Trade-in Remanufacturing, Customer Purchasing Behavior, and Government Policy

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Trade-in remanufacturing is a commonly adopted business practice under which firms collect used products for remanufacturing by allowing repeat customers to trade in used products for upgraded ones at a discount price. This paper studies how customer purchasing behavior and remanufacturing efficiency affect the economic and environmental values of such a business practice. We demonstrate a new benefit of trade-in remanufacturing: It helps exploit the forward-looking behavior of strategic customers, which could be much more significant than the widely recognized revenue-generating and environmental benefits of remanufacturing. High remanufacturing efficiency does not necessarily benefit a firm. With overly high remanufacturing efficiency, product durability is so high that repeat customers are reluctant to trade in and upgrade their used products. When customers are highly strategic, trade-in remanufacturing creates a tension between profitability and sustainability: On one hand, by exploiting the intensive forward-looking customer behavior, trade-in remanufacturing is quite valuable to the firm; on the other hand, with highly strategic customers, trade-in remanufacturing has a substantial negative impact on the environment and social welfare, since it may induce significantly higher production quantities without improving customer surplus. With nearly-myopic customers, however, trade-in remanufacturing benefits both the firm and the environment. Therefore, understanding the interactions between customer purchasing behavior and trade-in remanufacturing is important to both firms and policy-makers. Finally, to resolve the above tension, we study how a social planner (e.g., government) should design a public policy to maximize social welfare. The social optimum can be achieved by using a simple linear subsidy/tax scheme for both new production and remanufacturing. The proposed policy can also induce the firm to set the socially optimal remanufacturing efficiency.

Key words: customer behavior; trade-in rebates; remanufacturing; environment; government policy

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1. Introduction

Remanufacturing is the rebuilding of a product to specifications of the original manufactured product using a combination of reused, repaired, and new parts (Johnson and McCarthy 2014). The initial purpose of remanufacturing was to recover the residual value of the components and materials from used products (see, e.g., Guide and Van Wassenhove 2009, Debo et al. 2005). More recently, with growing awareness of sustainability, the environmental advantages of remanufacturing have also been widely recognized. As a result, remanufacturing has been increasingly adopted in practice to enhance a firm's competitive edge and public image on the market. For example, to facilitate the recycling and remanufacturing of used products, Apple recently invented Liam, a line of robots that can efficiently disassemble iPhones and sort the high-quality components that can be recycled for manufacturing new devices (Apple, Inc. 2017).

An important issue in remanufacturing is the remanufacturability or remanufacturing efficiency of the product, which depends on product design, durability, and other factors (Debo et al. 2006). A product with high remanufacturing efficiency normally requires more durable components and materials (Geyer et al. 2007). Thus, such product would be more cost effective and environmentally friendly to remanufacture. At the same time, a product of high remanufacturability is also more durable, so customers can derive higher residual values from used products. For instance, to ensure the environmental sustainability of its business, Apple has adopted high-grade materials (e.g., aluminum) in their electronic devices so that the devices have a longer life-time (i.e., more durable) and can be easily recycled (i.e., high remanufacturing efficiency, see Apple, Inc. 2017). As we will demonstrate below, the relationship between remanufacturing efficiency and product durability will play a significant role in how remanufacturing affects firms, the environment, and the society.

According to the U.S. International Trade Commission (2012), an integral component of closed-loop supply chains for remanufacturing is core collection, i.e., the process of obtaining used products from customers. A common practice for core collection is to provide trade-in rebates that encourage customers to return their used products. For example, Apple offers both in-store and online trade-in programs, which encourage customers to exchange their used iPhones, iPads, and Macs for credits to purchase new Apple products (Apple Online Store 2017). Analogously, Amazon allows Kindle owners to trade in their old products for newer versions at a discount price (Copy 2011). More examples of using trade-in rebates for core collection have been reported in industries such as furniture, carpets, power tools, etc. (see Ray et al. 2005).

It is quite common in practice that a customer needs to decide whether to make an immediate purchase or to wait for better future opportunities (e.g., a price mark-down or a new technology). A customer is called strategic or forward-looking if she strategizes the purchasing decision to maximize her long-run utilities. In contrast, a myopic customer does not consider future opportunities and bases her purchasing decision on the immediate utilities. Customer purchasing behavior can be quite complex in the real world. It has been empirically verified that customers exhibit a mixture of strategic and nonstrategic purchasing behaviors in various markets (see, e.g., Li et al. 2014, Hendel and Nevo 2013, Chevalier and Goolsbee 2009). So the actual customer purchasing behavior in a market should be somewhere between the two extremes of fully strategic and fully myopic. Despite its complexity, understanding the customer purchasing behavior in a market is important, since whether customers are strategic or not has significant impact on a firm's operations strategy. For example, whereas responsive pricing could effectively exploit customer segmentation with myopic customers, it has a potential adverse impact with strategic customers (Aviv and Pazgal 2008). As another example, in the durable goods and event ticketing markets, resale in the secondary market damages the profit when customers are myopic, but the profit will increase in the presence of the secondary/resale market if customers are forward-looking (see Chevalier and Goolsbee 2009, Su 2010, Cui et al. 2014).

Trade-in remanufacturing and customer purchasing behavior naturally interact with each other. The trade-in opportunity grants price discounts to repeat customers who return their used products, thus enabling the firm to price discriminate new and repeat customers (Van Ackere and Reyniers 1995). Under trade-in remanufacturing, strategic customers will anticipate a potential future price discount in the form of a trade-in rebate, which is ignored by myopic customers. As a consequence, different customer purchasing behaviors may lead to drastically different market outcomes under trade-in remanufacturing. Although both strategic and nonstrategic customer behaviors have been widely acknowledged in the literature, it is not clear what role they will play under the adoption of trade-in remanufacturing.

Recent years have witnessed an increasing number of government interventions of markets based on environmental issues. For instance, starting in 2011, the Chinese Ministry of Finance maintained a fund for the treatment of waste electrical and electronics equipment (WEEE); this fund is used to subsidize the recycling and remanufacturing of used electrical and electronic products (see Xie and Bai 2010, Chinese Ministry of Finance 2012). Similarly, the Department for Business, Innovation, and Skills in the UK established a fund of $775,000$ pounds to encourage the reuse of whole appliances, increase the tonnage of separately collected domestic WEEE for recycling, and improve the recycling rate of collected equipment (BIS 2015). As trade-in remanufacturing gains increasing popularity, it is both interesting and relevant to study how the government should design a public policy to regulate the market and enhance social welfare.

The primary goal of this paper is to deepen our understanding of the trade-in remanufacturing practice. Specifically, we analyze the impact of customer purchasing behavior and remanufacturing efficiency on the value of trade-in remanufacturing to different stakeholders. For this purpose, we develop a two-period model in which a profit-maximizing firm sells two generations of a product in a market. We use the customer discount factor to model the intensity of their forward-looking behavior. If this discount factor is large, the customers are highly strategic, and make their purchasing decisions with serious considerations of anticipated *future* utilities. Otherwise, the customer discount factor is low, so customers care *little* about future utilities and are more myopic. In the first period, the firm sells the first-generation product in the market. In the second period, the firm sells the second-generation product to new customers (who have not purchased in the first period); meanwhile, the firm offers trade-in rebates that allow repeat customers (who have purchased in the first period) to exchange used products for new second-generation ones at a discount price. The returned first-generation products are remanufactured by the firm to reduce the production costs of new second-generation products. We explicitly model two benefits from remanufacturing and recycling used products: First, it generates economic value (i.e., revenue) for the firm; second, it helps reduce the product's negative impact on the environment. It is worth noting that both benefits depend on the remanufacturing efficiency of the product. As discussed above, high remanufacturing efficiency normally corresponds to high product durability, i.e., high residual value of used products for customers (Geyer et al. 2007).

1.1. Main Contributions

1.1.1. Value of Trade-in Remanufacturing. A key message of our study is that both customer purchasing behavior and remanufacturing efficiency have important implications for the value of trade-in remanufacturing. Under trade-in remanufacturing, strategic customer behavior acts like a double-edged sword. On one hand, as recognized in the literature, strategic waiting of customers keeps them from purchasing early and is thus detrimental to firm profit. On the other hand, trade-in remanufacturing ensures a high surplus for repeat customers, which encourages strategic customers to purchase early, thus leading to a higher profit for the firm when customers are more strategic. We find that the profit improvement from trade-in remanufacturing with strategic customers can be much more significant than with myopic customers. Thus, our results indicate that a major benefit of trade-in remanufacturing for the firm is to exploit strategic customer behavior, which has not been identified in the previous literature. Moreover, we demonstrate that high remanufacturing efficiency does not necessarily improve the firm's profit: When remanufacturing is overly efficient, the product durability is so high that repeat customers would be discouraged from trading in and upgrading their used products. Therefore, the firm benefits the most when the remanufacturing efficiency is at a moderate level.

From the environmental and social perspectives, the impact of trade-in remanufacturing also depends critically on customer behavior and remanufacturing efficiency. We find that while trade-in remanufacturing generally benefits the environment and social welfare with myopic customers, the opposite is true with strategic customers. With strategic customers, trade-in remanufacturing offers customers a strong incentive to purchase early, which prompts the firm to increase production quantities. This may outweigh the environmental advantage of remanufacturing under general circumstances. Moreover, trade-in remanufacturing allows the firm to exploit the strategic customers, which may reduce the customer surplus and hence hurt the overall social welfare. Higher remanufacturing efficiency does not always help improve the environment and social welfare. In fact, the negative impact of trade-in remanufacturing on the environment is most significant when remanufacturing efficiency is moderate (i.e., either a low or high remanufacturing efficiency would help improve the environment). Our results call for caution when adopting the trade-in remanufacturing strategy. In particular, understanding customer purchasing behavior and remanufacturing efficiency is essential in evaluating this strategy, both for the firm and for the environment.

1.1.2. Government Policy Design. It follows from the above findings that trade-in remanufacturing may create a tension between profitability and sustainability. That is, it can greatly improve firm profit but meanwhile hurt the environment, especially under strategic customers. This motivates us to study how government intervention can resolve this tension and achieve the socially optimal outcome for a market where trade-in remanufacturing is commonly adopted and strategic customer behavior prevails (e.g., the electronics market). The government is modeled as a central planner who aims to maximize the social welfare, i.e., the sum of firm profit and customer surplus less environmental impact. We consider a linear subsidy/tax scheme that applies to the production of all product versions. We show that such a simple subsidy/tax scheme, if designed properly, can induce the social optimum regardless of the customer purchasing behavior. Interestingly, the proposed subsidy/tax policy can also induce the firm to set the socially optimal remanufacturing efficiency. In short, our proposed government policy helps resolve the tension between profitability and sustainability caused by trade-in remanufacturing.

The rest of the paper is organized as follows. Section 2 positions our work in the relevant literature. The model and equilibrium analysis are presented in Section 3. In Sections 4 and 5, we analyze the value of trade-in remanufacturing for the firm and the environment, respectively. Section 6 characterizes the socially optimal government policy. This paper concludes with Section 7. All proofs are given in the Appendix.

2. Literature Review

The impact of customer purchasing behavior upon a firm's operations decisions has received extensive attention in the literature. Bensako and Winston (1990) show that the presence of strategic customers drives a monopolist firm to charge a lower price at the initial stage of the sales season and to mark down less aggressively afterwards. In a revenue management framework, Aviv and Pazgal (2008) demonstrate that the responsive pricing strategy could effectively improve a monopolist's revenue with myopic customers, but this strategy could lead to significant revenue losses with strategic customers. In a follow-up work, Aviv et al. (2015) show that the benefits of responsive pricing and demand learning depend crucially on the nature of customer purchasing behavior: Their values tend to worsen when customers are strategic. Under a newsvendor framework, Cachon and Swinney (2009) show that quick response could deliver a significantly higher value to a retailer in the presence of strategic customers than without them. Caldentey et al. (2016) consider a robust formulation of a monopolist's pricing problem, and characterize the different pricing policies without knowing the customer valuation and arrival timing under different customer purchasing behaviors (i.e., fully strategic and fully myopic customers). The bottom line of this strand of research is that the effectiveness of an operations strategy is very sensitive to customer purchasing behavior. In addition to the above modeling works, several papers also empirically examine the customer purchasing behaviors in, for instance, the airline (Li et al. 2014), soft drink (Hendel and Nevo 2013), and textbook (Chevalier and Goolsbee 2009) industries. The empirical findings suggest that the market is likely to have a mixture of strategic and nonstrategic customers, and the overall customer purchasing behavior is complex. We contribute to this stream of research by investigating how the value of trade-in remanufacturing to the firm and the environment depends on different customer purchasing behaviors.

