Coopetition and Profit Sharing for Ride-sharing Platforms

Maxime C. Cohen
NYU Stern School of Business, New York, NY 10012, maxime.cohen@stern.nyu.edu

Renyu Zhang
NYU Shanghai, 1555 Century Avenue, Shanghai, China, 200122, renyu.zhang@nyu.edu

The introduction of on-demand ride-hailing platforms totally changed the way people commute. Recently, some of these platforms engaged in a profit sharing contract with one of their competitors by introducing a new joint service. For example, on June 6, 2017, an NYC-based online ride-sharing platform, Via, officially announced a partnership with the taxi-hailing platform Curb. This partnership allows riders to order a taxi and share some portion of the trip with other riders by using Via's efficient matching algorithm. These two platforms are both competing and cooperating with each other, so this form of partnership is often referred to as *coopetition*. This paper is motivated by such partnerships in the ride-sharing industry. We model the price competition between ride-hailing platforms using the Multinomial Logit choice model and analyze the impact of introducing the new joint service. First, we identify conditions under which the coopetition is beneficial for both platforms. Interestingly, we show that when both platforms are not over-congested, a well-designed *profit sharing contract* will benefit both platforms. This result admits a similar win-win outcome as in the supply chain contracts literature, even though the settings are very different. In addition, we show that one can design a profit sharing contract that also benefits riders and drivers. Consequently, such a coopetition partnership may benefit every single party (riders, drivers, and both platforms) when using a properly designed profit sharing contract.

Key words: Ride-sharing, coopetition, profit sharing contracts, choice models

1. Introduction

On-demand ride-hailing platforms totally changed the way people commute and travel for short distances. Several well-known players in this market are Uber, Lyft, Didi Chuxing, Grab, Go-Jek, Ola, Via, Gett, and Juno, just to name a few. In October 2016, it was reported that Uber had 40 million monthly riders worldwide.[1] Nowadays, using this type of transportation services has become the norm in most major cities (e.g., Uber now operates in more than 600 cities around the world). It is worth mentioning that during the first few years, the growth was moderate but within the last two years, one could see a very successful expansion. For example, it took Uber six years to complete their first billion rides (from 2009 to 2015) but only an additional six months to

[1] http://fortune.com/2016/10/20/uber-app-riders/

reach their two-billionth ride.[2] This means that during the first six months of 2016, the company was providing an average of 5.5 million rides a day (or 230,000 an hour).

Within the online ride-hailing market, a recent trend emerged on several platforms: *ride-sharing* or *carpooling* services. Several aforementioned companies offer an option which allows passengers heading in the same direction to be matched to the same vehicle and share their ride. In such a service, riders cannot select the people they are sharing with but instead an algorithm will match several riders to the same vehicle. In NYC, one can find at least three such services: uberPOOL, Lyft Line, and Via. These different companies bear several differentiating features such as price, waiting times, and other minor differences which are beyond the scope of this paper. One of the main arguments for sharing a ride is the low price paid by the rider.

Clearly, the taxi industry has suffered heavily from this recent market trend. It was reported in January 2017 that "Uber and Lyft cars outnumber yellow cabs in NYC 4 to 1."[3] In a response to this decline, taxi companies started to also offer on-demand services via mobile platforms to better fit in today's economy. For example, in several cities, taxi rides can now be directly ordered from a smartphone application and the payment (including the tip) can either be completed via the application or in person. One such company based in the U.S. is Curb.[4] On their website, one can read: "Curb is the #1 taxi app in the US that connects you to fast, convenient and safe rides in 65 cities (50,000 Cabs – 100,000 Drivers)." A screenshot of their homepage can be found in Figure 1.

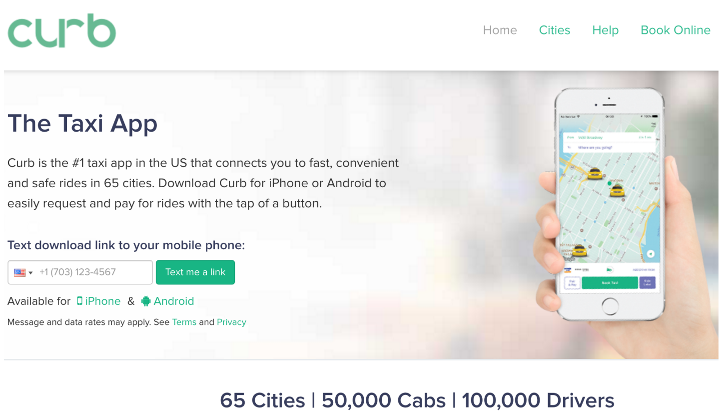


Figure 1 Homepage of the Curb website.

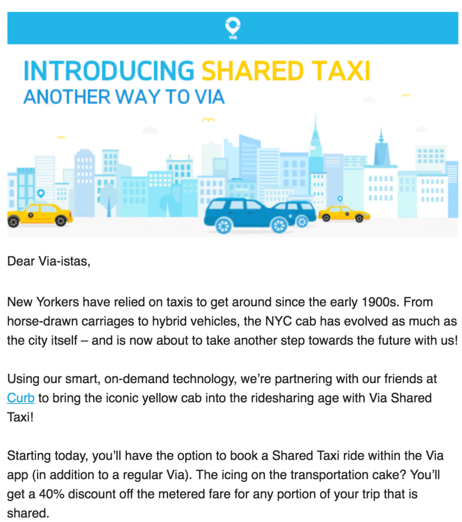
In the last two years, several partnerships between ride-hailing platforms have emerged. One such example is the partnership between Curb and Via in NYC. Via offers an affordable fare for riders who are willing to carpool, whereas Curb offers a private taxi ride while charging the

[2] https://techcrunch.com/2016/07/18/uber-has-completed-2-billion-rides/

[3] https://ny.curbed.com/2017/1/17/14296892/yellow-taxi-nyc-uber-lyft-via-numbers

[4] https://gocurb.com/

meter price plus an additional fixed fee per trip. One can definitely view these two platforms as competitors. Yet, they decided to collaborate and engage in a unique partnership. More precisely, on June 6, 2017, both platforms started to offer a joint service through a *profit sharing contract*, under which Curb and Via each earn a certain portion of the net profit from the joint service. This type of partnerships is sometimes referred to as *coopetition*, a term coined to describe cooperative competition (see, e.g., Brandenburger and Nalebuff 2011). The new joint service introduced by Curb and Via in NYC allows users to book a shared taxi from either platform. For example, when a user requests a ride through the Via smartphone application, s/he may be offered to ride with a nearby available taxi (this option is called Shared Taxi). Then, the rider can either accept the Shared Taxi or decline by requesting a regular Via ride. Shared Taxi fares are calculated using the meter price and paid directly to the driver. If the matching algorithm finds another rider heading in the same direction, the two riders will carpool and save 40% on any shared portion of the trip. Finally, the rider can pay (and tip) the taxi driver directly through either online platform. When introducing this new service, Via sent an e-mail advertising campaign to several NYC users (parts of the e-mail content can be seen in Figure 2).[5]


INTRODUCING SHARED TAXI
ANOTHER WAY TO VIA

Dear Via-istas,

New Yorkers have relied on taxis to get around since the early 1900s. From horse-drawn carriages to hybrid vehicles, the NYC cab has evolved as much as the city itself – and is now about to take another step towards the future with us!

Using our smart, on-demand technology, we're partnering with our friends at Curb to bring the iconic yellow cab into the ridesharing age with Via Shared Taxi!

Starting today, you'll have the option to book a Shared Taxi ride within the Via app (in addition to a regular Via). The icing on the transportation cake? You'll get a 40% discount off the metered fare for any portion of your trip that is shared.

Figure 2 **Advertising e-mail sent on June 6, 2017 on the new partnership between Curb and Via in NYC.**

The recent partnership between Curb and Via in NYC is definitely not an exception. Below we report other similar examples:

[5] The partnership between Curb and Via in NYC was the topic of extensive media coverage. See for example: https://www.nytimes.com/2017/06/06/nyregion/new-york-yellow-taxis-ride-sharing.html, https://techcrunch.com/2017/06/06/curb-and-via-bring-ride-sharing-to-nycs-yellow-taxis/ and https://qz.com/999132/can-shared-rides-save-the-iconic-new-york-city-yellow-cab/

• In December 2016, Uber partnered with Indonesia's second largest taxi operator PT Express Transindo Utama Tbk. This partnership allows for ride-sharing, and gave Uber access to Express fleet of more than 11,000 taxis and 17,000 drivers. Express drivers who participate in the program can now serve requests from the Uber application. On the other hand, Uber drivers can lease vehicles by making monthly payments, partly from the income generated from the Uber app. In this case, both pools of drivers are participating in the new service.

• In October 2014, Uber partnered with For Hire taxis to expand pick-up availability in Seattle. In this partnership, riders can select multiple options directly from the Uber app (UberX, UberXL, Black Car, SUV, and For Hire). The prices and availability of each option are different. As a result, the new service (operated through the Uber app) can be completed by both pools of drivers.

• In March 2017, Grab partnered with SMRT Taxis with the goal of building the largest car fleet (taxi and private-hire) in Southeast Asia. SMRT, Singapore's third largest taxi operator, was reported in April 2017 to have a fleet of 3,400 taxis. As of October 2017, Grab claims that its current market share is 95% in third-party taxi hailing and 72% in private vehicle hailing.[6] Since January 2017, the number of Grab drivers has nearly tripled (exceeding 1.8 million), making it the largest land transportation fleet in Southeast Asia. In this partnership, all SMRT drivers will use only Grab's ride-hailing application for third-party bookings (to complement street-hail pickups). As a result, customers can now be served by both platforms when booking a ride.

• On January 31, 2017, Go-Jek partnered with PT Blue Bird Tbk in Indonesia. In this partnership, riders will simply be served by the closest driver. Therefore, the new service uses both types of drivers (one platform has 23,000 vehicles, whereas the other has at least 11,000 vehicles).[7]

It is clear that both parties have their own incentives to engage in this type of partnerships. For example, it allows ride-hailing platforms to expand their number of drivers and complement their existing market share. In addition, platforms can potentially benefit from the technological advances developed by other platforms (e.g., efficient matching algorithms and online secured payments). Nevertheless, such partnerships can increase the congestion levels and cannibalize the original market shares. Indeed, by introducing the new joint service, each platform needs to be cautious about the potential elevated congestion and the market share losses (customers who were initially riding with one of the platforms may now switch to the new service). In addition, the platforms need to decide which pool of drivers will serve the new joint service as well as the terms of the profit sharing contract.

[6] http://www.straitstimes.com/business/companies-markets/grab-secures-record-us700m-in-debt-facilities-partnering-smrt-to-build

[7] https://www.techinasia.com/go-jek-launches-blue-bird-partnership-now-on-iphone

This paper is motivated by the type of partnerships described above. In particular, we are interested in studying the implications of introducing a new joint service between two competing ride-hailing platforms via a profit sharing contract. Our goal is to draw practical insights on the impact of the new joint service on both platforms (e.g., Curb and Via), drivers, and riders. We propose to model this problem using the Multinomial Logit (MNL) choice model to capture the fact that riders face several alternatives. We base our model on current practices in the ride-hailing industry and study the impact of introducing the coopetition partnership. In particular, we identify conditions under which a well-designed profit sharing contract increases the profits of both platforms. We then show that such a contract may be beneficial for riders and drivers, when properly designed. We note that the ideas, analysis, and insights presented in this paper are not limited to the ride-hailing industry. One can potentially apply a similar approach to any market with several competitors who decide to engage in a coopetition partnership through a profit sharing contract. However, to simplify the exposition and since this work was motivated by recent partnerships in the ride-hailing industry, we focus our presentation in this context.

1.1. Contributions

Given the recent popularity of ride-hailing and ride-sharing platforms, this paper studies a timely practical problem directly motivated by several recent partnerships. At a high level, our contributions can be summarized as follows.

- **Characterizing the equilibrium of a competitive ride-sharing market.** To the best of our knowledge, this paper is among the first to study the (price) competition between ride-sharing platforms. We use the MNL choice model to capture the decision process of potential riders and show that the competition between ride-sharing platforms reduces to a price competition under an MNL model with convex costs. Consequently, the equilibrium outcome is analytically tractable and can be computed efficiently.
- **Studying the impact of coopetition under a profit sharing contract.** We study how the introduction of the new joint service affects the competing platforms. First, we identify clear conditions under which the coopetition is beneficial for both platforms. In particular, the driver capacity congestion levels of both platforms should not be too high, as otherwise the coopetition will be detrimental to the total profits. We then consider several decision making dynamics depending on which platform sets the price of the new service. We identify three main effects induced by introducing the new service: new market share, cannibalization, and increased congestion.
- **Demonstrating that a profit sharing contract can yield a win-win outcome.** When initial congestion levels are not too high, we show that regardless of which platform sets the price of the new service, there always exists a profit sharing contract that increases the profits of both platforms. As a result, engaging in a coopetition could be a win-win strategy for both parties.

- **Showing that drivers (and riders) can also benefit.** As expected, riders also benefit from introducing the new service. Furthermore, we discuss two strategies that allow the drivers of both platforms to also benefit from the coopetition partnership. Consequently, when the coopetition terms are carefully designed, every single party will benefit (riders, drivers, and both platforms).

1.2. Related Literature

This paper is related to at least four streams of literature: the economics of ride-hailing platforms, coopetition models, choice models, and supply chain contracts.

First, the recent popularity of ride-hailing platforms triggered a great interest in studying pricing decisions in this context. Several papers consider the problem of designing incentives on prices and wages to coordinate supply with demand for on-demand service platforms (see, for example, Tang et al. 2017, Taylor 2017, Hu and Zhou 2017, Bimpikis et al. 2016, Benjaafar et al. 2018). Our work has a similar motivation but is among the first to explicitly capture the competition between two platforms using an MNL choice model. In Chen and Sheldon (2016), the authors analyze Uber data from 25 million trips and show empirically that dynamic wages (due to surge pricing) can entice drivers to work longer. In Chen and Hu (2016), the authors consider a single market with an intermediary who makes dynamic pricing and matching decisions. When consumers are forward-looking (i.e., they may wait strategically for better prices), the authors show that a simple pricing and matching policy is optimal while inducing buyers and seller to behave myopically. In Hu and Zhou (2017), the authors study the pricing decisions of an on-demand platform and demonstrate the good performance of a flat-commission contract. Banerjee et al. (2015) and Cachon et al. (2017) compare the impact of static versus dynamic prices and wages. By assuming that the payout ratio is exogenously given and that customers have heterogeneous valuations, Banerjee et al. (2015) demonstrate the good performance of static pricing. Under different modeling assumptions (endogenous payout ratio and homogeneous valuations), Cachon et al. (2017) found that dynamic pricing performs well.

A different related topic is the competition in the taxi industry (see, for example, Cairns and Liston-Heyes 1996). The recent work by Cramer and Krueger (2016) shows using data from five cities, that UberX drivers have captured a higher capacity utilization rate than taxi drivers. Although the concept of ride-sharing has existed for decades and can potentially provide several societal benefits such as reducing travel costs, congestion, and emissions, its adoption remained limited for some time (see, e.g., Furuhata et al. 2013). However, the recent ubiquity of mobile technology and the popularity of peer-to-peer services have led to an unprecedented recent growth.

As mentioned, when two competitors engage in some sort of cooperation, this is often referred to as *coopetition*, a term coined to describe cooperative competition (see, e.g., Brandenburger and

Nalebuff 2011). Closer to our work, there are several papers on coopetition in operations management settings. For example, Nagarajan and Sošić (2007) propose a model for coalition formation among competitors who set prices, and characterize the equilibrium behavior of the resulting strategic alliances. Casadesus-Masanell and Yoffie (2007) study the simultaneously competitive and cooperative relationship between two manufacturers of complementary products, such as Intel and Microsoft, on their R&D investment, pricing, and timing of new product releases. In a strategic alliance setting with capacity sharing, Roels and Tang (2017) show that an ex-ante capacity reservation contract always benefits both firms. In the revenue management literature, several papers have studied a commonly adopted form of coopetition among airline companies, called airline alliances. Netessine and Shumsky (2005) show that a well-designed revenue sharing contract can coordinate an airline alliance (i.e., achieve the first-best market outcome). Wright et al. (2010) further extend this result to a dynamic setting. Coopetition and its related contractual issues have been also studied in the context of service operations. For example, Roels et al. (2010) analyze the contracting issues that arise in collaborative services and identify the optimal contracts under different service environments. The marketing literature has also studied the impact of coopetition, e.g., in sharing the same advertising agency (Villas-Boas 1994). Our contribution with respect to this literature lies in the fact that our paper is among the first to study coopetition in the ride-sharing industry, motivated by recent partnerships.

The third stream of relevant literature is related to choice models (for a review on this topic, see Train 2009, and the references therein), and in particular the price competition under the MNL model and its extensions (see, e.g., Anderson et al. 1992, Gallego et al. 2006, Konovalov and Sándor 2010, Li and Huh 2011, Aksoy-Pierson et al. 2013). Using the MNL demand model, Gallego et al. (2006) show that a unique equilibrium exists when costs are increasing and convex in sales. In Li and Huh (2011), the authors consider the problem of pricing multiple products under the nested MNL choice model and show that characterizing the equilibrium outcome is analytically tractable. In this literature, the main focus is on showing the existence and/or uniqueness of the equilibrium outcome. In this paper, we extend the results of Gallego et al. (2006) and Li and Huh (2011) to show that a unique equilibrium exists in the ride-sharing competition when drivers are self-scheduled. More importantly, our emphasis is on drawing practical insights on how the coopetition impacts the different stakeholders of the ride-sharing market.

