

# Competition on Inventory Availability and Price

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Inventory availability has been widely recognized as an important leverage to enhance the competitive edge of a firm. We study the competition between two retailers who compete on both price and inventory availability. Competition on inventory availability may have important implications on firms' strategies, which has been under-explored in the literature. We develop a game-theoretic framework that integrates the newsvendor and Hotelling models to investigate such competition between retailers. We analyze the strategic interactions among the retailers and customers, and draw the following insights. First, contrary to intuition, the retailers may charge a higher price under competition than in the monopoly setting. A high price signals high product availability, thus facilitating the retailer to attract more customers in the presence of competition. It is well acknowledged that monetary compensation and inventory commitment can mitigate strategic customer behavior and improve a monopoly retailer's profit. In a competitive market, however, both monetary compensation and inventory availability would lead to a prisoner's dilemma. Although these strategies are preferred regardless of the competitor's price and inventory decisions, the equilibrium profit of each retailer could be lower in the presence of monetary compensation or inventory commitment because either strategy would intensify the competition between retailers. In contrast to the widely held belief that market competition improves customer surplus in general, it may hurt customers in our setting. This is because competition may drive the retailers to charge higher prices to signal inventory availability. We also show that monetary compensation and inventory commitment strategies provide incentives for customers to patronize the retailers, thus mitigating the customer surplus loss caused by competition.

*Key words:* Inventory availability, retail competition, strategic customer behavior, newsvendor, Hotelling model

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## 1. Introduction

How to ensure product availability has become an increasingly important issue in many industries. As reported by Grocery Manufacturers Association Trading Partner Alliance, the out-of-stock rate in U.S. supermarkets remains 8% on average, whereas the out-of-stocks for promoted items often

exceed 10% (Progressive-Grocer 2015). A consumer survey conducted by YouGov also revealed that 83% (resp. 70%) percent of UK consumers found the product they wanted to buy unavailable in-store (online) (Cavano 2015). This shocking data alarm firms as such a high stock-out rate implies potential revenue losses of at least 8% to 10% percent (Progressive-Grocer 2015). Additionally, the effect of stock-outs can be further amplified by consumers' vengeful response due to disappointing shopping experiences. In the healthcare industry, stock-out may even be life-threatening, thus triggering serious social problems. For example, due to drug stock-out in South Africa, 37% of the patients leave with insufficient medication or more burdensome treatment, 36% of the patients receive an alternative treatment, and another 27% of the patients leave with incomplete or no treatment at all (MSF 2016). Beyond potential health risks due to inadequate treatment, patients have to make repeated, costly trips to healthcare institutions to keep up with their prescriptions. Furthermore, pharmacists and nurses have to spend more time rationing drugs instead of caring for patients.

Firms in a competitive marketplace care more about inventory availability, because it is a key leverage to enhance their competitive advantage. On one hand, a high inventory availability enhances customers' surplus gain, and thus attracts more consumer visits. On the other hand, a high inventory availability reduces the direct potential profit loss due to stock-out. For instance, Corsten and Gruen (2003) empirically demonstrate that, when consumers cannot find the product they want, only 7% to 25% of them will continue shopping and purchase a substitute at the store. As a consequence, retailers could lose more than half of intended sales when customers encounter stock-outs. In the era of e-commerce and online shopping, abandoning an out-stocked retailer is unprecedentedly easy, so the business loss of stock-outs is higher than ever before. For example, in December 2011, BestBuy.com canceled some online orders due to the overwhelming demand for hot product offerings. Soon after the cancellation, many customers moved to Amazon.com with a click of button, as reported by TradeGecko (Tao 2014).

Being well aware of the importance of inventory availability, firms compete aggressively to win market shares by offering high service levels. For example, Target made an effort to decrease its stock-out probability by 40% during the 2015 holiday season, which led to a 34% increase in online sales (Wahba 2016). As a response, Walmart eliminated about 15% of its assortment to reduce the time to restock shelves and thus improve the fill rate. Walmart also engaged in another "unrelenting war" with Amazon in the e-commerce market in increasing the inventory of items on Walmart.com. Consequently, Walmart.com had more top-selling toys and electronics products in stock than Amazon.com and strengthened its position in the market (Davis 2016).

To relieve customers' anxiety about potential stock-outs, firms actively seek strategies such as monetary compensation and inventory commitment to improve their competitiveness in the market. Sloot et al. (2005) find that monetary compensations such as discount coupons, rain checks,

and additional services are effective in placating consumers upon stock-outs. Bhargava et al. (2006) report that MAP LINK (the U.S. largest map distributor), VERGE (a U.S. media network publisher), and IntelliHome (a U.S. smart home technology company) offer discounts of 2%, 5%, and 10%, respectively, for all backlogged items. Many car dealers provide price reductions if what automobile consumers choose is out of stock, whereas restaurants offer free dishes if consumers' original choices are sold out. In the context of online retailing, sellers usually waive delivery fees if items are backlogged. An alternative strategy is to make an inventory commitment, i.e., the retailer credibly reveals the inventory stocking quantity to its customers (see Su and Zhang 2009). Many e-commerce firms, including BestBuy.com and Neweggs.com, offer real-time availability information in the store and online. Target and Walmart also allow consumers to check the inventory availability of a particular product at local stores using the zip code and the DPCI number (Morgan 2015). Moreover, consumers have access to technologies and applications that help them track product availability information at competing retailers. For example, TrackAToy offers availability information for many retailers if consumers type the product's name on the web page<sup>1</sup>.

Although analyzing the inventory availability concerns of customers has received significant research attention, the majority of studies in the literature focus on a monopoly setting. We study the impact of inventory availability concerns under competition. In particular, we strive to understand how two competing retailers should optimize their pricing and inventory strategy when facing customers who are concerned about inventory availability. We also examine the effectiveness of the commonly adopted monetary compensation and inventory commitment strategies under competition. Finally, we study the impact of inventory availability competition on customer surplus.

We model two competing retailers as newsvendors located at the endpoints of a Hotelling line market. Customers are uniformly distributed on the Hotelling line. As in the standard Hotelling model, customers incur a travel cost to patronize a retailer. The closer a customer is located to a retailer, the less travel cost she will incur. Before demand is realized, each retailer sets its price and inventory order quantity to maximize the expected profit. The prices are observable to customers and the other retailer, whereas the inventory order quantity is each retailer's private information. Individual customers choose which retailer to patronize based on product price, travel cost, and belief about inventory availability. Under the monetary compensation strategy, a retailer compensates the customers who cannot get the product due to stock-out, whereas under the inventory commitment strategy, a retailer credibly reveals its inventory order information to the public.

The primary goal of this paper is to investigate the competition on inventory availability and price and evaluate its impact on firm profit and customer surplus. We also seek to evaluate the

<sup>1</sup> <http://www.trackatoy.com/>

commonly used monetary compensation and inventory commitment strategies under competition. To this end, we adopt the perfect Bayesian equilibrium (PBE) framework to study the strategic interactions between the retailers and the customers under competition on inventory availability and price. The main findings from this research can be summarized as follows:

First, depending on competition intensity, the equilibrium product availability and price under competition may be either higher or lower than those in the monopoly setting. In the presence of competition, the retailers are competing on both price and inventory availability. In particular, the retailers' trade-off is between decreasing price (which implies low inventory availability) and increasing inventory availability (which requires a high price). More specifically, when the competition is moderate (resp. intensive), the retailers compete more aggressively on inventory availability (resp. price), thus leading to higher (resp. lower) equilibrium price and inventory availability compared with those in the monopoly setting. Following the same intuition, we find that a more competitive market will increase (resp. decrease) the price and inventory availability if the competition is moderate (resp. intensive).

Second, monetary compensation and inventory commitment may decrease retailers' profit under market competition. In the monopoly setting, it has been shown that the monetary compensation and inventory commitment strategies benefit the retailer in the presence of strategic customers, because these strategies help reassure customers in the presence of stock-out risks (Su and Zhang 2009). Specifically, under market competition, monetary compensation may prompt the retailer to overcompensate customers so as to signal high product availability, thus backfiring on the retailers and hurting their profits. Likewise, if the inventory commitment option is allowed for each competing retailer, a prisoner's dilemma may arise. Specifically, although both retailers have incentives to commit to an inventory order quantity, the equilibrium profits of both retailers may decrease if inventory commitment is adopted. Revealing inventory information to customers intensifies market competition and results in inventory over-stocking, thus rendering lower profits for both retailers.

Third, competition may lead to lower average customer surplus. It has been well documented in the literature that competition generally benefits customers (see, e.g., Brynjolfsson et al. 2003). In our setting, one may intuit that competition will enhance the customer average surplus, because they will have more options and lower travel costs in the presence of market competition. Our results, however, indicate that market competition may actually increase the equilibrium price and, therefore, lower the average customer surplus. To help resolve this issue, we find that monetary compensation and inventory commitment provide incentives for customers to patronize the retailers and, thus, benefit the customers by increasing their surplus.

The remainder of this paper is organized as follows. Section 2 positions this paper in the relevant literature. The model is introduced in Section 3. We present the results of the base model in

Section 4. Sections 5 and 6, respectively, study the value of monetary compensation and inventory availability strategies under inventory availability competition. The analysis of customer surplus is given in Section 7. Finally, Section 8 concludes this paper. All the proofs are presented in the Appendix.

## 2. Literature Review

The impact of inventory availability has been extensively studied in the operations management literature. Dana and Petruzzi (2001) consider a newsvendor model where consumers are concerned about inventory availability and can choose whether to visit the firm. Su and Zhang (2008) introduce the strategic waiting behavior of customers into the newsvendor setting and investigate the impact of such behavior on a firm's pricing and stocking decisions. Liu and Van Ryzin (2008) demonstrate that one can mitigate the strategic waiting behavior by limiting inventory availability over repeated selling horizons. Since then there have been a growing number of operations studies that involve strategic customer behavior and availability considerations in various settings. For example, Su and Zhang (2009) and Cachon and Feldman (2014) further include a search cost for the stock-out-conscious customers. Cachon and Swinney (2009, 2011) focus on the value of quick response under strategic customer behavior. Prasad et al. (2014), Li and Zhang (2013), and Wei and Zhang (2017a) investigate the advance selling strategy where product availability may affect customers' optimal timing of purchase. Allon and Bassamboo (2011) use a cheap talk framework to quantify the value of providing inventory availability information to customers; Liang et al. (2014) examine a firm's product rollover strategies under consumers' forward-looking behavior. With variable assortment depth, Bernstein and Martínez-de Albéniz (2016) study the optimal dynamic product rotation strategy in the presence of strategic customers. Tereyagolu and Veeraraghavan (2012) study a retailer's problem when selling to conspicuous consumers whose consumption utility depends on the availability of the product. Finally, Gao and Su (2016) study the role of inventory availability in the context of omni-channel retailing. Wei and Zhang (2017b) provide a recent review of this line of research. Despite the fast growth of this topic, the majority of studies in the literature focus on single-firm settings; our paper, instead, contributes to the above literature by studying the impact of product availability in a competitive setting.

In a competitive marketplace, if a stock-out occurs at one firm, unsatisfied demand may switch to the other firms. Such stock-out-based substitution has also received significant attention in the operations management literature. Lippman and McCardle (1997) propose several ways to model demand allocation between competing newsvendors and show that competition leads to overstocking relative to the centralized solution. Netessine and Rudi (2003) develop a tractable model to compare inventory management under centralized vs. decentralized control. Several studies extend

the static substitution model to dynamic ones; see, for example, Bassok et al. (1999), Shumsky and Zhang (2009), and Yu et al. (2015). This line of research does not explicitly model individual customer behavior, which is a key focus of our work. Therefore, our paper differs from this research in terms of both the model setting and insights.

Another stream of papers study the competition on product availability in the economics literature. Carlton (1978) is among the first to formally consider the issue of product availability in a competitive market and argues that only equilibrium outcome with zero firm profit will arise. As a follow-up to Carlton's work, Deneckere and Peck (1995) consider a game where firms can decide on both price and capacity and demonstrate that a pure-strategy equilibrium exists if and only if the number of firms is sufficiently large. Lei (2015) studies a similar integrated newsvendor and Hotelling model but with asymmetric unit costs. He finds that firms with the lowest unit cost may survive in the long run. Along this line of research, Daughety and Reinganum (1991) and Dana Jr (2001) are closely related to our paper. Specifically, Daughety and Reinganum (1991) consider a setting where consumers have imperfect information on both price and stocking levels at firms. An important finding is that in equilibrium, the duopoly price is lower than the monopoly price if consumers' search cost is low, while the duopoly price is the same as the monopoly price if consumers' search cost is high. In contrast, we find that retailers may charge a strictly *higher* price to signal high product availability and thus attract more demand in the presence of market competition. Dana Jr (2001) adopts a newsvendor setup to model retailers competing on product availability. It has been shown that the retailers can enjoy a positive profit (i.e., they can charge a price higher than marginal cost) even though the products are perfectly substitutable because the retailers can signal a high probability of product availability using a high price. Our paper also uses a similar newsvendor paradigm, but with several important differences. First, we use the Hotelling setup to incorporate heterogeneous travel costs of customers, which which lead to different insights. Second, we examine the impact of availability competition on customer surplus, while Dana Jr (2001) focuses on the equilibrium outcome from the firms' perspectives. Finally, we also study the effectiveness of operational strategies such as stock-out compensations and inventory commitment, which are absent from the above economics literature.

Another literature related to our work is price competition under strategic consumer behavior. Rosenthal (1980) considers a competitive market setting with groups of loyal consumers and a group of switching consumers. Choi et al. (2018) use an oligopoly model to explore consumer sequential search behavior in market competition. Chen and Riordan (2007) find that competition can increase price when consumer valuations for products from competitors are perfectly negatively correlated. By comparing the monopoly and duopoly models, Chen and Riordan (2008) show that competition may drive a higher price when products are substitutes. Comparing to equilibrium

price in a monopoly model, the authors find that equilibrium price in the competition can be lower or higher than the monopoly price. Although our paper also examines the impact of competition on equilibrium prices, we are different from Chen and Riordan (2008) in significant ways. First, Chen and Riordan (2008) consider deterministic demand, so inventory availability is irrelevant in their context. However, because of demand uncertainty, inventory availability is a core trade-off in our model that generates new insights to the literature. Second, an outside market exists in Chen and Riordan (2008); thus, when a new product (firm) enters the market, the firms compete for market share from both their rivals as well as from the outside market. In our model, however, the market is closed and the firms have to compete for consumers with each other. As a result, the driving force behind the price comparison is quite different in our paper compared to that of Chen and Riordan (2008).

The dynamic price competition models with uncertain demand have also received considerable attention in the operations management literature. Levin et al. (2009) consider differentiated and perishable goods when consumers time their purchases. They find that firms can improve their benefits by limiting full information to consumers. Martínez-de Albéniz and Talluri (2011) investigate a Bertrand duopoly model in which sellers offer identical and fixed number of units and consumers choose to visit a seller with lower price. Gallego and Hu (2014) study a market setting with a mix of substitutable and complementary perishable assets. Different from the above dynamic price competition models with multiple periods, Kremer et al. (2017) study a two-period model with a mixture of myopic and strategic consumers. Their result shows that sellers offer small (resp. large) late season markdowns if the fraction of strategic consumers is low (resp. high). Fisher et al. (2017) conduct a field experiment to empirically study consumer choices among substitutable products under competition. The authors propose a best-response pricing strategy based on their estimates.

The economics literature has also studied the impact of competition on customer surplus. For example, Brynjolfsson et al. (2003) summarize two channels of how market competition of product variety improves consumer surplus. In their study of the on-line bookstores market, the increased product variety competition induces around three million dollars more consumer welfare in 2000. In the food industry, Hausman and Leibtag (2007) empirically verify that the entry of new business and the expansion of existing business improve average consumer surplus by approximately 25%. Goolsbee and Petrin (2004) show that the competition between broadcast satellites (DBS) and cable leads to a consumer welfare gain of \$2.5 billion for satellite buyers and \$3 billion for cable subscribers. Our contribution to this literature is that we demonstrate the adverse effect of market competition on customers if they are concerned about inventory availability.

### 3. Base Model

Our model is built upon the classical newsvendor and Hotelling frameworks. The newsvendor setup captures the key features of demand uncertainty and perishable inventory, which are common for a retail setting where inventory availability concern is most relevant. The Hotelling model highlights the spatial competition and customer heterogeneous tastes (preferences) for the retailers. These salient features of our model are often ignored in the literature studying inventory availability. We now introduce our model from the perspectives of retailers (he) and customers (she) separately.

**Retailers.** We model the market as a Hotelling line with a unit length, denoted by  $\mathcal{M} = [0, 1]$ . Two retailers,  $R_i$  ( $i = 1, 2$ ), are located at the two endpoints of the Hotelling market  $\mathcal{M}$ . Without loss of generality, we assume  $R_1$  is at 0 whereas  $R_2$  is at 1. Each retailer sells a substitutable product that has the same procurement cost  $c$ . Each retailer  $R_i$  stocks  $q_i$  units of inventory and charges a price  $p_i$  to maximize his own expected profit.

**Customers.** In the market  $\mathcal{M}$ , customers are uniformly distributed over the interval  $[0, 1]$ . Each customer has infinitesimal mass in  $\mathcal{M}$ , and purchases at most one unit of the product. The valuation of the product to all customers is homogeneous and denoted by  $v$ . Such a modeling setup helps single out the effect of competition. The aggregate market demand  $D$  (i.e., the total mass of the Hotelling line) is uncertain and follows a known distribution  $F(\cdot)$ . We assume that the demand distribution has an increasing failure rate, which can be satisfied by most commonly used distributions. To visit a retailer, each customer incurs a travel cost that increases linearly with her travel distance. More specifically, the travel cost of a customer located at  $x \in \mathcal{M}$  to visit  $R_1$  (resp.  $R_2$ ) is  $sx$  (resp.  $s(1 - x)$ ), where  $s$  is the unit distance travel cost. It is worth noting that the travel cost can also be interpreted as the search cost for customers. The longer the distance between a customer and her focal retailer, the more costly for her to patronize. The customer aims to maximize her expected payoff by choosing to visit (or not to visit) a retailer.

The sequence of events unfolds as follows. At the beginning of the sales season, each retailer  $R_i$  simultaneously decides his inventory stocking quantity  $q_i$  and announces the retail price  $p_i$ . Both the inventory level  $q_i$  and the price  $p_i$  cannot be adjusted throughout the sales horizon. Customers observe the prices  $(p_1, p_2)$ , but not the inventory levels  $(q_1, q_2)$ , and decide which retailer to visit (or not to visit any of them). The demand  $D_i$  for retailer is realized as a result of customers' cumulative purchasing decisions. If  $D_i \leq q_i$ , all customers requesting a product can get one. Otherwise,  $D_i > q_i$ , a stock-out occurs. Facing the stock-out, customers not receiving the product will leave the market immediately without trying the other retailer<sup>2</sup>. Finally, the transactions occur and the retailers collect the revenues.

<sup>2</sup> As we will explain below, under equilibrium, if one retailer faces a stock-out, the other one will also run out of inventory.



