{r*} \leq \mathfrak{p}_2^{u*} \leq p_2^{n*}$ with probability 1. It follows immediately that $S_c^{u*} = \delta\mathbb{E}[((1+\alpha)V - \mathfrak{p}_2^{u*})^+ X] \geq \delta\mathbb{E}[((1+\alpha)V - p_2^{n*})^+ X] = S_c^*$. In particular, if $k < 1$ and $Q_1^{u*} > 0$, $\mathfrak{p}_2^{u*} < p_2^{n*}$ with probability 1 and thus $S_c^{u*} > S_c^*$. This proves **part (c)**. $\square$

Proof of Lemma 3: Part (a). Let $W_2(p_2^n, p_2^r | X_2^n, X_2^r)$ be the expected social welfare in period 2 when the price for new customers is p_2^n, and that for repeat customers is p_2^r. Since all new (repeat) customers with valuation $(1+\alpha)V \geq p_2^n$ ($(k+\alpha)V \geq p_2^r$) will make a purchase (trade the used products in), the firm profit equals $(p_2^n - c_2)\bar{G}(\frac{p_2^n}{1+\alpha})X_2^n + (p_2^r - c_2 + r_2)\bar{G}(\frac{p_2^r}{k+\alpha})X_2^r$, the expected customer surplus equals $X_2^n\mathbb{E}((1+\alpha)V - p_2^n)^+ + X_2^r\mathbb{E}((k+\alpha)V - p_2^r)^+$, and the environmental impact equals $\kappa_2 X_2^n \bar{G}(\frac{p_2^n}{1+\alpha}) + (\kappa_2 - \iota_2)X_2^r\bar{G}(\frac{p_2^r}{k+\alpha})$. Therefore, $W_2(p_2^n, p_2^r | X_2^n, X_2^r) = X_2^n w_n(p_2^n) + X_2^r w_r(p_2^r)$, where

$$w_n(p_2^n) := (p_2^n - c_2 - \kappa_2)\bar{G}(\frac{p_2^n}{1+\alpha}) + \mathbb{E}((1+\alpha)V - p_2^n)^+ = \mathbb{E}((1+\alpha)V - c_2 - \kappa_2)1_{\{(1+\alpha)V \geq p_2\}},$$

and

$$w_r(p_2^r) := (p_2^r - c_2 + r_2 - \kappa_2 + \iota_2)\bar{G}(\frac{p_2^r}{k+\alpha}) + \mathbb{E}((k+\alpha)V - p_2^r)^+ = \mathbb{E}((k+\alpha)V - c_2 + r_2 - \kappa_2 + \iota_2)1_{\{(k+\alpha)V \geq p_2^r\}}.$$

Thus, $w_n'(p_2^n) = \frac{p_2^n - c_2 - \kappa_2}{1+\alpha} g(\frac{p_2^n}{1+\alpha})$ and $w_r'(p_2^r) = \frac{p_2^r - c_2 + r_2 - \kappa_2 + \iota_2}{k+\alpha} g(\frac{p_2^r}{k+\alpha})$. Thus, $w_n'(p_2^n) > 0$ if $p_2^n < c_2 + \kappa_2$ and $w_n'(p_2^n) < 0$ if $p_2^n > c_2 + \kappa_2$. Analogously, $w_r'(p_2^r) > 0$ if $p_2^r < c_2 + \kappa_2 - r_2 - \iota_2$ and $w_r'(p_2^r) < 0$ if $p_2^r > c_2 + \kappa_2 - r_2 - \iota_2$. Hence, the unique maximizer of $w_n(\cdot)$ is $c_2 + \kappa_2$, and the unique maximizer of $w_r(\cdot)$ is $c_2 + \kappa_2 - r_2 - \iota_2$. Finally, it is straightforward to check that $c_2 + \kappa_2 - r_2 - \iota_2 \leq c_2 + \kappa_2$, with the inequality being strict if and only if $r_2 > 0$ or $\iota_2 > 0$. Therefore, $p_{s,2}^n(X_2^n, X_2^r) \equiv p_{s,2}^{n*} = c_2 + \kappa_2$ and $p_{s,2}^r(X_2^n, X_2^r) \equiv p_{s,2}^{r*} = c_2 + \kappa_2 - r_2 - \iota_2$ for any realized (X_2^n, X_2^r). This proves **part (a)**.

Part (b). Under the equilibrium prices $(p_{s,2}^{n*}, p_{s,2}^{r*})$, a new customer will make a purchase if and only if her valuation $(1+\alpha)V \geq p_{s,2}^{n*}$, whereas a repeat customer will make a purchase (and join the trade-in program) if and only if her valuation $(k+\alpha)V \geq p_{s,2}^{r*}$. Therefore,

$$Q_{s,2}^n(X_2^n, X_2^r) = \mathbb{E}[X_2^n 1_{\{(1+\alpha)V \geq p_{s,2}^{n*}\}} | X_2^n] = X_2^n \bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right),$$

and

$$Q_{s,2}^r(X_2^n, X_2^r) = \mathbb{E}[X_2^r 1_{\{(k+\alpha)V \geq p_{s,2}^{r*}\}} | X_2^r] = X_2^r \bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right),$$

which proves **part (b)**.