Finally, our paper is related to the literature on risk sharing contracts in supply chains (see, e.g., Cachon 2003, Cachon and Lariviere 2005). This literature shows that one can coordinate the supply chain by using a well-designed contract (e.g., revenue sharing). Such a contract allows the supplier and the retailer to share the demand uncertainty risk, and hence will typically induce the retailer to order larger quantities. It is shown that a well-designed contract can achieve the first-best market outcome, i.e., the same profit as the centralized supply chain (when the supplier and the retailer collude to jointly optimize the total profits). In addition, such a contract can achieve a win-win outcome in which both supply chain partners earn higher profits. Our paper admits several similarities with this classical result even though the settings are very different. To the best of our knowledge, our paper is the first to study coopetition contracts in the context of ride-sharing.

Structure of the paper. Section 2 presents the MNL choice model and the characterization of the equilibrium outcome for the original setting. Section 3 considers the setting where the new joint service is introduced via a profit sharing contract. Section 4 studies the impact of coopetition on the profits of both platforms, on riders, and on drivers. We present computational experiments in Section 5 to illustrate and quantify our insights. Finally, our conclusions are reported in Section 6. The proofs of the technical results are relegated to the Appendix.

2. Model and Equilibrium Analysis

In this section, we first present our benchmark model that describes the competitive ride-sharing market without coopetition. We then characterize the equilibrium market outcome.

2.1. Benchmark Model

We consider two competing online platforms denoted by P_1 and P_2. Each platform P_i $(i = 1, 2)$ offers a ride-hailing (or ride-sharing) service through its mobile application.[8] Let q_1 and q_2 be the perceived value/quality (e.g., waiting time, safety, reliability, travel time) of taking a ride using P_1 and P_2 respectively. It is possible that one or both platforms offer ride-sharing services (e.g., Via, uberPool, and Lyft Line). Without loss of generality, we assume that $q_1 \geq q_2$, i.e., the quality of P_1's service is superior relative to P_2. Note that we do not explicitly model the waiting time of hailing a ride using P_i but instead, assume it is absorbed into q_i. We denote by $p_i \in [\underline{p}, \bar{p}]$ the average price per-trip charged by P_i. In practice, ride-sharing platforms (such as Via and Uber) typically charge a price which is independent of the number of riders who are sharing the trip. We therefore assume that p_i does not depend on the number of riders who share the same trip. A summary of the notation used in this paper can be found in Appendix A.

We assume that the riders' behavior follows the MNL discrete choice model. More specifically, a rider can choose between 3 options: P_1, P_2, and the outside option (other ride-sharing platforms, public transportation, etc.). The utility a customer derives from hailing a ride through P_i is $u_i = q_i - p_i + \xi_i$, where ξ_i represents the random unobserved utility terms for using P_i. The utility of the outside option is normalized so that $u_0 = \xi_0$. For each rider, ξ_1, ξ_2, and ξ_0 are assumed to be

[8] Ride-hailing refers to a platform that offers transportation services on-demand. We refer to ride-sharing (or car-pooling) when such a platform allows multiple unrelated riders to share the same vehicle.

independent and identically distributed with a Gumbel distribution. Therefore, after observing the prices (p_1, p_2), a rider selects P_1 with probability

$$s_1 = \frac{\exp(q_1 - p_1)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)},$$

selects P_2 with probability

$$s_2 = \frac{\exp(q_2 - p_2)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)},$$

and selects the outside option with probability

$$s_0 = \frac{1}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)}.$$

We remark that it is common to use MNL models to capture the different alternatives a consumer faces when selecting modes of transportation (see, for example, McFadden et al. (1977) and Bolduc (1999), as well as Brownstone and Golob (1992) who uses the MNL model in a ride-sharing context). Such models are widely used both in academia and in practice, as they are often analytically tractable and easy to estimate.

Let Λ be the total rider arrival rate in the market, i.e., the maximum number of potential riders arriving per unit time. We assume that Λ is deterministic and known to both platforms (the extension where Λ is stochastic is briefly discussed below). The demand faced by P_i per unit time is then given by Λs_i.

To simplify the analysis and to isolate the impact of the coopetition partnership, we impose several assumptions. In particular, we assume that each platform has a different pool of drivers, i.e., drivers work only on one platform (note that this assumption is satisfied in the Via and Curb example). Nevertheless, the same qualitative results hold if we consider the case where both platforms share the same pool of drivers. In addition, we abstract from the matching and pricing dynamics and we assume that the wage of each platform is set to match supply with demand. Imposing these simplifying assumptions makes the analysis tractable and does not interfere with our goal of studying the impact of coopetition. One can look at our analysis at a macro level in order to examine the impact of coopetition on ride-sharing platforms.

We next compute the total cost incurred by each platform. Given that the drivers of such platforms are self-scheduled (i.e., independent workers with flexible schedules), each platform will set the wage so as to attract enough drivers. Let K_i be the total number of drivers working for platform P_i, $r_i \sim G_i(\cdot)$ be the reservation wage of P_i's drivers per unit time (and $G_i(\cdot)$ its CDF), and w_i be the wage (per-unit time) offered by P_i to its drivers. Hence, the fraction of drivers working for P_i is given by $\mathbb{P}(r_i \leq w_i) = G_i(w_i)$ and the total number of active drivers in P_i is $K_i G_i(w_i)$, which is also the capacity of drivers for P_i. As mentioned, P_i seeks to match supply with demand, so that $K_i G_i(w_i) n_i = \Lambda s_i$, where n_i is the number of riders per trip for P_i (if there is no ride-sharing, $n_i = 1$). As a result, the wage offered by P_i to its drivers is given by:

$$w_i = G_i^{-1}\left(\frac{\Lambda s_i}{K_i n_i}\right),$$

and the total cost of P_i is equal to:

$$K_i G_i(w_i) w_i = \frac{\Lambda s_i}{n_i} G_i^{-1}\left(\frac{\Lambda s_i}{K_i n_i}\right).$$

Equivalently, the total cost of P_i can be written as $K_i \hat{f}_i\left(\frac{\Lambda s_i}{K_i n_i}\right)$, where $\hat{f}_i(y) := y G_i^{-1}(y)$. When the function $G_i(\cdot)$ satisfies the log-concave condition,[9] one can see that $\hat{f}_i(\cdot)$ is convex and strictly increasing in Λs_i. If $\Lambda s_i/(K_i n_i) > 1$, then P_i is over-congested. In this case, we assume that (i) the platform has to provide a higher wage to its drivers to compensate for overtime hours, and (ii) the congestion may lead to a reduction in the service quality (e.g., increased waiting times) which can potentially reduce future demand. To capture the congestion cost, we extend the function $\hat{f}_i(\cdot)$ to $f_i(\cdot)$ such that $f_i(y) = \hat{f}_i(y)$ for $y \in [0,1]$, and $f_i(\cdot)$ is convexly increasing and continuously differentiable on $[0, +\infty)$. The total cost of P_i is then expressed as $K_i f_i\left(\frac{\Lambda s_i}{K_i n_i}\right) = K_i f_i(u_i)$, where we define $u_i := \Lambda s_i/(K_i n_i)$ as the congestion level of P_i assuming that all its drivers are working. We impose the additional assumptions that $\lim_{y\to+\infty} f_i'(y) = +\infty$ and $\lim_{y\to+\infty} f_i(y)/y = +\infty$ to capture the fact that the marginal cost of paying drivers is very high when the congestion level is considerable.

Next, we compute the expected profit per unit time of each platform. Given the price vector (p_1, p_2), let $\pi_i(p_1, p_2)$ be the expected profit of P_i. We then have:

$$\pi_i(p_1, p_2) = p_i \Lambda s_i - K_i f_i(u_i), \text{ with } s_i = \frac{\exp(q_i - p_i)}{1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)} \text{ and } u_i = \frac{\Lambda s_i}{K_i n_i}.$$

As mentioned, our results and insights continue to hold in the case where Λ is stochastic and follows a given distribution $F(\cdot)$, which is common knowledge to both platforms. This setting applies to the case where customer demand for rides can vary under different scenarios (see Hu and Zhou 2017). With stochastic customer arrival rate, the expected profit of P_i is given by $\pi_i(p_1, p_2) = p_i s_i \mathbb{E}[\Lambda] - K_i \mathbb{E}[f_i(u_i)]$, where the expectation is taken with respect to the distribution of Λ.

[9] A distribution satisfies the log-concave condition, if the logarithm of its CDF is concave. Note that several commonly used distributions (e.g., uniform, normal, Gamma, Weibull, logistic) satisfy this condition (see Bagnoli and Bergstrom 2005). For the rest of the paper, we assume that $G_i(\cdot), i = 1, 2$ are log-concave.

2.2. Equilibrium Analysis

In the benchmark model, platforms P_1 and P_2 compete on price. More specifically, they engage in a simultaneous game in which P_1 sets p_1 to maximize $\pi_1(p_1,p_2)$ and P_2 sets p_2 to maximize $\pi_2(p_1,p_2)$. The equilibrium prices (p_1^*,p_2^*) satisfy the following:

$$p_1^* \in \arg\max_{p_1\in[\underline{p},\bar{p}]} \pi_1(p_1,p_2^*) \text{ and } p_2^* \in \arg\max_{p_2\in[\underline{p},\bar{p}]} \pi_2(p_1^*,p_2).$$

As reported in the next proposition, one can show the existence and uniqueness of the equilibrium market outcome.

Proposition 1. *There exists a unique equilibrium (p_1^*,p_2^*) which can be computed efficiently.*

Proposition 1 shows that there exists a unique equilibrium in the price competition game played by P_1 and P_2. In the Appendix, we provide a detailed procedure to efficiently compute the equilibrium outcome (i.e., to find the values of s_1^*, s_2^*, s_0^*, p_1^* and p_2^*, where (s_1^*,s_2^*,s_0^*) denote the equilibrium rider choice probabilities). Even though we cannot characterize the equilibrium outcome in closed form, the equilibrium can be solved efficiently (e.g., using binary search). Gallego et al. (2006) and Li and Huh (2011) characterize the equilibrium outcome for the price competition under the MNL and Nested MNL choice models, respectively. In this paper, we extend their solution technique to the competitive ride-sharing market with self-scheduled drivers.

3. Coopetition and Profit Sharing

In this section, we model the setting where the coopetition is introduced through a profit sharing contract between the two platforms. In particular, the two competing platforms P_1 and P_2 collaborate and offer a new joint service, which is available to riders from either platform. As mentioned before, one such recent example is the partnership between Curb and Via with the introduction of a taxi sharing service in NYC as of June 6, 2017. Such a partnership between competing firms is often referred to as *coopetition*. For the rest of this paper, we use the terms "new joint service", "new service", and "coopetition" interchangeably.

We use the superscript $\tilde{}$ to denote the different variables in the presence of coopetition. More specifically, we denote by $\tilde{p}_1$ and $\tilde{p}_2$ the prices of the original services offered by P_1 and P_2, respectively, after introducing the new joint service. The quality and price per trip of the new service are denoted by q_n and $\tilde{p}_n \in [\underline{p},\bar{p}]$ respectively. The (average) number of riders per trip in the new joint service is $\tilde{n}$ (as before, if the new joint service does not offer a ride-sharing option, $\tilde{n}=1$ and otherwise $\tilde{n}>1$).

As for the original services, the utility derived by a rider from choosing the new joint service is $u_n = q_n - \tilde{p}_n + \xi_n$, where ξ_n represents the random unobserved utility terms for using the new service. As in the benchmark case, we assume that for each rider, ξ_1, ξ_2, ξ_n, and ξ_0 are independent and identically distributed with a Gumbel distribution. After introducing the new joint service, a rider faces four different alternatives (P_1, P_2, the new joint service, and the outside option), and chooses the one that yields the highest utility. Therefore, in the presence of coopetition, the demand for P_1 is $\Lambda\tilde{s}_1$, where

$$\tilde{s}_1 = \frac{\exp(q_1 - \tilde{p}_1)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)}.$$

Analogously, in the presence of coopetition, the demand for P_2 is $\Lambda\tilde{s}_2$, where

$$\tilde{s}_2 = \frac{\exp(q_2 - \tilde{p}_2)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)}.$$

It is clear that the introduction of the new service cannibalizes the original services offered by both platforms (we will discuss this effect in greater detail in Section 4.1.4). Finally, demand for the new service is equal to $\Lambda\tilde{s}_n$, where $\tilde{s}_n$ is given by:

$$\tilde{s}_n = \frac{\exp(q_n - \tilde{p}_n)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)}.$$

We consider a *profit sharing contract* in which P_1 and P_2 split the net profit generated by the new joint service. More precisely, P_1 receives a fraction $0 < \gamma < 1$ of the profit generated by the new service, whereas P_2 receives $1 - \gamma$. In practice, the new joint service may be offered by (a) the pool of P_1's drivers, (b) the pool of P_2's drivers, or (c) the joint pool of both platforms. Since the third case includes the first two as special cases, we next evaluate the profit of each platform when the new joint service is offered by the joint pool of drivers.

Assume that α (resp. $1-\alpha$) of the new service is completed by P_1's (resp. P_2's) drivers. Hence, $\alpha = 1$ (resp. $\alpha = 0$) refers to the case where P_1's (resp. P_2's) drivers solely provide the new joint service. For example, in the coopetition partnership between Curb and Via, the new taxi-sharing service is solely provided by Curb's drivers. Consequently, the congestion level of P_1 (resp. P_2) is $\tilde{u}_1 = \frac{\Lambda(\tilde{s}_1/n_1 + \alpha\tilde{s}_n/\tilde{n})}{K_1}$ (resp. $\tilde{u}_2 = \frac{\Lambda[\tilde{s}_2/n_2 + (1-\alpha)\tilde{s}_n/\tilde{n}]}{K_2}$). In this case, P_i's drivers will fulfill the requests from P_i's original service as well as (some of) the requests from the new service. The total wage distributed to P_i's drivers to fulfill its original service is equal to $\frac{\Lambda\tilde{s}_i f_i(\tilde{u}_i)}{n_i\tilde{u}_i}$. The total wage distributed to P_i's drivers to fulfill the new joint service is $\frac{\alpha_i\Lambda\tilde{s}_n f_i(\tilde{u}_i)}{\tilde{n}\tilde{u}_i}$, where $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$.

Therefore, the profit earned by P_1 is given by:

$$\tilde{\pi}_1(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2|\gamma) = \Lambda\tilde{s}_1\tilde{p}_1 - \frac{\Lambda\tilde{s}_1 f_1(\tilde{u}_1)}{n_1\tilde{u}_1} + \gamma\left[\Lambda\tilde{s}_n\tilde{p}_n - \frac{\alpha\Lambda\tilde{s}_n f_1(\tilde{u}_1)}{\tilde{n}\tilde{u}_1} - \frac{(1-\alpha)\Lambda\tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2}\right],$$

and the profit earned by P_2 is given by:

$$\tilde{\pi}_2(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2|\gamma) = \Lambda\tilde{s}_2\tilde{p}_2 - \frac{\Lambda\tilde{s}_2 f_2(\tilde{u}_2)}{n_2\tilde{u}_2} + (1-\gamma)\left[\Lambda\tilde{s}_n\tilde{p}_n - \frac{\alpha\Lambda\tilde{s}_n f_1(\tilde{u}_1)}{\tilde{n}\tilde{u}_1} - \frac{(1-\alpha)\Lambda\tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2}\right],$$

where

$$\tilde{s}_i = \frac{\exp(q_i - \tilde{p}_i)}{1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)}, \ (i = 1, 2, n).$$

The total profit earned by both platforms is thus:

$$\tilde{\pi}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \tilde{\pi}_1(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2|\gamma) + \tilde{\pi}_2(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2|\gamma) = \Lambda\tilde{s}_1\tilde{p}_1 + \Lambda\tilde{s}_2\tilde{p}_2 + \Lambda\tilde{s}_n\tilde{p}_n - K_1 f_1(\tilde{u}_1) - K_2 f_2(\tilde{u}_2),$$

which is not explicitly dependent on γ as expected.

4. Impact of Coopetition

In this section, we analyze the impact of the coopetition partnership (i.e., introducing the new joint service). We first consider the profit implications on both platforms, and then examine the impact on riders and on drivers.

4.1. Impact on Platforms' Profits

Following the current business practice, we assume that P_1 and P_2 maintain the prices of their original services after the introduction of the new service, i.e., $\tilde{p}_1 = p_1^*$ and $\tilde{p}_2 = p_2^*$. This practice is mainly driven by branding and marketing considerations as well as market regulations (e.g., the meter price for taxis cannot be altered). In the example of Curb and Via in NYC, the prices for a Via ride and for a Curb ride did not change after the introduction of the new joint service.

