Coopetition and Profit Sharing for Ride-sharing Platforms

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The introduction of on-demand ride-hailing platforms totally changed the way people commute. In recent years, several firms entered this market to directly compete with traditional taxi companies. These online platforms often offer a carpooling service in which several passengers heading in the same direction can share a ride by being efficiently matched to an available vehicle. Examples of such services in NYC include uberPOOL, Lyft Line and Via. Recently, some of these platforms decided to engage in a profit sharing contract with one of their competitors by introducing a new hybrid service. For example, on June 6, 2017, Via officially announced a partnership with an online NYC taxi-hailing platform called Curb. This partnership allows riders to order a taxi, and share some portion of the trip with other riders by using Via's efficient matching algorithm. Since these two platforms are competing with each other, this form of partnership is often referred to as *coopetition*. This paper is motivated by this specific type of coopetition. We model the price competition between ride-hailing platforms by using the Multinomial Logit choice model, and show that a unique equilibrium exists. Then, we analyze the impact of introducing the new joint service to the market. Interestingly, we show that a well-designed *profit sharing contract* benefits both platforms. This result admits a similar win-win outcome as in the supply chain contract literature, even though these two settings are very different. In addition, we show that one can design a profit sharing contract that also benefits the riders and the drivers. Consequently, such a coopetition partnership may benefit every single party (riders, drivers and both platforms) when using a properly designed profit sharing contract.

Key words: Ride-sharing, Coopetition, Profit Sharing, Choice Models

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1. Introduction

On-demand ride-hailing platforms totally changed the way people commute and travel for short distances. Several well-known players in this market are Uber, Lyft, Didi Chuxing, Grab, Ola, Via, Gett and Juno, to name a few. In October 2016, it was reported that Uber had 40 million monthly riders worldwide.[1] Nowadays, using this type of transportation services has become the norm in most major cities (e.g., Uber now operates in more than 600 cities around the world). It is worth mentioning that during the first few years, the growth was moderate but within the last two years, one could see a very successful expansion. For example, it took Uber six years to complete the first

[1] http://fortune.com/2016/10/20/uber-app-riders/

billion rides (from 2009 to 2015), but only an additional six months to reach their two-billionth ride.[2] This means that during the first six months of 2016, the company was providing an average of 5.5 million rides a day (or 230,000 an hour). One can go on and on and readily quote impressive statistics on the scale of this industry.

Within the ride-hailing online market, a recent trend emerged in several cities: *carpooling* or *ride-sharing* services. Several aforementioned companies offer an option which allows passengers heading in the same direction to be matched to the same vehicle and share a ride. In such a service, riders cannot select the people they are sharing with, but instead an algorithm will match several riders to the same vehicle. In NYC, one can find at least three such services: uberPOOL, Lyft Line and Via. The first two platforms offer the ride-sharing service in addition to the regular (non-sharing) service, whereas the third platform offers exclusively the carpooling option. These different companies bear several differentiating features such as price, waiting times and other minor differences which are beyond the scope of this paper. One of the main arguments for sharing a ride is the low price paid by the rider. However, this attractive price comes at the expense of a (usually) lower quality of service: (i) Since the ride is potentially shared with other passengers, the rider loses his/her privacy, and (ii) The ride can take longer (as the driver may pick-up and/or drop-off other passengers). Interestingly, it was reported that some riders highly enjoy sharing rides, as they can meet new people and potentially be exposed to unexpected business opportunities (see, e.g., the NY Times article by Tell 2015).

Clearly, the taxi industry has suffered heavily from this recent market trend. First, these new online services are more efficient and generally cheaper for the rider. Second, the drivers are self-employed, and do not need to purchase an expensive license (medallion) in order to transport passengers. It was reported in January 2017 that "Uber and Lyft cars outnumber yellow cabs in NYC 4 to 1".[3] In a response to this decline, taxi drivers engaged in several complaints about this (perceived) unfair competition, and this was the topic of extensive media coverage. A different way to respond was by modernizing the taxi services so as to better fit into today's economy. For example, in several cities, taxi rides can now be directly ordered from a smartphone application, and the payment (including the tip) can either be completed via the application or in person. One such company based in the US is Curb.[4] On their website, one can read: "Curb is the #1 taxi app in the US that connects you to fast, convenient and safe rides in 65 cities (50,000 Cabs – 100,000 Drivers)". A screenshot of their homepage can be found in Figure 1. Some other local taxi companies also decided to build their own smartphone application (for example, Ace Taxi in

[2] https://techcrunch.com/2016/07/18/uber-has-completed-2-billion-rides/

[3] https://ny.curbed.com/2017/1/17/14296892/yellow-taxi-nyc-uber-lyft-via-numbers

[4] https://gocurb.com/

Cleveland, Ohio). In Europe, such online applications for ordering taxis are also available, examples include: Lecab in Paris, France, Talixo in Germany, and taxi.eu which operates in 100 European cities. These companies try to highlight the advantages of the taxi services, such as safety, reliability, comfortable cars, lack of surge pricing and low waiting times.

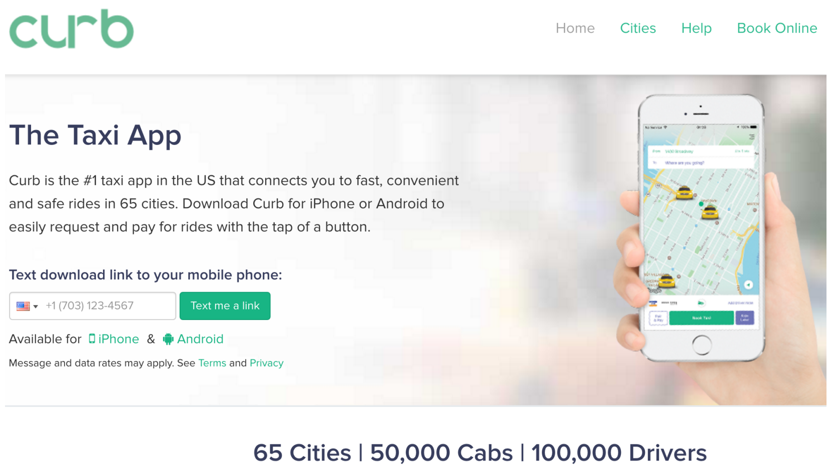


Figure 1 **Homepage of the Curb website**

It is worthwhile mentioning that even though ride-hailing platforms are very successful (both in raising capital and in capturing growing market shares), traditional taxis are still one of the most popular modes of transportation. For example, if we focus on the NYC market, the numbers are quite surprising. It was recently reported that the NYC taxis are still delivering twice as many rides as Uber.[5] Specifically, in April 2016, there were 11.1 million taxi trips, whereas Uber, Lyft and Via completed about 4.7 million, 750,000 and 450,000 rides respectively.

Consider for example two online platforms in the NYC market: Via (ride-sharing) and Curb (non-shared yellow taxis). Via offers a flat affordable fare for riders who are willing to carpool, whereas Curb offers a private taxi ride while charging the meter price plus an additional fixed fee per trip. One can definitely view them as two competitors. Yet, these two platforms decided to collaborate and engage in a unique partnership. More precisely, on June 6, 2017, the two platforms started to offer a joint service through a *profit sharing contract*, under which Curb and Via each earn a certain portion of the net profit from the joint service. This type of partnerships is sometimes referred to as *coopetition*, a term coined to describe cooperative competition (see, for example, Brandenburger and Nalebuff 2011). The new joint service introduced by Curb and Via in NYC allows users to book a shared taxi from either platform. For example, when a user requests a ride through the Via smartphone application, s/he may be offered to ride with a nearby available taxi

[5] https://techcrunch.com/2016/07/11/nyc-taxis-are-still-giving-twice-as-many-rides-as-uber/

(this option is called Shared Taxi). Then, the rider can either accept the Shared Taxi or decline by requesting a regular Via ride. Shared Taxi fares are calculated using the meter price, and paid directly to the driver. If the matching algorithm finds another rider heading in the same direction, the two riders will carpool and save 40% on any shared portion of the trip. Finally, the rider can pay (and tip) the taxi driver directly through either online platform. When introducing this new service, Via sent an e-mail advertising campaign to several NYC users (parts of the e-mail content can be seen in Figure 2).[6]

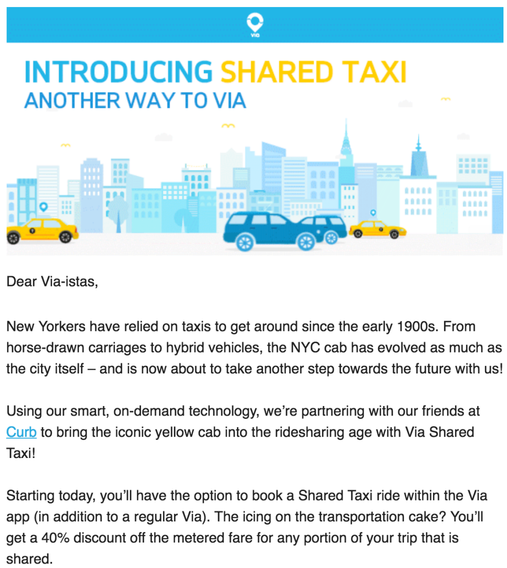


Figure 2 Advertising e-mail sent on June 6, 2017 on the new partnership between Curb and Via in NYC

This recent partnership between Curb and Via in NYC is definitely not an exception. Here are similar other examples:

- In December 2016, Uber partnered with Indonesia's second-biggest taxi operator PT Express Transindo Utama Tbk.
- In October 2014, Uber partnered with For Hire taxis to expand pick-up availability in Seattle.
- In March 2017, Grab partnered with SMRT Taxis in Southeast Asia.

It is clear that both parties (i.e., the ride-hailing platform and the traditional taxi company) have incentives to engage in such partnerships. For example, it allows online ride-hailing platforms to expand their number of available drivers as well as to complement their existing market share. In addition, local taxi companies can benefit from some technological advances (e.g., efficient matching

[6] The partnership between Curb and Via in NYC was the topic of extensive media coverage. See for example: https://www.nytimes.com/2017/06/06/nyregion/new-york-yellow-taxis-ride-sharing.html, https://techcrunch.com/2017/06/06/curb-and-via-bring-ride-sharing-to-nycs-yellow-taxis/ and https://qz.com/999132/can-shared-rides-save-the-iconic-new-york-city-yellow-cab/

algorithms and online secured payments). Note that the focus of this paper is mainly on the case where the new joint service offers a carpooling option (this was the case for Curb and Via and PT Express and Uber). In this case, it also allows the local taxi platform to be more cost effective. Nevertheless, such a partnership can induce an adverse cannibalization effect on the original market shares. Indeed, by introducing the new joint service, each platform needs to be cautious of the potential market share loss (customers who were initially riding with one of the platforms may now switch to the new service).

This paper is motivated by the type of partnerships described above. In particular, we are interested in studying the implications of introducing such a new ride-sharing service through a profit sharing contract. Our goal is to draw practical insights on the impact of the new joint service on both platforms (e.g., Curb and Via), the drivers and the riders. We propose to model this problem by using the Multinomial Logit (MNL) choice model in order to capture the fact that riders face several alternatives. We base our model on current practices in the ride-sharing industry, and study the impact of introducing the new joint service. In particular, we show that a well-designed profit sharing contract increases the profits of both platforms. Furthermore, we demonstrate that under some mild conditions, adding the new service may also benefit the riders and drivers of both platforms. We note that the ideas, analysis and insights presented in this paper are not limited to the ride-sharing industry. Instead, one can apply a similar approach to any market with several competitors who decide to engage in a coopetition partnership through a profit sharing contract. However, in order to simplify the exposition and since this work was motivated by recent partnerships in the ride-sharing industry, we focus our presentation in this context.

1.1. Contributions

Given the recent popularity of ride-hailing and ride-sharing platforms, this paper studies a timely practical problem directly motivated by several recent partnerships. At a high level, our contributions can be summarized as follows.

- **Characterizing the equilibrium of a competitive ride-sharing market.** To the best of our knowledge, this paper is among the first to study the (price) competition between ride-hailing platforms. We use the MNL choice model to capture the decision process of potential riders, and show that the competition between ride-sharing platforms reduces to a price competition under an MNL model with convex costs. Consequently, the equilibrium outcome is analytically tractable and can be computed efficiently.

- **Studying the impact of coopetition via a profit sharing contract.** We study how the introduction of the new joint ride-sharing service affects the competing platforms. In particular, we consider several decision making dynamics depending on which platform sets the price of the new service. We identify two key conflicting effects induced by introducing the new joint service: new market share and cannibalization, and draw insights on their interplay.

• **Demonstrating that a profit sharing contract can yield a win-win outcome.** We show that regardless of which platform sets the price of the new service, there always exists a profit sharing contract that increases the profits of both platforms. As a result, engaging in a coopetition could be a win-win strategy for both parties.

• **Showing that drivers (and riders) can also benefit.** As expected, riders also benefit from the introduction of the new service. Furthermore, we demonstrate that under mild and realistic conditions, the drivers of both platforms can also be better off in the presence of coopetition. Consequently, when the coopetition terms are carefully designed, every single party could benefit (riders, drivers and both platforms).

1.2. Related Literature

This paper is related to at least four streamlines of literature: the economics of ride-hailing platforms, coopetition models, choice models and supply chain contracts.

First, the recent popularity of ride-hailing platforms triggered a great interest in studying pricing decisions of such platforms. Several works consider the problem of designing the right incentives on prices and wages to coordinate supply and demand for on-demand service platforms (see, for example, Tang et al. 2017, Taylor 2017, Hu and Zhou 2017, Bimpikis et al. 2016). Our work has a similar motivation, but is among the first to explicitly capture the competition between two platforms by using an MNL choice model. In Chen and Sheldon (2016), the authors analyze Uber data from 25 million trips and show empirically that a dynamic wage (due to surge pricing) can entice drivers to work longer. In Chen and Hu (2016), the authors consider a single market with an intermediary who makes dynamic pricing and matching decisions. When consumers are forward-looking (i.e., they may wait strategically for better prices), the authors show that under their model and assumptions, a simple pricing and matching policy is optimal while inducing the buyers and the seller to behave myopically. In Hu and Zhou (2017), the authors study the pricing of an on-demand platform and demonstrate the good performance of a flat-commission contract (i.e., the platform takes a fixed cut). More precisely, they show that as long as supply curves are concave in the wage, the optimal flat-commission contract allows the platform to achieve at least 75% of the optimal profit (an extension with a welfare-maximization objective is also provided). Two recent papers, Banerjee et al. (2015) and Cachon et al. (2017), compare the impact of static versus dynamic prices and wages. By assuming that the payout ratio is exogenously given and that customers have heterogeneous valuations, Banerjee et al. (2015) show the good performance of static pricing. Under different modeling assumptions (endogenous payout ratio and homogeneous valuations), Cachon et al. (2017) found that dynamic (surge) pricing performs well.

A different related topic is the competition in the taxi industry (see, for example, Cairns and Liston-Heyes 1996). The recent article in Cramer and Krueger (2016) shows, by using data from five cities, that UberX drivers have captured a higher capacity utilization rate than taxi drivers. The authors suggest that this finding can be explained by four factors: (1) Uber's efficient matching technology, (2) Uber's larger scale, (3) Inefficient taxi regulations, and (4) Uber's flexible labor model and surge pricing. Although the concept of ride-sharing has existed for decades and can potentially provide several societal benefits, such as reducing travel costs, congestion and emissions, its adoption remained limited for some time (see, for example, Furuhata et al. 2013). However, the recent ubiquity of digital/mobile technology and the popularity of peer-to-peer services have led to an unprecedented growth in recent years. Several academic papers propose efficient scheduling algorithms to study the potential benefits of a real-time taxi sharing service via simulation (see, e.g., Ma et al. 2013, 2015).

As we mentioned, when two competitors engage in some sort of cooperation, this is often referred to as *coopetition*, a term coined to describe cooperative competition (see, for example, Brandenburger and Nalebuff 2011). Closer to our work, there are several papers on coopetition in operations settings. For example, Nagarajan and Sošić (2007) propose a model for coalition formation among competitors who set prices, and characterize the equilibrium behavior of the resulting strategic alliances. Casadesus-Masanell and Yoffie (2007) study the simultaneously competitive and cooperative relationship between two manufacturers of complementary products, such as Intel and Microsoft, on their R&D investment, pricing, and the timing of new product releases. In a strategic alliance setting with capacity sharing, Roels and Tang (2017) show that an *ex ante* capacity reservation contract always makes both firms better off. In the revenue management literature, several works have studied a commonly adopted form of coopetition among airline companies, called airline alliances. Netessine and Shumsky (2005) show that a well-designed revenue sharing contract can coordinate an airline alliance (i.e., achieve the first-best market outcome). Wright et al. (2010) further extend this result to a dynamic setting. Coopetition and its related contractual issues have also been studied in a service setting. For example, Roels et al. (2010) analyze the contracting issues that arise in collaborative services, and identify the optimal contracts under different service environments. The marketing literature has also studied the impact of coopetition, e.g., in sharing the same advertising agency (Villas-Boas 1994). Note, however, that our paper is among the first to study coopetition in the ride-sharing industry, motivated by such recent partnerships.

The third stream of relevant literature is related to choice models (for a review on this topic, see Train 2009, and the references therein), and in particular the price competition under the MNL model and its extensions (see, e.g., Anderson et al. 1992, Gallego et al. 2006, Konovalov and Sándor 2010, Li and Huh 2011, Aksoy-Pierson et al. 2013). Using the MNL demand model, Gallego et al.

(2006) show that a unique equilibrium exists when the costs are increasing and convex in the sales. In Li and Huh (2011), the authors consider the problem of pricing multiple products under the nested MNL choice model, and show that characterizing the equilibrium outcome is analytically tractable. In this literature, the main focus is on showing the existence and/or uniqueness of the equilibrium outcome. In this paper, we extend the results of Gallego et al. (2006) and Li and Huh (2011) to show that a unique equilibrium exists in the ride-sharing competition when the drivers can be self-scheduled. In addition, our emphasis is on drawing practical insights on how the coopetition impacts the different stakeholders of a ride-sharing market (online platforms, riders and drivers).