There is a rapidly growing stream of literature on remanufacturing and closed-loop supply chain management. A comprehensive review of this literature is given by Guide and Van Wassenhove (2009). Savaskan et al. (2004) study the optimal reverse channel structure for the collection of used products from customers. Ferguson and Toktay (2005) analyze the competition between new and remanufactured products (i.e., the cannibalization effect) and characterize the optimal recovery strategy. When remanufacturability is an endogenous decision, Debo et al. (2005) investigate a joint pricing and production technology selection problem of a manufacturer who sells a remanufacturable product to heterogeneous customers. Geyer et al. (2007) demonstrate that, to maximize the economic value of remanufacturing, production cost structure, product life cycle, and component durability need to be carefully coordinated. Atasu et al. (2008) show that remanufacturing could serve as a marketing strategy to target customers in the green segment and, hence, enhance the profitability of the OEM. Galbreth et al. (2013) study how the rate of product innovation affects the firm's reuse and remanufacturing decisions. Gu et al. (2015) investigate the quality design and environmental consequences of green consumerism with remanufacturing. The impact of trade-in rebates has also received some attention in the remanufacturing literature. For example, Ray et al. (2005) examine the value of price discrimination for new and repeat customers with differentiated ages (and qualities) of the products returned through trade-ins for remanufacturing. Our research reveals a new benefit of trade-in remanufacturing: to exploit the forward-looking behavior of strategic customers, which is most significant when remanufacturing efficiency is moderate. In addition, we deliver a new insight that, depending on the customer purchasing behavior and remanufacturing efficiency, trade-in remanufacturing may either create a tension between firm profit and environmental sustainability or simultaneously benefit both the firm and the environment.

Government regulations on remanufacturing and other environmentally relevant operations issues have also been studied in the literature, but for different problem settings. For instance, Calcott and Walls (2000) show that, in a supply chain, a downstream disposal fee charged by the government may not ensure the social optimum, unless supplemented with upstream instruments. Ma et al. (2013) study the impact of a government consumption-subsidy program on a dual-channel closed-loop supply chain. Cohen et al. (2015) characterize the impact of demand uncertainty on government subsidies for green technology adoption.

There are a few papers that investigate trade-in rebates or multiple product introductions in the presence of forward-looking customers. Fudenberg and Tirole (1998) study the monopoly pricing of overlapping generations of a durable good with and without a second-hand market. In an infinite-horizon model setting, Rao et al. (2009) demonstrate that trade-in rebates can alleviate the inefficiencies arising from the lemon problem. Liang et al. (2014) analyze the optimal product rollover strategies in the presence of strategic customers. Lobel et al. (2015) study the new product launch strategy, and show that the technology release pre-commitment can lead to significant profit improvement under forward-looking customer behavior. In this literature, Van Ackere and Reyniers (1995) is probably the closest to our work. They also consider the pricing problem in the presence of trade-ins and compare the market outcomes with strategic and myopic customers. The key difference between our work and theirs is that we highlight the critical role of remanufacturing, whose efficiency would substantially influence the value of trade-in remanufacturing to the firm and the environment. Moreover, we analyze how government should design regulatory policies to maximize social welfare, which is absent in their work.

Finally, our paper is also related to the literature on secondary markets, since the secondary market of a durable good also substantially influences customer purchases (Hendel and Lizzeri 1999). While this literature focuses on the benefits (e.g., Hendel and Lizzeri 1999) and harms (Chen et al. 2013) of secondary markets, our work highlights the impact of customer purchasing behavior and remanufacturing efficiency on both the economic and environmental values of trade-in remanufacturing. We also study the government policy that helps induce the social optimum. The focus and insights of our paper are, therefore, quite different from those in the secondary market literature.

3. Model and Analysis

3.1. Model Setup

We consider a monopoly firm (he) who sells a product to customers (she) in a two-period sales horizon. In the first period, the firm produces the first-generation product at a cost c_1. The potential market size X is *ex-ante* uncertain, and continuously distributed with a distribution function $F(\cdot)$ and a density function $f(\cdot) = F'(\cdot)$. We assume that all customers arrive at the beginning of period 1, but our results and insights continue to hold if there are new customers arriving in period 2. The customers are infinitesimal, each requesting at most one unit of the product in any period. Although demand uncertainty is prevalent with new product introduction, the firm can obtain more accurate demand information as the market matures. Hence, in period 2, the market uncertainty is resolved so the realized market size X becomes known to the firm.

Let V denote a customer's valuation of the first-generation product over the two-period horizon, which is an independent draw from a continuous distribution $G(\cdot)$ on a support $[\underline{v}, \bar{v}]$ $(0 \le \underline{v} < \bar{v})$. We call the customer with product valuation V the type-V customer. At the beginning of the sales horizon, each customer only knows the distribution of her own valuation $G(\cdot)$, but not the realization V. This assumption captures customers' uncertainties about product performance, and fits our setting where the product is brand new at the beginning. In period 2, all customers observe their own type V. Customers who purchased the product in period 1 learn their type V by consuming the product. Customers who did not purchase the product in period 1 learn its quality and fit (thus, their type V) through social learning platforms (e.g., the Amazon customer review system). Hence, the customers are homogeneous *ex ante* (i.e., in period 1), but heterogeneous *ex post* (i.e., in period 2). This is a common setting in models concerning customer purchasing behavior (see, e.g., Xie and Shugan 2001, Su 2009, Swinney 2011). We assume that the valuation distribution $G(\cdot)$ has an increasing failure rate, i.e., $h(v) := g(v)/\bar{G}(v)$ is increasing in v, where $g(\cdot) = G'(\cdot)$ is the density function and $\bar{G}(\cdot) = 1 - G(\cdot)$. This is a mild assumption and can be satisfied by most commonly used distributions. Let $\mu := \mathbb{E}(V) > c_1$, i.e., in expectation, a customer's valuation exceeds the production cost.

The firm offers an upgraded version of the product in period 2. This is a customary practice for product categories like consumer electronics, home appliances, and furniture. A type-V customer has a valuation $(1+\alpha)V$ of the upgraded second-generation product, where $\alpha \ge 0$ is exogenously given and captures the innovation level (e.g., the improved features) of the upgraded product. Accordingly, let the production cost of the second-generation product be c_2. To model product depreciation, we take the approach of Van Ackere and Reyniers (1995): If a type-V customer has already bought the product in period 1, her valuation from consuming the used product in period 2 is $(1-k)V$, where $k \in [0, 1)$ refers to the depreciation factor. Specifically, if k is small, the product is highly durable; if k approaches 1, the product is almost useless in period 2 (either the product is worn out or the technology is obsolete). It can also be computed that the willingness-to-pay of a type-V customer in period 2 is $(1+\alpha)V$ if she *did not* purchase the product in period 1 (i.e., a new customer), and is $(1+\alpha)V - (1-k)V = (k+\alpha)V$ if she *purchased* the product in period

1 (i.e., a repeat customer). We do not explicitly model the secondary market of the used first-generation products, but studying its impact in the presence of trade-in remanufacturing would be an interesting direction for future research.

As widely recognized in the literature, the firm can generate revenues by extracting materials and components from used products (see, e.g., Savaskan et al. 2004, Ray et al. 2005). We now model the revenue-generating effect of remanufacturing. Customers who bought the product in period 1 can trade in the used product for a new second-generation one at a discount price in period 2. Customers may incur an inconvenience cost to trade in used products. We do not explicitly model this cost, but it can be absorbed into the trade-in price for repeat customers without affecting model analysis. The unit net revenue (i.e., cost saving) of remanufacturing is r_2, where $r_2 \in [0, c_2]$. Following Savaskan et al. (2004), we assume all remanufactured products are upgraded to the quality standards of new ones so customers cannot distinguish them from newly made products. So our model best fits the setting where used products were recycled and decomposed so that its materials and components are reused in new product manufacturing. Including a valuation gap between the new and remanufactured products will not change the qualitative insights as long as the gap is sufficiently small. In our model, remanufacturing is performed by the firm itself (or by a third-party remanufacturer with a fixed cost). A potential avenue for future research is to incorporate a strategic third-party remanufacturer with pricing power into the current model setting.

The environmental impact of the product is the aggregate lifetime impact of the product on the environment. The total environmental impact is the production quantity multiplied by the per-unit impact (see, e.g., Thomas 2011, Agrawal et al. 2012). Let $\kappa_i > 0$ $(i = 1, 2)$ denote the unit environmental impact of the first and second generation products, respectively. Such impact may refer to the use of natural resources, emission of harmful gases, and generation of solid wastes. Moreover, the values of κ_i can be estimated by the conventional life-cycle analysis (see, e.g., Agrawal et al. 2012). Let $\iota_2 \in [0, \kappa_2]$ be the unit environmental benefit of remanufacturing the recycled first-generation products in period 2. Note that ι_2 measures the reductions in both the environmental impact of producing the second-generation product and the end-of-use/end-of-life disposals of the first-generation product, by recycling and reusing the materials and components.

A salient feature of our model is that the depreciation factor k (equivalently, the product durability) is correlated with other model primitives c_1, r_2, and ι_2. As discussed above, a small k implies high product durability (e.g., the firm uses better and more durable components and materials); as a result, the remanufacturing efficiency is also higher in this case (Geyer et al. 2007, Apple, Inc. 2017). To model this effect, we assume that the unit revenue generated by remanufacturing, r_2, and the unit environmental benefit of remanufacturing, ι_2, are both concavely decreasing in the depreciation factor k. More durable components and materials would also incur a higher production cost (Debo et al. 2006, Geyer et al. 2007). Thus, we assume c_1 is convexly decreasing in k. In our model, we take product durability as a long-term strategic choice constrained by the technology. Hence, the depreciation factor k is *exogenously* given before the firm makes the pricing and production decisions. However, along with the development of our results, we will briefly discuss how endogenizing the product durability/remanufacturing efficiency decision would affect our key insights.

The sequence of events unfolds as follows. At the beginning of period 1, the firm announces the price p_1 and decides the production quantity Q_1 . Each customer observes p_1, but not Q_1, and decides whether to make a purchase or to wait until period 2. The first-period demand $X_1 \le X$ is then realized, the firm collects his first-period revenue, and all customers stay in the market. Note that X_1 is determined by the collective effect of all customers' purchasing decisions. If $X_1 \le Q_1$, any customer who requests a product can get one in period 1. Otherwise, $X_1 > Q_1$, then the Q_1 products are randomly allocated to the demand, and $X_1 - Q_1$ customers have to wait due to the limited availability. At the end of period 1, the firm sells the leftover inventory at a (discounted) salvage price $s \in (r_2, c_1)$. At the beginning of period 2, the firm learns the realized total market size X, and each individual customer learns her type V. The firm then announces the price p_2^n for new customers as well as the trade-in price $p_2^r \le p_2^n$ ($p_2^n - p_2^r$ is the trade-in rebate); all new customers decide whether to purchase the second-generation product, whereas all repeat customers decide whether to trade in their used products for new second-generation ones. Finally, the firm produces the second-generation products, recycles and remanufactures the used products from repeat customers, and collects the second-period revenue.

To conclude this subsection, we remark that although this paper focuses on a two-period model, all results and insights can be generalized to an infinite-horizon setting where each customer stays in the market for two periods. For conciseness, we define $x \wedge y = \min(x, y)$ and $x^+ := \max(0, x)$. A summary of the notations is given in the Appendix.

3.2. Customer Purchasing Behavior and Equilibrium Analysis

We use $\delta \in (0, 1]$ to denote the risk-free discount factor of the market, which is also the discount factor of the firm. To study the impact of customer purchasing behavior, we denote $\delta_c \in [0, \delta]$ as the discount factor of customers, which measures the intensity of their forward-looking behavior. A large (resp. small) δ_c implies customers care a lot (resp. little) about future utilities, and they are more (resp. less) strategic. We call δ_c the *customer discount factor* and the *forward-looking behavior intensity* interchangeably hereafter. If $\delta_c = \delta$, customers are fully strategic and maximize their long-run utilities; if $\delta_c = 0$, customers are fully myopic and maximize their immediate utilities.

Using customer discount factor to capture their forward-looking behavior is a common approach in the literature (e.g., Levin et al. 2009, Chevalier and Goolsbee 2009). An alternative modeling approach is to assume there are two customer segments (strategic and myopic) in the market (e.g., Su 2007, Li et al. 2014). Under this approach, the intensity of forward-looking behavior is measured by the proportion of strategic customers. Both approaches generate the same qualitative insights, so we will focus on the former one for ease of analysis and exposition.

We adopt the rational expectation (RE) equilibrium framework to characterize the market outcome. Under the RE equilibrium, each player makes decisions based on individual beliefs, which are rationally formed and consistent with actual outcomes. By backward induction, we start with the subgame in period 2. There are $X_2^n = X - (X_1 \wedge Q_1)$ new customers and $X_2^r = X_1 \wedge Q_1$ repeat customers in the market. Since period 2 is the final period in our model, customers with different intensities of forward-looking behavior adopt the same purchasing strategy therein. Hence, regardless of the customer discount factor δ_c, the firm should adopt the same pricing strategy in period 2 as well. Given (X_2^n, X_2^r), let $p_2^n(X_2^n, X_2^r)$ and $Q_2^n(X_2^n, X_2^r)$ be the equilibrium price and production quantity for new customers in period 2. Analogously, we define $p_2^r(X_2^n, X_2^r)$ and $Q_2^r(X_2^n, X_2^r)$ as the equilibrium trade-in price and production quantity for repeat customers, and $\pi_2(X_2^n, X_2^r)$ as the equilibrium second-period profit of the firm.

