# Inventory Commitment and Monetary Compensation Under Competition

Junfei Lei
Foster School of Business, University of Washington, Seattle, WA 98195, jlei@uw.edu

Fuqiang Zhang
Olin Business School, Washington University in St. Louis, Campus Box 1156, 1 Brookings Drive, St. Louis, MO 63130, fzhang22@wustl.edu

Renyu Zhang
CUHK Business School, The Chinese University of Hong Kong, philipzhang@cuhk.edu.hk

Yugang Yu
School of Management, University of Science and Technology of China, 96 Jinzhai Road, Hefei, China, 230026, ygyu@ustc.edu.cn

July 6, 2023

**Problem definition:** Inventory commitment and monetary compensation have been widely recognized as effective strategies in monopoly settings when customers are concerned about stockouts. To attract more customer traffic, a firm reveals its inventory availability information to customers before the sales season, or offers monetary compensation to placate customers if the product is out of stock. This paper investigates these two strategies when retailers compete on both price and inventory availability.

**Methodology/results:** We develop a game-theoretic framework to analyze the strategic interactions among the retailers and customers and draw the following insights. First, both inventory commitment and monetary compensation may lead to a prisoner's dilemma. Although these strategies are preferred regardless of the competitor's price and inventory decisions, the equilibrium profit of each retailer could be lower in the presence of inventory commitment or monetary compensation, because they would intensify the competition between the retailers. Second, we find that market competition may hurt social welfare compared to a centralized setting by reducing the product availability in equilibrium. The inventory commitment and monetary compensation strategies further intensify the competition between the retailers, therefore causing an even lower social welfare.

**Managerial implications:** Our study shows that, although inventory commitment and monetary compensation improve retailers' profit and social welfare under monopoly, these strategies should be used with caution under competition.

*Key words*: Inventory availability, retail competition, inventory commitment, monetary compensation

---

## 1. Introduction

Inventory commitment and monetary compensation are widely-adopted marketing strategies for firms when consumers are worried about potential stockouts. For example, a retailer may choose to reveal its inventory stocking quantity to customers (see Su and Zhang 2009). Many e-commerce firms, including BestBuy.com and Neweggs.com, offer real-time availability information in the store and online. Target and Walmart also allow consumers to check the inventory availability of a particular product at local stores using the zip code and the DPCI number (Morgan 2015). Moreover, consumers have access to technologies and applications that help them track product availability information. For instance, TrackAToy offers availability information for many retailers if consumers enter the product's name on the web page (see `http://www.trackatoy.com/`). An alternative strategy to placate consumers is to offer monetary compensation if the product is out of stock when they visit the store. Sloot et al. (2005) find that monetary compensation such as discount coupons, rain checks, and additional services are effective in placating consumers experiencing stockouts. Bhargava et al. (2006) report that MAP LINK (the U.S.'s largest map distributor), VERGE (a U.S. media network publisher), and IntelliHome (a U.S. smart home technology company) offer discounts of 2%, 5%, and 10%, respectively, for all backlogged items. Many car dealers provide price reductions if the automobile consumers choose is out of stock, whereas restaurants offer free dishes if consumers' original choices are sold out. Online retailers usually waive delivery fees if items are backlogged.

The strategies of inventory commitment and monetary compensation improve firms' profits in a monopoly market environment (see, e.g., Su and Zhang 2009). In the first place, the two strategies attract demand as they signal an assurance of high inventory quantity in stock. Consumers would not accept necessary up-front costs (i.e., time) to patronize a firm if the product they are looking for is out of stock. Moreover, the monetary compensation reimburses consumers when the product is out of stock, which encourages them to visit the firms even if there is a certain probability that the product is unavailable. These two strategies become even more important in a competitive marketplace as product availability is a key leverage to capture market demand, especially in the era of e-commerce and online shopping. For example, in December 2011, BestBuy.com canceled some online orders due to the overwhelming demand for hot product offerings. Soon after the cancellation, many customers moved to Amazon.com with a click of button, as reported by TradeGecko (Tao 2014).

Being aware of the importance of inventory availability, firms may compete aggressively to attract market demand by committing to high inventory quantity in stock and/or offering high compensation upon stockouts. Although the strategies of inventory commitment and monetary compensation have been acknowledged to benefit firms in monopoly settings, there has been little research studying their effectiveness in a competitive marketplace. On one hand, these strategies provide incentives for customers to visit the retailers and enhance their competitive edge. On the other hand, it is also possible that firms will battle to overcommit inventory quantities and/or provide higher compensation in order to win a larger market share under competition. This phenomenon is analogous to the "price war" in many industries that has economically devastated many small businesses. For example, as major airlines went toe-to-toe in matching and exceeding one another's reduced fares, the whole industry recorded a higher volume of air travel as well as an alarming record of profit losses (see https://hbr.org/2000/03/how-to-fight-a-price-war). Similarly, the two marketing strategies, if adopted by competing firms, may result in an escalated competition on product availability, and eventually lead to excess inventory in stock throughout the whole industry.

In view of the potential alarming impact of over-competition on product availability, this paper examines the inventory commitment and monetary compensation strategies under market competition. We model two competing retailers as newsvendors located at the endpoints of a Hotelling line market. Customers are uniformly distributed on the Hotelling line. As in the standard Hotelling model, customers incur a search cost to patronize a retailer. The closer a customer is located to a retailer, the less search cost she will incur. Before demand is realized, each retailer sets its price and inventory order quantity to maximize the expected profit. The prices are observable to customers and the other retailer, whereas the inventory order quantity is each retailer's private information. Individual customers choose which retailer to patronize based on product price, search cost, and belief about inventory availability. Under the inventory commitment strategy, a retailer credibly reveals its inventory order information to the public, whereas under the monetary compensation strategy, a retailer compensates the customers who cannot get the product due to stockouts.

We adopt the Rational Expectation Equilibrium (REE) framework to study the strategic interactions between retailers and customers under competition. A fraction of customers may switch to the other retailer once the focal one runs out of inventory. The retailers are competing on both price and inventory availability. In particular, the retailers' trade-off is between decreasing price (which implies low inventory availability) and increasing inventory availability (which requires a high price). We characterize the market equilibrium and deliver the following insights.

First, inventory commitment and monetary compensation may decrease retailers' profit under market competition. In the monopoly setting, it has been shown that the inventory commitment and monetary compensation strategies benefit the retailer in the presence of strategic customers, because they help mitigate the stockout risk for customers (Su and Zhang 2009). However, under market competition, if the inventory commitment option is available to retailers, a prisoner's dilemma may arise. Specifically, although both retailers have incentives to commit to an inventory order quantity, the equilibrium profits of both retailers may decrease if inventory commitment is adopted. Revealing inventory information to customers intensifies market competition and results in inventory overcommitment (overstocking), reducing profits for both retailers. Likewise, monetary compensation may prompt the retailer to overcompensate customers so as to signal high product availability, thus backfiring on the retailers and hurting their profits.

Second, we find that market competition may hurt social welfare in this problem setting. It has been well documented in the economics literature that competition can increase social welfare (see, e.g., Stiglitz 1981). In our problem setting, one may intuit that competition will enhance social welfare as well because competition lowers equilibrium prices. Our results, however, indicate that market competition leads to lower product availability and, therefore, reduces the social welfare. Moreover, inventory commitment and monetary compensation strategies, while improving social welfare in a monopoly market (e.g., Su and Zhang 2009), induce more aggressive market competition between the retailers, which further decreases social welfare.

The remainder of this paper is organized as follows. Section 2 positions this paper in the relevant literature. The model is introduced in Section 3. Sections 4 and 5, respectively, study the value of the inventory commitment and monetary compensation strategies under inventory availability competition. The analysis of customer surplus and social welfare is given in Section 6. Finally, Section 7 concludes this paper. All the proofs are presented in the Appendix.

## 2. Literature Review

The impact of inventory availability has been extensively studied in the operations management literature. Dana and Petruzzi (2001) consider a newsvendor model where consumers are concerned about inventory availability and can choose whether to visit the firm. Su and Zhang (2008) introduce the strategic waiting behavior of customers into the newsvendor setting and investigate the impact of such behavior on a firm's pricing and stocking decisions. Liu and Van Ryzin (2008) demonstrate that one can mitigate the strategic waiting behavior by limiting inventory availability over repeated selling horizons. Since then, there have been a growing number of operations studies that involve strategic customer behavior and availability considerations in various settings. For example, Su and Zhang (2009) and Cachon and Feldman (2015) further include a search cost for the stockout-conscious customers. Cachon and Swinney (2009, 2011) focus on the value of quick response under strategic customer behavior. Prasad et al. (2014), Li and Zhang (2013), and Wei and Zhang (2017a) investigate the advance selling strategy where product availability may affect customers' optimal timing of purchase. Allon and Bassamboo (2011) use a cheap talk framework to quantify the value of providing inventory availability information to customers; Liang et al. (2014) examine a firm's product rollover strategies under consumers' forward-looking behavior. With variable assortment depth, Bernstein and Martínez-de Albéniz (2016) study the optimal dynamic product rotation strategy in the presence of strategic customers. Tereyagoglu and Veeraraghavan (2012) study a retailer's problem when selling to conspicuous consumers whose consumption utility depends on the availability of the product. Finally, Gao and Su (2016) study the role of inventory availability in the context of omni-channel retailing. Wei and Zhang (2017b) provide a recent review of this line of research. Despite the fast growth of this topic, the majority of studies in the literature focus on single-firm settings; our paper, instead, contributes to the above literature by studying the impact of product availability in a competitive setting.

In a competitive marketplace, if a stockout occurs at one firm, unsatisfied demand may switch to the other firms. Such stockout-based substitution has also received significant attention in the operations management literature. Lippman and McCardle (1997) propose several ways to model demand allocation between competing newsvendors and show that competition leads to overstocking relative to the centralized solution. Netessine and Rudi (2003) develop a tractable model to compare inventory management under centralized vs. decentralized control. Several studies extend the static substitution model to dynamic ones; see, for example, Bassok et al. (1999), Shumsky and Zhang (2009), and Yu et al. (2015). This line of research does not explicitly model individual customer behavior, which is a key focus of our work. Therefore, our paper differs from this research in terms of both the model setting and insights.

Another stream of papers studies the competition in product availability in the economics literature. Carlton (1978) is among the first to formally consider the issue of product availability in a competitive market and argues that only an equilibrium outcome with zero firm profit will arise. As a follow-up to Carlton's work, Deneckere and Peck (1995) consider a game where firms can decide on both price and capacity and demonstrate that a pure-strategy equilibrium exists if and only if the number of firms is sufficiently large. Lei (2015) studies a similar integrated newsvendor and Hotelling model but with asymmetric unit costs. He finds that firms with the lowest unit cost may survive in the long run. Along this line of research, Daughety and Reinganum (1991) and Dana (2001) are the closest to our paper. Specifically, Daughety and Reinganum (1991) consider a setting where consumers have imperfect information on both price and stocking levels at firms. An important finding is that in equilibrium, the duopoly price is lower than the monopoly price if consumers' search cost is low, while the duopoly price is the same as the monopoly price if consumers' search cost is high. In contrast, we find that retailers may charge a strictly *higher* price to signal high product availability and thus attract more demand in the presence of market competition. Dana (2001) adopts a newsvendor setup to model retailers competing on product availability. It has been shown that the retailers can enjoy a positive profit (i.e., they can charge a price higher than marginal cost) even though the products are perfectly substitutable because the retailers can signal a high probability of product availability using a high price. Our paper also uses a similar newsvendor paradigm, but with several important differences. First, we use the Hotelling setup to incorporate heterogeneous travel costs of customers, which leads to different insights. Second, we examine the impact of availability competition on customer surplus, while Dana (2001) focuses on the equilibrium outcome from the firms' perspectives. Finally, we also study the effectiveness of operational strategies such as stockout compensations and inventory commitment, which are absent from the above economics literature.

The economics literature has also studied the impact of competition on customer surplus. For example, Brynjolfsson et al. (2003) summarize two channels of how the market competition of product variety improves consumer surplus. In their study of the online bookstores market, the increased product variety competition induces around \$ 3 million more consumer welfare in 2000. In the food industry, Hausman and Leibtag (2007) empirically verify that the entry of new business and the expansion of existing business improve average consumer surplus by approximately 25%. Goolsbee and Petrin (2004) show that the competition between broadcast satellites (DBS) and cable leads to a consumer welfare gain of \$2.5 billion for satellite buyers and \$3 billion for cable subscribers. Our contribution to this literature is that we demonstrate the adverse effect of market competition on social welfare if customers are concerned about inventory availability.

## 3. Model

To study the inventory commitment and monetary compensation strategies under inventory availability competition, we build our model upon the classical newsvendor and Hotelling frameworks. The newsvendor setup captures the key features of demand uncertainty and perishable inventory, which are common for a retail setting where the inventory availability concern is most relevant. The Hotelling model highlights the competition between the retailers (he) under heterogeneous tastes/preferences of the customers (she). These salient features are often ignored in the literature studying inventory availability. Moreover, due to demand uncertainty, customers may patronize the other retailer upon the stockout of the first retailer she visits, which we refer to as the *customer switching behavior*. We will first study a *base model* without the customer switching behavior. Then, we extend the base model by considering the customer switching behavior, which we refer to as the *focal model*.

### 3.1. Base Model Without Customer Switching

We model the market as a Hotelling line with a unit length, denoted by $\mathcal{M} = [0,1]$. Two retailers, $R_i$ ($i = 1,2$), are located at the two endpoints of the Hotelling market $\mathcal{M}$. Without loss of generality, we assume $R_i$ is located at $i-1$ ($i = 1,2$). Each retailer sells a substitutable product that has the same procurement cost $c$. Retailer $R_i$ chooses a stock quantity $q_i$ and charges a price $p_i$ to maximize his own expected profit.

In the market $\mathcal{M}$, customers are uniformly distributed over the interval $[0,1]$. Each customer has an infinitesimal mass, and purchases at most one unit of the product. The valuation of the product to all customers is homogeneous and denoted $v$. The aggregate market demand $D$ (i.e., the total mass of the Hotelling line) is uncertain and follows a known distribution $F(\cdot)$. We assume that the demand distribution has an increasing failure rate, which can be satisfied by most commonly used distributions. For conciseness, we define $\mathbb{E}[\cdot]$ as the expectation operation and $x \wedge y := \min(x, y)$ as the minimum operation. To visit a retailer, each customer incurs a search cost that increases linearly with her distance. More specifically, the search cost of a customer located at $x \in \mathcal{M}$ to visit $R_1$ (resp. $R_2$) is $sx$ (resp. $s(1-x)$), where $s$ is the unit distance search cost. The search cost can also be interpreted as a disutility for a customer traveling to a retailer to obtain the product: The longer the distance between the customer and her focal retailer, the larger the disutility she incurs to purchase this product. Furthermore, to highlight the competition between the two retailers, we assume that the unit distance search cost $s$ is not too high such that all customers will consider visiting the focal retailer as well as switching to the alternative retailer upon stockout.[1] Finally, each customer aims to maximize her expected payoff by choosing a retailer to visit.

The sequence of events unfolds as follows. At the beginning of the sales season, each retailer $R_i$ simultaneously decides his stocking quantity $q_i$ and announces the retail price $p_i$. Both the inventory level and the price cannot be adjusted throughout the sales horizon. Customers observe the prices $(p_1, p_2)$, but not the inventory levels $(q_1, q_2)$, and decide which retailer to visit (or not to visit any of them). The demand $D_i$ for retailer $R_i$ is realized as a result of customers' cumulative purchasing decisions. If $D_i \leq q_i$, all customers requesting the product can get one. Otherwise, $D_i > q_i$, stockout occurs, and customers not receiving the product leave the market. Finally, the transactions occur and the retailers collect the revenues.

### 3.2. Base Model Equilibrium

Next, we analyze the equilibrium of the base model without customer switching. To this end, we adopt the Rational Expectations Equilibrium (REE) concept, which is commonly used in the game-theoretic analytical models in the operations literature (see, e.g., Cachon and Swinney 2009, Li and Jain 2016, Anand and Goyal 2019, Aviv et al. 2019). Under the REE, customers, upon observing the prices $(p_1, p_2)$, form beliefs about inventory availability and make purchasing decisions to maximize their own expected utilities, whereas retailers (at the beginning of the sales horizon) base their pricing and inventory decisions on the anticipations of customers' cumulative purchasing behaviors to maximize profits. Furthermore, under equilibrium, both the customers' beliefs about inventory availability and the retailers' anticipations should be consistent with the actual outcomes. The formal definition of REE will be specified below.

[1] If the unit search cost $s$ is too high, then either the model reduces to two monopoly markets without competition, or the customers will not consider switching upon stockout. The former situation is uninteresting, while the latter is essentially the base model. If $s$ is moderate, then only part of the customers will switch. The analysis and insights in this setting will be similar to those in our base model, so we omit the details for brevity.

**Customers' Problem.** We first analyze the customers' problem. Consider a customer located at $x \in \mathcal{M}$. Her surplus to visit $R_1$ is $v - p_1 - sx$ (resp. $-sx$) if the product is in stock (resp. out of stock). A similar analysis can be applied if she visits $R_2$. The customer gains zero surplus if she does not visit any retailer. Since customers cannot observe retailers' inventory status, they form a belief about it (see Dana 2001). To facilitate the analysis, we assume customers form beliefs about the (unobservable) inventory availability probability instead of order quantity because the influence of inventory stocking quantity on the expected utility (and thus the purchasing behavior) of a customer boils down to the availability probability it induces. Specifically, let $\theta_i(p_1, p_2) \in [0,1]$ be the in-stock probability of $R_i$ given the price, where $i = 1, 2$. Thus, the expected utility of a customer located at $x$ to visit $R_1$ (resp. $R_2$) is $\mathcal{U}_1(x) := (v - p_1)\theta_1(p_1, p_2) - sx$ (resp. $\mathcal{U}_2(x) := (v - p_2)\theta_2(p_1, p_2) - s(1 - x)$).

Customers base their purchasing decisions on the beliefs of product availability. More specifically, a customer chooses to visit the retailer from which she can earn a higher non-negative expected payoff. Since a customer is infinitesimal, without loss of generality, a customer located at $x$ will patronize $R_i$ if $\mathcal{U}_i(x) \geq \mathcal{U}_{-i}(x)$ $(i = 1, 2)$. Therefore, there exists a threshold $x(p_1, p_2)$ such that a customer located at $x$ will patronize $R_1$ if $x \leq x(p_1, p_2)$ and will patronize $R_2$ if $x \geq x(p_1, p_2)$. Simple algebraic manipulation yields

$$x(p_1, p_2) := \mathcal{P}_{[0,1]}\left(\frac{1}{2} - \frac{(v - p_2)\theta_2(p_1, p_2) - (v - p_1)\theta_1(p_1, p_2)}{2s}\right) \in [0, 1],$$

where $\mathcal{P}_{[0,1]}(x) = \max\{0, \min\{x, 1\}\}$ is the projection on to interval $[0, 1]$. Note that we focus on the more interesting case with market share competition in this paper by assuming that the search cost $s$ is not too high so that the market is fully covered in equilibrium.