Finally, our paper is related to the literature on risk sharing contracts in supply chains (see, e.g., Cachon 2003, Cachon and Lariviere 2005). This literature shows that one can coordinate the supply chain by using a well-designed contract (e.g., revenue sharing). Such a contract allows the supplier and the retailer to share the demand uncertainty risk, and hence will typically induce the retailer to order larger quantities. It is shown that a well-designed contract can achieve the first-best market outcome, i.e., the same profit as the centralized supply chain (when the supplier and the retailer collude and jointly optimize the total profit). In addition, such a contract can achieve a win-win outcome in which both supply chain partners earn higher profits. Our paper admits several similarities with this classical result, even though these two settings are very different. To the best of our knowledge, our paper is the first to study coopetition contracts in the context of ride-sharing.

Structure of the paper. Section 2 presents the MNL choice model and the characterization of the equilibrium outcome for the original setting. Section 3 considers the setting where the new joint service is introduced through a profit sharing contract between the two platforms. Section 4 studies the impact of introducing the new joint service on the profits of both platforms, on the riders and on the drivers. We report various computational experiments in Section 5 to illustrate and quantify our insights. Finally, our conclusions are presented in Section 6. The proofs of the technical results are relegated to the Appendix.

2. Model and Equilibrium Analysis

In this section, we first present our benchmark model that describes the competitive ride-sharing market of interest before introducing the new service. We then characterize the equilibrium market outcome.

2.1. Benchmark Model

We consider two competing online platforms denoted by P_1 and P_2. P_1 represents the platform that offers a taxi-hailing service (e.g., Curb), whereas P_2 offers a ride-sharing service (e.g., Via).

Let q_1 and q_2 be the perceived value/quality (e.g., service, waiting time, safety, reliability, traveling time, etc.) of taking a ride using P_1 and P_2 respectively. We assume that $q_1 > q_2$, as the expected waiting time of hailing a taxi may typically be smaller relative to requesting a ride through a ride-sharing platform (in addition, most riders will find taxis more reliable and more comfortable). We do not explicitly model the waiting time of hailing a ride using P_1 or P_2, but assume that it is absorbed into q_1 and q_2. We denote by p_1 and p_2 the per-trip average prices charged by P_1 and P_2 respectively. In practice, ride-sharing platforms (such as Via and Uber) typically charge a price which is independent of the number of riders who share the trip. We therefore assume that p_2 does not depend on the number of riders who are sharing the trip. A summary of the notation used in this paper can be found in Appendix A.

We assume that the rider's behavior follows the MNL discrete choice model. More specifically, a rider can choose between 3 options: P_1, P_2, and the outside option (other ride-sharing platforms, public transportation, etc.). The utility a customer derives from hailing a ride through P_i $(i=1,2)$ is $u_i = q_i - p_i + \xi_i$, where ξ_i represents the random unobserved utility terms for using P_i. The utility of the outside option is normalized such that $u_0 = \xi_0$. For each rider, ξ_1, ξ_2 and ξ_0 are assumed to be independent and identically distributed with a Gumbel distribution. Therefore, after observing the prices (p_1, p_2), a rider selects P_1 with probability

$$s_1 = \frac{\exp(q_1 - p_1)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)},$$

selects P_2 with probability

$$s_2 = \frac{\exp(q_2 - p_2)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)},$$

and selects the outside option with probability

$$s_0 = \frac{1}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)}.$$

We remark that it is common to use MNL models to capture the different alternatives a consumer faces when selecting modes of transportation (see, for example, McFadden et al. (1977), Bolduc (1999), and Brownstone and Golob (1992) who use the MNL model in a ride-sharing context). Such models are widely used both in academia and in practice, as they are often analytically tractable and easy to estimate/calibrate.

Let Λ be the total rider arrival rate in the market, i.e., the maximum average number of potential riders arriving per unit time. We assume that Λ is deterministic and known to both platforms (the extension where Λ is stochastic is briefly discussed below). The demand faced by P_i $(i=1,2)$ per unit time is given by $\lambda_i = \Lambda s_i$. Let $C_i(\lambda_i)$ be the total wage per unit time that P_i pays its drivers to serve the stream of riders with arrival rate λ_i.

Given the differences in driver availability and incentives, we derive the different functional forms of $C_i(\cdot)$ for the two platforms. As we mentioned in the introduction, in the NYC market, taxis still complete many more rides than other ride-hailing platforms. Furthermore, taxi drivers (or companies) need to purchase an expensive license, so they may tend to work for longer hours. We thus assume that P_1 has access to abundant taxi drivers, and that the drivers are compensated according to the meter price determined by regulations. Formally, $C_1(\lambda_1) = c_1\lambda_1$, where c_1 is the average meter price per trip, which is independent of the realized rider arrival rate λ_1. The difference $f_1 = p_1 - c_1$ is the fixed fee per trip charged to customers who request a ride through P_1. For example, the US company Curb charges its customers a fixed service fee of $f_1 = \$1.95$ per ride.

For platform P_2 (e.g., Via or Uber), the drivers are self-scheduled, and there may not be enough drivers working for P_2 if the rider arrival rate is high and the wage is low. In this case, the platform can set the drivers' wage to control its driver supply. Let k_2 be the total number of drivers partnering with P_2. We denote by $r_2 \sim G_2(\cdot)$ the per unit time reservation wage of P_2's drivers, which is assumed to be continuously distributed with CDF $G_2(\cdot)$. We further assume that $G_2(\cdot)$ is strictly increasing on the support of r_2. We denote by w_2 the per unit time wage paid by P_2 to its drivers. Hence, the fraction of drivers working for P_2 when the wage is set to w_2 is given by $\beta_2 = \mathbb{P}(r_2 \leq w_2) = G_2(w_2)$, and the total (average) number of active drivers on the road is $k_2^a = \beta_2 k_2 = k_2 G_2(w_2)$. We assume that P_2 is setting w_2 so as to match supply with demand (for example, a platform like Via or Uber often varies the wage dynamically depending on the supply and demand predictions). We denote by t_2 the average trip duration of a rider (including the pick-up time), and by n_2 the average number of riders per trip using P_2's ride-sharing service. Since P_2 may match several customers with similar origins and destinations to the same ride, it is often possible that $n_2 > 1$. Let $\alpha_2(m)$ denote the average proportion of a trip shared by m riders $\left(\sum_{m\geq 1}\alpha_2(m) = 1\right)$. We then have $n_2 = \sum_{m\geq 1} m\alpha_2(m)$. We note that, in practice, t_2 and n_2 may be dependent on the customer arrival rate λ_2 and on the number of active drivers k_2^a. Nevertheless, we assume for simplicity that t_2 and n_2 are exogenous, and independent of λ_2 and k_2^a. The expected number of riders served by P_2 is thus given by $N_2 = k_2^a n_2 = k_2 G_2(w_2) n_2$. On the other hand, by Little's Law, we have $N_2 = \lambda_2 t_2 = \Lambda s_2 t_2$. By matching supply with demand, we obtain: $k_2 G_2(w_2) n_2 = \lambda_2 t_2$. As a result, the wage per unit time for P_2's drivers is given by:

$$w_2 = G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right),$$

and the total wage per unit time paid by P_2 amounts to:

$$C_2(\lambda_2) = k_2^a w_2 = k_2 G_2(w_2) G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right) = \frac{\lambda_2 t_2}{n_2} G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right).$$

Since the probability that a rider (resp. driver) will select (resp. work for) P_2 lies in the interval $[0,1]$ (i.e., $0 \le s_2 \le 1$ and $0 \le \beta_2 \le 1$), the constraint $0 \le \lambda_2 \le \min\left\{\Lambda, \frac{n_2 k_2}{t_2}\right\}$ should hold. In other words, the realized arrival rate of P_2 is bounded by the total customer arrival rate (Λ) and by its supply capacity ($\frac{n_2 k_2}{t_2}$).

Lemma 1. *If $G_2(\cdot)$ satisfies the log-concave condition on its support,*[7] *then $C_2(\lambda_2)$ is convex and strictly increasing in λ_2.*

Lemma 1 establishes the convexity of the total wage paid by P_2 to its drivers, $C_2(\cdot)$, with respect to the arrival rate λ_2. The result in Lemma 1 will allow us to show the existence and uniqueness of the equilibrium market outcome (see Section 2.2).

We are now ready to compute the per unit time expected profit of both platforms. Given the price vector (p_1, p_2), let $\pi_i(p_1, p_2)$ be the expected profit of P_i $(i = 1, 2)$. We can write:

$$\pi_i(p_1, p_2) = p_i \Lambda s_i - C_i(\lambda_i), \text{ with } s_i = \frac{\exp(q_i - p_i)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)},$$

where $C_1(\lambda_1) = c_1 \lambda_1$ and $C_2(\lambda_2) = \frac{t_2}{n_2} \lambda_2 G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)$. The actual arrival rate of P_2 should not exceed its supply capacity, so we impose the constraint $\Lambda s_2 \le \frac{n_2 k_2}{t_2}$. As we previously mentioned, we consider the situation where P_1 has access to an abundant supply of drivers, whereas P_2 has a limited supply. Note, however, that most results and qualitative insights presented in this paper still hold for the cases where both platforms have abundant or limited supply, and the case where both platforms share the same pool of limited and self-scheduled drivers (e.g., drivers who work for several ride-hailing platforms). Moreover, it is worth mentioning that our results and insights continue to hold in the case where Λ is stochastic and follows a given distribution $F(\cdot)$, which is common knowledge to both platforms. This setting applies to the case where customer demand for rides can vary under different scenarios (see Hu and Zhou 2017). With stochastic customer arrival rate, the expected profit of P_i is given by $\pi_i(p_1, p_2) = p_i s_i \mathbb{E}[\Lambda] - \mathbb{E}[C_i(\Lambda s_i)]$, with $s_i = \frac{\exp(q_i - p_i)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)}$, where the expectation is taken with respect to the distribution of Λ.

2.2. Equilibrium Analysis

In the benchmark model, the platforms P_1 and P_2 compete with each other on price. More specifically, they engage in a simultaneous game in which P_1 sets p_1 to maximize $\pi_1(p_1, p_2)$, and P_2 sets p_2 to maximize $\pi_2(p_1, p_2)$. The equilibrium prices (p_1^*, p_2^*) satisfy:

$$p_1^* \in \arg\max_{p_1} \pi_1(p_1, p_2^*) \text{ and } p_2^* \in \arg\max_{p_2} \pi_2(p_1^*, p_2),$$

[7] A distribution satisfies the log-concave condition, if the logarithm of its CDF is concave. Note that several commonly used distributions (e.g., uniform, normal, Gamma, Weibull, logistic, etc.) satisfy this condition (see Bagnoli and Bergstrom 2005). For the rest of the paper, we assume that $G_2(\cdot)$ is log-concave.

with the constraint $\Lambda s_2^* t_2 \leq n_2 k_2$, i.e., $s_2^* \leq \frac{n_2 k_2}{\Lambda t_2}$. We denote the equilibrium rider choice probabilities by (s_1^*, s_2^*, s_0^*). As reported in the next proposition, one can show the existence and uniqueness of the equilibrium market outcome.

PROPOSITION 1. *There exists a unique equilibrium (p_1^*, p_2^*) which can be computed efficiently.*

Proposition 1 shows that there exists a unique equilibrium in the price competition game played by P_1 and P_2. In the Appendix, we provide a detailed procedure to efficiently compute the equilibrium outcome (i.e., to find the values of s_0^*, s_1^*, s_2^*, p_1^* and p_2^*). Even though we cannot characterize the equilibrium outcome in closed form, the equilibrium can be solved efficiently (e.g., through a binary search). Gallego et al. (2006) and Li and Huh (2011) characterize the equilibrium outcome for the price competition under the MNL and Nested MNL choice models, respectively. In this paper, we extend their modeling approach and solution technique to the competitive ride-sharing market with self-scheduled drivers.

3. Coopetition and Profit Sharing

In this section, we model the setting where the new service is introduced through a profit sharing contract between the two platforms. In particular, the two competing platforms P_1 and P_2 collaborate and offer a new joint service. In this new service, P_1's drivers provide shared rides, which are available to riders from either platform. As mentioned before, one such recent example is the partnership between Curb and Via with the introduction of a taxi sharing service in NYC since June 6, 2017. Such a partnership between competing firms is often referred to as *coopetition*. For the rest of this paper, we use the terms "new joint service" and "coopetition" interchangeably.

We use the symbol $\tilde{}$ to denote the different variables in the presence of coopetition. More specifically, we denote by $\tilde{p}_1$ and $\tilde{p}_2$ the prices of the original services offered by P_1 and P_2, respectively, after introducing the new joint service. The quality and price per trip of the new service are denoted by q_n and $\tilde{p}_n$ respectively. Since the new joint service allows riders to share some portion of the trip with passengers heading in the same direction, we naturally assume that $q_n < q_1$. On the other hand, we impose that $q_n > q_2$. Since the new joint service is fulfilled by P_1's drivers, the expected waiting time is typically lower (given that P_1's drivers are abundant), and the service may be more reliable and more comfortable. We note though that our results and insights continue to hold in the cases where $q_n \geq q_1$ or $q_n \leq q_2$.

As for the original services, the utility derived by a rider from choosing the new joint service is $u_n = q_n - \tilde{p}_n + \xi_n$, where ξ_n represents the random unobserved utility terms for using the new service. As in the benchmark case, we assume that for each rider, ξ_1, ξ_2, ξ_n and ξ_0 are independent and identically distributed with a Gumbel distribution. After introducing the new joint service, a rider faces four different alternatives (P_1, P_2, the new joint service, and the outside option), and chooses the one with the highest utility. Therefore, in the presence of coopetition, a rider selects P_1's original service with probability

$$\tilde{s}_1 = \frac{\exp(q_1 - \tilde{p}_1)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)},$$

selects P_2's original service with probability

$$\tilde{s}_2 = \frac{\exp(q_2 - \tilde{p}_2)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)},$$

selects the new joint service (through either platform) with probability

$$\tilde{s}_n = \frac{\exp(q_n - \tilde{p}_n)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)},$$

and selects the outside option with probability

$$\tilde{s}_0 = \frac{1}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)}.$$

Since the new joint service is fulfilled by P_1's drivers, the drivers for the new service are also abundant. We use $\tilde{n} \geq 1$ to denote the average number of riders per trip in the new joint service. Recall that the drivers of the new service are still paid according to the meter price c_1 for each completed trip (the current wage in a service like Shared Taxi in NYC is such that the drivers earn the meter price regardless of the number of riders who share the trip). The total wage per unit time of the new joint service is equal to c_1 multiplied by the number of active drivers (in this case, the number of drivers needed to serve the riders who request the new service) needed per unit time. The number of active drivers per unit time is $\tilde{\lambda}_n/\tilde{n}$, where $\tilde{\lambda}_n = \Lambda \tilde{s}_n$. Therefore, the total wage per unit time for the new joint service is given by: $\tilde{C}_n(\tilde{\lambda}_n) = \frac{c_1 \tilde{\lambda}_n}{\tilde{n}} = \tilde{c}_n \tilde{\lambda}_n$, where $\tilde{c}_n := \tilde{\lambda}_n/\tilde{n}$ denotes the average wage per trip per rider of the new joint service. It is worth noting that our analysis and insights can easily be generalized to the setting where the drivers of the new joint service are P_2's drivers, who endogenously decide whether and when to work for P_2.

As per the *profit sharing contract*, platforms P_1 and P_2 will split the net profit from the new joint service. More precisely, we consider a profit sharing contract in which P_1 receives a fraction γ of the profit generated by the new service, whereas P_2 receives $1-\gamma$ $(0 < \gamma < 1)$. Under such a contract, the total profits of P_1 and P_2 are given by:

$$\tilde{\pi}_1(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2 | \gamma) = \Lambda[(\tilde{p}_1 - c_1)\tilde{s}_1 + \gamma(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n],$$
$$\tilde{\pi}_2(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2 | \gamma) = \Lambda[\tilde{p}_2 \tilde{s}_2 + (1-\gamma)(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n] - C_2(\tilde{\lambda}_2),$$

where we have:

$$\tilde{s}_i = \frac{\exp(q_i - \tilde{p}_i)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)} \ (i = 1, 2, n), \text{ and } C_2(\tilde{\lambda}_2) = \frac{t_2}{n_2} \tilde{\lambda}_2 G_2^{-1}\left(\frac{\tilde{\lambda}_2 t_2}{n_2 k_2}\right).$$

Therefore, the total profits generated by both platforms is given by:

$$\tilde{\pi}(\tilde{p}_n,\tilde{p}_1,\tilde{p}_2)=\tilde{\pi}_1(\tilde{p}_n,\tilde{p}_1,\tilde{p}_2|\gamma)+\tilde{\pi}_2(\tilde{p}_n,\tilde{p}_1,\tilde{p}_2|\gamma)=\Lambda[(\tilde{p}_1-c_1)\tilde{s}_1+\tilde{p}_2\tilde{s}_2+(\tilde{p}_n-\tilde{c}_n)\tilde{s}_n]-C_2(\tilde{\lambda}_2).$$

As expected, note that the total profits does not explicitly depend on γ. Following the current business practice, we assume that both platforms will maintain the prices of their original services after the introduction of the new joint service, i.e., $\tilde{p}_1=p_1^*$ and $\tilde{p}_2=p_2^*$. This business practice is mainly driven by branding and marketing considerations as well as market regulations (e.g., the meter price for taxis cannot be altered). In the example of Curb and Via in NYC, the price charged for a Via ride has remained the same after the introduction of the new joint service.

The rest of this paper aims to study the impact of such a coopetition partnership. In particular, we are interested in designing the right profit sharing contract, and in quantifying its implications on both platforms as well as on riders and drivers. At a high level, the coopetition will induce two effects: (i) a new market share effect (i.e., capturing some new riders who were previously choosing the outside option), and (ii) a cannibalization effect (i.e., losing some existing market share to the new service). In the next section, we draw practical insights on the interplay of these two effects, and show that a well-designed profit sharing contract will lead to an overall positive benefit for both platforms.

To conclude this section, we remark that if the two platforms decide the price of the new joint service $\tilde{p}_n$ and the profit sharing parameter γ haphazardly, it may decrease the profits of both platforms. For example, consider a setting with $q_1=2,\ q_2=1,\ c_1=1.5,\ \Lambda=1{,}000,\ k_2=500,$ $n_2=3$, $t_2=1$, and assume that r_2 follows a uniform distribution on $[0,1]$. Then, by solving for the equilibrium outcome, we obtain: $p_1^*=2.79$ and $p_2^*=1.55$, with expected profits of $\pi_1(p_1^*,p_2^*)=289$ and $\pi_2(p_1^*,p_2^*)=422$. Consider that the new joint service satisfies $q_n=1.5$ and $\tilde{n}=1.5$ (i.e., an average of 1.5 passengers per ride), so that $\tilde{c}_n=c_1/\tilde{n}=1$. If we set $\tilde{p}_n=1.6$ and $\gamma=0.6$, we obtain: $\tilde{\pi}_1(\tilde{p}_n,p_1^*,p_2^*|\gamma)=273.6<\pi_1(p_1^*,p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n,p_1^*,p_2^*|\gamma)=406.9<\pi_2(p_1^*,p_2^*)$. As a result, when the coopetition terms are not carefully designed, introducing the new joint service may lead to an undesirable lose-lose outcome for both platforms.