Lemma 1. *(a) For any (X_2^n, X_2^r), $p_2^n(X_2^n, X_2^r) \equiv p_2^{n*}$ and $p_2^r(X_2^n, X_2^r) \equiv p_2^{r*}$, where $p_2^{r*} < p_2^{n*}$.*
(b) For any (X_2^n, X_2^r), $Q_2^n(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) X_2^n$, and $Q_2^r(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) X_2^r$.
(c) For all (X_2^n, X_2^r), $\pi_2(X_2^n, X_2^r) = \beta_n^* X_2^n + \beta_r^* X_2^r$ for some positive constants β_n^* and β_r^*.

Lemma 1 implies that both the equilibrium price for new customers and the equilibrium trade-in price are independent of the realized market size (X_2^n, X_2^r). In particular, the firm offers positive trade-in rebates to repeat customers (i.e., $p_2^{r*} < p_2^{n*}$). The equilibrium profit of the firm in period 2, $\pi_2(X_2^n, X_2^r)$, is linearly separable in X_2^n and X_2^r, with the coefficients β_n^* and β_r^* capturing the expected per unit profit from new and repeat customers, respectively.

We now analyze the equilibrium market outcome in period 1, starting with customers' purchasing behavior. Each customer rationally anticipates the second-period price for new customers p_2^{n*}, and the second-period trade-in price p_2^{r*}. The expected utility of a customer to purchase the product in period 1 is $\mathcal{U}_p = \mu + \delta_c \sigma_r^*$, where $\sigma_r^* = \mathbb{E}[(k+\alpha)V - p_2^{r*}]^+$. The expected utility of a customer to wait is $\mathcal{U}_w = \delta_c \sigma_n^*$, where $\sigma_n^* = \mathbb{E}[(1+\alpha)V - p_2^{n*}]^+$. Note that σ_n^* (resp. σ_r^*) is the expected surplus of a new (resp. repeat) customer in period 2. Therefore, a customer would opt to make a purchase in period 1 if and only if $\mathcal{U}_p - p_1 \geq \mathcal{U}_w$, i.e., $p_1 \leq \xi_r(\delta_c) := \mathcal{U}_p - \mathcal{U}_w = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$. Following the standard approach in the literature (Xie and Shugan 2001, Su and Zhang 2008, Cachon and Swinney 2011), we assume that all customers will make a purchase in period 1 if p_1 equals their reservation price $\xi_r(\delta_c)$. Thus, with customer discount factor δ_c, the first-period demand, X_1, is given by $X_1 = X \cdot \mathbf{1}_{\{p_1 \le \xi_r(\delta_c)\}}$.

Next, we consider the firm's problem in period 1. To maximize his total expected profit, the firm sets the first-period price $p_1(\delta_c)$ equal to the customer reservation price $\xi_r(\delta_c)$, which is the highest price customers are willing to pay in period 1. Thus, the firm believes that the first-period demand $X_1 = X$, the second-period market size of new customers is $X_2^n = (X - Q_1)^+$, and that of repeat customers is $X_2^r = X \wedge Q_1$. Given the customer discount factor δ_c, the firm sets the first-period production quantity Q_1 to maximize the total expected profit $\Pi_f(Q_1|\delta_c) = \mathbb{E}\{p_1(\delta_c)(X \wedge Q_1) - c_1 Q_1 + s(Q_1 - X)^+ + \delta\pi_2(X_2^n, X_2^r)\}$, where $p_1(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$, $X_2^n = (X - Q_1)^+$, and $X_2^r = X \wedge Q_1$.

To characterize the RE equilibrium, we define an auxiliary variable $m_1^*(\delta_c) := \mu + \delta(\beta_r^* - \beta_n^*) + \delta_c(\sigma_r^* - \sigma_n^*)$. As will be clear in our subsequent analysis, $m_1^*(\delta_c)$ is the first-period effective marginal revenue with customer discount factor δ_c. Based on Lemma 1, the following theorem characterizes the RE equilibrium market outcome.

THEOREM 1. *For any customer discount factor δ_c, there exists a unique RE equilibrium with (a) the price $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$; (b) the production quantity $Q_1^*(\delta_c) = \bar{F}^{-1}(\frac{c_1 - s}{m_1^*(\delta_c) - s})$; and (c) the expected total profit of the firm, $\Pi_f^*(\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}(X \wedge Q_1^*(\delta_c)) - (c_1 - s)Q_1^*(\delta_c) + \delta\beta_n^*\mathbb{E}(X)$.*

Theorem 1 shows that the equilibrium first-period price is the expected valuation of the first-generation product μ, plus the (discounted) expected surplus difference between a repeat customer and a new one in period 2 $\delta_c(\sigma_r^* - \sigma_n^*)$. The equilibrium first-period production quantity, on the other hand, can be determined by the solution of a corresponding newsvendor problem.

3.3. Benchmark Model Without Trade-in Remanufacturing

In the next two sections, we analyze the impact of customer purchasing behavior and remanufacturing efficiency on the value of trade-in remanufacturing, both from the firm's and from the environmental perspectives. To facilitate our comparison, we introduce a benchmark model where the firm does not adopt trade-in remanufacturing. As a consequence, the firm charges the same price for all customers and recycles no used products for remanufacturing in period 2. We call this the No Trade-in Remanufacturing (NTR) model, which is denoted by the superscript "u" hereafter. We use $p_2^u(X_2^n, X_2^r)$ to denote the equilibrium second-period pricing strategy of the firm in the NTR model, which does not depend on customer purchasing behavior. The characterization of $p_2^u(\cdot,\cdot)$ is given in Lemma 2 in the Appendix. As in the base model, customers form beliefs about the second-period prices, and time their purchases. The firm, on the other hand, forms a belief about customers' willingness-to-pay, and bases his (first-period) price and production decisions on this belief. One can show that a unique RE equilibrium exists with any customer forward-looking behavior intensity $\delta_c \in [0, \delta]$ in the NTR model (Theorem 11 in the Appendix). Let $(p_1^{u*}(\delta_c), Q_1^{u*}(\delta_c))$ denote the equilibrium first-period price and production decisions of the firm in the NTR model. Accordingly, the associated equilibrium total firm profit is denoted by $\Pi_f^{u*}(\delta_c)$, which depends on the customer discount factor δ_c.

We define a few notations that will prove useful throughout our analysis. Given the first-period production quantity Q_1, let $\sigma_r^u(Q_1) := \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+$ and $\sigma_n^u(Q_1) := \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+$ $(X_2^n = (X - Q_1)^+,\ X_2^r = X \wedge Q_1)$ denote the expected second-period surpluses for repeat and new customers in the NTR model. Clearly, $\sigma_n^u(\cdot)$ and $\sigma_r^u(\cdot)$ are the counterparts of σ_n^* and σ_r^* in the NTR model. If $Q_1^*(\delta_c) = 0$ or $Q_1^{u*}(\delta_c) = 0$, the problem is reduced to an uninteresting one with no repeat customer on the market in period 2. In this case, neither customer purchasing behavior nor the adoption of trade-in remanufacturing matters. Thus, without loss of generality, we assume $Q_1^*(\cdot) > 0$ and $Q_1^{u*}(\cdot) > 0$ for the rest of our paper.

4. Value of Trade-in Remanufacturing for the Firm

This section investigates the value of trade-in remanufacturing from the firm's perspective. To begin with, we perform a sensitivity analysis with respect to the customer discount factor δ_c, so as to unveil insights on the role of customer purchasing behavior.

THEOREM 2. *(a) Under trade-in remanufacturing, we have (i) $p_1^*(\delta_c)$, $Q_1^*(\delta_c)$, and $\Pi_f^*(\delta_c)$ are strictly increasing (resp. decreasing) in δ_c if $\sigma_r^* > \sigma_n^*$ (resp. $\sigma_r^* < \sigma_n^*$), and (ii) there exist two thresholds $\underline{k}$ and $\bar{k}$, such that $\sigma_r^* > \sigma_n^*$ (resp. $\sigma_r^* < \sigma_n^*$) if and only if $k \in (\underline{k}, \bar{k})$ (resp. $k < \underline{k}$ or $k > \bar{k}$).*

(b) Under no trade-in remanufacturing (i.e., the NTR model), we have (i) $p_1^{u*}(\delta_c)$ is strictly decreasing in δ_c on $[0, \delta_0]$ for some $\delta_0 > 0$, (ii) $Q_1^{u*}(\delta_c)$ and $\Pi_f^{u*}(\delta_c)$ are strictly decreasing in δ_c, and (iii) $\sigma_r^u(Q_1) < \sigma_n^u(Q_1)$ for all $Q_1 \geq 0$.

Interestingly, Theorem 2 demonstrates that the firm can earn a higher profit with more strategic customers if (1) trade-in remanufacturing is adopted, and (2) remanufacturing efficiency is moderate, i.e., $k \in (\underline{k}, \bar{k})$. It is worth noting that increasing remanufacturing efficiency has two effects on the firm: First, it enhances the revenue from remanufacturing (i.e., r_2 increases); second, it improves the durability of the product (i.e., k decreases), which discourages customers from trading in and upgrading their first-generation products. The former effect is to the benefit of the firm, whereas the latter is to the detriment. If remanufacturing efficiency is moderate, the above two effects are well-balanced so that remanufacturing could generate a high revenue without overly discouraging repeat customers from upgrading their products. In this case, repeat customers enjoy a higher expected surplus than new customers in period 2 (i.e., $\sigma_r^* > \sigma_n^*$), thus driving the firm to charge a higher price, produce more, and, consequently, earn a higher profit with more strategic customers (i.e., with a higher δ_c). Therefore, with moderate remanufacturing efficiency, trade-in remanufacturing allows the firm to exploit and benefit from strategic customer behavior. This insight complements the findings in the literature that strategic customer behavior may improve a seller's profit in some retail and airline settings (e.g., Su 2007, Li et al. 2014).

We emphasize that both the trade-in option and the remanufacturing process with proper efficiency are essential for the firm to benefit from strategic customer behavior: The former offers early purchase rewards to repeat customers, which can be well anticipated if customers are strategic, whereas the latter guarantees, in expectation, a repeat customer enjoys a higher surplus than a new customer in period 2. If the firm does not adopt trade-in remanufacturing or the remanufacturing efficiency is too high or too low, however, the expected repeat customer surplus will be lower than the expected new customer surplus (i.e., $\sigma_r^u(Q_1) < \sigma_n^u(Q_1)$ for all Q_1; and $\sigma_r^* < \sigma_n^*$ if $k < \underline{k}$ or $k > \bar{k}$). In these cases, the more strategic the customers, the more reluctant they are to make immediate purchases in period 1, and, as a consequence, the lower the firm profit.

From Theorem 2, we can derive some actionable insights for practitioners. In a market with frequent new product introductions and intensive strategic customer behavior (e.g., the electronics market, see Song and Chintagunta 2003, Plambeck and Wang 2009), the firm should neither completely abandon remanufacturing nor haphazardly improve remanufacturing efficiency. Instead, keeping remanufacturing moderately efficient (i.e., $k \in (\underline{k}, \bar{k})$) enables the firm to leverage the intensive forward-looking customer behavior via its trade-in remanufacturing program. On the other hand, if the firm does adopt trade-in remanufacturing with moderate remanufacturing efficiency, it would be a good idea to induce more intensive strategic customer behavior by extensively advertising the trade-in opportunities in the market.

Our next result characterizes how strategic customer behavior intensity and remanufacturing efficiency impact the value of trade-in remanufacturing to the firm.

Theorem 3. *(a)* $p_1^*(\delta_c) > p_1^{u*}(\delta_c)$ *and* $\Pi_f^*(\delta_c) > \Pi_f^{u*}(\delta_c)$ *for all* $\delta_c \in [0, \delta]$. *(b) There exists a threshold* K*, such that* $\Pi_f^*(\delta_c)$ *is increasing (resp. decreasing) in* k *when* $k \leq K$ *(resp.* $k \geq K$*). (c)* $|\frac{\partial p_1^*(\delta_c)}{\partial k}|$ *is increasing in* δ_c*.*

As one may expect, the firm earns a higher profit under trade-in remanufacturing, since it has higher pricing and production flexibilities. A particularly interesting implication of Theorem 3 is that the firm may not benefit from higher remanufacturing efficiency (note that $\Pi_f^*(\delta_c)$ is increasing in k for $k \leq K$). If remanufacturing is overly efficient ($k \leq K$), further improving remanufacturing efficiency would make the first-generation product so durable that repeat customers are reluctant to trade in and upgrade their used products, thus hurting the firm's profit. The threshold K can be interpreted as the optimal product durability/remanufacturing efficiency for the firm. Our analysis also reveals that the operational impact of remanufacturing efficiency is strengthened by strategic customer behavior. This is because strategic customers are more sensitive to anticipated future adjustments of trade-in price caused by changes in remanufacturing efficiency.