First, we study if and when it is beneficial for the platforms to engage in the coopetition partnership. We define $u_1^* := \frac{\Lambda s_1^*}{K_1 n_1}$ (resp. $u_2^* := \frac{\Lambda s_2^*}{K_2 n_2}$) as the congestion level of P_1 (resp. P_2) without coopetition (at the equilibrium). Namely, u_i^* measures how congested P_i is before introducing the new joint service. Recall that when introducing the new joint service, the platforms need to decide the price $\tilde{p}_n$ and the profit sharing parameter γ.

Proposition 2. *There exist four threshold curves $\underline{u}_1(\cdot), \bar{u}_1(\cdot), \underline{u}_2(\cdot)$, and $\bar{u}_2(\cdot)$, where $\underline{u}_1(\cdot)$ and $\bar{u}_1(\cdot)$ are decreasing in u_2^*, $\underline{u}_2(\cdot)$ and $\bar{u}_2(\cdot)$ are decreasing in u_1^*, $\underline{u}_1(\cdot) < \bar{u}_1(\cdot)$ for any u_2^*, and $\underline{u}_2(\cdot) < \bar{u}_2(\cdot)$ for any u_1^*.*

- *If $u_2^* < \underline{u}_2(u_1^*)$ and $u_1^* < \underline{u}_1(u_2^*)$, then $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$ for some $(\tilde{p}_n, \gamma)$ (i.e., both profits increase).*
- *If $\underline{u}_2(u_1^*) \le u_2^* \le \bar{u}_2(u_1^*)$ or $\underline{u}_1(u_2^*) \le u_1^* \le \bar{u}_1(u_2^*)$, then $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_1(p_1^*, p_2^*)$ or $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_2(p_1^*, p_2^*)$ for all $(\tilde{p}_n, \gamma)$ (i.e., at least one of the profits decreases).*
- *If $u_2^* > \bar{u}_2(u_1^*)$ or $u_1^* > \bar{u}_1(u_2^*)$, then $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$ for all $(\tilde{p}_n, \gamma)$ (i.e., both profits decrease).*

Proposition 2 shows that the congestion levels of both platforms have crucial implications on how the coopetition affects the profits. More specifically, if the congestion levels are not too high (i.e., $u_1^* < \underline{u}_1(u_2^*)$ and $u_2^* < \underline{u}_2(u_1^*)$), the platforms can design a profit sharing contract (by setting the price of the new service $\tilde{p}_n$ and the profit sharing parameter γ) that will make the coopetition partnership beneficial for both platforms (i.e., a Pareto improvement in both profits). If the congestion level of at least one platform is moderate (i.e., $\underline{u}_1(u_2^*) \leq u_1^* \leq \bar{u}_1(u_2^*)$ or $\underline{u}_2(u_1^*) \leq u_2^* \leq \bar{u}_2(u_1^*)$), at least one of the platforms will be hurt by the coopetition. In this case, introducing the new service will make (at least) one of the platforms over-congested thus reducing the profit of at least one platform. Depending on the value of γ, either platform can earn a lower profit. If the congestion level of at least one platform is high so that the original system is already over-congested (i.e., $u_1^* > \bar{u}_1(u_2^*)$ or $u_2^* > \bar{u}_2(u_1^*)$), the profits of both platforms will decrease in the presence of coopetition. Surprisingly, even if only one platform is over-congested (say P_1, i.e., $u_1^* > \bar{u}_1(u_2^*)$), the profit of the other platform will also be reduced by the coopetition. This follows from the two following effects. First, the cannibalization effect induced by the new service will decrease the profit earned from the original service of P_2. Second, the coopetition will potentially make P_1 so congested that the profit from the new service would be negative. Independent of which pool of drivers will fulfill the new service, these two effects together will reduce the profit earned by P_2 even if its own congestion level was low prior to the coopetition. When both platforms have low congestion levels, introducing the new joint service expands the market share of both platforms, hence increasing the revenues without imposing high additional wages. When congestion levels are higher, however, the coopetition will worsen the congestion of the platform(s) whose drivers fulfill the new joint service, thus reducing the profits of both platforms.

We next refine our findings by considering specific regimes of the parameters α and $\tilde{n}$. We define a threshold on the number of riders per trip for the new joint service (i.e., $\tilde{n}$): $\bar{n} := \left(\frac{s_1^*}{n_1} + \frac{s_2^*}{n_2}\right)^{-1}$. We also define two thresholds on the driver allocation policy (i.e., α): $\underline{\alpha} := \frac{s_1^* \tilde{n}}{n_1}$ and $\bar{\alpha} := 1 - \frac{s_2^* \tilde{n}}{n_2}$. The following result is an immediate corollary of Proposition 2.

COROLLARY 1. *The following statements hold:*

- *If $\alpha \leq \underline{\alpha}$ (resp. $\alpha \geq \bar{\alpha}$), the impact of the coopetition on the platforms does not depend on u_1^* (resp. u_2^*).*
- *Consider a setting with $\tilde{n} \geq \bar{n}$ (this implies that $\bar{\alpha} < \underline{\alpha}$). For any u_1^* and u_2^*, there exists a range of values $\alpha \in [\bar{\alpha}, \underline{\alpha}]$ such that $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^* | \gamma) \geq \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^* | \gamma) \geq \pi_2(p_1^*, p_2^*)$ for some $(\tilde{p}_n, \gamma)$.*

Corollary 1 suggests that the value of α (i.e., the way in which the new service is allocated to the drivers of both platforms) plays an essential role in determining the impact of coopetition on both platforms. More specifically, if most of the requests for the new service are allocated to one platform (i.e., α is close to 0 or close to 1), the impact of coopetition on the profits of both platforms only depends on the congestion level of the platform who fulfills the new service. In this case, the cannibalization effect of the new service will decrease the congestion level of the other platform. An additional implication of Corollary 1 is the following: If the new joint service has a significant carpooling effect (i.e., $\tilde{n} \geq \bar{n}$), engaging in the coopetition will always alleviate the congestion of both platforms as long as the drivers of both platforms fulfill a significant portion of the requests for the new service. Consequently, the coopetition can increase the profits of P_1 and P_2 regardless of the original congestion levels u_1^* and u_2^*.

In practice, it is often possible that one platform (say P_1) has a low congestion level, whereas the other platform (P_2) has a high congestion level. Then, by offering the new joint service using P_1's drivers (i.e., $\alpha = 1$), the profits of both platforms will improve in the presence of coopetition, as long as the congestion level of P_1, u_1^*, is not too high ($u_1^* < \underline{u}_1(u_2^*)$). Note that such a scenario has occurred in the coopetition between Via and Curb, in which Curb has access to a large number of (taxi) drivers, whereas Via may be in shortage of drivers and often over-congested. Consequently, the new taxi-sharing service was exclusively provided by Curb's drivers in this case. In other examples, such as the partnership between Uber and PT Express Transindo Utama Tbk, both pools of drivers are participating in the new service.

For the rest of this paper, we restrict our attention to the case in which $u_1^* < \underline{u}_1(u_2^*)$ and $u_2^* < \underline{u}_2(u_1^*)$, i.e., the driver capacity congestion levels are not too high. This case is the most relevant in the context of our paper as the coopetition partnership can potentially enhance the profits of both platforms. In practice, if the platforms are already over-congested, they will probably not be interested in a coopetition partnership that gives rise to even more demand (unless, they recruit additional drivers).

The rest of this paper aims to study the impact of such a coopetition partnership. In particular, we are interested in designing the right profit sharing contract and in quantifying its implications on both platforms as well as on riders and drivers. At a high level, the coopetition will induce two effects: (i) a new market share effect (i.e., capturing new riders who were previously choosing the outside option) and (ii) a cannibalization effect (i.e., losing some existing market share to the new service). In Section 4.1.4, we draw practical insights on the interplay of these two effects, and show that a well-designed profit sharing contract will lead to an overall positive benefit for both platforms.

To conclude this subsection, we remark that if the two platforms decide the price of the new joint service $\tilde{p}_n$ and the profit sharing parameter γ haphazardly, it may decrease the profits of both platforms (even if $u_1^* < \underline{u}_1(u_2^*)$ and $u_2^* < \underline{u}_2(u_1^*)$). For example, consider a setting with $q_1 = 2$, $q_2 = 1$, $G_1(\cdot)$ is concentrated at $r_1 = 1.5$, $\Lambda = 1,000$, $K_1 = +\infty$, $K_2 = 500$, $n_2 = 3$, and assume that r_2 follows a uniform distribution on $[0, 1]$. This setting may illustrate the coopetition between Curb and Via (P_1 is Curb whereas P_2 is Via). Then, by solving for the equilibrium outcome, we obtain: $p_1^* = 2.79$

and $p_2^* = 1.55$, with expected profits of $\pi_1(p_1^*, p_2^*) = 289$ and $\pi_2(p_1^*, p_2^*) = 422$. Consider that the new joint service satisfies $q_n = 1.5$ and $\tilde{n} = 1.5$ (i.e., an average of 1.5 passengers per ride). If we set $\tilde{p}_n = 1.6$, $\gamma = 0.6$, and $\alpha = 1$, we obtain: $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = 273.6 < \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = 406.9 < \pi_2(p_1^*, p_2^*)$. As a result, when the coopetition terms are not carefully designed, introducing the new service may lead to an undesirable lose-lose outcome for both platforms.

4.1.1. Joint Pricing Decision In this scenario, P_1 and P_2 jointly decide $\tilde{p}_n$ so as to maximize their total profits $\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$. We denote the optimal price by $\tilde{p}_n^* \in \arg\max_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$, which is the first-best pricing decision under coopetition.

Proposition 3. *When the price $\tilde{p}_n$ is jointly decided by both platforms so as to maximize the total profits, the following hold:*

- *There exists a unique price $\tilde{p}_n^*$ that maximizes $\tilde{\pi}(\cdot, p_1^*, p_2^*)$.*
- *There exists an interval $(\underline{\gamma}, \bar{\gamma}) \subset (0,1)$ such that, if and only if $\gamma \in (\underline{\gamma}, \bar{\gamma})$, then $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$.*
- *If $\gamma < \underline{\gamma}$, $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and if $\gamma > \bar{\gamma}$, $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.*

Note that as expected, $\tilde{p}_n^*$ does not depend on the value of γ. Proposition 3 conveys that if the price of the new service is jointly decided by P_1 and P_2 to maximize their total profits, a well-designed profit sharing contract (i.e., $\gamma \in (\underline{\gamma}, \bar{\gamma})$) will benefit both platforms. As discussed above, when the terms of the coopetition (i.e., $\tilde{p}_n$ and γ) are not carefully designed, introducing the new joint service can yield lower profits for both platforms. However, as we show in Proposition 3, if both platforms jointly decide the price of the new service and the profit split is not too extreme (i.e., $\gamma \in (\underline{\gamma}, \bar{\gamma})$), introducing the new service will lead to a win-win outcome for both platforms.

As mentioned, engaging in the coopetition under a profit sharing contract induces two conflicting effects: (i) a new market share that will be shared between the two platforms according to the profit sharing contract and (ii) an adverse cannibalization effect. Proposition 3 shows that under a well-designed profit sharing contract, the new market share effect dominates the cannibalization effect for each platform. We will discuss in greater detail this trade-off and the implications of these two effects in Section 4.1.4.

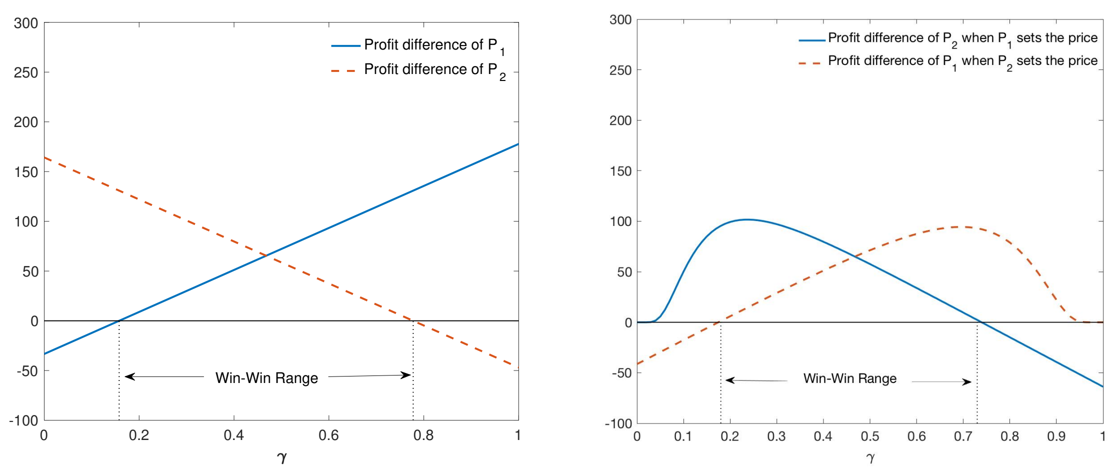
4.1.2. Price Set by One Platform In this scenario, Platform P_i ($i = 1$ or 2) sets the price of the new joint service $\tilde{p}_n$ so as to maximize its own profit $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$. We denote P_i's optimal price by $\tilde{p}_n^i(\gamma) \in \arg\max_{\tilde{p}_n} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$. It is clear that the resulting equilibrium price of the new joint service $\tilde{p}_n^i(\gamma)$ depends on which platform sets the price and on γ.

Proposition 4. *When $\tilde{p}_n$ is set by P_i ($i = 1, 2$) to maximize $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$, the following hold:*

- *For any γ, there exists a unique $\tilde{p}_n^i(\gamma)$ that maximizes $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$.*

- *If $i=1$ (resp. $i=2$), $\tilde{p}_n^i(\cdot)$ is decreasing (resp. increasing) in γ.*
- *There exists an interval $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$ such that for any $\gamma \in (\underline{\gamma}', \bar{\gamma}')$, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$, and $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$.*
- *If $\gamma < \underline{\gamma}'$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and if $\gamma > \bar{\gamma}'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.*

For any value of γ, regardless of which platform sets the price of the new service, there exists a unique optimal price that maximizes the price-setter's profit. In particular, if γ increases, the new market share effect for P_1 (resp. P_2) is strengthened (resp. weakened), which drives this platform to decrease (resp. increase) the price of the new service. An important implication of Proposition 4 is that, irrespective of who (P_1, P_2 or jointly) decides the price of the new joint service, a well-designed profit sharing contract (i.e., $\gamma \in (\underline{\gamma}', \bar{\gamma}')$) will yield higher profits for both platforms. We also find that $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$. Consequently, the range of profit sharing contracts that achieve a win-win outcome is more restricted when one platform sets the price of the new service relative to the case where both platforms jointly decide $\tilde{p}_n$. This bears a similar intuition as the fact that a decentralized decision making process generally induces a suboptimal outcome and introduces some market inefficiency. As illustrated in Figures 3a and 3b,[8] the range of profit sharing contracts that achieve a win-win outcome for both platforms can be substantial.



(a) Both platforms jointly decide $\tilde{p}_n$ (b) One of the platforms sets $\tilde{p}_n$

Figure 3 **Range of γ values that lead to a win-win outcome (Parameters:** $q_1 = 2$, $q_2 = 1$, $n_1 = 1$, $n_2 = 3$, $K_1 = +\infty$, $K_2 = 500$, $\Lambda = 1{,}000$, $q_n = 1.5$, $\tilde{n} = 1.5$, $r_1 = 1.5$, $r_2 \sim U[0,1]$**)**.

[8] In this example, we chose specific parameters to illustrate the range of γ values that lead to a win-win outcome. In our tests, this insight seems to be robust when varying the values of the different parameters.

So far, we considered two different scenarios depending on how $\tilde{p}_n$ is decided. A natural question is to inquire if one can attain the first-best outcome (i.e., the outcome under the joint decision) when a single platform sets $\tilde{p}_n$. The following proposition shows that this is indeed possible.

Proposition 5. *There exists a unique* $\gamma^* \in (\underline{\gamma}', \bar{\gamma}')$ *such that* $\tilde{p}_n^1(\gamma^*) = \tilde{p}_n^2(\gamma^*) = \tilde{p}_n^*$. *Moreover,* $\tilde{\pi}_1(\tilde{p}_n^*, p_1^*, p_2^* | \gamma^*) > \pi_1(p_1^*, p_2^*)$ *and* $\tilde{\pi}_2(\tilde{p}_n^*, p_1^*, p_2^* | \gamma^*) > \pi_2(p_1^*, p_2^*)$.

Proposition 5 suggests that the (system-wide) first-best price $\tilde{p}_n^*$ can still be achieved when one platform sets $\tilde{p}_n$, as long as the profit sharing contract is specified to be the (unique) parameter γ^*. In other words, a profit sharing contract with the parameter γ^* will induce the first-best market outcome regardless of which platform sets $\tilde{p}_n$. Under this profit sharing contract, both platforms face the same new market share/cannibalization tradeoff as they would have faced when $\tilde{p}_n$ is jointly decided. In addition, we have shown that this profit sharing contract will lead to higher profits for both platforms. As we illustrate in Figure 4, the profit sharing contract that leads to the first-best outcome (i.e., γ^*) is the unique solution of $\tilde{p}_n^1(\gamma) = \tilde{p}_n^2(\gamma)$.