4. Impact of Coopetition

In this section, we analyze the impact of the coopetition partnership (i.e., introducing the new joint service). We first consider the profit implications on both platforms, and then study the impact on riders and on drivers.

4.1. Impact on Platforms Profits

Since P_1 and P_2 do not change the prices of their original services after introducing the new joint service (i.e., $\tilde{p}_1 = p_1^*$ and $\tilde{p}_2 = p_2^*$), we are left with two decisions related to the coopetition partnership: the price of the new service $\tilde{p}_n$, and the profit sharing parameter $\gamma \in (0,1)$. We first consider the setting where the parameter γ is exogenous, and then extend our analysis to the case where γ is endogenous. We study three different scenarios: (a) P_1 and P_2 jointly decide $\tilde{p}_n$ to maximize their total profits $\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$; (b) P_1 decides $\tilde{p}_n$ to maximize its own profit $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma)$; and (c) P_2 decides $\tilde{p}_n$ to maximize its own profit $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma)$. We then consider a Stackelberg setting in which one platform strategically decides the profit sharing split parameter γ, and the other platform sets the price of the new service $\tilde{p}_n$ after observing the value of γ. Note that it is important to study the different decision making dynamics in this context, as it may lead to different incentives for the platforms, thus potentially yielding different outcomes.

4.1.1. Joint Pricing Decision In this scenario, P_1 and P_2 jointly decide $\tilde{p}_n$ so as to maximize their total profits $\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$. We denote the optimal price by $\tilde{p}_n^* \in \arg\max_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$, which is the first-best pricing decision under coopetition.

PROPOSITION 2. *When the price $\tilde{p}_n$ is jointly decided by both platforms, the following hold:*

- *There exists a unique price $\tilde{p}_n^*$ that maximizes $\tilde{\pi}(\cdot, p_1^*, p_2^*)$.*
- *There exists an interval $(\underline{\gamma}, \bar{\gamma}) \subset (0,1)$, such that, if and only if $\gamma \in (\underline{\gamma}, \bar{\gamma})$, then $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$.*
- *If $\gamma < \underline{\gamma}$, $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and if $\gamma > \bar{\gamma}$, $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.*

Note that as expected, $\tilde{p}_n^*$ does not depend on the value of γ. Proposition 2 conveys that if the price of the new service is jointly decided by P_1 and P_2 to maximize their total profits, a well-designed profit sharing contract (i.e., $\gamma \in (\underline{\gamma}, \bar{\gamma})$) will benefit both platforms. As discussed at the end of Section 3, when the terms of the coopetition (i.e., $\tilde{p}_n$ and γ) are not carefully designed, introducing the new joint service can yield lower profits for both platforms. However, as we show in Proposition 2, if both platforms jointly decide the price of the new service and the profit split of the new service is not too extreme (i.e., $\gamma \in (\underline{\gamma}, \bar{\gamma})$), introducing the new joint service will lead to a win-win outcome for both platforms.

As mentioned before, engaging in the coopetition under a profit sharing contract induces two conflicting effects: (i) a new market share that will be split between the two platforms according to the profit sharing contract, and (ii) an adverse cannibalization effect. Proposition 2 shows that under a well-designed profit sharing contract, the new market share effect dominates the cannibalization effect for each platform. We will discuss in greater detail this trade-off and the implications of these two effects in Section 4.1.4.

4.1.2. Price Set by One Platform In this scenario, Platform P_i ($i=1$ or 2) sets the price of the new joint service $\tilde{p}_n$ so as to maximize its own profit $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$. We denote P_i's optimal price by $\tilde{p}_n^i(\gamma) \in \arg\max_{\tilde{p}_n} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$. It is clear that the resulting equilibrium price of the new joint service $\tilde{p}_n^i(\gamma)$ depends on which platform sets the price, and on γ.

PROPOSITION 3. *When the price $\tilde{p}_n$ is set by P_i ($i=1,2$), the following hold:*

- *For any γ, there exists a unique $\tilde{p}_n^i(\gamma)$ that maximizes $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$.*
- *If $i=1$ (resp. $i=2$), $\tilde{p}_n^i(\cdot)$ is decreasing (resp. increasing) in γ.*
- *There exists an interval $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$, such that for any $\gamma \in (\underline{\gamma}', \bar{\gamma}')$, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$, and $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$.*
- *If $\gamma < \underline{\gamma}'$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and if $\gamma > \bar{\gamma}'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.*

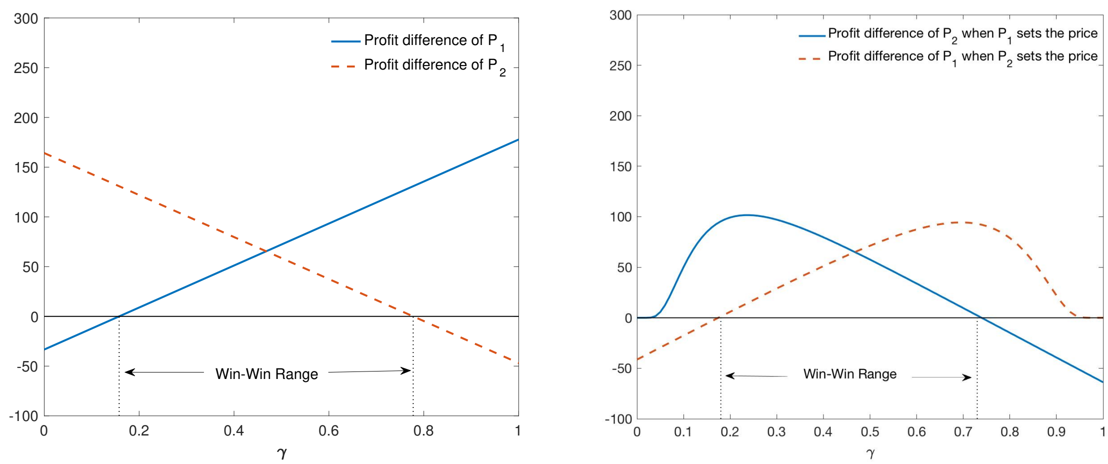
For any value of γ, regardless of which platform sets the price of the new service, there exists a unique optimal price that maximizes the price-setter's profit. In particular, if γ increases, the new market share effect for P_1 (resp. P_2) is strengthened (resp. weakened), which drives this platform to decrease (resp. increase) the price of the new service. An important implication of Proposition 3 is that, irrespective of who (P_1, P_2 or jointly) decides the price of the new joint service, a well-designed profit sharing contract (i.e., $\gamma \in (\underline{\gamma}', \bar{\gamma}')$) will yield higher profits for both platforms. We also find that $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$. Consequently, the range of profit sharing contracts that achieve a win-win outcome is more restricted when one platform sets the price of the new service relative to the case where both platforms jointly decide $\tilde{p}_n$. This bears a similar intuition as the fact that a decentralized decision making process generally induces a suboptimal outcome, and introduces some market inefficiency. As illustrated in Figures 3a and 3b,[8] the range of profit sharing contracts that achieve a win-win outcome for both platforms can be quite wide.

So far, we considered two different scenarios depending on how $\tilde{p}_n$ is decided. A natural question is to inquire if one can attain the first-best outcome (i.e., the market outcome under the joint decision) when a single platform sets $\tilde{p}_n$. The following proposition shows that this is indeed possible.

PROPOSITION 4. *There exists a unique $\gamma^* \in (\underline{\gamma}', \bar{\gamma}')$, such that $\tilde{p}_n^1(\gamma^*) = \tilde{p}_n^2(\gamma^*) = \tilde{p}_n^*$. Moreover, $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) > \pi_2(p_1^*, p_2^*)$.*

Proposition 4 suggests that the (system-wide) first-best price $\tilde{p}_n^*$ can still be achieved when one of the platforms sets $\tilde{p}_n$, as long as the profit sharing contract is specified to be the (unique) parameter

[8] In this example, we chose some specific parameters to illustrate the range of γ values that lead to a win-win outcome. In our tests, this insight seems to be robust when varying the values of the different parameters.



(a) Both platforms jointly decide $\tilde{p}_n$ (b) One of the platforms sets $\tilde{p}_n$

Figure 3 Range of γ values that lead to a win-win outcome (Parameters: $q_1 = 2$, $q_2 = 1$, $c_1 = 1.5$, $t_2 = 1$, $n_2 = 3$, $k_2 = 500$, $\Lambda = 1{,}000$, $q_n = 1.5$, $\tilde{n} = 1.5$, $\tilde{c}_n = 1$, $r_2 \sim U[0,1]$)

γ^*. In other words, a profit sharing contract with the parameter γ^* will induce the first-best market outcome regardless of which platform sets the price $\tilde{p}_n$. Under this profit sharing contract, both platforms face the same new market share/cannibalization tradeoff as they would have faced when $\tilde{p}_n$ is jointly decided. In addition, we have shown that this profit sharing contract will lead to higher profits for both platforms. As we illustrate in Figure 4, the profit sharing contract that leads to the first-best outcome (i.e., γ^*) is the unique solution of $\tilde{p}_n^1(\gamma) = \tilde{p}_n^2(\gamma)$.

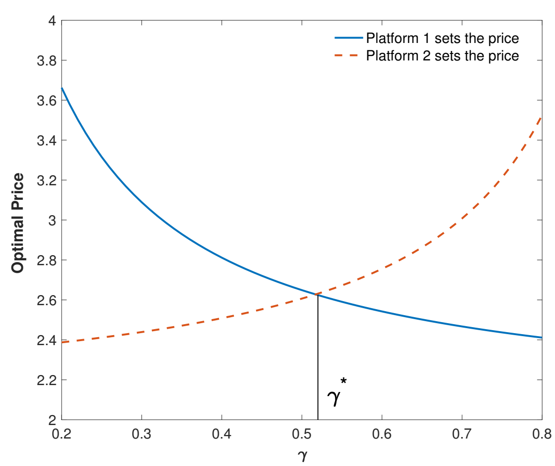


Figure 4 Optimal price as a function of γ when one platform decides $\tilde{p}_n$

Interestingly, the fact that γ^* coordinates both platforms and retrieves the market efficiency is analogous to a classical result in supply chain coordination (see, e.g., Cachon 2003), even though these two settings are very different. In both settings, the contract induces the first-best outcome by aligning the incentives of the decision makers to that of the entire system. In our model, this is achieved by adjusting the new market share/cannibalization trade-off, whereas in the supply chain literature, the coordinating contract aims to share the demand uncertainty risk between the supplier and the retailer.

4.1.3. Stackelberg Setting So far, we considered the case where the contract parameter γ was exogenous. In practice, when introducing the new joint service, the two platforms may have the flexibility to set both $\tilde{p}_n$ and γ. Next, we explore the Stackelberg setting under which one platform strategically decides the profit sharing contract γ, and the other platform sets $\tilde{p}_n$ after observing the value of γ. In particular, we consider both cases where either P_1 or P_2 is the Stackelberg leader to decide γ (while the other platform follows by setting $\tilde{p}_n$). We denote the Stackelberg game when P_1 (resp. P_2) is the leader by $\mathcal{G}_1$ (resp. $\mathcal{G}_2$). In $\mathcal{G}_1$ (resp. $\mathcal{G}_2$), it is clear that the subgame perfect equilibrium is such that for a given γ, P_2 (resp. P_1) selects $\tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n^1(\gamma)$), while P_1 (resp. P_2) sets $\gamma^1 \in \arg\max_{\gamma\in(0,1)} \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma)$ (resp. $\gamma^2 \in \arg\max_{\gamma\in(0,1)} \tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$).

Proposition 5. *In the Stackelberg setting, the following results hold:*

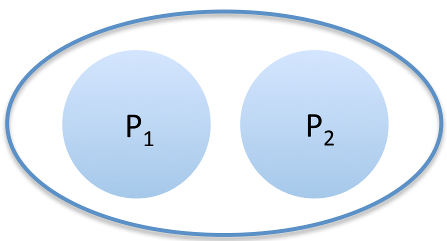
(a) In $\mathcal{G}_1$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) > \pi_1(p_1^*, p_2^*)$ *and* $\tilde{\pi}_2(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) > \pi_2(p_1^*, p_2^*)$. *Moreover,* $\gamma^1 > \gamma^*$ *and* $\tilde{p}_n^2(\gamma^1) > \tilde{p}_n^*$.

(b) In $\mathcal{G}_2$, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma^2), p_1^*, p_2^*|\gamma^2) > \pi_1(p_1^*, p_2^*)$ *and* $\tilde{\pi}_2(\tilde{p}_n^1(\gamma^2), p_1^*, p_2^*|\gamma^2) > \pi_2(p_1^*, p_2^*)$. *Moreover,* $\gamma^2 < \gamma^*$ *and* $\tilde{p}_n^1(\gamma^2) > \tilde{p}_n^*$.

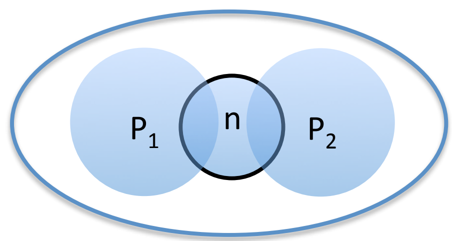
Proposition 5 shows that in the Stackelberg setting, regardless of which platform is the game leader, both platforms will earn higher profits after introducing the new joint service. In addition, the price of the new joint service will be higher in the Stackelberg setting when compared to the case where $\tilde{p}_n$ is jointly decided.

4.1.4. Insights In this section, we have considered various decision making dynamics. In practice, the decision making process may depend on the specific context, and often involves extensive negotiations between the two platforms. Interestingly, the results presented in this paper show that regardless of which platform is in charge of setting the price of the new service, a carefully designed profit sharing contract will increase the profits earned by both platforms. Furthermore, such a win-win outcome can also be achieved in a Stackelberg setting. Interestingly, this result admits a similar win-win outcome as in the supply-chain contracts literature.

As mentioned before, the win-win outcome was not obvious at the first glance, as introducing the new service induces an adverse cannibalization effect. More precisely, the new joint service will take away some customers from the original services. On the other hand, the new service will allow to increase the market share by capturing new customers who were choosing the outside option before the introduction of the new service. Figure 5 illustrates these two conflicting effects. Before the coopetition partnership, each platform captures its own profit share from the total pool of potential riders (Figure 5a). After the coopetition partnership, the new service captures a profit share denoted by n, while each platform's profit share decreases. More precisely, as we can see from Figure 5b, the profit share of the new service is composed of two parts: (i) a new market share (this represents the customers who were choosing the outside option, and now select the new joint service), and (ii) an overlap with the initial profit shares of both platforms (this represents the cannibalization effect of customers who switch from one of the platforms to the new service). As we have shown in this section, properly balancing these two conflicting effects through a profit sharing contract can yield a Pareto profit improvement for both platforms.



(a) Before introducing the new joint service



(b) After introducing the new joint service

Figure 5 **Illustration of the new market share and profit cannibalization effects**

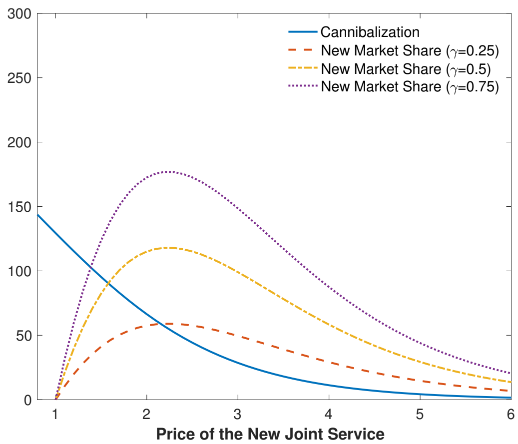
We next illustrate the new market share and cannibalization effects from a mathematical perspective. Recall that the profit difference of P_1 can be written as:

$$\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) - \pi_1(p_1^*, p_2^*) = \underbrace{\gamma\Lambda(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n}_{\text{New Market Share}} - \underbrace{\Lambda(p_1^* - c_1)(s_1^* - \tilde{s}_1)}_{\text{Profit Cannibalization}}.$$

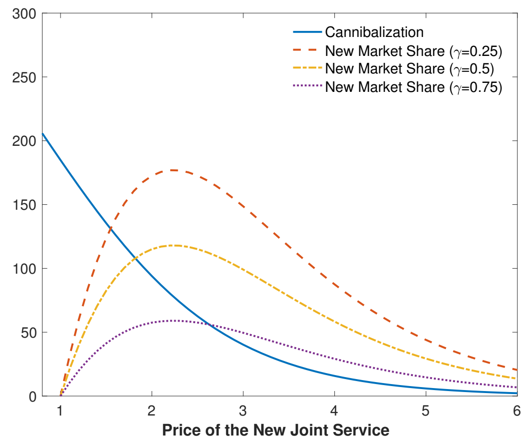
The first term corresponds to the profit increase of P_1 from the additional market share captured through the new joint service. The second term is negative, and represents the profit cannibalization effect from the introduction of the new joint service. Indeed, the market share of P_1 strictly decreases (i.e., $\tilde{s}_1 < s_1^*$), and hence the profit is also reduced. In this section, we have shown that under a well-designed profit sharing contract, the first term typically dominates the second. As a result, the overall profit earned by P_1 increases. Similarly, the profit difference of P_2 can be written as:

$$\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) - \pi_2(p_1^*, p_2^*) = \underbrace{(1-\gamma)\Lambda(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n}_{\text{New Market Share}} - \underbrace{\left[\{\Lambda p_2^* s_2^* - C_2(\Lambda s_2^*)\} - \{\Lambda p_2^* \tilde{s}_2 - C_2(\Lambda \tilde{s}_2)\}\right]}_{\text{Profit Cannibalization}}.$$

Note that the coopetition induces the same two conflicting effects for P_2. As before, the positive benefit of the new market share can overcome the adverse cannibalization effect, when using a properly designed profit sharing contract.