To conclude this section, we remark that our model is a generalization of the one studied by Dana Jr (2001). The model of Dana Jr (2001) is a special case of ours with zero travel cost ( $s = 0$ ). As we will show below, such generalization facilitates us to have a complete understanding for the impact of customers' concerns over availability in a competitive landscape, and derive sharper insights into the problem. Furthermore, though simple and parsimonious, our combined Hotelling and newsvendor model captures the key elements of interest such as demand uncertainty (thus inducing the inventory availability concern), strategic customer behavior, and competition. Moreover, all our results and insights are robust and readily generalizable to the models with multiple retailers (Chen and Riordan 2007), asymmetric transportation costs (van der Weijde et al. 2014), and/or heterogeneous product qualities (Hotz and Xiao 2013). Our model can be directly applied to an e-commerce scenario as well. The distance between a customer located on the Hotelling line and a retailer represents the customer's preferences to that e-retailer. The closer a customer to a retailer, the more the customer prefers the retailer. For conciseness, we define  $\mathbb{E}[\cdot]$  as the expectation operation, and  $x \wedge y := \min(x, y)$  as the minimum operation. All notations are summarized in Table 1 in the Appendix.

## 4. Analysis of Base Model

### 4.1. Equilibrium Characterization

We start with the equilibrium analysis of the base model and the monopoly benchmark. To this end, we adopt the Perfect Bayesian Equilibrium (PBE) concept. Under the PBE, customers, upon observing the prices  $(p_1, p_2)$ , form beliefs about inventory availability and make purchasing decisions to maximize their own expected utilities, whereas retailers (at the beginning of the sales horizon) base their pricing and inventory decisions on anticipations about customers' purchasing behaviors to maximize profits. Furthermore, under equilibrium, both the customers' beliefs about inventory availability and the retailers' anticipations should be consistent with the actual outcomes according to the Bayes rule. Following Dana Jr (2001), we restrict our attention to equilibria where the retailers play symmetric pure strategies.

**Customers' Problem.** First, by backward induction, we analyze the customers' problem. Consider a customer located at  $x \in \mathcal{M}$ . Her surplus to visit  $R_1$  is  $v - p_1 - sx$  (resp.  $-sx$ ) if the product is in stock (resp. out of stock). Similar analysis can be applied if she visits  $R_2$ . The customer gains zero surplus if she does not visit any retailer. Since customers cannot observe retailers' inventory status, they form a belief about it (see Dana Jr 2001). To facilitate the analysis, we assume customers form beliefs about the (unobservable) inventory availability probability instead of order quantity, because the influence of inventory stocking quantity on the expected utility (and thus the purchasing behavior) of a customer boils down to the availability probability it induces. Specifically,

let  $\theta_i(p_1, p_2) \in [0, 1]$  be the in-stock probability of  $R_i$  given the price  $(p_1, p_2)$ . Thus, the expected utility of a customer located at  $x$  to visit  $R_1$  (resp.  $R_2$ ) is  $\mathcal{U}_1(x) := (v - p_1)\theta_1(p_1, p_2) - sx$  (resp.  $\mathcal{U}_2(x) := (v - p_2)\theta_2(p_1, p_2) - s(1 - x)$ ).

Customers base their purchasing decisions on their beliefs about product availability. More specifically, a customer chooses to visit the retailer from which she can earn the highest non-negative expected payoff (otherwise, she will not visit anyone). Since a customer is infinitesimal, without loss of generality, a customer located at  $x$  will patronize  $R_i$  if  $\mathcal{U}_i(x) \geq \max\{0, \mathcal{U}_{-i}(x)\}$  and will not visit any retailer if  $\mathcal{U}_i(x) < 0$  ( $i = 1, 2$ ). Therefore, there exist two thresholds  $\underline{x}(p_1, p_2)$  and  $\bar{x}(p_1, p_2)$ <sup>3</sup> ( $0 \leq \underline{x}(p_1, p_2) \leq \bar{x}(p_1, p_2) \leq 1$ ), such that a customer located at  $x$  will patronize  $R_1$  if  $x \leq \underline{x}(p_1, p_2)$ , will patronize  $R_2$  if  $x \geq \bar{x}(p_1, p_2)$ , and will not visit any retailer if  $x \in (\underline{x}(p_1, p_2), \bar{x}(p_1, p_2))$ .

**Retailer's Problem.** Next, we analyze the retailer's pricing and inventory problem. Each retailer  $R_i$  strategizes his price and inventory decisions in anticipation of customers' purchasing decisions (thus his market share). Specifically, the demand for  $R_1$  (resp.  $R_2$ ) is  $\underline{x}(p_1, p_2)D$  (resp.  $(1 - \bar{x}(p_1, p_2))D$ ). Retailer  $R_i$ 's profit maximization problem is, given the competitor's price  $P_2$ ,

$$\max_{p_i} \max_{q_i} \{p_i \mathbb{E}(\alpha_i(p_1, p_2)D \wedge q_i) - cq_i\},$$

where  $\alpha_1(p_1, p_2) = \underline{x}(p_1, p_2)$  and  $\alpha_2(p_1, p_2) = 1 - \bar{x}(p_1, p_2)$  represent the respective market share. Therefore, given price  $p_i$ ,  $R_i$ 's optimal inventory order strategy is the newsvendor solution:  $q_i = \alpha_i \bar{F}^{-1}\left(\frac{c}{p_i}\right)$ .

To characterize the PBE, we need to specify the off-equilibrium customer belief on inventory availability (see, e.g., Dana Jr 2001). Moreover, we refine the off-equilibrium belief to rule out implausible equilibria. Following the equilibrium refinement strategy of Dana Jr (2001), we assume customers rationally believe that the retailers are stocking the optimal amount of inventory given any observed price. Specifically, given the price  $(p_1, p_2)$ , the customers believe that the inventory order quantity of  $R_i$  is  $Q_i(p_1, p_2) = \alpha_i(p_1, p_2) \bar{F}^{-1}\left(\frac{c}{p_i}\right)$ . We observe that conditioned on the existence of a customer, her belief about the total demand for  $R_i$  is a random variable with probability density function  $g_i(y|p_1, p_2) := \frac{y}{\alpha_i(p_1, p_2)\mu} f\left(\frac{y}{\alpha_i(p_1, p_2)}\right)$ , where  $\mu := \mathbb{E}[D]$  (see, e.g., Dana Jr 2001, Su and Zhang 2009). As shown by Dana Jr (2001), without competition (i.e., all customers have the same distance to the competing retailers), each customer holds an identical belief of inventory availability  $\int_{y=0}^{\infty} \frac{\min(Q_i(p_1, p_2), \alpha_i(p_1, p_2)y)}{\alpha_i(p_1, p_2)y} \cdot \frac{yf(y)}{\mu} dy = \int_{y=0}^{\infty} \frac{\min(Q_i(p_1, p_2), \alpha_i(p_1, p_2)y)f(y)}{\alpha_i(p_1, p_2)\mu} dy$ . In our model, however, customers have different distances to retailers and will arrive at different time, which requires us to carefully model how inventory is rationed to customers upon stock-outs.

<sup>3</sup> Algebraic manipulation yields that

$$\begin{cases} \underline{x}(p_1, p_2) = \mathcal{P}_{[0,1]} \left( \min \left\{ \frac{(v-p_1)\theta_1(p_1, p_2)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta_1(p_1, p_2) - (v-p_2)\theta_2(p_1, p_2)}{2s} \right\} \right), \\ \bar{x}(p_1, p_2) = \mathcal{P}_{[0,1]} \left( \max \left\{ 1 - \frac{(v-p_2)\theta_2(p_1, p_2)}{s}, \frac{1}{2} + \frac{(v-p_2)\theta_2(p_1, p_2) - (v-p_1)\theta_1(p_1, p_2)}{2s} \right\} \right), \end{cases}$$

where  $\mathcal{P}_{[0,1]}(x) := \min\{\max\{x, 0\}, 1\}$  is the projection operator onto the interval  $[0, 1]$ .

**Efficient Rationing.** A natural choice of the inventory rationing rule is uniform/random rationing, i.e., inventory will be allocated to each customer who visits a retailer with equal probability (see Section D for details). In this case, customers closer to a retailer will enjoy a higher expected surplus to visit this retailer. As shown in the behavioral economics literature, information of a stockout lowers customers satisfaction and reduces purchase motivation (Kim and Lennon 2011), and thus can be viewed as an exogenous imposed tax (Sicular 1988). Hence, customers with higher surpluses are more likely to visit the retailer earlier and would be served earlier as well. Even if the customers make retailer patronage decisions simultaneously, a customer closer to the focal retailer will arrive earlier due to the shorter travel distance. As a consequence, customers closer to a focal retailer will visit the retailer earlier and will be served with a higher priority. This phenomenon motivates us to adjust the inventory allocation rule that accounts for the effect of travel distance. Specifically, we adopt the efficient rationing rule which prioritizes customers closer to the retailer upon a stockout. We refer interested readers to a comprehensive discussion of efficient rationing by Tasnádi (1999).

We first specify the inventory availability probability for a customer located at the purchasing threshold. By the efficient rationing rule, the customer located at  $\underline{x}(p_1, p_2)$  (resp.  $\bar{x}(p_1, p_2)$ ) observes the lowest probability of product availability from  $R_1$  (resp.  $R_2$ ) among all customers located in the interval  $[0, \underline{x}(p_1, p_2)]$  (resp.  $[\bar{x}(p_1, p_2), 1]$ ). That is, the customers at the purchasing thresholds can receive the product if and only if the total demand for  $R_i$  does not exceed its inventory stocking quantity. Therefore, the belief of the customers (at the purchasing threshold) about  $R_i$ 's inventory availability probability satisfy:

$$\theta_i(p_1, p_2) = \mathbb{P}\left(\alpha_i(p_1, p_2)D \leq Q_i(p_1, p_2)\right) = \int_{y=0}^{Q_i(p_1, p_2)} g(y|p_1, p_2)dy = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i}\right)} yf(y)dy.$$

Interestingly, for a customer at the purchasing threshold, her belief of product availability only depends on the price of the focal retailer. We remark that this is driven by our equilibrium refinement rule that customers believe the retailers will stock the optimal newsvendor inventory, which induces a service level that depends on the price of the focal retailer only. For the subsequent analysis, we shall use  $\theta^*(p_1) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i}\right)} yf(y)dy$  to denote customers' belief of  $R_i$ 's inventory availability for those at the purchasing thresholds.

For customers who are not at the purchasing thresholds, their beliefs of product availability about  $R_i$  is a function of prices  $(p_1, p_2)$  and location  $x$ . However, we now demonstrate that specifying the equilibrium belief of customers at the purchasing thresholds is sufficient to characterize the purchasing behaviors of *all* customers.

LEMMA 1. If  $\mathcal{U}_1(\underline{x}(p_1, p_2)) = (v - p)\theta^*(p_1) - s\underline{x}(p_1, p_2) > 0$  (resp.  $\mathcal{U}_2(\bar{x}(p_1, p_2)) = (v - p)\theta^*(p_2) - s(1 - \bar{x}(p_1, p_2)) > 0$ ), then all customers located at  $x \in [0, \underline{x}(p_1, p_2)]$  (resp.  $x \in [\bar{x}(p_1, p_2), 1]$ ) will visit the retailer  $R_1$  (resp.  $R_2$ ).

Lemma 1 effectively simplifies the analysis of customer purchasing behavior. Assuming that customers hold identical beliefs of inventory availability probability as those at the purchasing thresholds, they will exhibit exactly the same purchasing behavior as they hold the “true” equilibrium beliefs. In other words, it suffices to use the beliefs of the customers at the purchasing thresholds to characterize the equilibrium decisions of all customers.

**Equilibrium.** We are now ready to characterize the symmetric equilibrium price and inventory decisions of the retailers. Under the symmetric PBE, both retailers charge the same equilibrium price  $p_d^*$ . Customers hold the same belief of equilibrium product availability, and we have

$$\theta^*(p_d^*) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)} y f(y) dy.$$

Accordingly, we obtain  $\underline{x}(p_d^*, p_d^*) = \min\{\frac{(v-p_d^*)\theta^*(p_d^*)}{s}, \frac{1}{2}\}$ ,  $\bar{x}(p_d^*, p_d^*) = \max\{1 - \frac{(v-p_d^*)\theta^*(p_d^*)}{s}, \frac{1}{2}\}$  and  $\alpha_i(p_i, p_d^*) = \min\{\frac{(v-p_i)\theta^*(p_i)}{s}, \frac{1}{2}\}$  for  $i = 1, 2$ . The travel cost  $s$  can also be viewed as a measure of competition intensity. In particular, if  $s$  is large enough, customers located at the middle of the Hotelling line  $\mathcal{M}$  will visit neither of the retailers. In this scenario, the market is fully separated and, hence, there is no competition between the retailers. On the other hand, if  $s$  is small enough, the surplus difference of customers to purchase from different retailers is small, so the two retailers compete on price and inventory availability to gain market shares.

If  $s$  is large, the market is not fully covered and, therefore, there is no competition between the two retailers. Then, the equilibrium price  $p_d^* = \arg \max_{0 \leq p \leq v} \Pi_d(p)$  is the optimal monopoly price solved by the following:

$$\begin{aligned} \max_{0 \leq p \leq v} \quad & \Pi_d(p) = p\mathbb{E}[\alpha(p)D \wedge Q(p)] - cQ(p) \\ \text{s.t.} \quad & Q(p) = \alpha(p)\bar{F}^{-1}\left(\frac{c}{p}\right), \\ & \alpha(p) = \frac{v-p}{s}\theta^*(p). \end{aligned} \tag{1}$$

The first constraint represents inventory order quantity driven by the newsvendor model. The second constraint follows from a retailer’s market share equation. Since there is no competition between two retailers, under equilibrium, we must have  $\alpha(p_d^*) < 1/2$ .

If  $s$  is small, the market is fully covered and the two retailers compete with each other. In this case, the equilibrium price  $p_d^* = \arg \max_{0 \leq p \leq v} \Pi_d(p, p_d^*)$  is solved by the following:

$$\begin{aligned}
 \max_{0 \leq p \leq v} \quad & \Pi_d(p, p_d^*) = p\mathbb{E}[\alpha(p, p_d^*)D \wedge Q(p, p_d^*)] - cQ(p, p_d^*) \\
 \text{s.t.} \quad & Q(p, p_d^*) = \alpha(p, p_d^*)\bar{F}^{-1}\left(\frac{c}{p}\right), \\
 & \alpha(p, p_d^*) = \frac{1}{2} + \frac{(v-p)\theta^*(p) - (v-p_d^*)\theta^*(p_d^*)}{2s}.
 \end{aligned} \tag{2}$$

It is worth noticing that the above two cases do not necessarily cover all the possible scenarios. It is also possible that although the two retailers cover the entire market, they do not compete with each other. This case occurs when the travel cost  $s$  is in the moderate range. More specifically, in this case, each retailer covers 50% of the market and the customer located at the middle of the market  $\mathcal{M}$  (i.e.,  $x = 0.5$ ) is indifferent between visiting either retailer or not visiting anyone, i.e., the equilibrium price  $p_d^*$  satisfies that  $\alpha(p_d^*) = \frac{(v-p_d^*)}{s}\theta^*(p_d^*) = \frac{1}{2}$ . By Su and Zhang (2009),  $p_d^*$  should be the larger root of the equation (of variable  $p$ )  $\frac{(v-p)}{s}\theta^*(p) = \frac{1}{2}$ . Indeed, we show in Lemma 2 that the condition  $p_d^* \in [\hat{p}, v]$  guarantees that  $p_d^*$  is the larger root of  $\frac{(v-p)}{s}\theta^*(p) = \frac{1}{2}$ . Finally, we note that, in this case,  $p_d^*$  is the solution to the following stochastic program:

$$\begin{aligned}
 \max_{0 \leq p \leq v} \quad & \Pi_d(p) = p\mathbb{E}[\alpha(p)D \wedge Q(p)] - cQ(p) \\
 \text{s.t.} \quad & Q(p) = \alpha(p)\bar{F}^{-1}\left(\frac{c}{p}\right), \\
 & \alpha(p) = \frac{v-p}{s}\theta^*(p).
 \end{aligned} \tag{3}$$

Furthermore, under equilibrium, we have  $\alpha(p_d^*) = \frac{1}{2}$  in this case. It should be noted that (3) and (1) share the same structure. The only difference is that we impose the constraint  $\alpha(p_d^*) = \frac{1}{2}$  for the case of full market coverage without competition.

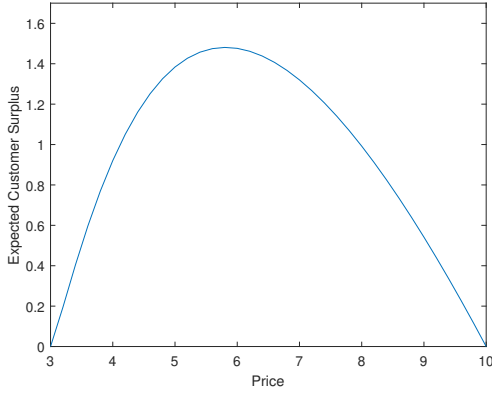
**LEMMA 2.** *The expected utility of a customer located at  $x$  to visit  $R_1$  (resp.  $R_2$ ),  $(v-p_1)\theta^*(p_1) - sx$  (resp.  $(v-p_2)\theta^*(p_2) - s(1-x)$ ), is concave in its price  $p_1 \in [\hat{p}, v]$  (resp.  $p_2$ ). Furthermore, the equilibrium price  $p_d^*$  satisfies that  $p_d^* \in [\hat{p}, v]$ , where  $\hat{p} = \arg \max_p (v-p)\theta^*(p)$ .*

As shown by Lemma 2 and illustrated in Figure 1, the expected customer utility is concave in price. Moreover, the market equilibrium price is lower bounded by  $\hat{p}$ , which is the price that maximizes the expected consumer surplus. Similar results have also been established by Dana Jr (2001), who proposes a special case of our model with zero travel cost (i.e.,  $s = 0$ ).

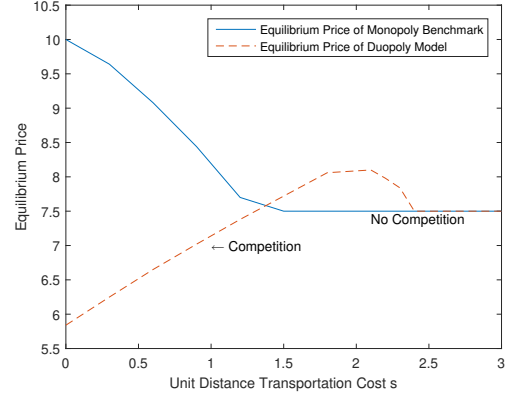
We are now ready to characterize the symmetric Perfect Bayesian equilibrium  $(p_d^*, q_d^*, \theta^*(\cdot))$  in the base model.