There are three beneficial effects of trade-in remanufacturing: (1) the revenue-generating effect of remanufacturing, i.e., remanufacturing can recover the residual value of used products, (2) the price-discrimination effect of trade-in rebates, i.e., the differentiated prices for new and repeat customers help the firm exploit customer segmentation, and (3) the early-purchase inducing effect of trade-in rebates, i.e., the price discount for repeat customers attracts strategic customers to purchase early. Note that the first two effects benefit the firm regardless of customer behavior, whereas the third effect improves its profit only if customers are strategic. Moreover, while the first effect will be enhanced if remanufacturing efficiency improves (i.e., k decreases), the second and third effects are not monotonic in k. The overall effect of the above three driving forces is that the firm profit $\Pi_f^*(\delta_c)$ is not monotonic in remanufacturing efficiency either.

We now use extensive numerical experiments to derive additional insights into the value of trade-in remanufacturing for the firm under different customer behaviors. The metric of interest is $\gamma(\delta_c) = (\Pi_f^*(\delta_c) - \Pi_f^{u*}(\delta_c))/\Pi_f^{u*}(\delta_c) \times 100\%$, which measures the relative profit improvement of trade-in remanufacturing with customer discount factor δ_c. We evaluate $\gamma(\delta_c)$ under a set of 3125 parameter combinations that cover a wide range of reasonable problem scenarios. The parameter values used in our numerical experiments are summarized in Table 1. Our focus is on examining the impact of customer discount factor δ_c, which corresponds to different customer purchasing behaviors, on $\gamma(\delta_c)$, the (relative) profit improvement of trade-in remanufacturing.

δ :	0.95;
δ_c :	$\{0, 0.25\delta, 0.5\delta, 0.75\delta, \delta\} = \{0, 0.2375, 0.475, 0.7125, 0.95\}$;
X :	Gamma distribution with mean 100 and standard deviation $\{50, 60, 70, 80, 90\}$;
V :	Uniformly distributed on $[0, 1]$;
k :	$\{0.3, 0.4, 0.6, 0.6, 0.7\}$;
c_1:	$\{0.05, 0.1, 0.15, 0.2, 0.25\}$;
κ_1:	1;
s:	0;
c_2 :	$\{0.25, 0.2625, 0.275, 0.2875, 0.3\}$;
κ_2 :	0.75;
r_2:	0;
α:	{0,0.05,0.1,0.15,0.2}
ι_2:	0.3;

Table 1 Summary of Numerical Setup

We present the summary statistics of $\gamma(\delta_c)$ in Table 2, and the box-plot of $\gamma(\delta_c)$ in Figure 1. There are a couple of interesting observations from Table 1 and Figure 1. First, the value of trade-in remanufacturing generally improves as the customers become more strategic. Second, and more importantly, compared to the case with myopic customers ($\delta_c = 0$), the value of trade-in remanufacturing could be much higher when customers are strategic (e.g., comparing the results of $\delta_c = 0$ and $\delta_c = 0.95$). This suggests that the benefits of exploiting strategic customer behavior could be much more significant than the benefits of revenue generating and price discrimination (the latter two benefits can be already captured with myopic customers). Thus, our numerical results deliver an important message that a major driver of using trade-in remanufacturing is to exploit strategic customer behavior, which has not been identified in the previous literature. In addition, such a benefit will be magnified if customers are more strategic.

	Min	25th percentile	Median	75th percentile	Max	Mean	Standard deviation
$\delta_c = 0$	0.008	1.0	2.5	4.6	11.7	3.1	2.5
$\delta_c = 0.2375$	3.2	7.4	10.5	13.6	18.2	10.4	3.7
$\delta_c = 0.475$	6.6	11.4	16.6	22.5	30.5	17.1	6.1
$\delta_c = 0.7125$	7.1	16.7	23.2	31.8	47.6	24.5	9.4
$\delta_c = 0.95$	5.8	19.9	28.3	40.1	61.6	30.2	13.1

Table 2 Summary Statistics of $\gamma(\delta_c)$ (%)

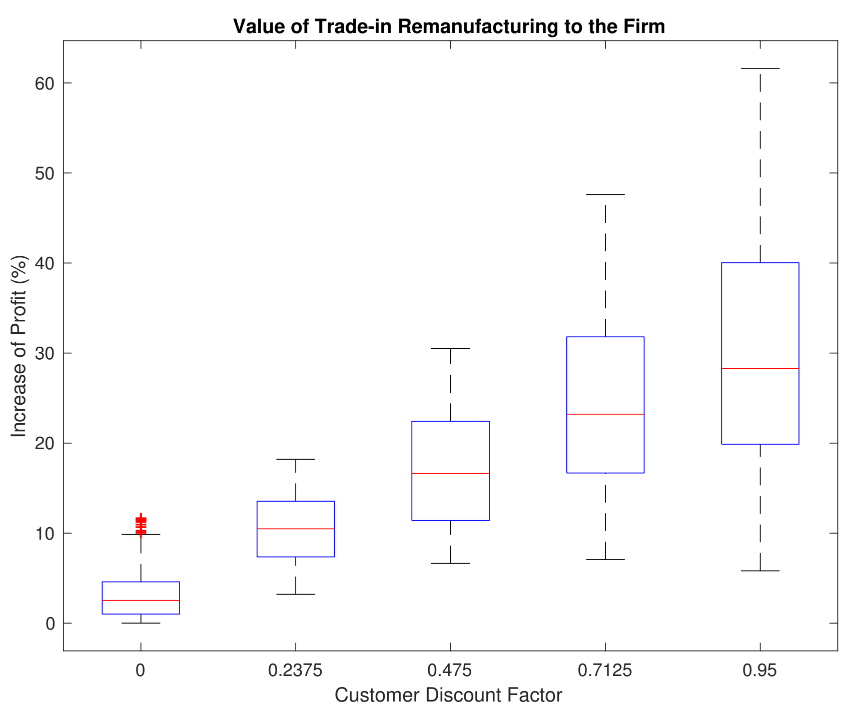

Figure 1 Box-Plot of $\gamma(\delta_c)$

It is worthwhile differentiating our results from the durable goods and secondary market literature that studies the value of trade-ins/resales in the presence of forward-looking customers (e.g., Van Ackere and Reyniers 1995, Fudenberg and Tirole 1998, Waldman 2003, Hendel and Lizzeri 1999). The main finding of this literature is that, when customers are forward-looking, trade-ins and resales in the secondary market could be beneficial to the firm, because customers can anticipate their values when making initial purchases. Our results complement this insight in the context of remanufacturing. Specifically, if complemented with a moderately efficient remanufacturing process, trade-ins could serve as a leverage for the firm to exploit strategic customer behavior (Theorem 2, Table 1 and Figure 1). While remanufacturing generates revenue, the firm does not benefit from overly efficient remanufacturing under trade-ins (Theorem 3). Both results highlight that, with moderate remanufacturing efficiency (thus moderate product durability), the value of trade-ins to the firm could be further elevated. Overly efficient remanufacturing, however, coincides with high product durability and production cost, and therefore discourages repeat customers from upgrading their used products. This mechanism, which is in line with the idea of planned obsolescence (e.g., Waldman 2003), dilutes the value of trade-in remanufacturing to the firm under high remanufacturing efficiency.

To conclude this section, we briefly discuss the setting with endogenized remanufacturing efficiency. In the literature, the design of remanufacturability is usually studied without taking into account customers' forward-looking behavior (e.g., Debo et al. 2005), which is a key element of our model. An immediate implication of Theorem 3 is that, if the firm can adjust its remanufacturing efficiency at no additional cost, then the optimal level of remanufacturing efficiency K will be chosen. In practice, however, higher remanufacturing efficiency requires higher investments in, for instance, recycling and remanufacturing technology (e.g., the robot Liam invented by Apple for remanufacturing; see Apple, Inc. 2017). Thus, one can model the upfront fixed cost of installing remanufacturing technology $c_t(k)$ as a convexly decreasing function of the depreciation factor k. The equilibrium depreciation level k^* should be the one that maximizes the profit with endogenized remanufacturing efficiency, i.e., $k^* = \arg\max_{k\in[0,1]}\{\Pi_f^*(\delta_c) - c_t(k)\}$. Since $\Pi_f^*(\delta_c) - c_t(k)$ is not necessarily quasiconcave in k, the analysis under endogenized remanufacturing efficiency is quite challenging. However, we can show that $k^* > K > 0$, so the insight that overly efficient remanufacturing could be detrimental to the firm will still hold. Furthermore, our numerical results suggest that how the value of trade-in remanufacturing depends on the customer discount factor δ_c is robust over different values of k. Thus, even under endogenized remanufacturing efficiency, trade-in remanufacturing could help exploit strategic customer behavior, and delivers higher value for the firm if customers are more strategic.

5. Value of Trade-in Remanufacturing for the Environment

We proceed to examine the environmental value of trade-in remanufacturing under different customer behaviors and remanufacturing efficiencies. To begin with, we study how strategic customer behavior intensity impacts the environmental value of trade-in remanufacturing. In equilibrium, the total environmental impact should be the difference between the total environmental impact of production/disposal and the total environmental benefit of remanufacturing. Hence, the equilibrium environmental impact with trade-in remanufacturing is $I_e^*(\delta_c) = \mathbb{E}\{\kappa_1 Q_1^*(\delta_c) + \delta\kappa_2(Q_2^n(X_2^{n*}, X_2^{r*}) + Q_2^r(X_2^{n*}, X_2^{r*})) - \delta\iota_2 Q_2^r(X_2^{n*}, X_2^{r*})\}$, where $X_2^{n*} = (X - Q_1^*(\delta_c))^+$ and $X_2^{r*} = X \wedge Q_1^*(\delta_c)$. In this section, we make an additional assumption that $\kappa_1 \geq \kappa_2 \bar{G}_2\left(\frac{p_2^{n*}}{1+\alpha}\right)$. This assumption is not restrictive in practice, and can be satisfied when the environmental impact of the first-generation product is not too low. In particular, it applies to the case where the newer generation product dominates the older generation in terms of environmental sustainability, i.e., $\kappa_1 \geq \kappa_2 > \delta\kappa_2 \bar{G}_2\left(\frac{p_2^{n*}}{1+\alpha}\right)$.

How does customer purchasing behavior affect the value of trade-in remanufacturing to the environment? The answer is given in the next theorem, where we define $I_e^{u*}(\delta_c)$ as the equilibrium total environmental impact in the NTR model.

THEOREM 4. *(a) There exists a threshold $\bar{\delta}_q < \delta$, such that if $\delta_c > \bar{\delta}_q$, $I_e^*(\delta_c) > I_e^{u*}(\delta_c)$ for $\delta_c > \bar{\delta}_q$ unless ι_2 is too large (formally specified in the Appendix).*

(b) Assume that c_1 is sufficiently small (formally specified in the Appendix). There exists a threshold $\underline{\delta}_q > 0$, such that $I_e^{u*}(\delta_c) > I_e^*(\delta_c)$ for $\delta_c < \underline{\delta}_q$ unless ι_2 is too small (formally specified in the Appendix).

Theorem 4 reveals a dichotomy on the environmental value of trade-in remanufacturing. With highly strategic customers, trade-in remanufacturing is likely to be detrimental to the environment, whereas, with mostly myopic customers, trade-in remanufacturing may benefit both the firm and the environment. Our next goal is to understand why the contrasting effects of trade-in remanufacturing with different customer behaviors would occur. Recall that trade-in remanufacturing offers strategic customers early-purchase rewards, so they purchase more in the first place and recycle used products more frequently. As a consequence, trade-in remanufacturing induces larger production quantities and accelerates product rollover, which places higher pressure on the environment. If customers are myopic, however, trade-in remanufacturing could not motivate them to purchase early. Instead, its price-discrimination effect enables the firm to earn a higher (per-customer) profit from new customers. Hence, the firm will lower the first-period production quantity to enlarge the (potential) second-period market size of new customers and to decelerate product rollover. The following theorem formalizes the above intuition and characterizes the contrasting effects of trade-in remanufacturing on the production decision under different customer behaviors.

Theorem 5. *(a) $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$ for all $\delta_c \in (\bar{\delta}_q, \delta]$.*

(b) Assume that c_1 is sufficiently small. $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$ for all $\delta_c \in [0, \underline{\delta}_q)$. In particular, if $\sigma_r^* > \sigma_n^*$, we have $\bar{\delta}_q = \underline{\delta}_q$, i.e., $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$ for all $\delta_c > \bar{\delta}_q = \underline{\delta}_q$ and $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$ for all $\delta_c < \bar{\delta}_q = \underline{\delta}_q$.

Theorem 5(a) and Theorem 4(a) echo the findings of a few related works in the literature (e.g., Debo et al. 2005, Galbreth et al. 2013, Gu et al. 2015) that remanufacturing may increase the production quantity and thus worsen the environment. Our analysis strengthens this insight and demonstrates that the environmental value of trade-in remanufacturing depends critically on customer purchasing behavior. If customers are mostly myopic, trade-in remanufacturing actually leads to lower production quantities and, thus, a better environment (Theorem 5(b) and Theorem 4(b)). This result highlights the necessity of understanding customer purchasing behavior in the market when evaluating the environmental value of trade-in remanufacturing.