(a) Platform 1



(b) Platform 2

Figure 6 Magnitudes of the new market share and profit cannibalization effects (Parameters: $q_1 = 2$, $q_2 = 1$, $c_1 = 1.5$, $t_2 = 1$, $n_2 = 3$, $k_2 = 500$, $\Lambda = 1,000$, $q_n = 1.5$, $\tilde{n} = 1.5$, $\tilde{c}_n = 1$, $r_2 \sim U[0,1]$**)**

Figure 6 illustrates the magnitudes of the new market share and profit cannibalization effects for both platforms under different values of γ and $\tilde{p}_n$. We make the following observations. First, one can see that depending on the price of the new joint service, either of the two effects can dominate. This conveys the importance of carefully designing the profit sharing contract and the associated price of the new service, as otherwise it may lead to an undesirable lose-lose outcome. Second, as long as the price of the new joint service is large enough, the new market share effect will dominate the profit cannibalization effect for both platforms. Finally, the cannibalization effect does not depend on how the profit from the new service is split, whereas the new market share effect gets strengthened for P_1 (resp. P_2) if γ increases (resp. decreases).

An additional perspective on the benefit of the coopetition partnership is related to the following trade-off. On the one hand, P_1 has access to unlimited supply, but may not have a large enough market share. On the other hand, P_2 typically has access to a large potential market, but often has a more limited number of drivers. By introducing the new joint service, it allows both platforms to take advantage of these asymmetries in supply and market share. In other words, P_1 can use its supply more efficiently by capturing a new market share. At the same time, P_2 can outsource some of its demand to the unlimited supply of drivers brought by P_1, and hence decrease the wage paid to its drivers. More precisely, this cost savings amount to: $C_2(\Lambda s_2^*) - C_2(\Lambda \tilde{s}_2) > 0$. Consequently, the introduction of the new joint service may be profitable to both platforms as it improves the matching of supply with demand.

4.2. Surpluses of Riders and Drivers

So far, we have focused on the profits earned by the two platforms. In this section, we turn our attention to the implications of the new joint service on the expected surpluses of riders and drivers.

It is worth noting that the expected surpluses of riders and drivers are not (explicitly) dependent on the profit sharing parameter γ. We denote by $RS(p_1, p_2)$ the total rider expected surplus of the benchmark setting before the coopetition partnership, when P_1 sets p_1 and P_2 sets p_2. We have:

$$RS(p_1, p_2) = \Lambda \mathbb{E}\Big[\max\{q_1 - p_1 + \xi_1, q_2 - p_2 + \xi_2, \xi_0\}\Big].$$

Let $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2)$ denote the total expected rider surplus after introducing the new joint service:

$$\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \Lambda \mathbb{E}\Big[\max\{q_1 - \tilde{p}_1 + \xi_1, q_2 - \tilde{p}_2 + \xi_2, q_n - \tilde{p}_n + \xi_n, \xi_0\}\Big].$$

We remark that the rider surpluses $RS(\cdot,\cdot)$ and $\tilde{RS}(\cdot,\cdot)$ are unique up to an additive constant. Indeed, for any rider, if the random utility terms $(\xi_1, \xi_2, \xi_n, \xi_0)$ are shifted to $(\xi_1 + c, \xi_2 + c, \xi_n + c, \xi_0 + c)$ for any constant c, then the probabilities that this rider will choose any of the four options (P_1, P_2, the new service, and the outside option) remain the same. Nevertheless, the change in the expected rider surplus generated by introducing the new joint service, $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) - RS(p_1, p_2)$, is independent of the constant c. More specifically, one can derive the following expressions: $RS(p_1, p_2) = \log[1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)] + c$, and $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \log[1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)] + c$. For more details on the consumer surplus under the MNL model, and on the derivation of the above expressions, we refer the reader to the literature on discrete choice models (see, e.g., Chapter 3.5 of Train 2009).

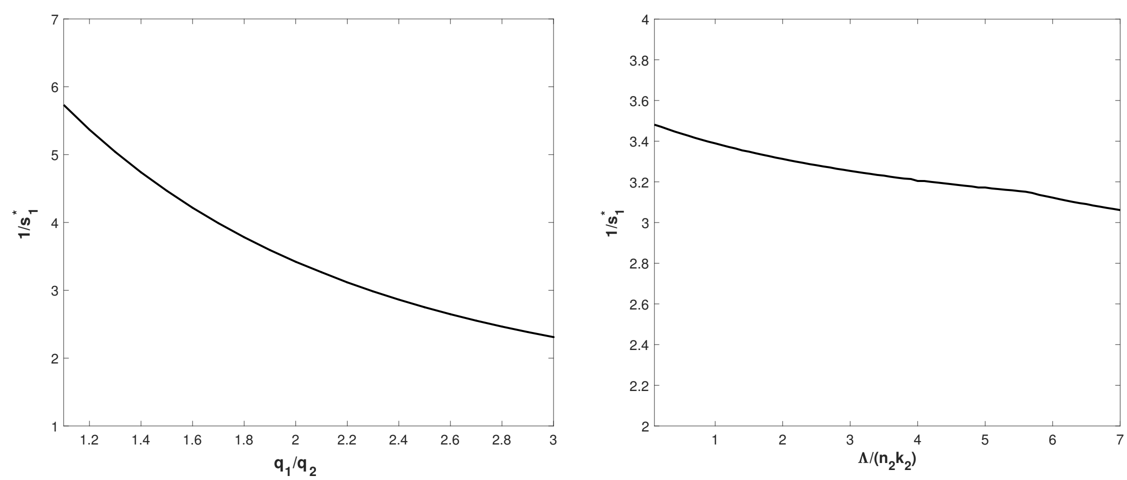
PROPOSITION 6. *For any price $\tilde{p}_n$, we have: $\tilde{RS}(\tilde{p}_n, p_1^*, p_2^*) > RS(p_1^*, p_2^*)$.*

The result in Proposition 6 shows that introducing the new joint service will improve the expected rider surplus, regardless of the specifics of the profit sharing contract and the price of the new service. This result is expected as riders can now enjoy an additional alternative for service.

The impact of the coopetition partnership on the drivers appears to be more subtle. We start by considering the surplus of P_1's drivers. Since the drivers partnering with P_1 are abundant on the market, we measure the expected driver surplus by the total wages paid to P_1's drivers per unit time. We denote by $DS_1(p_1, p_2)$ the total expected surplus of P_1's drivers before the coopetition partnership, given the prices (p_1, p_2). We have: $DS_1(p_1, p_2) = \Lambda c_1 s_1$. Analogously, the total expected surplus of P_1's drivers after introducing the new joint service, with prices $(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2)$ is given by: $\tilde{DS}_1(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \Lambda(c_1 \tilde{s}_1 + \tilde{c}_n \tilde{s}_n)$.

PROPOSITION 7. *For any price $\tilde{p}_n$, $\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) > DS_1(p_1^*, p_2^*)$ if and only if $\tilde{n} < \frac{1}{s_1^*}$. Moreover, s_1^* is increasing in q_1 and Λ, and decreasing in q_2, n_2, and k_2.*

In Proposition 7, we show that if the original market share of P_1 (i.e., s_1^*) is low or if the average number of riders per trip in the new joint service (i.e., $\tilde{n}$) is small, then introducing the new joint service will benefit P_1's drivers. An important implication of Proposition 7 is that P_1's drivers can benefit from the coopetition when a small number of riders share a ride in the new service ($\tilde{n} < 1/s_1^*$). Interestingly, P_1's drivers also face the trade-off between the new market share effect (some riders are switching from the outside option to the new service), and the cannibalization effect (some riders are switching from P_1's original service to the new service). If $\tilde{n}$ is small ($\tilde{n} < 1/s_1^*$), then the per-rider payment to P_1's drivers is not too low. In this case, the new market share effect will dominate the cannibalization effect, and hence P_1's drivers will enjoy a higher total expected surplus. On the other hand, if $\tilde{n}$ is big ($\tilde{n} > 1/s_1^*$), the cannibalization effect will dominate the new market share effect, and as a result, P_1's drivers will be worse off.



(a) Varying the service quality ratio q_1/q_2 ($q_1 \in [1,3]$ and $\Lambda = 1{,}000$)

(b) Varying the demand-supply ratio $\Lambda/(n_2k_2)$ ($q_1 = 2$ and $\Lambda \in [150, 10, 500]$)

Figure 7 Threshold values for $\tilde{n}$ (Parameters: $q_2 = 1$, $c_1 = 1$, $t_2 = 1$, $n_2 = 3$, $k_2 = 500$, $r_2 \sim U[0,1]$)

Proposition 7 also demonstrates how different market conditions affect P_1's market share, s_1^*, which in turn influences the threshold value of $\tilde{n}$ that determines whether the coopetition will benefit P_1's drivers. We illustrate this effect in Figure 7. Specifically, we examine the impact of the service differentiation (measured by the ratio q_1/q_2), and the demand-supply ratio of P_2, $\Lambda/(n_2k_2)$, on the threshold $1/s_1^*$. We draw three important conclusions from Proposition 7 and Figure 7. First, the value of $1/s_1^*$ seems to be larger than 2 in most of the cases we considered. Thus, as long as the average number of riders per trip for the new service is below 2, P_1's drivers will be better off under the coopetition. This condition is not very restrictive in some practical settings. For example, in the coopetition partnership between Curb and Via in NYC, there is a limit of two different parties at any time in a yellow cab (i.e., $\tilde{n} \leq 2$).[9] In addition, $\tilde{n}$ refers to the average number of riders who share the ride, and in many cases some (potentially large) portion of the trip is not shared. In Section 5, we present some extensive computational tests, which further suggest that the coopetition between the two platforms will benefit P_1's drivers in most cases. Second, as shown in Figure 7a, a higher service differentiation (captured by q_1/q_2) makes the original service offered by P_1 more attractive, and hence P_1's drivers are less likely to benefit from the introduction of the new service. Third, if the demand-supply ratio increases, P_1's drivers will be less likely to have a higher total surplus. When demand is too high relative to P_2's drivers capacity, the riders will be more likely to choose P_1 who has abundant drivers. Consequently, P_1's drivers will be less likely to benefit from introducing the new joint service.

Finally, we characterize the impact of the coopetition partnership on the total expected surplus of P_2's drivers. Let $DS_2(p_1, p_2)$ denote the total expected surplus of P_2's drivers before the coopetition partnership, given (p_1, p_2). Recall that P_2's drivers have a reservation wage r_2 distributed with $G_2(\cdot)$, and will decide to work for P_2 if and only if the offered wage w_2 exceeds r_2. As a result, we have: $DS_2(p_1, p_2) = k_2\mathbb{E}\big[\max\{r_2, w_2\}\big] = k_2\mathbb{E}\Big[\max\Big\{r_2, G_2^{-1}\Big(\frac{\Lambda s_2 t_2}{n_2 k_2}\Big)\Big\}\Big]$. Analogously, the total expected surplus of P_2's drivers after introducing the new joint service is given by: $\tilde{DS}_2(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = k_2\mathbb{E}\big[\max\{r_2, \tilde{w}_2\}\big] = k_2\mathbb{E}\Big[\max\Big\{r_2, G_2^{-1}\Big(\frac{\Lambda \tilde{s}_2 t_2}{n_2 k_2}\Big)\Big\}\Big]$.

PROPOSITION 8. *For any $\tilde{p}_n$, we have: $\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*) < DS_2(p_1^*, p_2^*)$.*

Proposition 8 shows that introducing the new joint service will always decrease the expected surplus of P_2's drivers, regardless of the price of the new service $\tilde{p}_n$. This follows from the fact that P_2's drivers are directly affected by the reduction in market share ($\tilde{s}_2 < s_2^*$) induced by the cannibalization effect. The coopetition will decrease the number of riders who ride with P_2, and consequently, P_2 can reduce the wage paid to its drivers.

In summary, we have shown that by properly designing the profit sharing contract, both platforms will increase their profits, riders will earn a higher surplus, and that P_1's drivers may also benefit. However, P_2's drivers will always be worse off. We next propose a simple and realistic way to address this issue: Platform 2 can decide to reallocate some of its profit gain to its drivers through promotions/bonuses or other monetary compensations. More precisely, instead of maximizing its own profit, P_2 can consider maximizing its total surplus (i.e., the sum of its own profit *and* its drivers' surplus). We next show that when P_2 modifies its objective function to account for its drivers' surplus, one can find a profit sharing contract that will guarantee a surplus gain for P_2 and for its drivers.

[9] https://www.nytimes.com/2017/06/06/nyregion/new-york-yellow-taxis-ride-sharing.html

Similarly as in Section 4.1, we consider three different scenarios depending on which platform sets the price of the new service. In each case, P_1 optimizes its own profit, whereas P_2 optimizes its total surplus. First, we assume that both platforms jointly decide the price of the new service, which is denoted by $\hat{p}_n^* \in \arg\max_{\tilde{p}_n}\{\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) + \tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)\}$. Second, P_1 maximizes its own profit for a given value of γ, which results in the pricing decision $\tilde{p}_n^1(\gamma)$ (see Proposition 4). Third, P_2 maximizes its total surplus (i.e., the sum of its profit and its drivers' surplus) for a given value of γ. The result of this optimization problem is denoted by $\hat{p}_n^2(\gamma) \in \arg\max_{\tilde{p}_n}\{\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_n^*|\gamma) + \tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)\}$.

PROPOSITION 9. *The following statements hold:*

- ***Joint decision:*** *There exists a unique* $\hat{p}_n^*$ *that maximizes* $\tilde{\pi}(\cdot, p_1^*, p_2^*) + \tilde{DS}_2(\cdot, p_1^*, p_2^*)$. *Furthermore, there exists an interval* $(\underline{\gamma}_d, \bar{\gamma}_d)$ *such that* $\tilde{\pi}_2(\hat{p}_n^*, p_1^*, p_2^*|\gamma) + \tilde{DS}_2(\hat{p}_n^*, p_1^*, p_2^*) > \pi_2(p_1^*, p_2^*) + DS_2(p_1^*, p_2^*)$ *and* $\tilde{\pi}_1(\hat{p}_n^*, p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ *for all* $\gamma \in (\underline{\gamma}_d, \bar{\gamma}_d)$.
- **$\mathbf{P_1}$*'s decision:*** *Assume that* P_1 *sets the price of the new joint service* $\tilde{p}_n^1(\gamma)$ *to maximize its own profit. Then, there exists a threshold* $\bar{\gamma}_d'$, *such that* $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) + \tilde{DS}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*) > \pi_2(p_1^*, p_2^*) + DS_2(p_1^*, p_2^*)$ *and* $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ *for all* $\gamma < \bar{\gamma}_d'$. *Moreover,* $\bar{\gamma}_d' < \bar{\gamma}_d$.
- **$\mathbf{P_2}$*'s decision:*** *Assume that* P_2 *sets the price of the new joint service* $\hat{p}_n^2(\gamma)$ *to maximize its total surplus. Then, there exists a threshold* $\underline{\gamma}_d'$, *such that* $\tilde{\pi}_2(\hat{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) + \tilde{DS}_2(\hat{p}_n^2(\gamma), p_1^*, p_2^*) > \pi_2(p_1^*, p_2^*) + DS_2(p_1^*, p_2^*)$ *and* $\tilde{\pi}_1(\hat{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ *for all* $\gamma > \underline{\gamma}_d'$. *Moreover,* $\underline{\gamma}_d < \underline{\gamma}_d' < \bar{\gamma}_d'$.
- ***Summary:*** *Regardless of which platform sets the price of the new joint service,* P_1 *will earn a higher profit and* P_2 *will increase its total surplus (the sum of its profit and its drivers' surplus) if and only if* $\gamma \in (\underline{\gamma}_d', \bar{\gamma}_d')$.

Proposition 9 demonstrates that when P_2 modifies its objective to account for its drivers' surplus, we retrieve our main insight: By carefully designing the terms of the coopetition partnership, all the parties (both platforms, riders and drivers) can be better off, as long as the expected number of riders per trip for the new service is not too large (i.e., $\tilde{n} < 1/s_1^*$). In Proposition 9, P_2 and the joint decision making explicitly take into account the surplus of P_2's drivers. Note that P_1 can similarly modify its objective to account for its drivers' surplus. In this case, we can show a similar result: There exists a profit sharing contract under which both platforms will earn higher total surpluses after introducing the new joint service. We next compare the market outcome in the case where P_2 adjusts its objective to include its drivers' surplus relative to the case where the objective is only composed of its own profit.

COROLLARY 1. *We have:*

- *The joint decision when* P_2 *takes into account its drivers' surplus yields a higher price relative to the case where the total profit is maximized, i.e.,* $\hat{p}_n^* > \tilde{p}_n^*$.

- *There exists a unique $\hat{\gamma}$, such that $\tilde{p}_n^1(\hat{\gamma}) = \hat{p}_n^2(\hat{\gamma}) = \hat{p}_n^*$. Moreover, $\tilde{\pi}_1(\hat{p}_n^*, p_1^*, p_2^*|\hat{\gamma}) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\hat{p}_n^*, p_1^*, p_2^*|\hat{\gamma}) + \tilde{DS}_2(\hat{p}_n^*, p_1^*, p_2^*) > \pi_2(p_1^*, p_2^*) + DS_2(p_1^*, p_2^*)$. Moreover, $\hat{\gamma} < \gamma^*$.*

The first result in Corollary 1 follows from the fact that P_2 seeks to protect its drivers from the cannibalization effect induced by the introduction of the new service. One way to mitigate the cannibalization effect is to increase the price of the new service in order to render it less attractive. The second result in Corollary 1 is analogous to Proposition 4, and establishes the existence of a unique profit sharing contract $\hat{\gamma}$ that coordinates the decision making process. In particular, under this specific contract, either platform has its incentive aligned with the joint system-wide objective and enjoys a higher surplus under coopetition. In addition, we will show computationally in Section 5 that, under the profit sharing contract γ^*, even if the profit-maximizing pricing decision $\tilde{p}_n^*$ is used, the sum of P_2's profit and its drivers' surplus will also typically increase after introducing the new joint service.

Finally, it is worth mentioning that a similar result as in Proposition 5 also holds when P_2 adjusts its objective to include its drivers' surplus. By considering the Stackelberg setting where one platforms decides γ and the other sets $\tilde{p}_n$ after observing the value of γ, we can show that regardless of which platform is the game leader, both platforms will earn a higher surplus after introducing the new joint service (the proof of this result is very similar to the proof of Proposition 5, and thus omitted for conciseness).

