Dynamic Pricing and Inventory Management under Fluctuating Procurement Costs

Philip (Renyu) Zhang

(Joint work with Guang Xiao and Nan Yang)

Olin Business School
Washington University in St. Louis

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  - $425 million cost reduction over a 6-year period.
- Portfolio Management Process
  - Regular price reviews and adjustments.
  - Price changes in response to production and supply chain costs, as well as global economic conditions, including currency volatility.

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- Executed by separate units of a firm (procurement and marketing).
- Goal of our paper: To understand how dynamic pricing and dual-sourcing can be coordinated under demand uncertainty and procurement cost fluctuation.

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3. What is the relationship between dynamic pricing and dual-sourcing?

Outline

- Related Literature
- Model
- Impact of Cost Volatility
- Strategic Relationship between Dynamic Pricing and Dual-Sourcing
- Conclusion: Takeaway Insights

Literature Review

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- Inventory management under fluctuating costs:
  - Kalymon (1971),
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- Our paper: Joint pricing and inventory management under demand uncertainty, cost fluctuation, and dual-sourcing.

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- No inventory resale:
  - No room for arbitrage.

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- Examples: GBMs, mean-reverting processes.

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- $f_t = \gamma c_t / \alpha$.
  - Effective unit cost: γc_t.
  - In reality, $f_t = F_t(c_t)$ is determined through bilateral negotiations.
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- The contract cannot be traded in the derivatives market.
  - Focus on the operational effect of forward-buying.

Demand Model

$$D_t(p_t) = d(p_t) + \epsilon_t.$$

- ϵ_t: independent continuous random variables, with $\mathbb{E}\{\epsilon_t\} = 0$.
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Assumption 1

$R(d_t) := p(d_t)d_t$ is continuously differentiable and strictly concave.

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- Demand D_t realized, revenue collected.
- Net inventory fully carried over to the next period:
  - Excess inventory fully carried over with unit cost h;
  - Unmet demand fully backlogged with unit cost b.

Bellman Equation

$$V_t(I_t|c_t) = \text{the maximal expected discounted profit in periods } t, t-1, \cdots, 1 \text{ with starting inventory level } I_t \text{ and cost } c_t \text{ in period } t.$$

Terminal condition: $V_0(I_0|c_0) = 0$.

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Bellman equation:

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\begin{aligned}
V_t(I_t|c_t) =& c_t I_t + \max_{x_t \geq I_t, q_t \geq 0, d_t \in [\underline{d}, \bar{d}]} J_t(x_t, q_t, d_t|c_t), \text{ where} \\
J_t(x_t, q_t, d_t|c_t) =& - c_t I_t + \mathbb{E}\{p(d_t)D_t - c_t(x_t - I_t) - \gamma c_t q_t - h(x_t - D_t)^+ \\
& - b(x_t - D_t)^- + \alpha V_{t-1}(x_t + q_t - D_t|s_t(c_t, \xi_t))|c_t\} \\
=& R(d_t) - c_t x_t - \gamma c_t q_t + \Lambda(x_t - d_t) + \Psi_t(x_t + q_t - d_t|c_t) \\
\text{with } \Lambda(y) =& \mathbb{E}\{-h(y - \epsilon_t)^+ - b(y - \epsilon_t)^-\}, \\
\text{and } \Psi_t(y|c_t) =& \alpha \mathbb{E}\{V_{t-1}(y - \epsilon_t|s_t(c_t, \xi_t))|c_t\}.
\end{aligned}
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Optimal Policy

- $(x_t^*(I_t, c_t), q_t^*(I_t, c_t), d_t^*(I_t, c_t))$: the optimal decisions in period t.
  - $\Delta_t^*(I_t, c_t) := x_t^*(I_t, c_t) - d_t^*(I_t, c_t)$: the optimal safety stock.

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- The cost-dependent order-up-to/pre-order-up-to list-price policy.
- If $I_t \leq x_t(c_t)$, order from both channels and charge a list price.
- If $I_t \in (x_t(c_t), I_t^*(c_t))$, order via the forward-buying contract only and charge a discounted price.
- If $I_t \geq I_t^*(c_t)$, order nothing.

Impact of Cost Volatility

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- Actually, the prediction is reversed:

Theorem 1

For two procurement cost processes $\{c_t\}_{t=T}^1$ and $\{\hat{c}_t\}_{t=T}^1$, assume that for every $t = T, T-1, \cdots, 1$, $s_t(c_t, \xi_t)$ and $\hat{s}_t(c_t, \xi_t)$ are concavely increasing in c_t for any realization of ξ_t. The following statements hold:

(a) *For any I_t, $V_t(I_t|c_t)$ is convexly decreasing in c_t.*

(b) *If $\{c_t\}_{t=T}^1$ and $\{\hat{c}_t\}_{t=T}^1$ are identical except that $\hat{s}_\tau(c_\tau, \xi_\tau) \geq_{cx} s_\tau(c_\tau, \xi_\tau)$ for some c_τ and τ, $\hat{V}_t(I_t|c_t) \geq V_t(I_t|c_t)$ for each (I_t, c_t) and t, where $\geq_{cx}$ refers to larger in convex order, and $\{\hat{V}_t(I_t|c_t)\}_{t=T}^1$ are the value functions associated with $\{\hat{c}_t\}_{t=T}^1$.*

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- The subtle timing issue:
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  - Respond to cost volatility.
- Capacity management and newsvendor network models with responsive/postponed pricing: Van Mieghem and Dada (1999), Chod and Rudi (2005) and Bish et al. (2012).

Impact of Cost Volatility: Assumptions

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Impact of Cost Volatility: Assumptions

- Risk neutrality is necessary for Theorem 1 to hold.
  - Opposite predictions in the OM-finance literature: risk aversion.
- The concavity of $s_t(c_t, \xi_t)$ generally can be satisfied (e.g., GBMs, mean-reverting processes).
- When $s_t(c_t, \xi_t)$ is not concave in c_t, the result holds for the majority of numerical cases (exceptions may exist when the initial cost is low), in particular when the initial cost follows the stationary distribution.

Optimal Response to Cost Volatility

$$J_t(x_t, q_t, d_t|c_t) = [R(d_t) - c_t d_t] + [\Lambda(\Delta_t) - (1-\gamma)c_t\Delta_t] + [\Psi_t(\Delta_t + q_t|c_t) - \gamma c_t(\Delta_t + q_t)].$$

- Three objectives: (a) generating revenue, (b) hedging against demand uncertainty, and (c) speculating on future costs.

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- Optimal safety-stock and spot-purchasing: $\Delta_t(c_t)$, $x_t(c_t) \downarrow c_t$, if $\gamma \leq 1$; $\Delta_t(c_t) \uparrow c_t$, if $\gamma > 1$.

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- Optimal forward-buying quantity: Generally not monotone in c_t.

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- Complements: if the additional sourcing channel is forward-buying.
- Substitutes: if the additional sourcing channel is spot-purchasing.
- Rationale: dynamic pricing mitigates the demand uncertainty risk, but the additional sourcing channel may dampen or intensify the demand uncertainty risk.

Conclusion: Takeaway Insights

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  - Timing of decision making and uncertainty realization.

- Dynamic pricing and dual-sourcing may be either complements or substitutes.
  - Dynamic pricing dampens both demand and cost risks, while dual-sourcing may either mitigate or intensify the demand risk.

Q&A

Thank you!

Questions?