We now numerically illustrate the environmental values of trade-in remanufacturing under different customer purchasing behaviors. We employ the same numerical setup as in Section 4 (see Table 1). We are interested in the following metric: $\eta(\delta_c) := (I_e^*(\delta_c) - I_e^{u*}(\delta_c))/I_e^{u*}(\delta_c) \times 100\%$, referring to the relative change in environmental impact after adopting trade-in remanufacturing. If $\eta(\delta_c) > 0$, trade-in remanufacturing increases the total negative impact and is, thus, detrimental to the environment. Otherwise, $\eta(\delta_c) \leq 0$, trade-in remanufacturing benefits the environment.

In our experiments, we evaluate $\eta(\delta_c)$ under the 3125 parameter combinations detailed in Table 1 and obtain the following findings: (1) Under each parameter combination, $\eta(\delta_c)$ is significantly higher as δ_c increases (i.e., customers become more strategic); and (2) it exhibits a clear pattern that, as δ_c increases, the proportion of problem instances with a positive $\eta(\delta_c)$ also becomes larger. For different values of δ_c, the summary statistics are presented in Table 3, whereas the box-plot is depicted in Figure 2.

	Min	25th percentile	Median	75th percentile	Max	Mean	Standard deviation
$\delta_c = 0$	-10.2	-6.9	-5.5	-3.8	4.5	-5.0	2.7
$\delta_c = 0.2375$	-5.6	-3.6	-1.8	7.9	39.9	2.6	8.7
$\delta_c = 0.475$	-3.9	-0.8	3.0	26.8	88.7	13.1	18.6
$\delta_c = 0.7125$	-2.3	4.3	15.8	58.2	129.1	30.9	30.5
$\delta_c = 0.95$	-1.2	10.0	37.8	87.1	172.0	49.2	41.4

Table 3 Summary Statistics of $\eta(\delta_c)$ (%)

Table 3 and Figure 2 confirm that the environmental value of trade-in remanufacturing is highly sensitive to customer purchasing behavior. Trade-in remanufacturing leads to much higher total environmental impact with more intensive strategic customer behavior ($\eta(\delta_c)$ is significantly higher with a larger δ_c). Though beneficial to the firm (see Table 1 and Figure 1), the early-purchase

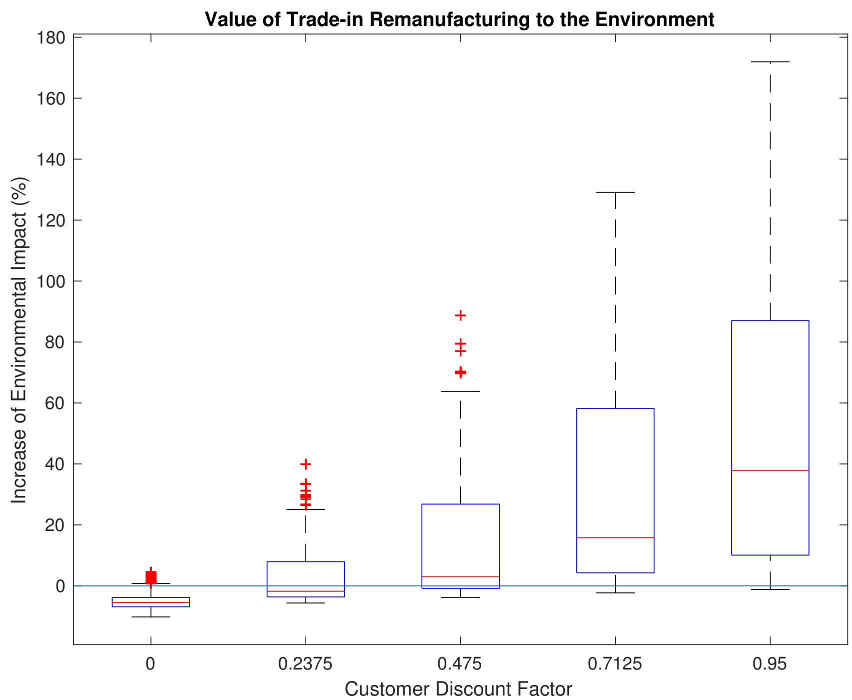

Figure 2 **Box-Plot of $\eta(\delta_c)$**

inducing effect of trade-in remanufacturing gives rise to much higher production quantities under more intensive forward-looking customer behavior, thus dominating the recycling effect of remanufacturing and leading to a much worse outcome from an environmental perspective.

The above results suggest that customer purchasing behavior has opposing effects on the value of trade-in remanufacturing: Intensive forward-looking behavior of customers makes this strategy attractive to the firm, but not desirable for the environment. In particular, trade-in remanufacturing may create a tension between firm profitability and environmental sustainability when customers are *highly strategic*, but benefits both the firm and the environment with *myopic customers*. When customers are highly strategic ($\delta_c \approx \delta$), the early-purchase inducing effect dominates the environmental benefit of remanufacturing. In this case, the firm benefits from trade-in remanufacturing significantly, but the environment suffers a lot. Thanks to the economic and environmental benefits of remanufacturing and the price-discrimination effect of trade-in rebates, both the firm and the environment, however, would benefit from trade-in remanufacturing when customers do not exhibit strong strategic purchasing behaviors (i.e., $\delta_c \approx 0$).

Our results reveal a tension between profitability and sustainability under the adoption of trade-in remanufacturing. The following theorem further shows that this tension is most prominent under moderate remanufacturing efficiency.

THEOREM 6. *(a) $I_e^*(\delta_c)$ is strictly increasing (resp. decreasing) in δ_c if $k \in (\underline{k}, \bar{k})$ (resp. $k < \underline{k}$ or $k > \bar{k}$). (b) There exists a threshold K_e such that $I_e^*(\delta_c)$ is increasing (resp. decreasing) in k if $k \leq K_e$ (reps. $k \geq K_e$).*

By comparing Theorem 6 with Theorems 2 and 3, one can see that trade-in remanufacturing has opposite impacts on the environment and firm profit. In the scenario where the impact on firm profit is most significant (i.e., remanufacturing efficiency is moderate), the associated environmental impact is also the highest. This observation further highlights the aforementioned profitability-sustainability tension under trade-in remanufacturing, and demonstrates that this tension is most significant when remanufacturing efficiency is moderate.

Although an increased production quantity means more pressure on the environment, it also increases the consumption level of the product. We next explore the impact of trade-in remanufacturing on total customer surplus under different customer purchasing behaviors. When customers are highly strategic (i.e., the profitability-sustainability tension is most intensive), there are two opposing effects of trade-in remanufacturing on customer surplus. First, the first-period production quantity is larger ($Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$ for $\delta_c > \bar{\delta}_q$), so that customers can earn a higher total surplus in period 1. Second, the second-period price for new customers is lower ($p_2^u(X_2^n, X_2^r) < p_2^{n*}$), so that the customer surplus will suffer in period 2 ($\sigma_n^* < \sigma_n^u(Q_1^{u*}(\delta_c))$). If the first (resp. second) effect plays a dominating role, trade-in remanufacturing enhances (resp. reduces) customer surplus. Let S_c stand for total customer surplus. The following theorem shows the latter effect actually dominates, and thus customer surplus is diminished by trade-in remanufacturing if customers are highly strategic.

THEOREM 7. *There exists a threshold $\bar{\delta}_s$ such that $S_c^*(\delta_c) < S_c^{u*}(\delta_c)$ for $\delta_c > \bar{\delta}_s$.*

Theorem 7 compares the total customer surpluses in the base model and the NTR model. In particular, we show that, when customers are sufficiently strategic (i.e., $\delta_c > \bar{\delta}_s$), they are worse off with the adoption of trade-in remanufacturing ($S_c^*(\delta_c) < S_c^{u*}(\delta_c)$). Strategic customers well perceive the potential price discounts of trade-in rebates, and, thus, are more willing to purchase in period 1. The firm, on the other hand, extracts their second-period surpluses by complementing trade-in remanufacturing with a wisely-designed first-period pricing and production strategy. Interestingly, Theorem 7 contrasts with our intuition that higher production quantities (see Theorem 5(a)) would lead to higher consumptions and thus a higher customer surplus. However, our analysis suggests that, under intensive strategic customer behavior, trade-in remanufacturing increases production quantities (thus hurting the environment) without improving customer surplus. Furthermore, our numerical study indicates that the social welfare (i.e., firm profit plus customer surplus less environmental impact) may decrease under trade-in remanufacturing as well.

To summarize, customer purchasing behavior and remanufacturing efficiency play important roles in the economic and environmental values of trade-in remanufacturing. In a market with not-so-strategic customers, trade-in remanufacturing benefits both the firm and the environment. With highly strategic customers, however, trade-in remanufacturing would be even more beneficial to the firm; meanwhile it seriously hurts the environment, decreases customer surplus, and possibly lowers social welfare. In this case, the value of trade-in remanufacturing is mainly about facilitating the firm to exploit strategic customer behavior, which is much more significant than the widely recognized revenue-generating and environmental benefits of remanufacturing. The tension between profitability and sustainability, in addition, is strengthened by moderate remanufacturing efficiency. In this case, remanufacturing generates revenue without overly discouraging repeat customers from upgrading used products through the trade-in program, thus prompting the firm to produce more and accelerate product rollover. In short, when making decisions related to trade-in remanufacturing, firms and policy-makers should keep in mind the customer purchasing behavior and remanufacturing efficiency in the focal market.

6. Social Optimum and Government Intervention

With increasing societal awareness of sustainability, the question of how to regulate a market with environmental concerns has attracted increasing attention from the government (e.g., Xie and Bai 2010, BIS 2015). As shown in Sections 4 and 5, adopting trade-in remanufacturing may create a tension between firm profitability and environmental sustainability under intensive strategic customer behavior. In this section, we analyze how a policy-maker (e.g., government) can design the public policy to resolve this tension and maximize the social welfare.

We first characterize the socially optimal market outcome by assuming that the government can set the prices and production levels in both periods, with an objective to maximize the social welfare. Let W_s denote the social welfare, which is defined by the expected profit of the firm Π_f, plus the expected customer surplus S_c, net the expected environmental impact I_e, i.e.,

$$W_s = \Pi_f + S_c - I_e.$$

Note that the government revenues and costs to regulate the market (i.e., taxes and subsidies) are not included in the computation of social welfare. This is because the taxes and subsidies are transactions between different stakeholders in a society, and, therefore, would *not* affect social welfare directly. All results and qualitative insights derived in this section are robust, and will continue to hold if the government has a budget constraint so that the total expected cost to implement the government policy cannot exceed the budget limit.

As in the base model, we start with the second-period pricing and production problem. For any given realized market size (X_2^n, X_2^r) in period 2, we use $(p_s^n(X_2^n, X_2^r), p_s^r(X_2^n, X_2^r))$ to denote the equilibrium second-period pricing strategy that maximizes the social welfare, and denote $w_2(X_2^n, X_2^r)$ as the equilibrium second-period social welfare. As shown by Lemma 4 (in the Appendix), under the socially optimal second-period pricing strategy, the prices for new and repeat customers are equal to the respective net total unit production and environmental cost (i.e., $p_s^n(X_2^n, X_2^r) \equiv p_s^{n*} = c_2 + \kappa_2$ and $p_s^r(X_2^n, X_2^r) \equiv p_s^{r*} = c_2 - r_2 + \kappa_2 - \iota_2$). Moreover, the equilibrium social welfare is linear in the realized market size (X_2^n, X_2^r). That is, $w_2(X_2^n, X_2^r) = \sigma_n^{s*} X_2^n + \sigma_r^{s*} X_2^r$, where the linear coefficient σ_n^{s*} (resp. σ_r^{s*}) is the equilibrium expected surplus of a new (resp. repeat) customer, which is also the equilibrium unit social welfare of selling to new (resp. repeat) customers in period 2.

In period 1, the government and customers base their decisions on rational beliefs. Let $(p_1^{s*}(\delta_c), Q_1^{s*}(\delta_c))$ denote the equilibrium first-period price and production quantity with customer discount factor δ_c. Analogous to the base model, we introduce the first-period effective marginal welfare, $m_1^{s*} := \mu + \delta(\sigma_r^{s*} - \sigma_n^{s*})$, which measures the marginal social welfare to produce in period 1. The following theorem characterizes the social welfare maximizing equilibrium outcome, and analyzes the impact of customer purchasing behavior and remanufacturing efficiency.

THEOREM 8. *(a) With customer discount factor δ_c, we have $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$; $Q_1^{s*}(\delta_c) = \bar{F}^{-1}(\frac{c_1+\kappa_1-s}{m_1^{s*}-s})$; and the equilibrium expected social welfare is $W_s^*(\delta_c) = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 + \kappa_1 - s)Q_1^{s*}(\delta_c) + \delta\sigma_n^{s*}\mathbb{E}[X]$.*

(b) $p_1^{s*}(\delta_c)$ is strictly increasing (resp. decreasing) in δ_c if and only if $\sigma_r^{s*} > \sigma_n^{s*}$ (resp. $\sigma_r^{s*} < \sigma_n^{s*}$). There exist two thresholds $\underline{k}_s$ and $\bar{k}_s$ ($\underline{k}_s < \bar{k}_s$), such that $\sigma_r^{s*} > \sigma_n^{s*}$ (resp. $\sigma_r^{s*} < \sigma_n^{s*}$) if and only if $k \in (\underline{k}_s, \bar{k}_s)$ (resp. $k < \underline{k}_s$ or $k > \bar{k}_s$). $Q_1^{s*}(\delta_c)$ and $W_s^*(\delta_c)$ are independent of δ_c.

