Original Article

POMS

Production and Operations Management
1–18

DOI: 10.1177/10591478251371267
journals.sagepub.com/home/pao
Sage

Optimal Growth of a Two-Sided Platform with Heterogeneous Agents

Yixin Zhu[1], Hongfan(Kevin) Chen[2], Renyu Zhang[2] and Sean X Zhou[2]

Abstract

We consider the dynamics of a two-sided platform, where the agent population on both sides experiences growth over time with heterogeneous growth rates. The compatibility between buyers and sellers is captured by a bipartite network. The platform sets commissions to optimize its total profit over T periods, considering the trade-off between short-term profit and growth as well as the spatial imbalances in supply and demand. We design an asymptotically optimal policy with the profit loss upper-bounded by a constant independent of T, in contrast with a myopic policy shown to be arbitrarily bad. To derive the policy, we first develop a benchmark problem that captures the platform's optimal steady state. We then identify the agent types with the lowest relative population ratio compared to the benchmark in each period, and adjust the service level of these types to be higher than or equal to their service level in the benchmark problem. A higher service level accelerates growth but requires substantial subsidies during the growth phase. Additionally, we provide the conditions under which the subsidy is necessary. We further examine the impact of the growth potential and the compatibility network structure on the platform's optimal profit, the agents' payment/income, and the optimal commissions at the optimal steady state. To achieve that, we introduce innovative metrics to quantify the long-run growth potential of each agent type. Using these metrics, we show that a "balanced" compatibility network, where the relative long-run growth potential between sellers and buyers for all submarkets is the same as that for the entire market, allows the platform to achieve maximum profitability. Our study provides insight into how the growth potential and compatibility network structure jointly influence the commission policy in the growth process and the optimal steady state.

Keywords

Two-Sided Market, Platform Growth, Heterogeneous Agent Types, Commission

Date received 26 August 2024; accepted 30 July 2025 after two revisions

Handling Editor: Michael Pinedo

1 Introduction

In recent years, the rapid growth of consumer-to-consumer platforms, such as Airbnb and Upwork, has transformed buyer-seller interactions. Their success relies on efficiently growing the agent base on both sides, which drives transaction volume and ultimately enhances platform profitability. Existing literature suggests that a pivotal strategy of the platform involves initially subsidizing agents to stimulate their growth and subsequently implementing charges to ensure long-term profitability (see Lian and Van Ryzin, 2021). Throughout this process, it is crucial to strike a balance between long-run growth and short-term profitability via a tailored commission structure. However, determining which agent segment to subsidize or charge higher fees becomes challenging, particularly considering the heterogeneity in their growth potentials and preferences or popularity on the platform.

In general, the growth of an active agent base over periods hinges on two primary factors: retaining current agents and encouraging new adoptions. The acquisition of new

[1]School of Information Management and Engineering, Shanghai University of Finance and Economics, Shanghai, China
[2]The Chinese University of Hong Kong, CUHK Business School, Hong Kong, China

Corresponding authors:
Hongfan(Kevin) Chen, The Chinese University of Hong Kong, CUHK Business School, Hong Kong, China.
Email: kevinchen@cuhk.edu.hk
Renyu Zhang, The Chinese University of Hong Kong, CUHK Business School, Hong Kong, China.
Email: philipzhang@cuhk.edu.hk
Sean X Zhou, The Chinese Univerity of Hong Kong, CUHK Business School, Hong Kong, China.
Email: zhoux@baf.cuhk.edu.hk

adopters is strongly influenced by word-of-mouth communication between potential adopters and current agents, facilitated through online reviews and comments. Different agent types exhibit varying retention rates and word-of-mouth effects. For example, tourists seeking vacation homes have lower retention rates than regular business travelers due to infrequent revisits (see Hamilton et al., 2017). However, they rely more on transaction histories and online reviews from previous guests when selecting properties in unfamiliar destinations (see Sundaram and Webster, 1999). Platforms could tailor their commission structures based on distinct growth patterns across different agent segments. For example, some platforms utilize machine learning algorithms to predict users' churn rate based on their past behaviors, and send coupons to those users with low usage frequency (see Yu and Zhu, 2021). This targeted promotion campaign can typically be viewed as an indirect way to implement personalized pricing to alleviate backlash from customers.

Furthermore, based on previous works on the cross-side network effect of a two-sided market (e.g., Chu and Manchanda, 2016; Eisenmann et al., 2006; Rochet and Tirole, 2003), the growth on one side of the market has a positive impact on the growth of the other side. However, the value contributed to the opposite side of the market differs across various agent segments, as buyers and sellers are horizontally differentiated in terms of their "popularity" and preferences for agents on the other side of the market. This compatibility difference arises from varying tastes, geographical constraints, or skill mismatches (see Birge et al., 2021). For instance, on Airbnb, listings located in popular tourist destinations or with a secure parking space tend to be more popular; on Upwork, freelancers who offer skills that match market demands and have flexible schedules tend to attract more companies. During the platform's growth phase, an increase in the number of "marquee users," typically prominent buyers or high-profile sellers, can attract more users on the other side to join the platform. Therefore, the platform can accelerate its growth by securing the participation of marquee users when setting commissions during the growth phase. For example, Airbnb charges different commissions based on the location of listings, room types, cancellation policies, and so on (see Airbnb, 2025).

With the intricate interplay of *intertemporal factors* marked by heterogeneous growth potentials and *spatial factors* characterized by the compatibility between agent types, it becomes challenging for the platform to find an optimal commission policy to grow the agent base and maximize its long-term profits. Furthermore, gaining insights into how both the intertemporal and spatial factors affect the platform's optimal policy and profit is of utmost importance. These are the two primary focal points of our study.

Results and Contributions. We consider a two-sided platform that charges commissions to sellers and buyers for facilitating their transactions. The compatibility between the buyers and sellers is captured by a bipartite graph, and the transaction quantities and prices between the agents are determined endogenously in a general equilibrium setting. The mass of each agent type in each period depends on the mass and transaction volume in the previous period, capturing the effect of retention and word-of-mouth communication. The platform determines the commissions in each period to maximize the total profit in T periods, taking into account the trade-off between the immediate revenue and the potential for future expansion. Our main findings are summarized as follows.

First, we formulate the platform's problem as a multi-period pricing optimization model, which, however, is challenging to solve due to its high-dimensional state space (determined by the sizes of different agent types) and lack of structural results. To overcome this challenge, we first construct a single-period problem and show that the gap between T times its optimal objective value and that of the original problem is upper bounded by a constant (see Proposition 1). Therefore, the solution to this single-period problem captures the optimal steady state (OSS) of the system, and we see it as a benchmark. We then develop a heuristic policy that is shown to be asymptotically optimal (see Theorem 1). During the growth phase, the policy identifies the scarcest agent type relative to the benchmark problem in each period and adjusts its service level to be equal to or higher than the corresponding value in the benchmark. The demand and supply quantities of other agent types are matched accordingly to ensure feasibility. We demonstrate that any service level within the proposed range guarantees exponential convergence of population toward the targeted level, but a higher service level accelerates growth and necessitates greater subsidies for users. Additionally, we identify which transactions should be subsidized during the growth stage (see Proposition 2). Once the population of the scarcest type surpasses the targeted level, our policy shifts to align its service level with the service level derived from the benchmark problems, helping to ensure that the population converges to the OSS. We also provide some numerical examples to illustrate possible growth trajectories of the platform and changes in commissions over time applying this policy.

In comparison, we show that the performance of a myopic policy (MP), which ignores the growth dynamics in the marketplace, can be arbitrarily poor (see Proposition 3). This result further emphasizes that the trade-off between short-term profit and long-term growth must be carefully managed, even for a monopoly platform. Our result provides managerial insights on platform growth: The platform should first identify the optimal state at which it can sustain and maximize profit. Instead of targeting the agent type with the lowest population, the platform should focus on ensuring the service level (e.g., by offering subsidies or lowering commissions) for agent types that lag behind the benchmark in each period during the growth phase.

Second, we focus on the platform's OSS characterized by the single-period benchmark problem. We analyze how the growth potential of agent types (*intertemporal factor*) and compatibility network structure (*spatial factor*) influence (1) the platform's profit, (2) the agents' payments/incomes, and (3) the optimal commissions. Regarding (1), previous literature (see Birge et al., 2021; Chou et al., 2011; Schrijver et al., 2003) considers static settings with exogenous agent bases and shows how the imbalance in supply–demand ratio across the market determines the platform's performance. However, we find that the metric of "balances" in the literature fails in the dynamic setting (see Example 2). To incorporate the intertemporal factor, we develop a novel metric that captures the long-run growth potential of each agent type. With a more specific growth function, we develop the intuition behind such a metric. We show that a "balanced" compatibility network, where the relative long-run growth potentials of sellers and buyers for all submarkets are the same as that for the entire market, leads to maximum platform profitability (see Theorem 2). In contrast, the extent of the "imbalance" of the compatibility network in terms of the relative long-run growth potentials between the two sides determines the lower bound of the platform's optimal profit.

Regarding (2), we show that the buyer (seller) type with a higher ratio of compatible sellers' (buyers') long-run growth potential to their own long-run growth potential receives lower payments (higher income) at the OSS (see Proposition 4). Based on this result, we conduct a sensitivity analysis to illustrate the impact of each agent type's long-run growth potential on its or others' income/payment (see Corollary 2). For (3), we show that the optimal commission charged from the submarket first decreases in the relative growth potential between sellers and buyers, and then increases (decreases) in it when the value distribution functions of both sides are convex (concave)(see Proposition 5). Our results suggest that the platform should strategically focus its marketing campaigns or loyalty programs on agents who exhibit relatively lower long-run growth potential compared with their compatible agents on the other side.

Organization of the Paper. The rest of the paper is organized as follows. After reviewing the relevant literature in Section 2, we introduce the model and discuss computational challenges in Section 3. In Section 4, we design a heuristic algorithm with provably good performance. In Section 5, we examine the impact of both the compatibility network structure and growth potential of agents on the platform's profit, agents' payments/incomes, and optimal commissions at the OSS. Section 6 concludes the paper.

Throughout the paper, we use "increasing" (and "decreasing") in a weak sense, that is, meaning "non-decreasing" (and "non-increasing") unless otherwise specified. In addition, we use $\mathbb{R}_+$ to denote the non-negative real number set.

2 Literature Review

Pricing in two-sided platforms has been extensively studied in the field of Economics and Operations Management. Based on Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2003), Rochet and Tirole (2006), a growing literature has explored the pricing and matching problems in the context of online platforms (e.g., Bai et al., 2019; Benjaafar et al., 2022, 2019; Cachon et al., 2017; Cohen and Zhang, 2022; Hagiu, 2009; Hu and Zhou, 2020; Taylor, 2018). Our work features network effects in a potentially incomplete two-sided market that evolves dynamically. Agents on one side of the market can only trade with a subset of agents on the other, and the platform's commissions influence the transactions and the growth of the agent base in the market. Therefore, our work is closely related to two streams of literature: (i) The growth of a marketplace and (ii) pricing in a networked market.

Early literature about the growth of a marketplace mainly focused on product diffusion, which provides a model to forecast the growth of the customer base for a new product, see for example, Bass (1969), Kalish (1985), Norton and Bass (1987). Based on these papers, more recent literature studies how to leverage discounts or investment incentives to influence the growth of new products (e.g., Ajorlou et al., 2018; Bass and Bultez, 1982; Shen et al., 2014) and that of two-sided platforms (e.g., He and Goh, 2022; Kabra et al., 2016; Lian and Van Ryzin, 2021). Specifically, Lian and Van Ryzin (2021) considered a two-sided market in which the platform can subsidize one or both sides to boost their growth. They show that the optimal policy is to employ a subsidy shock to rapidly steer the market towards its optimal long-term size. He and Goh (2022) studied the dynamics of a hybrid workforce comprising on-demand freelancers and traditional employees, both capable of fulfilling customer demands. They investigated how demand should be allocated between employees and freelancers, and under what conditions the system is sustainable in the long run. Our study differs from this stream of work in that agents have heterogeneous compatibility and growth potentials, which requires us to come up with a customized commission structure for different agent types; in addition, the transaction quantities and prices are both formed endogenously in a general equilibrium in each period.

Our study is also closely related to the literature on networked markets (e.g., Baron et al., 2022; Bimpikis et al., 2019; Chen and Wang, 2023; Kranton and Minehart, 2001; Zheng et al., 2023). In this line of work, the edges of the network capture the trading opportunities between agents, and the impacts of network effects on the market outcomes are analyzed. For example, Chen and Chen (2021) explored duopoly competition within a market involving network-connected buyers, and they showed that the existence of a symmetric market share equilibrium for two identical sellers depends on the intensity of network effects and the quality of the product. More closely, some recent studies explore how to improve operational efficiency in a two-sided market using centralized price controls (e.g., Banerjee et al., 2015; Ma et al., 2022; Varma et al., 2023) or non-pricing controls (e.g., Kanoria and Saban, 2021). For example, Hu and Zhou (2022) considered a platform that strategically matches buyers and sellers, who are categorized into distinct groups based on varying arrival rates and matching values. They provided sufficient conditions under which the optimal matching policy follows a priority hierarchy among matched pairs, determined by factors such as quality and distance. Our work adopts the framework proposed by Birge et al. (2021), in which a platform determines commission structure to maximize the total profit in a two-sided market, and the trades and prices are formed endogenously in a competitive equilibrium given the commissions. Differently, we delve into a dynamic setting and demonstrate that utilizing metrics for network imbalance from static settings in the prior studies to quantify the impact of network structure is inadequate. We introduce a novel metric that incorporates the intertemporal factor (i.e., the growth potentials of agents).