**Retailer's Problem.** Next, we analyze the retailer's pricing and inventory problem. Each retailer strategizes his price and inventory decisions in anticipation of customers' purchasing behaviors (thus his market share). Specifically, the demand for $R_1$ (resp. $R_2$) is $x(p_1, p_2)D$ (resp. $(1 - x(p_1, p_2))D$). Given the competitor's price, the retailer $R_i$'s profit maximization problem is:

$$\max_{(p_i, q_i)} \{p_i \mathbb{E}(\alpha_i(p_1, p_2)D \wedge q_i) - cq_i\},$$

where $\alpha_1(p_1, p_2) = x(p_1, p_2)$ and $\alpha_2(p_1, p_2) = 1 - x(p_1, p_2)$ represent the respective market shares. Therefore, given price $p_i$, retailer $R_i$'s optimal inventory order strategy is the newsvendor solution: $q_i = \alpha_i(p_1, p_2)F^{-1}\left(\frac{p_i - c}{p_i}\right)$.

To characterize the REE, we need to specify the off-equilibrium customer belief on inventory availability (see, e.g., Dana 2001). Moreover, we refine the off-equilibrium belief to rule out implausible equilibria. Consistent with the equilibrium refinement strategy of Dana (2001), customers rationally believe that the retailers are stocking the optimal amount of inventory given any observed price. Specifically, given the price $(p_1, p_2)$, the customers believe that the inventory order quantity of retailer $R_i$ is $q_i = \alpha_i(p_1, p_2)F^{-1}\left(\frac{p_i - c}{p_i}\right)$. Conditioned on the existence of a customer, her belief about the total demand for the retailer $R_i$ is a random variable with probability density function $g_i(y|p_1, p_2) := \frac{y}{\alpha_i(p_1,p_2)\mu} f\left(\frac{y}{\alpha_i(p_1,p_2)}\right)$, where $\mu := \mathbb{E}[D]$ (see, e.g., Dana 2001, Su and Zhang 2009). Since the customers simultaneously decide whether and which retailer to patronize, each customer holds an identical belief about the inventory availability for $R_i$. Therefore, the belief of the customers about $R_i$'s inventory availability probability is: $\theta_i(p_1, p_2) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_i - c}{p_i}\right)\right) \mathrm{d}F(y)$. This belief is also supported by the uniform rationing rule, i.e., upon stockout the retailer's inventory is randomly allocated to each customer who visits him. Note that the product availability belief only depends on the price of the focal retailer. We remark that this is driven by our equilibrium refinement rule that customers believe the retailers will stock the optimal newsvendor inventory, which induces a service level that depends on the price of the focal retailer only. For the subsequent analysis, we shall use $\theta^*(p_i) := \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_i - c}{p_i}\right)\right) \mathrm{d}F(y) = \theta_i(p_1, p_2)$ to denote customers' belief about $R_i$'s inventory availability.

**Equilibrium.** We are now ready to characterize the equilibrium price and inventory decisions of the retailers. Under the REE and given the competing retailer's price $p'$, the focal retailer's best price response, $p^*(p') = \arg\max_{0\le p\le v} \Pi_i(p, p')$, can be solved by the following:

$$\begin{aligned} \max_{0\le p\le v} \quad & \Pi(p, p') = p_i\mathbb{E}\left[\alpha(p, p')D \wedge q(p, p')\right] - cq(p, p') \\ \text{s.t.} \quad & q(p, p') = \alpha(p, p')F^{-1}\left(\frac{p - c}{p}\right), \\ & \alpha(p, p') = \frac{1}{2} + \frac{(v - p)\theta^*(p) - (v - p')\theta^*(p')}{2s}. \end{aligned} \tag{1}$$

In particular, if the equilibrium outcome is symmetric, the two retailers charge the same equilibrium price and, thus, we have $p^* = p^*(p^*)$. The next proposition characterizes the existence and uniqueness of the REE in the base model.

PROPOSITION 1. *There exists a unique REE in the base model. The equilibrium is symmetric and denoted $(p^*, q^*, \theta^*(\cdot))$. Moreover, we have $p^* = \arg\max_{0\le p_i\le v} \Pi_i(p_i, p^*)$, $q^* = \frac{1}{2}F^{-1}\left(\frac{p^* - c}{p^*}\right)$, and $\theta^*(p^*) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p^* - c}{p^*}\right)\right) \mathrm{d}F(y)$. Each retailer covers half of the entire market, i.e., $\alpha(p^*, p^*) = \frac{1}{2}$.*

Proposition 1 demonstrates that a unique REE exists. Furthermore, the REE is symmetric. In particular, the two retailers cover the entire market competitively, each with a 50% market share. Note that this symmetric equilibrium outcome shares a similar structure as in the standard Hotelling model without demand uncertainty (see Lemma 3 in Appendix B). We may compare the equilibrium price of our model to that of the standard Hotelling model.

Proposition 2. *The equilibrium price in our base model is higher than that in the Hotelling model, i.e., $p^* \geq p_d^*$, where $p_d^* = s + c$ is the equilibrium price of the Hotelling model (see Lemma 3 in Appendix B).*

Compared to the standard Hotelling model with deterministic demand, the retailers offer a higher equilibrium price when customers are concerned about product availability. The intuition of this result can be explained as follows. In the Hotelling model, the two retailers compete on offering low price to attract customers. However, in our base model, the retailers compete on both price and product availability simultaneously. Since a high price signals a high product availability, prompting the retailers to raise the price, the competition on price is alleviated. Therefore, the equilibrium price in our model is higher than that of the Hotelling model. This result also implies that inventory availability can serve as an operational lever to gain a competitive edge in the market, which softens the price competition between the retailers and thus leads to higher prices under equilibrium.

### 3.3. Focal Model with Customer Switching

In this subsection, we present our *focal model* with the customer switching behavior upon stockout. In this model, there are two customer segments in the market: non-switching customers and switching customers. We define these two customer segments as follows:

- *Non-switching customers.* The non-switching customers share identical behaviors as those characterized in the base model. If the product is out of stock at their focal retailer, the non-switching customers will leave the market directly.
- *Switching customers.* The switching customers consider visiting the competing retailer $R_{3-i}$ for substitutes upon the stockout of the focal retailer $R_i$ $(i = 1, 2)$.

The probability for a customer to be of the switching (resp. non-switching) type is denoted by $\gamma \in (0,1)$ (resp. $1-\gamma$), which is irrespective of her location $x$. Furthermore, to highlight the impact of customer switching, we assume in the focal model that the unit search cost $s$ is sufficiently small such that any switching customer will visit the competing retailer for substitutes upon the stockout of the focal retailer.

The switching behavior makes our problem more challenging. It has been documented in the literature that general dynamic demand substitution problems could be intractable and thus approximation approaches are needed (see, e.g., Mahajan and Van Ryzin 2001, Karaesmen and Van Ryzin 2004). One approach to analyzing our problem is to approximate it with a two-stage model accounting for the customer switching behavior upon stockout. Specifically, in the first stage, all customers visit their focal retailers who satisfy the demand using the initial in-stock inventory. In the second stage, the non-switching customers leave the market. In contrast, the switching customers will visit the competing retailer for substitutes if the focal retailer is out of stock. Finally, the retailers satisfy the demand from the customers who switch from their competitors using the remaining inventory left from the first stage. As can be seen, customers may hold two beliefs on the inventory availability probability for each retailer in the two-stage model. One is the *ex-ante* belief on retailer's inventory availability in the first stage when demand will be satisfied by the retailer's initial inventory in stock. The other is the *ex-post* belief on the retailer's inventory availability in the second stage with the knowledge that the demand will be satisfied by using the remaining inventory after satisfying the demand in the first stage. We provide a detailed analysis of the *ex-ante* and *ex-post* beliefs on the retailer's inventory availability in Appendix C. However, the market equilibrium analysis based on the two-stage model is technically intractable. To restore tractability, we make the following assumption for the focal model with customer switching.

ASSUMPTION 1. *(a) The travel time along the Hotelling line is negligible, i.e., a switching customer can immediately visit the other retailer upon the stockout of her focal retailer. (b) The retailers apply the same uniform rationing rule to all customers because they cannot distinguish the switching customers from the original customers.*

The key implication of Assumption 1 is that the switching and original customers arrive and make purchase decisions *simultaneously*, so they share the same probability of getting the product from a retailer. Therefore, all the customers hold the same belief on a retailer's product availability regardless of their location and switching status. As a consequence, given the open price information $(p_1, p_2)$, all customers hold the same inventory availability belief $\theta_i(p_1, p_2)$ for retailer $R_i$, $i = 1, 2$. Intuitively, Assumption 1 does not change the nature of the switching behavior but makes customers more likely to switch. Therefore, we conjecture that relaxing Assumption 1 would lead to weaker but similar qualitative insights.

We now examine switching customers' decision problems given their availability beliefs under Assumption 1. Similar to the base model, let $\theta_i(p_1, p_2)$ represent the in-stock probability of $R_i$ given prices, where $i = 1, 2$. Consider a representative switching customer at location $x$, facing stockout at $R_1$ (resp. $R_2$). She would then switch to $R_2$ (resp. $R_1$) for a substitute and earn a payoff $(v - p_2)\theta_2(p_1, p_2) - s(1 - x)$ (resp. $(v - p_1)\theta_1(p_1, p_2) - sx$). Therefore, the expected net surplus of the customer from switching to $R_2$ (resp. $R_1$) upon the stockout at $R_1$ (resp. $R_2$) is $\mathcal{U}_{12}(x) = (v - p_2)\theta_2(p_1, p_2) - s(1 - x)$ (resp. $\mathcal{U}_{21}(x) = (v - p_1)\theta_1(p_1, p_2) - sx$). Next, we examine the switching customer's choice of visiting the focal retailers by evaluating her expected *ex-ante* utility. For a switching customer at location $x$, her expected utility to visit $R_1$ (resp. $R_2$) with the product being available is $v - p_1 - sx$ (resp. $v - p_2 - s(1 - x)$). Instead, if the product is out of stock, the customer may switch to $R_2$ (resp. $R_1$) with an expected surplus $-sx + \mathcal{U}_{12}(x)$

(resp. $-s(1-x)+\mathcal{U}_{21}(x)$).[2] Hence, the expected total utility of a switching customer located at $x$ to visit $R_1$ (resp. $R_2$) first is $\mathcal{U}_1(x)=(v-p_1)\theta_1(p_1,p_2)-sx+(1-\theta_1(p_1,p_2))\mathcal{U}_{12}(x)$ (resp. $\mathcal{U}_2(x)=(v-p_2)\theta_2(p_1,p_2)-s(1-x)+(1-\theta_2(p_1,p_2))\mathcal{U}_{21}(x)$). Since a customer opts to first visit the focal retailer from which she can earn a higher expected (total) utility and will switch to the competing retailer upon stockout, the customer will patronize $R_i$ first if $\mathcal{U}_i(x)\geq\mathcal{U}_{3-i}(x)$. Based on the arguments above, there exists a threshold

$$x_s(p_1,p_2):=\mathcal{P}_{[0,1]}\left(\frac{\theta_1(p_1,p_2)}{\theta_1(p_1,p_2)+\theta_2(p_1,p_2)}\cdot\left(1+\frac{(p_2-p_1)\theta_2(p_1,p_2)}{s}\right)\right)\in[0,1],$$

such that a switching customer located at $x$ will first patronize $R_1$ (resp. $R_2$) and then switch to $R_2$ (resp. $R_1$) upon stockout if $x\leq x_s(p_1,p_2)$ (resp. $x>x_s(p_1,p_2)$), where $\mathcal{P}_{[0,1]}(x)=\max\{0,\min\{x,1\}\}$ is the projection on to interval $[0,1]$.

Finally, following the same paradigm of rational expectation equilibrium, we derive that a customer's belief on retailer $R_i$'s inventory availability probability is given by $\theta_i(p_1,p_2)=\theta^*(p_i)$ ($i=1,2$). For tractability, we focus on the *symmetric* REE in the presence of customer switching. Specifically, we consider the case where, under equilibrium, both retailers charge the same price $p_s^*$, capture the same market size $\alpha_s^*$, and the customers hold the same beliefs about product availability $\theta^*(p_s^*)$. Therefore, we have $\alpha_s^*=\alpha_s(p_s^*)=\gamma\left(1-\frac{1}{2}\theta^*(p_s^*)\right)+\frac{1-\gamma}{2}$.

We are now ready to characterize the symmetric equilibrium price and inventory decisions of the retailers for the focal model with customer switching. Under the REE, the equilibrium price $p_s^*=\underset{0<p<v}{\arg\max}\,\Pi_s(p,p_s^*)$ is solved by the following:

$$\begin{aligned}\max_{0\leq p\leq v}\quad & \Pi_s(p,p_s^*)=p\mathbb{E}\left[\alpha_s(p,p_s^*)D\wedge q(p,p_s^*)\right]-cq(p,p_s^*)\\ \text{s.t.}\quad & q(p,p_s^*)=\alpha_s(p,p_s^*)F^{-1}\left(\frac{p-c}{p}\right),\\ & \alpha_s(p,p_s^*)=\gamma\alpha_1(p,p_s^*)+(1-\gamma)\alpha_2(p,p_s^*),\\ & \alpha_1(p,p_s^*)=\frac{\theta^*(p)\theta^*(p_s^*)}{\theta^*(p)+\theta^*(p_s^*)}\left(1+\frac{(p_s^*-p)\theta^*(p_s)}{s}\right)+(1-\theta^*(p_s^*)),\\ & \alpha_2(p,p_s^*)=\frac{1}{2}+\frac{(v-p)\theta^*(p)-(v-p_s^*)\theta^*(p_s^*)}{2s}.\end{aligned}\tag{2}$$

Note that $\alpha_1(p,p_s^*)$ and $\alpha_2(p,p_s^*)$ represent the market sizes from switching customers and non-switching customers, respectively. In particular, $\alpha_2(p,p_s^*)$ is exactly the same as the market

[2] We detail the derivation of the customer search cost as follows. Assume a customer is at location $x$ and her focal retailer is $R_1$. (1) If the product is in stock, the total travel distance from the $x$ to the focal retailer and then back to $x$ is $2x$. (2) If the product is out of stock, the total travel distance should be: (i) the distance from the origin point $x$ to the first destination $R_1$ plus (ii) the distance from $R_1$ to the competing retailer $R_2$ and plus (iii) the distance from $R_2$ to the origin point $x$. Therefore, the total travel distance is $x+1+(1-x)=2$. Therefore, if the product is out of stock in the focal retailer, the extra travel distance for a customer located at $x$ is $2-2x=2(1-x)$. Without loss of generality, by redefining the unit search cost as $s/2$ per unit distance, the customer's extra search cost is $s(1-x)$ upon the stockout at the focal retailer.

share function, $\alpha(p, p^*)$, in (1). Compared to problem formulation (1) without customer switching, $\alpha_s(p, p^*)$ in problem (2) represents the total market size from both the switching customers and the non-switching customers. Therefore, the retailer's total demand includes non-switching customers, switching customers who first visit the retailer, and switching customers who switch to the retailer for substitutes. The next proposition characterizes the REE in the focal model.

PROPOSITION 3. *(a) There exists a unique symmetric REE $(p_s^*, q_s^*, \theta^*(\cdot))$ in the focal model.*
*(b) Under equilibrium, we have $p_s^* = \arg\max_{0\le p\le v} \Pi(p, p_s^*)$ (see (2) when $\frac{(v-p_s^*)\theta^*(p_s^*)}{s} \ge 1$), $q_s^* = \alpha_s^* F^{-1}\left(\frac{p_s^*-c}{p_s^*}\right)$, $\alpha_s^* = \gamma\left(1 - \frac{\theta^*(p_s^*)}{2}\right) + \frac{1-\gamma}{2}$ and $\theta^*(p_s^*) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_s^*-c}{p_s^*}\right)\right) f(y)\mathrm{d}y$.*

If $\gamma = 0$ (i.e., all customers belong to the non-switching type), the symmetric REE in Proposition 3 reduces to the one characterized by Proposition 1 in the base model without customer switching. Note that Proposition 3 focuses on the symmetric REE. The focal model with substitution-driven customer switching may have asymmetric equilibria. Consider the case where $R_1$ charges a high price and $R_2$ charges a low price. As a consequence, $R_1$ (resp. $R_2$) induces a small (resp. large) market size and attracts a fraction (resp. all) of the customers who face stockout at the other retailer (i.e., when $\frac{(v-p_s^*)\theta^*(p_s^*)}{s} < 1$). An asymmetric equilibrium may be sustained in this setting with certain model primitives. For example, the two retailers charge two different prices $p_1^* > p_2^*$ in an asymmetric equilibrium, with $\alpha_1^* < \alpha_2^*$. Moreover, it is possible that the switching customers search for substitutes from one retailer only, and thus, the profits of the two retailers may not necessarily be equal under the asymmetric REE. For the rest of this paper, we will focus our analysis on the inventory commitment and monetary compensation strategies under the symmetric REE in the model *with customer switching*. This enables us to capture the essential implications of customer switching without getting trapped by technical intractability. To conclude this section, we remark that Proposition 2 can be readily extended to the focal model with the same intuition, i.e., $p_s^* \ge p_d^*$. In the focal model with stockout-driven substitution, the inventory availability remains an operational lever that softens the price competition and, hence, increases the equilibrium prices.

## 4. Inventory Commitment

Inventory commitment is a commonly-used *ex-ante* strategy (i.e., it is used before demand realization) in the presence of availability-concerned customers (see, e.g., Su and Zhang 2009). Under this strategy, the retailer should credibly announce its order quantity to the public. For example, Amazon.com recently provided a lightning deal platform to allow retailers to promote their products. A salient feature of "lightning deals" is that sellers have to announce the amount of inventory to customers. In particular, a customer can see a real-time status bar on the web page of the seller indicating the current price, inventory, and percentage of units that have already been claimed by other customers. In some other circumstances, a retailer has to publicize its inventory information to customers, even if he is not willing to do so by himself. For instance, the affiliated stores of Great Clips (a hair salon franchise in the United States and Canada) post their real-time information of available slots online. Customers can check the anticipated waiting times of all stores in their area and add their names to the waitlist before actually visiting the salon. In this case, the competing franchised stores are forced to reveal their available inventory information.

It has been shown in the literature that the inventory commitment strategy benefits the monopoly retailer (e.g., Su and Zhang 2009). In this section, we strive to analyze this strategy in a competitive market. Our results imply that the inventory commitment strategy may lead to an undesirable prisoner's dilemma: Although both retailers will voluntarily reveal their inventory information under equilibrium, the equilibrium profit of each retailer will be lower than in the focal model where the retailers cannot credibly announce the order quantity information. Therefore, the inventory commitment strategy may not serve as an effective tool for retailers in a competitive market.

We now formally model the inventory commitment strategy in our duopoly market. We use subscript $v$ to represent the model with inventory commitment. At the beginning of the sales horizon, the competing retailers first decide whether to reveal the inventory information to the public (i.e., whether to adopt the inventory commitment strategy). Then, the retailers will announce prices and order inventory accordingly. If a retailer commits to publicizing its inventory information, he will truthfully announce its order quantity to the whole market. Finally, the customers observe the prices of the retailers and the amount of inventory ordered by the retailer who adopts the inventory commitment strategy, and decide which retailer to visit. As in the focal model, we adopt the REE framework to analyze the equilibrium market outcome. There are three cases to consider: (i) Both retailers do not reveal the inventory order quantities, which is essentially the focal model; (ii) Both retailers adopt the inventory commitment strategy; (iii) One retailer adopts the inventory commitment strategy whereas the other one does not reveal its inventory. Section 3 presents a detailed analysis for case (i). Below we analyze cases (ii) and (iii).