**PROPOSITION 1.** (a) *A unique symmetric PBE  $(p_d^*, q_d^*, \theta^*(\cdot))$  exists in the base model.*

(b) *There exists a threshold  $\underline{s}$  such that if  $s \leq \underline{s}$ , we have  $p_d^* = \arg \max_{0 \leq p \leq v} \Pi_d(p, p_d^*)$  (see (2)),  $q_d^* = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)$ , and  $\theta^*(p_d^*) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)} yf(y)dy$ . In this case, the two retailers compete with each other and each covers half of the entire market.*



**Figure 1** Customer Surplus as a Function of Price ( $v = 10, c = 3$ )



**Figure 2** Impact of Competition on Prices ( $D_i$  follows a gamma distribution with mean 90 and standard deviation 30,  $v = 10, c = 3, s \in [0, 3]$ )

(c) There exists another threshold  $\bar{s} > \underline{s}$ , such that if  $\underline{s} < s \leq \bar{s}$ , we have  $p_d^* = v - \frac{s}{2\theta^*(p_d^*)}$ ,  $q_d^* = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)$ , and  $\theta^*(p_d^*) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)} yf(y)dy$ . In this case, each retailer covers half of the entire market without competing with each other.

(d) If  $s > \bar{s}$ , we have  $p_d^* = \arg \max_{0 \leq p \leq v} \Pi_d(p)$  (see (1)),  $q_d^* = \frac{(v-p_d^*)\theta^*(p_d^*)}{s} \bar{F}^{-1}\left(\frac{c}{p_d^*}\right)$ , and  $\theta^*(p_d^*) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)} yf(y)dy$ . In this case, each retailer covers  $\frac{(v-p_d^*)\theta^*(p_d^*)}{s} < \frac{1}{2}$  of the entire market, so the market is partially covered by the retailers without competition.

Proposition 1 implies that a unique symmetric PBE exists in the duopoly base model. Furthermore, we find that if the travel cost is small ( $s \leq \underline{s}$ ), the competition is intensive and the two retailers will cover the entire market competitively, each with a 50% market share. If the travel cost is moderate ( $\underline{s} < s \leq \bar{s}$ ), although the two retailers remain covering the entire market (each still with a 50% market share), there is no competition in the market. Finally, if the travel cost is high ( $s > \bar{s}$ ), the market is completely segmented and some customers will opt not to visit any retailer, so the market share of each retailer is smaller than 50%. Such a market outcome structure is consistent with that in the standard Hotelling model without demand uncertainty (see Lemma 3 in the Appendix). We also remark that Proposition 1 also demonstrates that once one retailer faces stockout, the other one will also run out of inventory. This partially justifies our modeling assumption that, upon the stockout of one retailer, customers will not visit the other one.

**Monopoly Benchmark** To understand the strategic implications of market competition, we consider a benchmark model where there is only one retailer located at one endpoint of the Hotelling market. Specifically, the monopoly benchmark is identical to our base model except that  $R_2$  is removed from the market  $\mathcal{M}$ . As a consequence,  $\mathcal{M}$  is reduced to a monopolistic market and

customers only have one choice (i.e.,  $R_1$  located at point 0) if they opt to purchase the product. As in our base model, the retailer sets the price and the inventory stocking quantity to maximize his expected profit, whereas customers, without observing the inventory stocking quantity of  $R_1$ , decide whether to visit the retailer in order to maximize their expected payoff. In a similar spirit to the base model, we can characterize the PBE in the monopoly benchmark. In the monopoly benchmark, we denote the equilibrium price as  $p_m^*$  and the equilibrium order quantity as  $q_m^*$ . For conciseness, we relegate the equilibrium characterization of the monopoly benchmark to the Appendix.

#### 4.2. Impact of Retailer Competition on Availability

In this subsection, we analyze the impact of retailer competition on the market outcome by comparing the PBEs in the duopoly base model and the monopoly benchmark. Moreover, we characterize how the intensity of market competition (captured by the travel cost  $s$ ) would influence the structure of the competitive landscape and the market outcome.

A fundamental question in the economics literature is how will competition impact the equilibrium prices? The answer is in general unclear to the extent that competition may decrease or increase the equilibrium prices in different contexts. For example, Daughety and Reinganum (1991) show that market competition lowers the price since it decreases the search cost for customers. Basker and Noel (2009) empirically analyze grocery market response to Walmart's entry. The authors find that market response to the entry of a Walmart store, who has about 10% price advantage over other competitors, is a price reduction of 1% to 1.2% on average. As another example, Goolsbee and Syverson (2008) find that incumbent airlines cut fares significantly when threatened by Southwest's entry, and the fare-cutting occurs even before Southwest starts flying. On the other hand, one can also find empirical evidence in which market competition may increase price. For instance, Thomadsen (2007) finds that, in the fast food industry, positioning geographically close to a competitor could lead to prices higher than the monopoly level. In this paper, we are interested in exploring the impact of market competition on the equilibrium pricing and inventory strategy with customers concerned about inventory availability.

**PROPOSITION 2.** *The following statements hold:*

(a) (i)  $p_d^*$  is quasi-concave in  $s$ . More specifically,  $p_d^*$  is strictly increasing in  $s$  for  $s \in [0, \underline{s}]$ , strictly decreasing in  $s$  for  $s \in [\underline{s}, \bar{s}]$ , and independent of  $s$  for  $s \geq \bar{s}$ . (ii)  $q_d^*$  is strictly increasing in  $s$  for  $s \in [0, \underline{s}]$ , and strictly decreasing in  $s$  for  $s \geq \underline{s}$ .

(b) (i)  $p_m^*$  is non-increasing in  $s$ . More specifically,  $p_m^*$  is strictly decreasing in  $s$  for  $s \leq \bar{s}_m$  and independent of  $s$  for  $s > \bar{s}_m$ . (ii)  $q_m^*$  is strictly decreasing in  $s$  for all  $s \geq 0$ .

- (c) There exists a threshold  $\bar{s}_d$  ( $\bar{s}_d \leq \bar{s}$ ), such that (i)  $p_d^* \leq p_m^*$  and  $\theta^*(p_d^*) \leq \theta^*(p_m^*)$  for  $s \leq \bar{s}_d$ ; (ii)  $p_d^* > p_m^*$  and  $\theta^*(p_d^*) > \theta^*(p_m^*)$  for  $\bar{s}_d < s < \bar{s}$ ; and (iii)  $p_d^* = p_m^*$  and  $\theta^*(p_d^*) = \theta^*(p_m^*)$  for  $s \geq \bar{s}$ . (iv) Moreover,  $q_d^* < q_m^*$  for  $s < \bar{s}$  and  $q_d^* = q_m^*$  for  $s \geq \bar{s}$ .

Standard result of the Hotelling competition model predicts that, as the travel cost increases, it becomes easier for the retailers to secure the nearby customers, and thus the equilibrium price will increase (see Lemma 3 in the Appendix for details). Similar conclusion can also be obtained in our model with demand uncertainty and inventory availability concerns, as shown in Proposition 2(a) and illustrated in Figure 2. As a consequence of the quasi-concave structure of the equilibrium price in our duopoly model, low product differentiation and intense market competition ( $s \leq \bar{s}_d$ ) leads to a lower price than monopoly, whereas moderate differentiation and competition ( $\bar{s}_d < s < \bar{s}$ ) result in a higher price. To attract customers under competition, the retailers always face the trade-off between charging a lower price to offer price discounts (but with a low inventory availability) or ordering more inventory to ensure product availability (but with a high price). When  $s$  is small, customers are flexible to travel to either retailer and, thus, more sensitive to prices than to inventory availabilities. Hence, the retailers compete aggressively by cutting down the selling prices. When  $s$  is moderate, however, the travel cost and, thus, stock-out risk are high for customers, so customers are more sensitive to inventory availabilities than prices. Hence, the retailers compete on signaling inventory availabilities by increasing the prices. Our result suggests that, depending on the travel cost  $s$  (and thus the market competition intensity), competition may either increase or decrease the equilibrium price and product availability relative to the monopoly model. In terms of order quantity, we find that competition will always decrease the equilibrium stocking level of each retailer, as shown in Proposition 2(c).

It is useful to compare and differentiate our model and results from those of Dana Jr (2001). From the modeling perspective, Dana Jr (2001) analyzes a special case of our model with zero travel cost. Consistent with Dana Jr (2001), we show that when the competition is intensive (i.e., the travel cost  $s$  is small), retailers decrease their prices to gain competitive edge (Proposition 2(c.i)). As competition becomes moderate (i.e., the travel cost  $s$  increases), however, our result demonstrates that the prediction of Dana Jr (2001) is reversed (Proposition 2(c.ii)): Competition will drive the retailers to charge higher prices. On top of the equilibrium characterization, which is the focus of Dana Jr (2001), we have also investigated the effectiveness of the widely-adopted monetary compensation and inventory commitment strategies to address the inventory availability concerns of customers under competition. Our analysis suggests that while these strategies are effective in a monopoly market, they may not necessarily improve the retailers' profit under competition. See Sections 5 and 6 below for more details. In short, our model has extended Dana Jr (2001)



by introducing competition, which has led to interesting insights not yet well-understood in the literature.

One may wonder whether our main insight that competition may increase the equilibrium price is robust with respect to random rationing rule that is widely applied in a non-spatial competition model (see Dana Jr (2001)). Note that, on one hand, competition drives a retailer to decrease price so as to signal availability; on the other hand, competition also prompts a retailer to increase the price to signal inventory availability. Therefore, the equilibrium price  $\tilde{p}_d^*$  balances the aforementioned two effects and may be higher or lower than the monopoly benchmark. We summarize this result in Appendix D (Proposition 11), which demonstrates that our main insight remains valid under random rationing.

## 5. Monetary Compensation

We proceed to analyze the strategies the retailers could adopt to enhance their edge in a competitive market where customers have inventory availability concerns. A widely used marketing strategy is to compensate the customers who face stock-outs. This strategy could reassure the customers in the presence of potential stock-outs, thus motivating customers to visit the retailer more often. In practice, the compensation is offered in the form of coupons, gift cards, price discounts for future orders, and free shipping opportunities. For example, FoodLand offers consumers a rain check for the out-of-stock items<sup>10</sup>. The simplest and most direct compensation strategy is to placate customers for stock-outs with cash, which we refer to as the monetary compensation strategy. In this section, we focus on studying the effect of monetary compensation under competition.

The monetary compensation strategy has proven beneficial to a monopoly retailer (see Su and Zhang 2009), because it incentivizes the strategic customers to visit him. In a competitive market, however, the story is different. Our analysis in this section shows that, when monetary compensation is an option, competing retailers will (voluntarily) overcompensate customers to attract higher demand, which in turn decreases their profits compared with the baseline setting where monetary compensation is not allowed.

To model the monetary compensation strategy, we assume that each retailer offers a compensation  $m_i \geq 0$  ( $i = \{1, 2\}$ ) to customers who face stock-out. The special case where  $m_i = 0$  refers to that  $R_i$  does not offer monetary compensation. So both retailers have the flexibility to decide whether to offer monetary compensation upon stock-outs and the amount of compensation. As in the base model, customers observe the retailers' prices and monetary compensation terms, but not their inventory stocking quantities. The retailers set the price and inventory stocking quantity to maximize their profits, whereas customers decide where to make a purchase to maximize their

<sup>10</sup> <https://www.foodland.com/if-i-have-coupon-product-out-stock-may-i-receive-rain-check-product>

expected surplus. Following the same equilibrium analysis paradigm as in the base model, we consider the symmetric PBE in the model with monetary compensation. We use the subscript  $c$  to denote the model with monetary compensation.

For any customer located at  $x$ , she will visit the retailer that yields the highest non-negative expected payoff and receive monetary compensation upon stockout. Hence, the customer's expected payoff is  $(v - p_1)\theta_1 + m_1(1 - \theta_1) - sx$  (resp.  $(v - p_2)\theta_2 + m_2(1 - \theta_2) - s(1 - x)$ ), where  $\theta_1$  (resp.  $\theta_2$ ) is the customer's belief about  $R_1$ 's (resp.  $R_2$ 's) inventory availability probability.

The retailer  $R_i$ 's decision problem can be formalized as:

$$\max_{(p_i, m_i, q_i)} \Pi_i^c(p_i, m_i, q_i) = p_i \mathbb{E}(\alpha_i^c D \wedge q_i) - m_i \mathbb{E}(\alpha_i^c D - q_i)^+ - cq_i,$$

where  $\alpha_1^c = \underline{x}_c$  and  $\alpha_2^c = 1 - \bar{x}_c$ . Therefore,  $R_i$  orders the newsvendor quantity  $q_i^c = \alpha_i^c \bar{F}^{-1}\left(\frac{c}{p_i + m_i}\right)$ . Recall that the retailers adopt efficient rationing, the in-stock probability of a customer at the purchasing threshold is  $\theta_i^c = \theta^*(p_i + m_i) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_i + m_i}\right)} y f(y) dy$ . Here, the customer belief on inventory availability depends on  $(p_i, m_i)$  through the effective margin  $p_i + m_i$ .

Again we focus on the symmetric PBE  $(p_c^*, m_c^*, q_c^*, \theta^*(\cdot))$ , where  $p_c^*$  is the equilibrium price,  $m_c^*$  is the equilibrium compensation, and  $q_c^*$  is the equilibrium order quantity. Similar to the base model, the equilibrium market share of  $R_i$ ,  $\alpha_i^c$  takes one of the following three forms: (a) if  $s$  is large, the market is completely segmented without competition; (b) if  $s$  is small, the market is fully covered and the retailers compete with each other; and (c) if  $s$  is moderate, the market is fully covered but the retailers do not compete with each other.

The following proposition characterizes the PBE in the presence of monetary compensation.

**PROPOSITION 3.** *For the model with the monetary compensation option, the following statements hold:*

- (a) *A unique symmetric PBE  $(p_c^*, m_c^*, q_c^*, \theta^*(\cdot))$  exists.*
- (b) *There exists a threshold  $\underline{s}_c$ , such that if  $s \leq \underline{s}_c$ , we have  $(p_c^*, m_c^*) = \arg \max_{0 \leq p \leq v, m \geq 0} \Pi^c(p, m, q)$ , subject to  $q = \alpha(p, m) \bar{F}^{-1}\left(\frac{c}{p+m}\right)$  and  $\alpha(p, m) = \frac{1}{2} + \frac{(v-p-m)\theta^*(p+m)+m-(v-p_c^*-m_c^*)\theta^*(p_c^*+m_c^*)-m_c^*}{2s}$ . In equilibrium,  $q_c^* = \frac{1}{2} \bar{F}^{-1}\left(\frac{c}{p_c^*+m_c^*}\right)$ ,  $\theta_c^* = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_c^*+m_c^*}\right)} y f(y) dy$ , and each retailer covers half of the entire market with competition.*
- (c) *If  $\underline{s}_c < s \leq \bar{s}_c$  ( $\underline{s}_c < \bar{s}_c$ ), we have  $(p_c^*, m_c^*) = \arg \max_{0 \leq p \leq v, m \geq 0} \Pi^c(p, m, q)$ , subject to  $q = \frac{1}{2} \bar{F}^{-1}\left(\frac{c}{p+m}\right)$ ,  $\alpha = \frac{1}{2}$ , and  $p + m = v - \frac{s+2m}{2\theta^*(p+m)}$ . In this case, each retailer covers half of the entire market without competition.*
- (d) *If  $s > \bar{s}_c$ , we have  $(p_c^*, m_c^*) = \arg \max_{0 \leq p \leq v, m \geq 0} \Pi^c(p, m, q)$ , subject to  $q = \alpha(p, m) \bar{F}^{-1}\left(\frac{c}{p+m}\right)$  and  $\alpha(p, m) = \frac{(v-p-m)\theta^*(p+m)+m}{s}$ . In equilibrium,  $q_c^* = \alpha(p_c^* + m_c^*) \bar{F}^{-1}\left(\frac{c}{p_c^*+m_c^*}\right)$ ,*

$\theta_c^* = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p_c^* + m_c^*}\right)} y f(y) dy$ , and each retailer covers less than half of the entire market without competition.

Similar to Proposition 1, Proposition 3 shows the existence and uniqueness of PBE in the model with monetary compensation. Depending on the value of  $s$ , the retailers may cover the entire market or just a portion of it. If  $s$  is small ( $s \leq \underline{s}_c$ ), the retailers compete with each other and achieve full market coverage. Otherwise, the market may not be fully covered and there is essentially no market competition.

To examine the impact of the monetary compensation on the retailers' profit, we denote the equilibrium profit of  $R_i$  in this model with monetary compensation as  $\Pi_c^*$ . The following proposition shows that monetary compensation may hurt the retailers if market competition is intense.

**PROPOSITION 4.** *For the model with the monetary compensation option, the following statements hold:*

- (a) *There exists a critical threshold  $\bar{s}_{cd} < \bar{s}_c$ , such that if  $s < \bar{s}_{cd}$ , we have  $\Pi_c^* < \Pi_d^*$ .*
- (b) *Otherwise,  $s \geq \bar{s}_{cd}$ , we have  $\Pi_c^* \geq \Pi_d^*$ .*

Proposition 4 delivers an interesting message that, if the retailers have the option to offer monetary compensations upon stock-outs, they will earn a lower profit as long as the competition is sufficiently intense ( $s \leq \bar{s}_{cd}$ ). This is in contrast with the effect of monetary compensation in the monopoly setting, which always benefits the retailer (see Su and Zhang 2009). By offering a monetary compensation, the retailer, on one hand, is equipped with another lever in the competitive landscape; but, on the other hand, he competes more aggressively through direct subsidies to customers upon stock-outs. If  $s$  is large, the former effect dominates, which results in a higher profit in the presence of monetary compensation. If  $s$  is small, however, the latter effect dominates and monetary compensation leads to severe competition, which will in turn diminish the profit of each retailer. As a consequence, if the market competition is already fierce (i.e.,  $s$  is small), the monetary compensation option will further intensify the competition and hurt the retailers. In a similar spirit to the classical Hotelling model (see Lemma 3 in the Appendix C), the intensified competition induced by stock-out compensations drags the equilibrium effective margin ( $p_c^* + m_c^*$ ) down to the marginal cost  $c$  as  $s$  approaches zero. Hence, if the unit travel cost  $s$  is sufficiently small (i.e., the model proposed by Dana Jr 2001), both retailers may earn zero profit in the presence of the monetary compensation option.

In the existing literature, many studies have demonstrated that retailers can extract more profit by offering monetary compensation in a monopoly market. To convince customers of inventory availability, retailers adopt monetary compensation as a self-punishment mechanism upon stock-outs. With such a mechanism, customers will anticipate a high service level and increase their

willingness-to-pay, which in turn can boost the firm’s profit. For example, Su and Zhang (2009) show that monetary compensation can increase the retailer’s product availability in a monopoly model. Besides such a short-run effect, it is widely believed that the monetary compensation also has a long-run effect to expand a firm’s market share. Compensating customers upon stock-outs has a positive effect on customers’ shopping experience, and thus cultivates customer loyalty. In other words, by purposefully providing compensations for stock-outs, retailers have the potential to increase their demand in the long run (see, e.g., Bhargava et al. 2006). Albeit with all these benefits, our results (i.e., Proposition 4), nevertheless, deliver a new insight to our understanding of the monetary compensation strategy by demonstrating that this strategy may backfire and lead to profit losses for the retailers.