Some recent literature also explores the expansion of the platform's agent base in a network (e.g., Alizamir et al., 2022; Li et al., 2021). These studies assume a uniform retention and growth rate across agents from the same side or all agents in the network, with each agent's payoff determined by an exogenously specified function of the number of participants in the network. In contrast, we account for the heterogeneity of growth potentials among various agent types and introduce a novel metric that incorporates both spatial and intertemporal factors to assess the influence of the network structure on the platform's profitability.

3 Model

Consider a two-sided market in which a platform charges commissions to buyers and sellers for facilitating transactions. The compatibility between buyers and sellers is captured by a bipartite graph $(\mathcal{B} \cup \mathcal{S}, E)$, where $\mathcal{B} = \{1, 2, \dots, N_b\}$ denotes the set of buyer type and $\mathcal{S} = \{1, 2, \dots, N_s\}$ denotes the set of seller types; E is the set of edges that captures the potential trading opportunities between them. Specifically, $(i,j) \in E$ if and only if the service or product of type-i sellers can satisfy the demand of type-j buyers for $i \in \mathcal{S}$ and $j \in \mathcal{B}$.

This compatibility difference arises from varying tastes, geographical constraints, or skill mismatch. For example, the edge set on Airbnb reflects the preferences of leisure travelers over tourist destinations and business travelers preferring the city center or CBD; the edge set on Upwork/TaskRabbit is determined by the skill set of the freelancer and the demand of firms. This edge set is exogenous and remains stable throughout the decision horizon (see e.g., Hu and Zhou, 2022). In the rest of the paper, we will refer to an "incomplete market" as a bipartite network structure where there exists a pair (i,j) such that $i \in \mathcal{S}$ and $j \in \mathcal{B}$, but $(i,j) \notin E$. Otherwise, we call it "complete network".

Note that users may change their preferences over time and thus belong to different user segments at different times. For example, each traveler might occasionally act as a leisure traveler and at other times as a business traveler, and firms may want to outsource different tasks at different times. In this case, our model can treat them as two different users. In practice, some platforms utilize machine learning algorithms to infer user preferences based on their search behavior whenever they interact with listings or make inquiries (AI Business, 2017; Yu and Zhu, 2021).

In each period $t \in \{1, \dots, T\}$, the populations of type-i sellers and type-j buyers are respectively denoted by $s_i(t)$ for $i \in \mathcal{S}$ and $b_j(t)$ for $j \in \mathcal{B}$. The initial population of each type is finite, that is, $s_i(1) < \infty$ for $i \in \mathcal{S}$ and $b_j(1) < \infty$ for $j \in \mathcal{B}$. The buyers/sellers are infinitesimal, and each one of them supplies/demands at most one unit of product/service in one period. For $t \in \{1, \dots, T\}$, we use $q_i^s(t)$ and $q_j^b(t)$ respectively to denote the aggregate supply quantities of type-i sellers and the aggregate demand quantities of type-j buyers, where $q_i^s(t) \in [0, s_i(t)]$ for $i \in \mathcal{S}$ and $q_j^b(t) \in [0, b_j(t)]$ for $j \in \mathcal{B}$. Note that given the commission charged by the platform, the supply/demand vector $(\boldsymbol{q}^s(t), \boldsymbol{q}^b(t))$ is endogenously determined in equilibrium, with mechanism details discussed later.

Population Transition. A key feature of our model is that the mass of each agent type evolves dynamically at different rates (see the discussion in Section 1). For any $t \in \{1, \dots, T-1\}$, we consider the following population transition equations for all $i \in \mathcal{S}$ and $j \in \mathcal{B}$:

$$s_i(t+1) = \mathcal{G}_i^s(s_i(t), q_i^s(t)), \quad b_j(t+1) = \mathcal{G}_j^b(b_j(t), q_j^b(t)). \quad (1)$$

In (1), we assume that the mass of agents for the next period depends on the mass and transaction volume in the current period. A higher mass of agents in the current period contributes to a larger future agent base due to retention (see Lian and Van Ryzin, 2021). A higher transaction quantity leads to a higher future agent base due to the word-of-mouth effect or the imitation effect (see Bass, 1969; Mahajan and Peterson, 1985), that is, current agents who trade on the platform can share positive information about the platform with potential new adopters, attracting them to join the platform. For each type of agent (e.g., type-i sellers), the equilibrium transactions quantity in each period (e.g., $q_i^s(t)$) also depends on the mass of agents in other categories $(\boldsymbol{s}(t), \boldsymbol{b}(t))$, as discussed later in Definition 1. Therefore, these factors indirectly influence the agent's growth, reflecting the *network effects* mentioned in Section 1.

For the rest of the paper, $\mathcal{G}_i^s(\cdot, \cdot)$ and $\mathcal{G}_j^b(\cdot, \cdot)$ in (1) will be referred to as the *growth functions*. These growth functions can take various forms, such as $\mathcal{G}_i^s(q, s) = sf(q/s)$, with concave $f(\cdot)$ capturing the agent type's average surplus (see Lian and Van Ryzin, 2021). Instead of assuming a specific functional form, we impose only basic assumptions for the growth functions. For simplicity of notation, we let $(\mathcal{G}_i^s)'_1(s,q), (\mathcal{G}_i^s)'_2(s,q)$ denote the partial derivatives of $\mathcal{G}_i^s(s,q)$ with respect to $s \geq 0$ and $q \geq 0$; similarly, $(\mathcal{G}_j^b)'_1(b,q), (\mathcal{G}_j^b)'_2(b,q)$ denote the partial derivatives of $\mathcal{G}_j^b(b,q)$ respectively in $b \geq 0$ and $q \geq 0$.

Assumption 1 (**Growth Functions**). *For any $i \in \mathcal{S}$ and any $j \in \mathcal{B}$,*

(i) $\mathcal{G}_i^s(0,0) = 0$ *and* $\mathcal{G}_j^b(0,0) = 0$*;*

(ii) *$\mathcal{G}_i^s(s,q)$ is continuously differentiable, increasing and strictly concave in (s,q) for $0 \leq q \leq s$, and moreover, $\lim_{x\to\infty}[(\mathcal{G}_i^s)_1'(x,x) + (\mathcal{G}_i^s)_2'(x,x)] < 1$ for the seller side; the same properties hold for the buyer side with $\lim_{x\to\infty}[(\mathcal{G}_j^b)_1'(x,x) + (\mathcal{G}_j^b)_2'(x,x)] < 1$.*

Assumption 1(i) implies that if the population mass is zero and there is no transaction from the previous period, then there is no retention or word-of-mouth effect. Assumption 1(ii) requires that the future agent base increases in the current population mass and transaction volume, but the marginal effects of these two factors decrease because the total mass of potential agents in a market is finite. It also requires that the total marginal effects of these two factors are lower than one when the transaction volume and the population mass approach infinity, which implies that the number of agents in the system cannot grow infinitely large. We will delay the discussion about the class of examples under this assumption to Section 5.

We next discuss how the equilibrium supply/demand $(\boldsymbol{q}^s(t), \boldsymbol{q}^b(t))$ is formed in an (in)complete market given the commission by the platform in each period $t \in \{1, \ldots, T\}$.

Competitive Equilibrium. In period $t \in \{1, \ldots, T\}$, the platform charges commission $r_i^s(t)$ to type-i sellers and $r_j^b(t)$ to type-j buyers if they trade. The commissions are homogeneous within the same agent type but may vary across different types. When $r_i^s(t) < 0$ or $r_j^b(t) < 0$, the platform subsidizes the sellers or buyers. In practice, platforms could implement heterogeneous prices to different user types through personalized coupon distribution (see Amazon, 2020). Previous research also showed that revenue loss can be unbounded when using a uniform pricing strategy across types (see Birge et al., 2021). Therefore, in our setting, we consider type-dependent, heterogeneous pricing (see e.g., Varma et al., 2023).

Given the commissions, type-i sellers offer their product/service at price $p_i(t)$ and receive $p_i(t) - r_i^s(t)$; type-j buyers pay $p_i(t) + r_j^b(t)$ if they trade with type-i sellers. The market prices $\boldsymbol{p}(t)$ are endogenously formed in equilibrium to match supply and demand, rather than controlled by the platform (see Definition 1 later). For instance, hosts on Airbnb compete on their rental offers, and freelancers on Upwork compete on their hourly rates. We consider the case that a seller cannot charge different prices to different buyers, aligning with the standard practice of many online platforms, where seller prices are openly displayed on the web page. Finally, we assume for a type-j buyer, all compatible sellers (i.e., $i : (i,j) \in E$) provide perfectly substitutable products/services, and the type-j buyer does not have a preference over the compatible sellers' products if their prices are the same. Similarly, it is indifferent for a seller to trade with any compatible buyers, given that the market price is formed on the seller side. Note that vertical differentiation of sellers can be modeled by adding a quality term for each type of seller in the payoff function of buyers (see Birge et al., 2021), which does not fundamentally change our insights.

We use $F_{b_j} : [0, \bar{v}_{b_j}] \to [0,1]$ and $F_{s_i} : [0, \bar{v}_{s_i}] \to [0,1]$ to denote the cumulative distribution function of the (reservation) values respectively for type-j buyers and type-i sellers, in which $\bar{v}_{b_j}$ and $\bar{v}_{s_i}$ are finite for any $j \in \mathcal{B}$ and $i \in \mathcal{S}$. For simplicity, we refer to a seller by "he" and a buyer by "she". A type-i seller only engages in trades when the amount he receives from the transaction is weakly higher than his reservation value v, that is, $p_i(t) - r_i^s(t) \geq v$; similarly, a type-j buyer trades when the total payment is weakly lower than her value v, that is, $p_i(t) + r_j^b(t) \leq v$. To simplify our analysis later, we extend the domains of the value distributions to $\mathbb{R}$: let $F_{b_j}(v) = 1$ for $v \geq \bar{v}_{b_j}$ and $F_{b_j}(v) = 0$ for $v \leq 0$ for any $j \in \mathcal{B}$; similarly, for the seller side, we let $F_{s_i}(v) = 1$ for $v \geq \bar{v}_{s_i}$ and $F_{s_i}(v) = 0$ for $v \leq 0$ for any $i \in \mathcal{S}$. In addition, define $f_{b_j}(v)$ and $f_{s_i}(v)$ respectively as the derivative (or the density function) of $F_{b_j}(v)$ for $v \in [0, \bar{v}_{b_j}]$ and $F_{s_i}(v)$ for $v \in [0, \bar{v}_{s_i}]$. We impose the following assumption throughout the paper.

Assumption 2 (**Value Distribution**). *For any $j \in \mathcal{B}$ and $i \in \mathcal{S}$,*

(i) *$F_{b_j}(v)$ and $F_{s_i}(v)$ are strictly increasing in $v \in [0, \bar{v}_{b_j}]$ and $v \in [0, \bar{v}_{s_i}]$;*
(ii) *$F_{b_j}(v)$ and $F_{s_i}(v)$ are continuously differentiable respectively in $v \in [0, \bar{v}_{b_j}]$ and $v \in [0, \bar{v}_{s_i}]$; $f_{b_j}(v)$ and $f_{s_i}(v)$ are lower bounded by a positive constant.*

Under Assumption 2(i), we define the inverse function $F_{b_j}^{-1} : [0,1] \to [0, \bar{v}_{b_j}]$ and $F_{s_i}^{-1} : [0,1] \to [0, \bar{v}_{s_i}]$ such that $F_{b_j}^{-1}(F_{b_j}(v)) = v$ for $v \in [0, \bar{v}_{b_j}]$ and $F_{s_i}^{-1}(F_{s_i}(v)) = v$ for $v \in [0, \bar{v}_{s_i}]$. Under Assumption 2(ii), $F_{b_j}^{-1}(x)$ and $F_{s_i}^{-1}(x)$ are also continuous and differentiable in $x \in [0,1]$, and their density functions are also bounded. We further impose the following Assumption on $F_{b_j}^{-1}(x)$ and $F_{s_i}^{-1}(x)$.

Assumption 3 (**Concavity**). *$F_{b_j}^{-1}(1 - a/b)a$ and $-F_{s_i}^{-1}(a/b)a$ are strictly concave in (a,b) for $0 \leq a \leq b$.*

Assumptions 2 and 3 hold for many commonly used distributions such as uniform, truncated exponential, and truncated generalized Pareto distribution.

We can finally define the equilibrium in the incomplete market in each period, given the commission vector $(r_i^s(t) : i \in \mathcal{S},\ r_j^b(t) : j \in \mathcal{B})$ by the platform and the population vector $(\boldsymbol{s}(t), \boldsymbol{b}(t))$. We denote by $x_{ij}(t)$ the amount that type-j buyers purchase from type-i sellers.