Part (c). Plugging $p_{s,2}^{n*}$ and $p_{s,2}^{r*}$ into $w_2^n(\cdot)$ and $w_2^r(\cdot)$, respectively, we have $w_2^n(p_{s,2}^{n*}) = \mathbb{E}[(1+\alpha)V - p_{s,2}^{n*}]^+$ and $w_2^r(p_{s,2}^{r*}) = \mathbb{E}[(1+\alpha)V - p_{s,2}^{r*}]^+$. Therefore, $w_2(X_2^n, X_2^r) = X_2^n\mathbb{E}[(1+\alpha)V - p_{s,2}^{n*}]^+ + X_2^r\mathbb{E}[(1+\alpha)V - p_{s,2}^{r*}]^+$. This completes the proof of **part (c)**. $\square$

Proof of Lemma 4: Part (a). Let $W_s(Q_1)$ be the expected social welfare with first-period production quantity Q_1 under strategic customer behavior. Following the same argument as the proof of Theorem 1(a), we have

$$\begin{aligned} p_{s,1}^* &= \mu + \delta\mathbb{E}[(k+\alpha)V - \mathfrak{p}_{s,2}^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - \mathfrak{p}_{s,2}^{n*}]^+ \\ &= \mu + \delta\mathbb{E}[(k+\alpha)V - p_{s,2}^{r*}]^+ - \delta\mathbb{E}[(1+\alpha)V - p_{s,2}^{n*}]^+ \\ &= \mu + \delta(\beta_{s,r}^* - \beta_{s,n}^*) \\ &= m_{s,1}^*, \end{aligned}$$

which proves **part (a-i)**.

We now compute $W_s(Q_1)$. By Lemma 3(c), $w_2(X_2^n, X_2^r) = \beta_{s,n}^* X_2^n + \beta_{s,r}^* X_2^r$, so

$$\begin{aligned} W_s(Q_1) &= p_{s,1}^* \mathbb{E}(X \wedge Q_1) + (\mu - p_{s,1}^*)(X \wedge Q_1) - (c_1 + \kappa_1)Q_1 + (r_1 + \iota_1)\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{w_2(X - (X \wedge Q_1), X \wedge Q \\ &= (\mu - r_1 - \iota_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1 + \kappa_1 - \iota_1)Q_1 + \delta\mathbb{E}\{\beta_{s,n}^*(X - (X \wedge Q_1)) + \beta_{s,r}^*(X \wedge Q_1)\} \\ &= (m_{s,1}^* - r_1 - \iota_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1 + \kappa_1 - \iota_1)Q_1 + \delta\beta_{s,n}^*\mathbb{E}(X). \end{aligned}$$

Therefore, $Q_{s,1}^*$ is the solution to a newsvendor problem with marginal revenue $m_{s,1}^* - r_1 - \iota_1$, marginal cost $c_1 + \kappa_1 - r_1 - \iota_1$, and demand distribution $F(\cdot)$. Hence, $Q_{s,1}^* = \bar{F}^{-1}(\frac{c_1+\kappa_1-r_1-\iota_1}{m_{s,1}^*-r_1-\iota_1})$, and the equilibrium social welfare is

$$W_s^* = W_s(Q_{s,1}^*) = (m_{s,1}^* - r_1 - \iota_1)\mathbb{E}(X \wedge Q_{s,1}^*) - (c_1 - r_1 + \kappa_1 - \iota_1)Q_1 + \delta\beta_{s,n}^*\mathbb{E}(X).$$

This proves **part (a-ii,iii)**.

Part (b). Let $\tilde{W}_s(Q_1)$ be the expected social welfare with myopic customers, if the first-period production quantity is Q_1. The willingness-to-pay of myopic customers is their expected valuation of the first-generation product μ. Thus, $\tilde{p}_{s,1}^* = \mu$. This proves **part (b-i)**.

We now compute $\tilde{W}_s(Q_1)$. By Lemma 3(c), $w_2(X_2^n, X_2^r) = \beta_{s,n}^* X_2^n + \beta_{s,r}^* X_2^r$, so

$$\begin{aligned} \tilde{W}_s(Q_1) &= \tilde{p}_{s,1}^* \mathbb{E}(X \wedge Q_1) + (\mu - \tilde{p}_{s,1}^*)(X \wedge Q_1) - (c_1 + \kappa_1)Q_1 + (r_1 + \iota_1)\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{w_2(X - (X \wedge Q_1), X \wedge Q \\ &= (\mu - r_1 - \iota_1)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1 + \kappa_1 - \iota_1)Q_1 + \delta\mathbb{E}\{\beta_{s,n}^*(X - (X \wedge Q_1)) + \beta_{s,r}^*(X \wedge Q_1)\} \\ &= (\tilde{m}_{s,1}^* - r_1 - \iota_1)\mathbb{E}(X \wedge Q_1) - (c - r_1 + \kappa_1 - \iota_1)Q_1 + \delta\beta_{s,n}^*\mathbb{E}(X). \end{aligned}$$