(c) There exists a threshold K_s, such that $W_s^*(\delta_c)$ is increasing (resp. decreasing) in k for $k \leq K_s$ (resp. $k \geq K_s$).

Since the social planner balances firm profit, customer surplus, and environmental impact, whereas the firm maximizes his own profit only, the social-welfare-maximizing equilibrium outcome may be quite different from the profit-maximizing one, as shown by comparing Theorem 8 with Theorem 1. Observe that the equilibrium social-welfare-maximizing first-period production quantity $Q_1^{s*}(\delta_c)$ is *independent* of customer discount factor δ_c; so is the equilibrium social welfare $W_s^*(\delta_c)$. This is in sharp contrast to the equilibrium outcome in the base model, which depends critically on customer purchasing behavior (Theorem 2). Similar to the profit-maximizing equilibrium, the equilibrium social-welfare-maximizing first-period price $p_1^{s*}(\delta_c)$ depends on the forward-looking customer behavior intensity δ_c. As remanufacturing becomes more efficient (i.e., k decreases), the optimal social welfare, $W_s^*(\delta_c)$ first increases and then decreases. Therefore, higher remanufacturing efficiency does not necessarily improve the social welfare, even if the government tries to maximize the social welfare by directly exercising price and production decisions. Overly efficient remanufacturing is associated with an overly durable product, thus discouraging customers from trading in their used products. Such mechanism hurts the overall social welfare. This is in a similar spirit to the impact of remanufacturing efficiency in the base model where the firm seeks to maximize its own profit (Theorem 3(b)).

We proceed to study how the government, whose objective is to maximize the expected social welfare W_s, could induce the firm, whose objective is to maximize his expected profit Π_f, to set the socially optimal prices and production quantities. We consider a general class of policies under which the government subsidies (resp. taxes) are provided (resp. charged) for the sales of both generation products, as well as recycling/remanufacturing used products. Specifically, let $s_g := (s_1, s_2, s_r)$ denote the subsidy/tax scheme. The government offers the firm a per-unit subsidy s_1 for the production of the first-generation product, a per-unit subsidy s_2 for the production of the second-generation product, and a per-unit subsidy s_r for remanufacturing/recycling. If $s_i < 0$ $(i = 1, 2, r)$, the government taxes the production of respective product version or the recycling and remanufacturing of used products. This general subsidy/tax scheme is motivated by the current common practice in the electronics industry that the government establishes a fund for the treatment of WEEEs. OEMs contribute to this fund in the form of taxes, whereas firms that recycle and remanufacture used products get subsidies from this fund. See Chinese Ministry of Finance (2012) and BIS (2015) for examples in China and UK, respectively. In our model, the firm both manufactures new products and recycles used ones, so it may get both taxed and subsidized. The proposed government subsidization/taxation policy is quite general. Quite a few special forms of the proposed general policy have been discussed in the literature; see, e.g., Calcott and Walls (2000), Webster and Mitra (2007), Ma et al. (2013), and Wang et al. (2014).

Theorem 9. *For each customer discount factor $\delta_c \in [0, \delta]$, there exists a unique linear subsidy/tax scheme $s_g^*(\delta_c) = (s_1^*(\delta_c), s_2^*(\delta_c), s_r^*(\delta_c))$, under which the social-welfare maximizing RE equilibrium outcome is achieved.*

For any customer discount factor δ_c, the government can use a simple linear subsidy/tax scheme, $s_g^*(\delta_c)$, to induce the social optimum. The linear subsidy/tax policy s_g has great flexibilities in controlling the margin of the firm and the willingness-to-pay of customers. Hence, the government can use this incentive scheme, if well designed, to regulate the market and ensure that the firm sets the socially optimal prices and production quantities, and the customers make the socially optimal purchasing decisions accordingly. More specifically, the government should provide a combined subsidy/tax scheme for the production of both product generations and the recycling of used products. The components of $s_g^*(\delta_c)$ may have different signs, so it is possible that the government will tax the firm on one product generation and subsidize it for another (e.g., the Chinese government charges the OEM for the production of electrical and electronic products and subsidizes those who recycle and re-manufacture e-wastes). This phenomenon results from the government's goal of balancing the trade-off between firm profit, customer surplus, and environmental impact. We remark that, expectedly, the optimal subsidy/tax rate for the first-generation product $s_1^*(\delta_c)$ is sensitive to changes in customer purchasing behavior δ_c, whereas the remanufacturing efficiency k will affect $s_1^*(\delta_c)$ and the optimal subsidy/tax rate for remanufacturing $s_r^*(\delta_c)$.

To promote remanufacturing and leverage its environmental benefit, the government sometimes adopts the policy to subsidize for remanufacturing alone (e.g., Chen 2015, The Recycler 2015). This intuitive policy is a special case of our general subsidy/tax scheme with $s_1 = s_2 = 0$ and $s_r > 0$. However, this policy actually intensifies the aforementioned profitability-sustainability tension: It further increases the firm profit and environmental impact simultaneously. This is because subsidizing remanufactured products not only promotes the adoption of remanufacturing, but also increases the production levels of the first-generation product. The environment thus suffers from the increased production levels under the subsidization for remanufacturing alone. Therefore, the government should be careful about designing the subsidization policy, because haphazard subsidization for trade-in remanufacturing may result in an undesired outcome.

Finally, we examine the firm's profit under the subsidy/tax scheme that induces the social optimum. Specifically, we are interested in how customer purchasing behavior and remanufacturing efficiency will impact the firm's profit under the socially optimal government policy.

THEOREM 10. *(a) The profit of the firm under the subsidy/tax scheme, $\Pi_f^{s*}(\delta_c)$, is independent of δ_c. (b) $\Pi_f^{s*}(\delta_c)$ is increasing (resp. decreasing) in k if $k \le K_s$ (resp. $k \ge K_s$). Thus, under the subsidy/tax scheme $s_g^*(\delta_c)$, if the firm has the flexibility to set remanufacturing efficiency k, it will select the socially optimal one K_s.*

In contrast to the setting where government regulation is not explicitly considered (Theorem 2(a)), the firm profit under the socially optimal government policy is independent of customer purchasing behavior. This property further highlights the capability of the proposed subsidy/tax scheme to counter strategic customer behavior. Another salient feature of the socially optimal subsidy/tax scheme is that, if the firm can adjust remanufacturing efficiency without additional costs, this policy can also induce the firm to set the socially optimal remanufacturing efficiency, K_s. Therefore, the proposed subsidy/tax scheme also incentivizes the firm to make the socially optimal remanufacturing technology choice by setting remanufacturing efficiency at the socially optimal level K_s.

In summary, to alleviate the tension between profitability and sustainability and achieve the social optimum, it suffices for the government to use an easy-to-implement linear subsidy/tax scheme for the production of both product generations and remanufacturing. The proposed subsidy/tax scheme could induce the firm to both adopt the operational (i.e., production and pricing) strategies that maximize the social welfare, and to set the (socially) optimal remanufacturing efficiency level.

7. Conclusion

In this paper, we develop an analytical model to study how customer purchasing behavior and remanufacturing efficiency influence the economic and environmental values of trade-in remanufacturing. From the firm's perspective, we identify a new benefit of trade-in remanufacturing, i.e., it helps exploit the forward-looking behavior of strategic customers. A trade-in rebate essentially offers an early purchase reward and thus can deliver higher additional value when customers are highly strategic. In particular, under the adoption of trade-in remanufacturing with moderate remanufacturing efficiency, more intensive strategic customer behavior can increase the firm profit. However, more efficient remanufacturing may not necessarily enhance the firm profit, because high remanufacturing efficiency is accompanied with high product durability, which discourages repeat customers from trading in and upgrading their used products.

From an environmental perspective, the value of trade-in remanufacturing depends on the intensity of strategic customer behavior. With highly strategic customers, this business practice decreases the unit environmental impact, but increases the production quantities through the early-purchase inducing effect. Overall, trade-in remanufacturing may have a significant negative impact on the environment. Such negative impact is intensified by moderate remanufacturing efficiency, with which the firm benefits most from trade-in remanufacturing. Moreover, under intensive strategic customer behavior, adopting trade-in remanufacturing may decrease the customer surplus and social welfare. Hence, for a market with highly strategic customers, caution is needed when adopting trade-in remanufacturing, because it could be detrimental to the environment and the society. With not-so-strategic customers, however, trade-in remanufacturing leads to a lower first-period production quantity and, thus, generally improves the environment. Our results indicate that customer purchasing behavior and remanufacturing efficiency play important roles in evaluating the impact of trade-in remanufacturing on the environment. Specifically, under intensive strategic customer behavior and moderate remanufacturing efficiency, trade-in remanufacturing creates a tension between firm profitability and environmental sustainability; however, with (nearly) myopic customers, it generally benefits both the firm and the environment.

To resolve the tension caused by trade-in remanufacturing, we also study the government policy that balances firm profit, customer surplus, and environmental impact. To achieve the socially optimal outcome, it suffices for the government to employ a simple linear incentive scheme, which imposes either subsidies or taxes on the production of both product generations as well as remanufacturing. Such policy counters strategic customer behavior and induces a social optimum that does not depend on customer purchasing behavior. In addition, the proposed subsidy/tax scheme can induce the firm to set the socially optimal remanufacturing efficiency.

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Appendix A: Table of Notations

Table 4 Summary of Notations

X :	market size (total number of potential customers)	c_2 :	unit production cost of 1st-generation product
$F(\cdot)$:	distribution function of X	κ_2 :	unit environmental impact of 2nd-generation product
X_1 :	realized demand in period 1	r_2:	unit net revenue of remanufacturing for firm
X_2^n :	market size of new customers in period 2	ι_2:	unit environmental benefit of remanufacturing
X_2^r :	market size of repeat customers in period 2	e_2:	unit total benefit of remanufacturing, $e_2 = r_2 + \iota_2$
V :	customer valuation for 1st-generation product	p_1:	price for 1st-generation product
$G(\cdot)$:	distribution function of V, $\bar{G}(\cdot) = 1 - G(\cdot)$	Q_1:	production quantity in period 1
$g(\cdot)$:	density function of V	p_2^n :	price for new customers in period 2
$h(\cdot)$:	hazard rate function of V, i.e., $h(v) = g(v)/\bar{G}(v)$	p_2^r :	price for repeat customers in period 2
α:	innovation level of 2nd-generation product	Q_2^n :	production quantity for new customers in period 2
k :	product depreciation	Q_2^r :	production quantity for repeat customers in period 2
c_1:	unit production cost of 1st-generation product	δ :	discount factor for firm
κ_1:	unit environmental impact of 1st-generation product	δ_c :	discount factor for customers

Appendix B: Auxiliary Results

In this section, we present some auxiliary results in the NTR model and the model of social optimum. These results are building blocks of our subsequent analysis. The proofs of these results are available from the authors upon request. To begin with, we characterize the second-period equilibrium pricing and production strategy in the NTR model. Let $Q_u^n(X_2^n, X_2^r)$ and $Q_u^r(X_2^n, X_2^r)$ be the equilibrium production quantities for new and repeat customers, respectively.

LEMMA 2. *(a) For any (X_2^n, X_2^r), $p_2^u(X_2^n, X_2^r) = \arg\max_{p_2^u \geq 0} \Pi_2^u(p_2^u | X_2^n, X_2^r)$, where $\Pi_2^u(p_2^u | X_2^n, X_2^r) := X_2^n(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{1+\alpha}\right) + X_2^r(p_2^u - c_2)\bar{G}\left(\frac{p_2^u}{k+\alpha}\right)$.*

(b) For any (X_2^n, X_2^r), $Q_u^n(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) X_2^n$, and $Q_u^r(X_2^n, X_2^r) = \bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right) X_2^r$.

(c) $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r. Moreover, for any (X_2^n, X_2^r), $p_2^{r*} \leq p_2^u(X_2^n, X_2^r) \leq p_2^{n*}$, where the inequalities are strict if $X_2^n, X_2^r > 0$..

Let $\Pi_f^u(Q_1|\delta_c)$ $(p_1^u(Q_1|\delta_c))$ be the expected profit (equilibrium first-period price) of the firm to produce Q_1 products in period 1 in the NTR model with customer discount factor δ_c. We compute $\Pi_f^u(\cdot|\cdot)$ in the following lemma.