Definition 1 (**Competitive Equilibrium**). In period $t \in \{1, \ldots, T\}$, given the platform's commission profile $(\boldsymbol{r}^s(t), \boldsymbol{r}^b(t)) \in \mathbb{R}^{N_s} \times \mathbb{R}^{N_b}$ and the population vector of sellers and buyers $(\boldsymbol{s}(t), \boldsymbol{b}(t)) \in \mathbb{R}_+^{N_s} \times \mathbb{R}_+^{N_b}$, a competitive equilibrium is defined as the price-flow vector $(\boldsymbol{p}(t), \boldsymbol{x}(t), \boldsymbol{q}^s(t), \boldsymbol{q}^b(t))$ that satisfies the following conditions:

$$q_i^s(t) = s_i(t) F_{s_i}(p_i(t) - r_i^s(t)), \quad \forall i \in \mathcal{S}, \quad (2a)$$

$$q_j^b(t) = b_j(t)\left(1 - F_{b_j}\left(\min_{i':(i',j)\in E}\{p_{i'}(t)\} + r_j^b(t)\right)\right), \quad \forall j \in \mathcal{B}, \quad (2b)$$

$$q_i^s(t) = \sum_{j':(i,j')\in E} x_{i,j'}(t), \quad \forall i \in \mathcal{S}, \quad (2c)$$

$$q_j^b(t) = \sum_{i':(i',j)\in E} x_{i',j}(t), \quad \forall j \in \mathcal{B}, \quad (2d)$$

$$x_{ij}(t) \geq 0, \quad \forall (i,j) \in E, \quad (2e)$$

$$x_{ij}(t) = 0, \quad \forall i \notin \arg\min_{i':(i',j)\in E}\{p_{i'}\}, \ j \in \mathcal{B}. \quad (2f)$$

In Definition 1, Conditions (2a) and (2b) ensure that the total supply/demand quantities of type-i sellers and type-j buyers equal the mass of agents who can obtain nonnegative utilities from the transaction. Specifically, Condition (2b) assumes that type-j buyers only trade with compatible sellers with the lowest market price to maximize their utilities. Conditions (2c) and (2d) characterize the flow conservation conditions in the compatibility network. Finally, Condition (2e) requires that the transaction flow is non-negative, and Condition (2f) requires that the buyers only trade with their compatible sellers with the lowest prices.

The equilibrium concepts similar to Definition 1 have also been adopted in the two-sided market literature by, for example, Weyl (2010) and Birge et al. (2021). In our setting, the demand/supply quantities only depend on the prices and commissions in the current period, which is commonly seen in the literature about dynamic pricing, for example, Chen and Gallego (2019), Birge et al. (2024). With Definition 1, given any commission profile and the total mass of agents in each period, the equilibrium always exists, and the equilibrium supply–demand vector $(\boldsymbol{q^s}(t), \boldsymbol{q^b}(t))$ is always unique (see Proposition EC.1 in Section EC.1.1 of the E-companion).

Platform's Profit Optimization Problem. Given the mass of different types of agents in the first period $(\boldsymbol{s}(1), \boldsymbol{b}(1))$, the platform aims to maximize its total T-period profit by determining the commission for each type in each period. For simplicity of notation, we let $(\boldsymbol{s}, \boldsymbol{b}) := (\boldsymbol{s}(t), \boldsymbol{b}(t))_{t=2}^T$, and $(\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{x}, \boldsymbol{q^s}, \boldsymbol{q^b}) := (\boldsymbol{r}(t), \boldsymbol{p}(t), \boldsymbol{x}(t), \boldsymbol{q^s}(t), \boldsymbol{q^b}(t))_{t=1}^T$, then the platform's T-period profit maximization problem is as follows:

$$\mathcal{R}^*(T) = \max_{\boldsymbol{s},\boldsymbol{b},\boldsymbol{r},\boldsymbol{p},\boldsymbol{x},\boldsymbol{q^s},\boldsymbol{q^b}} \sum_{t=1}^{T}\left[\sum_{i\in\mathcal{S}} r_i^s(t) q_i^s(t) + \sum_{j\in\mathcal{B}} r_j^b(t) q_j^b(t)\right] \quad (3a)$$

$$\text{s.t.} \quad (\boldsymbol{s}(t), \boldsymbol{b}(t), \boldsymbol{r}(t), \boldsymbol{p}(t), \boldsymbol{x}(t), \boldsymbol{q^s}(t), \boldsymbol{q^b}(t)) \text{ satisfies (2)}, \quad \forall t \in \{1, \ldots, T\}, \quad (3b)$$

$$s_i(t+1) = \mathcal{G}_i^s(s_i(t), q_i^s(t)), \quad \forall i \in \mathcal{S}, t \in \{1, \ldots, T-1\}, \quad (3c)$$

$$b_j(t+1) = \mathcal{G}_j^b(b_j(t), q_j^b(t)), \quad \forall j \in \mathcal{B}, t \in \{1, \ldots, T-1\}. \quad (3d)$$

The platform's profit consists of the commissions from the sellers and buyers who trade in the market during the T periods. Constraint (3b) ensures that given the population vector $(\boldsymbol{s}(t), \boldsymbol{b}(t))$ and commission profile $(\boldsymbol{r^s}(t), \boldsymbol{r^b}(t))$ in period t, vector $(\boldsymbol{p}(t), \boldsymbol{x}(t), \boldsymbol{q^s}(t), \boldsymbol{q^b}(t))$ constitutes a competitive equilibrium; Constraints (3c)-(3d) dictate that the dynamics of populations follow the transition equations given in (1). Given that the equilibrium transaction quantities $(\boldsymbol{q^s}(t), \boldsymbol{q^b}(t))_{t=1}^T$ are unique under any commission $(\boldsymbol{r^s}(t), \boldsymbol{r^b}(t))_{t=1}^T$ (see Proposition EC.1(ii) in Section EC.1.1 in E-companion), the maximization problem (3) is well-defined.

It is worth noting that this optimization problem captures a fundamental trade-off between short-term profit and long-term user base expansion in platform growth. On one hand, increasing the platform's commission rate reduces user participation in transactions (see constraints (2a)-(2b)), which leads to a smaller user base in the future period (see constraints (3c)-(3d)). On the other hand, a higher commission rate may yield larger revenue in the current period, as indicated by the objective function (3a). Therefore, the platform must carefully balance this intertemporal trade-off when making decisions. In the rest of the paper, we refer to Problem (3) as OPT. Since OPT is non-convex (in $(\boldsymbol{r}, \boldsymbol{q})$), we will first reformulate it into a convex optimization problem and then discuss the challenges in solving it.

Reformulation and Challenges. For any period $t \in \{1, \ldots, T\}$, we deduce from (2a) that type-i sellers' incomes per unit are bounded below by the highest reservation value among those who participate in trading, that is, $p_i(t) - r_i^s(t) \geq F_{s_i}^{-1}\left(\frac{q_i^s(t)}{s_i(t)}\right)$ for i with $q_i^s(t) > 0$. Similarly, type-j buyers' payments are bounded above by the lowest value among them, that is, $p_i(t) + r_j^b(t) \leq F_{b_j}^{-1}\left(1 - \frac{q_j^b(t)}{b_j(t)}\right)$ for j with $q_j^b(t) > 0$. Therefore, the objective value of OPT is upper bounded by $\sum_{t=1}^T \left[\sum_{j\in\mathcal{B}} F_{b_j}^{-1}\left(1 - \frac{q_j^b(t)}{b_j(t)}\right) q_j^b(t) - \sum_{i\in\mathcal{S}} F_{s_i}^{-1}\left(\frac{q_i^s(t)}{s_i(t)}\right) q_i^s(t)\right]$, which is concave in $(\boldsymbol{q}, \boldsymbol{s}, \boldsymbol{b})$ under Assumption 3. By further relaxing some constraints of OPT, we can obtain a convex optimization problem where the decision variables only consist of $(\boldsymbol{s}, \boldsymbol{b}, \boldsymbol{q^s}, \boldsymbol{q^b}, \boldsymbol{x})$ but not commission $(\boldsymbol{r^s}, \boldsymbol{r^b})$. We present the formulation in Problem (EC.2a) and show that the relaxation is tight in Proposition EC.2 in Section EC.1.1 in the E-companion. After obtaining the optimal solution to this convex optimization problem, we can find a feasible commission profile $(\boldsymbol{r^s}, \boldsymbol{r^b})$ that can induce this equilibrium by solving a system of linear inequalities in each period (see Lemma EC.1 in Section EC.1.1 in E-companion). The feasible commissions always exist and are not necessarily unique, but the payments/incomes of agents with positive trades are uniquely determined in any equilibrium.

Even though the nonconvexity challenge of OPT can be circumvented by the reformulation, solving Problem (EC.2a-2h) remains computationally challenging when the problem's dimensionality is large (i.e., greater than $T\times(2N_s+2N_b+|E|)$). While Problem EC.2a-2h) can also be reformulated as a deterministic dynamic program (DP) with a high-dimensional state space, the lack of its structural properties makes it difficult to derive clear managerial implications for the growth strategy in the networked market. Therefore, we focus on designing a simpler policy with provable performance guarantees, which offers clear guidance for the platform's growth strategy.

4 Asymptotically Optimal Policy

We define an admissible policy as a sequence of functions $\pi =: \{\pi_t : \mathcal{H}_t \to \boldsymbol{r}^{N_s+N_b}(t)\}_{t=1}^T$ that prescribes the commission profile $(\boldsymbol{r}^s(t), \boldsymbol{r}^b(t))$ in each period t, where $\mathcal{H}_t$ is the history of population vectors $(\boldsymbol{s}(t'), \boldsymbol{b}(t'))_{t'=1}^t$ and transaction vectors $(\boldsymbol{x}(t'), \boldsymbol{q}^s(t'), \boldsymbol{q}^b(t'))_{t'=1}^{t-1}$. We denote $\mathcal{R}^\pi(T)$ as the platform's total profit in T periods for any admissible policy π. We evaluate the policy's performance by quantifying its profit loss relative to the optimal objective value $\mathcal{R}^*(T)$ of OPT, which is formally defined as

$$\mathcal{L}^\pi(T) \; = \; \mathcal{R}^*(T) - \mathcal{R}^\pi(T). \tag{4}$$

We aim to devise an admissible policy with good performance in the asymptotic setting as $T \to \infty$.

Notice that Flynn (1978) studies heuristic policies for solving infinite-horizon deterministic dynamic programming problems. He proposes a "steady-state policy", which involves solving a static problem to identify the OSS, steering the system to this state, and maintaining it there. Our algorithm shares a similar spirit of "steady-state policy." However, while he provides examples of constructing feasible rules to move the system from the initial state to the target steady state, most of them involve implementing the action at the OSS from the beginning or making straightforward modifications to it (Flynn, 1975a, 1981). We will see that those methods cannot be applied to our setting. In other words, it remains unclear how to construct a feasible solution that guides the system toward an OSS in a network problem with endogenous equilibrium constraints. This is challenging due to the flow conservation constraints in the networked market (i.e., Definition 1). To address this issue, we need to identify further which type within the network to prioritize during the growth stage and which metric to target for growth.

In this Section, after establishing the OSS, we introduce a novel approach called the Target-Ratio Policy (TRP). Inspired by the fundamental principle of Economics that "scarcity creates value" (Samuelson and Samuelson, 1980), the TRP prioritizes the agent type with the lowest population relative to the OSS, regulating its service level within specified ranges in each period. We prove that the TRP is asymptotically optimal, demonstrating that focusing solely on the scarcest agent type is sufficient for the platform to achieve strong performance. We also identify the conditions under which subsidization should be implemented. Finally, we show that a MP, which maximizes single-period profit while completely ignoring population growth, can perform poorly (see Proposition 3). This finding, again, highlights the critical need for carefully balancing short-term profitability with long-term growth, even for a monopolistic platform.

Long-run Average Value Problem (AVG). We begin by formulating a corresponding steady-state problem for OPT. This step is essential because deriving the exact optimal solution to OPT is challenging, while the optimal objective value of the steady-state problem offers a good approximation of the optimal objective value of OPT and can thus serve as a benchmark for evaluating the policy's profit loss in (4).

For convenience, we define

$$\widetilde{F}_{b_j}(q_j^b, b_j) := \begin{cases} 0, & q_j^b = b_j = 0, \\ F_{b_j}^{-1}\big(1 - \dfrac{q_j^b}{b_j}\big) q_j^b, & b_j > 0, 0 \le q_j^b \le b_j, \end{cases}$$

$$\widetilde{F}_{s_i}(q_i^s, s_i) := \begin{cases} 0, & q_i^s = s_i = 0, \\ F_{s_i}^{-1}\big(\dfrac{q_i^s}{s_i}\big) q_i^s, & s_i > 0, 0 \le q_i^s \le s_i. \end{cases}$$

Then we consider the following optimization problem which we refer to as AVG:

$$\overline{\mathcal{R}} \; = \; \max_{\boldsymbol{s},\boldsymbol{b},\boldsymbol{q}^s,\boldsymbol{q}^b,\boldsymbol{x}} \; \sum_{j\in\mathcal{B}} \widetilde{F}_{b_j}(q_j^b, b_j) - \sum_{i\in\mathcal{S}} \widetilde{F}_{s_i}(q_i^s, s_i), \tag{5a}$$

$$\text{s.t.} \quad q_i^s \; \le \; s_i, \; \sum_{j:(i,j)\in E} x_{ij} \; = \; q_i^s, \qquad \forall i \in \mathcal{S}, \tag{5b}$$

$$q_j^b \; \le \; b_j, \; \sum_{i:(i,j)\in E} x_{ij} \; = \; q_j^b, \qquad \forall j \in \mathcal{B}, \tag{5c}$$

$$x_{ij} \; \ge \; 0, \qquad \forall (i,j) \in E. \tag{5d}$$

$$s_i \; \le \; \mathcal{G}_i^s(s_i, q_i^s), \qquad \forall i \in \mathcal{S}, \tag{5e}$$

$$b_j \; \le \; \mathcal{G}_j^b(b_j, q_j^b), \qquad \forall j \in \mathcal{B}. \tag{5f}$$

We relax Constraint (3b) of OPT about equilibrium conditions to (5b)-(5d), and relax the population transition equations in Constraint (3c)-(3d) to inequalities in (5e)-(5f). As a result, AVG is a tractable convex optimization problem. We next characterize the properties of its optimal solution denoted by $(\overline{\boldsymbol{s}}, \overline{\boldsymbol{b}}, \overline{\boldsymbol{q}}^s, \overline{\boldsymbol{q}}^b, \overline{\boldsymbol{x}})$:

LEMMA 1 (**Optimal Solution to AVG**). *The optimal solution to Problem (5) exists, and*

(i) *the optimal population* $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}})$ *and the optimal supply–demand vector* $(\bar{\boldsymbol{q}}^s, \bar{\boldsymbol{q}}^b)$ *are unique;*
(ii) *the constraints (5e)-(5f) are tight at optimal.*

Lemma 1(ii) implies that the population can be sustained at $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}})$ by controlling the supply–demand at the level of $(\bar{\boldsymbol{q}}^s, \bar{\boldsymbol{q}}^b)$. We further show that the gap between T times the optimal objective value of AVG from (5) and that of OPT from (3) is upper bounded by a constant for any positive integer T.