Therefore, $\tilde{Q}_{s,1}^*$ is the solution to a newsvendor problem with marginal revenue $\tilde{m}_{s,1}^* - r_1 - \iota_1$, marginal cost $c_1 + \kappa_1 - r_1 - \iota_1$, and demand distribution $F(\cdot)$. Hence, $\tilde{Q}_{s,1}^* = \bar{F}^{-1}(\frac{c_1+\kappa_1-r_1-\iota_1}{\tilde{m}_{s,1}^*-r_1-\iota_1})$, and the equilibrium social welfare is

$$\tilde{W}_s^* = \tilde{W}_s(\tilde{Q}_{s,1}^*) = (\tilde{m}_{s,1}^* - r_1 - \iota_1)\mathbb{E}(X \wedge \tilde{Q}_{s,1}^*) - (c - r_1 + \kappa_1 - \iota_1)\tilde{Q}_1 + \delta\beta_{s,n}^*\mathbb{E}(X).$$

This proves **part (b-ii,iii)**.

Part (c). Since $p_{s,1}^* - \tilde{p}_{s,1}^* = \beta_{s,r}^* - \beta_{s,n}^* = e_s^*$, $p_{s,1}^* \geq \tilde{p}_{s,1}^*$ if and only if $e_s^* \geq 0$. The equalities $Q_{s,1}^* = \tilde{Q}_{s,1}^*$ and $W_s^* = \tilde{W}_s^*$ follow from the fact that $m_{s,1}^* = \tilde{m}_{s,1}^*$. This establishes **part (c)**. □

Proof of Theorem 7: Part (a). With the unit subsidy rate s_r for remanufactured products, the expected per demand profit from repeat customers $v_2^r(p_2^r) = (p_2^r + s_r + s_2 - c_2 + r_2)\bar{G}(\frac{p_2^r}{k+\alpha})$. Since $\partial_{p_2^r}\partial_{s_r} v_2^r(p_2^r) = -\frac{1}{1+\alpha} g(\frac{p_2^r}{1+\alpha}) \leq 0$, $v_2^r(p_2^r)$ is submodular in (p_2^r, s_r). Hence, $p_2^{r*} = \operatorname{argmax}_{p_2^r \geq 0} v_2^r(p_2^r)$ is continuously decreasing in s_r. This completes the proof of **part (a-i)**. Because $Q_2^r(X_2^n, X_2^r) = X_2^r \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)$ and p_2^{r*} is decreasing in s_r, $Q_2^r(X_2^n, X_2^r)$ is increasing in s_r, which proves **part (a-ii)**.

Part (b). By Theorem 1(a), $p_1^* = \mu + \delta[\mathbb{E}((k+\alpha)V - p_2^{r*})^+ - \mathbb{E}((1+\alpha)V - p_2^{n*})^+]$, which is decreasing in p_2^{r*}. Since p_2^{r*} is decreasing in s_r, p_1^* is increasing in s_r. With the unit subsidy rate s_r for remanufactured product,

$$\Pi_f(Q_1) = (p_1^* - r_1 - s_r)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1 - s_r)Q_1 + \delta\beta_n^*\mathbb{E}(X),$$

Hence, $Q_1^* = \bar{F}^{-1}\left(\frac{c_1-r_1-s_r}{p_1^*-r_1-s_r}\right)$. The critical fractile $\frac{c_1-r_1-s_r}{p_1^*-r_1-s_r}$ is decreasing in p_1^* and s_r. Therefore, Q_1^* is increasing in s_r. For each Q_1, $\Pi_f(Q_1)$ is increasing in s_r. Thus, $\Pi_f^* = \max_{Q_1 \geq 0} \Pi_f(Q_1)$ is increasing in s_r. By Lemma 5(a), $I_e^* = I_e(Q_1^*)$, which is increasing in Q_1^*. Thus, I_e^* is increasing in s_r as well. This establishes **part (b)**.

Part (c). By Theorem 1(b), $\tilde{p}_1^* = \mu$, which is independent of s_r. With the unit subsidy rate s_r for remanufactured product,

$$\tilde{\Pi}_f(Q_1) = (\tilde{p}_1^* - r_1 - s_r)\mathbb{E}(X \wedge Q_1) - (c_1 - r_1 - s_r)Q_1 + \delta\beta_n^*\mathbb{E}(X),$$