As mentioned at the end of Section 3, introducing the new joint service can decrease the profits earned by both platforms, if the profit sharing contract is poorly designed. However, as we have shown in this section, by carefully designing the terms of the profit sharing contract, both platforms will earn higher profits. In addition, the riders will always derive a higher expected surplus. The impact on the drivers appear to be a bit more subtle. P_1's drivers will usually benefit if the original market share of P_1 is low or if the average number of riders per trip in the new joint service is small. However, we show that P_2's drivers will be worse off in the presence of the new service, unless the platform deliberately allocates some of its profit gain to its drivers. In particular, by modifying P_2's objective function to be the total surplus (instead of the profit), we can find a profit sharing contract that benefits everyone (riders, drivers and both platforms).

5. Computational Experiments

In this section, we investigate computationally how three market features affect the impact of the coopetition: (a) Product differentiation, measured by q_1/q_2, (b) Demand-supply ratio of P_2, measured by $\Lambda/(n_2 k_2)$, and (c) The expected number of riders per trip in the new joint service, $\tilde{n}$. To this end, we set $q_2 = 1$, and vary q_1 so that $q_1/q_2 \in \{1.1, 1.4, 1.8, 2.2, 2.6, 3\}$. The wage paid by P_1 to its drivers (meter price) is set to $c_1 = 1$. We fix $n_2 = 3$, $k_2 = 500$, $t_2 = 1$, assume that r_2

is uniformly distributed on $[0,1]$, and vary Λ so that the demand-supply ratio of P_2 $\Lambda/(n_2k_2) \in \{0.5,1,1.5,2,5,7\}$. Finally, we consider several values of $\tilde{n} \in \{1,1.3,1.7,2,2.5,3\}$. Note that $\tilde{n}=1$ is the extreme case in which there is no carpooling in the new joint service. Recall that the partnership between Curb and Via in NYC is such that $\tilde{n} \leq 2$. However, we still consider the case where $\tilde{n}$ can be larger than 2 in order to test the robustness of our results. Note that the set of parameters we are using in this section encompasses a wide range of realistic instances and hence, this allows us to quantify the practical impact of the coopetition partnership.

It is natural to assume that the quality of the new joint service q_n increases with q_1 and decreases with $\tilde{n}$. To capture this behavior, we use $q_n = q_2 + (q_1 - q_2)(n_2 + 1 - \tilde{n})/n_2$. Note that $q_n = q_1$ when $\tilde{n}=1$ (in this case, the new service is equivalent to P_1's original service), and q_n is slightly larger than q_2 when $\tilde{n}=n_2$ (in this case, the new service is slightly better than P_2's original service). The average wage per trip per rider of the new joint service is equal to $\tilde{c}_n = c_1/\tilde{n} = 1/\tilde{n}$, which is decreasing in $\tilde{n}$, as expected. For all problem instances, we decided to use a profit sharing contract with the profit split parameter γ^*. This seems to be a desirable contract in practice, as it properly balances the incentives of both platforms (see the discussion after Proposition 4). Under the contract γ^*, the price of the new service (regardless of which platform sets it) is equal to $\tilde{p}_n^*$. In addition, we note that all the computational results presented in this section still qualitatively hold when using a different profit sharing contract (e.g., the Stackelberg setting studied in Section 4.1.3).

Table 1 summarizes the impact of the coopetition partnership on P_1, P_2, the drivers, and the riders for the problem instances discussed above. We compute the relative impact of introducing the new joint service for each party. For example, the relative profit difference of P_i $(i=1,2)$ is given by: $\Delta\pi_i/\pi_i = [\tilde{\pi}_i(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) - \pi_i(p_1^*, p_2^*)]/\pi_i(p_1^*, p_2^*)$.[10] Our computational tests convey that for the parameter values we consider, introducing the new joint service will in general substantially benefit all the stakeholders in the market. In particular, we can see from Table 1 that the average relative profit improvements for P_1 and P_2 are 25.38% and 23.45% respectively (and even in the worst case instances under consideration, the relative improvements amount to 13.37% and 13.13%). In addition, the average benefits of the drivers and the riders seem to be significant too. The only exception is a slight decrease in the expected surplus of P_1's drivers when $\tilde{n} > 2$ (i.e., every trip is shared by more than 2 riders on average), and q_1/q_2 is large. In this case, one can see from Table 1 that the surplus of P_1's drivers can be reduced by 3.78% in the worst case (this occurs for the instance with $q_1/q_2 = 3$ and $\tilde{n}=3$). This is consistent with Proposition 7, which shows that if $\tilde{n}$ is large, P_1's drivers will not necessarily benefit from the introduction of the new joint service.

[10] Since the rider surplus is unique up to an additive constant (see Chapter 3.5 of Train 2009), we report here the absolute (instead of the relative) differences in the expected rider surplus. The same comment applies to Tables 2-4.

However, in such cases, P_1 can still redistribute its profit gain to its drivers so that the coopetition will benefit the platform and its drivers together (see also the discussions after Proposition 9). We also note that even though we use a profit sharing contract with γ^* and $\tilde{p}_n^*$ (designed to maximize the platforms' profits), the total surplus of P_2 and its drivers still increases. Consequently, P_2 can also decide to redistribute some of its profit gain to its drivers.

	Average	Min	25th Percentile	Median	75th Percentile	Max
$\Delta\pi_1/\pi_1$	25.38	13.37	22.76	25.00	27.75	42.17
$\Delta\pi_2/\pi_2$	23.45	13.13	21.32	23.82	25.45	35.99
$\Delta DS_1/DS_1$	17.22	-3.78	5.94	14.85	27.36	48.02
$\Delta(\pi_2+DS_2)/(\pi_2+DS_2)$	20.30	9.38	18.37	20.96	22.69	27.93
ΔRS	1429.35	143.49	440.28	736.67	2627.44	4669.56

Table 1 Summary statistics of the impact of introducing the new joint service (%)

In Tables 2, 3 and 4, we report the average values of the relative impact when a single parameter is varied and the other two are set to specific values. This allows us to isolate the impact of a single market feature. One can see that in all cases, all the surpluses are increasing, suggesting that everyone benefits from the introduction of the new joint service. In Table 2, we study the effect of quality differentiation. We observe that as q_1/q_2 increases, the impact on the profits earned by both platforms is quite stable (the relative improvement remains around 20-30%). On the other hand, increasing the quality ratio will hurt P_1's drivers which will benefit less from the coopetition. In Table 3, we study the effect of the demand-supply ratio. When increasing $\Lambda/(n_2k_2)$, we can see that the impact on the profits of both platforms and on the drivers are quite stable, whereas the riders will benefit more from the coopetition. This follows from the fact that when the demand is high, introducing a new alternative will yield a larger benefit to the riders, as expected. In Table 4, we examine the effect of the expected number of riders per trip in the new joint service. In this case, increasing $\tilde{n}$ does not have a significant impact on the profits, on P_2's drivers and on the riders. However, it has a strong effect on P_1's drivers, as we have shown in Proposition 7. In summary, even though the impact of the coopetition partnership may be sensitive with respect to the different market conditions, it seems to be beneficial for all parties (both platforms, riders and drivers) in the vast majority of the instances we considered.

We observe in Tables 2-4 that there is not a clear monotonicity pattern, as when we vary a single parameter, the profit sharing parameter γ^* changes as well (since it is endogenously decided).

6. Conclusions

In many cities around the world, several online platforms offer ride-hailing and ride-sharing services. Even though the number of such platforms has increased significantly over the past few years,

q_1/q_2	1.1	1.4	1.8	2.2	2.6	3
$\Delta\pi_1/\pi_1$	34.67	34.07	32.33	29.52	26.27	22.78
$\Delta\pi_2/\pi_2$	29.92	29.82	28.56	26.45	23.77	21.02
$\Delta DS_1/DS_1$	25.55	19.87	13.68	8.77	5.00	2.10
$\Delta(\pi_2+DS_2)/(\pi_2+DS_2)$	24.04	23.92	22.68	20.83	18.44	16.13
ΔRS	2817.74	2826.93	2774.94	2688.51	2569.04	2438.02

Table 2 **Impact of the service quality ratio q_1/q_2 when $\Lambda/(n_2k_2)=5$ and $\tilde{n}=2$ (%)**

$\Lambda/(n_2k_2)$	0.5	1	1.5	2	5	7
$\Delta\pi_1/\pi_1$	24.69	25.80	26.85	27.81	32.33	34.52
$\Delta\pi_2/\pi_2$	24.20	24.89	25.49	26.05	28.56	29.73
$\Delta DS_1/DS_1$	14.17	14.14	14.05	14.01	13.68	13.51
$\Delta(\pi_2+DS_2)/(\pi_2+DS_2)$	23.35	23.26	23.18	23.08	22.68	22.49
ΔRS	220.21	456.14	708.88	972.93	2774.94	4123.17

Table 3 **Impact of the demand-supply ratio $\Lambda/(n_2k_2)$ when $q_1/q_2=2$ and $\tilde{n}=2$ (%)**

$\tilde{n}$	1	1.3	1.7	2	2.5	3
$\Delta\pi_1/\pi_1$	26.82	30.28	32.10	32.33	31.51	29.94
$\Delta\pi_2/\pi_2$	23.34	26.61	28.38	28.56	27.73	26.13
$\Delta DS_1/DS_1$	38.56	29.03	19.15	13.68	7.21	2.96
$\Delta(\pi_2+DS_2)/(\pi_2+DS_2)$	18.39	21.07	22.53	22.68	21.99	20.67
ΔRS	2358.98	2622.90	2759.60	2774.94	2712.94	2591.45

Table 4 **Impact of $\tilde{n}$ when $q_1/q_2=2$ and $\Lambda/(n_2k_2)=5$ (%)**

traditional taxis still often remain one of the main modes of transportation. Recently, several interesting partnerships emerged between ride-sharing and taxi platforms. Such examples includes Curb and Via in NYC and Uber and PT Express in Indonesia. This paper is motivated by such coopetition partnerships that can be implemented through a profit sharing contract.

It is not clear a-priori whether the competing platforms will benefit from such a coopetition partnership. In particular, by introducing the new joint service, two main conflicting factors arise: the new market share effect and the cannibalization effect. One of the key insights presented in this paper lies in understanding the interplay between these two effects.

We use the MNL choice model to capture the fact that potential riders face several alternatives, and we characterize the equilibrium market outcome. Our analysis extends previous approaches in the literature by accounting the fact that drivers are self-scheduled. We find that there always exists a profit sharing contract that increases the profits of both platforms, regardless of which platform sets the price of the new service. Consequently, under a well-designed profit sharing contract, the positive new market share effect dominates the adverse cannibalization effect, allowing both platforms to earn higher profits. Next, we study the impact of the coopetition on riders and on drivers. While it is straightforward to show that riders will be better off, the impact on drivers is more subtle. As expected, the surplus earned by the ride-sharing platform's drivers will decrease under the coopetition. However, we show analytically that under a well-designed profit sharing contract, the total surplus of the ride-sharing platform and its drivers will increase. In practice, the platform can readily allocate some of the additional profit to its drivers through promotions and/or other monetary compensations. In summary, the analysis and results in this paper suggest that when the coopetition terms are carefully designed, every single party will benefit (riders, drivers and both platforms). In addition, one can naturally expect additional societal and environmental benefits, as encouraging ride-sharing will potentially reduce congestion and pollution.

While we impose some assumptions to simplify the analysis, our main insights continue to hold when some of these assumptions are relaxed. For example, it is worth mentioning that the results and insights presented in this paper qualitatively extend to other choice models such as the mixed MNL. The mixed MNL choice model (see, for example, Hensher and Greene 2003) can be used to explicitly capture riders' heterogeneity in quality perception. In addition, one can relax the assumption that P_1's drivers are abundant, whereas P_2's drivers are limited and self-scheduled. Consequently, the main insights and results presented in this paper are robust to some of our modeling assumptions.

This paper is among the first to propose a tractable model to study competition and partnerships in the ride-sharing industry. It allows us to draw practical insights and to shed light on the impact of some recent business partnerships observed in practice. Several interesting extensions are left as future research. For example, what is the long-term impact of such partnerships? Shall the platforms consider more complicated contracts such as a two-part piecewise linear agreement (i.e., allowing two different profit portions depending on the scale of the joint service)? In order to study this problem, one needs to develop a dynamic model. A second potential direction for future research is to study an alternative form of coopetition, known as joint ownership of a subsidiary. For example, Uber and a local Russian taxi-hailing platform Yandex.Taxi recently merged their ride-hailing businesses in Russia (as well as several other Eastern European countries) under a new company.[11] It could be interesting to compare the two different forms of coopetition. A third interesting avenue for future research would be to empirically validate the use of a choice model for the ride-sharing market, and to estimate the impact of such partnerships using real data.

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[11] https://www.nytimes.com/2017/07/13/technology/uber-russia-yandex.html

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Appendix A: Summary of Notation

Table 5 Summary of Notation

P_1 :	Platform 1 (in our application, the taxi-hailing online platform)
P_2 :	Platform 2 (in our application, the ride-sharing online platform)
q_i :	Perceived quality of Platform i ($i=1,2$)
q_n :	Perceived quality of the new joint service
p_i :	Price per trip for Platform i without the new joint service
$\tilde{p}_i$:	Price per trip for Platform i with the new joint service
$\tilde{p}_n$:	Price per trip for the new joint service
s_i :	Probability that a rider selects Platform i ($i=1,2$) without the new joint service
s_0 :	Probability that a rider selects the outside option without the new joint service
$\tilde{s}_i$:	Probability that a rider selects Platform i ($i=1,2$) with the new joint service
$\tilde{s}_n$:	Probability that a rider selects the new joint service
$\tilde{s}_0$:	Probability that a rider selects the outside option with the new joint service
Λ :	Total rider arrival rate in the market
c_1 :	Average meter price per trip for P_1
k_2 :	Total number of drivers partnering with P_2
r_2 :	Per unit time reservation wage of P_2's drivers
$G_2(\cdot)$:	CDF of r_2, which is assumed to satisfy the log-concave condition
w_2 :	Per unit time wage paid by P_2 to a driver
t_2 :	Average trip duration of a rider (including the pick-up time)
n_2 :	Average number of riders per trip using P_2's service
$\tilde{n}$:	Average number of riders per trip using the new joint service
$C_i(\lambda_i)$:	Total wage per unit time that P_i pays its drivers to serve the stream of riders with arrival rate λ_i
γ :	Fraction of profit generated by the new joint service which is allocated to P_1

Appendix B: Proof of Statements

Auxiliary Lemma

Before presenting the proofs of our results, we state and prove an auxiliary lemma that is extensively used throughout this Appendix.

LEMMA 2. *For the model without coopetition, we have: $\partial_{p_i} s_i = -(1-s_i)s_i$ and $\partial_{p_j} s_i = s_i s_j$ ($i=1,2$ and $j \neq i$). For the model with coopetition, we have $\partial_{\tilde{p}_i} \tilde{s}_i = -(1-\tilde{s}_i)\tilde{s}_i$ and $\partial_{\tilde{p}_j} \tilde{s}_i = \tilde{s}_i \tilde{s}_j$.*

Proof. We only show the case without coopetition, as the case with coopetition follows the exact same argument. Since $s_i = \frac{\exp(q_i - p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)}$,

$$\begin{aligned}
\partial_{p_i} s_i =& \frac{-\exp(q_i-p_i)[1+\exp(q_1-p_1)+\exp(q_2-p_2)]+[\exp(q_i-p_i)]^2}{[1+\exp(q_1-p_1)+\exp(q_2-p_2)]^2} \\
=& -\frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} + \left(\frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)}\right)^2 \\
=& -s_i + s_i^2 = -(1-s_i)s_i,
\end{aligned}$$

and

$$\begin{aligned}
\partial_{p_j} s_i =& \frac{\exp(q_i-p_i)\exp(q_j-p_j)}{[1+\exp(q_1-p_1)+\exp(q_2-p_2)]^2} \\
=& \frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} \times \frac{\exp(q_j-p_j)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} = s_i s_j. \quad \square
\end{aligned}$$

Proof of Lemma 1

It suffices to show that $C_2'(\cdot)>0$, and is increasing on the support of $G_2(\cdot)$. Let $g_2(\cdot)=G_2'(\cdot)$ be the PDF of r_2. Since $G_2(\cdot)$ satisfies the log-concave condition, $[\log(G_2(\cdot))]'=g_2(\cdot)/G_2(\cdot)$ is decreasing on the support of r_2. Moreover, for any $x\in[0,1]$, $(G_2^{-1}(x))'=\frac{1}{g_2(G_2^{-1}(x))}$. Therefore,

$$C_2'(\lambda_2)=\frac{t_2}{n_2}G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)+\frac{(t_2)^2\lambda_2}{(n_2)^2 k_2 g_2\left[G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)\right]}>0.$$

Recall that we have $\lambda_2=\frac{n_2 k_2 G_2(w_2)}{t_2}$ and $w_2=G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)$. Therefore, we obtain:

$$\frac{t_2}{n_2}G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)+\frac{(t_2)^2\lambda_2}{(n_2)^2 k_2 g_2\left[G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)\right]}=\frac{t_2 w_2}{n_2}+\frac{t_2 G_2(w_2)}{n_2 g_2(w_2)}.$$

Note that w_2 is strictly increasing in λ_2. Since $g_2(\cdot)/G_2(\cdot)$ is decreasing, $C_2'(\lambda_2)$ is increasing in w_2, and hence in λ_2. $\square$

Proof of Proposition 1

Since the problem faced by P_1 has no constraint, the first order condition (FOC) $\partial_{p_1}\pi_1(p_1^*,p_2^*)=0$ applies. For P_2, if $\Lambda s_2^* t_2<n_2 k_2$, the driver capacity constraint is not binding, and the first order condition $\partial_{p_2}\pi_2(p_1^*,p_2^*)=0$ also applies. Otherwise, we have $\Lambda s_2^* t_2=n_2 k_2$, and the first order condition becomes $\partial_{p_2}\pi_2(p_1^*,p_2^*)-\mu^*\Lambda\partial_{p_2}s_2^*=0$, where $\mu^*\geq 0$ is the Lagrangian multiplier with respect to the constraint $\Lambda s_2^* t_2\leq n_2 k_2$. Since $\mu^*\geq 0$ and $\partial_{p_2}s_2^*=-s_2^*(1-s_2^*)<0$ (from Lemma 2), $\partial_{p_2}\pi_2(p_1^*,p_2^*)\leq 0$ if $\Lambda s_2^* t_2=n_2 k_2$. Therefore, the first order optimality conditions of the equilibrium can be summarized as:

$$\partial_{p_1}\pi_1(p_1^*,p_2^*)=0 \text{ and } \partial_{p_2}\pi_2(p_1^*,p_2^*)\begin{cases}=0, & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \leq 0, & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2}.\end{cases}$$

$\partial_{p_1}\pi_1(p_1^*,p_2^*)=0$ implies that $\Lambda[s_1^*+(p_1^*-c_1)\partial_{p_1}s_1^*]=0$. By using Lemma 2, $\partial_{p_1}s_1=-s_1+s_1^2$, we have:

$$\Lambda[s_1^*-(p_1^*-c_1)s_1^*(1-s_1^*)]=0,$$

which is equivalent to $1-(p_1^*-c_1)(1-s_1^*)=0$, i.e., $p_1^*=c_1+\frac{1}{1-s_1^*}$. On the other hand, by the definition of the MNL model, $\exp(q_1-p_1^*)=s_1^*/s_0^*$, i.e., $p_1^*=q_1+\log(s_0^*/s_1^*)$. Therefore,

$$c_1+\frac{1}{1-s_1^*}=q_1+\log(s_0^*/s_1^*),$$

i.e., $s_1^*\exp\left(\frac{s_1^*}{1-s_1^*}\right)=s_0^*\exp(q_1-c_1-1)$. We define $U_1(z_1):=z_1\exp\left(\frac{z_1}{1-z_1}\right)$. It is clear that $U_1(\cdot)$ is strictly increasing on $(0,1)$ with $U_1(0+)=0$ and $U_1(1-)=+\infty$. Note that $U_1^{-1}(\cdot)$ is strictly increasing with domain $(0,+\infty)$ and range $(0,1)$. We can write: $s_1^*=U_1^{-1}[s_0^*\exp(q_1-c_1-1)]$.