To conclude this section, we remark that offering monetary compensation may cause a free-rider issue. Specifically, customers who are not interested in purchasing the product may still visit the retailer with the hope of being compensated, as long as the travel cost is not too high. These customers are referred to as free-riders. The free-riding behavior creates a moral hazard issue, so that retailers can hardly recognize their true customers. Fortunately, many marketing strategies and new technology tools can be used to alleviate, or even eliminate, such free-riding issue. For example, retailers may ask customers to claim their desired product in order to be eligible for compensation upon stock-out. If the claimed product is out of stock and no substitute can match the customer’s need, then a monetary compensation is offered. Otherwise, the customers cannot receive the monetary compensation. Another mechanism the retailers can use is to illicit more information from customers through cheap talk. Once the retailer verifies a customer’s true motivation for purchasing the product, a monetary compensation can be awarded. Therefore, throughout our analysis, we assume that the free-riding behavior is negligible. This treatment is consistent with the business practice in various industries where retailers effectively compensates customers stock-outs to induce a higher demand. (see, e.g., Bhargava et al. 2006, Su and Zhang 2009)

## 6. Inventory Commitment

Inventory commitment is another commonly-used strategy to enhance a retailer’s competitive edge in the presence of availability-concerned customers. Under this strategy, the retailer credibly announces its order quantity to the market. For example, Amazon.com recently provided a lightning deal platform to allow retailers to promote their products. A salient feature of “lightning deals” is that sellers have to announce the amount of inventory to customers. In particular, a customer can see a real-time status bar on the webpage of the seller indicating the current price, inventory, and percentage of units that have already been claimed by other customers. In some other circumstances, a retailer has to publicize its inventory information to customers, even if he

is not willing to do so by himself. For instance, the affiliated stores of Great Clips (a hair salon franchise in the United States and Canada) must post their real-time information of available slots online. Customers can check the anticipated waiting times of all stores around and add their names to the waitlist before actually visiting the salon. In this case, the competing franchised stores are forced to reveal their available inventory information.

It has been shown in the literature that the inventory commitment strategy benefits the monopoly retailer (e.g., Su and Zhang 2009). In this section, we strive to analyze this strategy in a competitive market. Our results imply that the inventory commitment strategy may lead to an undesirable prisoner's dilemma: Although both retailers will voluntarily reveal their inventory information under equilibrium, the equilibrium profit of each retailer will be lower than in the base model where the retailers cannot credibly announce the order quantity information. Therefore, the inventory commitment strategy may not serve as an effective tool for retailers in a competitive market where customers are concerned about the inventory stock-out risk.

We now formally model the inventory commitment strategy in our duopoly market. We use subscript  $v$  to represent the model with inventory commitment. At the beginning of the sales horizon, the competing retailers first decide whether to reveal the inventory information to the public (i.e., whether to adopt the inventory commitment strategy). Then, the retailers will order inventory and announce the prices. If a retailer commits to publicizing its inventory information, he will truthfully announce its order quantity to the whole market. Finally, the customers observe the prices of the retailers and the amount of inventory ordered by the retailer who adopts the inventory commitment strategy, and decide which retailer to visit. As in the base model, we adopt the PBE framework to analyze the equilibrium market outcome. There are three cases to consider: (i) both retailers do not reveal the inventory order quantities, which is reduced to the base model; (ii) both retailers adopt the inventory commitment strategy; (iii) one retailer adopts the inventory commitment strategy whereas the other one does not reveal its inventory. Section 4 presents a detailed analysis for case (i). We now analyze cases (ii) and (iii).

**Both Retailers Adopt Inventory Commitment Strategy.** In the presence of inventory commitment, individual customers do not need to form beliefs about inventory availability, but directly optimize their purchasing decisions after observing both prices and inventory stocking quantities. Specifically, after observing  $R_i$ 's price  $p_i$  and stocking quantity  $q_i$ , customers estimate the in-stock probability of each retailer conditioned on her existence. Similar to the base model, there exists a threshold  $\underline{x}_v$  (resp.  $\bar{x}_v$ ) such that customers with locations  $x \leq \underline{x}_v$  (resp.  $x > \bar{x}_v$ ) will visit  $R_1$  (resp.  $R_2$ ). It suffices to characterize the perceived inventory availability probabilities at the purchasing thresholds  $\underline{x}_v$  and  $\bar{x}_v$ . We define  $\theta_1^v$  as the perceived inventory availability probability

for a customer located at  $\underline{x}_v$  to visit  $R_1$ , and  $\theta_2^v$  as the perceived inventory availability probability for a customer located at  $\bar{x}_v$  to visit  $R_2$ . Then, we have  $\theta_i^v = \frac{1}{\mu} \int_{y=0}^{q_i/\alpha_i^v} yf(y)dy$ , where  $\alpha_1^v := \underline{x}_v$  is the market share of  $R_1$  and  $\alpha_2^v := 1 - \bar{x}_v$  is the market share of  $R_2$ . Denote retailer  $R_i$ 's profit as  $\Pi_v(p_i, q_i) = p_i \mathbb{E}[\alpha_i(p_i, q_i)D \wedge q_i] - cq_i$ . As in the base model, we focus on the symmetric equilibrium  $(p_v^*, q_v^*, \alpha_v^*)$ , where  $p_v^*$  is the equilibrium price,  $q_v^*$  is the equilibrium order quantity, and  $\alpha_v^*$  is the equilibrium market share.

Similar to the base model, if  $s$  is large, the market is segmented and  $\alpha_v^* = \frac{(v-p_v^*)\theta_v^*}{s}$ . If  $s$  is small, the market is fully covered and the retailers compete on price and order quantity to win market share, so the market share of retailer  $R_i$ ,  $\alpha_i^v$ , is the unique solution to the following equation (of  $\alpha$ ):  $\frac{v-p_i}{\mu} \int_{y=0}^{q_i/\alpha} yf(y)dy - s\alpha = \frac{v-p_v^*}{\mu} \int_{y=0}^{q_v^*/(1-\alpha)} yf(y)dy - s(1-\alpha)$ . If  $s$  is moderate, the market is fully covered but the retailers do not compete with each other, so  $\alpha_v^* = 0.5$ . Summarizing the above three cases, the following proposition characterizes the equilibrium outcome if both retailers commit to revealing their inventory information to the market.

**PROPOSITION 5.** *If both retailers adopt the inventory commitment strategy, the following statements hold:*

- (a) *A unique symmetric PBE  $(p_v^*, q_v^*, \alpha_v^*)$  exists.*
- (b) *There exists a threshold  $\underline{s}_v$ , such that if  $s \leq \underline{s}_v$ , we have  $(p_v^*, q_v^*) = \arg \max_{0 \leq p \leq v, q \geq 0} \Pi_v(p, q)$  subject to the constraint  $\frac{v-p}{\mu} \int_{y=0}^{q_i/\alpha(p,q)} yf(y)dy - s\alpha(p, q) = \frac{v-p_v^*}{\mu} \int_{y=0}^{q_v^*/(1-\alpha(p,q))} yf(y)dy - s(1-\alpha(p, q))$ . In this case, each retailer covers half of the entire market and competes with each other.*
- (c) *There exists another threshold  $\bar{s}_v > \underline{s}_v$ , such that if  $\underline{s}_v < s \leq \bar{s}_v$ , we have  $(p_v^*, q_v^*) = \arg \max_{0 \leq p \leq v, q \geq 0} \Pi_v(p, q)$  subject to the constraints  $\frac{v-p}{s\mu} \int_{y=0}^{2q} yf(y)dy = \frac{1}{2}$  and  $\alpha(p, q) = \frac{1}{2}$ . In this case, each retailer covers half of the entire market without competition.*
- (c) *If  $s > \bar{s}_v$ , we have  $(p_v^*, q_v^*) = \arg \max_{0 \leq p \leq v, q \geq 0} \Pi_v(p, q)$  where  $\alpha(p, q) = \frac{v-p}{s\mu} \int_{y=0}^{q/\alpha} yf(y)dy$ . In this case, each retailer covers  $\alpha_v^* = \frac{v-p_v^*}{s\mu} \int_{y=0}^{q_v^*/\alpha_v^*} yf(y)dy < \frac{1}{2}$  of the entire market, so the market is partially covered and there is no competition between the retailers.*

Proposition 5 implies that the equilibrium outcome of the scenario where both retailers adopt the inventory commitment strategy shares the same structure as the base model. Specifically, Proposition 5(b) refers to the case with a low travel cost  $s$ , which results in full market coverage with competition, with each retailer serving half of the market and competing with each other. Proposition 5(c) refers to the case with complete market coverage without competition if the travel cost is moderate. Finally, the last part of Proposition 5 refers to the case with partial market coverage when the travel cost is high. Clearly, there is no competition in this case.

**Incentive for Inventory Commitment** In this subsection, we demonstrate that, regardless of whether the competitor adopts the inventory commitment strategy and the competitor's price and order quantity, the focal retailer will earn a higher profit by credibly revealing its inventory information to customers. This implies that inventory commitment is a dominating strategy for the retailers. As a consequence, the equilibrium outcome of the market with the inventory commitment option will be that both retailers voluntarily publicize their inventory order quantity, charge the price  $p_v^*$ , and order  $q_v^*$  units of inventory as prescribed in Proposition 5.

Assume  $R_1$  is the focal retailer, we shall consider both the case where  $R_2$  credibly announces  $q_2$  and the case where  $R_2$  does not reveal his order quantity. Given  $R_2$ 's price and inventory decision  $(p_2, q_2)$ , we use  $\Pi_{i,j}$  ( $i, j \in \{d, v\}$ ) to denote the maximum profit of  $R_1$  if he adopts strategy  $i$  and  $R_2$  adopts strategy  $j$ , where strategy  $d$  refers to no inventory commitment and strategy  $v$  refers to inventory commitment. For example,  $\Pi_{d,v}$  refers to the maximum profit of  $R_1$  if he does not adopt the inventory commitment strategy and  $R_2$  adopts this strategy. The calculations of  $\Pi_{i,j}$  ( $i, j \in \{v, d\}$ ) are given in the Appendix.

**PROPOSITION 6.** *For any  $(p_2, q_2)$  set by  $R_2$ , we have that  $\Pi_{v,d} > \Pi_{d,d}$  and  $\Pi_{v,v} > \Pi_{d,v}$ .*

An important implication of Proposition 6 is that, if the competing retailers can credibly reveal their inventory information to the market, adopting the inventory commitment strategy would be a *dominating* strategy for each of the retailers, regardless of the price and inventory decisions of the competitor. Therefore, the equilibrium outcome of the market under the inventory commitment *option* is that both retailers voluntarily reveal their inventory order quantity.

Our next result examines the profit implication of the inventory commitment strategy under Hotelling competition. We use  $\Pi_d^*$  (resp.  $\Pi_v^*$ ) to denote the equilibrium profit of a retailer without (resp. with) the inventory commitment strategy.

**PROPOSITION 7.** *If the retailers have the option to credibly announce their inventory information, the following statements hold:*

- (a) *Under equilibrium, both  $R_1$  and  $R_2$  adopt the inventory commitment strategy.*
- (b) *There exist a threshold  $\bar{s}_{vd}$  for the unit travel cost and a threshold  $\bar{c}_{vd}$  for the unit production cost. If  $s < \bar{s}_{vd}$  and  $c < \bar{c}_{vd}$ , then we have  $\Pi_v^* < \Pi_d^*$ . Otherwise,  $s \geq \bar{s}_{vd}$  or  $c \geq \bar{c}_{vd}$ , we have  $\Pi_v^* \geq \Pi_d^*$ .*

As shown in Proposition 7, if both the unit travel cost  $s$  and the ordering cost  $c$  are low (i.e.,  $s < \bar{s}_{vd}$  and  $c < \bar{c}_{vd}$ ), and if the inventory commitment strategy is adopted, the inventory stocking quantity can directly influence the purchasing behaviors of the customers. Therefore, the competition between retailers may be intensified by this strategy. The retailers may over-commit to inventory in a competitive market, thus reducing the profit of each retailer. Recall that in our base

model, the stocking quantity is not observable by customers but signaled by price, so the only competitive leverage of a retailer is the prevailing price he charges. However, if the retailers can commit to their pre-announced inventory order quantities, they have more flexibility to influence demand. Furthermore, the signaling power of price is diluted if the inventory information is directly available to customers. In particular, when the unit cost  $c$  is high, the inventory commitment strategy helps the retailers increase the willingness-to-pay of the customers, thus attracting higher demand. On the other hand, if the unit cost is low, this strategy may backfire by triggering an over-commitment of stocking quantity. If, in addition, the market competition is intensive (i.e.,  $s < \bar{s}_{vd}$ ), each retailer will aggressively order a large amount of inventory to attract customers, which in turn exacerbates market competition and decreases the profits of both retailers ( $\Pi_v^* < \Pi_d^*$  when  $c < \bar{c}_{vd}$  and  $s < \bar{s}_{vd}$ ). Therefore, whenever the cost  $c$  and customer travel cost  $s$  are both low, the retailers are actually worse off in the presence of the inventory commitment option due to the induced inventory over-commitment and intensified market competition. Proposition 6 and Proposition 7 together deliver a new and interesting insight that inventory commitment strategy may give rise to a prisoner's dilemma under market competition. Although this strategy is preferred by either retailer regardless of the competitor's inventory and price decisions, the retailers would be worse off if both of them adopt the inventory commitment strategy.

Our analysis demonstrates that the inventory commitment strategy is not always beneficial to the retailers under competition, which is in sharp contrast to the monopoly setting. There is a large body of research in the literature focusing on the inventory commitment strategy. A central message in this literature is that the inventory commitment strategy is beneficial for retailers. For example, Cachon and Swinney (2011) and Liu and Van Ryzin (2008) propose two-stage models to explore how to use availability information to manipulate customers' expectations and thus induce them to buy early. In a competitive market setting, revealing inventory information to customers may lead to a higher equilibrium price, and, as a result, improves the firms' profits (see Dana 2001, Carlton 1978, and Dana and Petruzzi 2001). In a supply chain setting, Su and Zhang (2008) demonstrate that the firm's profit can be improved by promising either that the available inventory will be limited (quantity commitment) or that the price will be kept high (price commitment). In a monopoly setting, Su and Zhang (2009) further shows that the inventory commitment strategy offers customers information to more accurately assess their chances of securing the product. Thus, the inventory commitment strategy increases customers' willingness-to-pay and improves the profit of a monopoly firm. In a Hotelling competition setting, however, our results demonstrate that the inventory commitment strategy may give rise to a prisoner's dilemma and hurt the retailers.

We complement our theoretical analysis with numerical experiments to further illustrate the effect of inventory commitment. We compare the equilibrium profits and stocking quantities in the



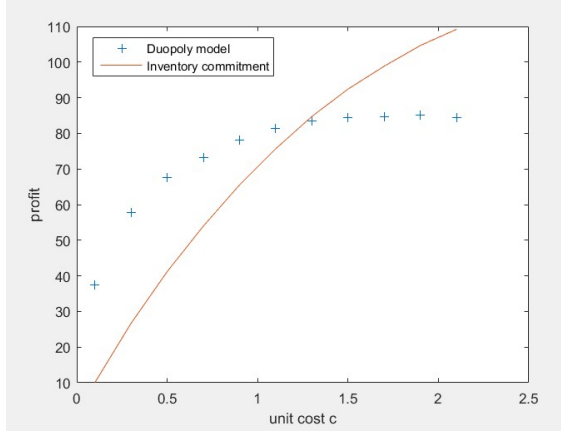


Figure 3 Equilibrium profits.

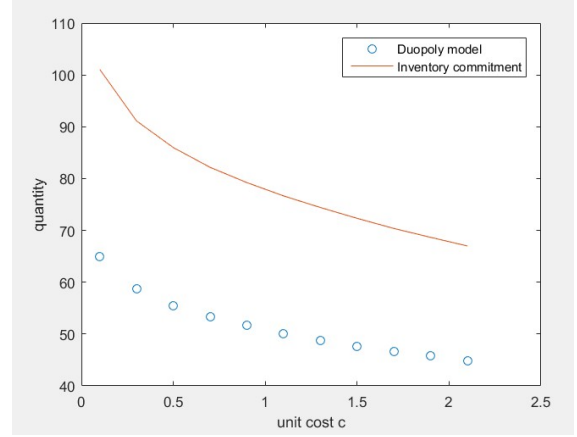


Figure 4 Equilibrium stocking quantity.

base model and the model with inventory commitment. In our numerical experiments, we set  $s = 0.1$ ,  $v = 10$ , and the market demand  $D$  to follow a Gamma distribution with mean 90 and standard deviation 30. Figures 3 and 4 plot the equilibrium profits and order quantities, respectively, for the base model and the model with inventory commitment. Figure 3 clearly shows that the equilibrium profit of a retailer will be lower in the presence of inventory commitment whenever the ordering cost  $c$  is low. Figure 4 further demonstrates that, with inventory commitment, the retailers will order much more than they would have without revealing the inventory information to the market.

To summarize, although monetary compensation and inventory commitment have been widely recognized to benefit a monopolist retailer facing customers with inventory availability concerns, our results demonstrate that these strategies may hurt retailers in a competitive market if the competition is intensive. In addition to the well-known effect that monetary compensation and inventory commitment could increase customers' willingness-to-pay and attract more demand, these two strategies may backfire by intensifying market competition. Our results deliver an important actionable insight that retailers should carefully evaluate their position in a competitive market before adopting these strategies to attract customers with availability concerns. Haphazardly employing monetary compensation or inventory commitment strategies may result in undesirable outcomes with lower profits for retailers under competition. Finally, we remark that, comparing the equilibrium outcomes of monetary compensation and inventory availability strategies is also interesting and we leave it as a direction for future research.

## 7. Customer Surplus

In this section, we take the customers' perspective to study the implications of inventory availability competition. More specifically, we seek to understand how competition between retailers will affect the *average* surplus of the customers in the market. A well-acknowledged insight in the economics literature is that competition will benefit customers (i.e., the average customer surplus

will increase in the presence of competition). Our results in this section, however, deliver an insight that contradicts this common wisdom: When customers are concerned about inventory availability, competition may hurt them when there is a moderate competition intensity.

We begin our analysis by quantifying the average customer surplus in different models, starting with the base (duopoly) model. Note that, in this model, the equilibrium outcome can be either whole market coverage or partial market coverage. We only consider the whole market coverage case (i.e.,  $s \leq \bar{s}$ ) in this section, since the partial market coverage case is trivial (the equilibrium average customer surplus of the two models is the same). In equilibrium, the customer *average* surplus is  $CS_d^* = \frac{(v-p_d^*)\mathbb{E}(D \wedge 2q_d^*)}{\mu} - \frac{s}{4}$ , where  $p_d^*$  is the equilibrium price and  $q_d^* = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p_d^*}\right)$  is the equilibrium inventory level. Analogously, the average customer surplus in the monopoly model is  $CS_m^* = \frac{(v-p_m^*)\mathbb{E}(D \wedge \frac{q_m^*}{\alpha_m})}{\mu} - \frac{s}{2}\alpha_m$ , where  $p_m^*$  and  $q_m^*$  are defined in Appendix B, and  $\alpha_m = \frac{(v-p_m^*)\theta^*(p_m^*)}{s}$  is the equilibrium market share of the retailer in the monopoly model.

How competition affects customer surplus is a well-studied problem in the economics literature. A general insight in this literature is that competition will improve customer surplus. For example, Brynjolfsson et al. (2003) summarize two channels of how market competition in product variety improves consumer surplus. Increased market competition improves consumer surplus by leading to lower average prices. In addition, increased market competition provides more products for sale, which allows consumers to maximize individual utility. In our setting of retailer competition on inventory availability, however, Proposition 2(c) demonstrates that competition may lead to higher equilibrium prices. This will further imply that, in contrast to the widely believed notion that competition enhances customer surplus, market competition may induce a lower average customer surplus, as shown in the next proposition.