Hence, $\tilde{Q}_1^* = \bar{F}^{-1}\left(\frac{c_1-r_1-s_r}{\tilde{p}_1^*-r_1-s_r}\right)$. The critical fractile $\frac{c_1-r_1-s_r}{\tilde{p}_1^*-r_1-s_r}$ is decreasing in s_r. Therefore, $\tilde{Q}_1^*$ is increasing in s_r. For each Q_1, $\tilde{\Pi}_f(Q_1)$ is increasing in s_r. Thus, $\tilde{\Pi}_f^* = \max_{Q_1\geq 0}\tilde{\Pi}_f(Q_1)$ is increasing in s_r. By Lemma 5(b), $\tilde{I}_e^* = \tilde{I}_e(Q_1^*)$, which is increasing in $\tilde{Q}_1^*$. Thus, $\tilde{I}_e^*$ is increasing in s_r as well. This establishes **part (c)**. □

Proof of Theorem 8: Part (a). If s_2^* is the solution to $p_{s,2}^{n*} = \operatorname{argmax}_{p_2^n\geq 0}(p_2^n + s_2 - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$, it is clear that the subsidy/tax scheme with $s_2 = s_2^*$ can induce the equilibrium price for new customers $p_{s,2}^{n*}$. We now show that s_2^* exists. Since $v_2^n(p_2^n)$ is quasiconcave in p_2^n for any s_2, the first-order condition $\partial_{p_2^n}v_2^n(p_2^n) = 0$ guarantees the optimal price for new customers. Moreover,

$$\partial_{p_2^n}v_2^n(p_{s,2}^{n*}) = \bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right) - \frac{p_{s,2}^{n*} + s_2 - c_2}{1+\alpha}g\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right),$$

which is strictly decreasing in s_2. Hence, there exists a unique s_2^*, such that $\partial_{p_2^n}v_2^n(p_{s,2}^{n*}) = 0$, thus inducing the socially optimal equilibrium price for new customers $p_{s,2}^{n*}$. This proves **part (a-i)**.

If s_r^* is the solution to $p_{s,2}^{r*} = \operatorname{argmax}_{p_2^r\geq 0}(p_2^r + s_2^* + s_r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$, the subsidy/tax scheme with $s_r = s_r^*$ can induce the equilibrium trade-in price for repeat customers $p_{s,2}^{r*}$. We now show that s_r^* exists. Since $v_2^r(p_2^r)$ is quasiconcave in p_2^r for any (s_2, s_r), the first-order condition $\partial_{p_2^r}v_2^r(p_2^r) = 0$ guarantees the optimal price for new customers. Moreover, if $s_2 = s_2^*$,

$$\partial_{p_2^r}v_2^r(p_{s,2}^{r*}) = \bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right) - \frac{p_{s,2}^{r*} + s_2^* + s_r - c_2 + r_2}{k+\alpha}g\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right),$$

which is strictly decreasing in s_r. Hence, there exists a unique s_r^*, such that $\partial_{p_2^r}v_2^r(p_{s,2}^{r*}) = 0$ if $s_2 = s_2^*$, thus inducing the socially optimal equilibrium trade-in price for repeat customers $p_{s,2}^{r*}$. This proves **part (a-ii)**.

Given the subsidy/tax scheme (s_1, s_2^*, s_r^*), as shown above, the firm adopts the same second-period pricing strategy as the social welfare maximizing one: $(p_{s,2}^{n*}, p_{s,2}^{r*})$. Hence, the first-period price should also be the same as the one which is socially optimal and characterized by Lemma 4(a): $p_{s,1}^* = \mu + \delta[\beta_{s,r}^* - \beta_{s,n}^*]$. Thus, the expected profit of the firm in period 1 is

$$\begin{aligned}\Pi_f^s(Q_1) &= (p_{s,1}^* + s_1 - r_1)\mathbb{E}(X\wedge Q_1) - (c_1 - r_1)Q_1 + \delta\mathbb{E}[(X - X\wedge Q_1)(p_{s,2}^{n*} + s_2^* - c_2)\bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right) \\ &\quad +(X\wedge Q_1)(p_{s,2}^{r*} + s_2^* + s_r^* - c_2 + r_2)\bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right)] \\ &= (m_1^s(s_1) - r_1)\mathbb{E}(X\wedge Q_1) - (c_1 - r_1)Q_1 + \delta(p_{s,2}^{n*} + s_2^* - c_2)\bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right)\mathbb{E}(X),\end{aligned}$$

where $m_1^s(s_1) = s_1 + m_{s,1}^* + \delta[(\kappa_2 + s_2^* + s_r^* - \iota_2)\bar{G}(\frac{p_{s,2}^{r*}}{k+\alpha}) - (\kappa_2 + s_2^*)\bar{G}(\frac{p_{s,2}^{n*}}{1+\alpha})]$. Thus, $\Pi_f^s(Q_1)$ has a unique optimizer $\bar{F}^{-1}(\frac{c_1-r_1}{m_1^s(s_1)-r_1})$. Moreover, as shown in Lemma 4, $Q_{s,1}^* = \bar{F}^{-1}(\frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*})$. Therefore, if s_1^* is the unique solution to $\frac{c_1-r_1}{m_1^s(s_1)-r_1} = \frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*}$, the optimal production quantity with the linear subsidy/tax scheme $s_g^* = (s_1^*, s_2^*, s_r^*)$ is $Q_{s,1}^*$, which is the socially optimal first-period production quantity. This proves **part (a-iii)**.