**Both Retailers Adopt the Inventory Commitment Strategy.** Under inventory commitment, individual customers do not need to form beliefs about inventory availability, but directly optimize their purchasing decisions after observing both prices and inventory stocking quantities. Specifically, after observing retailer $R_i$'s price $p_i$ and stocking quantity $q_i$, where $i \in \{1, 2\}$, customers estimate the in-stock probability of each retailer conditional on her existence. Similar to the focal model, there exists a threshold for non-switching customers, $x(p_1, q_1, p_2, q_2)$, such that the non-switching customers with $x \leq x(p_1, q_1, p_2, q_2)$ (resp. $x > x(p_1, q_1, p_2, q_2)$) will visit retailer $R_1$ only (resp. retailer $R_2$ only). For switching customers, there exists another threshold, $x_s(p_1, q_1, p_2, q_2)$, such that the switching customers with $x \le x_s(p_1,q_1,p_2,q_2)$ (resp. $x > x(p_1,q_1,p_2,q_2)$) visit retailer $R_1$ (resp. retailer $R_2$) first, and then switch to retailer $R_2$ (resp. retailer $R_1$) upon stockout. Here, we focus on the case where the search cost $s$ is sufficiently low to induce full market coverage with competition and customer switching. As in the focal model, a retailer's total market size includes non-switching customers who visit the retailer directly, switching customers who first visit the retailer, and switching customers who switch from the competing retailer due to stockout. Specifically, the market size of $R_1$ (resp. $R_2$) is $\alpha_1 = \gamma[x_s(p_1,q_1,p_2,q_2) + (1 - x_s(p_1,q_1,p_2,q_2))(1-\theta_2)] + (1-\gamma)x(p_1,q_1,p_2,q_2)$ (resp. $\alpha_2 = \gamma[1 - x_s(p_1,q_1,p_2,q_2) + x_s(p_1,q_1,p_2,q_2)(1-\theta_1)] + (1-\gamma)[1 - x(p_1,q_1,p_2,q_2)]$). Algebraic manipulation yields that

$$\begin{cases} \alpha_{1,v} = \gamma\left\{\frac{\theta_1\theta_2}{\theta_1+\theta_2}\left(1+\frac{\theta_2}{s}(p_2-p_1)\right) + (1-\theta_2)\right\} + (1-\gamma)\left\{\frac{1}{2} + \frac{(v-p_1)\theta_1-(v-p_2)\theta_2}{2s}\right\} \\ \alpha_{2,v} = \gamma\left\{1 - \frac{\theta_1^2}{\theta_1+\theta_2}\left(1+\frac{\theta_2}{s}(p_2-p_1)\right)\right\} + (1-\gamma)\left\{\frac{1}{2} + \frac{(v-p_2)\theta_2-(v-p_1)\theta_1}{2s}\right\} \end{cases} \tag{3}$$

where the subscript $v$ denotes the model under the inventory commitment strategy and $\gamma$ represents the portion of switching customers in the market. Similar to the focal model, it suffices to characterize the perceived inventory availability probabilities at the purchasing thresholds. Define $\theta_{1,v}$ (resp. $\theta_{2,v}$) as the perceived inventory availability probability for a customer located at the threshold to visit $R_1$ (resp. $R_2$). Then, we have $\theta_{i,v} = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \left(\frac{q_i}{\alpha_{i,v}}\right)\right) f(y)\mathrm{d}y$, where $\alpha_{i,v}$ is the market size of retailer $R_i$ defined by (3). Denote retailer $R_i$'s profit as $\Pi_v(p_i,q_i) = p_i\mathbb{E}[\alpha_{i,v}(p_i,q_i)D \wedge q_i] - cq_i$. As discussed above, we focus on the symmetric REE $(p_v^*, q_v^*, \theta_v^*(\cdot))$, where $p_v^*$ is the equilibrium price, $q_v^*$ is the equilibrium order quantity, and $\theta^*(\cdot)$ is the equilibrium belief of product availability.

We focus on the case when the search cost $s$ is sufficiently small, so the market is fully covered by the two retailers with competition, and all switching customers will switch to the competing retailer upon stockout. The two retailers compete on price and order quantity to win the market. The following proposition characterizes the equilibrium outcome if both retailers commit to revealing their inventory information under market competition.

Proposition 4. *If both retailers adopt the inventory commitment strategy, the following statements hold:*

*(a) There exists a unique symmetric REE* $(p_v^*, q_v^*, \theta_v^*(\cdot))$.

*(b) Under equilibrium, we have* $(p_v^*, q_v^*) = \underset{0\le p\le v, q\ge 0}{\arg\max}\, \Pi_v(p,q)$ *subject to the constraints* $(p_v^* - p)\theta_v = sx_s(1+\theta_v/\theta) - s/\theta$ *and* $(v-p)\theta - sx = (v-p_v^*)\theta_v - s(1-x)$, *where* $\theta = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \left(\frac{q}{\alpha}\right)\right) f(y)\mathrm{d}y$, $\theta_v = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \left(\frac{q_v^*}{\alpha'}\right)\right) f(y)\mathrm{d}y$, $\alpha = \gamma(x_s + (1-x_s)(1-\theta_v)) + (1-\gamma)x$, *and* $\alpha' = \gamma(1 - x_s + x_s(1-\theta)) + (1-\gamma)(1-x)$. *Moreover, each retailer's market size is* $\alpha_v^* = \gamma(1 - \frac{1}{2}\theta_v^*) + \frac{1-\gamma}{2}$, *where* $\theta_v^* = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \left(\frac{q_v^*}{\alpha_v^*}\right)\right) f(y)\mathrm{d}y$.

Proposition 4 implies that the equilibrium outcome of the scenario where both retailers adopt the inventory commitment strategy shares the same structure as that of the model with customer switching formulated by Eq. (2).

**Incentive for Inventory Commitment** Next, we show that inventory commitment is a dominating strategy for each retailer. As a consequence, the equilibrium outcome will be that both retailers voluntarily publicize their inventory, charge the price $p_v^*$, and order $q_v^*$ units of inventory as prescribed by Proposition 4.

With retailer $R_1$ as the focal retailer, we shall consider both the case where retailer $R_2$ credibly announces $q_2$ and the case where retailer $R_2$ does not reveal his order quantity. Given retailer $R_2$'s price and inventory decision $(p_2, q_2)$, we use $\Pi_{i,j}$ $(i,j \in \{d,v\})$ to denote the maximum profit of retailer $R_1$ if he adopts strategy $i$ and retailer $R_2$ adopts strategy $j$, where subscript $d$ refers to no inventory commitment and subscript $v$ refers to inventory commitment. For example, $\Pi_{d,v}$ refers to the maximum profit of retailer $R_1$ if he does not adopt the inventory commitment strategy and retailer $R_2$ adopts this strategy. The derivations of $\Pi_{i,j}$ $(i,j \in \{v,d\})$ are given in the proof of Lemma 1, Appendix D.

Lemma 1. *For any $(p_2, q_2)$ set by retailer $R_2$, we have $\Pi_{v,d} > \Pi_{d,d}$ and $\Pi_{v,v} > \Pi_{d,v}$.*

Lemma 1 suggests that, if the competing retailers can credibly reveal their inventory information to the market, adopting the inventory commitment strategy would be a *dominating* strategy for each of the retailers, regardless of the price and inventory decisions of the competitor. Therefore, the equilibrium outcome of the market under the inventory commitment *option* is that both retailers voluntarily reveal their inventory order quantity. Lemma 1 also reveals an important actionable insight for firms in a competitive market where customers are concerned about inventory availability: Credibly communicating the inventory stocking information to customers helps gain an edge for such firms. Our next result examines the profit implication of the inventory commitment strategy under market competition. We use $\Pi_v^*$ (resp. $\Pi^*$) to denote the equilibrium profit of a retailer with (resp. without) the inventory commitment strategy.

Proposition 5. *If the retailers have the option to credibly announce their inventory information, the following statements hold:*

*(a) Under equilibrium, both retailer $R_1$ and retailer $R_2$ adopt the inventory commitment strategy.*

*(b) There exist a threshold $\bar{s}_v$ for the search cost and a threshold $\bar{c}_v$ for the unit procurement cost such that if $s < \bar{s}_v$ and $c < \bar{c}_v$, then we have $\Pi_v^* < \Pi^*$.*

As shown in Proposition 5, if both the search cost $s$ and the procurement cost $c$ are low (i.e., $s < \bar{s}_v$ and $c < \bar{c}_v$), and if the inventory commitment strategy is adopted, the inventory stocking quantity can directly influence the purchasing behaviors of the customers. Therefore, the competition between retailers may be intensified by this strategy. The retailers may overcommit to inventory in a competitive market, thus reducing the profit of each retailer. Recall that in our focal model, the stocking quantity is not observable to customers but can be signaled by price, so the only competitive leverage of a retailer is the prevailing price he charges. However, if the retailers can commit to their preannounced inventory order quantities, they have more flexibility to influence demand. Furthermore, the signaling power of price is diluted if the inventory information is directly available to customers. In particular, when the unit cost $c$ is high, the inventory commitment strategy helps the retailers increase the willingness-to-pay of the customers, thus attracting higher demand. On the other hand, if the unit cost is low, this strategy may backfire by triggering an overcommitment of stocking quantity. If, in addition, the market competition is intense (i.e., $s < \bar{s}_v$), each retailer will aggressively order a large amount of inventory to attract customers, which in turn exacerbates market competition and decreases the profits of both retailers ($\Pi_v^* < \Pi^*$ when $c < \bar{c}_v$ and $s < \bar{s}_v$). Therefore, when the procurement cost $c$ and the search cost $s$ are both low, the retailers are actually worse off in the presence of the inventory commitment option due to the induced inventory overcommitment and intensified market competition. Lemma 1 and Proposition 5 together deliver a new and interesting insight that the inventory commitment strategy may give rise to a prisoner's dilemma under market competition. Although this strategy is preferred by either retailer regardless of the competitor's inventory and price decisions, the retailers would be worse off if both adopt the inventory commitment strategy.

Our analysis demonstrates that the inventory commitment strategy does not always benefit the retailers under competition, which is in sharp contrast to the monopoly setting. There is a large body of research focusing on the inventory commitment strategy. A central message in the literature is that the inventory commitment strategy is beneficial for retailers. For example, Cachon and Swinney (2011) and Liu and Van Ryzin (2008) propose two-stage models to explore how to use availability information to manipulate customers' expectations and thus induce them to buy early. In a competitive market setting, revealing inventory information to customers may lead to a higher equilibrium price, and, as a result, improves the firms' profits (see Dana 2001, Carlton 1978, and Dana and Petruzzi 2001). In a supply chain setting, Su and Zhang (2008) demonstrate that the firm's profit can be improved by promising either that the available inventory will be limited (quantity commitment) or that the price will be kept high (price commitment). In a monopoly setting, Su and Zhang (2009) further show that the inventory commitment strategy offers customers information to more accurately assess their chances of securing the product. Thus, the inventory commitment strategy increases customers' willingness-to-pay and improves the profit of a monopoly firm. In a Hotelling competition setting, however, our results demonstrate that the inventory commitment strategy may give rise to a prisoner's dilemma and hurt the retailers.

Our results also deliver actionable insights for e-tailers. In today's digitalized business environment, customers have easy access to extensive product information with almost zero search cost (i.e., a very small $s$ in our model) . For example, customers can easily search online for product alternatives as well as price and inventory availability information. Our analysis shows that, although such information transparency attracts customers to visit retailers more frequently, the retailers may hurt themselves by revealing too much inventory availability information as a consequence of intensified market competition. This is because the inventory availability information has to be disclosed to the entire market, instead of being limited to the intended customers of the retailers (i.e., the customers whose location is closest to the retailers). Granados and Gupta (2013) summarize two practical approaches to present inventory information: (i) A retailer may only disclose whether a product is in stock or not; or (ii) he may choose to publicize his inventory stocking level only when it is low. Both approaches reveal the retailer's inventory availability information in an imperfect way to prevent his competitors from using such information to improve their margins (i.e., stocking more inventory to attract demand, see Dewan et al. (2007)). We indeed strengthen this insight by demonstrating that retailers should be cautious about triggering the war of implementing the fully transparent inventory strategy (i.e., the inventory commitment strategy) because the other competitors will copy the strategy and, eventually, backfire for all retailers when the customer search cost is low.

We complement our theoretical analysis with numerical experiments to further illustrate the impact of the inventory commitment strategy. We compare the equilibrium profits and stocking quantities in models with and without inventory commitment. In our numerical experiments, we set $\gamma = s = 0.1$, $v = 10$, and the market demand $D$ follows a Gamma distribution with mean 90 and standard deviation 30. Figures 1 and 2 plot the equilibrium profits and order quantities, respectively, for the focal model and the model with inventory commitment. Figure 1 shows that the equilibrium profit of a retailer will be lower in the presence of inventory commitment whenever the ordering cost $c$ is low. Figure 2 further demonstrates that, with inventory commitment, the retailers will order much more than they would have without revealing the inventory availability information to the market.

To conclude this section, we remark that, implementing the inventory commitment strategy relies heavily on the retailers' credibility in the market. That is, the retailers should be able to credibly reveal their order quantity information to their competitors and their customers in the market. Otherwise, if the retailers fail to credibly convince the market, the effect of inventory commitment will be diluted. In the next section, we analyze an *ex post* monetary compensation strategy that is applicable even without such commitment power.

## 5. Monetary Compensation

In this section, we proceed to analyze the widely-used monetary compensation strategy, which is an *ex post* strategy. After customers visit a retailer and find that the product is out of stock, the

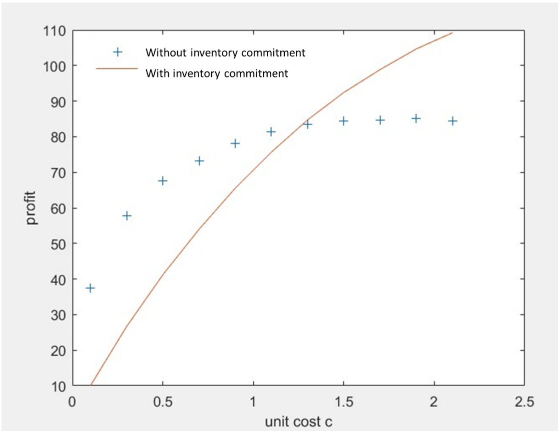


**Figure 1 Retailer profits in equilibrium.**

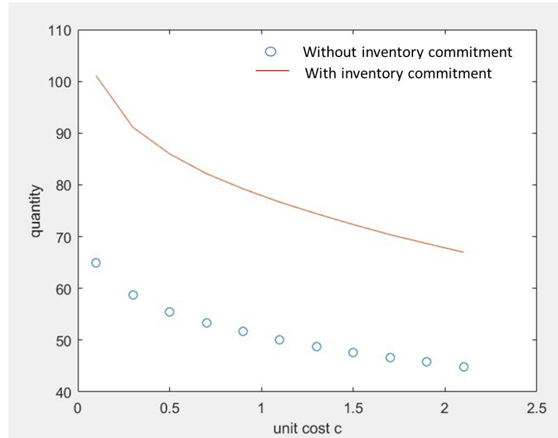


**Figure 2 Retailer order quantities in equilibrium.**

retailer will compensate them for such inconvenience. This strategy could reassure the customers in the presence of potential stockouts, thus motivating customers to visit the retailer. In practice, the compensation is offered in the form of coupons, gift cards, price discounts for future orders, and free shipping opportunities. For example, FoodLand offers consumers a rain check for the out-of-stock items. [3] The simplest and most direct compensation strategy is to placate customers for stockouts with cash, which we refer to as the monetary compensation strategy. In this section, we focus on studying the effect of monetary compensation under competition and substitution-based customer switching.

The monetary compensation strategy has proven beneficial to a monopoly retailer (see Su and Zhang 2009). In a competitive market, however, the story is different. Our analysis below shows that, when monetary compensation is an option, competing retailers will (voluntarily) overcompensate customers to attract higher demand, which in turn decreases their profits compared with the baseline setting where monetary compensation is not allowed.

To model the monetary compensation strategy, we assume that each retailer offers a compensation $m_i \geq 0$ ($i \in \{1,2\}$) to customers who face stockouts. The special case where $m_i = 0$ refers to that $R_i$ does not offer monetary compensation. So both retailers have the flexibility to decide whether to offer monetary compensation upon stockouts and the amount of compensation. As in the focal model, customers observe the retailers' prices and monetary compensation terms, but not their stocking quantities. The retailers set the price and stocking quantity to maximize their profits, whereas customers choose to purchase the product to maximize their expected surplus. In particular, each non-switching customer decides to visit a focal retailer only, and each switching customer chooses a focal retailer to make a purchase first and then switches to the other retailer for

[3] See `https://www.foodland.com/if-i-have-coupon-product-out-stock-may-i-receive-rain-check-product` for more details.

substitutes upon stockout. Following the same equilibrium analysis paradigm as in the focal model and the model with inventory commitment, we consider the symmetric REE in the model with monetary compensation. We use the subscript $c$ to denote the model with monetary compensation.

We first re-examine the purchase decisions of the non-switching customers. For a non-switching customer located at $x$, she will visit the retailer that yields a higher non-negative expected payoff and receive monetary compensation upon stockout. Hence, the customer's expected payoff is $(v-p_1)\theta_1+m_1(1-\theta_1)-sx$ (resp. $(v-p_2)\theta_2+m_2(1-\theta_2)-s(1-x)$), where $\theta_1$ (resp. $\theta_2$) represents the customer's belief about $R_1$'s (resp. $R_2$'s) inventory availability probability. Indeed, a rigorous definition of the inventory availability probability is $\theta_i(p_1,p_2,m_1,m_2)$, $i\in\{1,2\}$, which is a function of prices and monetary compensations. For conciseness, we drop the argument and use $\theta_i$ to represent retailer $R_i$'s inventory availability probability ($i\in\{1,2\}$) in the analysis.

Next, we examine the purchase decisions of the switching customers. If the product is out of stock at the focal retailer $R_1$ (resp. $R_2$), a switching customer (located at $x$) will switch to retailer $R_2$ for a substitute with an expected surplus $\mathcal{U}_{1,2}=(v-p_2)\theta_2+m_2(1-\theta_2)-s(1-x)$ (resp. $\mathcal{U}_{2,1}=(v-p_1)\theta_1+m_1(1-\theta_1)-sx$). Note that we have assumed a sufficiently small search cost $s$ to ensure that all switching customers will switch for substitutes upon stockout. Similarly, each switching customer chooses to first visit a focal retailer that yields a higher non-negative expected payoff and receive monetary compensation upon stockout. Hence, the customer's *total* expected payoff is $\mathcal{U}_1=(v-p_1)\theta_1+m_1(1-\theta_1)-sx+\mathcal{U}_{1,2}(1-\theta_1)$ (resp. $\mathcal{U}_2=(v-p_2)\theta_2+m_2(1-\theta_2)-s(1-x)+\mathcal{U}_{2,1}(1-\theta_2)$) if she visits $R_1$ (resp. $R_2$) first.