**PROPOSITION 8.** *The following statements hold:*

- (a) *There exists a threshold  $\bar{s}_w > \bar{s}_d$ , such that  $CS_d^* \geq CS_m^*$  if  $s \leq \bar{s}_w$ .*
- (b) *If  $\bar{s}_w < s < \bar{s}$ , we have  $CS_d^* < CS_m^*$ .*
- (c) *If  $s \geq \bar{s}$ , we have  $CS_d^* = CS_m^*$ .*

The most interesting implication of Proposition 8 is that, if the travel cost is moderate (i.e.,  $\bar{s}_w < s < \bar{s}$ ), market competition actually hurts customers by decreasing the average customer surplus. From the customers' perspective, there are two effects associated with a competitive market. The first is a travel cost effect, under which the average travel cost of the customers is lower if market competition exists ( $\frac{s}{4} < \frac{s}{2}\alpha_m$  when  $\alpha_m > 0.5$ , which is the case where competition is prevalent in the duopoly model). Thus, the travel cost effect benefits the customers. The second is the pricing effect, under which competition drives the retailers to charge higher prices if the travel cost  $s$  is moderate (see also Proposition 2). The pricing effect hurts the customers. Proposition 8(b) demonstrates the

latter effect dominates when the travel cost is moderate. As a result, moderate market competition ( $\bar{s}_w < s < \bar{s}$ ) would reduce the average customer surplus.

In Sections 5 and 6, we study the strategies of monetary compensation and inventory commitment from the perspective of the retailers. We now shift our attention to explore whether these two strategies can be used to improve the *average* consumer surplus in a competitive market. The equilibrium *average* consumer surplus under the monetary compensation strategy is  $CS_c^* = \frac{(v - p_c^* - m_c^*)\mathbb{E}(D \wedge 2q_c^*)}{\mu} - \frac{s}{4} + m_c^*$ , where  $p_c^*$  is the equilibrium price,  $q_c^*$  is the equilibrium order quantity, and  $m_c^*$  is the equilibrium compensation rate in the presence of monetary compensation strategy (see Proposition 3). Similarly, in the presence of the inventory commitment strategy, the *average* consumer surplus is given by  $CS_v^* = (v - p_v^*)\frac{\mathbb{E}(D \wedge 2q_v^*)}{\mu} - \frac{s}{4}$ , where  $p_v^*$  and  $q_v^*$  are the equilibrium inventory level in the presence of inventory commitment.

**PROPOSITION 9.** *Under the monetary compensation and inventory commitment strategies, we have that (a)  $CS_c^* \geq CS_d^*$ , and (b)  $CS_v^* \geq CS_d^*$ .*

Proposition 9 shows that, although monetary compensation and inventory commitment do not necessarily benefit the retailers under competition, these strategies are always beneficial to the customers. Both strategies provide incentives to attract customers to patronize the retailers and, as a consequence, benefit the customers once adopted by the retailers.

In summary, our analysis in this section shows that, contrary to the widely-held belief, competition may decrease the average customer surplus when customers have inventory availability concerns. This is because competition will make retailers charge higher prices to signal inventory availability, leading to lower average customer surplus. On the other hand, monetary compensation and inventory commitment strategies provide incentives for customers to patronize the retailers, thus improving the average customer surplus.

## 8. Conclusion

In this paper we study two retailers competing on price and inventory availability. Customers do not observe the inventory status of the retailers, but can form beliefs about product availability based on the price vector observed. The retailers sell substitutable products and are located at the endpoints of a Hotelling line market. Using the perfect Bayesian equilibrium paradigm, we characterize the strategic interactions among the retailers and the customers. By comparing such a duopoly model to a monopoly benchmark, we investigate the implication of availability competition on market outcome, firm profitability, and average consumer surplus.

There are several main results from this research. First, we find that, depending on competition intensity, market competition may give rise to either higher or lower equilibrium price and inventory availability compared with those in the monopoly setting. This is because, when the competition

is intense (resp. moderate), firms mainly compete on price (resp. product availability), and they use low prices (resp. high inventory availability) to attract customers.

Second, monetary compensation and inventory commitment strategies have been proposed in the literature to mitigate strategic customer behavior and enhance firm profit in a monopoly setting. However, in a competitive setting, we find that both strategies lead to a prisoner's dilemma: Although a retailer would benefit either strategy regardless of the competitor's price and inventory decisions, both monetary compensation and inventory commitment will intensify market competition and hurt the retailers in a competitive market. Specifically, the monetary compensation strategy tends to overcompensate customers, while the inventory commitment strategy may dilute the signaling power of price, thus leading to over-stock of inventory for the competing retailers.

Third, it is widely believed that competition normally improves the customer surplus. In contrast, we find that with customers' product availability concerns, competition may increase equilibrium retail prices, which decreases the average customer surplus. On the other hand, monetary compensation and inventory commitment can incentivize the customers to visit both retailers, thus enhancing the average customer surplus in a competitive market.

This research can be extended in several directions. First, it has been assumed in this paper that a customer will leave the market if she experiences a stock-out at the chosen retailer. An alternative assumption is that the customer will try the other retailer after experiencing a stock-out. How would the results change after incorporating such dynamic customer choice is an open question. Second, this paper considers a static price setting where the retailer cannot change price over time. It would be interesting to study how firms should manage inventory availability in a competitive marketplace with dynamic pricing. Finally, omni-channel retailing has attracted substantial attention recently. A promising direction for future research is to investigate the role of inventory availability competition in omni-channel settings.

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## Appendices to “Competition on Inventory Availability and Price under Customer Heterogeneity”

### Appendix A: Summary of Notations

**Table 1** Summary of Notations

$R_i$	Retailer $i$ ( $i = 1, 2$ )
$p_i$	Price of Retailer $i$
$q_i$	Stocking quantity of Retailer $i$
$\alpha_i$	Market share of Retailer $i$
$\Pi_i$	Profit of Retailer $i$
$D$	Market aggregate demand
$s$	Unit travel cost
$c$	Procurement cost
$v$	Product valuation
$F(\cdot)$	Distribution function of demand ( $\bar{F}(x) := 1 - F(x)$ )
$f(\cdot)$	Density function demand distribution
$x$	Consumer's location on the Hotelling line, $x \in \mathcal{M}$ and $\mathcal{M} = [0, 1]$
$\mathbb{E}[\cdot]$	Taking expectation
$x \wedge y$	Taking the minimum
$\theta_i$	Customers' (rational) expectation of $R_i$ 's inventory availability probability ( $i = 1, 2$ )

### Appendix B: Equilibrium in Monopoly Benchmark

In this section, we characterize the PBE in the monopoly benchmark. Observe that the purchasing decisions of the customers will be governed by a threshold policy, under which a customer will visit the monopoly retailer if and only if she is located below a threshold  $\underline{x}_m(p)$ , where  $p$  is the price charged by the retailer. In a similar spirit to the base model, we assume that efficient rationing is adopted when demand exceeds supply for the retailer. Thus, customers located at the purchasing threshold  $\underline{x}_m(p)$  hold the equilibrium belief  $\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_i})} y f(y) dy$  about the retailer's inventory availability probability. This implies that a customer located at  $x$  will purchase the product from  $R_1$  if and only if  $x \leq \underline{x}_m(p) := \min\{\frac{(v-p)\theta_m(p)}{s}, 1\}$ , and the retailer anticipates his market share  $\alpha_m = \min\{\frac{(v-p)\theta_m(p)}{s}, 1\}$  given the price he charges,  $p$ . Therefore, the equilibrium price in the monopoly benchmark,  $p_m^*$ , can be characterized by the following:

$$p_m^* = \arg \max_{0 \leq p \leq v} \left\{ \min \left( \frac{(v-p)\theta^*(p)}{s}, 1 \right) \right\} \left\{ p \mathbb{E} \left[ D \wedge \bar{F}^{-1} \left( \frac{c}{p} \right) \right] - c \bar{F}^{-1} \left( \frac{c}{p} \right) \right\},$$

subject to the constraint

$$\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p})} y f(y) dy.$$

Similar to Proposition 1, we characterize the market equilibrium for the monopoly benchmark.

PROPOSITION 10. (a) A unique PBE  $(p_m^*, q_m^*, \theta^*(\cdot))$  exists in the monopoly benchmark model.

(b) There exists a critical threshold  $\bar{s}_m$  such that if  $s \leq \bar{s}_m$ ,

$$p_m^* = \arg \max_{(v-p)\theta^*(p) \geq s} \left\{ p \mathbb{E} \left[ D \wedge \bar{F}^{-1} \left( \frac{c}{p} \right) \right] - c \bar{F}^{-1} \left( \frac{c}{p} \right) \right\},$$

$q_m^* = \bar{F}^{-1} \left( \frac{c}{p_m^*} \right)$ , and  $\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p})} y f(y) dy$ . In this case, the entire market is covered by  $R_1$ .



(c) If  $s > \bar{s}_m$ , we have

$$p_m^* = \arg \max_{0 \leq p \leq v} \frac{(v-p)\theta^*(p)}{s} \left\{ p \mathbb{E} \left[ D \wedge \bar{F}^{-1} \left( \frac{c}{p} \right) \right] - c \bar{F}^{-1} \left( \frac{c}{p} \right) \right\},$$

$q_m^* = \frac{(v-p_m^*)\theta^*(p_m^*)}{s} \bar{F}^{-1} \left( \frac{c}{p_m^*} \right)$ , and  $\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p})} y f(y) dy$ . In this case, the retailer covers  $\frac{(v-p_m^*)\theta^*(p_m^*)}{s} < 1$  of the entire market, so the market is partially covered by  $R_1$ .

With a small transportation cost ( $s \leq \bar{s}_m$ ), a monopoly retailer can cover the entire market  $\mathcal{M}$ . In this case, the monopoly benchmark is reduced to the model studied in Su and Zhang (2009), which does not incorporate customer differentiation in location. If the transportation cost is large ( $s > \bar{s}_m$ ), the market share of the retailer is less than 100%. A comparison of Proposition 1(d) and Proposition 10(c) implies that when the travel cost  $s$  is sufficiently large, the equilibrium of the base model is identical to the monopoly benchmark, i.e.,  $(p_d^*, q_d^*) = (p_m^*, q_m^*)$ .

### Appendix C: Deterministic Hotelling Model Benchmark

In this section, we introduce the classic Hotelling competition model with deterministic demand as the benchmark. The comparison between our focal model and the deterministic benchmark could help us crystallize the impact of demand uncertainty and customers' availability concern.

We consider the same Hotelling line setup as the base model presented in Section 3 but with deterministic total market size. Specifically, we assume the aggregate market demand  $D$  is deterministic and known to everyone in the market. Without loss of generality, we normalize  $D = 1$ . In the absence of demand uncertainty, the retailers will order exactly the amount of their respective market share, so every customer will be able to obtain her requested product. The two retailers  $R_1$  and  $R_2$  determine their respective prices  $p_1$  and  $p_2$  to maximize their own profits, whereas each retailer choose whether and where to visit. As in the base model, we focus on the symmetric equilibrium  $(\tilde{p}_d^*, \tilde{q}_d^*)$ , where  $\tilde{p}_d^*$  is the equilibrium price and  $\tilde{q}_d^*$  is the equilibrium order quantity of each retailer. Here, we use “~” to represent the deterministic benchmark model. It is then straightforward to observe that if  $s$  is small,  $R_1$  and  $R_2$  can serve the entire market, each covering 50% of the customers. If, otherwise,  $s$  is large, there is essentially no competition between the two retailers and the market is not completely covered. Formally, we characterize the equilibrium prices of the deterministic benchmark in the following lemma.

LEMMA 3. Assume that  $D = 1$  with certainty. The following statements hold:

- (a) If  $s < \frac{2(v-c)}{3}$ ,  $\tilde{p}_d^* = s + c$  and  $\tilde{q}_d^* = \frac{1}{2}$ . Hence,  $R_1$  and  $R_2$  each cover 50% of the market.
- (b) If  $\frac{2(v-c)}{3} \leq s \leq v - c$ ,  $\tilde{p}_d^* = v - \frac{1}{2}s$  and  $\tilde{q}_d^* = \frac{1}{2}$ . Hence,  $R_1$  and  $R_2$  each cover 50% of the market.
- (c) If  $s > v - c$ ,  $\tilde{p}_d^* = \frac{1}{2}(v + c)$  and  $\tilde{q}_d^* = \frac{1}{2s}(v - c) < \frac{1}{2}$ . Hence,  $R_1$  and  $R_2$  each cover less than 50% of the market.

As shown in Lemma 3, when the competition is intensive ( $s < \frac{2(v-c)}{3}$ ), the equilibrium price is increasing in  $s$ . If the competition is moderate ( $\frac{2(v-c)}{3} \leq s \leq v - c$ ), the equilibrium price is decreasing in  $s$ . Finally, if the competition is mild ( $s > v - c$ ), the equilibrium price is independent of  $s$ .

## Appendix D: Random Rationing

In our base model, we have assumed efficient rationing of inventory upon stock-out. As discussed earlier, we make this assumption to capture the effect that customers closer to the focal retailer will visit the retailer earlier and will therefore be served with a higher priority. In some circumstances, inventory will be uniformly randomly allocated to customers if demand exceeds supply, which we refer to as random rationing. We conclude this section by discussing how random rationing would affect our results. Under random rationing, all customers hold the same belief on the availability probability. Similar to the case of efficient rationing, under random rationing, customers form belief on inventory availability based on the observed prices  $(p_1, p_2)$ . Specifically, based on the same argument as Su and Zhang (2009), the belief of  $R_i$ 's product availability for any customer is:

$$\theta_i(p_1, p_2) = \int_0^{+\infty} \left( \frac{y \wedge Q_i(p_1, p_2)}{y} \right) g(y|p_1, p_2) dy = \frac{1}{\mu} \mathbb{E} \left[ \bar{F}^{-1} \left( \frac{c}{p_i} \right) \wedge D \right],$$

where the last equality follows from that the retailer will adopt the newsvendor order quantity that maximizes its expected profit. Hence, the inventory availability probability only depends on the price of the focal retailer, but not that of the other one, which we denote as  $\theta_i(p_i)$ . It can be checked that  $\theta_i(p_i)$  is concavely increasing in price  $p_i$  (see Dana Jr 2001), so is the customer,  $\mathcal{U}_i(p_i) = (v - p)\theta(p_i) - sx$ . This will also restore the conclusion of Lemma 2 (equivalently, Figure 1) and, consequently, the existence and uniqueness of equilibrium (i.e., Proposition 1).

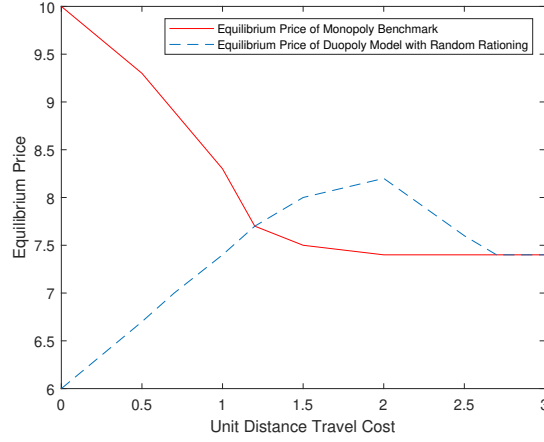
If the retailers adopt the random rationing rule, customers at different locations form identical beliefs of product availability. The following Proposition characterizes the impact of competition under random rationing.

**PROPOSITION 11.** *If the customers belief of product availability follows the rule of random rationing, there exists a threshold  $\tilde{s}_d$  ( $\tilde{s}_d \leq \bar{s}$ ), such that (i)  $p_d^* \leq p_m^*$  and  $\theta^*(p_d^*) \leq \theta^*(p_m^*)$  for  $s \leq \tilde{s}_d$ ; (ii)  $p_d^* > p_m^*$  and  $\theta^*(p_d^*) > \theta^*(p_m^*)$  for  $\tilde{s}_d < s < \bar{s}$ ; and (iii)  $p_d^* = p_m^*$  and  $\theta^*(p_d^*) = \theta^*(p_m^*)$  for  $s \geq \bar{s}$ .*

Proposition 11 shows that the impact of competition shares the same qualitative structure under random rationing as under efficient rationing. Intensive competition will decrease the equilibrium price whereas moderate competition will raise the price. We also present a numerical example to illustrate this insight (see Figure 5).

## Appendix E: Proof of Statements

**Proof of lemma 1** For a customer located at  $x \in [0, \underline{x})$  and desires to visit  $R_1$ , the efficient rationing scheme implies that  $\theta(\vec{p}, x) > \theta^*(p)$ , since a shorter distance to the focal retailer implies a higher probability of inventory availability. Similarly, if  $x \in (\underline{x}, 1]$ , we have  $\theta(\vec{p}, x) < \theta^*(p)$ , since a longer distance to the focal retailer implies a lower probability of inventory availability. Therefore, (1) on one side, for  $x \in [0, \underline{x})$ ,  $\mathcal{U}(\vec{p}, \theta) = (v - p)\theta(\vec{p}, x) - sx > (v - p)\theta^*(p) - s\underline{x} = 0$ . In other words, the customers earn positive payoff and they will visit the retailer  $R_1$ . (2) On the other side, for  $x \in (\underline{x}, 1]$ ,  $\mathcal{U}(\vec{p}, x) = (v - p)\theta(\vec{p}, x) - sx < (v - p)\theta^*(p) - s\underline{x} = 0$ . In other words, the customers earn a negative payoff and they won't visit the retailer  $R$ . Summarizing the above two conditions, using the threshold  $\underline{x}$  as a reference point, if a customer is closer


 Figure 5 Random Rationing ( $v = 10$ ,  $c = 3$ )

to  $R_1$  than the threshold, she will definitely visit the retailer; while if a customer is farther away from  $R_1$  than the threshold, she will definitely not visit the retailer. That is to say, if a customer holds the same belief of product availability as that under the purchasing threshold, i.e.,  $\theta^*(p)$ , her purchasing behavior will remain the same as that under the “true” belief of product availability. The analysis of  $R_2$  is similar to the above analysis.  $\square$

**Proof of Lemma 2.** For a customer located at  $x$ , her expected payoff is  $(v - p_i)\theta_i^*(p) - sx$  if she chooses to buy from retailer  $R_i$ . Under the PBE equilibrium, a customer’s expected inventory availability probability coincides with the actual product availability induced by the retailer’s inventory stocking decision. Recall that  $\theta_i^*(p_i) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_i})} yf(y)dy$ . Hence, a customer located at  $x$  has an expected payoff of  $U(p_i) := (v - p_i) \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_i})} yf(y)dy - sx$ . We have

$$U'(p_i) = -\frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_i})} yf(y)dy + \frac{c(v - p_i)\bar{F}^{-1}(\frac{c}{p_i})}{\mu p_i^2 f(\frac{c}{p_i})},$$

which is strictly decreasing in  $p_i$  given that  $D$  follows a distribution with increasing failure rate. By setting  $U'(p) = 0$ , the expected payoff  $U(p)$  is maximized at  $p = \hat{p}$ , where  $\hat{p} = \arg \max_p (v - p)\theta^*(p)$ .