We now show that s_2^* is increasing in κ_2. As shown in part (a-i), s_2^* satisfies $\bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right)-\frac{p_{s,2}^{n*}+s_2^*-c_2}{1+\alpha}g\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right)=0$, i.e.,

$$s_2^* = \frac{(1+\alpha)\bar{G}\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right)}{g\left(\frac{p_{s,2}^{n*}}{1+\alpha}\right)} - p_{s,2}^{n*} + c_2 = \frac{(1+\alpha)\bar{G}\left(\frac{c_2+\kappa_2}{1+\alpha}\right)}{g\left(\frac{c_2+\kappa_2}{1+\alpha}\right)} - \kappa_2.$$

Because $g(v)/\bar{G}(v)$ is increasing in v, s_2^* is strictly decreasing in κ_2. Analogously, by part (a-ii), s_r^* satisfies $\bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right)-\frac{p_{s,2}^{r*}+s_2^*+s_r-c_2+r_2}{k+\alpha}g\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right)=0$, i.e.,

$$s_r^* = \frac{(k+\alpha)\bar{G}\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right)}{g\left(\frac{p_{s,2}^{r*}}{k+\alpha}\right)} - p_{s,2}^{r*} - s_2^* + c_2 - r_2 = \frac{(k+\alpha)\bar{G}\left(\frac{c_2-r_2+\kappa_2-\iota_2}{k+\alpha}\right)}{g\left(\frac{c_2-r_2+\kappa_2-\iota_2}{k+\alpha}\right)} - s_2^* - \kappa_2 + \iota_2.$$

Because $g(v)/\bar{G}(v)$ is increasing in v, s_r^* is strictly increasing in ι_2.

By part (a-iii), s_1^* satisfies $\frac{c_1-r_1}{m_1^s(s_1^*)-r_1}=\frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*}$, the left-hand-side of which is strictly decreasing in s_1^*, whereas the right-hand-side of which is strictly increasing in κ_1. Therefore, s_1^* is strictly decreasing in κ_1. This proves **part (a-iv)**.

Define $\bar{\kappa}_2^s$ as the solution to $\frac{(1+\alpha)\bar{G}\left(\frac{c_2+\kappa_2}{1+\alpha}\right)}{g\left(\frac{c_2+\kappa_2}{1+\alpha}\right)}=\kappa_2$, $\bar{\iota}_2^s$ as the solution to $\frac{(k+\alpha)\bar{G}\left(\frac{c_2-r_2+\kappa_2-\iota_2}{k+\alpha}\right)}{g\left(\frac{c_2-r_2+\kappa_2-\iota_2}{k+\alpha}\right)}-s_2^*-\kappa_2+\iota_2=0$, and $\bar{\kappa}_1^s$ as the solution to $\frac{c_1-r_1}{m_1^s(0)-r_1}=\frac{c_1+\kappa_1-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*}$. Since $g(v)/\bar{G}(v)$ is increasing in v, $\bar{\kappa}_2^s$, $\bar{\iota}_2^s$, and $\bar{\kappa}_1^s$ are well-defined and unique. By the proof of part (a-iv), s_2^* is strictly decreasing in κ_2, s_r^* is strictly increasing in ι_2, and s_1^* is strictly decreasing in κ_1. Therefore, $s_1^*\geq 0$ if and only if $\kappa_1\leq\bar{\kappa}_1^s$, $s_2^*\geq 0$ if and only if $\kappa_2\leq\bar{\kappa}_2^s$, and $s_r^*\geq 0$ if and only if $\iota_2\geq\bar{\iota}_2^s$. This proves **part (a-v)**.

Part (b). By Lemma 3 and Lemma 4 (c), the social-welfare-maximizing equilibrium outcome is the same with strategic customers and with myopic customers, except that $p_{s,1}^*=m_{s,1}^*$ and $\tilde{p}_{s,1}^*=\mu$. Therefore, exactly the same argument as the proof of part (a) proves **part (b)** as well. In particular, since the second-period decisions should be independent of whether the customers are strategic or myopic, $s_2^*=\tilde{s}_2^*$ and $s_r^*=\tilde{s}_r^*$.