Now, we are ready to formulate the retailer $R_i$'s decision problem, where $i=1,2$:

$$\max_{(p_i,m_i,q_i)} \Pi_{i,c}(p_i,m_i,q_i)=p_i\mathbb{E}(\alpha_{i,c}D\wedge q_i)-m_i\mathbb{E}(\alpha_{i,c}D-q_i)^+-cq_i,$$

where market size $\alpha_{1,c}$ and $\alpha_{2,c}$ are the following:

$$\begin{cases} \alpha_{1,c}= & \gamma\left\{\frac{\theta_2\theta_1}{\theta_1+\theta_2}\left(\frac{(p_2+m_2)-(p_1+m_1)}{s}\theta_2+1\right)+\frac{\theta_2(m_1\theta_2-m_2\theta_1)}{s(\theta_1+\theta_2)}-(1-\theta_2)\right\} \\ & +(1-\gamma)\left\{\frac{1}{2}+\frac{[v-(p_1+m_1)]\theta_1-[v-(p_2+m_2)]\theta_2+(m_1-m_2)}{2s}\right\}, \\ \alpha_{2,c}= & \gamma\left\{\frac{\theta_2\theta_1}{\theta_1+\theta_2}\left(\frac{(p_1+m_1)-(p_2+m_2)}{s}\theta_1+1\right)+\frac{\theta_1(m_2\theta_1-m_1\theta_2)}{s(\theta_1+\theta_2)}-(1-\theta_1)\right\} \\ & +(1-\gamma)\left\{\frac{1}{2}+\frac{[v-(p_2+m_2)]\theta_2-[v-(p_1+m_1)]\theta_1+(m_2-m_1)}{2s}\right\}. \end{cases} \tag{4}$$

Thus, retailer $R_i$ orders the newsvendor quantity $q_{i,c}=\alpha_i^cF^{-1}\left(\frac{p_i+m_i-c}{p_i+m_i}\right)$, where $i=1,2$. Recall that the retailers adopt uniform rationing, so the in-stock probability for a customer is $\theta_{i,c}=\theta_c^*(p_i+m_i)=\frac{1}{\mu}\int_{y=0}^{\infty}\min\left(y\wedge F^{-1}\left(\frac{p_i+m_i-c}{p_i+m_i}\right)\right)\mathrm{d}F(y)$. Here, the customer belief in retailer $R_i$'s inventory availability depends on $(p_i,m_i)$ through the effective margin $p_i+m_i$.

We denote the symmetric REE as $(p_c^*,m_c^*,q_c^*,\theta_c^*(\cdot))$, where $p_c^*$ is the equilibrium price, $m_c^*$ is the equilibrium compensation, $q_c^*$ is the equilibrium order quantity, and $\theta_c^*(\cdot)$ is the equilibrium product availability. Moreover we focus on the case where the search cost $s$ is sufficiently small. The following proposition characterizes the REE in the presence of monetary compensation.

PROPOSITION 6. *For the model with monetary compensation, the following statements hold:*

*(a) There exists a unique symmetric REE, which we denote as* $(p_c^*, m_c^*, q_c^*, \theta_c^*(\cdot))$*.*

*(b) We have* $(p_c^*, m_c^*) = \underset{0\le p\le v, m\ge 0}{\arg\max}\ \Pi_c(p,m,q)$*, subject to* $q = \alpha(p,m)F^{-1}\left(\frac{p+m-c}{p+m}\right)$ *and* $\alpha(p,m)$ *(see 5). In equilibrium, we have* $q_c^* = \alpha_c^* F^{-1}\left(\frac{p_c^*+m_c^*-c}{p_c^*+m_c^*}\right)$, $\theta_c^* = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}\left(\frac{p_c^*+m_c^*-c}{p_c^*+m_c^*}\right)\right)\mathrm{d}F(y)$*, and each retailer has market size* $\alpha_c^* = \gamma\left(1-\frac{1}{2}\theta_c^*\right) + \frac{1-\gamma}{2}$*.*

To examine the impact of the monetary compensation on the retailers' profit, we denote the equilibrium profit of a retailer in the model with monetary compensation as $\Pi_c^*$ (the equilibrium profit of the retailers in the focal model is $\Pi^*$). The following proposition shows that monetary compensation may hurt the retailers if the market competition is intense.

PROPOSITION 7. *For the model with monetary compensation, there exists a critical threshold* $\bar{s}_c$ *such that if* $s < \bar{s}_c$*, we have* $\Pi_c^* < \Pi^*$*.*

Proposition 7 delivers an interesting message that, if the retailers have the option to offer monetary compensations upon stockout, they will earn a lower profit as long as the competition is sufficiently intense ($s \le \bar{s}_c$). This is in contrast with the effect of monetary compensation in the monopoly setting, which always benefits the retailer (see Su and Zhang 2009). By offering monetary compensation, the retailer, on one hand, is equipped with another lever in the competitive landscape; however, on the other hand, he competes more aggressively through direct subsidies to customers upon stockout. If the search cost, $s$, is large, the former effect dominates, which results in a higher profit in the presence of monetary compensation. If the search cost, $s$, is small, however, the latter effect dominates and monetary compensation leads to severe competition, which will in turn diminish the profit of each retailer. As a consequence, if the market competition is already fierce (i.e., $s$ is small), the monetary compensation option will further intensify the competition and hurt the retailers. In a similar spirit to the classical Hotelling model (see Lemma 3 in the Appendix D), the intensified competition induced by stockout compensations drags the equilibrium effective margin $p_c^* + m_c^*$ down to the marginal cost $c$ as $s$ approaches zero. Hence, if the unit travel cost $s$ is sufficiently small (i.e., the model proposed by Dana 2001), both retailers may earn zero profit in the presence of the monetary compensation option.

We next complement the finding in Proposition 7 with numerical results to further illustrate the impact of the monetary compensation strategy. In Figure 3, we compare the two retailers' equilibrium profits in models with and without monetary compensation. In Figure 4, we plot the retailers' monetary compensation in the equilibrium with monetary compensation. In our numerical

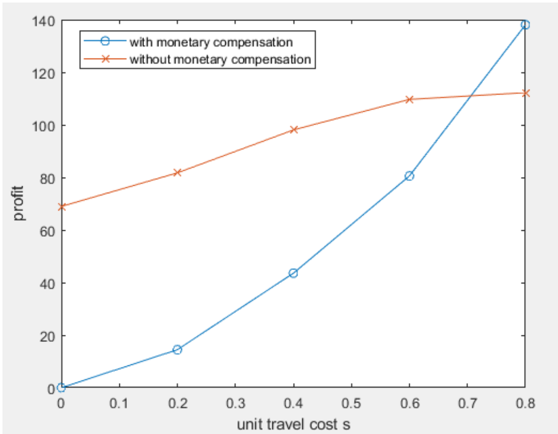


**Figure 3** **Retailer profits in equilibrium.**

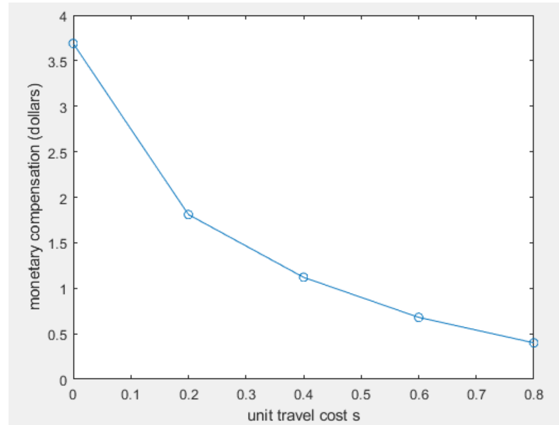


**Figure 4** **Equilibrium monetary compensation.**

studies, we set $\gamma = 0.1$, $c = 5$, $v = 10$, and the market demand $D$ follows a Gamma distribution with mean 90 and standard deviation 30. Figure 3 shows that the equilibrium profit will be lower in the presence of monetary compensation when the unit search cost $s$ is low. Figure 4 further shows that the retailers will compensate customers with a considerable amount of monetary compensation when the unit search cost $s$ is low, which indicates that the monetary compensation leads to overcompensation when the market is highly competitive.

In the existing literature, many studies have demonstrated that retailers can extract more profit by offering monetary compensation in a monopoly market. To convince customers of inventory availability, retailers adopt monetary compensation as a self-punishment mechanism upon stockouts. With such a mechanism, customers will anticipate a high service level and increase their willingness-to-pay, which in turn boosts the firm's profit. For example, Su and Zhang (2009) show that monetary compensation can increase the retailer's product availability in a monopoly model. For a competitive market environment, Kim et al. (2004) demonstrate that a capacity-reward program benefits the firms when market demands are non-stationary across periods. By offering this program, firms can effectively reduce excess capacities when market demand is low, and thus avoid intense price competition. Besides such a short-run effect, it is widely believed that monetary compensation also has a long-run effect to expand a firm's market share. Compensating customers upon stockouts has a positive effect on customers' shopping experience, and thus cultivates customer loyalty. In other words, by purposefully providing compensations for stockouts, retailers have the potential to increase their demand in the long run (see, e.g., Bhargava et al. 2006). Kim et al. (2001) further validate this viewpoint by showing that the firms should apply the most inefficient rewards (i.e., monetary compensation) if the market consists of a small portion of price-sensitive customers. Albeit the monetary compensation strategy has all these benefits, our results (i.e., Proposition 7), nevertheless, deliver a new insight that this strategy may backfire and lead to profit losses for the retailers. Similar results are also shown by Kopalle and Neslin (2001) when firms compete in a market with relatively fixed sizes.

We also remark that offering monetary compensation may cause a free-rider issue. Specifically, customers who are not interested in purchasing the product may still visit the retailer with the hope of being compensated, as long as the travel cost is not too high. These customers are referred to as free-riders. The free-riding behavior creates a moral hazard issue, so that retailers can hardly recognize their true customers. Fortunately, many marketing strategies and new technology tools can be used to alleviate, or even eliminate, the free-riding issue. For example, retailers may ask customers to claim their desired product in order to be eligible for compensation upon stockout. If the claimed product is out of stock and no substitute can match the customer's need, then a monetary compensation is offered. Otherwise, the customers cannot receive the monetary compensation. Another mechanism the retailers can use is to solicit more information from customers through cheap talk. Once the retailer verifies a customer's true motivation for purchasing the product, a monetary compensation can be awarded. Therefore, throughout our analysis, we assume that the free-riding behavior is negligible. This is consistent with the business practice in various industries where retailers effectively compensate customers' stockouts to induce a higher demand (see, e.g., Bhargava et al. 2006, Su and Zhang 2009).

Finally, we note that both inventory commitment and monetary compensation can be viewed as offering options that appeal customers. Other business strategies offered by competing firms to attract customers and induce higher demand have also been studied in the literature. For example, Chen et al. (2001) show that individual marketing by two competing firms can lead to a win-win competition even if the firms behave non-cooperatively and the market does not expand. Shin and Sudhir (2010) examine whether a firm should use behavior-based pricing (BBP) to discriminate between its own and competitors' customers in a competitive market. The paper finds that it is optimal to reward one's own customers under symmetric competition and BBP can increase profits with fully strategic and forward-looking consumers. Kim et al. (2001) shows that reward (promotion) programs weaken price competition because firms gain less from undercutting their prices, so the equilibrium prices go up in this case. In sum, whereas the strategies may benefit the competing firms for various reasons, we show that inventory commitment and monetary compensation intensify competition and may lead to a prisoner's dilemma and a lose-lose outcome.

## 6. Social Welfare Implications

In this section, we study two important questions regarding the social welfare of a competitive market. First, how does market competition impact social welfare? Second, what are the social welfare implications of inventory commitment and monetary compensation under competition?

We begin our analysis by quantifying the average customer surplus and social welfare in different models, starting with the focal model. Note that we will focus on the setting with full market competition and customer switching. Now, we introduce the *average* customer surplus for switching customers and non-switching customers, respectively. The switching customers will first visit their focal retailers and then switch to the competing retailer for substitutes. Under equilibrium, the expected surplus for a switching customer at $x$ is $\mathcal{U}_s(x) = (v-p_s^*)\theta(p_s^*) - sx + [(v-p_s^*)\theta(p_s^*) - s(1-x)](1-\theta(p_s^*))$. Since the two retailers are symmetric and the customers are uniformly distributed along the Hotelling line, the average surplus for switching customers is $2\int_0^{1/2}\mathcal{U}_s(x)dx = (v-p_s^*)\theta(p_s^*)(2-\theta(p_s^*)) - s\left(1-\frac{3}{4}\theta(p_s^*)\right)$. In contrast, the non-switching customers will visit their focal retailers only and leave the market upon stockout. Therefore, the expected surplus for a non-switching customer at $x$ is $\mathcal{U}(x) = (v-p_s^*)\theta(p_s^*) - sx$, which provides the non-switching customers' average surplus $2\int_0^{1/2}\mathcal{U}(x)dx = (v-p_s^*)\theta(p_s^*) - \frac{s}{4}$. Recall that the market consists of $\gamma$ portion of switching customers and $1-\gamma$ portion of non-switching customers, the average surplus for all customers is $CS^* = 2\gamma\int_0^{1/2}\mathcal{U}_s(x)dx + 2(1-\gamma)\int_0^{1/2}\mathcal{U}(x)dx = (v-p_s^*)\theta(p_s^*) - \frac{s}{4} + \gamma(1-\theta(p_s^*))\left[(v-p_s^*)\theta(p_s^*) - \frac{3s}{4}\right]$, where $p_s^*$ is the equilibrium price characterized by Proposition 3. The social welfare is the summation of the two retailers' profits and total customers' surplus, therefore, we have social welfare $SW^* = 2\Pi(p_s^*) + \mu \times CS^* = v\mu\theta(p_s^*) - cF^{-1}\left(\frac{p_s^*-c}{p_s^*}\right) - \frac{\mu s}{4} + \gamma(1-\theta(p_s^*))\left[v\mu\theta(p_s^*) - cF^{-1}\left(\frac{p_s^*-c}{p_s^*}\right) - \frac{3\mu s}{4}\right]$. Note that the equilibrium price $p_s^*$ plays a key role in determining the average customer surplus and social welfare as it explicitly influences the order quantity and inventory availability.

To explore the impact of inventory availability competition, we introduce a benchmark model where retailers at the two endpoints of the Hotelling line belong to a single firm and are managed in a centralized fashion. The firm optimizes price and inventory decisions of the two retailers to maximize their total profits. To ensure fair comparison, again the firms engage in the full market coverage and customers switch upon stockout because of the small unit search cost $s$. In the subsequent analysis, we will use subscript $b$ to denote the benchmark model. Analogous to the analysis of the focal model, the average customer surplus in the benchmark model is $CS_b^* = (v-p_b^*)\theta(p_b^*) - \frac{s}{4} + \gamma(1-\theta(p_b^*))\left[(v-p_b^*)\theta(p_b^*) - \frac{3s}{4}\right]$ and the social welfare is $SW_b^* = v\mu\theta(p_b^*) - cF^{-1}\left(\frac{p_b^*-c}{p_b^*}\right) - \frac{\mu s}{4} + \gamma(1-\theta(p^*))\left[v\mu\theta(p_b^*) - cF^{-1}\left(\frac{p_b^*-c}{p_b^*}\right) - \frac{3\mu s}{4}\right]$, where $p_b^* = v - \frac{s}{\theta(p_b^*)}$. It is worth noting that the customer surplus and social welfare share the same structure for the cases with and without competition, but with different equilibrium prices. Therefore, the key to understanding the impact of competition boils down to analyzing how it affects the equilibrium prices. The following lemma characterizes the impact of equilibrium price on customer surplus and social welfare:

LEMMA 2. *The following statements hold:*

*(a) The average customer surplus functions, $CS^*(p)$ and $CS_b^*(p)$, are concave in price $p$. In particular, the equilibrium price in the model with competition and customer switching, $p_s^*$, satisfies the condition $p_s^* \in [\hat{p}, v)$, where $\hat{p} = \arg\max_p CS^*(p)$.*

*(b) The social welfare functions, $SW^*(p)$ and $SW_b^*(p)$, are concave in price $p$.*

As shown by Lemma 2(a), the expected customer surplus functions in both models are concave in price. Moreover, the equilibrium price in the focal model is lower bounded by $\hat{p}$, which is the price that maximizes the average expected customer surplus. As a result, the customer's expected surplus is concavely decreasing in price under equilibrium. Lemma 2(b) shows that the social welfare functions are concave in price. Hence, as price increases, it will first improve social welfare as the high price signals a high product availability; later, social welfare declines as the retailers may overstock the product.

The impact of competition on customer surplus and social welfare is a well-studied topic in the economics literature. A general insight from this literature is that competition would improve customer surplus. For example, Brynjolfsson et al. (2003) summarize two mechanisms that drive market competition on product variety to improve consumer surplus. Increased market competition lowers market prices and expands product lines, both of which lead to increased customer surplus. However, the economics literature does not have a conclusive answer on how competition affects social welfare. Although many researchers have shown that competition may potentially improve social welfare, how market competition between firms could influence social welfare is still an open question because the benefits from customer surplus may not dominate the losses from firm profits (e.g., Stiglitz 1981). Our model incorporates the competition on both price and inventory availability. Recall that a high price can signal high product availability under equilibrium. Therefore, it is unclear apriori whether competition will drive retailers to lower prices to directly attract customers or to increase prices to indirectly signal high product availability. The following proposition addresses this question and characterizes the conditions under which either effect dominates.

Proposition 8. *Given full market coverage with competition and customer switching, we have (a) $CS^* \geq CS_b^*$ and (b) $SW^* \leq SW_b^*$.*

Proposition 8 shows that market competition benefits customers but hurts social welfare. This differs from the insight in some economics literature that competition will increase social welfare (see Stiglitz 1981) . To understand the rationale of Proposition 8, we identify two opposing effects. The first effect is referred to as the pricing effect, under which competition drives retailers to charge lower prices as a promotion to attract customers. The second effect is called the product availability effect, under which competition drives retailers to signal high inventory availability by increasing the prices. Specifically, as shown in Proposition 8(a), the retailers compete on capturing more market share by offering higher customer surplus and, thus, the market competition is beneficial to the customers. However, since the average customer surplus is decreasing in equilibrium price (see Lemma 2(a)), the retailers compete to offer lower prices in the market competition (the pricing effect dominates). In contrast, the social welfare might be increasing in equilibrium price (see Lemma 2(b)) as a high equilibrium price signals a high equilibrium product availability. Consequently, when retailers are competing on offering a lower price to attract more market share, the product availability decreases and, thus, social welfare decreases. In other words, although market competition improves the average customer surplus, the loss from retailers dominates the benefit from customers, so social welfare declines.