Before proving that the expected payoff function is concave, we first show that the equilibrium price falls into the interval  $[\hat{p}, v)$ . First, we prove that  $\Pi(\hat{p}) > \Pi(p')$  for any  $p' < \hat{p}$ . Without loss of generality, we use retailer  $R_1$  for illustration. Suppose retailer  $R_1$  decreases its price from  $\hat{p}$  to  $p'$ , then some customers will not visit the retailer, because the expected payoff of the customers is maximized at the price  $p = \hat{p}$ . As a result, by decreasing price from  $\hat{p}$  to  $p'$ , the retailer will induce a lower demand and a strictly lower profit margin. This implies that  $\Pi(\hat{p}) > \Pi(p')$ . Thus, the retailer must charge a price higher than  $p \geq \hat{p}$ . Next, we show that the equilibrium price cannot exceed  $v$ . If the price is greater than or equal to the product valuation, i.e.,  $p \geq v$ , no customer can afford the product, which further implies that the demand is zero and the retailer earns zero profit. Therefore, the retailer’s optimal price must be within the range of  $[\hat{p}, v)$ . This completes proof of Lemma 2.  $\square$

**Proof of Lemma 3.** In a deterministic Hotelling model, two retailers compete on market share by charging prices. Demand is determined and is an open knowledge to all players in the market, so there is no issue of product availability. Without loss of generality, we shall use retailer  $R_1$  as an example in the analysis.

*Case I. Full Market Coverage with competition.* When the travel cost  $s$  is small, the two retailers cover the entire market. Given price  $p_1$  and  $p_2$ , consumers located at  $x \in [0, 1]$  visit the retailer  $R_1$  if  $v - p_1 - sx \geq v - p_2 - s(1 - x) > 0$ . Thus, the retailer  $R_1$  earns market share  $\frac{-p_1 + p_2 + s}{2s}$ , and accordingly profit  $\pi_1(p_1) = (p_1 - c) \frac{-p_1 + p_2 + s}{2s}$ . Taking first derivative of the profit function yields  $p_1^* = \frac{p_2 + s + c}{2}$ . Since the two retailers are symmetric, retailer  $R_2$  asks the same optimal price  $p_2^* = \frac{p_1 + s + c}{2}$  to maximize his own profit. In equilibrium, the two retailers have the same optimal solutions:  $p_d^* = s + c$ ,  $q_d^* = \frac{1}{2}$ , and each covers half market share. Finally, to guarantee  $v - p_d^* - \frac{s}{2} > 0$ , we obtain  $s < \frac{2(v-c)}{3}$ .

*Case II. Partial Market Coverage without competition.* When the travel cost  $s$  is large, each retailer covers less than half market share. Given price  $p_1$ , consumers visit the retailer  $R - 1$  if  $v - p_1 - sx > 0$ . In other words, the retailer earns market share  $\frac{v - p_1}{s}$  and profit  $\pi_1(p_1) = (p_1 - c) \frac{v - p_1}{s}$ . Taking first derivative of the profit function yields  $p_1^* = \frac{v + c}{2}$ . Similarly, by symmetry of the game, we obtain  $p_2^* = \frac{v + c}{2}$ . Thus, we have  $p_d^* = \frac{v + c}{2}$ ,  $q_d^* = \frac{v - c}{2s}$ , and each retailer covers market share  $\frac{v - c}{2s}$ . Finally, to guarantee that equilibrium market share is less than half, we obtain  $s > v - c$ .

*Case III. Full Market Coverage without competition.* To make our analysis complete, we still need to analyze the case when  $v - c \leq s \leq \frac{2(v-c)}{3}$ . In this case, each retailer charges a price to cover just half market share (consumers in the middle of the Hotelling line earn zero surplus from either retailer, which ensures zero market competition), so we have  $v - p_1 - \frac{s}{2} = 0$ . Thus, we obtain  $p_d^* = v - \frac{s}{2}$  and  $q_d^* = \frac{1}{2}$ . From here, we have completed proof of Lemma 2.  $\square$

For ease of exposition, below we first present the proof of Proposition 10 (the monopoly benchmark model) before the proof of Proposition 1 (the duopoly base model).

**Proof of Proposition 10.** In the monopoly market, individual customers located at  $x$  ( $x \in [0, 1]$ ) obtain an expected payoff  $(v - p)\theta^*(p) - sx$ , where  $\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_i})} y f(y) dy$ , if she visits  $R_1$ . We consider two cases: (i) the market is partially covered, i.e.,  $\alpha < 1$ , and (ii) the market is fully covered, i.e.,  $\alpha = 1$ .

*Case I. Partial market coverage.* If the market is not fully covered,  $R_1$ 's profit function can be expressed as  $\Pi(p) = p\mathbb{E}(\alpha(p)D \wedge q_o(p)) - cq_o(p)$ , where  $q_o(p) = \alpha(p)F^{-1}(\frac{p-c}{p})$ ,  $\alpha(p) = \frac{U(p)}{s}$ , and  $U(p) = (v - p)\theta^*(p)$ . Hence, we have

$$\Pi'(p) = \frac{U'(p)}{s} \pi(p) + \frac{U(p)}{s} \pi'(p),$$

where  $\pi(p) = p\mathbb{E}\left(D \wedge F^{-1}\left(\frac{p-c}{p}\right)\right) - cF^{-1}\left(\frac{p-c}{p}\right)$ . By Lemma 2, we have that  $U(p)$  is a decreasing and concave function in  $p$  for  $p \in [\hat{p}, v]$ . Moreover, we know that  $U'(p) = 0$  at  $p = \hat{p}$ ;  $U'(p) < 0$  and  $U(p) = 0$  at  $p = v$ . Since  $\pi(p)$  is an increasing function, we have  $\Pi'(p) > 0$  at  $p = \hat{p}$  and  $\Pi'(p) < 0$  at  $p = v$ . Hence, the first-order condition,  $\Pi'(p) = 0$ , results in a unique optimal price  $p_m^* \in [\hat{p}, v]$ . This proves the first case of

partial market coverage.

*Case II. Full market coverage.* In this case, the retailer's profit function is  $\Pi(p) = pE(D \wedge q_o(p)) - cq_o(p)$ , where  $q_o(p) = F^{-1}(\frac{p-c}{p})$  and the market share of the monopolist is 100%. Observe that this profit function is always increasing in price  $p$ . Furthermore, to ensure the full market coverage condition, we must have  $(v-p)\theta^*(p) - s \geq 0$ . By the proof of Lemma 2,  $\theta^*(p)$  is decreasing in  $p$  for  $p \in [\hat{p}, v)$ . Thus, under equilibrium, the retailer charges a price  $p_m^*$  that satisfies  $(v-p_m^*)\theta^*(p_m^*) - s = 0$ . The equilibrium price  $p_m^*$  is the highest price the retailer could charge to ensure full market coverage. This proves the second case of full market coverage.

Finally, we show the existence of  $\bar{s}_m$ . Define  $\bar{s}_m$  as  $(v-p_m^*)\theta^*(p_m^*) = s_m$ , where  $p_m^*$  is the optimal monopoly price obtained in Case I above. It is straightforward from the above analysis that if  $s < \bar{s}_m$ , Case II emerges with a full market coverage; whereas if  $s \geq \bar{s}_m$ , Case II emerges with a partial market coverage. This completes the proof of Proposition 10.  $\square$

**Proof of Proposition 11.** Suppose the two retailers compete on both price and availability in the duopoly model, each retailer should cover more than half of the market size if we remove the other retailer (this is the monopoly case). However, in the equilibrium of the duopoly competition, each retailer covers only half of the market size. Next, let us see the retailer's profit maximization problems in the two cases, respectively. In the monopoly model, the retailer maximizes profit function  $\Pi(p) = \alpha_m(p)\pi(p)$ , where  $\alpha_m(p) = U(p)/s$ ,  $U(p) = (v-p)\theta(p)$ , and  $\pi(p) = pE\left[\bar{F}^{-1}\left(\frac{c}{p}\right) \wedge D\right] - c\bar{F}^{-1}\left(\frac{c}{p}\right)$ . In the duopoly competition model, the retailer maximizes profit function  $\Pi(p) = \alpha_d(p)\pi(p)$ , where  $\alpha_d(p) = (U(p) - U(p_d^*))/2s + \frac{1}{2}$ ,  $U(p) = (v-p)\theta(p)$ , and  $\pi(p) = pE\left[\bar{F}^{-1}\left(\frac{c}{p}\right) \wedge D\right] - c\bar{F}^{-1}\left(\frac{c}{p}\right)$ . Recall conclusion of lemma 2, the function of  $U(p)$  is concavely decreasing in price  $p$ . It is clearly to see that  $\frac{d\alpha_m(p)}{dp} = 2\frac{d\alpha_d(p)}{dp} < 0$ . In other words, market share in the monopoly model is more elastic than that in the duopoly model. Therefore, to maximize total profit, the retailer in the duopoly may charge a higher price. For a complete mathematical proof, one can refer to the proof of Proposition 2.  $\square$

**Proof of Proposition 1.** Given the equilibrium retailer decisions  $(p_d^*, q_d^*)$ , a customer located at  $x$  has an expected payoff of  $(v-p_d^*)\theta^*(p_d^*) - sx$ , where  $x \in [0, 1]$ . Note that, if the unit travel cost  $s$  is small, retailers compete on both price and inventory availability and the market  $\mathcal{M}$  is fully covered under equilibrium. If the unit travel cost  $s$  is large,  $\mathcal{M}$  is not fully covered in equilibrium and, thus, the retailers do not directly compete with each other. In this case, the equilibrium outcome satisfies  $(v-p_d^*)\theta^*(p_d^*) - s\alpha^* = 0$ , where  $\alpha^*$  is the equilibrium market share of a retailer. Hence, the expected payoff of the customers located at  $x = \alpha^*$  and  $x = 1 - \alpha^*$  should be 0. Finally, when the unit travel cost  $s$  is in a medium range,  $\mathcal{M}$  is fully covered but the two retailers do not compete with each other. In this case, each retailer covers half of the market share under equilibrium. Thus, we have that  $(v-p_d^*)\theta^*(p_d^*) - \frac{1}{2}s = 0$ . For the rest of our proof, we use  $R_1$  as the focal retailer.

*Case I. Full Market Coverage with competition.* Let  $p_i$  be the price charged by retailer  $R_i$  and  $\alpha_i$  be the market share of  $R_i$ . Since the two retailers cover the entire market, a customer at the intersection of their

respective market segments should be indifferent between visiting either retailer, i.e.,  $(v - p_1)\theta^*(p_1) - s\alpha_1 = (v - p_2)\theta^*(p_2) - s(1 - \alpha_1) \geq 0$ . Under equilibrium,  $R_2$  charges equilibrium price  $p_d^*$ , and we next analyze  $R_1$ 's best response given  $R_2$ 's price  $p_d^*$ , which is denoted as  $p_1(p_d^*)$ . We write  $R_1$ 's profit as  $\Pi(p_1, p_d^*) := p_1 \mathbb{E}(\alpha_1(p_1, p_d^*)D \wedge q_{1o}) - cq_{1o}$ , where  $q_{1o} = \alpha_1(p_1, p_d^*)F^{-1}(\frac{p_1 - c}{p_1})$ , and its market share  $\alpha_1(p_1, p_d^*)$  satisfies the following equilibrium condition (the expected payoff to visit  $R_1$  is the same as that to visit  $R_2$ ):  $(v - p_1)\theta^*(p_1) - s\alpha_1(p_1, p_d^*) = (v - p_d^*)\theta^*(p_d^*) - s(1 - \alpha_1(p_1, p_d^*))$ . For simplicity, we rewrite the equilibrium condition as  $U(p_1) - s\alpha_1(p_1, p_d^*) = U(p_d^*) - s(1 - \alpha_1(p_1, p_d^*))$ , where  $U(p) = (v - p)\theta^*(p)$ . According to Lemma 2, we know that  $U(p)$  is a decreasing concave function when  $p \in [\hat{p}, v)$ . Under equilibrium, by symmetry, the market share satisfies the condition  $\alpha_1^* = \frac{1}{2}$  and the price satisfies  $p_1(p_d^*) = p_d^*$ , i.e.,

$$p_d^* = p_1(p_d^*) := \arg \max_{0 \leq p \leq v} \left( \frac{1}{2} + \frac{(v - p)\theta^*(p) - (v - p_d^*)\theta^*(p_d^*)}{2s} \right) \left\{ p \mathbb{E} \left[ D \wedge \bar{F}^{-1} \left( \frac{c}{p} \right) \right] - c \bar{F} \left( \frac{c}{p} \right) \right\}$$

To find  $R_1$ 's best response  $p_1(p_d^*)$ , we take derivative of the profit function  $\Pi(p_1, p_d^*)$  with respect to price  $p_1$ , which yields

$$\frac{\partial \Pi(p_1, p_d^*)}{\partial p_1} = \frac{1}{2s} U'(p_1) \pi(p_1) + \left( \frac{1}{2} + \frac{U(p_1) - U(p_d^*)}{2s} \right) \pi'(p_1),$$

where  $\pi(p_1) = p_1 \mathbb{E} \left( D \wedge F^{-1} \left( \frac{p_1 - c}{p_1} \right) \right) - c F^{-1} \left( \frac{p_1 - c}{p_1} \right)$ . By the proof of the Proposition 10, we have that a unique best-response price  $p_1(p_d^*)$  exists. Moreover, the first-order condition implies that  $U'(p_d^*)\pi(p_d^*) + s\pi'(p_d^*) = 0$  in equilibrium.

Next, we prove the existence and uniqueness of the equilibrium. The implicit function theorem and the envelope theorem together yield that  $\frac{d^2 p_1(p_d^*)}{d(p_d^*)^2} = \left\{ -\frac{\partial}{\partial p_d^*} \frac{\partial^2 \Pi(p_1)}{\partial p_1^2} \cdot \frac{\partial^2 \Pi(p_1)}{\partial p_1 \partial p_d^*} + \frac{\partial^2 \Pi(p_1)}{\partial p_1^2} \cdot \frac{\partial}{\partial p_d^*} \frac{\partial^2 \Pi(p_1)}{\partial p_1 \partial p_d^*} \right\} / \left( \frac{\partial^2 \Pi(p_1)}{\partial p_1 \partial p_d^*} \right)^2$ . Thus, it can be easily verified that  $\frac{dp_1(p_d^*)}{dp_d^*} > 0$  and  $\frac{d^2 p_1(p_d^*)}{d(p_d^*)^2} < 0$ , i.e.,  $p_1(p_d^*)$  is concavely increasing in  $p_d^*$ . In addition, observe that  $\lim_{p_d^* \rightarrow v} p_1(p_d^*) < v$  and  $\lim_{p_d^* \rightarrow \hat{p}} p_1(p_d^*) \geq \hat{p}$  (see Lemma 2). Thus, the function  $p_1(p) - p$  has a unique root on  $[\hat{p}, v)$ , which implies that the best-response function  $p_1(\cdot)$  has a unique fixed point on the interval  $[\hat{p}, v)$ . This proves the existence and uniqueness of the equilibrium. By the symmetry of the equilibrium outcome, we have  $\alpha_d^* = \frac{1}{2}$  and  $q_d^* = \frac{1}{2} \bar{F}^{-1} \left( \frac{c}{p_d^*} \right)$  under equilibrium. This finishes the proof of part (b).

*Case II. Partial Market Coverage.* In this case, the two retailers have no direct competition and the market is partially covered. It is straightforward to observe that the equilibrium outcome of each retailer is identical to that of a monopoly retailer with partial market coverage as characterized by Proposition 10. This completes the proof of part (d).

*Case III. Full Market Coverage without Competition.* In this case, the two retailers have no direct competition, but each covers half of the market share. Hence, we have  $U(p_d^*) = \frac{1}{2}s$ , i.e.,  $p_d^* = v - \frac{s}{2\theta^*(p_d^*)}$ . Since the equilibrium market share if  $R_1$  is  $\frac{1}{2}$ , the equilibrium order quantity is given by  $q_d^* = \frac{1}{2} \bar{F}^{-1} \left( \frac{c}{p_d^*} \right)$ . This completes the proof of part (c).

Finally, to complete the proof, we show the existence of the two critical thresholds  $\underline{s}$  and  $\bar{s}$ . Define the threshold  $\underline{s}$  as the unit travel cost satisfying  $U(p_d^*) = \frac{1}{2}\underline{s}$ , where  $p_d^*$  is the equilibrium price characterized in Case I. Thus, the threshold  $\underline{s}$  represents the situation under which the two retailers are barely competing with each other. Similarly, we define the threshold  $\bar{s}$  as the unit travel cost satisfying  $U(p_d^*) = \frac{1}{2}\bar{s}$ , where  $p_d^*$  is the equilibrium price characterized in Case II, i.e., the threshold  $\bar{s}$  represents the situation under which the two retailers barely cover the entire market. Since the objective function is concavely decreasing in  $p$  for

$p \in [\hat{p}, v)$ , we have that  $\underline{s} < \bar{s}$ . This completes the proof of Proposition 1.  $\square$

**Proof of Proposition 2.** To begin with, we prove that  $p_d^*$  is quasi-concave in the searching cost  $s$ . More specifically, we shall prove that the equilibrium price  $p_d^*$  increases in  $s$  when  $s$  is small, decreases in  $s$  when  $s$  is medium, and independent of  $s$  when  $s$  is large.

First, if  $s \leq \underline{s}$ , the equilibrium price  $p_d^*$  satisfies the first-order condition

$$U'(p_d^*)\pi(p_d^*) + s\pi'(p_d^*) = 0,$$

where  $U(p) = (v - p)\theta^*(p)$  and  $\pi(p) = pE\left[D \wedge \bar{F}^{-1}\left(\frac{c}{p}\right)\right] - c\bar{F}^{-1}\left(\frac{c}{p}\right)$ . Now, we consider the equilibrium price as a function of  $s$ :  $p_d^*(s)$ . Taking the derivative of the above first order equation with respect to  $s$ , we obtain

$$\frac{dp_d^*(s)}{ds} = -\pi'(p_d^*(s)) / \frac{d}{dp}(U'(p_d^*(s))\pi(p_d^*(s)) + s\pi'(p_d^*(s))).$$

Since  $\pi'(p) > 0$  for all  $p$ , the numerator of the right side is negative. Since the profit function is concave, we have  $\frac{d(U'(p)\pi(p) + s\pi'(p))}{dp} < 0$ . Hence, we obtain  $\frac{dp_d^*(s)}{ds} > 0$ . Therefore, the equilibrium price  $p_d^*(s)$  is increasing in  $s$  for  $s \leq \underline{s}$ . In this case,  $q_d^*(s) = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p_d^*(s)}\right)$  is increasing in  $s$  as well.

If  $s \in [\underline{s}, \bar{s}]$ , we have  $p_d^*(s) = v - \frac{s}{2\theta^*(p_d^*(s))}$ . Since  $\theta^*(p)$  decreases in  $p$  for the range  $p \in [\hat{p}, v)$ , we have that  $p_d^*(s)$  is decreasing in  $s$  for  $s \in [\underline{s}, \bar{s}]$ . In this case,  $q_d^*(s) = \frac{1}{2}\bar{F}^{-1}\left(\frac{c}{p_d^*(s)}\right)$  is decreasing in  $s$  as well.