Part (c). Since $m_{s,1}^*=\tilde{m}_{s,1}^*$, parts (a) and (b) imply that $\frac{c_1-r_1}{m_1^s(s_1^*)-r_1}=\frac{c_1-r_1}{\tilde{m}_1^s(\tilde{s}_1^*)-r_1}$. Thus, $m_1^s(s_1^*)=\tilde{m}_1^s(\tilde{s}_1^*)$ and, hence, $s_1^*+m_{s,1}^*=\tilde{s}_1^*+\mu$, i.e., $s_1^*-\tilde{s}_1^*=\mu-m_{s,1}^*=-e_s^*$. Therefore, $s_1^*\geq\tilde{s}_1^*$ if and only if $e_s^*\leq 0$. Moreover, since $\bar{\kappa}_1^s$ satisfies $\frac{c_1-r_1}{m_1^s(0)-r_1}=\frac{c_1+\bar{\kappa}_1^s-r_1-\iota_1-s_r^*}{m_{s,1}^*-r_1-s_r^*}$ and $\tilde{\bar{\kappa}}_1^s$ satisfies $\frac{c_1-r_1}{\tilde{m}_1^s(0)-r_1}=\frac{c_1+\tilde{\bar{\kappa}}_1^s-r_1-\iota_1-s_r^*}{\tilde{m}_{s,1}^*-r_1-s_r^*}$. Because $m_{s,1}^*=\tilde{m}_{s,1}^*$, $\bar{\kappa}_1^s\geq\tilde{\bar{\kappa}}_1^s$ if and only if $m_1^s(0)\leq\tilde{m}_1^s(0)$, i.e., $e_s^*\leq 0$. This proves **part (c)**. □

Proof of Theorem 9: Part (a). We first compute $C_g^*=C_g(s_g^*)$ and $\tilde{C}_g^*=\tilde{C}_g(\tilde{s}_g^*)$, observe that

$$C_g(s_g^*)=\mathbb{E}\{s_1^*(X\wedge Q_{s,1}^*)+s_r^*(Q_{s,1}^*-X)^++\delta[s_2^*Q_{s,2}^n((X-Q_{s,1}^*)^+,X\wedge Q_{s,1}^*)+(s_r^*+s_2^*)Q_{s,2}^r((X-Q_{s,1}^*)^+,X\wedge Q_{s,1}^*)]\},$$

and

$$\tilde{C}_g(\tilde{s}_g^*)=\mathbb{E}\{\tilde{s}_1^*(X\wedge\tilde{Q}_{s,1}^*)+\tilde{s}_r^*(\tilde{Q}_{s,1}^*-X)^++\delta[\tilde{s}_2^*\tilde{Q}_{s,2}^n((X-\tilde{Q}_{s,1}^*)^+,X\wedge\tilde{Q}_{s,1}^*)+(s_r^*+s_2^*)\tilde{Q}_{s,2}^r((X-\tilde{Q}_{s,1}^*)^+,X\wedge\tilde{Q}_{s,1}^*)]\}.$$

By Lemma 4(c) and Theorem 8(c), $Q_{s,1}^*=\tilde{Q}_{s,1}^*$, $s_2^*=\tilde{s}_2^*$, and $s_r^*=\tilde{s}_r^*$, it follows immediately that $C_g(s_g^*)-\tilde{C}_g(\tilde{s}_g^*)=(s_1^*-\tilde{s}_1^*)\mathbb{E}(X\wedge Q_{s,1}^*)$, which proves **part (a)**.

Part (b). By part (a), $C_g^*\geq\tilde{C}_g^*$ if and only if $s_1^*\geq\tilde{s}_1^*$. By Theorem 8(c), $s_1^*\geq\tilde{s}_1^*$ if and only if $e_s^*\leq 0$. Since $e_s^*=\mathbb{E}((k+\alpha)V-c_2-\kappa_2+r_2+\iota_2)^+-\mathbb{E}((1+\alpha)V-c_2-\kappa_2)^+$ is strictly increasing in $r_2+\iota_2$. Hence, let $\bar{\mathcal{V}}:=\min\{r_2+\iota_2:e_s^*\geq 0\}$. It follows immediately that $e_s^*\leq 0$ if and only if $r_2+\iota_2\leq\bar{\mathcal{V}}_2$. We observe that $\mathbb{E}((k+\alpha)V-c_2-\kappa_2)^+-\mathbb{E}((1+\alpha)V-c_2-\kappa_2)^+<0$. Thus, $\bar{\mathcal{V}}_2>0$. This establishes **part (b)**. □

Appendix C: Model with Inventory Carryover

In our base model, we assume that the firm recycles and remanufactures all the excess inventory of the first-generation product at the end of the first period. In some scenarios, the firm carries the leftover first-generation products and offer both generations of the product in the second period. Note that the firm may offer two product generations to the market only if the realized first-period demand X_1 is lower than the first-period production quantity Q_1. Otherwise, the firm only sells the second-generation product in period 2. For tractability, we assume that V follows the uniform distribution on $[0, 1]$.

We begin our analysis with the pricing and production decisions in the second period. As in the base model, the optimal price and purchasing decisions are independent of whether the customers are strategic or myopic. If $X_1 \geq Q_1$, there is no leftover first-generation products to sell in period 2. Hence, the equilibrium decisions are identical to those characterized in Lemma 1. On the other hand, if $X_1 < Q_1$, there are $X_2^r = X_1$ repeat customers and $Y_2 = Q_1 - X_1$ leftover inventory in period 2. For a type-V customer, her valuation for the first-generation product is $(k + \alpha)V$, and that for the second-generation product is kV. For any given (X_2^r, Y_2), let $p_2^1(X_2^r, Y_2)$ and $p_2^2(X_2^r, Y_2)$ be the optimal prices for the first-generation and the second-generation products in period 2, respectively. Accordingly, the associated optimal sales of the first- and second- generation product is $Q_2^1(X_2^r, Y_2)$ and $Q_2^2(X_2^r, Y_2)$, respectively. The following lemma characterizes the second-period equilibrium decisions in the model with inventory carryover.