LEMMA 3. *In the NTR model, we have $p_1^u(Q_1|\delta_c) = \mu + \delta_c(\sigma_r^u(Q_1) - \sigma_n^u(Q_1))$ and $\Pi_f^u(Q_1|\delta_c) = (m_1^u(Q_1|\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta R_2^u(Q_1)$, where $m_2^u(Q_1|\delta_c) = \mu + \delta(\beta_r^u(Q_1) - \beta_n^u(Q_1)) + \delta_c(\sigma_r^u(Q_1) - \sigma_n^u(Q_1))$, $\beta_n^u(Q_1) := \mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r))]$, $\beta_n^u(Q_1) := \mathbb{E}[\hat{v}_2^r(p_2^u(X_2^n, X_2^r))]$, and $R_2^u(Q_1) = \mathbb{E}[v_2^n(p_2^u(X_2^n, X_2^r))X]$ $(X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1)$. Moreover, $\beta_r^u(\cdot)$ is increasing, whereas $\sigma_r^u(\cdot)$, $\sigma_n^u(\cdot)$, $\beta_n^u(\cdot)$ and $R_2^u(\cdot)$ are decreasing in Q_1, respectively.*

It is clear that $\beta_n^u(Q_1)$ and $\beta_r^u(Q_1)$ are the expected second-period unit profit from new and repeat customers in the NTR model, respectively, whereas $m_2^u(Q_1|\delta_c)$ is the effective first-period marginal revenue. $\beta_n^u(\cdot)$, $\beta_r^u(\cdot)$, and $m_1^u(\cdot|\cdot)$ are the counterparts of β_n^*, β_r^*, and $m_1(\cdot)$ in the NTR model. The following theorem summarizes the equilibrium price and production quantity $(p_1^{u*}(\delta_c), Q_1^{u*}(\delta_c))$ in the NTR model.

THEOREM 11. *In the NTR model, for any customer discount factor δ_c, a unique RE equilibrium exists with (a) $Q_1^{u*}(\delta_c)=\arg\max_{Q_1\ge 0}\Pi_f^u(Q_1|\delta_c)$; (b) $p_1^{u*}(\delta_c)=\mu+\delta_c(\sigma_r^u(Q_1^{u*}(\delta_c))-\sigma_n^u(Q_1^{u*}(\delta_c)))$; and (c) the expected profit of the firm $\Pi_f^{u*}(\delta_c)=(m_1^u(Q_1^{u*}(\delta_c)|\delta_c)-s)\mathbb{E}(X\wedge Q_1^{u*}(\delta_c))-(c_1-s)Q_1^{u*}(\delta_c)+\delta R_2^u(Q_1^{u*}(\delta_c))$.*

Finally, we have the following lemma that characterizes the equilibrium second-period pricing strategy in the model of social optimum.

LEMMA 4. *(a) $p_s^n(X_2^n,X_2^r)\equiv p_s^{n*}$ and $p_s^r(X_2^n,X_2^r)\equiv p_s^{r*}$, where $p_s^{n*}=c_2+\kappa_2$ and $p_s^{r*}=c_2-r_2+\kappa_2-\iota_2$. Hence, $p_s^{n*}>p_s^{r*}$ if and only if $r_2>0$ or $\iota_2>0$.*

(b) $w_2(X_2^n,X_2^r)=\sigma_n^{s*}X_2^n+\sigma_r^{s*}X_2^r$, where $\sigma_n^{s*}=\mathbb{E}((1+\alpha)V-p_s^{n*})^+$ and $\sigma_r^{s*}=\mathbb{E}((k+\alpha)V_2-p_s^{r*})^+$.

Appendix C: Proofs of Statements

[1]**Proof of Lemma 1: Part (a).** Given (p_2^n,p_2^r) with $p_2^r\le p_2^n$, the *ex-ante* probability that a new customer will purchase the second-generation product is $\bar G\left(\frac{p_2^n}{1+\alpha}\right)$, whereas the probability that a repeat customer will join the trade-in program is $\bar G\left(\frac{p_2^r}{k+\alpha}\right)$. Therefore, conditioned on the realized market size (X_2^n,X_2^r), the expected profit of the firm in period 2 is given by: $\Pi_2(p_2^n,p_2^r|X_2^n,X_2^r):=X_2^n(p_2^n-c_2)\bar G\left(\frac{p_2^n}{1+\alpha}\right)+X_2^r(p_2^r-c_2+r_2)\bar G\left(\frac{p_2^r}{k+\alpha}\right)=X_2^nv_2^n(p_2^n)+X_2^rv_2^r(p_2^r)$, where $v_2^n(p_2^n):=(p_2^n-c_2)\bar G(\frac{p_2^n}{1+\alpha})$ and $v_2^r(p_2^r):=(p_2^r-c_2+r_2)\bar G(\frac{p_2^r}{k+\alpha})$. We now show that $v_2^n(\cdot)$ is quasiconcave in p_2^n, and $v_2^r(\cdot)$ is quasiconcave in p_2^r. Note that $\partial_{p_2^n}v_2^n(p_2^n)=-\left(\frac{p_2^n-c_2}{1+\alpha}\right)g\left(\frac{p_2^n}{1+\alpha}\right)+\bar G\left(\frac{p_2^n}{1+\alpha}\right)$ and $\partial_{p_2^r}v_2^r(p_2^r)=-\left(\frac{p_2^r-c_2+r_2}{k+\alpha}\right)g\left(\frac{p_2^r}{k+\alpha}\right)+\bar G\left(\frac{p_2^r}{k+\alpha}\right)$. Because $h(v)=g(v)/\bar G(v)$ is continuously increasing in v, $g(\frac{p_2^n}{1+\alpha})/\bar G(\frac{p_2^n}{1+\alpha})$ is continuously increasing in p_2^n and $g(\frac{p_2^r}{k+\alpha})/\bar G(\frac{p_2^r}{k+\alpha})$ is continuously increasing in p_2^r. Hence, $\partial_{p_2^n}v_2^n(p_2^n)=0$ has a unique solution p_2^{n*} and $\partial_{p_2^r}v_2^r(p_2^r)=0$ has a unique solution p_2^{r*}. Clearly, for $i=n,r$, $v_2^i(\cdot)$ is strictly increasing on $[0,p_2^{i*})$ and strictly decreasing on $(p_2^{i*},+\infty)$. Therefore, $\Pi_2(\cdot,\cdot|X_2^n,X_2^r)$ is quasiconcave in (p_2^n,p_2^r), and $(p_2^n(X_2^n,X_2^r),p_2^r(X_2^n,X_2^r))=(p_2^{n*},p_2^{r*})$.

It remains to show that $p_2^{n*}>p_2^{r*}$. Note that p_2^{n*} satisfies $\left(\frac{p_2^{n*}-c_2}{1+\alpha}\right)g\left(\frac{p_2^{n*}}{1+\alpha}\right)/\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)=1$, and p_2^{r*} satisfies $\left(\frac{p_2^{r*}-c_2+r_2}{k+\alpha}\right)g\left(\frac{p_2^{r*}}{k+\alpha}\right)\Big/\bar G\left(\frac{p_2^{r*}}{k+\alpha}\right)=1$. Since $k<1$, $\frac{p_2^{n*}-c_2+r_2}{k+\alpha}>\frac{p_2^{n*}-c_2}{1+\alpha}$, and the increasing failure rate condition implies that $g\left(\frac{p_2^{n*}}{k+\alpha}\right)/\bar G\left(\frac{p_2^{n*}}{k+\alpha}\right)\ge g\left(\frac{p_2^{n*}}{1+\alpha}\right)/\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)$. Thus, $\left(\frac{p_2^{n*}-c_2+r_2}{k+\alpha}\right)g\left(\frac{p_2^{n*}}{k+\alpha}\right)/\bar G\left(\frac{p_2^{n*}}{k+\alpha}\right)>\left(\frac{p_2^{n*}-c_2}{1+\alpha}\right)g\left(\frac{p_2^{n*}}{1+\alpha}\right)/\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)=1$, and, hence, $\partial_{p_2^r}v_2^r(p_2^{n*})<0$. Since $v_2^r(\cdot)$ is quasiconcave, $p_2^{r*}<p_2^{n*}$.

Part (b). Because all new customers with willingness-to-pay $(1+\alpha)V$ greater than $p_2^n(X_2^n,X_2^r)\equiv p_2^{n*}$ would make a purchase. Hence, $Q_2^n(X_2^n,X_2^r)=\mathbb{E}[X_2^n1_{\{(1+\alpha)V\ge p_2^{n*}\}}|X_2^n]=\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)X_2^n$. Analogously, all repeat customers with willingness-to-pay $(k+\alpha)V$ greater than $p_2^r(X_2^n,X_2^r)\equiv p_2^{r*}$ would make a purchase. Hence, $Q_2^r(X_2^n,X_2^r)=\mathbb{E}[X_2^r1_{\{(k+\alpha)V\ge p_2^{r*}\}}|X_2^r]=\bar G\left(\frac{p_2^{r*}}{k+\alpha}\right)X_2^r$.

Part (c). Since $\pi_2(X_2^n,X_2^r):=\max\{\Pi_2(p_2^n,p_2^r|X_2^n,X_2^r):0\le p_2^r\le p_2^n\}$, it follows that $\pi_2(X_2^n,X_2^r)=[\max v_2^n(p_2^n)]X_2^n+[\max v_2^r(p_2^r)]X_2^r$. To complete the proof, it remains to show that $\beta_n^*=[\max v_2^n(p_2^n)]>0$ and $\beta_r^*=[\max v_2^r(p_2^r)]>0$. It is straightforward to check that $p_2^{n*}-c_2>0$, $\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)>0$, $p_2^{r*}-c_2+r_2>0$, and $\bar G\left(\frac{p_2^{r*}}{k+\alpha}\right)>0$. Hence, $\beta_n^*=(p_2^{n*}-c_2)\bar G\left(\frac{p_2^{n*}}{1+\alpha}\right)>0$ and $\beta_r^*=(p_2^{r*}-c_2+r_2)\bar G\left(\frac{p_2^{r*}}{k+\alpha}\right)>0$. *Q.E.D.*

Proof of Theorem 1: Part (a). This part has already been shown by the discussions before the theorem.

[1] Due to the page limit requirement, we only provide a sketch of the proof. The complete proof is available from the authors upon request.

Part (b,c). Plugging $p_1^*(\cdot)$ into $\Pi_f(\cdot|\cdot)$ and, with some algebraic manipulations, we have $\Pi_f(Q_1|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta\beta_n^*\mathbb{E}(X)$. Therefore, $Q_1^*(\delta_c)$ is the solution to a newsvendor problem with marginal revenue $m_1^*(\delta_c) - s$, marginal cost $c_1 - s$, and demand distribution $F(\cdot)$. Hence, $Q_1^*(\delta_c) = \bar{F}^{-1}(\frac{c_1-s}{m_1^*(\delta_c)-s})$ and $\Pi_f^*(\delta_c) = \Pi_f(Q_1^*(\delta_c)|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}(X \wedge Q_1^*(\delta_c)) - (c_1 - s)Q_1^*(\delta_c) + \delta\beta_n^*\mathbb{E}(X)$. *Q.E.D.*

Proof of Theorem 2: Part (a). It follows from Theorem 1(a) that $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$ and $m_1^*(\delta_c) = \mu + \delta(\beta_r^* - \beta_n^*) + \delta_c(\sigma_r^* - \sigma_n^*)$ are strictly increasing (decreasing) in δ_c if $\sigma_r^* > \sigma_n^*$ ($\sigma_r^* < \sigma_n^*$). By Theorem 1(b), $Q_1^*(\delta_c) = \bar{F}^{-1}(\frac{c_1-s}{m_1^*(\delta_c)-s})$ is increasing (decreasing) in δ_c if and only if $\sigma_r^* > \sigma_n^*$ ($\sigma_r^* < \sigma_n^*$). Moreover, for any Q_1 and any $\hat{\delta}_c > \delta_c$, $\Pi_f(Q_1|\hat{\delta}_c) - \Pi_f(Q_1|\delta_c) = (\hat{\delta}_c - \delta_c)(\sigma_r^* - \sigma_n^*)\mathbb{E}(X \wedge Q_1) > 0$ if and only if $\sigma_r^* > \sigma_n^*$. Therefore, $\Pi_f^*(\hat{\delta}_c) = \max \Pi_f(Q_1|\hat{\delta}_c) > \max \Pi_f(Q_1|\delta_c) = \Pi_f^*(\delta_c)$ if and only if $\sigma_r^* > \sigma_n^*$. If, on the other hand, $\sigma_r^* < \sigma_n^*$, it follows immediately from the same argument that $\Pi_f^*(\hat{\delta}_c) < \Pi_f^*(\delta_c)$.