If  $s \geq \bar{s}$ , is large such that the two retailers have no market competition,  $p_d^*(s)$  satisfies the first-order condition

$$U'(p_d^*(s))\pi(p_d^*(s)) + U(p_d^*(s))\pi'(p_d^*(s)) = 0,$$

which is independent of  $s$ . Therefore, the equilibrium price  $p_d^*(s)$  is independent of  $s$  for  $s \geq \bar{s}$ . Furthermore, it follows immediately that  $q_d^*(s) = \frac{(v - p_d^*(s))\theta^*(p_d^*(s))}{s}\bar{F}^{-1}\left(\frac{c}{p_d^*(s)}\right)$  is decreasing  $s$ . This concludes the proof of part (a).

For the monopoly model, if  $s \leq \bar{s}_m$ ,  $p_m^*(s)$  satisfies  $p_m^*(s) = v - \frac{s}{\theta^*(p_m^*(s))}$ . Thus,  $p_m^*(s)$  is strictly decreasing in  $s$  for  $s \leq \bar{s}_m$ , so is  $q_m^*(s) = \bar{F}^{-1}\left(\frac{c}{p_m^*(s)}\right)$ . If  $s \geq \bar{s}_m$ ,  $p_m^*(s)$  satisfies the first-order condition

$$U'(p_m^*(s))\pi(p_m^*(s)) + U(p_m^*(s))\pi'(p_m^*(s)) = 0,$$

which is independent of  $s$ . Therefore, the equilibrium price  $p_m^*(s)$  is independent of  $s$  for  $s \geq \bar{s}$ . Furthermore, it follows immediately that  $q_m^*(s) = \frac{(v - p_m^*(s))\theta^*(p_m^*(s))}{s}\bar{F}^{-1}\left(\frac{c}{p_m^*(s)}\right)$  is decreasing  $s$ . This concludes the proof of part (b).

To show part (c), we observe that  $\bar{s}_m$  satisfies  $(v - p^*)\theta^*(p^*) = \bar{s}_m$  whereas  $\bar{s}$  satisfies  $(v - p^*)\theta^*(p^*) = \frac{1}{2}\bar{s}$ , where  $p^*$  is the solution to the FOC  $U'(p^*)\pi(p^*) + U(p^*)\pi'(p^*) = 0$ . Hence, we have  $\bar{s}_m < \bar{s}$ . Therefore, if  $\max\{\bar{s}_m, \underline{s}\} \leq s \leq \bar{s}$ ,  $p_d^*(s) > p^* = p_m^*(s)$ . In particular,  $p_d^*(s^*) > p_m^*(s^*)$  for  $s^* = \max\{\bar{s}_m, \underline{s}\}$ . Since  $p_d^*(s)$  is increasing in  $s$  whereas  $p_m^*(s)$  is decreasing in  $s$  for  $s \leq \underline{s}$ , it suffices to show that if  $s = 0$ ,  $p_m^*(s) > p_d^*(s)$ . This follows immediately from that  $p_m^*(0) = v > p_d^*(0)$ , (see, also, Dana Jr 2001). Therefore, there exists a threshold  $\bar{s}_d < s^*$ , such that  $p_d^*(\bar{s}_d) = p_m^*(\bar{s}_d)$ . Therefore, if  $s < \bar{s}_d$ ,  $p_d^*(s) < p_m^*(s)$ ; whereas if  $s \in (\bar{s}_d, \bar{s})$ ,  $p_d^*(s) > p_m^*(s)$ ; and  $p_d^*(s) = p_m^*(s)$  for  $s \geq \bar{s}$ . As a consequence, we also have that if  $s < \bar{s}_d$ ,  $\theta^*(p_d^*(s)) < \theta^*(p_m^*(s))$ ; if  $s \in (\bar{s}_d, \bar{s})$ ,  $\theta^*(p_d^*(s)) > \theta^*(p_m^*(s))$ ; and  $\theta^*(p_d^*(s)) = \theta^*(p_m^*(s))$  for  $s \geq \bar{s}$ .

Finally, we compare the equilibrium order quantity of the base model and that of the monopoly benchmark. If  $s \geq \bar{s}$ ,  $p_d^* = p_m^*$  and, thus,  $q_d^* = \frac{(v-p_d^*)\theta^*(p_d^*)}{s} \bar{F}^{-1}\left(\frac{c}{p_d^*}\right) = \frac{(v-p_m^*)\theta^*(p_m^*)}{s} \bar{F}^{-1}\left(\frac{c}{p_m^*}\right) = q_m^*$ . If  $s \leq \bar{s}$ ,  $p_d^* < p_m^*$  and, thus,  $q_m^* = \alpha_m^* \bar{F}^{-1}\left(\frac{c}{p_m^*}\right) > \frac{1}{2} \bar{F}^{-1}\left(\frac{c}{p_d^*}\right) = q_d^*$ . If  $\bar{s}_d \leq s \leq \bar{s}$ , we have that  $p_d^* = \frac{1-F(2q_d^*)}{c}$  in the duopoly base model and  $p_m^* = \frac{1-F\left(\frac{q_d^*}{\alpha_m^*}\right)}{c}$  in the monopoly benchmark, where  $\alpha_m^* > \frac{1}{2}$  is the equilibrium market coverage for the monopoly model. Since  $p_d^* \geq p_m^*$ , we have that

$$F(2q_d^*) < F\left(\frac{q_d^*}{\alpha_m^*}\right).$$

Since  $F(\cdot)$  is an increasing function, we obtain  $\alpha_m^* < \frac{q_d^*}{2q_m^*}$ . If  $q_m^* > q_d^*$ , then we must have  $\alpha_m^* < \frac{1}{2}$ , which contradicts our previous conclusion that  $\alpha_m^* > \frac{1}{2}$ . Therefore, we have  $q_m^* > q_d^*$  for  $\bar{s}_d \leq s < \bar{s}$ . This concludes the proof of part (c) and, thus, Proposition 2.  $\square$

**Proof of Proposition 3.** We continue to use retailer  $R_1$  as the focal retailer. Analogous to the proof of Proposition 1, we need to consider three cases: (I) when the unit travel cost  $s$  is small, the two retailers cover the entire market and compete to each other under equilibrium; (II) when the unit travel cost  $s$  is medium, the two retailers cover the entire market, but have no market competition; and (III) when the unit travel cost  $s$  is large, the market is not fully covered under equilibrium and, thus, the retailers have no direct competition. In case (III), the problem can be decomposed into two separate monopoly problems.

We first focus on the case (I): complete market coverage. Given equilibrium price  $p^*$  and equilibrium compensation  $m^*$ , the expected profit of retailer  $R_1$  is

$$(p_1 + m_1)\mathbb{E}(\alpha_1 D \wedge q_o) - cq_o - m_1\mathbb{E}(\alpha_1 D),$$

where  $q_o = \alpha_1 F^{-1}\left(\frac{p_1 + m_1 - c}{p_1 + m_1}\right)$ . Since the two retailers cover the entire market, we have the equilibrium condition  $U(p_1 + m_1) - s\alpha_1 + m_1 = U(p^* + m^*) - s(1 - \alpha_1) + m^*$ , where  $U(p + m) = (v - p - m)\theta^*(p + m)$  and  $\theta^*(p + m) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}\left(\frac{c}{p+m}\right)} y f(y) dy$ . In other words, the consumers at the intersection of their respective market coverages are indifferent between visiting either retailer. For ease of exposition, we define  $t_1 = p_1 + m_1$ , which refers to the effective marginal revenue of the product. Hence, the equilibrium condition on the customer's choice behavior at the intersection of the retailers' respective market coverage (i.e.,  $x = \alpha_1$ ) can be rewritten as

$$U(t_1) - s\alpha_1 + m_1 = U(t^*) - s(1 - \alpha_1) + m^*.$$

Therefore, the profit function of  $R_1$  can be rewritten as

$$t_1\mathbb{E}(\alpha_1 D \wedge q_o) - cq_o - (A^* + 2s\alpha_1 - U(t_1))\mathbb{E}(\alpha_1 D),$$

where  $A^* = U(t^*) + m^* - s$ . Following the same argument in the proof of Proposition 1, it can be shown that the expected profit of  $R_1$  is concave in  $t_1 \in [\hat{p}, v]$ , which further implies that there exists a unique best response for  $R_1$ ,  $t_1(t^*)$ . Given  $t_1(t^*)$ , we have best compensation response  $m_1(m^*) = A^* + 2s\alpha_1 - (v - t_1(t^*))\theta^*(t_1(t^*))$  and best price response  $p_1(p^*) = t_1(t^*) - m_1(m^*)$ . Therefore, a unique equilibrium  $(p_c^*, m_c^*)$  exists. Exchanging the roles of  $R_1$  and  $R_2$ , we have that, given  $R_1$ 's decisions  $(p^*, m^*)$ , the expected profit of  $R_2$  is concave in



$t_2$ . The concavity of the expected profit function also implies that an equilibrium exists. This concludes the proof of the case with full market coverage.

We then consider the case (III): partial market coverage, i.e.,  $\alpha_1 + \alpha_2 < 1$ . In this case,  $R_1$  makes the price and monetary compensation decisions to maximize its profit  $\Pi(p_1, m_1) = (t_1)\mathbb{E}(\alpha_1 D \wedge q_{1o}) - cq_{1o} - m_1\mathbb{E}(\alpha_1 D)$ , where  $q_{1o} = \alpha_1 F^{-1}(\frac{t_1 - c}{t_1})$ , and  $\alpha_1 = \frac{U(t_1) + m_1}{s}$ . Thus, the expected profit can be rewritten as

$$t_1 \mathbb{E}(\alpha_1 D \wedge q_o) - cq_o - \{s\alpha_1 - U(t_1)\} \mathbb{E}(\alpha_1 D).$$

Following the same argument in the proof of Proposition 1, we know this expected profit function is concave in  $t_1$ . Hence, the equilibrium  $(p_c^{**}, m_c^{**})$  exists if  $s$  is sufficiently large.

Next, we turn to case (II): when the entire market is fully covered, but without competition. Obviously, each retailer in this case charges a price such that half market share is just covered, so we have constraint  $\frac{U(t_1) + m_1}{s} = \frac{1}{2}$ . Given this constraint, the retailer balances  $t_1$  and  $m_1$  to maximize profit  $t_1 \mathbb{E}(\frac{1}{2} D \wedge q_o) - cq_o - \frac{1}{2} m_1 \mathbb{E}(D)$ , where  $q_o = \frac{1}{2} F^{-1}(\frac{t_1 - c}{t_1})$ . Similar as analysis above, the profit function is concave in  $t_1 \in [\hat{p}, v]$ , so we have a unique solution  $t^*$ . By the constraint function, we have  $m^* = \frac{1}{2}s - U(t^*)$ . Therefore, we obtain a unique equilibrium solution  $(p_c^{***}, m_c^{***})$ .

Finally, we find thresholds  $\underline{s}_c$  and  $\bar{s}_c$  to separate the above three cases. When  $s = \underline{s}_c$ , the two retailers just cover the whole market with competition. When  $s = \bar{s}_c$ , the two retailers are monopolists and just cover the whole market. Similar as results in the Proposition 1, we have  $\underline{s}_c \leq \bar{s}_c$ . Therefore, the equilibrium outcome will be  $(p_c^*, m_c^*)$  when  $s \leq \underline{s}_c$ ; be  $(p_c^{**}, m_c^{**})$  when  $s > \bar{s}_c$ ; and be  $(p_c^{***}, m_c^{***})$  when  $\underline{s}_c < s \leq \bar{s}_c$ . This concludes the proof of Proposition 3.  $\square$

**Proof of Proposition 4.** We start our proof by considering two extreme cases.

Case I. Zero searching cost (i.e.,  $s = 0$ ). In this case, the two retailers compete on offering higher consumer expected payoff, because  $\alpha'(p, m) \rightarrow -\infty$ . In the model of monetary compensation, the expected payoff function is  $U(p + m) + m - sx$  for consumers located at  $x \in [0, 1]$ . The first term,  $U(p + m)$ , is concave with its maximum value,  $U(\hat{p})$ , at  $p + m = \hat{p}$ . The second term is linearly increasing in  $m$ . In other words, given  $p + m = \hat{p}$ , retailers can always capture the entire market by continuously increasing compensation  $m$ . However, each retailer's profit function is strictly decreasing in compensation  $m$ , so the retailers have to stop raising compensation at zero profit. Therefore, each retailer obtains zero profit under equilibrium when  $s = 0$ . In contrast, each retailer charges price  $p = \hat{p}$  in the duopoly base model, because  $\hat{p}$  implies the highest expected payoff. Since we always have  $\hat{p} > c$ , each retailer must have a positive profit in the duopoly base model. Therefore, we have  $\Pi_d^* > \Pi_c^*$  when  $s = 0$ .

Case II. Infinitely large search cost (i.e.,  $s \rightarrow \infty$ ). In this case, the two retailers have no direct competition (i.e., partial market coverage). In the model of monetary compensation, each retailer maximizes its profit  $\alpha(p + m) \left\{ (p + m) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p+m})) - c \bar{F}^{-1}(\frac{c}{p+m}) - m \mathbb{E}(D) \right\}$ , where  $\alpha(p + m) = \frac{U(p+m)}{s}$ . In the duopoly base model, each retailer maximizes its profit  $\alpha(p) \left\{ (p) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p})) - c \bar{F}^{-1}(\frac{c}{p}) \right\}$ , where  $\alpha(p) = \frac{U(p)}{s}$ . The profit function in the model of monetary compensation restores to the profit function in the duopoly base

model when  $m = 0$ . Since  $m$  is a free variable, the duopoly base model is a special case of the monetary compensation model when  $s = 0$ . In other words,

$$\Pi_c^* = \max_{(p,m)} \Pi_c(p,m) \geq \max_p \Pi_c(p,0) = \max_p \Pi_d(p) = \Pi_d^*.$$

Therefore, we have  $\Pi_c^* \geq \Pi_d^*$  when  $s \rightarrow \infty$ .

Finally, recall that  $\Pi_d^*$  and  $\Pi_c^*$  are quasi-concave in  $s$ . By proofs above, we have already obtained  $\Pi_d^* > \Pi_c^*$  when  $s = 0$ ; and  $\Pi_c^* \geq \Pi_d^*$  when  $s \rightarrow \infty$ . Therefore, there exists a threshold  $\bar{s}_{cd}$  such that  $\Pi_c^* < \Pi_d^*$  if  $s < \bar{s}_{cd}$ ;  $\Pi_c^* \geq \Pi_d^*$  if  $s \geq \bar{s}_{cd}$ . This completes the proof of Proposition 4.  $\square$

**Proof of Proposition 5.** Similar to the previous proofs, we set  $R_1$  as the focal retailer. As in the proof of Proposition 1 and Proposition 4, we consider three cases: (I)  $s$  is small so that the two retailers cover the entire market and compete, (II)  $s$  is medium so that the two retailers cover the entire market without competition, and (III)  $s$  is big so that the two retailers partially cover the market. We start our analysis with case I, the case of full market coverage, i.e.,  $s$  is small.

Assume that retailer  $R_2$  charges the equilibrium price  $p^*$  and stocks the equilibrium inventory quantity  $q^*$ . Retailer  $R_1$  maximizes its profit  $\Pi(p, q) := p\mathbb{E}(\alpha(p, q)D \wedge q) - cq$ . Note that  $R_1$ 's market share  $\alpha(p, q)$  satisfies the equilibrium condition  $(v - p)\theta(\frac{q}{\alpha(p, q)}) - s\alpha(p, q) = (v - p^*)\theta(\frac{q^*}{1 - \alpha(p, q)}) - s(1 - \alpha(p, q)) \geq 0$ , where  $\theta(\frac{q}{\alpha}) = \frac{1}{\mu} \int_{y=0}^{\frac{q}{\alpha}} yf(y)dy$ . Taking derivative for both sides regards to quantity, we obtain that the market share  $\alpha(p, q)$  is concavely increasing in the stocking quantity  $q$ . Then, we take the derivative of the profit function with respect to  $q$ , the first-order condition then implies that

$$q^*(p) = \alpha(p, q)F^{-1} \left( \frac{p - c}{p} + \frac{d\alpha(p, q)}{dq} \int_0^{\frac{q}{\alpha(p, q)}} x dF(x) \right).$$

Under equilibrium, we have  $p = p^*$ ,  $q = q^*(p^*) = 0.5F^{-1} \left\{ \frac{p^* - c}{p^*} + \frac{d\alpha(p^*, q^*)}{dq^*} \int_0^{2q^*} x dF(x) \right\}$ , where  $\frac{d\alpha(p, q)}{dq} > 0$ .

Comparing the equilibrium order quantity in the base model,  $q_o(p) = 0.5F^{-1}(\frac{p-c}{p})$ , and the equilibrium order quantity in the model with inventory commitment,  $q^*(p) = 0.5F^{-1} \left\{ \frac{p-c}{p} + \frac{d\alpha(p, q)}{dq} \int_0^{2q} x dF(x) \right\} := g(p)$ , we find that  $g(p)$  shares the same functional properties as  $F^{-1}(\frac{p-c}{p})$ , which is concavely increasing in  $p$ . Moreover, given the same price as in the duopoly base model, the retailer in the inventory commitment model has a tendency to increase inventory stock.

Next, we examine how  $R_1$  would determine the price given the optimal quantity decision  $q^*(p)$ . Note that the expected profit of  $R_1$  is

$$p_1 \mathbb{E}[(\alpha_1(p_1)D) \wedge q^*(p_1)] - cq^*(p_1),$$

where  $q^*(p) = \alpha(p)F^{-1} \left( \frac{p-c}{p} + \frac{d\alpha(p, q)}{dq} \int_0^{\frac{q}{\alpha(p)}} y dF(y) \right)$  and  $\alpha(p) = \frac{1}{2} + \frac{1}{2s} \left\{ (v - p) \int_0^{\frac{q^*(p)}{\alpha(p)}} y dF(y) - (v - p^*) \int_0^{\frac{q^*}{1 - \alpha(p)}} y dF(y) \right\}$ . Following the same argument in the proof of Proposition 1, we know the best-response function for price  $p(p^*)$  is concavely increasing in  $R_2$ 's price decision  $p^*$ . As in the proof of Proposition 1, this implies that a unique price equilibrium exists and we denote the equilibrium price as  $p_v^*$ . Putting everything together, we have that a unique symmetric equilibrium  $(p_v^*, q_v^*)$  exists for the case where the retailers cover the entire market.

Next, we consider case III: partial market coverage. The case II will be discussed later. In this case,  $(v-p)\theta(p,q) - s\alpha(p,q) = 0$ , where  $\theta(p,q) = \frac{1}{\mu} \int_0^{\frac{q}{\alpha(p,q)}} yf(y)dy$ . In other words, we have  $p = v - \frac{s\alpha(p,q)}{\theta(p,q)}$ . Hence, the optimization problem of  $R_1$  is given by

$$\max_{(p,q)} \{p\mathbb{E}(\alpha(p,q)D \wedge q) - cq\},$$

subject to the constraint  $p = v - \frac{s\alpha(p,q)}{\theta(p,q)}$ . We use  $(p_v^{**}, q_v^{**})$  to denote the equilibrium outcome with partial market coverage.