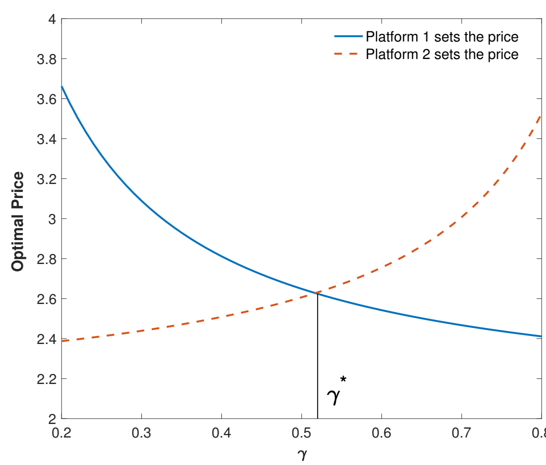


Figure 4 Optimal price as a function of γ when one platform decides $\tilde{p}_n$.

Interestingly, the fact that γ^* coordinates both platforms and retrieves the market efficiency is analogous to a classical result in supply chain coordination (see, e.g., Cachon 2003), even though the settings are very different. In both settings, the contract induces the first-best outcome by aligning the incentives of the decision makers to that of the entire system. In our model, this is achieved by adjusting the new market share/cannibalization tradeoff, whereas in the supply chain literature, the coordinating contract aims to share the demand uncertainty risk between the supplier and the retailer.

4.1.3. Stackelberg Setting So far, we considered the case where the contract parameter γ was exogenous. In practice, when introducing the new joint service, the two platforms may have the flexibility to set both $\tilde{p}_n$ and γ. Next, we explore the Stackelberg setting under which one platform strategically decides the profit sharing parameter γ, and the other platform sets $\tilde{p}_n$ after observing the value of γ. In particular, we consider both cases where either P_1 or P_2 is the Stackelberg leader to decide γ (while the other platform follows by setting $\tilde{p}_n$). We denote the Stackelberg game when P_1 (resp. P_2) is the leader by $\mathcal{G}_1$ (resp. $\mathcal{G}_2$). In $\mathcal{G}_1$ (resp. $\mathcal{G}_2$), it is clear that the subgame perfect equilibrium is such that for a given γ, P_2 (resp. P_1) selects $\tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n^1(\gamma)$), while P_1 (resp. P_2) sets $\gamma^1 \in \arg\max_{\gamma\in(0,1)} \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma)$ (resp. $\gamma^2 \in \arg\max_{\gamma\in(0,1)} \tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$).

Proposition 6. *In the Stackelberg setting, the following results hold:*

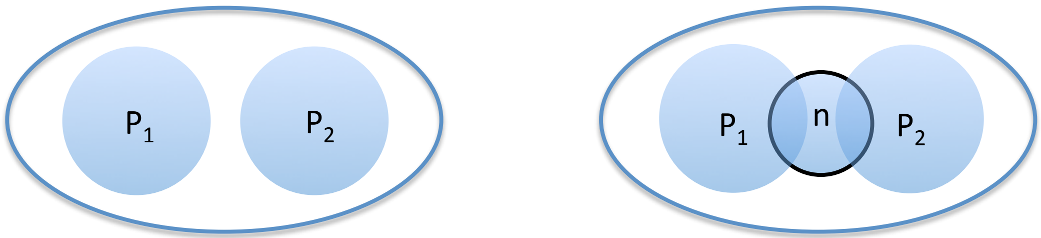
(a) In $\mathcal{G}_1$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) > \pi_2(p_1^*, p_2^*)$. Moreover, $\gamma^1 > \gamma^*$ and $\tilde{p}_n^2(\gamma^1) > \tilde{p}_n^*$.

(b) In $\mathcal{G}_2$, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma^2), p_1^*, p_2^*|\gamma^2) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n^1(\gamma^2), p_1^*, p_2^*|\gamma^2) > \pi_2(p_1^*, p_2^*)$. Moreover, $\gamma^2 < \gamma^*$ and $\tilde{p}_n^1(\gamma^2) > \tilde{p}_n^*$.

Proposition 6 shows that in the Stackelberg setting, regardless of which platform is the game leader, both platforms will earn higher profits after introducing the new joint service. In addition, the price of the new joint service will be higher in the Stackelberg setting when compared to the case where $\tilde{p}_n$ is jointly decided.

4.1.4. Insights So far, we considered various decision making dynamics. In practice, the decision making process may depend on the specific context and often involves extensive negotiations between the two platforms. Interestingly, the results presented in this paper show that regardless of which platform is in charge of setting the price of the new service, a carefully designed profit sharing contract will increase the profits earned by both platforms. Furthermore, such a win-win outcome can also be achieved in a Stackelberg setting. Interestingly, this result admits a similar win-win outcome as in the supply-chain contracts literature.

As mentioned before, the win-win outcome was not obvious at first sight, as introducing the new service induces an adverse cannibalization effect (and potentially an increased congestion level). More precisely, the new joint service will take away some customers from the original services. On the other hand, the new service allows to increase the market share by attracting new customers who were choosing the outside option before the introduction of the new service. Figure 5 illustrates these two conflicting effects. Before the coopetition partnership, each platform captures its own profit share from the total pool of potential riders (Figure 5a). After the coopetition partnership, the new service captures a profit share denoted by n, while each platform's profit share decreases. More precisely, as we can see from Figure 5b, the profit share of the new service is composed of two parts: (i) a new market share (this represents the customers who were choosing the outside option, and now select the new service) and (ii) an overlap with the initial profit shares of both platforms (this represents the cannibalization effect of customers who switch from one of the platforms to the new service). As we have shown in this section, properly balancing these two conflicting effects through a profit sharing contract can yield a Pareto profit improvement for both platforms.



(a) Before introducing the new joint service (b) After introducing the new joint service

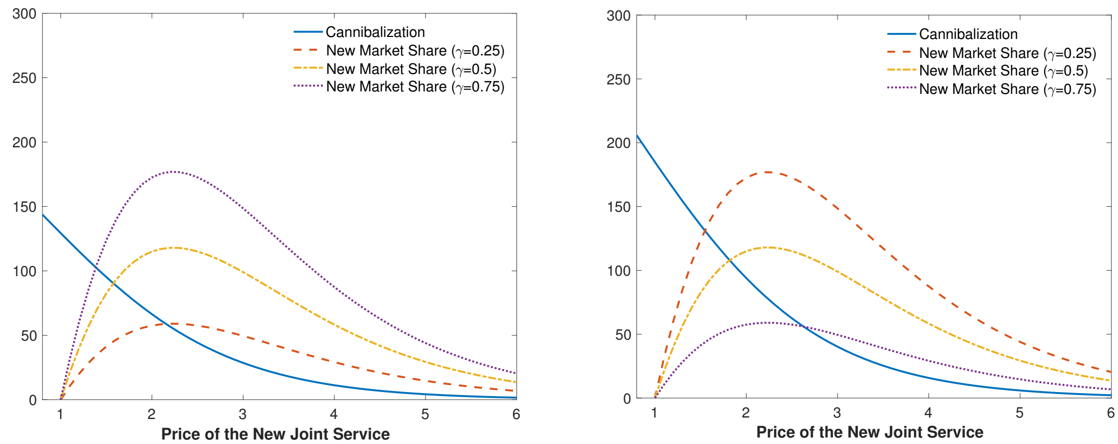
Figure 5 Illustration of the new market share and profit cannibalization effects.

We next illustrate the new market share, cannibalization, and congestion effects from a mathematical perspective. Define the profit sharing parameter $\gamma_1 = \gamma$ and $\gamma_2 = 1-\gamma$. We can write the profit difference of P_i as follows:

$$\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma) - \pi_i(p_1^*, p_2^*) = \underbrace{\gamma_i \Lambda \tilde{p}_n \tilde{s}_n}_{\text{New Market Share}} - \underbrace{\Lambda p_i^*(s_i^* - \tilde{s}_i)}_{\text{Profit Cannibalization}} \\ \underbrace{- \left[\frac{\Lambda \tilde{s}_i f_i(\tilde{u}_i)}{n_i \tilde{u}_i} + \gamma_i \left(\frac{\alpha \Lambda \tilde{s}_n f_1(\tilde{u}_1)}{\tilde{n}\tilde{u}_1} + \frac{(1-\alpha)\Lambda \tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2} \right) - K_i f_i(u_i^*) \right]}_{\text{Change in Congestion Level}}.$$

The first term corresponds to the profit increase of P_i from the additional market share captured via the new joint service. The second term is negative and represents the profit cannibalization effect from the introduction of the new joint service. Indeed, the market share of P_i strictly decreases (i.e., $\tilde{s}_i < s_i^*$), and hence the profit is also reduced. The third term represents the profit impact from the change in congestion level. This term quantifies the additional wages that P_i needs to pay in the presence of coopetition. In this section, we have shown that, as long as the original congestion levels of both platforms are not too high (i.e., $u_1^* < \underline{u}_1(u_2^*)$ and $u_2^* < \underline{u}_2(u_1^*)$), under a well-designed profit sharing contract, the first term typically dominates the last two. Consequently, the overall profits earned by both platforms will increase in the presence of coopetition.

Figure 6 illustrates the magnitudes of the new market share and profit cannibalization effects for both platforms under different values of γ and $\tilde{p}_n$. We make the following observations. First, one can see that depending on the price of the new joint service, either of the two effects can dominate. This conveys the importance of carefully designing the profit sharing contract and the associated price of the new service, as otherwise it may lead to an undesirable lose-lose outcome.



(a) Platform 1 (b) Platform 2

Figure 6 Magnitudes of the new market share and profit cannibalization effects (Parameters: $q_1 = 2$, $q_2 = 1$, $n_1 = 1$, $n_2 = 3$, $K_1 = +\infty$, $K_2 = 500$, $\Lambda = 1{,}000$, $q_n = 1.5$, $\tilde{n} = 1.5$, $\alpha = 1$, $r_1 = 1.5$, $r_2 \sim U[0,1]$**).**

Second, as long as the price of the new joint service is large enough, the new market share effect will dominate the profit cannibalization effect for both platforms. Finally, the cannibalization effect does not depend on how the profit from the new service is split, whereas the new market share effect gets strengthened for P_1 (resp. P_2) if γ increases (resp. decreases).

An additional perspective on the benefit of the coopetition partnership is related to the following tradeoff. Consider a situation where both platforms have an asymmetry in their market share and supply. Then, introducing the new joint service may help both platforms by taking advantage of these asymmetries in market share and supply. When designed properly, the coopetition partnership will improve the matching of supply with demand, and hence will be profitable for both platforms.

Finally, as mentioned at the beginning of Section 4.1, when at least one platform has a high initial congestion level, engaging in the coopetition is not beneficial due to the elevated congestion level caused by the new joint service.

4.2. Which Driver Pool Should Provide the New Service

As mentioned, the new service can be fulfilled by either pool of drivers (or by both). In this section, we investigate which driver pool should provide the new joint service, assuming that the platforms engage in the coopetition.

Proposition 7. *Assume that P_1 and P_2 engage in the coopetition and that $\tilde{p}_n$ is given. Then, there exist two thresholds on the congestion level ratio $\underline{\theta}$ and $\bar{\theta}$ such that:*

- *If $u_1^*/u_2^* \leq \underline{\theta}$, both platforms will earn higher profits when the new joint service is fulfilled by P_1's drivers (i.e., $\alpha = 1$);*
- *If $\underline{\theta} < u_1^*/u_2^* < \bar{\theta}$, both platforms will earn higher profits when the new joint service is fulfilled by the drivers of both platforms (i.e., $0 < \alpha < 1$);*

- *If $u_1^*/u_2^* \geq \bar{\theta}$, both platforms will earn higher profits when the new joint service is fulfilled by P_2's drivers (i.e., $\alpha = 0$).*

Proposition 7 shows that, if the congestion levels of both platforms are highly asymmetric (i.e., $u_1^*/u_2^* \leq \underline{\theta}$ or $u_1^*/u_2^* \geq \bar{\theta}$), it is best for both platforms to have the drivers of the platform with low congestion level to offer the new service (i.e., $\alpha = 0$ or $\alpha = 1$). On the other hand, if the driver capacities of both platforms are close together (i.e., $\underline{\theta} \leq u_1^*/u_2^* \leq \bar{\theta}$), both platforms could earn a higher profit when the new service is jointly offered (i.e., $0 < \alpha < 1$). Interestingly, a well-designed profit sharing contract allows to align the incentives of both platforms so that they strive to reach a good market outcome for both parties. In this case, if P_1 is more congested, it will be better (for both platforms) if the new joint service is fulfilled by P_2's drivers.

Proposition 7 also implies that when one platform is congested while the other has some available driver capacity, the new service should be offered by the drivers of the latter platform. Interestingly, this insight seems consistent with the practice of several ride-hailing platforms. In the Via and Curb example, the new service is offered by Curb drivers (Via's drivers are highly utilized as Via uses mechanisms to regulate the number of drivers, whereas Curb has typically a larger number of available drivers). On the other hand, in other partnerships such as the partnership between Uber and PT Express Transindo Utama Tbk, both pools of drivers are participating in the new service.

To conclude this section, we show that the driver allocation parameter α that maximizes the total profit of both platforms can be obtained by solving a one-dimensional convex program. The following result follows immediately from Proposition 7.

Corollary 2. *Assume that P_1 and P_2 engage in the coopetition and that $\tilde{p}_n$ is given. Let α^* be the driver allocation parameter that maximizes the total profit $\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$. Then, we have:*

$$\alpha^* = \underset{\alpha \in [0,1]}{\arg\min} \left\{ K_1 f_1\left(u_1^*(1-\tilde{s}_n) + \frac{\alpha \Lambda \tilde{s}_n}{\tilde{n} K_1} \right) + K_2 f_2 \left(u_2^*(1-\tilde{s}_n) + \frac{(1-\alpha)\Lambda \tilde{s}_n}{\tilde{n} K_2} \right) \right\}.$$

Note that this is a convex program in α, as the functions $f_1(\cdot)$ and $f_2(\cdot)$ are assumed to be convex and their arguments are linear in α.

4.3. Surpluses of Riders and Drivers

So far, we focused on the impact of coopetition on the profits earned by the platforms. In this section, we investigate the impact of coopetition on the surpluses of riders and drivers. It is worth noting that the surpluses of riders and drivers are not (explicitly) dependent on the profit sharing parameter γ. We use $RS(p_1, p_2)$ to denote the total rider surplus of the benchmark setting (i.e., without coopetition), when P_1 (resp. P_2) sets p_1 (resp. p_2) for its original service. We have:

$$RS(p_1, p_2) = \Lambda \mathbb{E}\Big[\max\{q_1 - p_1 + \xi_1, q_2 - p_2 + \xi_2, \xi_0\} \Big].$$

Let $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2)$ denote the total expected rider surplus after introducing the new joint service:

$$\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \Lambda \mathbb{E}\Big[\max\{q_1 - \tilde{p}_1 + \xi_1, q_2 - \tilde{p}_2 + \xi_2, q_n - \tilde{p}_n + \xi_n, \xi_0\}\Big].$$

We remark that the rider surpluses $RS(\cdot,\cdot)$ and $\tilde{RS}(\cdot,\cdot)$ are unique up to an additive constant. Indeed, for any rider, if the random utility terms $(\xi_1, \xi_2, \xi_n, \xi_0)$ are shifted to $(\xi_1 + c, \xi_2 + c, \xi_n + c, \xi_0 + c)$ for any constant c, then the probabilities that this rider will choose any of the four options (P_1, P_2, the new service, and the outside option) remain the same. Nevertheless, the change in the expected rider surplus generated by introducing the new joint service, $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) - RS(p_1, p_2)$, is independent of the constant c. More specifically, one can derive the following expressions: $RS(p_1, p_2) = \log[1 + \exp(q_1 - p_1) + \exp(q_2 - p_2)] + c$ and $\tilde{RS}(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = \log[1 + \exp(q_1 - \tilde{p}_1) + \exp(q_2 - \tilde{p}_2) + \exp(q_n - \tilde{p}_n)] + c$. For more details on the consumer surplus under the MNL model and on the derivation of the above expressions, we refer the reader to the literature on discrete choice models (see, e.g., Chapter 3.5 of Train 2009).

Proposition 8. *For any price $\tilde{p}_n$, $\tilde{RS}(\tilde{p}_n, p_1^*, p_2^*) > RS(p_1^*, p_2^*)$.*

Proposition 8 shows that introducing the new joint service will improve the expected rider surplus, regardless of the specifics of the profit sharing contract and of the new service price. This result is expected as riders can now enjoy an additional alternative for service.

The impact of coopetition on the drivers appears to be more subtle. Let $DS_i(p_1, p_2)$ denote the total expected surplus of P_i's drivers before the coopetition partnership, given (p_1, p_2). Recall that P_i's drivers have a reservation wage r_i distributed with $G_i(\cdot)$ and will decide to work for P_i if and only if the offered wage w_i exceeds r_i. As a result, we have: $DS_i(p_1, p_2) = K_i \mathbb{E}\big[\max\{r_i, w_i\}\big] = K_i \mathbb{E}\Big[\max\Big\{r_i, G_i^{-1}\Big(\frac{\Lambda s_i}{n_i K_i}\Big)\Big\}\Big]$. Analogously, the total expected surplus of P_i's drivers after introducing the new joint service is given by: $\tilde{DS}_i(\tilde{p}_n, \tilde{p}_1, \tilde{p}_2) = K_i \mathbb{E}\big[\max\{r_i, \tilde{w}_i\}\big] = K_i \mathbb{E}\Big[\max\Big\{r_i, G_i^{-1}\Big(\frac{\Lambda \tilde{s}_i / n_i + \alpha_i \Lambda \tilde{s}_n / \tilde{n}}{K_i}\Big)\Big\}\Big]$, where $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$.