PROPOSITION 1 (**Alternative Benchmark**). *There exists a positive constant* C_1 *such that for any* $T = 1, 2, \ldots$, $|\mathcal{R}^*(T) - T\overline{\mathcal{R}}| \leq C_1$.

Proposition 1 dictates that the difference between $\frac{1}{T}\mathcal{R}^*(T)$ and $\overline{\mathcal{R}}$ converges to zero as T approaches infinity. Therefore, the optimal solution to AVG $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}}, \bar{\boldsymbol{q}}^s, \bar{\boldsymbol{q}}^b)$ captures a steady state where the long-run average profit is maximized, and therefore we call it the *OSS* in the rest of the paper (see Flynn, 1975a, 1992). In addition, as we previously mentioned, in contrast to the high-dimensional problem OPT, AVG is a much more tractable static convex optimization problem. Therefore, we will consider $T\overline{\mathcal{R}}$, instead of $\mathcal{R}^*(T)$ as the benchmark to quantify the policy's profit loss in (4). We next propose the TRP that achieves fast convergence to OSS and formally establish its asymptotic optimality.

Target Ratio Policy. For ease of illustration, we refer to $\frac{s_i(t)}{\bar{s}_i}$ for $i \in \mathcal{S}$ and $\frac{b_j(t)}{\bar{b}_j}$ for $j \in \mathcal{B}$ as the *population ratio* of type-i seller and type-j buyer, respectively. In addition, we notice that $\frac{q_j^b(t)}{b_j(t)}$ $\left(\frac{q_i^s(t)}{s_i(t)}\right)$ is the fraction of type-j buyers (type-i sellers) who trade in period t, and we refer to this fraction as the *service level* of the corresponding agent type. Recall that the service level also determines the payment/income of agents (i.e., $F_{b_j}^{-1}\left(1 - \frac{q_j^b(t)}{b_j(t)}\right)$ and $F_{s_i}^{-1}\left(\frac{q_i^s(t)}{s_i(t)}\right)$).

Inspired by Proposition 1, we design our algorithm to guide the population of all agent types toward the levels at OSS $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}})$ and maintain them there. A straightforward approach is to align the service level (equivalently, the income/payment) of each type with its corresponding level at the OSS, that is, $\frac{q_i^s(t)}{s_i(t)} \approx \frac{\bar{q}_i^s}{\bar{s}_i}$ for any $i \in \mathcal{S}$ and $\frac{q_j^b(t)}{b_j(t)} \approx \frac{\bar{q}_j^b}{\bar{b}_j}$ for any $j \in \mathcal{B}$ for $t \in \{1, \ldots, T\}$. However, a significant challenge arises: such a policy is not always feasible in a network. Consider, for instance, a simple scenario with one buyer and one seller type. Given the flow conservation constraint $q^s(1) = q^b(1)$, if we target the service level for the seller side at OSS (i.e., $\frac{q^s(1)}{s(1)} = \frac{\bar{q}^s}{\bar{s}}$), the service level for the buyer side may be different from that of OSS in general (e.g., $\frac{q^b(1)}{b(1)} < \frac{\bar{q}^b}{\bar{b}}$ if $\frac{s(1)}{\bar{s}} < \frac{b(1)}{\bar{b}}$). A critical question remains: Which type should be prioritized to foster growth during the early stages, particularly in a market with multiple user segments? If the principle that "scarcity creates value" holds, what is the proper metric to measure "scarcity" in this context?

From the above example, we observe that the type with the lower population ratio constrains the transaction volume of the type with the higher ratio, thereby further hindering its growth. Therefore, in each period, we should prioritize the type with the lowest population ratio and seek to boost its growth (i.e., $q^s(1) = s(1)\frac{\bar{q}^s}{\bar{s}}$ if $\frac{s(1)}{\bar{s}} < \frac{b(1)}{\bar{b}}$), while we match the transaction quantities of other types to guarantee the feasibility of the policy (i.e., $q^b(1) = q^s(1) = s(1)\frac{\bar{q}^b}{\bar{s}}$ if $\frac{s(1)}{\bar{s}} < \frac{b(1)}{\bar{b}}$). As the type with the lowest population ratio may change over time, the focus dynamically shifts to different types in different periods, until the system's state converges to the OSS.

We will demonstrate that the above method guarantees strong performance without requiring the platform to provide subsidies throughout the entire planning horizon. However, for platforms prioritizing rapid growth over short-term profitability, we find that they can further accelerate the growth of all types during the early stages by raising the service level of the most constrained type above its OSS value. One way to implement this is by involving as many agents as possible from the scarcest type in transactions, $q_i^s(1) = q_j^b(1) = \min\{s(1), b(1)\}$, until the system shows signs of overexpansion. As we will show later, this approach may, under certain conditions, require the platform to provide subsidies. Importantly, we will see that any policy falling between these two extremes is asymptotically optimal, but they require different levels of subsidy/commission and lead to varying growth rates. Building on this idea, we formally define the class of Target Ratio Policies parameterized by the acceleration weight w as in Algorithm 1.

One key advantage of TRP is its computational efficiency: It only requires solving the single-period optimization problem AVG once. Subsequently, in each period, it identifies two critical values: the lowest population ratio, $m(t)$, and the lowest ratio between population and targeted transaction quantity, $M(t)$. It can be easily seen that $m(t) \leq M(t)$. By setting $q_i^s(t) = \bar{q}_i^s m(t)$ and $q_j^b(t) = \bar{q}_j^b m(t)$, the policy sets the service level of the scarcest type at its OSS value. In contrast, by setting $q_i^s(t) = \bar{q}_i^s M(t)$ and $q_j^b(t) = \bar{q}_j^b M(t)$, the policy maximizes the service level of the scarcest type. The platform can select any value between these two extremes $\hat{m}(t)$ by adjusting the acceleration weight $w \in [0, 1]$. The parameter w reflects how much the platform can trade off short-term profit for accelerated growth, which we will formally demonstrate later.

In the early stages of growth, if $w > 0$, we continue to raise the service level of the scarcest type above its service level at OSS, as long as this policy does not push the population of the scarcest type above that at OSS (i.e.,

Algorithm 1: Target Ratio Policy (TRP)

Input: Optimal solution to AVG $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}}, \bar{\boldsymbol{q}}^s, \bar{\boldsymbol{q}}^b, \bar{\boldsymbol{x}})$; initial mass of agents $(\boldsymbol{s}(1), \boldsymbol{b}(1))$; acceleration weight $w \in [0,1]$.*OverExpansion* $\leftarrow$ *False*;**for** $t = 1$ *to* T **do**

  $m(t) \leftarrow \min_{i,j:\bar{s}_i>0,\bar{b}_j>0} \left\{ \frac{s_i(t)}{\bar{s}_i}, \frac{b_j(t)}{\bar{b}_j} \right\}$; $M(t) \leftarrow \min_{i,j:\bar{q}_i^s,q_j^b>0} \left\{ \frac{s_i(t)}{\bar{q}_i^s}, \frac{b_j(t)}{\bar{q}_j^b} \right\}$; $\hat{m}(t) \leftarrow (1-w)m(t) + wM(t)$**if** $\min_{i,j:\bar{s}_i>0,\bar{b}_j>0} \left\{ \frac{\mathcal{G}_i^s(s_i(t),\bar{q}_i^s\hat{m}(t))}{\bar{s}_i}, \frac{\mathcal{G}_j^b(b_j(t),\bar{q}_j^b\hat{m}(t))}{\bar{b}_j} \right\} \le 1$ *and OverExpansion = False* **then**

    for $(i,j) \in E$ **do**

      $q_i^s(t) \leftarrow \bar{q}_i^s\hat{m}(t)$; $q_j^b(t) \leftarrow \bar{q}_j^b\hat{m}(t)$; $x_{ij}(t) \leftarrow \bar{x}_{ij}\hat{m}(t)$;

  else

    OverExpansion $\leftarrow$ *True*;**for** $(i,j) \in E$ **do**

      $q_i^s(t) \leftarrow \bar{q}_i^s m(t)$; $q_j^b(t) \leftarrow \bar{q}_j^b m(t)$; $x_{ij}(t) \leftarrow \bar{x}_{ij} m(t)$;

  Solve (EC.1a-1d) in Section EC.1.1 in E-companion to obtain $(\boldsymbol{r}^s(t), \boldsymbol{r}^b(t))$;

  if there are multiple feasible solutions, select one arbitrarily;

  update population profile $(\boldsymbol{s}(t+1), \boldsymbol{b}(t+1))$ by the system dynamics in (1).

Output: $(\boldsymbol{r}^s(t), \boldsymbol{r}^b(t))_{t=1}^T$.

$\min_{i,j:\bar{s}_i>0,\bar{b}_j>0} \left\{ \frac{\mathcal{G}_i^s(s_i(t),\bar{q}_i^s\hat{m}(t))}{\bar{s}_i}, \frac{\mathcal{G}_j^b(b_j(t),\bar{q}_j^b\hat{m}(t))}{\bar{b}_j} \right\} \le 1$). However, once this increases its population beyond the targeted population level (i.e. $\min_{i,j:\bar{s}_i>0,\bar{b}_j>0} \left\{ \frac{\mathcal{G}_i^s(s_i(t),\bar{q}_i^s\hat{m}(t))}{\bar{s}_i}, \frac{\mathcal{G}_j^b(b_j(t),\bar{q}_j^b\hat{m}(t))}{\bar{b}_j} \right\} > 1$), our policy shifts to match its service level to that at OSS, that is, $\frac{q_i^s(t)}{s_i(t)} = \frac{\bar{q}_i^s}{\bar{s}_i}$ or $\frac{q_j^b(t)}{b_j(t)} = \frac{\bar{q}_j^b}{\bar{b}_j}$, which helps ensure that the population converges to the OSS.

Notably, for other types with higher population ratios, their demand/supply quantities are matched correspondingly to guarantee feasibility in the networked market. As a result, their service level will be lower than that at OSS (i.e., $\frac{q_i^s(t)}{s_i(t)} < \frac{\bar{q}_i^s}{\bar{s}_i}$ or $\frac{q_j^b(t)}{b_j(t)} < \frac{\bar{q}_j^b}{\bar{b}_j}$). This may lead to slower population growth or even a decline at the beginning. Consequently, the scarcest agent type may change over time within the market, prompting the platform to focus on enhancing the growth of different types throughout the planning horizon. Perhaps surprisingly, by always guaranteeing the growth of the agent types with the *lowest* population ratio in each period, the entire market could converge to the OSS quickly. Let $\mathcal{L}^{TR}(T)$ denote the profit loss of TRP relative to the optimal objective value $\mathcal{R}^*(T)$, and let $m^w(t)$ denote the lowest population ratio under the policy with acceleration weight w. Then the following result gives a theoretical performance guarantee for TRP:

THEOREM 1 (**Performance of TRP**). *There exists a constant C_2 such that for all $T = 1, 2..$ $\mathcal{L}^{TR}(T) \le C_2$. Furthermore, for any $w \in (0,1]$ and $t = \{1, \dots, T\}$, we have $|m^w(t) - 1| \le |m^0(t) - 1|$.*

Theorem 1 shows that the profit loss of TRP relative to the optimal policy is uniformly bounded (with respect to T) by a constant, which further suggests that TRP is asymptotically optimal in the networked market. Moreover, it reaffirms the fundamental principle that "scarcity creates value." Notably, our proof technique for showing that the policy is asymptotically optimal differs significantly from the methods used in the literature on "steady-state policy" (Flynn, 1975a, 1975b, 1992). We first show that under TRP, even though the type with the lowest ratio may change over time, the lowest ratio $m(t)$ *monotonically converges* to one at an exponential rate, that is, $|m(t+1) - 1| \le \gamma|m(t) - 1|$ for some $\gamma \in (0,1)$. Therefore, for each type, the transaction quantity $q_i^s(t) = \bar{q}_i^s m(t)$ or $q_j^b(t) = \bar{q}_j^b m(t)$ converges to the optimal level $\bar{q}_i^s$ or $\bar{q}_j^b$ for any $i \in \mathcal{S}$ and $j \in \mathcal{B}$, which ensures that the population profile $(\boldsymbol{s}(t), \boldsymbol{b}(t))$ also converges to OSS $(\bar{\boldsymbol{s}}, \bar{\boldsymbol{b}})$. By establishing the fast convergence rate, we observe that there exists a constant C_1' such that $|T\bar{\mathcal{R}} - \mathcal{R}^{TR}(T)| \le C_1'$. Together with the result in Proposition 1, we conclude that there exists a constant C_2 such that $|\mathcal{R}^*(T) - \mathcal{R}^{TR}(T)| \le C_2$. The detailed proof of Theorem 1 is relegated to Section EC.2 in the E-companion.