Another question we wish to address in this paper is how inventory commitment and monetary compensation strategies impact social welfare under competition. We now explore whether these two strategies can improve the *average* consumer surplus and social welfare under market competition. The equilibrium average consumer surplus and social welfare functions under the inventory commitment strategy are given by $CS_v^* = (v - p_v^*)\theta(p_v^*) - \frac{s}{4} + \gamma(1-\theta(p_v^*))\left[(v-p_v^*)\theta(p_v^*) - \frac{3s}{4}\right]$ and $SW_v^* = v\mu\theta(p_v^*) - cF^{-1}\left(\frac{p_v^*-c}{p_v^*}\right) - \frac{\mu s}{4} + \gamma(1-\theta(p_v^*))\left[v\mu\theta(p_v^*) - cF^{-1}\left(\frac{p_v^*-c}{p_v^*}\right) - \frac{3\mu s}{4}\right]$, respectively, where $v$ represents the case of inventory commitment strategy. Similarly, the equilibrium average consumer surplus and social welfare functions under the monetary compensation strategy are given by $CS_c^* = (v - p_c^*)\theta(p_c^*) - \frac{s}{4} + \gamma(1-\theta(p_c^*))\left[(v-p_c^*)\theta(p_c^*) - \frac{3s}{4}\right]$ and $SW_c^* = v\mu\theta(p_c^*) - cF^{-1}\left(\frac{p_c^*-c}{p_c^*}\right) - \frac{\mu s}{4} + \gamma(1-\theta(p_c^*))\left[v\mu\theta(p_c^*) - cF^{-1}\left(\frac{p_c^*-c}{p_c^*}\right) - \frac{3\mu s}{4}\right]$, respectively, where $c$ represents the case of monetary compensation strategy. Note that the compensation term $m_c^*$ will not directly affect the social welfare as it is a cash transfer between the retailers and customers. However, the compensation $m_c^*$ does impact the equilibrium average consumer surplus because customers who face stockout will be compensated.

PROPOSITION 9. *Under the inventory commitment or monetary compensation strategies, we have (a) $CS_v^* \geq CS^*$, and (b) $CS_c^* \geq CS^*$.*

Proposition 9 shows that, although inventory commitment and monetary compensation do not necessarily benefit retailers under competition, these strategies are always beneficial to customers. Both strategies provide incentives to attract customers to patronize the retailers and, as a consequence, benefit the customers once adopted by the retailers. It has also been shown in the operations literature that inventory commitment and monetary compensation strategies improve social welfare in a monopoly market (e.g., Su and Zhang 2009). However, we demonstrate in the following proposition that these strategies may induce the retailers to compete more aggressively on inventory availability, which turns out to further decrease social welfare under market competition.

PROPOSITION 10. *The following statements hold:*

*(a) Under the inventory commitment strategy, there exists a threshold $s_{vw}$ such that $SW_v^* < SW^*$ for $s < s_{vw}$.*

*(b) Under the monetary compensation strategy, there exists a threshold $s_{cw}$ such that $SW_c^* < SW^*$ for $s < s_{cw}$.*

Different from Proposition 9, Proposition 10 shows that inventory commitment and monetary compensation strategies may hurt social welfare under intense competition. Recall from Propositions 5 and 7 that, under intense competition, both strategies will backfire and decrease the profit and inventory availability probability of the retailers. A similar rationale applies to Proposition 10 as well. Since the inventory commitment and monetary compensation strategies provide an alternative channel in which the retailers could compete for market share, the equilibrium price and product availability may decline when market competition is intense. As a result, social welfare will decrease as well. Combining Propositions 5, 7, 9, and 10, we find that inventory commitment and monetary compensation strategies will always make customers better off but retailers worse off under intense market competition, with the former dominating the latter, so social welfare will decrease under these strategies in this case.

The above findings provide some practical insights for the central planner (e.g., industry regulator or the government). According to Lemma 2(b), social welfare is concave in price, so the central planner could set a price floor to restore the maximum social welfare (i.e., the maximum price in the market is set at the social-welfare-maximizing one).[4] Indeed, a properly set price floor may increase the retailer profit by mitigating price competition, which also induces higher equilibrium product availability and eventually improves social welfare. It is worth noting that the price floor also benefits the customers in the long run. Since the market competition lowers the retailer profit under equilibrium, the retailers may tacitly coordinate to avoid marketing competition and thus charge a high price in the repeated game (e.g., the benchmark equilibrium price without demand uncertainty, $p_b^*$), which will eventually hurt the customer's surplus. A carefully chosen price floor ensures the retailer profit under competition and, consequently, increases the cost of deviating to the "tacit coordination" (see Dufwenberg et al. 2007).

## 7. Conclusion

Inventory commitment and monetary compensation have been widely observed in practice. The literature has shown that these strategies could mitigate strategic customer behavior and enhance

[4] Let $p^*$ be the equilibrium price in the base model (the model without customer switching) and let $p_b^*$ be the price that achieves the maximum social welfare. According to the proof of Proposition 8, we have $p^* < p_b^*$. Since the social welfare function is concave in price (see Lemma 3), to restore the maximum social welfare, the market planner will set a price floor that equals $p_b^*$. As a result, the retailers will stop competing on offering lower prices to attract more market share at price $p = p_b^*$.

firm profit in a monopoly setting. This paper examines these strategies in a competitive setting when retailers compete on both price and inventory availability. Customers concerned about inventory availability may choose which retailer to patronize. Combining the newsvendor and Hotelling frameworks, we investigate the strategic interactions among the retailers and the customers. We derive market equilibrium price and inventory availability and quantify the impact of these strategies on firms' profitability, average consumer surplus, and social welfare. There are two main results from this research.

First, we find that both strategies may lead to a prisoner's dilemma: Although a retailer would benefit from either strategy regardless of the competitor's price and inventory decisions, both inventory commitment and monetary compensation will intensify market competition and hurt the retailers in a competitive market. This is in contrast to the common wisdom that these strategies improve the retailer profit under monopoly. Specifically, the inventory commitment strategy may dilute the signaling power of price, thus leading to overstock of inventory for the competing retailers, while the monetary compensation strategy tends to overcompensate customers. That is, both strategies will intensify market competition and, thus, reduce the profit of both retailers.

Second, our results indicate that with customers' product availability concerns, competition may decrease equilibrium retail prices compared to a centralized setting, which decreases product availability and social welfare. This contrasts the insight in the literature that competition generally improves social welfare. Furthermore, inventory commitment and monetary compensation may further intensify competition between the retailers and, as a consequence, decrease product availability and hurt social welfare. Therefore, although inventory commitment and monetary compensation are beneficial in monopoly settings, both retail firms and social planners should exert caution when applying these strategies in competitive market environments.

## References

Allon G, Bassamboo A (2011) Buying from the babbling retailer? the impact of availability information on customer behavior. *Management Science* 57(4):713–726.

Anand KS, Goyal M (2019) Ethics, bounded rationality, and ip sharing in it outsourcing. *Management Science* 65(11):5252–5267.

Aviv Y, Wei MM, Zhang F (2019) Responsive pricing of fashion products: The effects of demand learning and strategic consumer behavior. *Management Science* 65(7):2982–3000.

Bassok Y, Anupindi R, Akella R (1999) Single-period multiproduct inventory models with substitution. *Operations Research* 47:632– 642.

Bernstein F, Martínez-de Albéniz V (2016) Dynamic product rotation in the presence of strategic customers. *Management Science* 63(7):2092–2107.

Bhargava HK, Sun D, Xu SH (2006) Stockout compensation: Joint inventory and price optimization in electronic retailing. *INFORMS Journal on Computing* 18(2):255–266.

Brynjolfsson E, Hu Y, Smith MD (2003) Consumer surplus in the digital economy: Estimating the value of increased product variety at online booksellers. *Management Science* 49(11):1580–1596.

Cachon GP, Feldman P (2015) Price commitments with strategic consumers: Why it can be optimal to discount more frequently... than optimal. *Manufacturing & Service Operations Management* 17(3):399–410.

Cachon GP, Swinney R (2009) Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* 55(3):497–511.

Cachon GP, Swinney R (2011) The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior. *Management science* 57(4):778–795.

Carlton DW (1978) Market behavior with demand uncertainty and price inflexibility. *The American Economic Review* 68(4):571–587.

Chen Y, Narasimhan C, Zhang ZJ (2001) Individual marketing with imperfect targetability. *Marketing Science* 20(1):23–41.

Dana JD (2001) Competition in price and availability when availability is unobservable. *Rand Journal of Economics* 497–513.

Dana JD, Petruzzi NC (2001) Note: The newsvendor model with endogenous demand. *Management Science* 47(11):1488–1497.

Daughety AF, Reinganum JF (1991) Endogenous availability in search equilibrium. *The Rand Journal of Economics* 287–306.

Deneckere R, Peck J (1995) Competition over price and service rate when demand is stochastic: A strategic analysis. *The RAND Journal of Economics* 148–162.

Dewan RM, Freimer ML, Jiang Y (2007) A temporary monopolist: Taking advantage of information transparency on the web. *Journal of Management Information Systems* 24(2):167–194.

Dufwenberg M, Gneezy U, Goeree JK, Nagel R (2007) Price floors and competition. *Economic Theory* 33(1):211–224.

Gao F, Su X (2016) Omnichannel retail operations with buy-online-and-pick-up-in-store. *Management Science* .

Goolsbee A, Petrin A (2004) The consumer gains from direct broadcast satellites and the competition with cable tv. *Econometrica* 72(2):351–381.

Granados N, Gupta A (2013) Transparency strategy: Competing with information in a digital world. *MIS quarterly* 637–641.

Hausman J, Leibtag E (2007) Consumer benefits from increased competition in shopping outlets: Measuring the effect of wal-mart. *Journal of Applied Econometrics* 22(7):1157–1177.

Karaesmen I, Van Ryzin G (2004) Overbooking with substitutable inventory classes. *Operations Research* 52(1):83–104.

Kim BD, Shi M, Srinivasan K (2001) Reward programs and tacit collusion. *Marketing Science* 20(2):99–120.

Kim BD, Shi M, Srinivasan K (2004) Managing capacity through reward programs. *Management Science* 50(4):503–520.

Kopalle PK, Neslin S (2001) The economic viability of frequency reward programs in a strategic competitive environment. *Available at SSRN 265431* .

Lei J (2015) Market equilibrium and social welfare when considering different costs. *International Journal of Economics Research* 6(2):60–77.

Li C, Zhang F (2013) Advance demand information, price discrimination, and preorder strategies. *Manufacturing & Service Operations Management* 15(1):57–71.

Li KJ, Jain S (2016) Behavior-based pricing: An analysis of the impact of peer-induced fairness. *Management Science* 62(9):2705–2721.

Liang C, Cakanyildirim M, Sethi SP (2014) Analysis of product rollover strategies in the presence of strategic customers. *Management Science* 60(4):1022–1056.

Lippman SA, McCardle KF (1997) The competitive newsboy. *Operations Research* 45(1):54–65.

Liu Q, Van Ryzin GJ (2008) Strategic capacity rationing to induce early purchases. *Management Science* 54(6):1115–1131.

Mahajan S, Van Ryzin G (2001) Stocking retail assortments under dynamic consumer substitution. *Operations Research* 49(3):334–351.

Morgan C (2015) How to check inventory at your local target and walmart stores without leaving home. http://hip2save.com/2015/02/10/how-to-check-inventory-at-your-local-target-walmart-stores-without-leaving-home/.

Netessine S, Rudi N (2003) Centralized and competitive inventory models with demand substitution. *Operations Research* 51:329–335.

Prasad A, Stecke K, Zhao X (2014) Advance selling by a newsvendor retailer. *Production Oper. Management* 20(1):129–142.

Shin J, Sudhir K (2010) A customer management dilemma: When is it profitable to reward one's own customers? *Marketing Science* 29(4):671–689.

Shumsky R, Zhang F (2009) Dynamic capacity management with substitution. *Operations Research* 57:671–684.

Sloot LM, Verhoef PC, Franses PH (2005) The impact of brand equity and the hedonic level of products on consumer stock-out reactions. *Journal of Retailing* 81(1):15–34.

Stiglitz JE (1981) Potential competition may reduce welfare. *The American Economic Review* 71(2):184–189.

Su X, Zhang F (2008) Strategic customer behavior, commitment, and supply chain performance. *Management Science* 54(10):1759–1773.

Su X, Zhang F (2009) On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science* 55(5):713–726.

Tao R (2014) Out of stock problems? walmart, nike and best buy had them too, but here's how you can do better. https://www.tradegecko.com/blog/out-of-stock-problems-and-solutions-walmart-nike-bestbuy-case-studies.

Tereyagoglu N, Veeraraghavan S (2012) Selling to conspicuous consumers: Pricing, production, and sourcing decisions. *Management Science* 58(12):2168–2189.

Wei MM, Zhang F (2017a) Advance selling to strategic consumers: Preorder contingent production strategy with advance selling target. *Production Oper. Management* .

Wei MM, Zhang F (2017b) Recent research developments of strategic consumer behavior in operations management. *Computers & Operations Research* .

Yu Y, Chen X, Zhang F (2015) Dynamic capacity management with general upgrading. *Operations Research* 63:1372–1389.

# Online Appendices to "Inventory Commitment and Monetary Compensation under Competition"

## Appendix A: Summary of Notations

**Table 1 Summary of Notations**

| | |
|---|---|
| $R_i$ : | Retailer $i$ ($i = 1, 2$) |
| $p_i$ : | Price of Retailer $i$ |
| $q_i$ : | Inventory stocking quantity of Retailer $i$ |
| $\alpha_i$ : | Market share/size of Retailer $i$ |
| $\Pi_i$ : | Expected profit of Retailer $i$ |
| $D$ : | Market aggregate demand |
| $s$ : | Unit search cost |
| $c$ : | Procurement cost |
| $v$ : | Product valuation |
| $F(\cdot)$ : | Cumulative distribution function of demand; $\bar{F}(x) := 1 - F(x)$ |
| $f(\cdot)$ : | Density function of demand distribution |
| $x$ : | Customer location on the Hotelling line; $x \in \mathcal{M}$ and $\mathcal{M} = [0, 1]$ |
| $\mathbb{E}[\cdot]$ : | Expectation operation |
| $x \wedge y$ : | Minimum operation |
| $\theta_i$ : | Customers' (rational) expectation of $R_i$'s inventory availability probability |

## Appendix B: Deterministic Hotelling Model Benchmark

In this section, we introduce the classic Hotelling competition model with deterministic demand as the benchmark. The comparison between our base model and the deterministic benchmark could help us crystallize the impact of demand uncertainty and customers' availability concern.

We consider the same Hotelling line setup as the base model presented in Section 3.1 but with deterministic total market size. Specifically, we assume the aggregate market demand $D$ is deterministic and known to everyone in the market. Without loss of generality, we normalize $D = \mu$. In the absence of demand uncertainty, the retailers will order exactly the amount of their respective market share, so every customer will be able to obtain her requested product. The two retailers $R_1$ and $R_2$ determine their respective prices $p_1$ and $p_2$ to maximize their own profits, whereas each retailer choose whether and where to visit. As in the base model, we focus on the equilibrium under competition. Let $(p_d^*, q_d^*)$ be the equilibrium outcomes, where $p_d^*$ is the equilibrium price and $q_d^*$ is the equilibrium order quantity of each retailer. Similar to the base model in the main paper, it is straightforward to show that if $s$ is small, $R_1$ and $R_2$ can serve the entire market, each covering 50% of the customers. If, otherwise, $s$ is too large, there is essentially no competition between the two retailers and the market is not completely covered. Formally, we characterize the equilibrium prices (with competition) of the deterministic benchmark in the following lemma, which shows that the equilibrium price is increasing in $s$.

LEMMA 3. *Assume that $D = \mu$ with certainty. If $s < \frac{2(v-c)}{3}$, $p_d^* = s + c$ and $q_d^* = \frac{\mu}{2}$. Each retailer covers 50% of the market.*

**Proof of Lemma 3**

In a deterministic Hotelling model, the retailers compete on market share by charging their prices. Demand is determined and public knowledge to all players in the market, so there is no issue of product availability. Without loss of generality, we shall use retailer $R_1$ as an example in the analysis.

When the search cost $s$ is small, the two retailers cover the entire market. Given price $p_1$ and $p_2$, consumers located at $x \in [0,1]$ visit the retailer $R_1$ if $v - p_1 - sx \geq v - p_2 - s(1-x) \geq 0$. Thus, the retailer $R_1$ earns a market share $\frac{-p_1+p_2+s}{2s}$, and accordingly a profit $\pi_1(p_1) = (p_1 - c)\frac{-p_1+p_2+s}{2s}\mu$. Taking the first derivative of the profit function yields the retailer's best response function: $p_1^*(p_2) = \frac{p_2+s+c}{2}$. Since the two retailers are symmetric, retailer $R_2$ asks the same optimal price $p_2^*(p_1) = \frac{p_1+s+c}{2}$ to maximize his own profit. Note that the best response function, $p_i^*(p_{3-i})$, is increasing in price $p_{3-i}$, where $i \in \{1,2\}$, so there exists a unique equilibrium. In particular, the two retailers have the same optimal solutions in the equilibrium: $p_d^* = s + c$, $q_d^* = \frac{\mu}{2}$, and each covers half of the market. Finally, to guarantee $v - p_d^* - \frac{s}{2} > 0$, we obtain $s < \frac{2(v-c)}{3}$. □

## Appendix C: A Two-Stage Model with Customer Switching

In this section, we introduce a two-stage model with customer switching behavior upon stockout (i.e., without Assumption 1). There are two customer segments in the market: the switching customers (with proportion $\gamma$) and the non-switching customers (with proportion $1-\gamma$). In the first stage, both switching and non-switching customers visit their focal retailers and purchase the product if it is in stock. In the second stage, if the focal retailer is out of stock, the non-switching customers will directly leave the market, whereas the switching customers will switch to the other retailer for substitutes.

**Customers' Problem.** We first analyze the customers' problem in the second stage. Since the non-switching customers leave the market in the second stage, we only need to analyze the switching customers' decision problem. Consider a representative switching customer (at location $x$) who finds the product out of stock at her focal retailer, $R_1$ (resp. $R_2$). The customer would then switch to $R_2$ (resp. $R_1$) for a substitute in the second stage or leave the market. To avoid trivial analysis, we assume that the search cost, $s$, is small enough such that all switching customers will search for substitutes upon stockout (i.e., the switching customers earn a non-negative expected utility if switch to the competing retailers for substitutes). Therefore, the expected net surplus for the customer from switching to $R_2$ (resp. $R_1$) upon stockout at $R_1$ (resp. $R_2$) is: $U_{12}(x) = (v - p_2)\hat{\theta}_2(p_1, p_2) - s(1-x)$ (resp. $U_{21}(x) = (v - p_1)\hat{\theta}_1(p_1, p_2) - sx$), where $\hat{\theta}_i(p_1, p_2)$ is customers' belief on retailer $R_i$'s inventory availability in the second stage.

We next examine the switching customers' choice of visiting the focal retailers by evaluating her expected utility in the first stage. For a switching customer at location $x$, her utility to visit $R_1$ (resp. $R_2$) with the product being available is $v - p_1 - sx$ (resp. $v - p_2 - s(1-x)$). Instead, if the product is out of stock, the customer switches to $R_2$ (resp. $R_1$) with an expected surplus $-sx + U_{12}(x)$ (resp. $-s(1-x) + U_{21}(x)$). Hence, the expected total utility of a switching customer located at $x$ to visit $R_1$ (resp. $R_2$) in the first stage is $U_1(x) = (v - p_1)\theta_1(p_1, p_2) - sx + (1 - \theta_1(p_1, p_2))U_{12}(x)$ (resp. $U_2(x) = (v - p_2)\theta_2(p_1, p_2) - s(1 - x) + (1 - \theta_2(p_1, p_2))U_{21}(x))$, where $\theta_i(p_1, p_2)$ is customers' belief on retailer $R_i$'s inventory availability in the first stage. The customer chooses to first visit a focal retailer from which she can earn a higher total expected utility, i.e., $U_i(x) \geq U_{3-i}(x)$, and then switches to the competing retailer upon stockout. It is worth noting that the consumers' beliefs on retailer's inventory availability probabilities in two stages are different, $\theta_i(p_1,p_2) \neq \hat{\theta}_i(p_1,p_2)$, as the switching customers can update their beliefs upon stockout.