Proposition 9. *For any $\tilde{p}_n$, we have:*

- *If $\tilde{n} \leq \bar{n}$, $\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) \geq DS_1(p_1^*, p_2^*)$ and $\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*) \geq DS_2(p_1^*, p_2^*)$ for $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha}$ and $\bar{\alpha}$ are defined in Corollary 1.*
- *If $\tilde{n} > \bar{n}$, for any α, at least one of the following holds: $\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*) < DS_1(p_1^*, p_2^*)$ or $\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*) < DS_2(p_1^*, p_2^*)$.*

As shown in Proposition 9, the drivers of both platforms may not necessarily benefit from the coopetition. When the average number of riders per trip for the new service is small (i.e., $\tilde{n} \leq \bar{n}$), there exists a range of driver allocation parameters (i.e., $\alpha \in [\underline{\alpha}, \bar{\alpha}]$) for which both pools of drivers are better off under coopetition. Indeed, when $\tilde{n}$ is small, the platforms need to increase the wages to attract additional drivers to fulfill the demand from the new joint service. When $\tilde{n}$ is large, however, at least one pool of drivers will be worse off in the presence of coopetition. In this case, fewer drivers are needed, so the platforms will reduce the wages. This finding explains partially why several coopetition partnerships either have no ride-sharing option for the new service (i.e., $\tilde{n}=1$), or impose a restriction on the number of riders per trip. For example, in the case of Curb and Via, the platforms imposed a limit of at most 2 riders who can share a ride for the new taxi-sharing service (i.e., $\tilde{n} \leq 2$). Finally, it is worth noting that, if the proportion of rides from the new service allocated to the drivers of one platform is small enough (i.e., α is close to 0 or close to 1), the drivers of this platform will be hurt by the coopetition. In this case, the new joint service cannibalizes the original service of this platform without compensating its drivers.

We next propose a simple and realistic way to address the issue that some drivers may be hurt by the coopetition. In particular, P_1 and P_2 can reallocate some of their profit gains to their drivers through promotions/bonuses or other monetary compensations. More precisely, instead of maximizing its own profit, P_i $(i=1,2)$ can consider maximizing its total surplus (i.e., the sum of its own profit *and* its drivers' surplus). We next show that when each platform modifies its objective function to account for its drivers' surplus, one can find a profit sharing contract that will guarantee a surplus gain for both platforms together with their drivers.

Proposition 10. *There exist a pair* $(\tilde{p}_n, \gamma)$ *such that* $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^* | \gamma) + \tilde{DS}_i(\tilde{p}_n, p_1^*, p_2^*) > \pi_i(p_1^*, p_2^*) + DS_i(p_1^*, p_2^*)$ *for* $i=1,2$ *and any* $\alpha \in [0,1]$.

Proposition 10 shows that one can design a profit sharing contract that will benefit all the players (i.e., both platforms, both pools of drivers, and riders). Furthermore, this holds for any allocation of drivers to the new service. The results presented in this section have some important practical implications for ride-sharing platforms who are considering coopetition partnerships. First, the platforms need to make sure that they are not overly congested, and if they are, they should either recruit new drivers or avoid engaging in a coopetition partnership. Assuming that congestion levels are not too high, one can design a coopetition partnership in which all the players are better off. To this end, we have two possible strategies: (1) Allocating enough drivers from each platform to serve the new joint service and restricting the number of passengers per ride, or (2) Redistributing some of the profit gains to the drivers by offering bonuses. Such strategies will ensure that all the players benefit from the coopetition.

5. Computational Experiments

In this section, we investigate computationally how three market features affect the impact of coopetition: (a) Product differentiation, measured by q_1/q_2, (b) Demand-supply ratio of P_2, measured by $\Lambda/(n_2K_2)$, and (c) The expected number of riders per trip in the new joint service, $\tilde{n}$.

To this end, we set $q_2 = 1$ and vary q_1 so that $q_1/q_2 \in \{1.1, 1.4, 1.8, 2.2, 2.6, 3\}$. To illustrate the coopetition partnership between Curb (P_1) and Via (P_2), we assume that $K_1 = +\infty$, the distribution $G_1(\cdot)$ is concentrated at $r_1 = 1$, and the new joint service is fulfilled exclusively by P_1's drivers (i.e., $\alpha = 1$). For the original taxi-hailing service of Curb, the average riders per trip is $n_1 = 1$. We fix $n_2 = 3$, $k_2 = 500$, assume that r_2 is uniformly distributed on $[0, 1]$, and vary Λ so that $\Lambda/(n_2 k_2) \in \{0.5, 1, 1.5, 2, 5, 7\}$. Finally, we consider several values of $\tilde{n} \in \{1, 1.3, 1.7, 2, 2.5, 3\}$. Note that $\tilde{n} = 1$ is the extreme case in which there is no carpooling in the new joint service. Recall that the partnership between Curb and Via in NYC is such that $\tilde{n} \leq 2$. However, we still consider the case where $\tilde{n}$ can be larger than 2 in order to test the robustness of our results. Note that the set of parameters we are using in this section encompasses a wide range of realistic instances and hence, this allows us to quantify the practical impact of the coopetition partnership.

It is natural to assume that the quality of the new joint service q_n increases with q_1 and decreases with $\tilde{n}$. To capture this behavior, we use $q_n = q_2 + (q_1 - q_2)(n_2 + 1 - \tilde{n})/n_2$. Note that $q_n = q_1$ when $\tilde{n} = 1$ (in this case, the new service is equivalent to P_1's original service) and q_n is slightly larger than q_2 when $\tilde{n} = n_2$ (in this case, the new service is slightly better than P_2's original service). For all problem instances, we decided to use a profit sharing contract with the profit sharing parameter γ^*. This seems to be a desirable contract in practice, as it properly balances the incentives of both platforms (see the discussion after Proposition 5). Under the contract γ^*, the the new service price (regardless of which platform sets it) is equal to $\tilde{p}_n^*$. In addition, we note that all the computational results presented in this section still qualitatively hold when using a different profit sharing contract (e.g., the Stackelberg setting studied in Section 4.1.3).

Table 1 summarizes the impact of the coopetition partnership on P_1, P_2, drivers, and riders for the problem instances discussed above. We compute the relative impact of introducing the new joint service for each party. For example, the relative profit difference of P_i $(i = 1, 2)$ is given by: $\Delta\pi_i/\pi_i = [\tilde{\pi}_i(\tilde{p}_n^*, p_1^*, p_2^* | \gamma^*) - \pi_i(p_1^*, p_2^*)]/\pi_i(p_1^*, p_2^*)$.[10] Our computational tests convey that for the parameter values we consider, introducing the new joint service will in general substantially benefit all the stakeholders in the market. In particular, we can see from Table 1 that the average relative profit improvements for P_1 and P_2 are 25.38% and 23.45% respectively (and even in the worst case instances under consideration, the relative improvements amount to 13.37% and 13.13%). In addition, the average benefits of the drivers and the riders seem also to be significant. The only exception is a slight decrease in the expected surplus of P_1's drivers when $\tilde{n} > 2$ (i.e., every trip is shared by more than 2 riders on average) and q_1/q_2 is large. In this case, one can see from Table 1 that the surplus of P_1's drivers can be reduced by 3.78% in the worst case (this occurs for

[10] Since the rider surplus is unique up to an additive constant (see Chapter 3.5 of Train 2009), we report here the absolute (instead of the relative) differences in the expected rider surplus. The same comment applies to Tables 2-4.

the instance with $q_1/q_2 = 3$ and $\tilde{n} = 3$). This is consistent with Proposition 9, which shows that if $\tilde{n}$ is large, at least one platform's drivers will not necessarily benefit from the introduction of the new joint service. However, in such cases, P_1 can still redistribute its profit gain to its drivers so that the coopetition will benefit the platform and its drivers together (see the discussion after Proposition 10). We also note that even though we use a profit sharing contract with γ^* and $\tilde{p}_n^*$ (designed to maximize the platforms' profits), the total surplus of P_2 and its drivers still increases. Consequently, P_2 can also decide to redistribute some of its profit gain to its drivers.

Table 1 Summary statistics of the impact of introducing the new joint service (%).

	Average	Min	25th Percentile	Median	75th Percentile	Max
$\Delta\pi_1/\pi_1$	25.38	13.37	22.76	25.00	27.75	42.17
$\Delta\pi_2/\pi_2$	23.45	13.13	21.32	23.82	25.45	35.99
$\Delta DS_1/DS_1$	17.22	-3.78	5.94	14.85	27.36	48.02
$\Delta(\pi_2+DS_2)/(\pi_2+DS_2)$	20.30	9.38	18.37	20.96	22.69	27.93
ΔRS	1429.35	143.49	440.28	736.67	2627.44	4669.56

In Tables 2, 3, and 4, we report the average values of the relative impact when a single parameter is varied and the other two are set to specific values. This allows us to isolate the impact of a single market feature. One can see that in all cases, all the surpluses are increasing, suggesting that everyone benefits from the introduction of the new joint service. In Table 2, we study the effect of quality differentiation. We observe that as q_1/q_2 increases, the impact on the profits earned by both platforms is quite stable (the relative improvement remains around 20-30%). On the other hand, increasing the quality ratio will hurt P_1's drivers which will benefit less from the coopetition. In Table 3, we study the effect of the demand-supply ratio. When increasing $\Lambda/(n_2k_2)$, we can see that the impact on the profits of both platforms and on drivers are quite stable, while riders will benefit more from the coopetition. This follows from the fact that under high demand, introducing a new alternative will yield a larger benefit to the riders, as expected. In Table 4, we examine the effect of the expected number of riders per trip in the new joint service. In this case, increasing $\tilde{n}$ does not have a significant impact on the profits, on P_2's drivers, and on the riders. However, it has a strong effect on P_1's drivers, who exclusively provide the new joint service in our example. In summary, even though the impact of the coopetition partnership may be sensitive with respect to the different market conditions, it seems to be beneficial for all parties (both platforms, drivers, and riders) in the vast majority of the instances we considered.

We observe in Tables 2-4 that there is not a clear monotonicity pattern, as when we vary a single parameter, the profit sharing parameter γ^* changes as well (since it is endogenously decided).

Table 2 Impact of the service quality ratio q_1/q_2 when $\Lambda/(n_2k_2) = 5$ and $\tilde{n} = 2$ (%).

q_1/q_2	1.1	1.4	1.8	2.2	2.6	3
$\Delta\pi_1/\pi_1$	34.67	34.07	32.33	29.52	26.27	22.78
$\Delta\pi_2/\pi_2$	29.92	29.82	28.56	26.45	23.77	21.02
$\Delta DS_1/DS_1$	25.55	19.87	13.68	8.77	5.00	2.10
$\Delta(\pi_2 + DS_2)/(\pi_2 + DS_2)$	24.04	23.92	22.68	20.83	18.44	16.13
ΔRS	2817.74	2826.93	2774.94	2688.51	2569.04	2438.02

Table 3 Impact of the demand-supply ratio $\Lambda/(n_2k_2)$ when $q_1/q_2 = 2$ and $\tilde{n} = 2$ (%).

$\Lambda/(n_2k_2)$	0.5	1	1.5	2	5	7
$\Delta\pi_1/\pi_1$	24.69	25.80	26.85	27.81	32.33	34.52
$\Delta\pi_2/\pi_2$	24.20	24.89	25.49	26.05	28.56	29.73
$\Delta DS_1/DS_1$	14.17	14.14	14.05	14.01	13.68	13.51
$\Delta(\pi_2 + DS_2)/(\pi_2 + DS_2)$	23.35	23.26	23.18	23.08	22.68	22.49
ΔRS	220.21	456.14	708.88	972.93	2774.94	4123.17

Table 4 Impact of $\tilde{n}$ when $q_1/q_2 = 2$ and $\Lambda/(n_2k_2) = 5$ (%).

$\tilde{n}$	1	1.3	1.7	2	2.5	3
$\Delta\pi_1/\pi_1$	26.82	30.28	32.10	32.33	31.51	29.94
$\Delta\pi_2/\pi_2$	23.34	26.61	28.38	28.56	27.73	26.13
$\Delta DS_1/DS_1$	38.56	29.03	19.15	13.68	7.21	2.96
$\Delta(\pi_2 + DS_2)/(\pi_2 + DS_2)$	18.39	21.07	22.53	22.68	21.99	20.67
ΔRS	2358.98	2622.90	2759.60	2774.94	2712.94	2591.45

6. Conclusions

The number of online ride-hailing platforms has increased significantly over the past few years. Recently, several coopetition partnerships emerged between such platforms. Examples includes Curb and Via in NYC and Uber and PT Express in Indonesia. This paper is motivated by such coopetition partnerships that can be implemented through a profit sharing contract.

It is not clear a-priori whether the competing platforms will benefit from a coopetition partnership. In particular, by introducing the new joint service, three main conflicting factors arise: the new market share effect, the cannibalization effect, and the increased congestion level. One of the key insights presented in this paper lies in understanding the interplay between these effects.

We use the MNL choice model to capture the fact that potential riders face several alternatives and we characterize the equilibrium market outcome. We then identify clear conditions under which the coopetition is beneficial for both platforms. In particular, the driver capacity congestion levels of both platforms should not be too high, as otherwise the coopetition will be detrimental to the total profits. Under this regime, we find that there always exists a profit sharing contract that increases the profits of both platforms, regardless of which platform sets the price of the new service. Next, we study the impact of the coopetition on riders and on drivers. While it is straightforward to show that riders will be better off, the impact on drivers is more subtle. To ensure that all the players benefit from the coopetition, one needs to either (i) allocate enough drivers from each platform to serve the new joint service and restrict the number of passengers per ride, or (ii) redistribute some of the profit gains to drivers by offering bonuses.

This paper is among the first to propose a tractable model to study competition and partnerships in the ride-sharing industry. It allows us to draw practical insights on the impact of some recent business partnerships observed in practice. Several interesting extensions are left as future research. For example, what is the long-term impact of such partnerships? Shall the platforms consider more complicated contracts such as two-part piecewise linear agreements (i.e., allowing two different profit portions depending on the scale of the joint service)? A second potential direction for future research is to study an alternative form of coopetition, known as joint ownership of a subsidiary. For example, Uber and a local Russian taxi-hailing platform Yandex.Taxi recently merged their ride-hailing businesses in Russia under a new company.[11] It could be interesting to compare the two different forms of coopetition.

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Appendix A: Summary of Notation

Table 5 Summary of Notation.

P_1 :	Platform 1
P_2 :	Platform 2
q_i :	Perceived quality of Platform i $(i=1,2)$
q_n :	Perceived quality of the new joint service
p_i :	Price per trip for P_i without the new joint service
$\tilde{p}_i$:	Price per trip for P_i with the new joint service
$\tilde{p}_n$:	Price per trip for the new joint service
s_i :	Probability that a rider selects P_i without the new joint service
s_0 :	Probability that a rider selects the outside option without the new joint service
$\tilde{s}_i$:	Probability that a rider selects P_i with the new joint service
$\tilde{s}_n$:	Probability that a rider selects the new joint service
$\tilde{s}_0$:	Probability that a rider selects the outside option with the new joint service
Λ :	Total rider arrival rate in the market
K_i :	Total number of drivers partnering with P_i
r_i :	Per unit time reservation wage of P_i's drivers
$G_i(\cdot)$:	CDF of r_i which is assumed to satisfy the log-concave condition
w_i :	Per unit time wage paid by P_i to its drivers
n_i :	Average number of riders per trip using P_i's service
$\tilde{n}$:	Average number of riders per trip using the new joint service
u_i :	Congestion level of P_i without the new joint service
$\tilde{u}_i$:	Congestion level of P_i with the new joint service
$K_i f_i(u_i)$:	Total wage per unit time that P_i pays its drivers under the congestion level u_i
γ :	Fraction of profit generated by the new joint service allocated to P_1
α :	Fraction of the new joint service allocated to P_1's drivers

Appendix B: Proof of Statements

Auxiliary Lemma

Before presenting the proofs of our results, we state and prove an auxiliary lemma that is extensively used throughout this Appendix.

LEMMA 1. *For the model without coopetition, we have:* $\partial_{p_i} s_i = -(1-s_i)s_i$ *and* $\partial_{p_j} s_i = s_i s_j$ $(i=1,2$ *and* $j \neq i)$. *For the model with coopetition, we have* $\partial_{\tilde{p}_i} \tilde{s}_i = -(1-\tilde{s}_i)\tilde{s}_i$ *and* $\partial_{\tilde{p}_j} \tilde{s}_i = \tilde{s}_i \tilde{s}_j$.