Lemma **6** *Assume that* $0 < X_1 < Q_1$ *(i.e.,* $X_2^r > 0$ *and* $Y_2 > 0$*).*

(a) There exist two thresholds $\theta_1(X_2^r, Y_2)$ *and* $\theta_2(X_2^r, Y_2)$ $(0 \leq \theta_1(X_2^r, Y_2) \leq \theta_2(X_2^r, Y_2) \leq 1)$*, such that customers with valuations* $V \in [\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2)]$ *will purchase the first-generation product, and those with valuations* $V \in [\theta_2(X_2^r, Y_2), 1]$ *will purchase the second-generation product. Hence,* $Q_2^1(X_2^r, Y_2) = X_2^r(\theta_2(X_2^r, Y_2) - \theta_1(X_2^r, Y_2))$ *and* $Q_2^2(X_2^r, Y_2) = X_2^r(1 - \theta_2(X_2^r, Y_2))$*.*

(b) $p_2^1(X_2^r, Y_2) = k\theta_1(X_2^r, Y_2)$ *and* $p_2^2(X_2^r, Y_2) = k\theta_1(X_2^r, Y_2) + \alpha\theta_2(X_2^r, Y_2)$*.*

(c) Let $\lambda := Y_2/X_2^r$*. There exists a threshold* $\bar{\lambda}$*, such that if* $\lambda > \bar{\lambda}$*, (i)* $\theta_1(X_2^r, Y_2) = \theta_1^*$ *and* $\theta_2(X_2^r, Y_2) = \theta_2^*$ *for some constants* θ_1^* *and* θ_2^**; and (ii)* $Q_2^1(X_2^r, Y_2) = X_2^r(\theta_2^* - \theta_1^*) < Y_2$*. Otherwise,* $\lambda \leq \bar{\lambda}$*,* $Q_2^1(X_2^r, Y_2) = Y_2$*.*

Lemma 6 implies that, in the model with inventory carryover, the repeat customers with high valuations will purchase the second-generation product (i.e., $V \geq \theta_2(X_2^r, Y_2)$), whereas those with moderate valuations (i.e., $V \in [\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2)]$) will purchase the first-generation one. Moreover, if the leftover inventory level is high (i.e., $\lambda > \bar{\lambda}$), the customers purchase a fraction of the leftover first-generation products ($Q_2^1(X_2^r, Y_2) < Y_2$), and the purchasing thresholds $\theta_1(X_2^r, Y_2)$ and $\theta_2(X_2^r, Y_2)$ are independent of the (X_2^r, Y_2). Otherwise, the customers purchase all of the leftover first-generation products ($Q_2^1(X_2^r, Y_2) = Y_2$).

We now study the first-period RE equilibrium. If the customers are strategic and the first-period production quantity is Q_1, the first-period equilibrium price $p_1^{ic}(Q_1) = \mu + \delta[(k+\alpha)\mathbb{E}(V - \theta_2(X \wedge Q_1, (Q_1 - X)^+))^+ + (k + \alpha)\mathbb{E}\{(V - \theta_1(X \wedge Q_1, (Q_1 - X)^+))^+ : V \in [\theta_1(X \wedge Q_1, (Q_1 - X)^+), \theta_2(X \wedge Q_1, (Q_1 - X)^+)]\} - \mathbb{E}((1 + \alpha)V - p_2^{n*})^+]$. Therefore, given the production quantity Q_1, the expected total profit $\Pi_f^{ic}(Q_1)$ is given by

$$\begin{aligned}\Pi_f^{ic}(Q_1) \quad = \quad & p_1^{ic}(Q_1)\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + \delta\{\mathbb{E}[(1 - \theta_2(X \wedge Q_1, (Q_1 - X)^+))(p_2^2(X \wedge Q_1, (Q_1 - X)^+) - c_2 + r_2)] \\ & + \mathbb{E}[(\theta_2(X \wedge Q_1, (Q_1 - X)^+) - \theta_1(X \wedge Q_1, (Q_1 - X)^+))p_2^1(X \wedge Q_1, (Q_1 - X)^+)] + \beta_n^* \mathbb{E}(X - Q_1)^+\}\end{aligned}$$