Finally, we examine the non-switching customer's choice of visiting the focal retailers. Similar to the base model, for a non-switching customer at location $x$, her expected utility to visit $R_1$ (resp. $R_2$) is $U_1(x) = (v-p_1)\theta_1(p_1,p_2) - sx$ (resp. $U_2(x) = (v-p_2)\theta_2(p_1,p_2) - s(1-x)$), where $\theta_i(p_1,p_2)$ is customers' belief on retailer $R_i$'s inventory availability in the first stage. Note that the non-switching and switching customers' beliefs on retailer's inventory availability probability are the same in the first stage, as they arrive at the focal retailers at the same time.

To summarize, there exists a threshold for the switching customers:

$$x_s(p_1,p_2) = \frac{\theta_1(p_1,p_2)}{\theta_1(p_1,p_2)+\theta_2(p_1,p_2)}\Bigg(1 + \frac{(v-p_1)\left[\theta_1(p_1,p_2) - (1-\theta_2(p_1,p_2))\hat{\theta}_1(p_1,p_2)\right]}{s} - \frac{(v-p_2)\left[\theta_2(p_1,p_2) - (1-\theta_1(p_1,p_2))\hat{\theta}_2(p_1,p_2)\right]}{s}\Bigg),$$

such that a switching customer with location $x$ will first patronize $R_1$ (resp. $R_2$) if $x \le x_s(p_1,p_2)$ (resp. $x > x_s(p_1,p_2)$) and then switch to the other retailer upon stockout. Moreover, there exists another threshold for the non-switching customers:

$$x(p_1,p_2) = \frac{1}{2} + \frac{(v-p_1)\theta_1(p_1,p_2) - (v-p_2)\theta_2(p_1,p_2)}{2s},$$

such that a non-switching customer with location $x$ will patronize $R_1$ (resp. $R_2$) only if $x \le x(p_1,p_2)$ (resp. $x > x(p_1,p_2)$). The total market size for retailer $R_1$ (resp. $R_2$) is $\alpha_1(p_1,p_2) = \gamma(x_s(p_1,p_2) + (1-\theta_2(p_1,p_2))(1-x_s(p_1,p_2))) + (1-\gamma)x(p_1,p_2)$ (resp. $\alpha_2(p_1,p_2) = \gamma(1-x_s(p_1,p_2) + (1-\theta_1(p_1,p_2))x_s(p_1,p_2)) + (1-\gamma)(1-x(p_1,p_2))$).

**Retailer's Problem.** We next analyze the retailer's pricing and inventory problem. The retailer satisfies demands from non-switching and switch customers in the first stage. In the second stage, the retailer satisfies the switching customers' demand via the remaining on-hand stock. Given market size $\alpha_i(p_1,p_2)$ defined above, retailer $R_i$'s profit maximization problem is:

$$\max_{(p_i,q_i)} \{p_i \mathbb{E}(\alpha_i(p_1,p_2)D \wedge q_i) - cq_i\}.$$

Therefore, given a price $p_i$, the retailer $R_i$'s optimal ordering strategy is the newsvendor solution: $q_i = \alpha_i(p_1,p_2)F^{-1}\left(\frac{p_i-c}{p_i}\right)$.

**Consumers' Belief on Inventory Availability Probability.** Next, we model the customers' beliefs on retailer's inventory availability probability, staring from the first stage. Similar to the analysis of the base model, conditioned on the existence of a customer, her belief about retailer $R_i$'s demand $y_i$ in the first stage is a random variable with probability density function $g_i(y_i|p_1,p_2) := \frac{y}{\tilde{\alpha}_i(p_1,p_2)\mu} f\left(\frac{y}{\tilde{\alpha}_i(p_1,p_2)}\right)$, where $\tilde{\alpha}_1(p_1,p_2) = \gamma x_s(p_1,p_2) + (1-\gamma)x(p_1,p_2)$ and $\tilde{\alpha}_2(p_1,p_2) = \gamma(1-x_s(p_1,p_2)) + (1-\gamma)(1-x(p_1,p_2))$. Note that $\tilde{\alpha}_i(p_1,p_2)$ represents retailer $R_i$'s market size in the first stage (i.e., customers who choose retailer $R_i$ as their focal retailer). Each customer holds an identical belief about the inventory availability for $R_i$ so we have:

$$\theta_i(p_1,p_2) = \int_y \frac{\min\{q_i, \tilde{\alpha}_i y\}}{\tilde{\alpha}_i y} g_i(y|p_1,p_2)dy.$$

In the second stage, switching customers switch for substitutes if their focal retailers are out of stock. The competing retailers satisfy demand from the switching customers using the stock left from the first stage. For example, the demand for retailer $R_i$ is $[\tilde{\alpha}_{3-i}(p_1,p_2)D - q_{3-i}]^+$ and his remaining stock is $[q_i - \tilde{\alpha}_i(p_1,p_2)D]^+$ in the second stage. Therefore, the customers' belief about retailer's inventory availability probability is

$$\hat{\theta}_i(p_1,p_2) = \int_y \frac{\min\{(\tilde{\alpha}_{3-i}(p_1,p_2)y - q_{3-i})^+, (q_i - \tilde{\alpha}_i(p_1,p_2)y)^+\}}{(\tilde{\alpha}_{3-i}(p_1,p_2)y - q_{3-i})^+} g(y|p_1,p_2)dy.$$

Now, we are ready to characterize the symmetric equilibrium price and inventory decisions of the retailers. The equilibrium price can be obtained through the following maximization problem:

$$\begin{aligned}
\max_{0\le p\le v} \quad & \Pi(p,p^*) = p\mathbb{E}\left[\alpha(p,p^*)D \wedge q(p,p^*)\right] - cq(p,p^*) \\
\text{s.t.} \quad & q(p,p^*) = \alpha(p,p^*)F^{-1}\left(\frac{p-c}{p}\right), \\
& \alpha(p,p^*) = \gamma\alpha_1(p,p^*) + (1-\gamma)\alpha_2(p,p^*), \\
& \alpha_1(p,p^*) = x_s(p,p^*) + (1-\theta(p,p^*))(1-x_s(p,p^*)), \\
& \alpha_2(p,p^*) = x(p,p^*),
\end{aligned}$$

where $\alpha_1(p,p^*)$ and $\alpha_2(p,p^*)$ represent market size from switching and non-switching customers, respectively.

## Appendix D: Proof of Statements

### Proof of Proposition 1

Given the equilibrium retailer decisions $(p^*,q^*)$, a customer located at $x$ has an expected payoff of $(v-p^*)\theta^*(p^*) - sx$, where $x\in[0,1]$. Note that, if the search cost $s$ is small, the retailers compete on both price and inventory availability and the market $\mathcal{M}$ is fully covered under equilibrium. If the search cost $s$ is large, $\mathcal{M}$ is not fully covered in equilibrium and, thus, the retailers do not directly compete with each other. In this case, the equilibrium outcome satisfies $(v-p^*)\theta^*(p^*) - s\alpha^* = 0$, where $\alpha^*$ is the equilibrium market share of a retailer. Hence, the expected payoff of the customers located at $x=\alpha^*$ and $x = 1-\alpha^*$ should be 0. Finally, when the search cost $s$ is in a medium range, $\mathcal{M}$ is fully covered but the two retailers do not compete with each other. In this case, each retailer covers half of the market share under equilibrium. Thus, we have that $(v-p^*)\theta^*(p^*) - \frac{1}{2}s = 0$. For the rest of our proof, we use $R_1$ as the focal retailer and we shall focus on the first case where the two retailers compete with each other.

Let $p$ be the price charged by retailer $R_1$ (the focal retailer), $p'$ be the price charged by retailer $R_2$, $\alpha$ be the market share of $R_1$, and $\alpha'$ be the market share of $R_2$. Since the two retailers cover the entire market, a customer at the intersection of their respective market segments should be indifferent between visiting either retailer, i.e., $(v-p)\theta^*(p) - s\alpha = (v-p')\theta^*(p') - s(1-\alpha') \ge 0$. Recall that retailer $R_2$ charges price $p'$, we next analyze $R_1$'s best response function given price $p'$, which will be denoted as $p^*(p')$. We write $R_1$'s profit as $\Pi(p,p') := p\mathbb{E}(\alpha(p,p')D \wedge q^*(p,p')) - cq^*(p,p')$, where $q^*(p,p') = \alpha(p,p')F^{-1}(\frac{p-c}{p})$, and its market share $\alpha(p,p')$ satisfies the following equilibrium condition (the expected payoff to visit $R_1$ is the same as that to visit $R_2$): $(v-p)\theta^*(p) - s\alpha(p,p') = (v-p')\theta^*(p') - s(1-\alpha(p,p'))$. For simplicity, we rewrite the equilibrium condition as $U(p) - s\alpha(p,p') = U(p') - s(1-\alpha(p,p'))$, where $U(p) = (v-p)\theta^*(p)$. Therefore, for any given price $p'$ from retailer $R_2$, the focal retailer's best price response satisfies $p^*(p')$, i.e.,

$$\begin{aligned}
p^*(p') :&= \arg\max_{0\le p\le v} \Pi(p,p') \\
&= \arg\max_{0\le p\le v} \left(\frac{1}{2} + \frac{(v-p)\theta^*(p) - (v-p')\theta^*(p')}{2s}\right)\left\{p\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p-c}{p}\right)\right] - cF^{-1}\left(\frac{p-c}{p}\right)\right\}.
\end{aligned}$$

To find $R_1$'s best response $p^*(p')$, we take derivative of the profit function $\Pi(p,p')$ with respect to price $p$, which yields

$$\frac{\partial \Pi(p,p')}{\partial p} = \frac{1}{2s}U'(p)\pi(p) + \left(\frac{1}{2} + \frac{U(p)-U(p')}{2s}\right)\pi'(p),$$

where $\pi(p) := p\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) - cF^{-1}\left(\frac{p-c}{p}\right)$.

According to Lemma 2, we know that $U(p)$ is decreasing and concave in $p$ for $p \in [\hat{p}, v)$, where $\hat{p}$ maximizes $U(p)$. Moreover, we know that $U'(p) = 0$ at $p = \hat{p}$; $U'(p) < 0$ and $U(p) = 0$ at $p = v$. Since $\pi(p)$ is increasing in $p$, we have $\Pi'(p) > 0$ at $p = \hat{p}$ and $\Pi'(p) < 0$ at $p = v$. Hence, the first-order condition, $\Pi'(p) = 0$, results in a unique optimal price $p^*(p) \in [\hat{p}, v)$ when the search cost is sufficiently small. Furthermore, if the two retailers charge the same equilibrium price $p^*$, the equilibrium price satisfies the condition $U'(p^*)\pi(p^*) + s\pi'(p^*) = 0$.

Next, we prove the existence and uniqueness of the equilibrium. The implicit function theorem and the envelope theorem together yield $\frac{d^2p^*(p')}{d(p')^2} = \{-\frac{\partial}{\partial p}\frac{\partial^2\Pi(p,p')}{\partial p^2}\cdot\frac{\partial^2\Pi(p,p')}{\partial p\partial p'} + \frac{\partial^2\Pi(p,p')}{\partial p^2}\cdot\frac{\partial}{\partial p'}\frac{\partial^2\Pi(p,p')}{\partial p\partial p'}\}/(\frac{\partial^2\Pi(p,p')}{\partial p\partial p'})^2$. Thus, it can be easily verified that $\frac{\mathrm{d}p^*(p')}{\mathrm{d}p'} > 0$ and $\frac{\mathrm{d}^2p^*(p')}{\mathrm{d}(p')^2} < 0$, i.e., $p^*(p')$ is concavely increasing in $p'$. In addition, observe that $\lim_{p'\to v} p^*(p') < v$ and $\lim_{p'\to\hat{p}} p^*(p') \geq \hat{p}$. Thus, the function $p^*(p') - p'$ has a unique root on $[\hat{p}, v)$, which implies that the best-response function $p^*(p')$ has a unique fixed point on the interval $[\hat{p}, v)$. In other words, the equilibrium price $p^*$ satisfies the equation $p^*(p^*) - p^* = 0$, which also implies that the equilibrium is symmetric. This proves the existence, uniqueness, and symmetry of the equilibrium. By the symmetry of the equilibrium outcome, we have $\alpha^* = \frac{1}{2}$ and $q^* = \frac{1}{2}F\left(\frac{p^*-c}{p^*}\right)$ under equilibrium. Finally, to complete the proof, we need to guarantee that the two retailers compete on market share under the equilibrium price $p^*$. That is, $U(p^*) \geq \frac{s}{2}$, where $p^*$ is the equilibrium price characterized above. □

**Proof of Proposition 2**

For the Hotelling model with deterministic demand, the retailer's market share function is $\frac{-p+p_d^*+s}{2s}$ (see the proof of Lemma 3). Hence, the equilibrium price $p_d^*$ can be obtained by the following first-order condition:

$$-\frac{1}{2s}(p-c)\mu + \frac{1}{2}\mu = 0.$$

Therefore, we obtain the equilibrium price $p_d^* = c + s$.

In our base model with demand uncertainty, we have $\theta(p) = \frac{1}{\mu}\int_0^\infty \left(y \wedge F^{-1}\left(\frac{p-c}{p}\right)\right)dF(y)$ and $\pi(p) = p\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) - cF^{-1}\left(\frac{p-c}{p}\right) = p\mu\theta(p) - cF^{-1}\left(\frac{p-c}{p}\right)$. Therefore, under equilibrium, the price satisfies the following first-order condition (also see the proof of Proposition 1):

$$\frac{-\theta(p) + (v-p)\theta'(p)}{2s}\pi(p) + \frac{1}{2}\mu\theta(p) = 0. \tag{5}$$

Clearly, we have $\theta(p) < 1$ and $\theta'(p) > 0$ for any $p \in [c, v)$. Moreover, we have $\pi(p) \leq (p-c)\mu$. The first-order condition (5) gives the equilibrium price $p^*$.

Next, we show $p^* > p_d^* = c + s$. Define

$$g(p) := \frac{-\theta(p) + (v-p)\theta'(p)}{2s}(p-c)\mu + \frac{1}{2}\mu\theta(p) \text{ and } g(\hat{p}^*) = 0.$$

Hence, $\hat{p}^* = c + s\frac{\theta(\hat{p}^*)}{\theta(\hat{p}^*)-(v-\hat{p}^*)\theta'(\hat{p}^*)} > c + s = p_d^*$. Recall that $\pi(p) \leq (p-c)\mu$, so (5) implies that $p^* > \hat{p}^* > p_d^*$, which concludes the proof. □

**Proof of Proposition 3**

We first examine the case when $\gamma = 1$ (i.e., all customers are switching customers). Assuming that the competing retailer charges the equilibrium price $p_s^*$, we know the focal retailer's (i.e., retailer $R_1$) market size is:

$$\alpha(p_1,p_s^*) = \frac{\theta^*(p_1)\theta^*(p_s^*)}{\theta^*(p_1)+\theta^*(p_s^*)}\left(1+\frac{(p_s^*-p_1)\theta^*(p_s^*)}{s}\right)+\min\left(\frac{(v-p_1)\theta^*(p_1)}{s},1\right)(1-\theta^*(p_s^*)).$$

Let $\phi_1(p_1,p_s^*) := \frac{\theta^*(p_1)\theta^*(p_s^*)}{\theta^*(p_1)+\theta^*(p_s^*)}$ and $\phi_2(p_1,p_s^*) := \left(1+\frac{(p_s^*-p_1)\theta^*(p_s^*)}{s}\right)$, we have

$$\frac{d\phi_1(p_1,p_s^*)}{dp_1} \frac{(\theta^*(p_s^*))^2}{(\theta^*(p_1)+\theta^*(p_s^*))^2}\frac{d\theta^*(p_1)}{dp_1} > 0$$

$$\frac{d^2\phi_1(p_1,p_s^*)}{dp_1^2} = \frac{(\theta^*(p_s^*))^2}{(\theta^*(p_1)+\theta^*(p_s^*))^3}\left\{\frac{d^2\theta^*(p_1)}{dp_1^2}[\theta^*(p_s^*)+\theta^*(p_1)]-2\left(\frac{d\theta^*(p_1)}{dp_1}\right)^2\right\} < 0,$$

as $\frac{d^2\theta^*(p_1)}{dp_1^2} < 0$. Similarly, we can also obtain $\frac{d\phi_2(p_1,p_s^*)}{dp_1} < 0$ and $\frac{d^2\phi_2(p_1,p_s^*)}{dp_1^2} = 0$. Therefore, the term $\phi_1(p_1,p_s^*)\phi_2(p_1,p_s^*)$ is concave in $p_1$.

Now, we study the second term of the market size function. Let $\phi_3(p_1) = \frac{(v-p_1)\theta^*(p_1)}{s}$. As shown in Proposition 1, $\phi_3(p_1)$ is concave in $p_1$. Hence, the retailer's market size:

$$\alpha(p_1,p_s^*) = \phi_1(p_1,p_s^*)\phi_2(p_1,p_s^*)+\min(1,\phi_3(p_1))(1-\theta^*(p_s^*)),$$

is concave in price $p_1$.

Next, we examine a more general case when a fraction $\gamma$ of the customers are switching customers and the rest, $1-\gamma$, are no-switching customers. Note that $R_1$'s market size from the non-switch customers

$$\alpha(p_1,p_s^*) = (1-\gamma)\left\{\frac{1}{2}+\frac{(v-p)\theta(p)-(v-p_s^*)\theta^*(p_s^*)}{2s}\right\}$$

is convex in price $p_1$. Thus, the retailer's total market size from the switching customers and non-switching customers is convex in price $p_1$. Similar to the proof of Proposition 1, the retailer's profit function can be written as

$$\Pi(p_1,p_s^*) = \alpha(p_1,p_s^*)\left\{p\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p-c}{p}\right)\right]-cF^{-1}\left(\frac{p-c}{p}\right)\right\},$$

where $\alpha_1(p_1,p_s^*)$ represents the total market size in the presence of both switching customers and non-switching customers.

Since $s$ is sufficiently small so that all switching customers will switch upon stockout, we have $(v-p_s^*)\theta^*(p_s^*) > s$. In this case, the equilibrium price satisfies the first-order condition as follows:

$$\left\{\gamma\left(\frac{1}{4}\frac{d\theta^*(p_s^*)}{dp_s^*}-\frac{(\theta^*(p_s^*))^2}{2s}\right)+\frac{(1-\gamma)U'(p_s^*)}{2s}\right\}\pi(p_s^*)+\left\{\gamma\left(1-\frac{\theta^*(p_s^*)}{2}\right)+\frac{1-\gamma}{2}\right\}\pi'(p_s^*) = 0.$$

Similar to the proof of Proposition 1, when the search cost $s$ is sufficiently small, the symmetric equilibrium price $p_s^*$ will the unique root of the above first-order conditions. This concludes the proof. $\square$

**Proof of Proposition 4**

Similar to the other proofs, we set $R_1$ as the focal retailer and focus on the case where $s$ is sufficiently small so that all switching customers will switch to the other retailer for substitutes upon stockout.