Furthermore, Theorem 1 demonstrates that increasing the service level for the most limited type beyond its value at OSS (by choosing a positive w) accelerates the convergence of its population to the targeted level (will be illustrated in Figure 1(b)). Next, we will discuss how the commission or subsidy structure under TRP changes with the parameter w to elucidate the costs associated with achieving this accelerated growth. As mentioned, the commissions used to induce the desired transaction quantities in each period can be obtained by solving a system of linear inequalities (see (EC.1a- 1d) in Lemma EC.1, Section EC.1.1 in E-companion).

Commission Structure: Subsidy and Surcharge. Notice that the commission charged from each type of user is not unique, but the total commission collected from both sides for each transaction is unique (see Proposition EC.1 and Lemma EC.1). Therefore, we focus on the total commission for each transaction with a positive flow x_{ij}. Recall that a negative commission can be interpreted as the platform subsidizing the users.

PROPOSITION 2 (**Conditions for Subsidy**). *For all (i,j) with $x_{ij} > 0, t \in \{1, \ldots, T\}$,*

(i) *under TRP with $w = 0$, $r_i^s(t) + r_j^b(t) \geq 0$;*
(ii) *under TRP with $w > 0$, there exist positive constants z_{ij} and $\tilde{t}$ such that $r_i^s(t) + r_j^b(t) \leq 0$ if $\frac{\max\{s_i(t),b_j(t)\}}{\min_{i' \in S, j' \in B}\{s_{i'}(t), b_{j'}(t)\}} < z_{i,j}w$ and $t < \tilde{t}$.*

Proposition 2(i) highlights that subsidization during the growth stage is not necessary, particularly in a monopoly environment. Instead, by lowering the commission for appropriate user segments, the platform can already effectively guide the user base toward the OSS. Therefore, a policy with a small w is an ideal option for startup platforms with limited budgets. Proposition 2(ii) identifies the specific user types that should be subsidized during the early growth phase when the platform seeks to accelerate growth and capture market share quickly by choosing a positive w. Subsidies become necessary when the ratio of buyers and sellers involved in transactions to the minimum population of types drops below a certain threshold. This indicates that the number of participants on both sides of a transaction is close to the population of the scarcest type, aligning with real-world practices where platforms often provide coupons or discounts to attract scarce users during the initial growth stages. A higher w is more likely to render subsidization.

To better understand this condition, consider a simple example with one buyer and one seller type. If the buyer type has a lower initial population (i.e., $b(1) < s(1)$), the condition is simplified to $b(1) \ll s(1)$

Finally, we identify how population levels, growth potentials, and network structures influence our policy design. For expositional ease, we denote by $Y_j(t) := \min_{i':(i',j) \in E}\{p_{i'}(t) + r_j^b(t)\}$ the type-j buyer's payment at time t, and denote by $I_i(t) := p_i(t) - r_i^s(t)$ the type-i seller's income at time t.

COROLLARY 1. *Suppose that $F_{s_i} = F_s$ for any $i \in \mathcal{S}$ and $F_{b_j} = F_b$ for any $j \in \mathcal{B}$, then under TRP, for any $t \in \{1, \ldots, T\}$,*

(i) *for the buyer side, if $\frac{\bar{q}_j^b}{\bar{b}_j} / \frac{b_j(t)}{\bar{b}_j} < \frac{\bar{q}_l^b}{\bar{b}_l} / \frac{b_l(t)}{\bar{b}_l}$, we have $Y_j(t) > Y_l(t)$, and there exists one feasible commission such that $r_j^b(t) > r_l^b(t)$ for any $j, l \in \mathcal{B}$;*
(ii) *for the seller side, if $\frac{\bar{q}_i}{\bar{s}_i} / \frac{s_i(t)}{\bar{s}_i} < \frac{\bar{q}_l}{\bar{s}_l} / \frac{s_l(t)}{\bar{s}_l}$, we have $I_i(t) < I_l(t)$, and there exists one feasible commission such that $r_i^s(t) > r_l^s(t)$ for any $i, l \in \mathcal{S}$.*

Our findings indicate that the payment or income for each type depends only on the targeted service level at the OSS and the current population ratio. In Section 5, we will construct a novel metric to demonstrate how the targeted service level is affected by the network structure and the growth potential of different types (see Proposition 4). Essentially, the targeted service level is lower for a type with higher growth potential but is compatible with types with lower growth potential. Consequently, the platform should charge higher commissions from them. On the other hand, the platform should charge lower commissions from user types with lower population ratios.

We will numerically illustrate the profit (see Figure 1(a)), population growth trajectory (see Figure 1(b)), and feasible commission structure (see Figure 1(c)-Figure 1(d)) for $w \in \{0, 0.05, 0.1, \ldots, 1\}$ (a total of 21 cases). We use the superscript $\{w\}$ to denote the value under a policy with an acceleration weight w.

EXAMPLE 1. Consider a one-buyer-one-seller example. The seller side has a lower initial population ratio (i.e., $30\% = \frac{s(1)}{\bar{s}} > \frac{b(1)}{\bar{b}} = 50\%$). The retention rate of the seller side is assumed to be lower than the buyer side: $s(t+1) = 0.7s(t) + 2(q^s(t))^{0.9}$ and $b(t+1) = 0.8b(t) + 2(q^b(t))^{0.9}$.

Figure 1(a) shows that the platform's profit is neither monotonic nor concave with respect to the parameter w, but it can be seen that the profit reaches its maximum value when $w = 0.15$. Therefore, we highlight the population and commission trajectories for $w = 0.15$ using a solid line and use a dashed line to highlight the case when $w = 0$ for comparison. In Figure 1(b), the mass of both types grows over time and eventually stabilizes at OSS under both policies. Specifically, compared to the policy with $w = 0$, the population under the case with $w = 0.15$ approaches the targeted goal more quickly in the early stages. Figures 1(c) and 1(d) illustrate the evolution of feasible commissions for buyers and sellers, respectively, before stabilizing. Under this class of policies, the platform initially charges lower commissions on the seller side to stimulate growth, as sellers are relatively scarce in the early stages. As the seller side's population ratio surpasses that of the buyer side, the platform then increases the commission for sellers while decreasing it for buyers. Specifically, for $w = 0$, the platform charges a positive commission throughout the decision horizon, which is consistent with Proposition 2(i). In contrast, for $w = 0.15$, the strategy involves subsidizing the type with a lower population ratio rather than merely lowering prices, especially when the population of both sides is close (see the condition in Proposition 2(ii)). This class of policy mirrors Airbnb's growth strategy: The company initially maintained low commission rates for hosts to attract new listings and build a strong inventory. Over time, Airbnb reduced its guest service fees to enhance competitiveness with traditional hotels while increasing host commissions (see Airbnb, 2020).

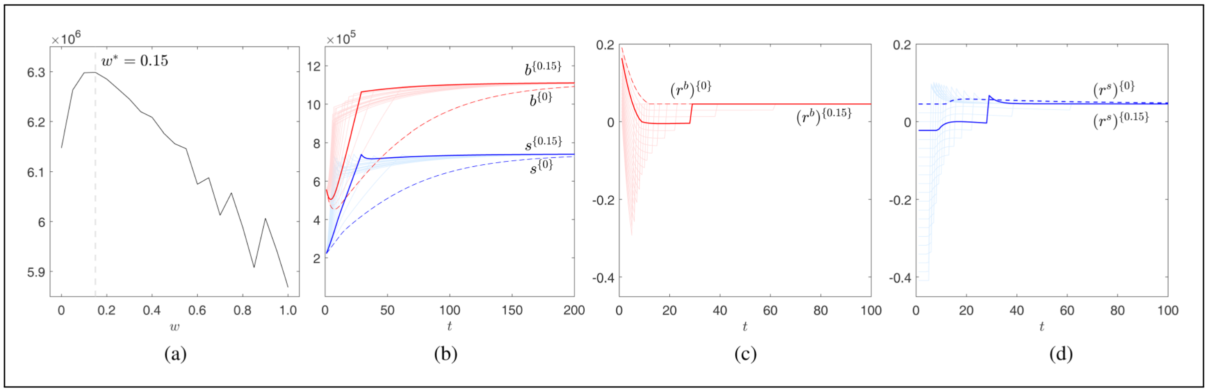

Figure 1. Performance of FRP. (a) Profit, (b) Population growth, (c)Feasible r^b and (d) Feasible r^s.

Myopic Policy. Some prior studies have examined the effectiveness of the MP in the product diffusion process of a monopoly seller, and they draw varying results under different diffusion functions. Robinson and Lakhani (1975) showed that MP results in significant profit loss relative to the optimal policy if a lower current price could stimulate future demand. In contrast, Bass and Bultez (1982) considered a different diffusion function and showed by a numerical study that the difference in the discounted profits between the MP and optimal policies is small. We now examine how MP performs in our model.

Under MP, in each period t, the platform determines the commissions $(\boldsymbol{r}^b(t), \boldsymbol{r}^s(t))$ to maximize its profit in the current period (i.e., $\sum_{i\in\mathcal{S}} r_i^s(t)q_i^s(t) + \sum_{j\in\mathcal{B}} r_j^b(t)q_j^b(t)$) subject to the equilibrium constraints in (2), without considering the population dynamics in (1) and its impact on future profit. The formal definition of the MP is given by Definition EC.1 in Section EC.3 in E-companion. We let $\mathcal{R}^{MP}(t)$ denote the platform's profit under MP in period t, and recall that $\overline{\mathcal{R}}$ is the optimal value of AVG and could be achieved under TRP. The following result shows that the performance of MP could be arbitrarily bad.

PROPOSITION 3 (**Performance of MP**). *Under MP, for any $\epsilon > 0$, there exists a problem instance such that $\lim_{t\to\infty} \mathcal{R}^{MP}(t) := \overline{\mathcal{R}}^{MP} < \infty$ and $\overline{\mathcal{R}}^{MP} < \epsilon\overline{\mathcal{R}}$. Hence, there exists $C_3 > 0$ such that $\mathcal{L}^{MP}(T) \geq C_3 T$.*

Proposition 3 suggests that ignoring the commissions' impact on population growth could lead to significant profit loss even if the platform serves as a monopoly intermediary. In the proof of Proposition 3, we show that the commissions set by the platform under MP at the steady state are higher than those under TRP. This result, again, highlights that the platform must sacrifice some short-term margin to achieve long-term profitability.

Since the TRP requires controlling the service level (or equivalently, their payment/income) of different agent types at the level of the OSS, we next examine how this service level is determined by both the compatibility network structure $G(\mathcal{S} \cup \mathcal{B}, E)$ and population dynamics from (1).

5 Impact of Population Dynamics and Compatibility Network Structure

In this section, we investigate how the intertemporal factors marked by heterogeneous growth potentials and the spatial factors characterized by the compatibility influence the platform's profit (see Section 5.1) as well as the incomes/payments of agents and optimal commission (see Section 5.2) at OSS. Investigating the impacts of these spatial-temporal factors can provide insights into the platform's revenue management strategy.

The prior studies showed that in a static setting, a network that more efficiently matches supply with demand achieves a better performance from the platform's perspective, and the agent types connected to a larger population on the other side would gain higher surplus (see Birge et al., 2021; Chou et al., 2011; Schrijver et al., 2003). For example, Chou et al. (2011) showed that a bipartite network, in which every subset of nodes is linked to a sufficiently large number of neighboring nodes, is optimal for the system. Similarly, Birge et al. (2021) showed that supply–demand imbalance across the network, measured by the lowest seller-to-buyer population ratio among all submarkets, determines the lower bounds of the platform's achievable profit relative to that with a complete network. Interestingly, we see from the following numerical example that in a dynamic setting, the imbalance in terms of the equilibrium population ratio at OSS can no longer provide a profit guarantee for the platform.

EXAMPLE 2. Consider a compatibility network with $\mathcal{S} = \{1, 2\}, \mathcal{B} = \{1, 2\}, E = \{(1, 1), (2, 2)\}$. Suppose that buyers' and sellers' (reservation) values are uniformly distributed between $[0, 1]$. Consider $s_i(t+1) = \alpha s_i(t) + \beta_i^s q_i^s(t)^\xi$ and $b_j(t+1) = \alpha b_j(t) + \beta_j^b q_j^b(t)^\xi$ for $i \in \{1, 2\}, j \in \{1, 2\}$ with parameters $\alpha = 0.5$, $\xi = 0.8$, $\beta_1^s = \beta_2^b = 2$, $\beta_2^s = \beta_1^b = 1$. The population ratio in AVG satisfies $\frac{\sum_{i\in N_E(\widetilde{\mathcal{B}})}\bar{s}_i}{\sum_{j\in\widetilde{\mathcal{B}}}\bar{b}_j} \geq 50\% \times \frac{\sum_{i\in\mathcal{S}}\bar{s}_i}{\sum_{j\in\mathcal{B}}\bar{b}_j}$ for any $\widetilde{\mathcal{B}} \subseteq \mathcal{B}$, but the platform's optimal profit at OSS is only about 36 % of that in a complete market, that is, $\overline{\mathcal{R}}(E, \psi^s, \psi^b) = 36\% \times \overline{\mathcal{R}}(\overline{E}, \psi^s, \psi^b)$,

Hence, it becomes crucial to incorporate temporal factors into the "imbalance" measure for the compatibility network, which requires us to first measure the growth potential of each agent type. As we mentioned in Section 1, the growth of an active agent base consists of retaining previous agents and encouraging word-of-mouth effect to attract new adoption. Therefore, to better quantify these two effects, we consider the following class of growth functions $\mathcal{G}^s(\cdot)$ and $\mathcal{G}^b(\cdot)$ in (1):

$$\mathcal{G}_i^s(s,q) = \alpha_i^s s + \beta_i^s g_s(s,q), \quad 4$$
$$\mathcal{G}_j^b(b,q) = \alpha_j^b b + \beta_j^b g_b(b,q). \quad (6)$$

where we consider homogeneous-degree-ξ_s (and ξ_b) functions $g_s(\cdot,\cdot)$ (and $g_b(\cdot,\cdot)$) with $\xi_s \in (0,1)$ (and $\xi_b \in (0,1)$) for any $q, s, b \geq 0$. A function $g(\cdot,\cdot)$ is homogeneous of degree ξ means that $g(ns, nq) = n^\xi g(s,q)$ for any $s \geq q \geq 0$, $n > 0$.