The first order condition for P_2 implies that:

$$\Lambda(s_2^*+p_2^*\partial_{p_2}s_2^*)-\Lambda C_2'(\Lambda s_2^*)\,\partial_{p_2}s_2^*\begin{cases}=0, & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \leq 0, & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2}.\end{cases}$$

By using Lemma 2, we have $\partial_{p_2}s_2=-s_2+s_2^2$. Hence, we obtain:

$$\Lambda[s_2^*-p_2^*s_2^*(1-s_2^*)]+C_2'(\Lambda s_2^*)\,s_2^*(1-s_2^*)\begin{cases}=0, & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \leq 0, & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2},\end{cases}$$

which is equivalent to

$$1-p_2^*(1-s_2^*)+C_2'\left(\Lambda s_2^*\right)(1-s_2^*)\begin{cases}=0, & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \leq 0, & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2},\end{cases}$$

i.e.,

$$p_2^*\begin{cases}=\frac{1}{1-s_2^*}+C_2'\left(\Lambda s_2^*\right), & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \geq \frac{1}{1-s_2^*}+C_2'\left(\Lambda s_2^*\right), & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2}.\end{cases}$$

As before, by the definition of the MNL model, we have $\exp(q_2-p_2^*)=s_2^*/s_0^*$, i.e., $p_2^*=q_2+\log(s_0^*/s_2^*)$. Therefore,

$$q_2+\log(s_0^*/s_2^*)\begin{cases}=\frac{1}{1-s_2^*}+C_2'\left(\Lambda s_2^*\right), & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \geq \frac{1}{1-s_2^*}+C_2'\left(\Lambda s_2^*\right), & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2},\end{cases}$$

which is equivalent to

$$s_2^*\exp\left(\frac{s_2^*}{1-s_2^*}\right)\exp\left[C_2'\left(\Lambda s_2^*\right)\right]\begin{cases}=s_0^*\exp(q_2-1), & \text{if } s_2^*<\frac{n_2k_2}{\Lambda t_2};\\ \leq s_0^*\exp(q_2-1), & \text{if } s_2^*=\frac{n_2k_2}{\Lambda t_2}.\end{cases} \tag{1}$$

We next show that for any given s_0^*, there exists a unique s_2^* that satisfies equation (1). We define $U_2(z_2):=z_2\exp\left(\frac{z_2}{1-z_2}\right)\exp\left[C_2'\left(\Lambda z_2\right)\right]$, where $z_2\in(0,1)$ and $z_2\leq\frac{n_2k_2}{\Lambda t_2}$. Since $C_2(\cdot)$ is increasing and convex in λ_2 (from Lemma 1), then $U_2(\cdot)$ is strictly increasing in z_2 on its domain. We define:

$$V_2(x):=\min\left\{U_2^{-1}(x),\frac{n_2k_2}{\Lambda t_2}\right\}.$$

We divide the analysis in two cases: (a) $U_2^{-1}[s_0^*\exp(q_2-1)]<\frac{n_2k_2}{\Lambda t_2}$ and (b) $U_2^{-1}[s_0^*\exp(q_2-1)]\geq\frac{n_2k_2}{\Lambda t_2}$.

(a) If $U_2^{-1}[s_0^*\exp(q_2-1)]<\frac{n_2k_2}{\Lambda t_2}$, then $V_2[s_0^*\exp(q_2-1)]=U_2^{-1}[s_0^*\exp(q_2-1)]$ and $U_2\left(\frac{n_2k_2}{\Lambda t_2}\right)>s_0^*\exp(q_2-1)$. Thus, the first order condition in (1) is not satisfied for $s_2^*=\frac{n_2k_2}{\Lambda t_2}$. Hence, $s_2^*<\frac{n_2k_2}{\Lambda t_2}$ and the FOC

$$s_2^*\exp\left(\frac{s_2^*}{1-s_2^*}\right)\exp\left[C_2'\left(\Lambda s_2^*\right)\right]=s_0^*\exp(q_2-1)$$

is satisfied if $U_2^{-1}[s_0^*\exp(q_2-1)]<\frac{n_2k_2}{\Lambda t_2}$. Equivalently, $s_2^*=U_2^{-1}[s_0^*\exp(q_2-1)]=V_2[s_0^*\exp(q_2-1)]$.

(b) If $U_2^{-1}[s_0^*\exp(q_2-1)]\geq\frac{n_2k_2}{\Lambda t_2}$, then $V_2[s_0^*\exp(q_2-1)]=\frac{n_2k_2}{\Lambda t_2}$ and $U_2\left(\frac{n_2k_2}{\Lambda t_2}\right)\leq s_0^*\exp(q_2-1)$, i.e., $s_2^*=\frac{n_2k_2}{\Lambda t_2}$ satisfies the following FOC:

$$s_2^*\exp\left(\frac{s_2^*}{1-s_2^*}\right)\exp\left[C_2'\left(\Lambda s_2^*\right)\right]\leq s_0^*\exp(q_2-1).$$

Therefore, by using equation (1) we obtain: $s_2^*=\frac{n_2k_2}{\Lambda t_2}=V_2[s_0^*\exp(q_2-1)]$.

Since $s_0^*+s_1^*+s_2^*=1$, $s_1^*=U_1^{-1}[s_0^*\exp(q_1-c_1-1)]$ and $s_2^*=V_2[s_0^*\exp(q_2-1)]$, s_0^* must satisfy $s_0^*+U_1^{-1}[s_0^*\exp(q_1-c_1-1)]+V_2[s_0^*\exp(q_2-1)]=1$. Since $U_1^{-1}(\cdot)$ is strictly increasing, $V_2(\cdot)$ is increasing, and $U_1^{-1}(0+)=V_2(0+)=0$, there exists a unique s_0^* that satisfies:

$$s_0^*+U_1^{-1}[s_0^*\exp(q_1-c_1-1)]+V_2[s_0^*\exp(q_2-1)]=1.$$

Note that one can solve for s_0^* efficiently (e.g., through binary search).

Since $p_1^*=q_1+\log(s_0^*/s_1^*)$ and $p_2^*=q_2+\log(s_0^*/s_2^*)$, we have: $p_1^*=q_1+\log(s_0^*/s_1^*)=q_1+\log(s_0^*/U_1^{-1}[s_0^*\exp(q_1-c_1-1)])$ and $p_2^*=q_2+\log(s_0^*/s_2^*)=q_2+\log(s_0^*/V_2[s_0^*\exp(q_2-1)])$. $\square$

Proof of Proposition 2

Existence of $\tilde{p}_n^*$

The optimization problem can be formulated as:

$$\tilde{p}_n^* = \arg\max_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^* | \gamma),$$
$$\text{s.t. } \tilde{s}_2 \leq \frac{n_2 k_2}{\Lambda t_2}.$$

First, we observe that since P_1 and P_2 do not change p_1^* and p_2^*, we have:

$$\tilde{s}_2 = \frac{\exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n)} < \frac{\exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = s_2^* \leq \frac{n_2 k_2}{\Lambda t_2} \text{ for any } \tilde{p}_n.$$

Hence, the constraint $\Lambda \tilde{s}_2 t_2 \leq n_2 k_2$ is not binding, and the first order condition with respect to $\tilde{p}_n$ is satisfied with equality, i.e., $\partial_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n^*, p_1^*, p_2^*) = 0$. Therefore,

$$\Lambda[(p_1^* - c_1)\partial_{\tilde{p}_n} \tilde{s}_1^* + p_2^* \partial_{\tilde{p}_n} \tilde{s}_2^* + \tilde{s}_n^* + (\tilde{p}_n^* - \tilde{c}_n)\partial_{\tilde{p}_n} \tilde{s}_n^*] - \Lambda C_2'(\Lambda \tilde{s}_2^*)\, \partial_{\tilde{p}_n} \tilde{s}_2^* = 0.$$

From Lemma 2, we have $\partial_{\tilde{p}_n} \tilde{s}_1 = \tilde{s}_1 \tilde{s}_n$, $\partial_{\tilde{p}_n} \tilde{s}_2 = \tilde{s}_2 \tilde{s}_n$ and $\partial_{\tilde{p}_n} \tilde{s}_n = -\tilde{s}_n(1 - \tilde{s}_n)$, and hence,

$$(p_1^* - c_1)\tilde{s}_n^* \tilde{s}_1^* + p_2^* \tilde{s}_n^* \tilde{s}_1^* + \tilde{s}_n^* - (\tilde{p}_n^* - \tilde{c}_n)(1 - \tilde{s}_n^*)\tilde{s}_n^* - C_2'(\Lambda \tilde{s}_2^*)\, \tilde{s}_n^* \tilde{s}_2^* = 0,$$

i.e.,

$$\tilde{p}_n^* = \tilde{c}_n + (p_1^* - c_1)\frac{\tilde{s}_1^*}{1 - \tilde{s}_n^*} + \frac{1}{1 - \tilde{s}_n^*} + p_2^* \frac{\tilde{s}_2^*}{1 - \tilde{s}_n^*} - C_2'(\Lambda \tilde{s}_2^*)\, \frac{\tilde{s}_2^*}{1 - \tilde{s}_n^*}.$$

Since the prices of the original services (p_1^*, p_2^*) remain the same, we have:

$$(1 - \tilde{s}_n^*)s_1^* = \frac{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n^*)} \times \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = \tilde{s}_1^*.$$

Thus, $s_1^* = \frac{\tilde{s}_1^*}{1 - \tilde{s}_n^*}$. Analogously, $\frac{\tilde{s}_2^*}{1 - \tilde{s}_n^*} = s_2^*$, so that we have:

$$\tilde{p}_n^* = \tilde{c}_n + (p_1^* - c_1)s_1^* + \frac{1}{1 - \tilde{s}_n^*} + p_2^* s_2^* - s_2^* C_2'(\Lambda \tilde{s}_2^*)\,. \tag{2}$$

We define $f_n(\tilde{p}_n) := \tilde{c}_n + (p_1^* - c_1)s_1^* + \frac{1}{1 - \tilde{s}_n} + p_2^* s_2^* - s_2^* C_2'(\Lambda \tilde{s}_2)$, where $\tilde{s}_n$ and $\tilde{s}_2$ depend on $\tilde{p}_n$. Then, $\tilde{p}_n^*$ is a positive fixed point of $f_n(\cdot)$. Since $\tilde{s}_2$ is strictly increasing in $\tilde{p}_n$, whereas $\tilde{s}_n$ is strictly decreasing in $\tilde{p}_n$, then $f_n(\cdot)$ is strictly decreasing in $\tilde{p}_n$. Thus, $f_n(\tilde{p}_n) - \tilde{p}_n$ is strictly decreasing in $\tilde{p}_n$ and $\lim_{\tilde{p}_n \to +\infty}(f_n(\tilde{p}_n) - \tilde{p}_n) = -\infty$. By the proof of Proposition 1, we have: $p_1^* = c_1 + \frac{1}{1 - s_1^*} > c_1$, and $p_2^* \geq \frac{1}{1 - s_2^*} + C_2'(\Lambda s_2^*) > C_2'(\Lambda s_2^*) > C_2'(\Lambda \tilde{s}_2^*)$, where the last inequality follows from $\tilde{s}_2^* = s_2^*(1 - \tilde{s}_n^*) < s_2^*$. Hence, $(p_1^* - c_1)s_1^* > 0$ and $p_2^* s_2^* - s_2^* C_2'(\Lambda \tilde{s}_2) > 0$. Thus, $f_n(0) > \tilde{c}_n + \frac{1}{1 - \tilde{s}_n} > 0$. Together with $\lim_{\tilde{p}_n \to +\infty}(f_n(\tilde{p}_n) - \tilde{p}_n) = -\infty$, we have that $f_n(\cdot)$ has a unique fixed point $\tilde{p}_n^* > 0$ such that equation (2) holds. Moreover, since $f_n(\tilde{p}_n) - \tilde{p}_n$ is strictly decreasing in $\tilde{p}_n$, if $\tilde{p}_n < \tilde{p}_n^*$ (resp. $\tilde{p}_n > \tilde{p}_n^*$), $f_n(\tilde{p}_n) > f_n(\tilde{p}_n^*) = \tilde{p}_n^* > \tilde{p}_n$ (resp. $f_n(\tilde{p}_n) < f_n(\tilde{p}_n^*) = \tilde{p}_n^* < \tilde{p}_n$), which implies that $\partial_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) > 0$ (resp. < 0) for $\tilde{p}_n < \tilde{p}_n^*$ (resp. $\tilde{p}_n > \tilde{p}_n^*$), i.e., $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is strictly increasing (resp. decreasing) in $\tilde{p}_n$. Therefore, $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is maximized at $\tilde{p}_n = \tilde{p}_n^*$.

Existence of the interval $(\underline{\gamma}, \bar{\gamma})$

Note that as $\tilde{p}_n \to +\infty$, we have $\tilde{s}_n \to 0$, $\tilde{s}_1 \to s_1^*$ and $\tilde{s}_2 \to s_2^*$. Therefore,

$$\lim_{\tilde{p}_n \to +\infty} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = \Lambda[(p_1^* - c_1)s_1^* + p_2^* s_2^*] - C_2(\Lambda s_2^*) = \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*).$$

Since $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is strictly decreasing in $\tilde{p}_n$ for $\tilde{p}_n > \tilde{p}_n^*$,

$$\tilde{\pi}(\tilde{p}_n^*, p_1^*, p_2^*) > \lim_{\tilde{p}_n \to +\infty} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*).$$

We denote $\tilde{\pi}_1^* := \Lambda(p_1^* - c_1)\tilde{s}_1^*$, $\tilde{\pi}_n^* := \Lambda(\tilde{p}_n^* - \tilde{c}_n)\tilde{s}_n^*$, and $\tilde{\pi}_2^* := \Lambda p_2^* \tilde{s}_2^* - C_2(\Lambda \tilde{s}_2^*)$. We have $\tilde{\pi}(\tilde{p}_n^*, p_1^*, p_2^*) = \tilde{\pi}_1^* + \tilde{\pi}_2^* + \tilde{\pi}_n^*$. We observe that:

$$\tilde{s}_1 = \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n)} < \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = s_1^* \text{ for any } \tilde{p}_n.$$

Since $p_1^* > c_1$ and $\tilde{s}_1^* < s_1^*$, then $\tilde{\pi}_1^* = \Lambda(p_1^* - c_1)\tilde{s}_1^* < \Lambda(p_1^* - c_1)s_1^* = \pi_1^*$. By using the proof of Proposition 1:

$$p_2^* \geq \frac{1}{1 - s_2^*} + C_2'(\Lambda s_2^*) > C_2'(\Lambda s_2^*) > C_2'(\Lambda \tilde{s}_2^*),$$

where the third inequality follows from the convexity of $C_2(\cdot)$ and the fact that $\tilde{s}_2^* < s_2^*$ for any $\tilde{p}_n$. We define $f(x) := \Lambda p_2^* x - C_2(\Lambda x)$, which is a concave function of x. For all $x \in [\tilde{s}_2^*, s_2^*]$, $f'(x) \geq f'(s_2^*) = \Lambda[p_2^* - C_2'(\Lambda s_2^*)] > 0$. Therefore,

$$\tilde{\pi}_2^* - \pi_2^* = -\int_{\tilde{s}_2^*}^{s_2^*} f'(x)\,\mathrm{d}x < 0, \text{ i.e., } \tilde{\pi}_2^* < \pi_2^*.$$

Since $\tilde{\pi}_1^* + \tilde{\pi}_2^* + \tilde{\pi}_n^* > \pi_1^* + \pi_2^*$, $\pi_1^* > \tilde{\pi}_1^*$ and $\pi_2^* > \tilde{\pi}_2^*$, then $\tilde{\pi}_n^* > \pi_1^* - \tilde{\pi}_1^* > 0$, $\tilde{\pi}_n^* > \pi_2^* - \tilde{\pi}_2^* > 0$ and $\tilde{\pi}_n^* > (\pi_1^* - \tilde{\pi}_1^*) + (\pi_2^* - \tilde{\pi}_2^*) > 0$. We define $\underline{\gamma} := \frac{\pi_1^* - \tilde{\pi}_1^*}{\tilde{\pi}_n^*}$ and $\bar{\gamma} := \frac{\tilde{\pi}_n^* - \pi_2^* + \tilde{\pi}_2^*}{\tilde{\pi}_n^*}$. One can easily check that $0 < \underline{\gamma} < \bar{\gamma} < 1$. Furthermore, if $\gamma \in (\underline{\gamma}, \bar{\gamma})$, then $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^* | \gamma) = \tilde{\pi}_1^* + \gamma \tilde{\pi}_n^* > \pi_1^*$ and $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^* | \gamma) = \tilde{\pi}_2^* + (1-\gamma)\tilde{\pi}_n^* > \pi_2^*$. If $\gamma < \underline{\gamma}$, $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^* | \gamma) < \pi_1^*$ and if $\gamma > \bar{\gamma}$, $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^* | \gamma) < \pi_2^*$. □

Proof of Proposition 3

We use the parameter γ to denote the dependence of the market outcome on γ, and the superscript ^{1} to specify that Platform 1 sets the price of the new joint service. As in Proposition 2, we have $\tilde{s}_j^i(\gamma) < s_j^*$ $(i, j = 1, 2)$ for any $\gamma > 0$ and any $\tilde{p}_n > 0$.