Assume that retailer $R_2$ charges the equilibrium price $p_v^*$ and stocks the equilibrium inventory quantity $q_v^*$. The focal retailer $R_1$ maximizes his profit $\Pi(p,q) := p\mathbb{E}(\alpha(p,q)D \wedge q) - cq$. First of all, we drive the equilibrium condition of the market size given that all switching customers will switch upon stocks out. Recall that $R_1$'s market size is

$$\alpha(p,q) = \gamma \left\{x_s(p,q) + [1 - x_s(p,q)]\,(1-\theta_v)\right\} + (1-\gamma)x(p,q),$$

where $x_s(p,q) = \frac{\theta}{\theta+\theta_v}\left\{\frac{(p_v^*-p)\theta_v}{s}+1\right\}$, $x(p,q) = \frac{1}{2} + \frac{(v-p)\theta-(v-p_v^*)\theta_v}{2s}$, $\theta = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \frac{q}{\alpha(p,q)}\right)\mathrm{d}F(y)$, $\theta_v(p,q) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge \frac{q^*}{\alpha_v}\right)f(y)\mathrm{d}y$, and $\alpha(p,q)+\alpha_v(p,q) = 1+\gamma[1-\theta_v+x_s(p,q)(\theta_v-\theta)]$. Clearly, the market size function is decreasing in price $p$, i.e., $\frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}p} < 0$, and increasing in quantity $q$, i.e., $\frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q} > 0$.

Then, we take the derivative of the profit function, $\Pi_v(p,q)$, with respect to $q$. The first-order condition implies that

$$q_v^*(p) = \alpha(p, q_v^*(p))F^{-1}\left(\frac{p-c}{p} + \frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q}\bigg|_{q=q_v^*(p)} \int_0^{\frac{q_v^*(p)}{\alpha(p,q_v^*(p))}} x dF(x)\right).$$

Under equilibrium, we have $p = p_v^*,\ q = q_v^*(p^*) = \alpha(p_v^*, q_v^*)F^{-1}\left(\frac{p_v^*-c}{p_v^*} + \frac{\mathrm{d}\alpha(p_v^*,q_v^*)}{\mathrm{d}q_v^*}\int_0^{\frac{q_v^*}{\alpha(p_v^*,q_v^*)}} x\mathrm{d}F(x)\right)$, where $\alpha(p_v^*, q_v^*) = \gamma\left\{1 - \frac{1}{2}\theta^*\left(\frac{q_v^*}{\alpha(p_v^*,q_v^*)}\right)\right\} + \frac{1-\gamma}{2}$.

Comparing the equilibrium order quantity in the focal model, $q^*(p)/\alpha(p) = F^{-1}(\frac{p-c}{p})$, and the equilibrium order quantity in the model with inventory commitment, $q^*(p)/\alpha(p,q^*(p)) = F^{-1}\left(\frac{p-c}{p} + \frac{\mathrm{d}\alpha(p,q)}{\mathrm{d}q}\big|_{q=q^*(p)}\int_0^{\frac{q^*(p)}{\alpha(p,q^*(p))}} x\mathrm{d}F(x)\right) := g_v(p)$, we find that $g_v(p)$ shares the same functional properties as $F^{-1}(\frac{p-c}{p})$, which is concavely increasing in $p$. Moreover, given the same price as in the focal model, the retailer in the inventory commitment model has a tendency to increase inventory stock.

Next, we examine the equilibrium price given the optimal quantity decision $q^*(p)$ following the path of symmetric equilibrium. Given $R_2$'s decision, $(p, q^*(p))$, $R_1$ maximizes his expected profit:

$$\Pi(p) = p\mathbb{E}[(\alpha(p)D) \wedge q^*(p)] - cq^*(p),$$

subject to:

$$\begin{aligned}
\alpha(p) &= \gamma\left\{x_s(p) + (1-x_s(p))\left(1-\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right)\right\} + (1-\gamma)x(p),\\
x_s(p) &= \frac{\theta\left(\frac{q^*(p)}{\alpha(p)}\right)}{\theta\left(\frac{q^*(p)}{\alpha(p)}\right)+\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)}\left(\frac{p^*-p}{s}\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)+1\right) + \left(1-\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right),\\
x(p) &= \frac{1}{2} + \frac{1}{2s}\left((v-p)\theta\left(\frac{q^*(p)}{\alpha(p)}\right) - (v-p^*)\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)\right),\\
\alpha_v(p) &= 1-\alpha(p)+\gamma\left\{1-\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)+x_s(p)\left(\theta\left(\frac{q^*(p)}{\alpha_v(p)}\right)-\theta\left(\frac{q^*(p)}{\alpha(p)}\right)\right)\right\}.
\end{aligned}$$

To conclude this proof, we need to argue the uniqueness of the symmetric equilibrium price $p^*$ when the search cost $s$ is sufficiently small. Due to the complexity of the problem, we examine the first-order condition that determines the equilibrium price as follows:

$$\Pi'(p) = \alpha'(p)\pi(p) + \alpha(p)\pi'(p) = 0,$$

where $\pi(p) = p\mathbb{E}\left(D \wedge \frac{q^*(p)}{\alpha(p)}\right) - c\frac{q^*(p)}{\alpha(p)}$. Following the same argument as in the proof of Proposition 1, we have $\pi'(p) > 0$, $\alpha'(p) \propto \frac{1}{s}$, and $\alpha'' \propto \frac{1}{s}$. As a result, when the search cost $s$ is sufficiently small, there is a unique solution of the first-order condition and we denote the equilibrium price as $p_v^*$. Putting everything together, we know there is a unique symmetric equilibrium $(p_v^*, q_v^*, \alpha_v^*)$. □

**Proof of Lemma 1**

To compare the profit of $R_1$ under different strategies, we first calculate his profit in different circumstances. We first examine the case where $R_2$ does not reveal his inventory information. In this case, a customer at the purchasing threshold forms a belief $\theta_2 = \theta^*(p_2) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y \wedge F^{-1}(\frac{p_2-c}{p_2})\right) f(y)\mathrm{d}y$ of the inventory availability probability at $R_2$. A customer will visit $R_1$ first if and only if her expected utility of visiting $R_1$ dominates that of visiting $R_2$. Therefore, if $R_1$ also does not reveal its inventory information to the market, his market size $\alpha_1$ is given by

$$\begin{aligned}\alpha_1(p_1) =& \gamma\left\{\frac{\theta_1^*(p_1)\theta_2^*(p_2)}{\theta_1^*(p_1)+\theta_2^*(p_2)}\left(1+\frac{\theta_2^*(p_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta^*(p_1)-(v-p_2)\theta^*(p_2)}{2s}\right\}.\end{aligned} \tag{6}$$

Thus, the maximum profit of $R_1$ if he does not adopt the inventory commitment strategy is

$$\Pi_{d,d} := \max_{0\le p_1\le v}\left\{\alpha_1(p_1)\cdot\left\{p_1\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p_1-c}{p_1}\right)\right]-cF^{-1}\left(\frac{p_1-c}{p_1}\right)\right]\right\}.$$

Similarly, if $R_1$ adopts the inventory commitment strategy, his market share $\alpha_1$ satisfies the following equation:

$$\begin{aligned}\alpha_1(p_1,q_1) =& \gamma\left\{\frac{\theta_1^*(p_1,q_1)\theta_2^*(p_2)}{\theta_1^*(p_1,q_1)+\theta_2^*(p_2)}\left(1+\frac{\theta_2^*(p_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1,q_1)-(v-p_2)\theta_2^*(p_2)}{2s}\right\},\end{aligned} \tag{7}$$

where $\theta_1^*(p_1,q_1) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_1}{\alpha_1}\right)f(y)\mathrm{d}y$ and $\theta_2^*(p_2) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}(\frac{p_2-c}{p_2})\right)f(y)\mathrm{d}y$. Therefore, the maximum profit of $R_1$ if he adopts the inventory commitment strategy is

$$\Pi_{v,d} := \max_{0\le p_1\le v, q_1\ge 0}\left\{p_1\mathbb{E}[\alpha_1(p_1,q_1)D\wedge q_1]-cq_1\right\},$$

where $\alpha_1$ satisfies equation (7).

We now turn our attention to the case where $R_2$ adopts the inventory commitment strategy. If $R_1$ does not reveal his inventory information to the market, customers form a belief $\theta_1 = \theta^*(p_1) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}(\frac{p_1-c}{p_1})\right)f(y)\mathrm{d}y$ about the inventory availability probability of the retailer. Therefore, the market size $\alpha_1$ is

$$\begin{aligned}\alpha_1(p_1) =& \gamma\left\{\frac{\theta_1^*(p_1)\theta_2^*(p_2,q_2)}{\theta_1^*(p_1)+\theta_2^*(p_2,q_2)}\left(1+\frac{\theta_2^*(p_2,q_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2,q_2))\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1)-(v-p_2)\theta_2^*(p_2,q_2)}{2s}\right\},\end{aligned} \tag{8}$$

where $\theta_2^*(p_2,q_2)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_2}{\alpha_2}\right)f(y)\mathrm{d}y$. The maximum profit of $R_1$ is

$$\Pi_{d,v}:=\max_{0\le p_1\le v}\left\{\alpha_1(p_1)\cdot\left\{p_1\mathbb{E}\left[D\wedge F^{-1}\left(\frac{p_1-c}{p_1}\right)\right]-c\bar{F}^{-1}\left(\frac{c}{p_1}\right)\right]\right\}.$$

If $R_1$ adopts the inventory commitment strategy, his market share $\alpha_1$ satisfies the following equation:

$$\begin{aligned}\alpha_1(p_1,q_1)=&\gamma\left\{\frac{\theta_1^*(p_1,q_1)\theta_2^*(p_2,q_2)}{\theta_1^*(p_1,q_1)+\theta_2^*(p_2,q_2)}\left(1+\frac{\theta_2^*(p_2,q_2)(p_2-p_1)}{s}\right)+(1-\theta_2^*(p_2,q_2))\right\}\\&+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-p_1)\theta_1^*(p_1,q_1)-(v-p_2)\theta_2^*(p_2,q_2)}{2s}\right\},\end{aligned}\tag{9}$$

where $\theta_1^*(p_1,q_1)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_1}{\alpha_1}\right)f(y)\mathrm{d}y$ and $\theta_2^*(p_2,q_2)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q_2}{\alpha_2}\right)f(y)\mathrm{d}y$. The maximum profit of $R_1$ if he adopts the inventory commitment strategy is

$$\Pi_{v,v}:=\max_{0\le p_1\le v,q_1\ge 0}\left\{p_1\mathbb{E}[\alpha_1(p_1,q_1)D\wedge q_1]-cq_1\right\}.$$

By comparing the profit functions of $R_1$ under different strategy profiles, it is straightforward to observe that the equilibrium market share of $R_1$ is larger if he commits to an inventory order quantity, regardless of whether $R_2$ reveals his inventory information. Hence, the profit of $R_1$ will be higher under the inventory commitment strategy if the retailer commits to an inventory level that leads to the same in-stock probability. Therefore, regardless of the price and order quantity decisions for $R_2$ and regardless of whether $R_2$ adopts the inventory commitment strategy, the profit of $R_1$ is higher if he adopts the inventory commitment strategy, i.e., $\Pi_{v,d}>\Pi_{d,d}$ and $\Pi_{v,v}>\Pi_{d,v}$. □

**Proof of Proposition 5**

First, we prove $\Pi_v^*\ge\Pi^*$ when there is no competition (i.e., $s$ is sufficiently large). In the monopoly model (without commitment), a retailer's profit function is

$$\Pi(p)=p\mathbb{E}(\alpha(p)D\wedge q^*(p))-cq^*(p),$$

where $q^*(p)=\alpha(p)F^{-1}(\frac{p-c}{p})$, $\alpha(p)=\frac{v-p}{s}\theta^*(p)$ and $\theta^*(p)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}\left(\frac{p-c}{p}\right)\right)f(y)\mathrm{d}y$. In the model with inventory commitment, a retailer's profit function is

$$\Pi_v(p,q)=p\mathbb{E}(\alpha(p,q)D\wedge q)-cq,$$

where $\alpha(p,q)=\frac{v-p}{s}\theta(p,q)$ and $\theta(p,q)=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge\frac{q}{\alpha(p,q)}\right)f(y)\mathrm{d}y$. It is clear from the profit functions that, the two models have the same profit functions but the model without inventory commitment has an additional constraint $q=\alpha(p)F^{-1}(\frac{p-c}{p})$. Hence, $\Pi_v^*=\max_{(p,q)}\Pi_v(p,q)\ge\max_v\Pi(p,q(p))=\max\Pi(p)=\Pi^*$. Therefore, if the search cost $s$ is large such that the market is partially covered by the two retailers, we have $\Pi_v^*\ge\Pi^*$.

Now we turn to the case of full market coverage with customer switching. To begin with, we analyze the equilibrium pricing policies of both models when $s=0$, starting with the focal model. First, we examine the purchase decision of non-switching customers. When $s=0$, $R_1$ attracts demand from all non-switching customers if

$$(v-p_1)\theta^*(p_1)\ge(v-p_2)\theta^*(p_2).$$

Since the expected utility function, $(v-p)\theta^*(p)$, is concave in price $p$, to attract demand from all non-switching customers, the two retailers compete on offering lower prices when $p > \hat{p}$ and on offering higher prices when $p \leq \hat{p}$, where $\hat{p} = \max_p (v-p)\theta^*(p)$.

Next, we examine the purchase decision of switching customers when $s = 0$. Consider a switching customer located at $x$, he will visit $R_1$ first if

$$(v-p_1)\theta^*(p_1) + (v-p_2)(1-\theta^*(p_1))\theta^*(p_2) \geq (v-p_2)\theta^*(p_2) + (v-p_1)(1-\theta^*(p_2))\theta^*(p_1).$$

Simplifying the above condition, we obtain $p_1 \leq p_2$. That is, if $p_1 < p_2$, $R_1$'s market size from the switching customers is 1; otherwise, if $p_1 > p_2$, $R_1$'s market size from the switching customers is $1-\theta(p_2)$. Hence, to compete for the switching customers, the retailers will compete on offering lower prices until $p = c$.

Now, we analyze the equilibrium price, $p^*$, for the general case where the market consists of $\gamma$ portion of switching customers and $1-\gamma$ non-switching customers. First, the equilibrium price should be no greater than $\hat{p}$ ($p^* \leq \hat{p}$), as a lower price signals a higher market size thus a higher profit (for example, the retailers will compete on offering lower prices). Second, given that $R_2$ charges the price $\hat{p}$, $R_1$'s market size is $\alpha(\hat{p},\hat{p}) = \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta(\hat{p}))\right) + (1-\gamma)\frac{1}{2}$ if he charges the price $p = \hat{p}$. However, if $R_1$ decreases price to $p = \hat{p} - \epsilon$, his market size will be $\alpha(\hat{p}-\epsilon, \hat{p}) = \gamma$ (all the switching customers will visit $R_1$, while all the non-switching customers will vist $R_2$.). Therefore, the equilibrium price will be $\tilde{p}^* = \hat{p}$ when $\alpha(\hat{p},\hat{p}) \geq \alpha(\hat{p}-\epsilon,\hat{p})$, which gives the condition $\hat{p} \leq \theta^{-1}\left(\frac{1-\gamma}{\gamma}\right)$. Finally, we examine the region where $R_2$ charges a price $p_2 < \hat{p}$. Similarly, $R_1$'s market size is $\alpha(p_2,p_2) = \gamma\left(\frac{1}{2} + \frac{1}{2}(1-\theta(p_2))\right) + (1-\gamma)\frac{1}{2}$ if he charges the same price $p = p_2$. However, if $R_1$ decreases price to $p_2 - \epsilon$, $R_1$'s the market size is $\alpha(p_2-\epsilon, p_2) = \gamma$. Clearly, the equilibrium price is the one that gives a zero marginal increase in market size. Hence, we have $p^* = \theta^{-1}\left(\frac{1-\gamma}{\gamma}\right)$. Combining all the cases above, we have

$$\tilde{p}^* = \min\left\{\hat{p}, (\theta^*)^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right\},$$

where $\hat{p} = \max_p (v-p)\theta^*(p)$. Since $\tilde{p}^* > c$, we have $\Pi^* > 0$ in the focal model when $s = 0$.

We now consider the model with inventory commitment given $s = 0$. According to the first-order condition with respect to $q$, the retailer's optimal order quantity $q_v^*(p)$ satisfies the equation $q_v^*(p) = \alpha_v^* F^{-1}\left(\frac{p-c}{p} + \frac{d\alpha(p,q)}{dq}|_{q=q_v^*(p)} \int_0^{\frac{q_v^*(p)}{\alpha_v^*}} xf(x)dx\right)$. Since we have $\frac{d\alpha(p,q)}{dq}|_{q=q_v^*(p)} > 0$, for any given price $p$, the retailer tends to stock more under inventory commitment, i.e., $q_v^*(p) \geq q^*(p)$.

Similar to the analysis of the equilibrium price in the focal model when $s = 0$, we have:

$$p_v^* = \min\left\{\hat{p}_v, (\theta_v^*)^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right\},$$

where $\hat{p}_v = \max_p (v-p)\theta_v^*(p, q^*(p))$. Clearly, we have $p_v^* \leq p^*$ since the same price in the model with inventory commitment signals a higher quantity and thus a higher inventory availability in the equilibrium. Further, note that if $c\to 0$, the optimal inventory availability in the model of inventory commitment is no less than that in the base model, $\theta_v^*\geq\theta^*$. Therefore, we have

$$\begin{aligned}\Pi^* &= p^*\mathbb{E}\left(\alpha^* D\wedge q^*\right)-cq^*\\ &=\alpha^*\left\{p^*\mathbb{E}\left(D\wedge F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)-cF^{-1}\left(\frac{p^*-c}{p^*}\right)\right\}\\ &\geq\alpha_v^*\left\{p^*\mathbb{E}\left(D\wedge F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)-cF^{-1}\left(\frac{p^*-c}{p^*}\right)\right\}\\ &\geq\alpha_v^*\left\{p_v^*\mathbb{E}\left(D\wedge F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)\right)-cF^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)\right\}\\ &\geq p_v^*\mathbb{E}\left(\alpha_v^* D\wedge q_v^*\right)-cq_v^*=\Pi_v^*.\end{aligned}$$

The first inequality follows that $\alpha^*=\gamma\left(\frac{1}{2}+\frac{1}{2}(1-\theta^*)\right)+(1-\gamma)\frac{1}{2}\geq\gamma\left(\frac{1}{2}+\frac{1}{2}(1-\theta_v^*)\right)+(1-\gamma)\frac{1}{2}=\alpha_v^*$ as $\theta^*\leq\theta_v^*$. The second inequality follows from $p^*\geq p_v^*$. The third inequality follows from $q_v^*\neq\alpha_v^*F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)$ (where the quantity $q_v^*=\alpha_v^*F^{-1}\left(\frac{p_v^*-c}{p_v^*}\right)$ is the optimal quantity that maximizes the profit function given the price $p_v^*$). Therefore, the inventory commitment strategy results in a lower profit when $s=0$ and $c\to 0$.