In (6), $\alpha_i^s \in (0,1)$ and $\alpha_j^b \in (0,1)$ respectively represent the retention rate of type-i sellers and type-j buyers (Alizamir et al., 2022; He and Goh, 2022; Lian and Van Ryzin, 2021). Then $1-\alpha_i^s$ or $1-\alpha_j^b$ can be seen as the attrition rate of the agent type. In practice, platforms can track retention data across different user groups and aim to maintain specific retention rates. For instance, Upwork has an overall retention rate of 58%, while the retention rate for core clients who have spent over 5,000 is 83% (see Upwork, 2018). Airbnb's overall retention rate is approximately 30%–40% from 2012 to 2017 (see Bloomberg, 2019). The second term captures new agents' adoption. $\beta_i^s \in (0,1)$ and $\beta_j^b \in (0,1)$ measure the type-specific impact of the current user base and transactions on new adoption. Some previous studies on the growth of two-sided platforms assume that the new adoption depends on the transaction volume/price/surplus in the last period, and the growth rates are homogeneous for all agents from one side (see He and Goh, 2022; Lian and Van Ryzin, 2021). Instead, we impose a property known as the homogeneous degree of ξ, which measures the elasticity of the future user base with respect to current transactions and user base. $\xi < 1$ suggests a decreasing marginal effect of transaction quantity and user base on the increase in new adoptions. A similar pattern can be seen from the buyer side. We present the main notations used in this section in E-companion (See Table EC.1 in Section EC.3) for the reader's reference.

Furthermore, to isolate the impact of compatibility network structure and growth potential, we assume that different types of sellers/buyers have homogeneous value distributions.

Assumption 4. *$F_{s_i}(v) = F_s(v)$ for any $i \in \mathcal{S}$ and $F_{b_j}(v) = F_b(v)$ for any $j \in \mathcal{B}$.*

We next construct a metric to measure the long-run growth potential of each agent type and then measure the imbalance of the compatibility network using this metric.

Agents' Long-run Growth Potential. To obtain an intuitive expression of the long-run growth potential, we use, throughout this section, a simple polynomial term for g_s and g_b as an illustrative example. For $t \in \{1, \ldots, T-1\}$, $i \in \mathcal{S}$, $j \in \mathcal{B}$,

$$s_i(t+1) = \alpha_i^s s_i(t) + \beta_i^s (q_i^s(t))^{\xi_s},$$
$$b_j(t+1) = \alpha_j^b b_j(t) + \beta_j^b (q_j^b(t))^{\xi_b}. \quad (7)$$

Under this form, we can provide a closed-form expression for long-run growth potential, based on which we further deduce all the following results and managerial insights. However, it is worth pointing out that our proof technique does not rely on the exact expressions of (7). Given type-i sellers' service level $\frac{\bar{q}_i^s}{\bar{s}_i}$ induced by the platform, the population of type-i seller converges to $\bar{s}_i$ that satisfies $\bar{s}_i = \alpha_i^s \bar{s}_i + \beta_i^s (\bar{q}_i^s)^{\xi_s}$. Algebraic manipulations suggest that

$$\bar{s}_i = \left(\frac{\beta_i^s}{1-\alpha_i^s}\right)^{\frac{1}{1-\xi_s}} \left(\frac{\bar{q}_i^s}{\bar{s}_i}\right)^{\frac{\xi_s}{1-\xi_s}},$$
$$\bar{q}_i^s = \left(\frac{\beta_i^s}{1-\alpha_i^s}\right)^{\frac{1}{1-\xi_s}} \left(\frac{\bar{q}_i^s}{\bar{s}_i}\right)^{\frac{1}{1-\xi_s}} \text{ where } i \in \mathcal{S}, \quad (8)$$

Eqn. (8) reveals that given the service level $\frac{\bar{q}_i^s}{\bar{s}_i}$ for type-i sellers, the population of an agent type and the transaction quantities at OSS are proportional to the coefficients $\left(\frac{\beta_i^s}{1-\alpha_i^s}\right)^{\frac{1}{1-\xi_s}}$. The same equations hold for the buyer side. Based on this, we formally define the long-run growth potential as follows:

$$\psi_i^s := \left(\frac{\beta_i^s}{1-\alpha_i^s}\right)^{\frac{1}{1-\xi_s}},\ i \in \mathcal{S}, \quad 5$$
$$\psi_j^b := \left(\frac{\beta_j^b}{1-\alpha_j^b}\right)^{\frac{1}{1-\xi_b}},\ j \in \mathcal{B}. \quad (9)$$

We next provide some intuitive explanations for (ψ^s, ψ^b). For simplicity, we omit the superscripts (s, b) and subscripts (i, j). In the population dynamics in (7), β captures the impact of transaction quantities on the population growth, and only a fraction $\alpha < 1$ of agents stay in the system after each period. As $\frac{\beta}{1-\alpha} = \sum_{t=0}^{\infty} \beta\alpha^t$, it captures the net present value for the long-run marginal impact of the transaction quantity q^ξ. Similarly, the impact of the population elasticity ξ after t periods can be captured by ξ^t. As $\frac{1}{1-\xi} = \sum_{t=0}^{\infty} \xi^t$, it represents the net present value of the long-term impact of the elasticity ξ. Therefore, we refer to ψ_i^s for $i \in \mathcal{S}$ and ψ_j^b for $j \in \mathcal{B}$ in (9) as the long-run growth potential of each agent type.

Rankings of Relative Growth Potential. Based on the long-run growth potential, we introduce a ranking of different types of agents. Let $N_E(X)$ denote the set of all neighbors of agent types $X \subseteq \mathcal{B} \cup \mathcal{S}$ in the graph $G(\mathcal{S} \cup \mathcal{B}, E)$ such that $N_E(X) = \{i \notin X : (i,j) \in E \text{ for } j \in X\}$. Given a compatibility network $G(\mathcal{S} \cup \mathcal{B}, E)$ and the long-run growth potential vector (ψ^s, ψ^b), we first let $\mathcal{B}^0 = B$, $\mathcal{S}^0 = S$ and $E^0 = E$. For $\tau = 0, 1, \ldots$, we define $\mathcal{B}_\tau$ and $\mathcal{S}_\tau$ iteratively as follows:

$$\mathcal{B}_{\tau+1} = \arg\min_{\widetilde{\mathcal{B}} \subseteq \mathcal{B}^\tau} \frac{\sum_{i \in N_{E^\tau}(\widetilde{\mathcal{B}})} \psi_i^s}{\sum_{j \in \widetilde{\mathcal{B}}} \psi_j^b}, \tag{10a}$$

$$\mathcal{S}_{\tau+1} = N_{E^\tau}(\mathcal{B}_{\tau+1}). \tag{10b}$$

where $\mathcal{B}^\tau = \mathcal{B}^{\tau-1} \backslash \mathcal{B}_\tau$, $\mathcal{S}^\tau = \mathcal{S}^{\tau-1} \backslash \mathcal{S}_\tau$, $E^\tau = \{(i,j) \in E : i \in \mathcal{S}^\tau \text{ and } j \in \mathcal{B}^\tau\}$ are the remaining buyer set, seller set, and edge set after removing the subgraph labeled in step τ. If multiple sets achieve the minimum, the arg min operator returns the largest one.

In (10a), for each subset of buyer types $\widetilde{\mathcal{B}}$ of $\mathcal{B}^\tau$, $\frac{\sum_{i \in N_{E^\tau}(\widetilde{\mathcal{B}})} \psi_i^s}{\sum_{j \in \widetilde{\mathcal{B}}} \psi_j^b}$ is the ratio between the total long-run growth potential of its (remaining) compatible sellers and its own. We refer to the ratio as the *relative growth potential* between $N_{E^\tau}(\widetilde{\mathcal{B}})$ and $\widetilde{\mathcal{B}}$. This metric, similar to those used for comparing two economies, for example, in Krugman (1989), captures the relative growth potential of sellers and buyers. In (10), we can iteratively identify a subgraph such that the relative growth potential of sellers is the lowest. Subsequently, we label it and remove this subgraph from the network, and then B^τ and S^τ are the remaining agent types, and E^τ is the remaining graph after τ iterations. We repeat the procedure until the remaining subgraph is empty. As a result, the subnetwork with a higher index τ has a higher relative growth potential of sellers against buyers in the graph. This ranking incorporates both intertemporal factors captured by the long-run growth potential ψ and spatial factors captured by the graph structure $G(\mathcal{B} \cup \mathcal{S}, E)$.

We use the example below to illustrate the rankings of relative growth potential. This example illustrates the compatibility between freelance coders and clients in need of IT services on Upwork. Specifically, clients needing AI Services can only be served by coders with AI skills, and clients requiring immediate delivery of work can only choose coders with flexible working hours. By enumeration, we can obtain the index of each type, and the solid (dotted) line represents the lines between sets with the same (different) index. For a large-scale network, we can obtain the ranking by solving a convex optimization problem This procedure borrows the algorithmic idea to characterize the lexicographically optimal bases of polymatroids from Fujishige (1980).

EXAMPLE 3. Consider a compatibility network as shown in Figure 2.

Suppose that $\psi_i^s = \psi_j^b = 1$ for $i = \{1, 2, 3\}$ and $j = \{1, 2, 3\}$. Then by enumeration, we know $\{1, 2\} = \arg\min_{\widetilde{\mathcal{B}} \subseteq \mathcal{B}} \frac{\sum_{i \in N_E(\widetilde{\mathcal{B}})} \psi_i^s}{\sum_{j \in \widetilde{\mathcal{B}}} \psi_j^b}$, which means $\mathcal{B}_1 = \{1, 2\}$ and $\mathcal{S}_1 = \{1\}$ (blue nodes). After eliminating $\mathcal{B}_1$ and $\mathcal{S}_1$ from the network E, we have $\mathcal{B}^1 = \{3\}, \mathcal{S}^1 = \{2, 3\}, E^1 = \{(2, 3), (3, 3)\}$. Since there is only one buyer type left, we know $\mathcal{B}_2 = \{3\}$ and $\mathcal{S}_2 = \{2, 3\}$ (black nodes). Finally, all agent types are labeled with an index.

We will next show the connection between the lowest relative growth potential in the compatibility network and the platform's profit at OSS.

5.1 Optimal Compatibility Network for the Platform's Profit

To signify the dependence on the compatibility network structure $G(\mathcal{S} \cup \mathcal{B}, E)$ and long-run growth potential (ψ^s, ψ^b), we let $\overline{\mathcal{R}}(E, \psi^s, \psi^b)$ denote the platform's optimal steady-state profit. Given that the feasible region for a complete graph is the largest in Problem (3), the platform can achieve the maximum optimal profit in a complete graph. Therefore, we let $\overline{E}$ denote the edge set for the complete graph with the set of seller types $\mathcal{S}$ and that of buyer types $\mathcal{B}$, and use $\overline{\mathcal{R}}(\overline{E}, \psi^s, \psi^b)$ to benchmark the impact of compatibility network structure E on the platform's profit. The following theorem establishes a connection between the temporal-spatial factors and the platform's optimal profit in network $G(\mathcal{S} \cup \mathcal{B}, E)$.

THEOREM 2 (**($1-\epsilon$)-optimal Network Structure**). *For any $\epsilon \in [0, 1]$, if $G(\mathcal{B} \cup \mathcal{S}, E)$ satisfies*

$$\frac{\sum_{i \in \mathcal{S}^1} \psi_i^s}{\sum_{j \in \mathcal{B}^1} \psi_j^b} \;\geq\; (1-\epsilon) \frac{\sum_{i \in \mathcal{S}} \psi_i^s}{\sum_{j \in \mathcal{B}} \psi_j^b}, \tag{11a}$$

then

$$\overline{\mathcal{R}}(E, \psi^s, \psi^b) \;\geq\; (1-\epsilon) \overline{\mathcal{R}}(\overline{E}, \psi^s, \psi^b). \tag{11b}$$

In Condition (11a), the right-hand-side expression $\frac{\sum_{i \in \mathcal{S}} \psi_i^s}{\sum_{j \in \mathcal{B}} \psi_j^b}$ represents the relative long-run growth potential of all sellers to all buyers within the entire compatibility network $G(\mathcal{B} \cup \mathcal{S}, E)$. Likewise, the left-hand side is the relative growth potential of the compatible sellers to a subset of buyers $\mathcal{B}^1$, whose relative long-run growth potential is the lowest (see (10)). Therefore, ϵ quantifies the degree of imbalance: a positive value of ϵ indicates that there exists no submarket in which the relative growth potential is ϵ lower than that of the entire market. Then (11b) implies that the degree of imbalance ϵ in the compatibility network does not cause more than ϵ optimal profit loss for the platform. When $\epsilon = 0$, the condition in (11a) ensures that the relative growth potential for all submarkets is equal to that for the entire market. In other words, the long-run growth potentials are "balanced" in the compatibility network. In this case, even though the market E may be incomplete, the lower bound in (11b) is tight, and the platform's optimal profit achieves the maximum possible optimal profit, that is, $\overline{\mathcal{R}}(E, \psi^s, \psi^b) = \overline{\mathcal{R}}(\overline{E}, \psi^s, \psi^b)$.