Existence of $\tilde{p}_n^1(\gamma)$

We first show the existence of $\tilde{p}_n^1(\gamma)$. For any $\gamma \in (0, 1)$, the first order condition of Platform 1 is $\partial_{\tilde{p}_n} \tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^* | \gamma) = 0$, i.e.,

$$\Lambda[(p_1^* - c_1)\partial_{\tilde{p}_n} \tilde{s}_1^1(\gamma) + \gamma\{\tilde{s}_n^1(\gamma) + (\tilde{p}_n^1(\gamma) - \tilde{c}_n)\partial_{\tilde{p}_n} \tilde{s}_n^1(\gamma)\}] = 0.$$

By using Lemma 2, $\partial_{\tilde{p}_n} \tilde{s}_1^1(\gamma) = \tilde{s}_1^1(\gamma)\tilde{s}_n^1(\gamma)$ and $\partial_{\tilde{p}_n} \tilde{s}_n^1(\gamma) = -\tilde{s}_n^1(\gamma)[1 - \tilde{s}_n^1(\gamma)]$, and therefore,

$$(p_1^* - c_1)\tilde{s}_1^1(\gamma)\tilde{s}_n^1(\gamma) - \gamma[1 - \tilde{s}_n^1(\gamma)]\tilde{s}_n^1(\gamma)[\tilde{p}_n^1(\gamma) - \tilde{c}_n] + \gamma \tilde{s}_n^1(\gamma) = 0.$$

We obtain:

$$\tilde{p}_n^1(\gamma) = \tilde{c}_n + \frac{\tilde{s}_1^1(\gamma)(p_1^* - c_1)}{\gamma[1 - \tilde{s}_n^1(\gamma)]} + \frac{1}{1 - \tilde{s}_n^1(\gamma)}.$$

Since we have:

$$[1 - \tilde{s}_n^1(\gamma)]s_1^* = \frac{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp[q_n - \tilde{p}_n^1(\gamma)]} \times \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = \tilde{s}_1^1(\gamma),$$

then we obtain: $\frac{\tilde{s}_1^1(\gamma)}{1 - \tilde{s}_n^1(\gamma)} = s_1^*$. Therefore,

$$\tilde{p}_n^1(\gamma) = \tilde{c}_n + \frac{\tilde{s}_1^1(\gamma)(p_1^* - c_1)}{\gamma[1 - \tilde{s}_n^1(\gamma)]} + \frac{1}{1 - \tilde{s}_n^1(\gamma)} = \tilde{c}_n + \frac{(p_1^* - c_1)s_1^*}{\gamma} + \frac{1}{1 - \tilde{s}_n^1(\gamma)}. \tag{3}$$

We define $f_1(\tilde{p}_n) := \tilde{c}_n + \frac{(p_1^* - c_1)s_1^*}{\gamma} + \frac{1}{1-\tilde{s}_n^1(\gamma)}$. Since $\tilde{s}_n^1$ is strictly decreasing in $\tilde{p}_n$, then $f_1(\cdot)$ is strictly decreasing in $\tilde{p}_n$, and so is $f_1(\tilde{p}_n) - \tilde{p}_n$. Since $p_1^* = \frac{1}{1-s_1^*} + c_1^* > c_1^*$ (see the proof of Proposition 1), we have $f_1(0) = \tilde{c}_n + \frac{(p_1^* - c_1)s_1^*}{\gamma} + \frac{1}{1-\tilde{s}_n^1(\gamma)} > \tilde{c}_n > 0$. Moreover, $\lim_{\tilde{p}_n \to +\infty} f_1(\tilde{p}_n) = -\infty$. Thus, for any given γ, $f_1(\tilde{p}_n) - \tilde{p}_n$ has a unique root $\tilde{p}_n^1(\gamma)$, i.e., equation (3) holds. Furthermore, if $\tilde{p}_n < \tilde{p}_n^1(\gamma)$ (resp. $\tilde{p}_n > \tilde{p}_n^1(\gamma)$), $f_1(\tilde{p}_n) > f_1(\tilde{p}_n^1(\gamma)) = \tilde{p}_n^1(\gamma) > \tilde{p}_n$ (resp. $f_1(\tilde{p}_n) < f_1(\tilde{p}_n^1(\gamma)) = \tilde{p}_n^1(\gamma) > \tilde{p}_n$), i.e., $\partial_{\tilde{p}_n}\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \Lambda[(p_1^* - c_1)\partial_{\tilde{p}_n}\tilde{s}_1^1(\gamma) + \gamma(\tilde{s}_n^1(\gamma) + (\tilde{p}_n^1(\gamma) - \tilde{c}_n)\partial_{\tilde{p}_n}\tilde{s}_n^1(\gamma))] > 0$ (resp. $\partial_{\tilde{p}_n}\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \Lambda[(p_1^* - c_1)\partial_{\tilde{p}_n}\tilde{s}_1^1(\gamma) + \gamma(\tilde{s}_n^1(\gamma) + (\tilde{p}_n^1(\gamma) - \tilde{c}_n)\partial_{\tilde{p}_n}\tilde{s}_n^1(\gamma))] < 0$). Hence, if $\tilde{p}_n < \tilde{p}_n^1(\gamma)$ (resp. $\tilde{p}_n > \tilde{p}_n^1(\gamma)$), $\tilde{\pi}_1(\cdot, p_1^*, p_2^*|\gamma)$ is strictly increasing (resp. decreasing) in $\tilde{p}_n$. Hence, $\tilde{\pi}_1(\cdot, p_1^*, p_2^*|\gamma)$ is maximized at $\tilde{p}_n = \tilde{p}_n^1(\gamma)$.

Existence of $\tilde{p}_n^2(\gamma)$

We next show that $\tilde{p}_n^2(\gamma)$ exists and is unique for any γ. For any γ, the first order condition for P_2 is $\partial_{\tilde{p}_n}\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) = 0$, i.e.,

$$\Lambda[p_2^*\partial_{\tilde{p}_n}\tilde{s}_2^2(\gamma) + (1-\gamma)\{\tilde{s}_n^2(\gamma) + (\tilde{p}_n^2(\gamma) - \tilde{c}_n)\partial_{\tilde{p}_n}\tilde{s}_n^2(\gamma)\}] - \Lambda C_2'(\Lambda\tilde{s}_2^2(\gamma))\partial_{\tilde{p}_n^2}\tilde{s}_2^2(\gamma) = 0.$$

By using Lemma 2, $\partial_{\tilde{p}_n}\tilde{s}_2^2(\gamma) = \tilde{s}_2^2(\gamma)\tilde{s}_n^2(\gamma)$ and $\partial_{\tilde{p}_n}\tilde{s}_n^2(\gamma) = -\tilde{s}_n^2(\gamma)(1 - \tilde{s}_n^2(\gamma))$, and therefore,

$$p_2^*\tilde{s}_2^2(\gamma)\tilde{s}_n^2(\gamma) - (1-\gamma)[1 - \tilde{s}_n^2(\gamma)]\tilde{s}_n^2(\gamma)[\tilde{p}_n^2(\gamma) - \tilde{c}_n] + (1-\gamma)\tilde{s}_n^2(\gamma) - C_2'(\Lambda\tilde{s}_2^2(\gamma))\tilde{s}_2^2(\gamma)\tilde{s}_n^2(\gamma) = 0.$$

We obtain:

$$\tilde{p}_n^2(\gamma) = \tilde{c}_n + \frac{\tilde{s}_2^2(\gamma)p_2^*}{(1-\gamma)[1 - \tilde{s}_n^2(\gamma)]} + \frac{1}{1 - \tilde{s}_n^2(\gamma)} - \frac{C_2'(\Lambda\tilde{s}_2^2(\gamma))\tilde{s}_2^2(\gamma)}{(1-\gamma)[1 - \tilde{s}_n^2(\gamma)]}.$$

Since we have:

$$[1 - \tilde{s}_n^2(\gamma)]s_2^* = \frac{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp[q_n - \tilde{p}_n^2(\gamma)]} \times \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = \tilde{s}_2^2(\gamma),$$

then we obtain: $\frac{\tilde{s}_2^2(\gamma)}{1 - \tilde{s}_n^2(\gamma)} = s_2^*$. Therefore,

$$\begin{aligned}\tilde{p}_n^2(\gamma) =& \tilde{c}_n + \frac{\tilde{s}_2^2(\gamma)p_2^*}{(1-\gamma)[1 - \tilde{s}_n^2(\gamma)]} + \frac{1}{1 - \tilde{s}_n^2(\gamma)} - \frac{C_2'(\Lambda\tilde{s}_2^2(\gamma))\tilde{s}_2^2(\gamma)}{(1-\gamma)[1 - \tilde{s}_n^2(\gamma)]} \\ =& \tilde{c}_n + \frac{p_2^* s_2^*}{1-\gamma} + \frac{1}{1 - \tilde{s}_n^2(\gamma)} - \frac{C_2'(\Lambda\tilde{s}_2^2(\gamma))s_2^*}{1-\gamma}. \end{aligned} \tag{4}$$

We define $f_2(\tilde{p}_n) := \tilde{c}_n + \frac{p_2^* s_2^*}{1-\gamma} + \frac{1}{1-\tilde{s}_n} - \frac{C_2'(\Lambda\tilde{s}_2)s_2^*}{1-\gamma}$. Since $\tilde{s}_n$ is strictly decreasing in $\tilde{p}_n$ whereas $\tilde{s}_2$ is strictly increasing in $\tilde{p}_n$, then $f_2(\cdot)$ is strictly decreasing in $\tilde{p}_n$. Moreover, since $p_2^* \geq \frac{1}{1-s_2^*} + C_2'(\Lambda s_2^*) > C_2'(\Lambda s_2^*) > C_2'(\Lambda\tilde{s}_2)$ for any $\tilde{p}_n$ (see the proof of Proposition 1), as $\tilde{p}_n \downarrow 0$, $f_2(\tilde{p}_n)$ is lower bounded by $\tilde{c}_n > 0$. Thus, for any given $\gamma \in (0,1)$, $f_2(\cdot)$ has a unique fixed point $\tilde{p}_n^2(\gamma)$ such that $\tilde{p}_n^2(\gamma) = f_2(\tilde{p}_n^2(\gamma))$, i.e., equation (4) holds. Furthermore, if $\tilde{p}_n < \tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n > \tilde{p}_n^2(\gamma)$), $\partial_{\tilde{p}_n}\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > 0$ (resp. $\partial_{\tilde{p}_n}\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < 0$). Thus, if $\tilde{p}_n > \tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n < \tilde{p}_n^2(\gamma)$), $\tilde{\pi}_2(\cdot, p_1^*, p_2^*|\gamma)$ is strictly increasing (resp. decreasing) in $\tilde{p}_n$. Hence, $\tilde{\pi}_2(\cdot, p_1^*, p_2^*|\gamma)$ is maximized at $\tilde{p}_n = \tilde{p}_n^2(\gamma)$.

Monotonicity of $\tilde{p}_n^i(\gamma)$ in γ

We next show that $\tilde{p}_n^1(\gamma)$ (resp. $\tilde{p}_n^2(\gamma)$) is decreasing (resp. increasing) in γ. Recall that $\tilde{p}_n^i(\gamma)$ is obtained as the fixed point of the following equation: $\tilde{p}_n^i(\gamma) = f_i(\tilde{p}_n^i(\gamma))$. We use $f_i(\tilde{p}_n|\gamma)$ to denote the dependence of the function $f_i(\cdot)$ on γ. Since $p_1^* > c_1$, then $f_1(\tilde{p}_n|\gamma) = \tilde{c}_n + \frac{(p_1^*-c_1)s_1^*}{\gamma} + \frac{1}{1-\tilde{s}_n}$ is strictly decreasing in γ for any given $\tilde{p}_n$. Assume that $\hat{\gamma} > \gamma$. Thus, we have:

$$f_1(\tilde{p}_n^1(\gamma)|\hat{\gamma}) < f_1(\tilde{p}_n^1(\gamma)|\gamma) = \tilde{p}_n^1(\gamma),$$

where the inequality follows from the fact that $f_1(\tilde{p}_n|\gamma)$ is strictly decreasing in γ for any fixed $\tilde{p}_n$. Since $f_1(\tilde{p}_n|\hat{\gamma}) - \tilde{p}_n$ is strictly decreasing in $\tilde{p}_n$ for any fixed $\hat{\gamma}$, $f_1(\tilde{p}_n^1(\gamma)|\hat{\gamma}) < \tilde{p}_n^1(\gamma)$ implies that $\tilde{p}_n^1(\hat{\gamma}) < \tilde{p}_n^1(\gamma)$.

By the proof of Proposition 1, $p_2^* \geq \frac{1}{1-s_2^*} + C_2'(\Lambda s_2^*) > C_2'(\Lambda s_2^*)$. Thus, for any given γ, $f_2(\tilde{p}_n|\gamma) = \tilde{c}_n + \frac{p_2^* s_2^*}{1-\gamma} + \frac{1}{1-\tilde{s}_n} - \frac{C_2'(\Lambda \tilde{s}_2)s_2^*}{1-\gamma}$ is strictly increasing in γ for any fixed $\tilde{p}_n$. Therefore,

$$f_2(\tilde{p}_n^2(\gamma)|\hat{\gamma}) > f_2(\tilde{p}_n^2(\gamma)|\gamma) = \tilde{p}_n^2(\gamma),$$

where the inequality follows from the fact that $f_2(\tilde{p}_n|\gamma)$ is strictly increasing in γ for any fixed $\tilde{p}_n$. Since $f_2(\tilde{p}_n|\hat{\gamma}) - \tilde{p}_n$ is strictly decreasing in $\tilde{p}_n$ for any fixed $\hat{\gamma}$, $f_2(\tilde{p}_n^2(\gamma)|\hat{\gamma}) > \tilde{p}_n^2(\gamma)$ implies that $\tilde{p}_n^2(\hat{\gamma}) > \tilde{p}_n^2(\gamma)$.

Existence of $(\underline{\gamma}', \bar{\gamma}')$

First, we note that $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n} \tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n}\{\Lambda[(p_1^* - c_1)\tilde{s}_1 + \gamma(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n]\}$ is strictly increasing in γ, and that $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n} \tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n}\{\Lambda[p_2^*\tilde{s}_2 + (1-\gamma)(\tilde{p}_n - \tilde{c}_n)\tilde{s}_n] - C_2(\Lambda\tilde{s}_2)\}$ is strictly decreasing in γ. Hence, for any $\gamma \in (0,1)$, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \lim_{\gamma'\to 0} \tilde{\pi}_1(\tilde{p}_n^1(\gamma'), p_1^*, p_2^*|\gamma') = \pi_1(p_1^*, p_2^*)$, and $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \lim_{\gamma'\to 1} \tilde{\pi}_2(\tilde{p}_n^2(\gamma'), p_1^*, p_2^*|\gamma') = \pi_2(p_1^*, p_2^*)$.

Note that $\lim_{\gamma\to 0} \tilde{p}_n^1(\gamma) = +\infty$ and $\lim_{\gamma\to 1} \tilde{p}_n^2(\gamma) = +\infty$. Since $\tilde{p}_n^1(\gamma)$ (resp. $\tilde{p}_n^2(\gamma)$) is strictly decreasing (resp. increasing) in γ, $\tilde{p}_n^1(\gamma) = \tilde{p}_n^2(\gamma)$ has a unique solution, which we denote by γ'. We also denote $\tilde{p}_n' := \tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma')$. Thus, if $\gamma > \gamma'$ (resp. $\gamma < \gamma'$), $\tilde{p}_n^1(\gamma) > \tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n^1(\gamma) < \tilde{p}_n^2(\gamma)$).

Now, we show that $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$ is decreasing in γ for $\gamma > \gamma'$. Note that if $\gamma > \gamma'$, then $\tilde{p}_n^2(\gamma) > \tilde{p}_n' > p_n^1(\gamma)$. By using the above argument, $\tilde{\pi}_2(\cdot, p_1^*, p_2^*|\gamma)$ is strictly increasing in $\tilde{p}_n$, for $\tilde{p}_n \leq \tilde{p}_n^2(\gamma)$. Therefore, for $\hat{\gamma} > \gamma > \gamma'$,

$$\tilde{\pi}_2(\tilde{p}_n^1(\hat{\gamma}), p_1^*, p_2^*|\hat{\gamma}) < \tilde{\pi}_2(\tilde{p}_n^1(\hat{\gamma}), p_1^*, p_2^*|\gamma) < \tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma),$$

where the first inequality follows from the fact that $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma)$ is decreasing in γ for any given $\tilde{p}_n$, and the second inequality follows from $\tilde{p}_n^1(\hat{\gamma}) < \tilde{p}_n^1(\gamma) < \tilde{p}_n^2(\gamma)$. Thus, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$ is decreasing in γ for $\gamma > \gamma'$.

We next show that there exists a threshold $\bar{\gamma}' \in (\gamma', \bar{\gamma})$, such that $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$ for all $\gamma > \bar{\gamma}'$. It suffices to show that $\tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) < \pi_2(p_1^*, p_2^*)$. Observe that, if $\gamma = \bar{\gamma}$, then $\tilde{\pi}_1(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) > \tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma})$ by the optimality of $\tilde{p}_n^1(\bar{\gamma})$. Therefore,

$$\tilde{\pi}_1(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) + \tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) < \tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) + \tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) = \tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) + \pi_2(p_1^*, p_2^*),$$

where the inequality follows from the fact that $\tilde{p}_n^*$ maximizes $\tilde{\pi}(\cdot, p_1^*, p_2^*)$, and the equality follows from $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) = \pi_2(p_1^*, p_2^*)$. Indeed, in the proof of Proposition 2, we defined $\bar{\gamma} := \frac{\tilde{\pi}_n^* - \pi_2^* + \tilde{\pi}_2^*}{\tilde{\pi}_n^*}$. By re-arranging this definition, we obtain $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) = \pi_2(p_1^*, p_2^*)$. Hence, $\tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) - \pi_2(p_1^*, p_2^*) < \tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\bar{\gamma}) - \tilde{\pi}_1(\tilde{p}_n^1(\bar{\gamma}), p_1^*, p_2^*|\bar{\gamma}) < 0$. Without loss of generality, we define $\bar{\gamma}' \in (\gamma', \bar{\gamma})$ such that $\tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}'), p_1^*, p_2^*|\bar{\gamma}') = \pi_2(p_1^*, p_2^*)$.