For case II, the two retailers have no competition but cover the entire market. Thus, we have  $p = v - \frac{s}{2\theta(p,q)}$  by full market coverage. Each retailer maximizes profit

$$\max_{(p,q)} \{p\mathbb{E}(\frac{1}{2}D \wedge q) - cq\},$$

subject to the constraint  $p = v - \frac{s}{2\theta(p,q)}$ . We use  $(p_v^{***}, q_v^{***})$  to denote the equilibrium outcome of this case.

To conclude our proof, we still need to show that there exists two threshold  $\underline{s}_v$  and  $\bar{s}_v$ , such that an equilibrium with partial market coverage exists if  $s > \bar{s}_v$  and an competitive equilibrium with full market coverage exists if  $s \leq \underline{s}_v$ . Define  $\bar{s}_v$  as  $(v - p_v^{**})\theta(p_v^{**}, q^*(p_v^{**})) = 0.5\bar{s}_v$  and  $(v - p_v^*)\theta(p_v^*, q^*(p_v^*)) = 0.5\underline{s}_v$ . Similar as in the Proposition 3, we have  $p_v^* > p_v^{**}$  in equilibrium, which implies  $\underline{s}_v \leq \bar{s}_v$ . Therefore, if  $s \leq \underline{s}_v$ , each retailer covers half market and competes; if  $\underline{s}_v < s \leq \bar{s}_v$ , each retailer covers half market but has no competition; otherwise, if  $s > \bar{s}_v$ , each retailer covers partial market and behaves as a local monopolist. This completes the proof of Proposition 5.  $\square$

**Proof of Proposition 6.** To compare the profit of  $R_1$  under different strategies, we first calculate its profit in different circumstances.

We first examine the case where  $R_2$  does not reveal its inventory information. In this case, a customer at the purchasing threshold forms the belief  $\theta_2 = \theta^*(p_2) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_2})} yf(y)dy$  the inventory availability probability of  $R_2$ . A customer will visit  $R_1$  if and only if her utility of visiting  $R_1$  dominates that of visiting  $R_2$  and that of visiting no one (i.e., 0). Therefore, if  $R_1$  also does not reveal its inventory information to the market, its market share  $\alpha_1$  is given by

$$\alpha_1 = \min \left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1) - (v-p_2)\theta^*(p_2)}{2s} \right\} \quad (4)$$

Thus, the maximum profit of  $R_1$  if he does not adopt the inventory commitment strategy is

$$\Pi_{d,d} := \max_{0 \leq p_1 \leq v} \left\{ \min \left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1) - (v-p_2)\theta^*(p_2)}{2s} \right\} \cdot \left\{ p_1 \mathbb{E} \left[ D \wedge \bar{F}^{-1} \left( \frac{c}{p_1} \right) \right] - c\bar{F}^{-1} \left( \frac{c}{p_1} \right) \right\} \right\}$$

Similarly, if  $R_1$  adopts the inventory commitment strategy, its market share  $\alpha_1$  satisfies the following equation:

$$\alpha_1 = \min \left\{ \frac{v-p_1}{s\mu} \int_{y=0}^{q_1/\alpha_1} yf(y)dy, \frac{1}{2} + \frac{v-p_1}{2s\mu} \int_{y=0}^{q_1/\alpha_1} yf(y)dy - \frac{(v-p_2)\theta^*(p_2)}{2s} \right\}. \quad (5)$$

Therefore, the maximum profit of  $R_1$  if he adopts the inventory commitment strategy is

$$\Pi_{v,d} := \max_{0 \leq p_1 \leq v, q_1 \geq 0} \{p_1 \mathbb{E}[\alpha_1 D \wedge q_1] - cq_1\},$$

where  $\alpha_1$  satisfies equation (5).

We now turn our attention to the case where  $R_2$  adopts the inventory commitment strategy. If  $R_1$  does not reveal its inventory information to the market, customers at the purchasing threshold for  $R_1$  form belief  $\theta_1 = \theta^*(p_1) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p_1})} yf(y)dy$  about the inventory availability probability of the retailer. Therefore, the market share  $\alpha_1$  is the solution to the following equation

$$\alpha_1 = \min \left\{ \frac{(v-p_1)\theta^*(p_1)}{s}, \frac{1}{2} + \frac{(v-p_1)\theta^*(p_1)}{2s} - \frac{v-p_2}{2s\mu} \int_{y=0}^{q_2/(1-\alpha_1)} yf(y)dy \right\}. \quad (6)$$

The maximum profit of  $R_1$  if he does not reveal its inventory information is

$$\Pi_{d,v} := \max_{0 \leq p_1 \leq v, q_1 \geq 0} \{p_1 \mathbb{E}[\alpha_1 D \wedge q_1] - cq_1\},$$

where  $\alpha_1$  satisfies equation (6).

If  $R_1$  adopts the inventory commitment strategy, its market share  $\alpha_1$  satisfies the following equation:

$$\alpha_1 = \min \left\{ \frac{v-p_1}{s\mu} \int_{y=0}^{q_1/\alpha_1} yf(y)dy, \frac{1}{2} + \frac{v-p_1}{2s\mu} \int_{y=0}^{q_1/\alpha_1} yf(y)dy - \frac{v-p_2}{2s\mu} \int_{y=0}^{q_2/(1-\alpha_1)} yf(y)dy \right\}. \quad (7)$$

The maximum profit of  $R_1$  if he adopts the inventory commitment strategy is

$$\Pi_{v,v} := \max_{0 \leq p_1 \leq v, q_1 \geq 0} \{p_1 \mathbb{E}[\alpha_1 D \wedge q_1] - cq_1\},$$

where  $\alpha_1$  satisfies equation (7).

By comparing the profit function of  $R_1$  under different strategy profiles, it is straightforward to observe that the equilibrium market share of  $R_1$  is larger if he commits to an inventory order quantity, regardless of whether  $R_2$  reveals his inventory order. Hence, the profit of  $R_1$  will be higher under the inventory commitment strategy if the retailer commits to ordering an inventory level that leads to the same in-stock probability. Therefore, regardless of the price and inventory order quantity decisions for  $R_2$  and regardless of whether  $R_2$  adopts the inventory commitment strategy, the profit of  $R_1$  is higher if he adopts the inventory commitment strategy, i.e.,  $\Pi_{v,d} > \Pi_{d,d}$  and  $\Pi_{v,v} > \Pi_{d,v}$ . This completes the proof of Proposition 6.  $\square$

**Proof of Proposition 7.** First, we prove  $\Pi_v^* \geq \Pi_d^*$  when the market has no competition (i.e.,  $s$  is sufficiently large). In the base (monopoly) model, a retailer's profit function is

$$\Pi_d(p) = p\mathbb{E}(\alpha(p)D \wedge q_o(p)) - cq_o(p),$$

where  $q_o(p) = \alpha(p)\bar{F}^{-1}(\frac{c}{p})$ ,  $\alpha(p) = \frac{v-p}{s}\theta^*(p)$  and  $\theta^*(p) = \frac{1}{\mu} \int_{y=0}^{\bar{F}^{-1}(\frac{c}{p})} yf(y)dy$ . In the model with inventory commitment, a retailer's profit function is

$$\Pi_v(p, q) = p\mathbb{E}(\alpha(p, q)D \wedge q) - cq,$$

where  $\alpha(p, q) = \frac{v-p}{s}\theta(p, q)$  and  $\theta(p, q) = \frac{1}{\mu} \int_{y=0}^{\frac{q}{\alpha(p, q)}} yf(y)dy$ . It is clear from the formulation of the profit functions that, the two models have the same profit functions but the model without inventory commitment has an additional constraint  $q = \alpha(p)\bar{F}^{-1}(\frac{c}{p})$ . Hence,  $\Pi_v^* = \max_{(p, q)} \Pi_v(p, q) \geq \max_v \Pi_d(p, q(p)) = \max_d \Pi_d(p) = \Pi_d^*$ . Therefore, if  $s$  is large such that the market is partially covered by the two retailers, we have  $\Pi_v^* \geq \Pi_d^*$ .

Now we turn to the case of full market coverage. To begin with, we analyze the equilibrium pricing policies of both models, starting with the duopoly base model. First observe that as  $s = 0$ , an infinitely small increase of expected payoff could attract all customers to visit. As a result, under equilibrium, both retailers will set the price at the customer-surplus maximizing one:  $\hat{p}$  (see the proof of Lemma 2). Hence, the retailer's profit in the base model is

$$\Pi_d^* = p_d^* \mathbb{E} \left( \frac{1}{2} D \wedge q_d(p_d^*) \right) - c q_d(p_d^*),$$

where  $q_d(p_d^*) = \frac{1}{2} \bar{F} \left( \frac{p_d^* - c}{p_d^*} \right)$  and  $p_d^* = \hat{p}$ . It is clear that  $\Pi_d^* > 0$ .

We now consider the model with inventory commitment. When  $s = 0$ , by the first-order condition with respect to  $q$ , we have that the equilibrium order quantity  $q_v^*$  satisfies the equation  $q_v^* = \frac{1}{2} F \left( \frac{p_v^* - c}{p_v^*} + \frac{\int_0^{2q_v^*} x f(x) dx}{4q_v^*} \right)$ . Also note that if  $c \rightarrow 0$ , we have  $\frac{\partial \alpha_v^*}{\partial p_v^*} = \frac{1}{2\mu} \frac{\theta(p_v^*, q_v^*)}{s + 2(v - p_v^*)(2q_v^*)^2 f(2q_v^*)}$  in the model of inventory commitment, which is smaller than  $\frac{\partial \alpha_d^*}{\partial p_1} |_{p_1 = p_v^*}$  in the duopoly base model. Thus, we have that  $p_v^* < p_d^*$ . Therefore,

$$\begin{aligned} \Pi_d^* &= p_d^* \mathbb{E} \left( \frac{1}{2} D \wedge q_d^* \right) - c q_d^* = \frac{1}{2} \left\{ \hat{p} \mathbb{E} \left( D \wedge \bar{F}^{-1} \left( \frac{c}{\hat{p}} \right) \right) - c \bar{F}^{-1} \left( \frac{c}{\hat{p}} \right) \right\} \\ &> \frac{1}{2} \left\{ p_v^* \mathbb{E} \left( D \wedge \bar{F}^{-1} \left( \frac{c}{p_v^*} \right) \right) - c \bar{F}^{-1} \left( \frac{c}{p_v^*} \right) \right\} \geq p_v^* \mathbb{E} \left( \frac{1}{2} D \wedge q_v^* \right) - c q_v^* = \Pi_v^*, \end{aligned}$$

where the first inequality follows from  $p_d^* = \hat{p} > p_v^*$ , and the second follows from  $q_v^* \neq \bar{F}^{-1} \left( \frac{c}{p_v^*} \right)$  (where the quantity  $q = \bar{F}^{-1} \left( \frac{c}{p_v^*} \right)$  is the optimal quantity that maximizes the profit function given the price  $p_v^*$ ). Therefore, the inventory commitment strategy results in a lower profit if  $s = 0$  and  $c \rightarrow 0$ .

Finally, the two equilibrium profits,  $\Pi_d^*$  and  $\Pi_v^*$ , are both continuous in  $s$  and  $c$ . Moreover, we have just shown that  $\lim_{s, c \rightarrow 0} \Pi_d^* > \lim_{s, c \rightarrow 0} \Pi_v^*$ . Thus, there exist two thresholds  $\bar{s}_{vd}$  (for  $s$ ) and  $\bar{c}_{vd}$  (for  $c$ ), such that if  $s < \bar{s}_{vd}$  and  $c < \bar{c}_{vd}$ ,  $\Pi_v^* < \Pi_d^*$ ; otherwise  $\Pi_v^* \geq \Pi_d^*$ . This concludes the proof of Proposition 7.  $\square$

**Proof of Proposition 8.** To begin with, we analyze properties of monotonicity for the two average consumer surplus functions. In this proof, we focus on conditions when market share exists in a duopoly base model, and we only discuss conditions when price falls into the range of  $[\hat{p}, v)$ . The average consumer surplus in the duopoly model is

$$CS_d^* = \frac{1}{\mathbb{E}(D)} \{ (v - p_d^*) \mathbb{E}(D \wedge 2q_d^*) \} - \frac{s}{4},$$

where  $2q_d^* = \bar{F}^{-1} \left( \frac{c}{p_d^*} \right)$ . Observe the above function, the first term is decreasingly concave in  $s$  and the second term is strictly decreasing in  $s$ , so  $CS_d^*$  is concavely decreasing in  $s$ . The average consumer surplus function in the monopoly model is

$$CS_m^* = \frac{1}{\mathbb{E}(D)} \left\{ (v - p_m^*) \mathbb{E} \left( D \wedge \frac{q_m^*}{\alpha_m} \right) \right\} - \frac{s}{2} \alpha_m,$$

where  $\alpha_m = \frac{(v - p_m^*) \theta^*(p_m^*)}{s}$  and  $\frac{q_m^*}{\alpha_m} = \bar{F}^{-1} \left( \frac{c}{p_m^*} \right)$ . Recall that  $p_m^*$  is non-increasing in  $s$  in the monopoly model (see Proposition 3), so the first term is non-decreasing. The second term, by plugging  $\alpha_m$  in to the function, is  $\frac{(v - p_m^*) \theta^*(p_m^*)}{2}$ , so it is also a non-decreasing function of  $s$ . Hence, the  $CS_m^*$  is non-decreasing (weakly increasing) in  $s$ .

Now, we focus on analyzing two extreme cases.

*Case I. A small searching cost (i.e.,  $s \downarrow 0$ ).* In this case, the entire market is fully covered in both the duopoly base model and the monopoly model. More specifically,  $CS_d^* \rightarrow \frac{1}{\mathbb{E}(D)} \left\{ (v - p_d^*) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p_d^*})) \right\}$  and  $CS_m^* = \frac{1}{\mathbb{E}(D)} \left\{ (v - p_m^*) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p_m^*})) \right\}$ . The two functions have the same structure, but different equilibrium prices. Since we have shown that  $p_m^* > p_d^*$  when  $s \downarrow 0$  in the Proposition 3 (c), so  $CS_m^* < CS_d^*$ .

*Case II. A large searching cost (i.e.,  $s \uparrow \bar{s}$ ).* In this case, each retailer covers half of the entire market in both the duopoly base model and the monopoly model. More specifically,  $CS_d^* \rightarrow \frac{1}{\mathbb{E}(D)} \left\{ (v - p_d^*) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p_d^*})) \right\} - \frac{s}{4}$  and  $CS_m^* = \frac{1}{\mathbb{E}(D)} \left\{ (v - p_m^*) \mathbb{E}(D \wedge \bar{F}^{-1}(\frac{c}{p_m^*})) \right\} - \frac{s}{4}$ . The first terms in the two functions have the same structure and the second terms are the same. Since we have proved shown that  $p_m^* < p_d^*$  when  $s \uparrow \bar{s}$  in the Proposition 3 (c), so  $CS_m^* > CS_d^*$ .

Recall that we have proved that  $CS_m^*$  is weakly increasing in  $s$  and  $CS_d^*$  is decreasing in  $s$ , combine the above two extreme cases, there exists a threshold  $\bar{s}_w$ , such that  $CS_m^* \leq CS_d^*$  for  $s \leq \bar{s}_w$ ; and  $CS_m^* > CS_d^*$  for  $\bar{s}_w < s < \bar{s}$ . Before ending our proof, we shall also show that  $\bar{s}_w > \bar{s}_d$ . When  $s = \bar{s}_d$ , equilibrium prices in the duopoly model is the same as equilibrium price in the monopoly model, which implies the first terms in the two average social welfare functions are the same. However, the average search cost in the duopoly base model,  $\frac{s}{4}$ , is smaller than the average search cost in the duopoly base model,  $\frac{s}{2}\alpha_m$  (because  $\alpha_m > \frac{1}{2}$ ). Hence, we have  $CS_m^* < CS_d^*$  when  $s = \bar{s}_d$ , and therefore  $\bar{s}_w > \bar{s}_d$ . This completes our proof.  $\square$

**Proof of Proposition 9.** To begin with, we show  $SC_c^* \geq SC_d^*$ . Suppose the market follows the equilibrium path of standard duopoly competition model and achieves equilibrium solutions  $(p_d^*, q_d^*)$ . In this case, the monetary compensation  $m_d^* = 0$ . Now, we allow the retailers to pay compensation to consumers. Accordingly, the equilibrium compensation switches from  $m_d^* = 0$  to  $m_c^* \geq 0$ . A higher compensation rate increases consumer surplus and thus helps retailers earn more market share (but decreases its marginal revenue). If  $m_c^* = 0$ , the retailers have no incentive to compete more in market share, so the two models result in the same consumer surplus. If  $m_c^* > 0$ , the two retailers have incentives to compete more in market share, so a positive compensation rate raises consumer surplus. In short, since we always have  $m_c^* \geq m_d^* = 0$ , offering non-negative monetary compensation to customers upon stock can always increases the equilibrium average customer surplus, i.e.,  $SC_c^* \geq SC_d^*$ .

Next, we show  $SC_v^* \geq SC_d^*$ . Similarly, suppose the market follows the equilibrium path of standard duopoly model and achieves equilibrium solution  $(p_d^*, q_d^*)$ . Now, we allow the retailers to announce quantity information to the market. As a result, the market switches to a new equilibrium path  $(p_v^*, q_v^*)$  under inventory commitment. In the case of inventory commitment, the retailers are motivated to increase quantity and decrease price.

First, the retailers have incentives to increase quantity. Assume the equilibrium price  $p_d^*$  is unchanged. Once the retailers commit inventory to the market, the inventory stock must not decrease. The argument is as follows. On one side, decreasing quantity decreases consumer surplus and thus decreases market share. On the other side, deviating from the critical fractile quantity ( $q = \frac{1}{2}F^{-1}(\frac{v-c}{p})$ ) decreases marginal revenue. As a result, by decreasing inventory quantity, the retailers must earn less profit, so the inventory quantity must not be decreased. However, the retailers may increase quantity. Although increasing stock quantity also

deviates from the critical fractile quantity and thus decreases marginal revenue, it raises market share by offering higher product availability. Thus, the retailers may earn higher profit by increasing stock quantity. Therefore, given a duopoly equilibrium price, the retailers may choose to increase quantity.

Second, the retailers have incentives to decrease price. Similarly, assume the equilibrium quantity  $q_d^*$  is unchanged, the retailers have no incentive to increase retail price. The argument is as follows. On one side, increasing price decreases consumer surplus and thus decreases market share. On the other side, deviating from the critical fractile price ( $p = c/\bar{F}(2q)$ ) decreases marginal revenue. As a result, by increasing price, the retailers must earn less profit, so the retail price must not be increased. However, retailers may decrease price. Although decreasing price also deviates from the critical fractile price and thus shrinks marginal revenue, it raises market share. In other words, the retailers may earn higher profit by decreasing price. Therefore, given a duopoly equilibrium quantity, the retailers may choose to decrease price.

In sum, once retailers apply inventory commitment, we have  $q_v^* \geq q_d^*$  and(or)  $p_v^* \leq p_d^*$ . Since both increasing quantity and decreasing price are beneficial to consumers' surplus, we have  $SC_v^* \geq SC_d^*$ .  $\square$