The managerial insight derived from Theorem 2 suggests that the platform should aim to enhance the balance of the

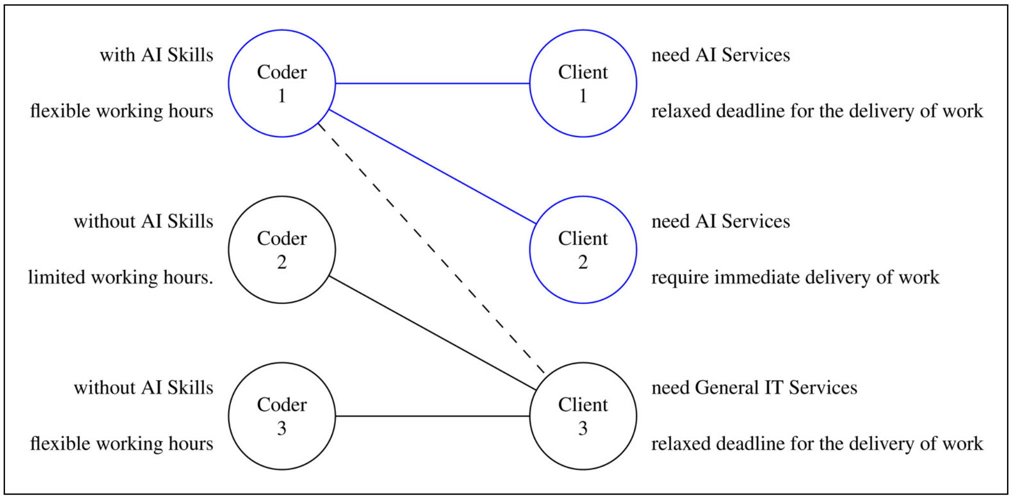

Figure 2. Compatibility between freelance coders and clients in need of IT services on upwork.

compatibility network in terms of long-run growth potential to maximize its steady-state optimal profit. Specifically, the platform could target its marketing campaign on agent types with relatively low long-run growth potential to increase their retention and attract new users.

Connection to Agent Heterogeneity. The parameter ϵ captures the extent of network imbalance, which is closely associated with the heterogeneity among agents in terms of their growth potentials and compatibility. Specifically, when the growth potentials and the number of compatible types are homogeneous for all agent types from the same side, we have $\epsilon = 0$, which suggests that the network is balanced. To better understand ϵ, we can consider two extreme examples: One with homogeneous growth potential and another with homogeneous compatibility.

(1). If $\psi_i^s = \psi^s$ for any $i \in \mathcal{S}$ and $\psi_j^b = \psi^b$ for any $j \in \mathcal{B}$, then (11a) becomes $\min_{\tilde{\mathcal{B}} \subseteq \mathcal{B}} \frac{|N_E(\tilde{\mathcal{B}})|}{|\tilde{\mathcal{B}}|} \geq (1-\epsilon)\frac{|\mathcal{S}|}{|\mathcal{B}|}$. That is, when the growth potentials are homogeneous for all agent types from the same side, $1-\epsilon$ captures the disparity between the minimum seller-buyer ratio and the average seller-buyer ratio. In other words, it captures the heterogeneity of buyer types' preferences.

(2). If $|\mathcal{B}| = |\mathcal{S}|$ and $(i,j) \in E$ if and only if $i = j$, then (11a) becomes $\min_{l \in \mathcal{B}} \frac{\psi_l^s}{\psi_l^b} \geq (1-\epsilon)\frac{\sum_{i\in\mathcal{S}} \psi_i^s}{\sum_{j\in\mathcal{B}} \psi_j^b}$. When each type of buyer corresponds uniquely to a type of seller and vice versa, the term $1-\epsilon$ reflects only the differences in relative growth potentials in each combination.

5.2 Agent Payments/Incomes and Platform Commissions

In this subsection, we analyze the impact of agents' growth potential on the platform's commission decisions. Recall that the optimal commission $(\overline{\boldsymbol{r}}^s, \overline{\boldsymbol{r}}^b)$ at OSS is not necessarily unique, but any optimal commission profile induces the same (net) payments and incomes for agent types engaged in transactions (see Proposition EC.1 and Lemma EC.1). Furthermore, the total commission generated from a transaction (i.e., $r_i^s + r_j^b$ for $(i,j) \in E$), which represents the difference between buyers' payments and sellers' incomes, is inherently unique. Therefore, in this subsection, we will first study the impact of compatibility network structure and growth potentials on (net) payments and incomes for agent types and then analyze their impact on the total optimal commission.

Buyers' Payments and Sellers' Incomes. We next establish that the ranking of the relative growth potentials of sellers to buyers given in (10) determines the ranking of buyers' payments and sellers' incomes at OSS. We denote by $Y_j = \min_{i':(i',j)\in E}\{\overline{p}_{i'} + \overline{r}_j^b\}$ the payment of any type-j buyers, and denote by $I_i = \overline{p}_i - \overline{r}_i^s$ the income of any type-i sellers at OSS.

Proposition 4 (**Ranking of Buyers' Payments and Sellers' Incomes**). *In the compatibility network $G(\mathcal{S} \cup \mathcal{B}, E)$, under any platform's optimal commission profile $(\overline{\boldsymbol{r}}^s, \overline{\boldsymbol{r}}^b)$ at the steady state,*

(1) *for any $j_1 \in \mathcal{B}_{\tau_1}$ and $j_2 \in \mathcal{B}_{\tau_2}$ with $\tau_1 \leq \tau_2$, $Y_{j_1} \geq Y_{j_2}$ and $\frac{\overline{q}_{j_1}^b}{\overline{b}_{j_1}} \leq \frac{\overline{q}_{j_2}^b}{\overline{b}_{j_2}}$;*
(2) *for any $i_1 \in \mathcal{S}_{\tau_1}$ and $i_2 \in \mathcal{S}_{\tau_2}$ with $\tau_1 \leq \tau_2$, $I_{i_1} \geq I_{i_2}$ and $\frac{\overline{q}_{i_1}^b}{\overline{s}_{i_1}} \geq \frac{\overline{q}_{i_2}^b}{\overline{s}_{i_2}}$.*

Proposition 4 posits that at the OSS, with a higher relative long-run growth potential of sellers to buyers (i.e., higher index τ indicates higher $\frac{\sum_{i\in\mathcal{S}_\tau} \psi_i^s}{\sum_{j\in\mathcal{B}_\tau} \psi_j^b}$ in (10)), the buyers pay less and experience a higher service level, while the sellers earn a lower income and experience a lower service level in equilibrium. By using the Example 3 to illustrate, the payments on the buyer (i.e., client) side satisfy $Y_1 = Y_2 > Y_3$ given that $\mathcal{B}_1 = \{1,2\}$ and $\mathcal{B}_2 = \{3\}$; the incomes on the seller (i.e., coder) side satisfy that $I_1 > I_2 = I_3$ given that $\mathcal{S}_1 = \{1\}$ and $\mathcal{S}_2 = \{2,3\}$. The managerial implication from Proposition 4 is that while determining the service level, the platform needs to consider not only the retention rate and growth potential of the focal agent types but also their trading partners on the other side of the market. Specifically, the platform should incentivize the agent types with higher relative growth potential of the other side by offering them lower commissions and extract a higher surplus from those with lower relative growth potential.

Proposition 4 offers a theoretical explanation on why platforms should prioritize securing the participation of marquee users (i.e., popular sellers or less-selective buyers) to accelerate growth, as discussed in Section 1, within the TRP framework. To isolate the effect of user popularity or preference structure, suppose all user types share the same growth potential (i.e., $\psi_i^s = \psi^s$ and $\psi_j^b = \psi^b$). Under this assumption, the definition in equation (10a) is simplified to $\mathcal{B}_{\tau+1} = \arg\min_{\widetilde{\mathcal{B}} \subseteq \mathcal{B}^\tau} \frac{|N_{E^\tau}(\widetilde{\mathcal{B}})|}{|\widetilde{\mathcal{B}}|}$ and Proposition 4 reveals that user types with more connections to the other side of the market have a higher service level at OSS, and thus achieve a higher population level at OSS, given the same growth potential. When all types have equal or close population sizes in the current period, the type with the highest population level at OSS exhibits the lowest population ratio, and thus should be prioritized for growth under TRP. In this sense, the "marquee users" mentioned in Section 1, typically high-demand buyers or high-profile sellers, are more likely to be strategically targeted during the platform's early-stage growth phase.

Proposition 4 suggests that any change in the values of (ψ^s, ψ^b) induces changes in the service level of each agent type, ultimately affecting the equilibrium demand, supply, and population at OSS. Lastly, we examine the influence of the long-run growth potential (ψ^s, ψ^b) to offer guidance for the platform's commission decisions.

COROLLARY 2 (**Impact of the Long-run Growth Potential**). *Given any $\xi_s \in (0,1)$ and $\xi_b \in (0,1)$,*

(1) *for the service levels,*
  (i) *given $j \in \mathcal{B}$, $\overline{q}_j^b / \overline{b}_j$ decreases in $\psi_{j'}^b \geq 0$ for $\forall j' \in \mathcal{B}$ and increases in $\psi_{i'}^s \geq 0$ for $\forall i' \in \mathcal{S}$;*
  (ii) *given $i \in \mathcal{S}$, $\overline{q}_i^s / \overline{s}_i$ decreases in $\psi_{i'}^s \geq 0$ for $\forall i' \in \mathcal{S}$ and increases in $\psi_{j'}^b \geq 0$ for $\forall j' \in \mathcal{B}$;*

(2) *for the transaction quantities and populations,*
  (i) *given $j \in \mathcal{B}$, $(\overline{q}_j^b, \overline{b}_j)$ increases in $\psi_j^b \geq 0$, decreases in $\psi_{j'}^b \geq 0$ for $\forall j' \in \mathcal{B}$ with $j' \neq j$, and increases in $\psi_{i'}^s \geq 0$ for $\forall i' \in \mathcal{S}$;*
  (ii) *given $i \in \mathcal{S}$, $(\overline{q}_i^s, \overline{s}_i)$ increases in $\psi_i^s \geq 0$, decreases in $\psi_{i'}^s \geq 0$ for $\forall i' \in \mathcal{S}$ with $i' \neq i$ and increases in $\psi_{j'}^b \geq 0$ for $\forall j' \in \mathcal{B}$.*

Note that for any $\xi_s \in (0,1)$ and $\xi_b \in (0,1)$, the vectors (ψ^s, ψ^b) are determined by the retention rates $(\boldsymbol{\alpha}^s, \boldsymbol{\alpha}^b)$ and the growth coefficients $(\boldsymbol{\beta}^s, \boldsymbol{\beta}^b)$. Corollary 2(1) suggests that the service level of any agent decreases in the growth potential of all types from the same side but increases in those on the other side of the market. Corollary 2(2) implies that the transaction volume and population of each type are increasing in their own growth potential and those on the other side of the network, but decreasing in those of other types on the same side.

We discuss the intuition using the buyer side as an example. Both a high long-run growth potential and a high service level contribute to an increase in the population of a buyer type at OSS. Consequently, when the long-run growth potential of a buyer type is high, the platform can maintain a high population by inducing a relatively lower service level. However, if other buyer types have higher long-run growth potential, their equilibrium demand will rise, resulting in increased prices for the sellers and a reduced service level for our focal buyer type. Conversely, if the corresponding sellers have higher long-run growth potential, their supply will increase, leading to lower prices and benefiting all buyers.

Platform's Optimal Commissions. We now focus on the total commission charged by the platform from one transaction, viz., the difference between the buyers' payments and the sellers' incomes. Note that under the optimal commission, type-i sellers with $i \in \mathcal{S}_\tau$ only trade with type-j buyers with $j \in \mathcal{B}_\tau$. Therefore, we will examine how the total commission charged from one transaction between sellers in $\mathcal{S}_\tau$ and buyers in $\mathcal{B}_\tau$ depends on the ranking of the relative growth potential of sellers to buyers τ given in (10). Here, we assume $\xi_s = \xi_b$ to isolate the impact of value distribution.

PROPOSITION 5 (**Ranking the Platform's Optimal Commissions**). *Assume that F_s and F_b are twice differentiable in their domains and $\xi_s = \xi_b$. There exists $\widetilde{\tau}$ such that*

(1) *$r_i^s + r_j^b$ for $i \in \mathcal{S}_\tau, j \in \mathcal{B}_\tau$ is decreasing in τ for $\tau < \widetilde{\tau}$;*
(2) *$r_i^s + r_j^b$ for $i \in \mathcal{S}_\tau, j \in \mathcal{B}_\tau$ is decreasing in τ for $\tau \geq \widetilde{\tau}$ if $F_s(v)$ and $F_b(v)$ are concave; whereas it is increasing in τ for $\tau \geq \widetilde{\tau}$ if $F_s(v)$ and $F_b(v)$ are convex.*

In Proposition 5(1), when the relative growth potential of sellers to buyers falls below a threshold, the total commission charged from the transaction decreases with the relative growth potential between sellers and buyers. In Proposition 5(2), the concavity of $F_s(v)$ and $F_b(v)$ implies a higher density of agents with lower (reservation) value. In this case, when the relative growth potential of sellers to buyers is higher, the optimal total commission charged by the platform should be lower. Similarly, the convexity of $F_s(v)$ and $F_b(v)$ implies that the number of agents with higher (reservation) value is higher. In this scenario, the platform charges lower total commissions for transactions involving agents with moderate relative growth potentials of sellers to buyers.