Hence, $Q_1^{ic*} = \text{argmax}_{Q_1 \geq 0} \Pi_f^{ic}(Q_1)$ is the equilibrium production quantity, and $p_1^{ic*} = p_1^{ic}(Q_1^{ic*})$ is the equilibrium first-period price, with strategic customers. If the customers are myopic, the first-period equilibrium price $\tilde{p}_1^{ic*}$ should equal to the expected valuation μ. Therefore, given the production quantity Q_1, the expected total profit $\tilde{\Pi}_f^{ic}(Q_1)$ is given by

$$\begin{aligned}\tilde{\Pi}_f^{ic}(Q_1) \quad = \quad & \mu\mathbb{E}(X \wedge Q_1) - c_1 Q_1 + \delta\{\mathbb{E}[(1 - \theta_2(X \wedge Q_1, (Q_1 - X)^+))(p_2^2(X \wedge Q_1, (Q_1 - X)^+) - c_2 + r_2)] \\ & + \mathbb{E}[(\theta_2(X \wedge Q_1, (Q_1 - X)^+) - \theta_1(X \wedge Q_1, (Q_1 - X)^+))p_2^1(X \wedge Q_1, (Q_1 - X)^+)] + \beta_n^* \mathbb{E}(X - Q_1)^+\}\end{aligned}$$

Hence, $\tilde{Q}_1^{ic*} = \text{argmax}_{Q_1 \geq 0} \tilde{\Pi}_f^{ic}(Q_1)$ is the equilibrium production quantity with myopic customers.

Without trade-in remanufacturing, we consider two cases: (a) $X \geq Q_1$ and (b) $X < Q_1$. In case (a) (i.e., $X \geq Q_1$), there's no leftover inventory in period 1, the firm should adopt the same pricing strategy as $p_2^u(\cdot, \cdot)$. In case (b) (i.e., $X > Q_1$), there is no new customer in the market in period 2, so the pricing strategy is the same as the one characterized in Lemma 6. Therefore, in the model with inventory carryover, the impact of strategic customer behavior should be qualitatively the same as the model without: Strategic customer behavior makes trade-in remanufacturing a lot more attractive to the firm, but also a lot more detrimental to the environment. Because the second-period equilibrium pricing strategy is contingent with the realized market sizes (X_2^n, X_2^r), and leftover inventory Y_2, the socially optimal (second-period) subsidy/tax scheme also depends on (X_2^n, X_2^r, Y_2). Moreover, since the first-period production quantity optimization is no longer a newsvendor problem, the optimal subsidy/tax policy should be contingent on the production quantity Q_1 as well.

Proof of Lemma 6: Part (a). For any prices (p_2^1, p_2^2), the customers with valuation $\{V \in [0,1] : (k+\alpha)V - p_2^2 \geq \max\{0, kV - p_2^1\}\} = \{V \in [0,1] : V \geq \max\{p_2^2/(k+\alpha), (p_2^2 - p_2^1)/\alpha\}\}$ will purchase the second-generation product, whereas those with valuation $\{V \in [0,1] : kV - p_2^1 \geq \max\{0, (k+\alpha)V - p_2^2\}\} = \{V \in [0,1] : V \in [p_2^1/k, (p_2^2 - p_2^1)/\alpha]\}$ will purchase the first-generation product. Since V is uniformly distributed on $[0,1]$, **part (a)** follows immediately.

Part (b). By the proof of part (a), $\theta_1(X_2^r, Y_2) = p_2^1(X_2^r, Y_2)/k$ and $\theta_2(X_2^r, Y_2) = (p_2^2(X_2^r, Y_2) - p_2^1(X_2^r, Y_2))/\alpha$. Direct algebraic manipulation yields that $p_2^1(X_2^r, Y_2) = k\theta_1(X_2^r, Y_2)$ and $p_2^2(X_2^r, Y_2) = k\theta_1(X_2^r, Y_2) + \alpha\theta_2(X_2^r, Y_2)$. This proves **part (b)**.

Part (c). The optimal thresholds $(\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2))$ is the solution to the following convex program:

$$\begin{aligned}(\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2)) =& \text{argmax}_{0 \leq \theta_1 \leq \theta_2 \leq 1}[k\theta_1(\theta_2 - \theta_1) + (k\theta_1 + \alpha\theta_2 - c_2)(1 - \theta_2)]X_2^r \\ \text{s.t. } (\theta_2 - \theta_1)X_2^r \leq& Y_2.\end{aligned} \tag{4}$$

It is straightforward to show that $(\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2))$ exists and is unique, and is determined by the ratio $\lambda = Y_2/X_2^r$. In particular, let (θ_1^*, θ_2^*) be the solution to (4) without the constraint $(\theta_2 - \theta_1) \leq \lambda$. Let $\bar{\lambda} := \theta_2^* - \theta_1^* \geq 0$. If $Y_2/X_2^r > \bar{\lambda}$, the constraint $(\theta_2 - \theta_1) \leq \lambda$ is non-binding and thus $(\theta_1(X_2^r, Y_2), \theta_2(X_2^r, Y_2)) = (\theta_1^*, \theta_2^*)$. If $Y_2/X_2^r \leq \bar{\lambda}$, the constraint $(\theta_2 - \theta_1) \leq \lambda$ is binding and thus $Q_2^1(X_2^r, Y_2) = (\theta_2(X_2^r, Y_2) - \theta_1(X_2^r, Y_2))X_2^r = Y_2$. This establishes **part (c)**. □