Proof. We only present the proof for the case without coopetition as the case with coopetition follows the exact same argument. Since $s_i = \frac{\exp(q_i - p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)}$, we have:

$$\begin{aligned}
\partial_{p_i} s_i =& \frac{-\exp(q_i-p_i)[1+\exp(q_1-p_1)+\exp(q_2-p_2)]+[\exp(q_i-p_i)]^2}{[1+\exp(q_1-p_1)+\exp(q_2-p_2)]^2} \\
=& -\frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} + \left(\frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)}\right)^2 \\
=& -s_i + s_i^2 = -(1-s_i)s_i
\end{aligned}$$

and

$$\begin{aligned}
\partial_{p_j} s_i =& \frac{\exp(q_i-p_i)\exp(q_j-p_j)}{[1+\exp(q_1-p_1)+\exp(q_2-p_2)]^2} \\
=& \frac{\exp(q_i-p_i)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} \times \frac{\exp(q_j-p_j)}{1+\exp(q_1-p_1)+\exp(q_2-p_2)} = s_i s_j. \quad \square
\end{aligned}$$

Proof of Proposition 1

For P_i, the first order condition (FOC) can be written as follows:

$$\partial_{p_i}\pi_i(p_i^*,p_j^*)\begin{cases}=0, & \text{if } p_i^*\in(\underline{p},\bar{p});\\ =-\underline{\mu}^*\le 0, & \text{if } p_i^*=\underline{p};\\ =\bar{\mu}^*\ge 0, & \text{if } p_i^*=\bar{p},\end{cases}$$

where $\underline{\mu}^*\ge 0$ (resp. $\bar{\mu}^*\ge 0$) is the Lagrangian multiplier with respect to the constraint $p_i\ge\underline{p}$ (resp. $p_i\le\bar{p}$). $\partial_{p_i}\pi_i(p_1^*,p_2^*)=\Lambda[s_i^*+p_i^*\partial_{p_i}s_i^*-f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\partial_{p_i}s_i^*]=\Lambda s_i^*[1-p_i^*(1-s_i^*)+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)(1-s_i^*)]$. Therefore,

$$p_i^*\begin{cases}=\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\in(\underline{p},\bar{p});\\ \ge\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\le\underline{p};\\ \le\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\ge\bar{p}.\end{cases}$$

On the other hand, by the definition of the MNL model, $\exp(q_i-p_i^*)=s_i^*/s_0^*$, i.e., $p_i^*=q_i+\log(s_0^*/s_i^*)$. As a result, we can write:

$$q_i+\log(s_0^*/s_i^*)\begin{cases}=\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\in(\underline{p},\bar{p});\\ \ge\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\le\underline{p};\\ \le\frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\ge\bar{p},\end{cases}$$

which is equivalent to

$$s_i^*\exp\left(\frac{s_i^*}{1-s_i^*}\right)\exp\left[f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\right]\begin{cases}=s_0^*\exp(q_i-1), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\in(\underline{p},\bar{p});\\ \ge s_0^*\exp(q_i-1), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\le\underline{p};\\ \le s_0^*\exp(q_i-1), & \text{if } \frac{1}{1-s_i^*}+f_i'\left(\frac{\Lambda s_i^*}{K_in_i}\right)\ge\underline{p}.\end{cases}\tag{1}$$

We define $U_i(z):=z\exp\left(\frac{z}{1-z}\right)\exp\left[f_i'\left(\frac{\Lambda z}{K_in_i}\right)\right]$. Since $f_i(\cdot)$ is convexly increasing, $U_i(z)$ is strictly increasing in z on its domain. Furthermore, we define $\underline{s}$ as the solution of $\frac{1}{1-\underline{s}}+f_i'\left(\frac{\Lambda\underline{s}}{K_in_i}\right)=\bar{p}$ and $\bar{s}$ as the solution of $\frac{1}{1-\bar{s}}+f_i'\left(\frac{\Lambda\bar{s}}{K_in_i}\right)=\underline{p}$. We also define $V_i(x):=\max\{\min\{U_i^{-1}(x),\bar{s}\},\underline{s}\}$. As a result, we have $s_i^*=V_i(s_0^*)$. Since $s_0^*+s_1^*+s_2^*=1$ and $s_i^*=V_i(s_0^*)$, then s_0^* must satisfy $s_0^*+V_1[s_0^*\exp(q_1-1)]+V_2[s_0^*\exp(q_2-1)]=1$. Moreover, $V_i(\cdot)$ is an increasing function, so there exists a unique s_0^* that satisfies:

$$s_0^*+V_1[s_0^*\exp(q_1-1)]+V_2[s_0^*\exp(q_2-1)]=1.$$

Note that one can solve for s_0^* efficiently (e.g., through binary search).

Since $p_1^*=q_1+\log(s_0^*/s_1^*)$ and $p_2^*=q_2+\log(s_0^*/s_2^*)$, we have $p_1^*=q_1+\log(s_0^*/s_1^*)=q_1+\log(s_0^*/V_1[s_0^*\exp(q_1-1)])$ and $p_2^*=q_2+\log(s_0^*/s_2^*)=q_2+\log(s_0^*/V_2[s_0^*\exp(q_2-1)])$. □

Proof of Proposition 2

Case 1. $\tilde{n}<\bar{n}$ and $\alpha\in(\underline{\alpha},\bar{\alpha})$. We first consider the case with $\tilde{n}<\bar{n}$ and $\alpha\in(\underline{\alpha},\bar{\alpha})$ (the definitions of $\bar{n}$, $\underline{\alpha}$, and $\bar{\alpha}$ can be found in the discussion preceding Corollary 1). The proof for other cases are simpler and follow a similar argument.

Since $\tilde{n}<\bar{n}$ and $\alpha\in\left(\frac{s_1^*\tilde{n}}{n_1},1-\frac{s_2^*\tilde{n}}{n_2}\right)$, it follows from algebraic manipulations that $\tilde{u}_1=u_1^*+u_1^*\tilde{s}_n\left(\frac{\alpha n_1}{\tilde{n}s_1^*}-1\right)>u_1^*$ and $\tilde{u}_2=u_2^*+u_2^*\tilde{s}_n\left(\frac{(1-\alpha)n_2}{\tilde{n}s_2^*}-1\right)>u_2^*$.

First, we show that there exists a threshold $\underline{u}_1$ that depends on u_2^*, such that $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi^* := \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$ (resp. $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) < \pi^*$) for a fixed u_2^* and for all $u_1^* < \underline{u}_1$ (resp. $u_1^* > \underline{u}_1$).

From the definition of the MNL choice model, we have $\tilde{s}_n + \tilde{s}_1^* + \tilde{s}_2^* > s_1^* + s_2^*$. Since $\tilde{p}_n = \bar{p} \geq p_i^*$, we have:

$$\begin{aligned} \Lambda(\tilde{s}_1^* p_1^* + \tilde{s}_2^* p_2^* + \tilde{s}_n \bar{p}_n) - \Lambda(s_1^* p_1^* + s_2^* p_2^*) =& \Lambda[\tilde{p}_n \tilde{s}_n - p_1^*(s_1^* - \tilde{s}_1^*) - p_2^*(s_2^* - \tilde{s}_2^*)] \\ \geq& \Lambda[\tilde{p}_n \tilde{s}_n - \tilde{p}_n(s_1^* - \tilde{s}_1^*) - \tilde{p}_n(s_2^* - \tilde{s}_2^*)] \\ =& \Lambda \tilde{p}_n(\tilde{s}_n + \tilde{s}_1^* + \tilde{s}_2^* - s_1^* - s_2^*) > 0, \end{aligned} \tag{2}$$

where the first inequality follows from $\tilde{p}_n = \bar{p} \geq p_i^*$ $(i = 1, 2)$ and the second inequality from $\tilde{s}_n + \tilde{s}_1^* + \tilde{s}_2^* > s_1^* + s_2^*$. We define $C := \Lambda(\tilde{s}_1^* p_1^* + \tilde{s}_2^* p_2^* + \tilde{s}_n \bar{p}_n) - \Lambda(s_1^* p_1^* + s_2^* p_2^*) > 0$.

It suffices to show that there exists a threshold $\underline{u}_1$, such that $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi^*$ if and only if $u_1^* < \underline{u}_1$, which is equivalent to the following condition:

$$K_1 f_1(\tilde{u}_1) + K_2 f_2(\tilde{u}_2) < K_1 f_1(u_1^*) + K_2 f_2(u_2^*) + C, \text{ if and only if } u_1^* < \underline{u}_1.$$

Since $f_1(\cdot)$ is a strictly convex function, $f_1'(u_1) > 0$ for all u_1. We have $K_1 f_1(\tilde{u}_1) - K_1 f_1(u_1^*) = K_1 \int_{u_1^*}^{u_1^*+\delta} f_1'(u) du > 0$, where $\delta = u_1^* \tilde{s}_n \left(\frac{\alpha n_1}{\tilde{n} s_1^*} - 1\right) > 0$ and $\delta \to +\infty$ as $u_1^* \to +\infty$. Furthermore, since $f_1'(u) \to +\infty$ as $u \to \infty$, by the monotone convergence Theorem, $K_1 \int_{u_1^*}^{u_1^*+\delta} f_1'(u) du \to +\infty$ as $u_1^* \to +\infty$. Putting everything together, we obtain that $\underline{u}_1$ (which depends on the value of u_2^*) can be defined as

$$\begin{aligned} &\underline{u}_1 := \max\{u_1^* : K_1 f_1(\tilde{u}_1) - K_1 f_1(u_1^*) < K_2 f_2(u_2^*) + C - K_2 f_2(\tilde{u}_2)\}, \\ &\text{where } \tilde{u}_2 = u_2^* + u_2^* \tilde{s}_n \left[\frac{(1-\alpha) n_2}{\tilde{n} s_2^*} - 1\right] \text{ is fixed.} \end{aligned} \tag{3}$$

If the right-hand-side of (3) is an empty set, we define $\underline{u}_1 = 0$. This completes the existence proof of $\underline{u}_1$. Since our model is symmetric with respect to both platforms, there also exists a threshold $\underline{u}_2$, which depends on the value of u_1^*, such that $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi^*$ if and only if $u_2^* < \underline{u}_2$.

If $u_1^* < \underline{u}_1$ and $u_2^* < \underline{u}_2$, we have just shown that $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi^*$, i.e., $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^* | \gamma) + \tilde{\pi}_2(\bar{p}, p_1^*, p_2^* | \gamma) > \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$. Furthermore, after some calculations we obtain:

$$\tilde{\pi}_1(\bar{p}, p_1^*, p_2^* | 0) = \Lambda\left[p_1^* - \frac{f_1(\tilde{u}_1)}{\tilde{u}_1}\right] \tilde{s}_1 < \Lambda\left[p_1^* - \frac{f_1(u_1^*)}{u_1^*}\right] s_1^* = \pi_1(p_1^*, p_2^*),$$

where the inequality follows from $\tilde{u}_1 > u_1^*$ and $s_1^* > \tilde{s}_1$. Similarly, we have $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^* | 1) < \pi_2(p_1^*, p_2^*)$. We define the net profit from the new joint service $\tilde{\pi} := \Lambda \bar{p} \tilde{s}_n - \frac{\alpha \Lambda \tilde{s}_n f_1(\tilde{u}_1)}{\tilde{n} \tilde{u}_1} - \frac{(1-\alpha) \Lambda \tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n} \tilde{u}_2} > 0$, where the inequality follows from the fact that the total profit of both platforms increases, that is, $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi^*$. Therefore, if $\gamma \in (\underline{\gamma}, \bar{\gamma})$, then $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^* | \gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^* | \gamma) > \pi_2(p_1^*, p_2^*)$, where $\underline{\gamma} := \frac{\pi_1(p_1^*, p_2^*) - \tilde{\pi}_1(\bar{p}, p_1^*, p_2^* | 0)}{\tilde{\pi}_n}$ and $\bar{\gamma} := \frac{\tilde{\pi}_n - \pi_2(p_1^*, p_2^*) + \tilde{\pi}_2(\bar{p}, p_1^*, p_2^* | 1)}{\tilde{\pi}_n}$. This completes the proof of the first part of the Proposition.

We next show that $\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$ is quasi-concave in $\tilde{p}_n$. We first compute the derivative $\partial_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)$:

$$\partial_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = \Lambda(p_1^* \partial_{\tilde{p}_n} \tilde{s}_1 + p_2^* \partial_{\tilde{p}_n} \tilde{s}_2 + \tilde{s}_n + \tilde{p}_n \partial_{\tilde{p}_n} \tilde{s}_n) - K_1 f_1'(\tilde{u}_1) \partial_{\tilde{p}_n} \tilde{u}_1 - K_2 f_2'(\tilde{u}_2) \partial_{\tilde{p}_n} \tilde{u}_2.$$

Since $\partial_{\tilde{p}_n} \tilde{s}_i = \tilde{s}_i \tilde{s}_n$ $(i = 1, 2)$ and $\partial_{\tilde{p}_n} \tilde{s}_n = -(1 - \tilde{s}_n) \tilde{s}_n$ (see Lemma 1), we also have:

$$\partial_{\tilde{p}_n} \tilde{u}_i = \frac{\Lambda[\partial_{\tilde{p}_n} \tilde{s}_i / n_i + \alpha_i \tilde{\partial}_{\tilde{p}_n} s_n / \tilde{n}]}{K_i}, \text{ with } \alpha_1 = \alpha \text{ and } \alpha_2 = 1 - \alpha.$$

Therefore:

$$\begin{aligned} \partial_{\tilde{p}_n} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) =& \Lambda\left[p_1^* \tilde{s}_1 \tilde{s}_n + p_2^* \tilde{s}_2 \tilde{s}_n + \tilde{s}_n - \tilde{p}_n (1 - \tilde{s}_n) \tilde{s}_n\right] \\ &- \Lambda f_1'(\tilde{u}_1)\left[\frac{\tilde{s}_2 \tilde{s}_n}{n_2} - \frac{\alpha(1 - \tilde{s}_n) \tilde{s}_n}{\tilde{n}}\right] - \Lambda f_2'(\tilde{u}_2)\left[\frac{\tilde{s}_1 \tilde{s}_n}{n_2} - \frac{(1-\alpha)(1 - \tilde{s}_n) \tilde{s}_n}{\tilde{n}}\right]. \end{aligned}$$

We define $\tilde{p}_n^*$ as the unconstrained maximizer of $\tilde{\pi}(\cdot, p_1^*, p_2^*)$. Hence, $\tilde{p}_n^*$ satisfies the first order condition $\partial_{\tilde{p}_n}\tilde{\pi}(\tilde{p}_n^*, p_1^*, p_2^*) = 0$, which further allows us to express $\tilde{p}_n^*$ as follows:

$$\tilde{p}_n^* = p_1^* s_1^* + p_2^* s_2^* + \frac{1}{1-\tilde{s}_n} - f_1'(\tilde{u}_1)\Big[\frac{s_1^*}{n_1} - \frac{\alpha}{\tilde{n}}\Big] - f_2'(\tilde{u}_2)\Big[\frac{s_2^*}{n_2} - \frac{1-\alpha}{\tilde{n}}\Big], \tag{4}$$

where we used identity $\tilde{s}_i = (1-\tilde{s}_n)s_i^*$ $(i=1,2)$. Therefore, $\tilde{p}_n^*$ is the fixed point of the function $F(\tilde{p}_n)$ which is defined as

$$F(\tilde{p}_n) = p_1^* s_1^* + p_2^* s_2^* + \frac{1}{1-\tilde{s}_n} - f_1'(\tilde{u}_1)\Big[\frac{s_1^*}{n_1} - \frac{\alpha}{\tilde{n}}\Big] - f_2'(\tilde{u}_2)\Big[\frac{s_2^*}{n_2} - \frac{1-\alpha}{\tilde{n}}\Big].$$

Since $\alpha \in (\underline{\alpha}, \bar{\alpha})$, $\frac{s_1^*}{n_1} - \frac{\alpha}{\tilde{n}} > 0$ and $\frac{s_2^*}{n_2} - \frac{1-\alpha}{\tilde{n}} > 0$. Note that by the (strict) convexity of $f_i(\cdot)$, $\tilde{s}_n$, $f_i'(\tilde{u}_i)$ $(i=1,2)$ are both decreasing in $\tilde{p}_n$. Thus, $F(\tilde{p}_n)$ is strictly (and continuously) decreasing in $\tilde{p}_n$ with $F(0) > 0$. Consequently, $F(\cdot)$ has a unique fixed point $\tilde{p}_n^*$, which further implies that (4) is satisfied. Therefore, $\partial_{\tilde{p}_n}\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = 0$ for $\tilde{p}_n = \tilde{p}_n^*$. If $\tilde{p}_n < \tilde{p}_n^*$ (resp. $\tilde{p}_n > \tilde{p}_n^*$), $F(\tilde{p}_n) < \tilde{p}_n$ (resp. $F(\tilde{p}_n) > \tilde{p}_n$), and thus $\partial_{\tilde{p}_n}\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) > 0$ (resp. $\partial_{\tilde{p}_n}\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) < 0$). In other words, $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is strictly increasing (resp. decreasing) in $\tilde{p}_n$ for $\tilde{p}_n < \tilde{p}_n^*$ (resp. $\tilde{p}_n > \tilde{p}_n^*$). As a result, $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is quasi-concave in $\tilde{p}_n$ and is maximized at $\tilde{p}_n = \tilde{p}_n^*$. Similarly, we can show that $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$ is quasi-concave in $\tilde{p}_n$ for $i=1,2$ and any $\gamma \in [0,1]$.