Intuitively, when the relative growth potential between sellers and buyers is below a threshold, the number of sellers is significantly smaller than that of buyers. In such cases, the platform uses its commission to keep the sellers' income at a sufficiently high level to ensure the participation of sellers. As the relative growth potential increases, the number of sellers rises, prompting the platform to gradually reduce buyer payments to stimulate demand. As a result, the total commission charged from the transaction, which is the difference between buyer payments and seller incomes, decreases with the relative growth potential between sellers and buyers.

When the relative growth potential of sellers compared to buyers exceeds a certain threshold, the market becomes saturated with sellers, and the platform no longer needs to offer substantial subsidies to ensure their participation. In this setting, further increases in sellers' relative growth potential indicate that the platform should strategically reduce the service level for sellers and increase it for buyers to better balance supply and demand. In markets where most agents have relatively low valuations for the product or service, the platform must offer significant price reductions to buyers to stimulate demand, while even a slight reduction in sellers' earnings may lead to a noticeable decline in supply. Consequently, the total commission revenue from transactions tends to decrease as the sellers' relative growth potential rises. In contrast, when agents generally place a high value on the product or service, a modest price cut is sufficient to attract buyers, and the platform can considerably reduce sellers' earnings without substantially affecting supply. In this case, the total transaction-based commission increases with the relative growth potential of sellers compared to buyers.

6 Conclusion

In this study, we consider a two-sided platform that facilitates transactions between buyers and sellers with heterogeneous growth potentials. The compatibility between buyer and seller types is captured by a bipartite graph, which is not necessarily complete. The platform sets the commissions to maximize its T-period profit. To address the complexity of the platform's profit optimization problem, we consider the long-run average problem (AVG) as a benchmark and propose an algorithm called TRP with a provable performance guarantee. We show that the platform should prioritize boosting the growth of the agent type with the lowest population ratio relative to the long-run average benchmark in each period. Additionally, we demonstrate that providing subsidies to users can accelerate the growth of the user base, highlighting the tradeoff between short-term profit and long-run growth. We also outline the conditions under which subsidies should be implemented.

Furthermore, we delve into the OSS obtained via AVG and explore how the growth potentials of agents and network structure influence the agents' income/payment in the market and the platform's profit. We begin by introducing a set of metrics designed to capture the growth potential of agents. Based on it, we show that a balanced network, in which sellers with relatively high (low) growth potentials trade with buyers with relatively high (low) growth potentials, results in maximum profitability, while the degree of imbalance in the network establishes a lower bound for the platform's optimal profit (relative to that under the complete graph). We then show that buyer (seller) types compatible with higher sellers' (buyers') growth potentials experience lower payments (higher income). A sensitivity analysis demonstrates the impact of the agent type's long-run growth potential on income/payment. Finally, the commission charged by the platform in a submarket depends on the relative growth potential of the two sides of the market.

Acknowledgment

We are grateful to the Senior Editor and two anonymous referees for their constructive and insightful comments that have helped greatly improve the paper.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The authors received the following financial support for the research, authorship, and/or publication of this article: Hongfan(Kevin) Chen is partially supported by the Hong Kong Research Grants Council [Grant GRF 14506622, GRF 14501023, ECS 24502224]. Renyu Zhang is partially supported by the National Natural Science Foundation of China [Grant 72422004] and Hong Kong Research Grants Council [Grant GRF 14502722, GRF 14504123, and GRF 14503224]. Sean X. Zhou is partially supported by the National Natural Science Foundation of China [Grant 72394395], Hong Kong Research Grant Council [Grant GRF 14500323], and the Asian Institute of Supply Chains and Logistics.

ORCID iDs

Yixin Zhu https://orcid.org/0009-0002-5170-4430
Renyu Zhang https://orcid.org/0000-0003-0284-164X
Sean X Zhou https://orcid.org/0000-0001-5314-2676

Supplemental Material

Supplemental material for this article is available online (doi: 10.1177/10591478251371267).

References

AI Business. (2017) Airbnb: Machine learning promises ultra-personalized travel. Availabel at: https://aibusiness.com/ml/airbnb-machine-learning-promises-ultra-personalized-travel (accessed 15 April 2024).

Airbnb. (2020) Simplified pricing. Availabel at: https://www.airbnb.com/e/commissione-semplificata-guida (accessed 07 May 2024).

Airbnb. (2025) Airbnb service fees. Availabel at: https://www.airbnb.com/help/article/1857?_set_bev_on_new_domain=1714622366_NjIzNGU3NTViNWNm&locale=en#section-heading-7-0 (accessed 15 April 2024).

Ajorlou A, Jadbabaie A and Kakhbod A (2018) Dynamic pricing in social networks: The word-of-mouth effect. *Management Science* 64(2): 971–979.

Alizamir S, Chen N, Kim SH, et al. (2022) Impact of network structure on new service pricing. *Mathematics of Operations Research* 47(3): 1999–2033.

Amazon. (2020) Increasing customer engagement and loyalty with personalized coupon recommendations using amazon personalize. Availabel at: https://aws.amazon.com/cn/blogs/machine-learning/increasing-customer-engagement-and-loyalty-with-personalized-coupon-recommendations-using-amazon-personalize/ (accessed 15 April 2024).

Armstrong M (2006) Competition in two-sided markets. *The RAND Journal of Economics* 37(3): 668–691.

Bai J, So KC, Tang CS, et al. (2019) Coordinating supply and demand on an on-demand service platform with impatient customers. *Manufacturing & Service Operations Management* 21(3): 556–570.

Banerjee S, Johari R and Riquelme C (2015) Pricing in ride-sharing platforms: A queueing-theoretic approach. In: *Proceedings of the Sixteenth ACM conference on economics and computation*, pp.639–639.

Baron O, Hu M and Malekian A (2022) Revenue volatility under uncertain network effects. *Operations Research* 70(4): 2254–2263.

Bass FM (1969) A new product growth for model consumer durables. *Management Science* 15(5): 215–227.

Bass FM and Bultez AV (1982) Technical note—a note on optimal strategic pricing of technological innovations. *Marketing Science* 1(4): 371–378.

Benjaafar S, Ding JY, Kong G, et al. (2022) Labor welfare in on-demand service platforms. *Manufacturing & Service Operations Management* 24(1): 110–124.

Benjaafar S, Kong G, Li X, et al. (2019) Peer-to-peer product sharing: Implications for ownership, usage, and social welfare in the sharing economy. *Management Science* 65(2): 477–493.

Bimpikis K, Ehsani S and Ilkılıç R (2019) Cournot competition in networked markets. *Management Science* 65(6): 2467–2481.

Birge J, Candogan O, Chen H, et al. (2021) Optimal commissions and subscriptions in networked markets. *Manufacturing & Service Operations Management* 23(3): 569–588.

Birge JR, Chen H, Keskin NB, et al. (2024) To interfere or not to interfere: Information revelation and price-setting incentives in a multiagent learning environment. *Operations Research* 72(6): 2391–2412.

Bloomberg. (2019) Ahead of IPO, Airbnb's consumer sales surpass most hotel brands. Availabel at: https://secondmeasure.com/datapoints/airbnb-sales-surpass-most-hotel-brands/ (accessed 07 May 2024).

Cachon GP, Daniels KM and Lobel R (2017) The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management* 19(3): 368–384.

Caillaud B and Jullien B (2003) Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics* 34(2): 309–328.

Chen H and Wang H (2023) When a platform competes with third-party sellers in networked markets: A revenue management perspective. *Working Paper* CUHK Business School, Hong Kong, China.

Chen N and Chen YJ (2021) Duopoly competition with network effects in discrete choice models. *Operations Research* 69(2): 545–559.

Chen N and Gallego G (2019) Welfare analysis of dynamic pricing. *Management Science* 65(1): 139–151.

Chou MC, Chua GA, Teo CP, et al. (2011) Process flexibility revisited: The graph expander and its applications. *Operations Research* 59(5): 1090–1105.

Chu J and Manchanda P (2016) Quantifying cross and direct network effects in online consumer-to-consumer platforms. *Marketing Science* 35(6): 870–893.

Cohen MC and Zhang R (2022) Competition and coopetition for two-sided platforms. *Production and Operations Management* 31(5): 1997–2014.

Eisenmann T, Parker G and Van Alstyne MW (2006) Strategies for two-sided markets. *Harvard Business Review* 84(10): 92.

Flynn J (1975a) Retaining productive units: A dynamic programming model with a steady state solution. *Management Science* 21(7): 753–764.

Flynn J (1975b) Steady state policies for a class of deterministic dynamic programming models. *SIAM Journal on Applied Mathematics* 28(1): 87–99.

Flynn J (1978) Steady-state policies, dynamic programming, and optimal economic growth. In: *Dynamic programming and its applications*, pp.173–200. Elsevier.

Flynn J (1981) A dynamic programming model of fractional flows with application to maintenance and replacement problems. *Mathematics of Operations Research* 6(3): 405–419.

Flynn J (1992) Computing efficient steady state policies for deterministic dynamic programs, i. *Journal of Mathematical Analysis and Applications* 166(1): 175–191.

Fujishige S (1980) Lexicographically optimal base of a polymatroid with respect to a weight vector. *Mathematics of Operations Research* 5(2): 186–196.

Hagiu A (2009) Two-sided platforms: Product variety and pricing structures. *Journal of Economics & Management Strategy* 18(4): 1011–1043.

Hamilton RW, Rust RT and Dev CS (2017) Which features increase customer retention. *MIT Sloan Management Review* 58(2): 79–84.

He EJ and Goh J (2022) Profit or growth? dynamic order allocation in a hybrid workforce. *Management Science* 68(8): 5891–5906.

Hu M and Zhou Y (2020) Price, wage, and fixed commission in on-demand matching. *Working Paper* University of Toronto, Toronto, Ontario, Canada.

Hu M and Zhou Y (2022) Dynamic type matching. *Manufacturing & Service Operations Management* 24(1): 125–142.

Kabra A, Belavina E and Girotra K (2016) Designing promotions to scale marketplaces. *Working Paper* INSEAD, Fontainebleau, France.

Kalish S (1985) A new product adoption model with price, advertising, and uncertainty. *Management Science* 31(12): 1569–1585.

Kanoria Y and Saban D (2021) Facilitating the search for partners on matching platforms. *Management Science* 67(10): 5990–6029.

Kranton RE and Minehart DF (2001) A theory of buyer-seller networks. *American Economic Review* 91(3): 485–508.

Krugman P (1989) Differences in income elasticities and trends in real exchange rates. *European Economic Review* 33(5): 1031–1046.

Li H, Shen Q and Bart Y (2021) Dynamic resource allocation on multi-category two-sided platforms. *Management Science* 67(2): 984–1003.

Lian Z and Van Ryzin G (2021) Optimal growth in two-sided markets. *Management Science* 67(11): 6862–6879.

Ma H, Fang F and Parkes DC (2022) Spatio-temporal pricing for ridesharing platforms. *Operations Research* 70(2): 1025–1041.

Mahajan V and Peterson RA (1985) *Models for Innovation Diffusion*. 48. California: Sage.

Norton JA and Bass FM (1987) A diffusion theory model of adoption and substitution for successive generations of high-technology products. *Management Science* 33(9): 1069–1086.

Robinson B and Lakhani C (1975) Dynamic price models for new-product planning. *Management Science* 21(10): 1113–1122.

Rochet JC and Tirole J (2003) Platform competition in two-sided markets. *Journal of the European Economic Association* 1(4): 990–1029.

Rochet JC and Tirole J (2006) Two-sided markets: A progress report. *The RAND Journal of Economics* 37(3): 645–667.

Samuelson P and Samuelson W (1980) *Economics*. International student edition. New York: McGraw-Hill.

Schrijver A (2003) *Combinatorial Optimization: Polyhedra and Efficiency*. Vol. 24. Berlin, Heidelberg: Springer.

Shen W, Duenyas I and Kapuscinski R (2014) Optimal pricing, production, and inventory for new product diffusion under supply constraints. *Manufacturing & Service Operations Management* 16(1): 28–45.

Sundaram D and Webster C (1999) The role of brand familiarity on the impact of word-of-mouth communication on brand evaluations. *Advances in Consumer Research* 26(1): 664.

Taylor TA (2018) On-demand service platforms. *Manufacturing & Service Operations Management* 20(4): 704–720.

Upwork. (2018) Prospectus. Availabel at: https://www.sec.gov/Archives/edgar/data/1627475/000119312518267594/d575528ds1.htm (accessed 07 May 2024).

Varma SM, Bumpensanti P, Maguluri ST, et al. (2023) Dynamic pricing and matching for two-sided queues. *Operations Research* 71(1): 83–100.

Weyl EG (2010) A price theory of multi-sided platforms. *American Economic Review* 100(4): 1642–1672.

Yu S and Zhu Y (2021) Algorithm awareness in repeated marketing campaigns: A field experiment. *Working Paper* Massachusetts Institute of Technology, Cambridge, MA.

Zheng R, Shou B and Chen Y (2023) Differential pricing in social networks with strategic consumers. *Production and Operations Management* 32(8): 2638–2655.

How to cite this article

Zhu Y, Chen H, Zhang R and Zhou SX (2025) Optimal Growth of a Two-Sided Platform with Heterogeneous Agents. *Production and Operations Management* XX(XX): 1–18.