Next, we show that $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$ for all $\gamma < \gamma'$. For $\gamma < \gamma'$, since $\tilde{p}_n^2(\cdot)$ is increasing in γ, whereas $\tilde{p}_n^1(\cdot)$ is decreasing in γ, we have $\tilde{p}_n^2(\gamma) < \tilde{p}_n' < \tilde{p}_n^1(\gamma)$ if $\gamma < \gamma'$. Recall that $\tilde{\pi}_2(\cdot, p_1^*, p_2^*|\gamma)$ is strictly decreasing in $\tilde{p}_n$ for $\tilde{p}_n \geq p_n^2(\gamma)$. Hence, if $\gamma \leq \gamma'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \lim_{\tilde{p}_n \to +\infty} \tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \pi_2(p_1^*, p_2^*)$. Therefore, for any $\gamma < \bar{\gamma}'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$, whereas for any $\gamma > \bar{\gamma}'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.

By exchanging the roles of P_1 and P_2 and repeating the above argument, we show the existence of a threshold $\underline{\gamma}' \in (\underline{\gamma}, \gamma^*)$, such that $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ if and only if $\gamma > \underline{\gamma}'$. Putting everything together, it follows that for any $\gamma \in (\underline{\gamma}', \bar{\gamma}')$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$. □

Proof of Proposition 4

As in the proof of Proposition 3, we define γ' such that $\tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma')$.

$\underline{\tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma') = \tilde{p}_n^*}$

Note that $\tilde{p}_n^1(\gamma')$ satisfies:

$$\gamma'[\tilde{p}_n^1(\gamma') - \tilde{c}_n] = (p_1^* - c_1)s_1^* + \frac{\gamma'}{1 - \tilde{s}_n^1(\gamma')}, \tag{5}$$

and $\tilde{p}_n^2(\gamma')$ satisfies:

$$(1-\gamma')[\tilde{p}_n^2(\gamma') - \tilde{c}_n] = p_2^* s_2^* + \frac{1-\gamma'}{1 - \tilde{s}_n^2(\gamma')} - C_2'(\Lambda \tilde{s}_n^2(\gamma'))s_2^*. \tag{6}$$

Since $\tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma') = \tilde{p}_n'$, by summing up equations (5) and (6) we obtain:

$$\tilde{p}_n' = \tilde{c}_n + (p_1^* - c_1)s_1^* + \frac{1}{1 - \tilde{s}_n^1(\gamma')} + p_2^* s_2^* - C_2'(\Lambda \tilde{s}_n')s_2^*, \tag{7}$$

where

$$\tilde{s}_n' = \frac{\exp(q_n - \tilde{p}_n')}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2) + \exp(q_n - \tilde{p}_n')}.$$

By comparing equations (7) and (2), we conclude that $\tilde{p}_n' = \tilde{p}_n^*$. As a result, $\tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma') = \tilde{p}_n^*$, and $\gamma' = \gamma^*$.

$\underline{\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) > \pi_1(p_1^*, p_2^*) \textit{ and } \tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) > \pi_2(p_1^*, p_2^*)}$

Since $\tilde{p}_n^1(\gamma^*) = \tilde{p}_n^2(\gamma^*) = \tilde{p}_n^*$, then $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) = \tilde{\pi}_1(\tilde{p}_n^1(\gamma^*), p_1^*, p_2^*|\gamma^*) > \lim_{\gamma \to 0} \tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) = \pi_1(p_1^*, p_2^*)$, where the inequality follows from the fact that $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$ is strictly increasing in γ. Analogously, $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma^*) = \tilde{\pi}_2(\tilde{p}_n^2(\gamma^*), p_1^*, p_2^*|\gamma^*) > \lim_{\gamma \to 1} \tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) = \pi_2(p_1^*, p_2^*)$, where the inequality follows from the fact that $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma)$ is strictly decreasing in γ. □

Proof of Proposition 5

Part (a)

In $\mathcal{G}_1$, given γ, P_2's optimal pricing policy has already been characterized in Proposition 3. Using the proof of Proposition 3, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma)$ is strictly increasing in γ for $\gamma \leq \gamma^*$. Thus, $\gamma^1 = \arg\max_\gamma \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \gamma^*$. We then obtain:

$$\tilde{\pi}_1(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) = \max_\gamma \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) \geq \tilde{\pi}_1(\tilde{p}_n^2(\gamma^*), p_1^*, p_2^*|\gamma^*) > \pi_1(p_1^*, p_2^*),$$

where the second inequality follows from Proposition 4.

Part(b)

The proof follows the same argument as the proof of part (a). □

Proof of Proposition 6

Since the platforms do not change the prices of their original services (p_1^*, p_2^*), we have:

$$\begin{aligned}\tilde{RS}(\tilde{p}_n, p_1^*, p_2^*) =& \Lambda \mathbb{E}\Big[\max\{q_1 - p_1^* + \xi_1, q_2 - p_2^* + \xi_2, q_n - \tilde{p}_n + \xi_n, \xi_0\}\Big] \\ >& \Lambda \mathbb{E}\Big[\max\{q_1 - p_1^* + \xi_1, q_2 - p_2^* + \xi_2, \xi_0\}\Big] = RS(p_1^*, p_2^*). \quad \square\end{aligned}$$

Proof of Proposition 7

$\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) > DS_1(p_1^*, p_2^*)$ *if and only if* $\tilde{n} < \frac{1}{s_1^*}$

First, we observe that:

$$\begin{aligned}s_1^*(1 - \tilde{s}_n) =& \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} \times \frac{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n)} \\ =& \frac{\exp(q_1 - p_1^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n)} = \tilde{s}_1^*.\end{aligned}$$

Therefore,

$$\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) - DS_1(p_1^*, p_2^*) = \Lambda(c_1 \tilde{s}_1^* + \tilde{c}_n \tilde{s}_n) - \Lambda c_1 s_1^* = \Lambda\left[c_1 s_1^*(1 - \tilde{s}_n) + \frac{c_1}{\tilde{n}}\tilde{s}_n\right] - \Lambda c_1 s_1^* = \Lambda c_1 \tilde{s}_n\left[\frac{1}{\tilde{n}} - s_1^*\right],$$

where the second equality follows from $\tilde{s}_1^* = s_1^*(1 - \tilde{s}_n)$ and $\tilde{c}_n = c_1/\tilde{n}$. As a result, $\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) > DS_1(p_1^*, p_2^*)$ if and only if $\tilde{n} < \frac{1}{s_1^*}$.

s_1^* *is increasing in* q_1

As we have shown in Proposition 1, s_0^* satisfies:

$$s_0^* + U_1^{-1}[s_0^* \exp(q_1 - c_1 - 1)] + V_2[s_0^* \exp(q_2 - 1)] = 1.$$

Since $U_1^{-1}(\cdot)$ is a strictly increasing function, whereas $V_2(\cdot)$ is an increasing function, s_0^* is strictly decreasing in q_1, and so is $s_2^* = V_2[s_0^* \exp(q_2 - 1)]$. Therefore, $s_1^* = 1 - s_0^* - s_2^*$ is strictly increasing in q_1 as well.

s_1^* *is increasing in* Λ

We use $U_2(z_2|\Lambda)$ to denote the dependence of $U_2(\cdot)$ on Λ. Note that $U_2(z_2|\Lambda) = z_2 \exp\left(\frac{z_2}{1-z_2}\right)\exp[C_2'(\Lambda z_2)]$ is increasing in Λ for any given z_2. Assume that $\hat{\Lambda}_2 > \Lambda_2$. We then have $U_2(z_2|\hat{\Lambda}) > U_2(z_2|\Lambda)$. For any given Λ, we denote by $\hat{V}_2(x|\Lambda)$ the inverse of $U_2(z_2|\Lambda)$, i.e., $U_2(\hat{V}_2(x|\Lambda)|\Lambda) = x$. Since $U_2(z_2|\Lambda)$ is increasing in Λ, $U_2(\hat{V}_2(x|\Lambda)|\hat{\Lambda}) \geq U_2(\hat{V}_2(x|\Lambda)|\Lambda) = x$. Since $U_2(z_2|\Lambda)$ is strictly increasing in z_2, $\hat{V}_2(x|\hat{\Lambda}) \leq \hat{V}_2(x|\Lambda)$ for any given x. Therefore, $V_2(x) = \min\left\{\hat{V}_2(x), \frac{n_2 k_2}{\Lambda t_2}\right\}$ is decreasing in Λ for any given x. Since $s_0^* + U_1^{-1}[s_0^* \exp(q_1 - c_1 - 1)] + V_2[s_0^* \exp(q_2 - 1)] = 1$, we conclude that s_0^* is increasing in Λ. As a result, $s_1^* = U_1^{-1}[s_0^* \exp(q_1 - c_1 - 1)]$ is also increasing in Λ.

s_1^* *is decreasing in* q_2

Since $s_0^* + U_1^{-1}[s_0^* \exp(q_1 - c_1 - 1)] + V_2[s_0^* \exp(q_2 - 1)] = 1$ and $V_2(\cdot)$ is an increasing function, s_0^* is decreasing in q_2. Thus, $s_1^* = U_1^{-1}[s_0^* \exp(q_1 - c_1 - 1)]$ is also decreasing in q_2 given that $U_1^{-1}(\cdot)$ is an increasing function.

s_1^* *is decreasing in* n_2

Recall that we have:

$$C_2'(\lambda) = \frac{t_2}{n_2} G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right) + \frac{(t_2)^2 \lambda_2}{(n_2)^2 k_2 g_2 \left[G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)\right]} = \frac{t_2 w_2}{n_2} + \frac{t_2 G_2(w_2)}{n_2 g_2(w_2)}, \tag{8}$$

where we used $w_2 = G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)$. Therefore, w_2 is decreasing in n_2 for any λ_2. Since $g_2(\cdot)/G_2(\cdot)$ is a decreasing function, then $C_2'(\lambda_2)$ is also decreasing in n_2 for any λ_2. As a result, $U_2(z_2) = z_2 \exp\left(\frac{z_2}{1-z_2}\right)\exp[C_2'(\Lambda z_2)]$ is also decreasing in n_2 for any z_2. We use $U_2(z_2|n_2)$ to denote the dependence of $U_2(\cdot)$ on n_2. Note that $U_2(z_2|n_2) = z_2 \exp\left(\frac{z_2}{1-z_2}\right)\exp[C_2'(\Lambda z_2)]$ is decreasing in n_2 for any given z_2. Assume that $\hat{n}_2 > n_2$. We then have $U_2(z_2|\hat{n}_2) < U_2(z_2|n_2)$. For any given n_2, we denote by $\hat{V}_2(x|n_2)$ the inverse of $U_2(z_2|n_2)$, i.e., $U_2(\hat{V}_2(x|n_2)|n_2) = x$. Since $U_2(z_2|n_2)$ is decreasing in n_2, $U_2(\hat{V}_2(x|n_2)|\hat{n}_2) \le U_2(\hat{V}_2(x|n_2)|n_2) = x$. Since $U_2(z_2|n_2)$ is strictly increasing in z_2, $\hat{V}_2(x|\hat{n}_2) \ge \hat{V}_2(x|n_2)$ for any given x. Therefore, $V_2(x) = \min\left\{\hat{V}_2(x), \frac{n_2k_2}{\Lambda t_2}\right\}$ is increasing in n_2 for any given x. Since $s_0^* + U_1^{-1}[s_0^*\exp(q_1 - c_1 - 1)] + V_2[s_0^*\exp(q_2 - 1)] = 1$, we conclude that s_0^* is decreasing in n_2. As a result, $s_1^* = U_1^{-1}[s_0^*\exp(q_1 - c_1 - 1)]$ is also decreasing in n_2.

s_1^* is decreasing in k_2

Recall that $g_2(\cdot)/G_2(\cdot)$ is a decreasing function. By using equation (8), $C_2'(\lambda_2)$ is decreasing in k_2 for any λ_2. Therefore, $U_2(z_2) = z_2 \exp\left(\frac{z_2}{1-z_2}\right)\exp[C_2'(\Lambda z_2)]$ is decreasing in k_2 for any z_2, its inverse $\hat{V}_2(\cdot)$ is increasing in k_2, and so is $V_2(x) = \min\left\{\hat{V}_2(x), \frac{n_2k_2}{\Lambda t_2}\right\}$. Since $s_0^* + U_1^{-1}[s_0^*\exp(q_1 - c_1 - 1)] + V_2[s_0^*\exp(q_2 - 1)] = 1$, we conclude that s_0^* is decreasing in k_2. As a result, $s_1^* = U_1^{-1}[s_0^*\exp(q_1 - c_1 - 1)]$ is decreasing in k_2 as well. $\square$

Proof of Proposition 8

Note that:

$$\tilde{s}_2 = \frac{\exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*) + \exp(q_n - \tilde{p}_n)} < \frac{\exp(q_2 - p_2^*)}{1 + \exp(q_1 - p_1^*) + \exp(q_2 - p_2^*)} = s_2^*.$$

Then, for any $\tilde{p}_n$, we have:

$$\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*) = k_2\mathbb{E}\Big[\max\Big\{r_2, G_2^{-1}\left(\frac{\Lambda \tilde{s}_2 t_2}{n_2k_2}\right)\Big\}\Big] < k_2\mathbb{E}\Big[\max\Big\{r_2, G_2^{-1}\left(\frac{\Lambda s_2^* t_2}{n_2k_2}\right)\Big\}\Big] = DS_2(p_1^*, p_2^*). \quad \square$$

Proof of Proposition 9

First, we define the new objective functions: $\hat{\pi}(\tilde{p}_n) := \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) + \tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)$ as the sum of the total profits and P_2's driver surplus, and $\hat{\pi}_2(\tilde{p}_n|\gamma) := \tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) + \tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)$. We have:

$$\hat{\pi}(\tilde{p}_n) = \Lambda(p_1^* - c_1)\hat{s}_1 + \Lambda p_2^*\hat{s}_2 + \Lambda(\tilde{p}_n - \tilde{c}_n)\hat{s}_n - E_2(\Lambda\hat{s}_2),$$

where $\hat{s}_1 = \frac{\exp(q_1 - p_1^*)}{1+\exp(q_2-p_2^*)+\exp(q_n-\tilde{p}_n)}$, $\hat{s}_2 = \frac{\exp(q_2 - p_2^*)}{1+\exp(q_2-p_2^*)+\exp(q_n-\tilde{p}_n)}$, $\hat{s}_n = \frac{\exp(q_n - \tilde{p}_n)}{1+\exp(q_2-p_2^*)+\exp(q_n-\tilde{p}_n)}$, and

$$E_2(\lambda_2) := C_2(\lambda_2) - k_2\mathbb{E}\Big[\max\Big\{r_2, G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2k_2}\right)\Big\}\Big].$$

Analogously, $\hat{\pi}_2(\tilde{p}_n|\gamma) = \Lambda p_2^*\hat{s}_2 + (1-\gamma)\Lambda(\tilde{p}_n - \tilde{c}_n)\hat{s}_n - E_2(\Lambda\hat{s}_2)$.

We next show that $E_2(\cdot)$ is increasing and convex in λ_2. We have:

$$E_2'(\lambda_2) = \frac{t_2}{n_2}G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2k_2}\right) + \frac{(t_2)^2\lambda_2}{(n_2)^2k_2g_2\left[G_2^{-1}\left(\frac{\lambda_2t_2}{n_2k_2}\right)\right]} - \frac{(t_2)^2\lambda_2}{(n_2)^2k_2g_2\left[G_2^{-1}\left(\frac{\lambda_2t_2}{n_2k_2}\right)\right]} = \frac{t_2}{n_2}G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2k_2}\right) > 0,$$

which is increasing in λ_2. Therefore, $E_2(\cdot)$ is increasing and convex in λ_2. As a result, $\hat{\pi}(\cdot)$ and $\hat{\pi}_2(\cdot|\gamma)$ have exactly the same structure as $\pi(\cdot, p_1^*, p_2^*)$ and $\pi_2(\cdot, p_1^*, p_2^*|\gamma)$, respectively. Consequently, Proposition 9 follows from the same argument as the proofs of Propositions 2 and 3. $\square$

Proof of Corollary 1

$\hat{p}_n^* > \tilde{p}_n^*$

The first order condition implies that $\hat{p}_n^*$ needs to satisfy the following equation:

$$\hat{p}_n^* = \tilde{c}_n + (p_1^* - c_1)s_1^* + p_2^* s_2^* + \frac{1}{1-\hat{s}_n^*} - E_2'(\Lambda \hat{s}_2^*)s_2^*. \tag{9}$$

By comparing equations (9) and (2), and using the fact that $C_2'(\lambda_2) - E_2'(\lambda_2) = \frac{(t_2)^2\lambda_2}{(n_2)^2 k_2 g_2\left[G_2^{-1}\left(\frac{\lambda_2 t_2}{n_2 k_2}\right)\right]} > 0$, we conclude that $\hat{p}_n^* > \tilde{p}_n^*$.

There exists $\hat{\gamma} < \gamma^*$ such that $\tilde{p}_n^1(\hat{\gamma}) = \hat{p}_n^2(\hat{\gamma}) = \hat{p}_n^*(\hat{\gamma})$

The existence of $\hat{\gamma}$ follows from the same argument as in the proof of Proposition 4. We know that $\tilde{p}_n^1(\hat{\gamma}) = \hat{p}_n^*$ and $\tilde{p}_n^1(\gamma^*) = \tilde{p}_n^*$. By using $\hat{p}_n^* > \tilde{p}_n^*$ and the fact that $\tilde{p}_n^1(\gamma)$ is strictly decreasing in γ, we conclude that $\hat{\gamma} < \gamma^*$. $\square$