Finally, the two equilibrium profits, $\Pi^*$ and $\Pi_v^*$, are both continuous in $s$ and $c$. Moreover, we have just shown that $\lim_{s,c\to 0}\Pi^*>\lim_{s,c\to 0}\Pi_v^*$. Thus, there exist two thresholds $\bar{s}_v$ (for $s$) and $\bar{c}_v$ (for $c$), such that $\Pi_v^*<\Pi^*$ if $s<\bar{s}_v$ and $c<\bar{c}_v$. □

**Proof of Proposition 6**

We continue to use retailer $R_1$ as the focal retailer. Analogous to the proof of Proposition 1, we will focus on the case when the unit travel cost $s$ is small such that the retailers compete with each other with full customer switching in equilibrium.

Given the equilibrium price $p^*$ and equilibrium compensation $m^*$, the expected profit of retailer $R_1$ is

$$\Pi(p_1,m_1)=(p_1+m_1)\mathbb{E}(\alpha_1 D\wedge q^*(p_1+m_1))-cq^*(p+m_1)-\alpha_1 m_1\mu,$$

where $q^*(p_1+m_1)=\alpha_1F^{-1}(\frac{p_1+m_1-c}{p_1+m_1})$ and $\mu=\mathbb{E}(D)$. The market size is

$$\begin{aligned}\alpha_1=&\gamma\left\{\frac{\theta_1\theta^*}{\theta_1+\theta^*}\left(\frac{(p^*+m^*)-(p_1+m_1)}{s}\theta^*+1\right)+\frac{\theta^*(m_1\theta^*-m^*\theta_1)}{s(\theta_1+\theta^*)}+(1-\theta^*)\right\}\\ &+(1-\gamma)\left\{\frac{1}{2}+\frac{[v-(p_1+m_1)]\theta_1-[v-(p^*+m^*)]\theta^*+(m_1-m^*)}{2s}\right\},\end{aligned}$$

where $\theta=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}\left(\frac{p+m-c}{p+m}\right)\right)\mathrm{d}F(y)$. For ease of exposition, we define $t_1=p_1+m_1$, which refers to the effective marginal revenue of the product. Hence, the problem can be rewritten as

$$\begin{aligned}&\Pi(t_1,m_1)=\alpha_1\left\{t_1\mathbb{E}\left(D\wedge F^{-1}\left(\frac{t_1-c}{t_1}\right)\right)-cF^{-1}\left(\frac{t_1-c}{t_1}\right)-m_1\mu\right\}\\ s.t.\quad &\alpha_1(t_1,m_1)=\gamma\left\{\frac{\theta_1\theta^*}{\theta_1+\theta^*}\left(\frac{t^*-t_1}{s}\theta^*+1\right)+\frac{\theta^*(m_1\theta^*-m^*\theta_1)}{s(\theta_1+\theta^*)}+(1-\theta^*)\right\}\\ &\qquad+(1-\gamma)\left\{\frac{1}{2}+\frac{(v-t_1)\theta_1-(v-t^*)\theta^*+(m_1-m^*)}{2s}\right\},\end{aligned}$$

where $\theta=\frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}\left(\frac{t-c}{t}\right)\right)\mathrm{d}F(y)$. It can be shown that the market size function is concave in $t_1$. Let $\pi(t_1,m_1)=t_1\mathbb{E}\left(D\wedge F^{-1}\left(\frac{t_1-c}{t_1}\right)\right)-cF^{-1}\left(\frac{t_1-c}{t_1}\right)-m_1\mu$. Following the path of symmetric equilibrium and the same argument in the proof of Proposition 1, when the search cost is sufficiently small, we can show that the expected profit of $R_1$ is concave in $t_1$, which further implies a unique equilibrium $t^*$. Given $t^*$, we next examine the retailer's best compensation response. Since $\pi(t^*, m_1)$ is linearly decreasing in $m_1$ and the market size is linearly increasing in $m_1$, the expected profit of $R_1$ is concave in $m_1$. Hence, we have a unique best compensation response $m^*(t^*)$. Therefore, we obtain a unique symmetric equilibrium $(p_c^*, m_c^*)$. □

**Proof of Proposition 7**

We start the proof by verifying two extreme cases. We first consider the case when the search cost is zero (i.e., $s=0$). In this case, the two retailers compete on offering higher consumer expected payoff, because $\alpha'(p,m) \to -\infty$. For example, the expected payoff is $U(p_1+m_1)+m_1-sx$ for a non-switching customer located at $x \in [0,1]$ who chooses to visit $R_1$. The first term, $U(p_1+m_1)$, is concave with its maximum value, $U(\hat{p})$, at $p_1+m_1=\hat{p}$. The second term is linearly increasing in $m_1$. Similarly, the expected payoff is $U(p_1+m_1)+m_1-sx+(U(p_2+m_2)+m_2-s(1-x))(1-\theta_1)$ for a switching customer located at $x \in [0,1]$ who visits $R_1$ first. According to the expected utility of customers (switching customers and non-switching customers), retailers can always capture the entire market by continuously increasing compensation $m$. However, each retailer's profit function is strictly decreasing in compensation $m$, so the retailers have to stop raising compensation at zero profit. Therefore, each retailer obtains zero profit under equilibrium when $s=0$. In contrast, each retailer charges price $p=p^*$ in the base model, as shown in the proof of Proposition 5. Since we always have $p^*>c$, each retailer must have a positive profit in the base model. Therefore, we have $\Pi^* > \Pi_c^*$ when $s=0$.

We next examine the case when the search cost is large. In this case, the two retailers have no direct competition (i.e., partial market coverage). In the model with monetary compensation, each retailer maximizes his profit $\alpha(p+m)\left\{(p+m)\mathbb{E}(D\wedge F^{-1}(\frac{p+m-c}{p+m})))-cF^{-1}(\frac{p+m-c}{p+m})-m\mathbb{E}(D)\right\}$, where $\alpha(p+m)=\frac{U(p+m)}{s}$. In the base model, each retailer maximizes its profit $\alpha(p)\left\{(p)\mathbb{E}(D\wedge F^{-1}(\frac{p-c}{p}))-cF^{-1}(\frac{p-c}{p})\right\}$, where $\alpha(p)=\frac{U(p)}{s}$. The profit function in the model of monetary compensation restores to the profit function in the base model when $m=0$. Since $m$ is a free variable, the base model is a special case of the monetary compensation model when $s=0$. In other words,

$$\Pi_c^* = \max_{(p,m)} \Pi_c(p,m) \geq \max_p \Pi_c(p,0) = \max_p \Pi(p) = \Pi^*.$$

Therefore, we have $\Pi_c^* \geq \Pi^*$ when $s \to \infty$.

Finally, recall that $\Pi^*$ and $\Pi_c^*$ are continuous in $s$. We have already obtained that $\Pi^* > \Pi_c^*$ when $s=0$; and that $\Pi_c^* \geq \Pi^*$ when $s\to\infty$. Therefore, there exists a threshold $\bar{s}_c$ such that $\Pi_c^* < \Pi^*$ if $s < \bar{s}_c$. □

**Proof of Lemma 2**

We first show that the customer's expected payoff function is concave in price $p$. We start by examining the average surplus for non-switching customers, $CS_1(p) = (v-p)\theta^*(p) - \frac{s}{4}$, where $\theta^*(p) = \frac{1}{\mu}\int_{y=0}^{\infty}\left(y\wedge F^{-1}\left(\frac{p-c}{p}\right)\right)f(y)\mathrm{d}y$. We have $\frac{d^2CS_1(p)}{dp^2} = -2\frac{d\theta^*(p)}{dp} + (v-p)\frac{d^2\theta^*(p)}{dp^2}$. Clearly, if $\frac{d^2\theta^*(p)}{dp^2}<0$, then $CS_1(p)$ is concave in price $p$. Note that

$$\mu\frac{d\theta^*(p)}{dp} = \frac{\frac{c}{p}}{f\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}\frac{c}{p^2} = \frac{1-F\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}{f\left(F^{-1}\left(\frac{p_i-c}{p_i}\right)\right)}\frac{c}{p^2},$$

which is strictly decreasing in $p$ given that $D$ follows a distribution with an increasing failure rate. Hence, $\theta^*(p)$ is concave in $p$ and thus, the average surplus for non-switching customers is also concave in $p$.

Next, we examine the average surplus for switching customers, $CS_2(p)=(v-p)(2-\theta^*(p))\theta^*(p)-s(1-\frac{3}{4}\theta^*(p))$. Clearly, the second term, $-s[1-(1-x)\theta^*(p)]$, is concave in $p$. Hence, the concavity of surplus function $CS_2(p)$ boils down to the concavity of term $(v-p)(2-\theta^*(p))\theta^*(p)$. Taking the second derivative of the term yields:

$$\frac{d^2}{dp^2}[(v-p)(2-\theta^*(p))\theta^*(p)]=-2p\frac{dg(\theta^*)}{d\theta^*}\frac{d\theta^*(p)}{dp}+(v-p)\left(\frac{d^2g(\theta^*)}{d(\theta^*)^2}\frac{d\theta^*(p)}{dp}+\frac{d\theta^*(p)}{dp}\frac{d^2\theta^*(p)}{dp^2}\right),$$

where $g(\theta^*)=(2-\theta^*(p))\theta^*(p)$ and $\theta^*(p)\in[0,1]$. Since $\frac{dg(\theta^*)}{d\theta^*}>0$ and $\frac{d^2g(\theta^*)}{d(\theta^*)^2}<0$, the first term is concave in price $p$. Thus, the average surplus for switching customers is also concave in price $p$. Finally, recall that the total average customers' surplus is a weighted summation of the average surplus functions of the two customer segments, therefore, the total average customers' surplus, $CS(p)$, is concave in price $p$. From $CS'(p)=0$, the expected payoff function $CS(p)$ is maximized at $p=\hat{p}$.

Next, we show that the equilibrium price in the focal model falls into the interval $[\hat{p},v)$. First, we prove that $\Pi(\hat{p})>\Pi(p')$ for any $p'<\hat{p}$. Without loss of generality, we use retailer $R_1$ for illustration. Suppose retailer $R_1$ decreases the price from $\hat{p}$ to $p'$, his market size will decrease, because the expected payoff of the customers is maximized at the price $p=\hat{p}$. As a result, by decreasing price from $\hat{p}$ to $p'$, the retailer will induce a lower demand and a strictly lower profit margin. This implies that $\Pi(\hat{p})>\Pi(p')$. Thus, the retailer should charge a price $p\geq\hat{p}$. Next, we show that the equilibrium price cannot exceed $v$. If the price is greater than or equal to the product valuation, i.e., $p\geq v$, no customer can afford the product, which further implies that the demand is zero and the retailer earns zero profit. Therefore, the retailer's optimal price must be within the range of $[\hat{p},v)$.

Finally, we show that the social welfare function is concave in price $p$. Similar to the proof of the average customers' surplus, we fist examine the social welfare from non-switching customers. We have $SW_1(p)=v\mu\theta(p^*)-cF^{-1}\left(\frac{p^*-c}{p^*}\right)-\frac{\mu s}{4}$ and

$$\frac{dSW_1(p)}{dp}=\frac{c^2}{p^2}\left(\frac{v-p}{p}\right)\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}=\frac{c}{p}\left(\frac{v-p}{p}\right)\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}>0.$$

Since the demand, $D$, has an increasing failure rate, $SW_1(p)$ is increasing and concave in $p$. Now, we examine the social welfare function from switching customers: $SW_2=(2-\theta(p))\left[v\mu\theta(p)-cF^{-1}\left(\frac{p-c}{p}\right)\right]-\frac{\mu s}{4}-(1-\theta(p))\frac{3\mu s}{4}$. We have

$$\begin{aligned}\frac{dSW_2(p)}{dp}&=\frac{c^2}{\mu p^3}\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}\left\{(2-\theta^*(p))(v-p)\mu-\left(v\mu\theta^*(p)-cF^{-1}\left(\frac{p-c}{p}\right)\right)+\frac{3\mu s}{4}\right\}\\&=\frac{c}{\mu p^2}\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}\left\{(2-\theta^*(p))(v-p)\mu-\left(v\mu\theta^*(p)-cF^{-1}\left(\frac{p-c}{p}\right)\right)+\frac{3\mu s}{4}\right\}.\end{aligned}$$

Note that the term in the bracket, $(2-\theta^*(p))(v-p)\mu-\left(v\mu\theta^*(p)-cF^{-1}\left(\frac{p-c}{p}\right)\right)+\frac{3\mu s}{4}$, is decreasing in price $p$. Since the term $\frac{1-F\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}$ is decreasing in $p$ due to the increasing failure rate of the demand, the social welfare function is concave in $p$. Finally, since the total social welfare function is a weighted summation of the social welfare functions from the two customer segments, the total social welfare function $SW(p)$ is concave in price $p$. □

**Proof of Proposition 8**

We first compare the two social welfare functions. Clearly, we have $p^* \le p_b^*$ because $p_b^*$ is the maximum price that allows full market coverage and customer switching. Recall that the average consumer surplus function is decreasing in price $p \in [\hat{p}, v)$ and $p^* \in [\hat{p}, v)$, thus we have $CS^* \ge CS_b^*$.

Now, we compare the two social welfare functions. We start by examining the social welfare when the market has non-switching customers only (i.e., $\gamma = 0$). According to the proof of lemma 2, we have

$$\frac{dSW(p)}{dp} = \frac{c^2}{p^2}\left(\frac{v-p}{p}\right)\frac{1}{f\left(F^{-1}\left(\frac{p^*-c}{p^*}\right)\right)}.$$

Clearly, we have $\frac{dSW(p)}{dp} > 0$. Therefore, the social welfare function is strictly increasing in $p$. Recall that we have $p^* \le p_d^*$, $SW^* \le SW_b^*$ when $\gamma = 0$.

Next, we examine the case when the market has switching customers only (i.e., $\gamma = 1$). As shown in the proof of Proposition 5, the switching customers will always visit the retailer with lower price first, so the retailer who charges a lower price attract more demand. As a result, the retailers compete on offering lower prices when $\gamma = 1$. In equilibrium, we have $p^* = c$, so the retailers stock zero inventory and thus we have $SW^* = 0$. In contrast, the social welfare in the benchmark model is $SW_b^* > 0$ as $\theta(p_b^*) > 0$. Hence, we have $SW^* \le SW_b^*$ when $\gamma = 1$.

Finally, when the market consists of both customer types, the equilibrium price will be lower than the equilibrium price when $\gamma = 0$ and higher than that when $\gamma = 1$. Since we have proved that the market competition will lead to a lower social welfare when the market has non-switching customers and switching customers, respectively, we have $SW^* \le SW_b^*$. □

**Proof of Proposition 9**

First, we show $SC_c^* \ge SC^*$. Suppose the market follows the equilibrium path of the model without monetary compensation and achieves equilibrium solutions $(p^*, q^*)$. In this case, the monetary compensation $m^* = 0$. Now, we allow the retailers to pay compensation to consumers. Accordingly, the equilibrium compensation switches from $m^* = 0$ to $m_c^* \ge 0$. A higher compensation rate increases consumer surplus and thus helps retailers earn more market share (but decreases its marginal revenue). If $m_c^* = 0$, the retailers have no incentives to compete more in market share, so the two models result in the same consumer surplus. If $m_c^* > 0$, the two retailers have incentives to compete more in market share, so a positive compensation rate raises consumer surplus. In short, since we always have $m_c^* \ge m^* = 0$, offering non-negative monetary compensation to customers upon stock can always increases the equilibrium average customer surplus, i.e., $SC_c^* \ge SC^*$.

Next, we show $SC_v^* \ge SC^*$. Similarly, suppose the market follows the equilibrium path of the base model and achieves equilibrium solution $(p^*, q^*)$. Now, we allow the retailers to announce quantity information to the market. As a result, the market switches to a new equilibrium path $(p_v^*, q_v^*)$ under inventory commitment. In the case of inventory commitment, the retailers are motivated to increase quantity and decrease price.

First, the retailers have incentives to increase quantity. Assume the equilibrium price $p^*$ is unchanged. Once the retailers commit inventory to the market, the inventory quantity must not decrease. The argument is as follows. On one side, decreasing quantity decreases consumer surplus and thus decreases market share.

One the other side, deviating from the critical fractile quantity ($q = \alpha F^{-1}(\frac{p-c}{p})$) decreases marginal revenue. As a result, by decreasing inventory quantity, the retailers must earn less profit, so the inventory quantity must not be decreased. However, the retailers may increase quantity. Although increasing stock quantity also deviates from the critical fractile quantity and thus decreases the marginal revenue, it raises market share by offering higher product availability. Thus, the retailers may earn a higher profit by increasing stock quantity. Therefore, given the equilibrium price, the retailers may choose to increase quantity.

Second, the retailers have incentives to decrease price. Similarly, assume the equilibrium quantity $q^*$ is unchanged, the retailers have no incentives to increase retail price. The argument is as follows. On one side, increasing price decreases consumer surplus and thus decreases market share. One the other side, deviating from the critical fractile price ($p = c/(1-F(\frac{q}{\alpha}))$) decreases marginal revenue. As a result, by increasing price, the retailers must earn less profit, so the retail price must not be increased. However, retailers may decrease price. Although decreasing price also deviates from the critical fractile price and thus reduces the marginal revenue, it raises market share. In other words, the retailers may earn a higher profit by decreasing price. Therefore, given the equilibrium quantity, the retailers may choose to decrease price.

In sum, once the retailers adopt the inventory commitment strategy, we have $q_v^* \geq q^*$ and(or) $p_v^* \leq p^*$. Since increasing quantity and decreasing price are both beneficial to the consumers' surplus, we have $SC_v^* \geq SC^*$. □

**Proof of Proposition 10**

We focus on analyzing two extreme cases.

Case I. A sufficiently small search cost (i.e., $s \downarrow 0$). In this case, the entire market is fully covered with customer switching. As shown in the proof of Proposition 5, the equilibrium profit and quantity are positive in the focal model, so $SW^* > 0$. However, the retailer's profit under the inventory commitment and monetary compensation strategies are close to zero as $s \downarrow 0$, so the retailer will stock zero quantity, provide zero product availability, which leads to zero social welfare. Therefore, we have $SW_v^* < SW^*$ and $SW_c^* < SW^*$.

Case II. A large search cost. In this case, the entire market has no competition (i.e., the customer at the border has a surplus exactly equal to 0 to patronage the focal retailer) and customer switching. Moreover, each retailer can be viewed as a monopolist that serves a separate market (with a market size $\alpha < \frac{1}{2}$). Su and Zhang (2008) have proved that the inventory commitment and monetary compensation strategies provide a higher order quantity in the monopoly setting, so we have $\theta^*(p_v^*) > \theta^*(p^*)$ and $\theta^*(p_c^*) > \theta^*(p^*)$. Recall the social fare function is increasing in product availability without customer switching, so $SW_v^* > SW^*$ and $SW_c^* > SW^*$.

Finally, recall that the social welfare functions are continuous in equilibrium price $p^*$ and the equilibrium price $p^*$ is continuous in $s$, we conclude that: (1) there exists a threshold $s_{vw}$ and we have $SW_v^* < SW^*$ if $s < s_{vw}$; and (2) there exists a threshold $s_{cw}$ and we have $SW_c^* < SW^*$ if $s < s_{cw}$. □