Note that, if $\tilde{p}_n \uparrow +\infty$, then $\tilde{s}_i \downarrow s_i^*$ and $\tilde{s}_n \downarrow 0$. Hence, $\lim_{\tilde{p}_n \uparrow +\infty} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$. Thus, if $\tilde{p}_n^* \le \bar{p}$, then $\tilde{\pi}(\tilde{p}_n^*, p_1^*, p_2^*) \ge \tilde{\pi}(\bar{p}, p_1^*, p_2^*) > \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$, where the first inequality follows from the quasi-concavity of $\tilde{\pi}(\cdot, p_1^*, p_2^*)$. Therefore, if $\tilde{\pi}(\bar{p}, p_1^*, p_2^*) \le \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$, we have $\tilde{p}_n^* \ge \bar{p}$. Since $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ is strictly increasing in $\tilde{p}_n$ for $\tilde{p}_n \le \bar{p} \le \tilde{p}_n^*$, $\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) = \tilde{\pi}(\bar{p}, p_1^*, p_2^*) < \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$. As a result, if $u_1^* \ge \underline{u}_1$ or $u_2^* \ge \underline{u}_2$, $\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) \le \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$. Since $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) + \tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*) \le \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*)$, it must be the case that $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_1(p_1^*, p_2^*)$ or $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_2(p_1^*, p_2^*)$ for all $(\tilde{p}_n, \gamma)$ ($\tilde{p}_n \in [\underline{p}, \bar{p}]$ and $\gamma \in [0,1]$), that is, at least one of the platforms is worse off in the presence of coopetition.

Next, we show that there exists a threshold $\bar{u}_1$ that depends on the value of u_2^*, such that if $u_1^* > \bar{u}_1$, $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_1^* := \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_2^* := \pi_2(p_1^*, p_2^*)$. Since $\tilde{\pi}_i(\bar{p}, p_1^*, p_2^*|\gamma)$ $(i=1,2)$ is linear in γ, it is enough to show the inequalities for $\gamma = 1$ and $\gamma = 0$.

If $\gamma = 1$, we obtain:

$$\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|1) - \pi_1^* = \Lambda(\tilde{s}_n\bar{p} + \tilde{s}_1 p_1^* - s_1^* p_1^*) + \Big[K_1 f_1(u_1^*) - K_1 f_1(\tilde{u}_1) - \frac{(1-\alpha)\Lambda\tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2}\Big].$$

As we have shown, $K_1 f_1(u_1^*) - K_1 f_1(\tilde{u}_1)$ is strictly decreasing in u_1^* with $\lim_{u_1^* \uparrow +\infty}[K_1 f_1(u_1^*) - K_1 f_1(\tilde{u}_1)] = -\infty$. We then define $\bar{u}_1^1$ as

$$\bar{u}_1^1 := \max\left\{u_1^* : K_1 f_1(u_1^*) - K_1 f_1(\tilde{u}_1) > \frac{(1-\alpha)\Lambda\tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2} + \Lambda s_1^* p_1^* - \Lambda(\tilde{s}_n\bar{p} + \tilde{s}_1^* p_1^*)\right\},$$
$$\text{where } \tilde{u}_2 = u_2^* + u_2^*\tilde{s}_n\Big[\frac{(1-\alpha)n_2}{\tilde{n}s_2^*} - 1\Big] \text{ is fixed.}$$

Therefore, we have $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|1) - \pi_1^* < 0$ for $u_1^* > \bar{u}_1^1$. Furthermore, we have $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1) - \pi_2^* = \Lambda(\tilde{s}_2 - s_2^*)p_2^* + K_2 f_2(u_2^*) - \frac{\Lambda\tilde{s}_2 f_2(\tilde{u}_2)}{n_2\tilde{u}_2}$. Hence, $\pi_2^* = \Lambda s_2^* p_2^* - K_2 f_2(u_2^*) \ge \Lambda\tilde{s}_2 p_2^* - K_2 f_2\left(\frac{\Lambda\tilde{s}_2}{K_2 n_2}\right) > \Lambda\tilde{s}_2 p_2^* - \frac{\Lambda\tilde{s}_2 f_2(\tilde{u}_2)}{n_2\tilde{u}_2} = \tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1)$, where the first inequality follows from the fact that (p_1^*, p_2^*) are the equilibrium prices in the model without coopetition and the second inequality from $\tilde{u}_2 > \frac{\Lambda\tilde{s}_2}{K_2 n_2}$ if $\alpha < \bar{\alpha}$. As a result, $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1) - \pi_2^* < 0$, i.e., $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1) < \pi_2^*$.

We now fix u_1^* and thus, $\tilde{u}_1 = u_1^* + u_1^*\tilde{s}_n\left(\frac{\alpha n_1}{\tilde{n}s_1^*} - 1\right)$. We define

$$\bar{u}_2^1 := \max\left\{u_2^* : \frac{(1-\alpha)\Lambda\tilde{s}_n f_2(\tilde{u}_2)}{\tilde{n}\tilde{u}_2} < K_1 f_1(\tilde{u}_1) - K_1 f_1(u_1^*) + \Lambda s_1^* p_1^* - \Lambda(\tilde{s}_n\bar{p} + \tilde{s}_1^* p_1^*)\right\},$$
$$\text{where } \tilde{u}_2 = u_2^* + u_2^*\tilde{s}_n\Big[\frac{(1-\alpha)n_2}{\tilde{n}s_2^*} - 1\Big].$$

Since $f_2(u_2)/u_2$ is increasing in u_2 with $\lim_{u_2\uparrow+\infty} f_2(u_2)/u_2 = +\infty$ from our modeling assumption, we have $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|1) - \pi_1^* < 0$ for $u_2^* > \bar{u}_2^1$. Therefore, if $u_1^* > \bar{u}_1^1$ or $u_2^* > \bar{u}_1^2$, then $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|1) - \pi_1^* < 0$ and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1) - \pi_2^* < 0$.

If $\gamma = 0$, we exchange the roles of P_1 and P_2 in the proof for $\gamma = 1$. Then, we obtain that $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|0) < \pi_1^*$ for any u_1^* and u_2^*. Furthermore, there exist two thresholds $\bar{u}_1^0$ and $\bar{u}_2^0$ such that if $u_1^* > \bar{u}_1^0$ or $u_2^* > \bar{u}_2^0$, then $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|0) < \pi_2^*$.

We define $\bar{u}_1 := \max\{\bar{u}_1^0, \bar{u}_1^1\}$ and $\bar{u}_2 := \max\{\bar{u}_2^0, \bar{u}_2^1\}$. Hence, it follows that if $u_1^* > \bar{u}_1$ or $u_2^* > \bar{u}_2$, then $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|1) < \pi_1^*$, $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|1) < \pi_2^*$, $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|0) < \pi_1^*$, and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|0) < \pi_2^*$. Since $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|\gamma)$ and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|\gamma)$ are linear in γ, then $\tilde{\pi}_1(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_1^*$ and $\tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_2^*$ for any $\gamma \in [0,1]$. As we have shown, if $\tilde{\pi}_i(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_i^*$, then $\max_{\tilde{p}_n \in [\underline{p},\bar{p}]} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \tilde{\pi}_i(\bar{p}, p_1^*, p_2^*|\gamma) < \pi_i^*$. Therefore, if $u_1^* > \bar{u}_1$ or $u_2^* > \bar{u}_2$, then $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma) < \pi_i^*$ $(i = 1, 2)$ for all $(\tilde{p}_n, \gamma)$ ($\tilde{p}_n \in [\underline{p}, \bar{p}]$ and $\gamma \in [0,1]$). This completes the proof of all three parts for the case with $\tilde{n} < \bar{n}$ and $\alpha \in (\underline{\alpha}, \bar{\alpha})$.

Case 2. $\tilde{n} < \bar{n}$ and $\alpha \le \underline{\alpha}$. In this case, $\tilde{u}_1 < u_1^*$. Hence, $K_1 f_1(\tilde{u}_1) < K_1 f_1(u_1^*)$ and the coopetition will always decrease the wage distributed by P_1, which further implies that $\underline{u}_1 = +\infty$. Consequently, we only need to consider the congestion level of P_2, u_2^*. Following the same argument as in the proof of Case 1, we can find two thresholds $\underline{u}_2$ and $\bar{u}_2$, both of which depend on u_1^*, such that: If $u_2^* < \underline{u}_2$, we can find some $(\tilde{p}_n, \gamma)$, such that the profit of each platform will increase in the presence of coopetition (i.e., $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) > \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) > \pi_2(p_1^*, p_2^*)$); if $u_2^* \in [\underline{u}_2, \bar{u}_2]$, for all $(\tilde{p}_n, \gamma)$, the profit of at least one platform will decrease in the presence of coopetition (i.e., $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_1(p_1^*, p_2^*)$ or $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_2(p_1^*, p_2^*)$); and if $u_2^* > \bar{u}_2$, for all $(\tilde{p}_n, \gamma)$, the profit of each platform will decrease in the presence of coopetition (i.e., $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_1(p_1^*, p_2^*)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) \le \pi_2(p_1^*, p_2^*)$ for all $\tilde{p}_n \in [\underline{p}, \bar{p}]$ and $\gamma \in [0,1]$).

Case 3. $\tilde{n} < \bar{n}$ and $\alpha \ge \bar{\alpha}$. The proof of this case follows the same argument as in the proof of Case 2 and is omitted for conciseness.

Case 4. $\tilde{n} \ge \bar{n}$ (thus, $\bar{\alpha} \le \underline{\alpha}$) and $\alpha \le \bar{\alpha}$. In this case, $\tilde{u}_1 < u_1^*$. This case is the same as Case 2, so the proof is omitted for conciseness.

Case 5. $\tilde{n} \ge \bar{n}$ (thus, $\bar{\alpha} \le \underline{\alpha}$) and $\alpha \ge \underline{\alpha}$. In this case, $\tilde{u}_2 < u_2^*$. This case is the same as Case 3, so the proof is omitted for conciseness.

Case 6. $\tilde{n} \ge \bar{n}$ (thus, $\bar{\alpha} \le \underline{\alpha}$) and $\alpha \in (\bar{\alpha}, \underline{\alpha})$. In this case, $\tilde{u}_1 < u_1^*$ and $\tilde{u}_2 < u_2^*$. Therefore, $K_1 f_1(\tilde{u}_1) < K_1 f_1(u_1^*)$ and $K_2 f_2(\tilde{u}_2) < K_2 f_2(u_2^*)$. Hence, the coopetition will always decrease the total wage distributed by both platforms. We have shown in Case 1 that the total revenue will be increased by the coopetition for $\tilde{p}_n = \bar{p}$, i.e., $\Lambda(\bar{p}\tilde{s}_n + p_1^*\tilde{s}_1^* + p_2^*\tilde{s}_2^*) > \Lambda(p_1^* s_1^* + p_2^* s_2^*)$. As a result, the total profit of both platforms in the presence of coopetition dominates the total profit without coopetition, i.e.,

$$\begin{aligned}\tilde{\pi}(\bar{p}, p_1^*, p_2^*) =& \Lambda(\bar{p}\tilde{s}_n + p_1^*\tilde{s}_1^* + p_2^*\tilde{s}_2^*) - K_1 f_1(\tilde{u}_1) - K_2 f_2(\tilde{u}_2) \\ >& \Lambda(p_1^* s_1^* + p_2^* s_2^*) - K_1 f_1(u_1^*) - K_2 f_2(u_2^*) = \pi_1(p_1^*, p_2^*) + \pi_2(p_1^*, p_2^*).\end{aligned}$$

Following the same argument as in the proof of Case 1, we can always find $(\tilde{p}_n, \gamma)$ such that the profit of each platform is higher under coopetition, i.e., $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma) > \pi_i(p_1^*, p_2^*)$. In this case, $\underline{u}_1 = \underline{u}_2 = +\infty$. $\square$

Proof of Proposition 3

Proposition 3 follows directly from the proof of Proposition 2. $\square$

Proof of Proposition 4

We use the parameter γ to denote the dependence of the market outcome on γ and the superscript ^{i} to specify that the price of the new joint service is set by P_i.

Existence and Monotonicity of $\tilde{p}_n^i(\gamma)$

As shown in the proof of Proposition 2, $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$ is quasi-concave in $\tilde{p}_n$ for any γ and there exists a unique price $\tilde{p}_n^i(\gamma) = \arg\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$. We rewrite the profit function of each platform as $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \tilde{\pi}_1(\tilde{p}_n) + \gamma\tilde{\pi}_n(\tilde{p}_n)$ and $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \tilde{\pi}_2(\tilde{p}_n) + (1-\gamma)\tilde{\pi}_n(\tilde{p}_n)$, where given the price $\tilde{p}_n$ for the new joint service, $\tilde{\pi}_i(\tilde{p}_n)$ is the net profit of P_i from its original service and $\tilde{\pi}_n(\tilde{p}_n)$ is the net profit from the new joint service. Note that $\tilde{\pi}_i(\tilde{p}_n)$ is increasing in $\tilde{p}_n$ for $i = 1, 2$, i.e., $\tilde{\pi}_i'(\tilde{p}_n) > 0$. Hence, in the region where $\tilde{\pi}_n'(\tilde{p}_n) \geq 0$, $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$ is strictly increasing in $\tilde{p}_n$. Since $\tilde{\pi}_i(\tilde{p}_n)$ is quasi-concave in $\tilde{p}_n$, the optimal price for P_i, $\tilde{p}_n^i(\gamma)$, lies in the interval $[p_{\min}, \bar{p}]$, where $p_{\min} := \min\{\tilde{p}_n : \tilde{\pi}_n'(\tilde{p}_n) < 0\}$. Consequently, $\tilde{p}_n^i(\gamma) = \arg\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \arg\max_{\tilde{p}_n \in [p_{\min}, \bar{p}]} \tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$, i.e., we can confine the optimization to the interval $[p_{\min}, \bar{p}]$.

One can easily check that $\partial_{\tilde{p}_n}\partial_\gamma \tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma) = \tilde{\pi}_n'(\tilde{p}_n) \leq 0$ if $\tilde{p}_n \in [p_{\min}, \bar{p}]$, i.e., $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma)$ is submodular in $(\tilde{p}_n, \gamma)$ on $[p_{\min}, \bar{p}] \times [0,1]$. Similarly, we have $\partial_{\tilde{p}_n}\partial_\gamma \tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma) = -\tilde{\pi}_n'(\tilde{p}_n) \geq 0$ if $\tilde{p}_n \in [p_{\min}, \bar{p}]$, i.e., $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma)$ is supermodular in $(\tilde{p}_n, \gamma)$ on $[p_{\min}, \bar{p}] \times [0,1]$. Therefore, by Topkis' Theorem, $\tilde{p}_n^1(\gamma)$ is decreasing in γ and $\tilde{p}_n^2(\gamma)$ is increasing in γ.

Existence of $(\underline{\gamma}', \bar{\gamma}')$

First, since $u_1^* < \underline{u}_1$ and $u_2^* < \underline{u}_2$, the profit from the new joint service is strictly positive. Hence, $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n \in [\underline{p}, \bar{p}]}[\tilde{\pi}_1(\tilde{p}_n) + \gamma\tilde{\pi}_n(\tilde{p}_n)]$ is strictly increasing in γ and $\tilde{\pi}_2(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) = \max_{\tilde{p}_n \in [\underline{p}, \bar{p}]}[\tilde{\pi}_2(\tilde{p}_n) + (1-\gamma)\tilde{\pi}_n(\tilde{p}_n)]$ is strictly decreasing in γ.

Note that $\tilde{p}_n^1(0) = \arg\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}_1(\tilde{p}_n) = \bar{p}$ and that $\tilde{\pi}_1(\cdot)$ is increasing in $\tilde{p}_n$. Similarly, $\tilde{p}_n^2(1) = \arg\max_{\tilde{p}_n \in [\underline{p}, \bar{p}]} \tilde{\pi}_2(\tilde{p}_n) = \bar{p}$. Since $\tilde{p}_n^1(\gamma)$ (resp. $\tilde{p}_n^2(\gamma)$) is decreasing (resp. increasing) in γ, $\tilde{p}_n^1(\gamma) - \tilde{p}_n^2(\gamma)$ is decreasing in γ with $\tilde{p}_n^1(0) - \tilde{p}_n^2(0) \geq 0$ and $\tilde{p}_n^1(1) - \tilde{p}_n^2(1) \leq 0$. As a result, there exists a $\gamma' \in [0,1]$ such that $\tilde{p}_n^1(\gamma) = \tilde{p}_n^2(\gamma)$. We also denote $\tilde{p}_n' := \tilde{p}_n^1(\gamma') = \tilde{p}_n^2(\gamma')$. Thus, if $\gamma > \gamma'$ (resp. $\gamma < \gamma'$), then $\tilde{p}_n^1(\gamma) < \tilde{p}_n' < \tilde{p}_n^2(\gamma)$ (resp. $\tilde{p}_n^1(\gamma) > \tilde{p}_n' > \tilde{p}_n^2(\gamma)$). If $\tilde{p}_n' < \bar{p}$, the first order condition $\partial_{\tilde{p}_n}\pi_i(\tilde{p}_n', p_1^*, p_2^*|\gamma) = 0$ (for $i = 1, 2$) implies that $\partial_{\tilde{p}_n}\tilde{\pi}(\tilde{p}_n', p_1^*, p_2^*) = \partial_{\tilde{p}_n}\pi_1(\tilde{p}_n', p_1^*, p_2^*|\gamma) + \partial_{\tilde{p}_n}\pi_2(\tilde{p}_n', p_1^*, p_2^*|\gamma) = 0$. Hence, $\tilde{p}_n'$ also maximizes $\tilde{\pi}(\cdot, p_1^*, p_2^*)$ over $\tilde{p}_n \in [\underline{p}, \bar{p}]$. Similarly, using again the first order condition, we can show that if $\tilde{p}_n' = \bar{p}$, then $\partial_{\tilde{p}_n}\pi(\tilde{p}_n', p_1^*, p_2^*) = \partial_{\tilde{p}_n}\pi_1(\tilde{p}_n', p_1^*, p_2^*) + \partial_{\tilde{p}_n}\pi_2(\tilde{p}_n', p_1^*, p_2^*) \geq 0$. Therefore, $\pi(\cdot, p_1^*, p_2^*)$ is maximized at $\tilde{p}_n = \tilde{p}_n' = \bar{p}$ as well.

In addition, we have $\tilde{\pi}_i(\tilde{p}_n', p_1^*, p_2^*|\gamma') \geq \tilde{\pi}_i(\bar{p}, p_1^*, p_2^*|\gamma') > \pi_i(p_1^*, p_2^*)$, where the first inequality follows from the fact that $\tilde{p}_n'$ maximizes $\tilde{\pi}_i(\cdot, p_1^*, p_2^*|\gamma)$ over $\tilde{p}_n \in [\underline{p}, \bar{p}]$, and the second inequality from $u_j^* < \underline{u}_j$ $(j = 1, 2)$. Therefore, $\tilde{\pi}_i(\tilde{p}_n^j(\gamma), p_1^*, p_2^*|\gamma') > \pi_i(p_1^*, p_2^*)$ for $i, j \in \{1, 2\}$. Since $\tilde{\pi}_i(\tilde{p}_n^j(\gamma), p_1^*, p_2^*|\gamma)$ is continuous in γ, there exists an interval $(\underline{\gamma}', \bar{\gamma}')$ such that $\tilde{\pi}_i(\tilde{p}_n^j(\gamma), p_1^*, p_2^*|\gamma) > \pi_i(p_1^*, p_2^*)$ for $i, j \in \{1, 2\}$ and $\gamma \in (\underline{\gamma}', \bar{\gamma}')$.

Next, we show that $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$. It suffices to show that $\underline{\gamma}' \geq \underline{\gamma}$ and $\bar{\gamma}' \leq \bar{\gamma}$. Observe that $\pi_1(\tilde{p}_n', p_1^*, p_2^*|\underline{\gamma}) = \pi_1(p_1^*, p_2^*)$. Furthermore, since $\tilde{p}_n^1(\gamma)$ is decreasing in γ whereas $\tilde{p}_n^2(\gamma)$ is increasing in γ, we then have $\tilde{p}_n^1(\underline{\gamma}) \geq \tilde{p}_n^1(\gamma') = \tilde{p}_n' = \tilde{p}_n^2(\gamma') \geq \tilde{p}_n^2(\underline{\gamma})$. Since $\tilde{\pi}_1(\cdot, p_1^*, p_2^*|\underline{\gamma})$ is quasi-concave in $\tilde{p}_n$, $\tilde{\pi}_1(\tilde{p}_n^2(\underline{\gamma}), p_1^*, p_2^*|\underline{\gamma}) \leq \pi_1(\tilde{p}_n', p_1^*, p_2^*|\underline{\gamma}) =$

$\pi_1(p_1^*, p_2^*)$. Hence, $\underline{\gamma} \notin (\underline{\gamma}', \bar{\gamma}')$. Therefore, $\underline{\gamma}' \geq \underline{\gamma}$. Analogously, exchanging the roles of P_1 and P_2, we obtain $\bar{\gamma}' \leq \bar{\gamma}$. Putting everything together, we conclude that $(\underline{\gamma}', \bar{\gamma}') \subset (\underline{\gamma}, \bar{\gamma})$.

If $\gamma < \underline{\gamma}'$, $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < \pi_1(p_1^*, p_2^*)$ and if $\gamma > \bar{\gamma}'$, $\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \pi_2(p_1^*, p_2^*)$.

First, note that $\underline{\gamma}'$ satisfies $\tilde{\pi}_1(\tilde{p}_n^2(\underline{\gamma}'), p_1^*, p_2^*|\underline{\gamma}') = \pi_1(p_1^*, p_2^*)$ and that $\bar{\gamma}'$ satisfies $\tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}'), p_1^*, p_2^*|\bar{\gamma}') = \pi_2(p_1^*, p_2^*)$. Observe that if $\gamma < \underline{\gamma}'$,

$$\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\underline{\gamma}') < \tilde{\pi}_1(\tilde{p}_n^2(\underline{\gamma}'), p_1^*, p_2^*|\underline{\gamma}') = \pi_2(p_1^*, p_2^*),$$

where the first inequality follows from the fact that $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*, |\gamma)$ is strictly increasing in γ and the second inequality from $\tilde{p}_n^2(\gamma) < \tilde{p}_n^2(\underline{\gamma}') < \tilde{p}_n^1(\underline{\gamma}')$ and using that $\tilde{\pi}_1(\cdot, p_1^*, p_2^*, |\gamma)$ is quasi-concave in $\tilde{p}_n$. Analogously, if $\gamma > \bar{\gamma}'$,

$$\tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) < \tilde{\pi}_2(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\bar{\gamma}') < \tilde{\pi}_2(\tilde{p}_n^1(\bar{\gamma}'), p_1^*, p_2^*|\bar{\gamma}') = \pi_2(p_1^*, p_2^*),$$

where the first inequality follows from the fact that $\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*, |\gamma)$ is strictly decreasing in γ and the second inequality from $\tilde{p}_n^1(\gamma) < \tilde{p}_n^1(\bar{\gamma}') < \tilde{p}_n^2(\bar{\gamma}')$ and using that $\tilde{\pi}_2(\cdot, p_1^*, p_2^*, |\gamma)$ is quasi-concave in $\tilde{p}_n$. This concludes the proof of Proposition 4. □

Proof of Proposition 5

Proposition 5 follows directly from the proof of Proposition 4 by setting $\gamma^* = \gamma'$. □

Proof of Proposition 6

Part (a)

In $\mathcal{G}_1$, given γ, P_2's optimal pricing policy has already been characterized in Proposition 4. If $\gamma \leq \gamma^*$, then $\tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) < \tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma) \leq \tilde{\pi}_1(\tilde{p}_n^1(\gamma^*), p_1^*, p_2^*|\gamma^*)$, where the second inequality follows from the fact that $\tilde{\pi}_1(\tilde{p}_n^1(\gamma), p_1^*, p_2^*|\gamma)$ is strictly increasing in γ (see the proof of Proposition 4). Therefore, $\gamma^1 = \arg\max_\gamma \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) > \gamma^*$. We then obtain:

$$\tilde{\pi}_1(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) = \max_\gamma \tilde{\pi}_1(\tilde{p}_n^2(\gamma), p_1^*, p_2^*|\gamma) \geq \tilde{\pi}_1(\tilde{p}_n^2(\gamma^*), p_1^*, p_2^*|\gamma^*) > \pi_1(p_1^*, p_2^*),$$

where the second inequality follows from Proposition 5. Furthermore, for P_2, $\tilde{\pi}_2(\tilde{p}_n^2(\gamma^1), p_1^*, p_2^*|\gamma^1) \geq \tilde{\pi}_2(\bar{p}, p_1^*, p_2^*|\gamma^1) > \pi_2(p_1^*, p_2^*)$, where the second inequality follows from the assumptions that $u_1^* < \underline{u}_1$ and $u_2^* < \underline{u}_2$.

Part(b)

The proof follows from the same argument as in the proof of part (a). □

Proof of Proposition 7

Since $\tilde{p}_n$ is fixed, the total revenue of both platforms is also fixed and equals to $\Lambda(\tilde{s}_1 p_1^* + \tilde{s}_2 p_2^* + \tilde{s}_n \tilde{p}_n)$. Hence, maximizing the total profit of both platforms is equivalent to optimizing the driver allocation policy α so as to minimize the total wage $K_1 f_1(\tilde{u}_1) + K_2 f_2(\tilde{u}_2)$, where $\tilde{u}_i = \Lambda(\tilde{s}_i/n_1 + \alpha_i \tilde{s}_n/\tilde{n})/K_i$ ($\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha_1$). By rewriting $\tilde{u}_i$ in terms of u_i^*, we obtain $\tilde{u}_i = u_i^* \left(1 + \tilde{s}_n \left[\frac{\alpha_i n_i}{\tilde{n} s_i^*} - 1\right]\right)$. Therefore, the driver allocation policy that minimizes the total wage is given by:

$$\alpha^* = \arg\min_{\alpha \in [0,1]} \left\{ K_1 f_1 \left(u_1^* \left(1 + \tilde{s}_n \left[\frac{\alpha n_1}{\tilde{n} s_1^*} - 1 \right] \right) \right) + K_2 f_2 \left(u_2^* \left(1 + \tilde{s}_n \left[\frac{(1-\alpha) n_2}{\tilde{n} s_2^*} - 1 \right] \right) \right) \right\}.$$

Since $f_i(\cdot)$ $(i=1,2)$ is convex, one can easily solve for α^*. In particular, observe that $K_1 f_1\left(u_1^*\left(1+\tilde{s}_n\left[\frac{\alpha n_1}{\tilde{n} s_1^*}-1\right]\right)\right)+K_2 f_2\left(u_2^*\left(1+\tilde{s}_n\left[\frac{(1-\alpha) n_2}{\tilde{n} s_2^*}-1\right]\right)\right)$ is supermodular in (α, u_1^*) and submodular in (α, u_2^*). Therefore, by Topkis' Theorem, α^* is decreasing in u_1^* and increasing in u_2^*. In other words, there exist two thresholds $\underline{\theta}$ and $\bar{\theta}$, such that $\alpha^*=1$, if $u_1^*/u_2^* \leq \underline{\theta}$; $\alpha^* \in (0,1)$ if $u_1^*/u_2^* \in (\underline{\theta}, \bar{\theta})$; and $\alpha^*=0$ if $u_1^*/u_2^* \geq \bar{\theta}$.

Under the optimal driver allocation scheme α^*, the total cost of both platforms $K_1 f_1(\tilde{u}_1)+K_2 f_2(\tilde{u}_2)$ is minimized, i.e., the total profit $\tilde{\pi}_1(\tilde{p}_n, p_1^*, p_2^*|\gamma)+\tilde{\pi}_2(\tilde{p}_n, p_1^*, p_2^*|\gamma)$ is maximized. Then, by using the same argument as in the proof of Proposition 2, we can find a range of profit sharing parameters $(\gamma_l, \gamma_h) \subset (0,1)$, such that if $\gamma \in (\gamma_l, \gamma_h)$, then $\tilde{\pi}_i(\tilde{p}_n, p_1^*, p_2^*|\gamma)$ $(i=1,2)$ is maximized with $\alpha=\alpha^*$. This concludes the proof of Proposition 7. $\square$

Proof of Proposition 8

Since the platforms do not change the prices of their original services (p_1^*, p_2^*), we have:

$$\begin{aligned} \tilde{RS}(\tilde{p}_n, p_1^*, p_2^*) =& \Lambda \mathbb{E}\Big[\max\{q_1-p_1^*+\xi_1, q_2-p_2^*+\xi_2, q_n-\tilde{p}_n+\xi_n, \xi_0\}\Big] \\ >& \Lambda \mathbb{E}\Big[\max\{q_1-p_1^*+\xi_1, q_2-p_2^*+\xi_2, \xi_0\}\Big]=RS(p_1^*, p_2^*). \quad \square \end{aligned}$$

Proof of Proposition 9

First, we consider the case with $\tilde{n} \leq \bar{n}$. This implies that $\underline{\alpha} \leq \bar{\alpha}$. Note that $\tilde{DS}_i(\tilde{p}_n, p_1^*, p_2^*) \geq DS_i(p_1^*, p_2^*)$ if and only if $\frac{\Lambda \tilde{s}_i/n_i+\alpha_i \Lambda \tilde{s}_n/\tilde{n}}{K_i} \geq \frac{\Lambda s_i}{n_i K_i}$. On the other hand, we have:

$$\frac{\Lambda \tilde{s}_i/n_i+\alpha_i \Lambda \tilde{s}_n/\tilde{n}}{K_i}-\frac{\Lambda s_i}{n_i K_i}=\frac{\Lambda \tilde{s}_n}{K_i}\left(\frac{\alpha_i}{\tilde{n}}-\frac{s_i^*}{n_i}\right), \text{ where } \alpha_1=\alpha \text{ and } \alpha_2=1-\alpha.$$

Since $\tilde{n} \leq \bar{n}$ and $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $\frac{\alpha_i}{\tilde{n}}-\frac{s_i^*}{n_i} \geq 0$ and therefore, $\frac{\Lambda \tilde{s}_i/n_i+\alpha_i \Lambda \tilde{s}_n/\tilde{n}}{K_i} \geq \frac{\Lambda s_i}{n_i K_i}$ for $i=1,2$, which implies that $\tilde{DS}_i(\tilde{p}_n, p_1^*, p_2^*) \geq DS_i(p_1^*, p_2^*)$ for $i=1,2$.

Second, when $\tilde{n}>\bar{n}$, we have $\underline{\alpha}>\bar{\alpha}$. Furthermore, if $\alpha<\underline{\alpha}$, $\frac{\alpha}{\tilde{n}}<\frac{s_1^*}{n_1}$. On the other hand, if $\alpha>\bar{\alpha}$, $\frac{1-\alpha}{\tilde{n}}<\frac{s_2^*}{n_2}$. As a result, $\underline{\alpha}>\bar{\alpha}$ implies that either $\alpha<\underline{\alpha}$ or $\alpha>\bar{\alpha}$, which further suggests that at least one of the following holds $\frac{\alpha}{\tilde{n}}<\frac{s_1^*}{n_1}$ or $\frac{1-\alpha}{\tilde{n}}<\frac{s_2^*}{n_2}$. Consequently, at least one of the following holds: $\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*)<DS_1(p_1^*, p_2^*)$ or $\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)<DS_2(p_1^*, p_2^*)$ $\square$

Proof of Proposition 10

First, we define the new objective function $\hat{\pi}(\tilde{p}_n):=\tilde{\pi}(\tilde{p}_n, p_1^*, p_2^*)+\tilde{DS}_1(\tilde{p}_n, p_1^*, p_2^*)+\tilde{DS}_2(\tilde{p}_n, p_1^*, p_2^*)$ as the sum of the total profits and driver surpluses. We then have:

$$\begin{aligned} \hat{\pi}(\tilde{p}_n) =& \Lambda p_1^* \tilde{s}_1+\Lambda p_2^* \tilde{s}_2+\Lambda \tilde{p}_n \tilde{s}_n-K_1 \tilde{u}_1 G_1^{-1}(\tilde{u}_1)+K_1 \mathbb{E} \max\{r_1, G_1^{-1}(\tilde{u}_1)\}-K_2 \tilde{u}_2 G_2^{-1}(\tilde{u}_2)+K_2 \mathbb{E} \max\{r_2, G_2^{-1}(\tilde{u}_2)\} \\ =& \Lambda p_1^* \tilde{s}_1+\Lambda p_2^* \tilde{s}_2+\Lambda \tilde{p}_n \tilde{s}_n-K_1 E_1(\tilde{u}_1)-K_2 E_2(\tilde{u}_2), \end{aligned}$$

where we define $E_i(\tilde{u}_i):=K_i \tilde{u}_i G_i^{-1}(\tilde{u}_i)-K_i \mathbb{E} \max\{r_i, G_i^{-1}(\tilde{u}_i)\}$ $(i=1,2)$. We next show that $E_i(\tilde{u}_i)$ is increasing and convex in $\tilde{u}_i$. We have: $E_i'(\tilde{u}_i)=K_i\Big[G_i^{-1}(\tilde{u}_i)+\frac{\tilde{u}_i}{g_i\left(G_i^{-1}(\tilde{u}_i)\right)}-\frac{\tilde{u}_i}{g_i\left(G_i^{-1}(\tilde{u}_i)\right)}\Big]=K_i G_i^{-1}(\tilde{u}_i)>0$, which is increasing in $\tilde{u}_i$. Therefore, $E_i(\cdot)$ is increasing and convex in $\tilde{u}_i$. As a result, $\hat{\pi}(\cdot)$ has exactly the same structure as $\tilde{\pi}(\cdot, p_1^*, p_2^*)$. Consequently, Proposition 10 follows from the same argument as in the proof of <u>Case 1</u> in Proposition 2. $\square$