To show that $\sigma_r^* > \sigma_n^*$ (resp. $\sigma_r^* < \sigma_n^*$) if $k \in (\underline{k}, \bar{k})$ (resp. $k < \underline{k}$ or $k > \bar{k}$), it suffices to prove that if σ_r^* is increasing in k at $k = k_0$, it is increasing in k when $k \leq k_0$. σ_r^* is increasing in k at $k = k_0$ implies that, for $\epsilon > 0$ and small enough, $\mathbb{E}[(k_0 + \alpha)V - p_2^{r*}(k_0)]^+ > \mathbb{E}[(k_0 - \epsilon + \alpha)V - p_2^{r*}(k_0 - \epsilon)]^+$, where we use $p_2^{r*}(\cdot)$ to denote the dependence of p_2^{r*} on the depreciation factor k. Since r_2 is concavely decreasing in k, $p_2^{r*}(k) - p_2^{r*}(k - \epsilon)$ is increasing in k. Therefore, for $k < k_0$, $\mathbb{E}[(k + \alpha)V - p_2^{r*}(k)]^+ > \mathbb{E}[(k - \epsilon + \alpha)V - p_2^{r*}(k - \epsilon)]^+$ for $\epsilon > 0$ small enough, where the inequality follows from $\mathbb{E}[(k_0 + \alpha)V - p_2^{r*}(k_0)]^+ > \mathbb{E}[(k_0 - \epsilon + \alpha)V - p_2^{r*}(k_0 - \epsilon)]^+$ and that $p_2^{r*}(k) - p_2^{r*}(k - \epsilon)$ is increasing in k. Therefore, σ_r^* is increasing in k for all $k \leq k_0$. As an implication, we have also established that σ_r^* is quasiconcave in k. Hence, there exist two thresholds $\underline{k}$ and $\bar{k}$, such that $\sigma_r^* > \sigma_n^*$ if and only if $k \in (\underline{k}, \bar{k})$, and $\sigma_r^* < \sigma_n^*$ if and only $k < \underline{k}$ or $k > \bar{k}$.

Part (b). We first show (b-iii). By definition, $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) = \mathbb{E}[(k + \alpha)V - p_2^u(X_2^n, X_2^r)]^+ - \mathbb{E}[(1 + \alpha)V - p_2^u(X_2^n, X_2^r)]^+$. Since $k < 1$ and $p_2^u(X_2^n, X_2^r) \in (p_2^{r*}, p_2^{n*})$ (Lemma 2), $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) < 0$ for all $Q_1 \geq 0$.

By Theorem 11, $p_1^{u*}(\delta_c) = \mu + \delta_c(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)))$ is continuously differentiable in δ_c. Since the right derivative of $p_1^{u*}(\cdot)$ at 0 is $\partial_{\delta_c}^+ p_1^{u*}(0) = \sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)) < 0$, there exists a positive threshold $\delta_0 > 0$ such that $p_1^{u*}(\cdot)$ is strictly decreasing on $[0, \delta_0]$.

To show that $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c, it suffices to show that $\Pi_f^u(Q_1|\delta_c)$ is strictly submodular on a neighborhood of $(Q_1^{u*}(\delta_c), \delta_c)$. Direct computation yields $\partial_{\delta_c}\Pi_f^u(Q_1(\delta_c)|\delta_c) = (\sigma_r^u(Q_1) - \sigma_n^u(Q_1))\mathbb{E}(X \wedge Q_1)$. Note that $\sigma_r^u(Q_1) - \sigma_n^u(Q_1) < 0$ and is decreasing in Q_1, whereas $\mathbb{E}(X \wedge Q_1) > 0$ and is strictly increasing in Q_1 in a neighborhood of $Q_1^{u*}(\delta_c)$. It follows immediately that $\partial_{\delta_c}\Pi_f^u(Q_1|\delta_c)$ is strictly decreasing in Q_1 on a neighborhood of $Q_1^{u*}(\delta_c)$. Therefore, $\Pi_f^u(Q_1|\delta_c)$ is strictly submodular on a neighborhood of $(Q_1^{u*}(\delta_c), \delta_c)$ and, thus, $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c. By the envelope theorem, $\partial_{\delta_c}\Pi_f^{u*}(\delta_c) = (\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)))\mathbb{E}(X \wedge Q_1^{u*}(\delta_c)) > 0$. Hence, $\Pi_f^{u*}(\delta_c)$ is strictly increasing in δ_c. *Q.E.D.*

Proof of Theorem 3: Part (a). By Lemma 2, $p_2^{r*} < p_2^u(X_2^n, X_2^r) < p_2^{n*}$ with probability 1. Thus, if $Q_1 > 0$, $\sigma_r^* = \mathbb{E}[(k + \alpha)V - p_2^{r*}]^+ \geq \mathbb{E}[(k + \alpha)V - p_2^u((X - Q_1)^+, X \wedge Q_1)]^+ = \sigma_r^u(Q_1)$, and $\sigma_n^* = \mathbb{E}[(1 + \alpha)V - p_2^{n*}]^+ < \mathbb{E}[(1 + \alpha)V - p_2^u((X - Q_1)^+, X \wedge Q_1)]^+ = \sigma_n^u(Q_1)$.

Part (b). By Theorem 1(b) and Theorem 11(b), for all $\delta_c > 0$, $p_1^*(\delta_c) - p_1^{u*}(\delta_c) = \delta_c[\sigma_r^* - \sigma_n^*] - \delta_c[\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))] = \delta_c[\sigma_r^* - \sigma_r^u(Q_1^{u*}(\delta_c))] + \delta_c[\sigma_n^u(Q_1^{u*}(\delta_c)) - \sigma_n^*] > 0$. Since $\partial_{p_2^u}(\mathbb{E}[(k+\alpha)V - p_2^u]^+ -$

$\mathbb{E}[(1+\alpha)V - p_2^u]^+) = \mathbb{P}[\frac{p_2^u}{1+k} \le V \le \frac{p_2^u}{k+\alpha}] > 0$ and $p_2^{r*} < p_2^u(X_2^n, X_2^r) < p_2^{n*}$ with probability 1, $\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c)) < \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \mathbb{E}[(1+\alpha)V - p_2^{n*}]^+ = \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \sigma_n^*$. Hence, $p_1^*(\delta_c) - p_1^{u*}(\delta_c) > \delta_c[\sigma_r^* - \sigma_n^*] - \delta_c\{\mathbb{E}[(k+\alpha)V - p_2^{n*}]^+ - \sigma_n^*\} = \delta_c(\sigma_r^* - \mathbb{E}[(k+\alpha)V - p_2^{n*}]^+)$ for all $\delta_c > 0$. It is also straightforward to check that, for any $\delta_c \in [0, \delta]$ and $Q_1 > 0$, $\Pi_f(Q_1|\delta_c) > \Pi_f^u(Q_1|\delta_c)$ for all $Q_1 > 0$ and $\delta_c \in [0, \delta]$. Therefore, $\Pi_f^*(\delta_c) = \max_{Q_1} \Pi_f(Q_1|\delta_c) > \max_{Q_1} \Pi_f^u(Q_1|\delta_c) = \Pi_f^{u*}(\delta_c)$.

Part (c). Note that $\Pi_f(Q_1|\delta_c) = (m_1^*(\delta_c) - s)\mathbb{E}[Q_1 \wedge X] - (c_1 - s)Q_1 + \delta\beta_n^*\mathbb{E}[X]$. By the proof of Theorem 2(a), it is easy to check that, if $m_1^*(\delta_c) = \mu + \delta(\beta_r^* - \beta_n^*) + \delta_c(\sigma_r^* - \sigma_n^*)$ is increasing in k at $k = k_0$, it is increasing in k for all $k \le k_0$. In other words, $m_1^*(\delta_c)$ is quasiconcave in k. Furthermore, since c_1 is convexly decreasing in k, following the same argument as the proof of Theorem 2(a), direct computation yields that the critical fractile $\frac{c_1 - s}{m_1^*(\delta_c) - s}$ is decreasing in k at k_0, so it is decreasing in k for all $k \le k_0$. Thus, $\frac{c_1 - s}{m_1^*(\delta_c) - s}$ is quasiconvex in k, and, therefore, there exists a K such that $Q_1^*(\delta_c)$ is increasing in k if $k \le K$, and decreasing in k if $k \ge K$. Next we show that $\Pi_f^*(\delta_c)$ is also increasing in k when $k \le K$ and decreasing in k when $k \ge K$. It is clear that $K = \arg\min_k \left[\frac{c_1 - s}{m_1^*(\delta_c) - s}\right]$. Since c_1 is convexly decreasing in k, for any realization of X and production quantity $Q_1 = Q_1^*(\delta_c) = \bar{F}^{-1}\left(\frac{c_1 - s}{m_1^*(\delta_c) - s}\right)$, $(m_1^*(\delta_c) - s)(Q_1 \wedge X) - (c_1 - s)Q_1$ is increasing in k for $k \le K$, and decreasing in k for $k \ge K$. Therefore, $\Pi_f^*(\delta_c) = \mathbb{E}[(m_1^*(\delta_c) - s)(Q_1^*(\delta_c) \wedge X) - (c_1 - s)Q_1^*(\delta_c) + \delta\beta_n^* X]$ is increasing in k when $k \le K$, and it is decreasing in k when $k \ge K$.

Part (d). Since $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$, $|\partial_k p_1^*(\delta_c)| = \delta_c|\partial_k \sigma_r^*|$, which is clearly increasing in δ_c. *Q.E.D.*

Before showing Theorem 4, we first prove Theorem 5 and Theorem 6.

Proof of Theorem 5: Part (a). Since $Q_1^*(\cdot)$ and $Q_1^{u*}(\cdot)$ are continuous in δ_c, it suffices to show that $Q_1^*(\delta) > Q_1^{u*}(\delta)$. We first show that $m_1^u(Q_1|\delta)$ is decreasing in Q_1. Observe that $m_1^u(Q_1|\delta) = \mu + \delta[U_r(Q_1) - U_n(Q_1)]$, where $U_r(Q_1) := \mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{k+\alpha}\right)\right] + \mathbb{E}((k+\alpha)V - p_2^u(X_2^n, X_2^r))^+$, and $U_n(Q_1) := \mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)\right] + \mathbb{E}((1+\alpha)V - p_2^u(X_2^n, X_2^r))^+$. Let $u_r(p) := (p - c_2)\bar{G}(\frac{p}{k+\alpha}) + \mathbb{E}((k+\alpha)V - p)^+ = \mathbb{E}[(k+\alpha)V - c_2]1_{\{(k+\alpha)V \ge p\}}$ and $u_n(p) := (p - c_2)\bar{G}(\frac{p}{1+\alpha}) + \mathbb{E}((1+\alpha)V - p)^+ = \mathbb{E}[(1+\alpha)V - c_2]1_{\{(1+\alpha)V \ge p\}}$. It's clear that $u_r(\cdot)$ and $u_n(\cdot)$ are continuously decreasing in p. Moreover, $U_r(Q_1) = \mathbb{E}[u_r(p_2^u(X_2^n, X_2^r))]$ and $U_n(Q_1) = \mathbb{E}[u_n(p_2^u(X_2^n, X_2^r))]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. Since $p_2^u(X_2^n, X_2^r)$ is increasing in X_2^n and decreasing in X_2^r, it is stochastically decreasing in Q_1. Hence, it suffices to show that $u_r(p) - u_n(p)$ is increasing in p. Observe that $u_r(p) - u_n(p) = -[\int_{p/(1+\alpha)}^{\bar{v}}((1+\alpha)V - \max(p, (k+\alpha)V))g(V)\,\mathrm{d}V]$, which is continuously increasing in p. Therefore, $m_1^u(Q_1|\delta) = \mu + \delta(U_r(Q_1) - U_n(Q_1)) = \mu + \delta\{\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_r(p_2^u(X_2^n, X_2^r))]\}$ is continuously decreasing in Q_1.

We now show that $m_1^u(Q_1|\delta) < m_1^*(\delta)$ for all Q_1. Observe that $m_1^u(Q_1|\delta) - m_1^*(\delta) = \delta\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] - \delta\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})]$. Because $p_2^{r*} \le p_2^u(X_2^n, X_2^r) \le p_2^{n*}$ and $u_r(\cdot)$ and $u_n(\cdot)$ are decreasing in p, $\delta\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\delta\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$. Hence, $m_1^u(Q_1|\delta) \le m_1^*(\delta)$. Since $k < 1$, $p_2^{r*} < p_2^{n*}$, one of the inequalities $\mathbb{E}[u_r(p_2^u(X_2^n, X_2^r)) - u_r(p_2^{r*})] \le 0$ and $\mathbb{E}[u_n(p_2^u(X_2^n, X_2^r)) - u_n(p_2^{n*})] \ge 0$ must be strict. Therefore, $m_1^u(Q_1|\delta) < m_1^*(\delta)$ for all $Q_1 \ge 0$.

Next, we show that $Q_1^*(\delta) > Q_1^{u*}(\delta)$. Observe that $\Pi_f^u(Q_1|\delta) - \Pi_f(Q_1|\delta) = (m_1^u(Q_1|\delta) - m_1^*(\delta))\mathbb{E}(X \wedge Q_1) + \delta\mathbb{E}\left[(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right) - \beta_n^*\right]X$. Let $\Pi(Q_1, 1) = \Pi_f(Q_1|\delta)$ and $\Pi(Q_1, 0) = \Pi_f^u(Q_1|\delta)$. Then, $\Pi(Q_1, 1) - \Pi(Q_1, 0) = (m_1^*(\delta) - m_1^u(Q_1|\delta))\mathbb{E}(X \wedge Q_1) + \delta\mathbb{E}[\beta_n^* - (p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)]X$. Since $m_1^*(\delta) \geq m_1^u(Q_1|\delta)$ and $m_1^u(Q_1|\delta)$ is decreasing in Q_1, $(m_1^*(\delta) - m_1^u(Q_1|\delta))\mathbb{E}(X \wedge Q_1)$ is increasing in Q_1. Also note that $p_2^u(X_2^n, X_2^r)$ and thus $(p_2^u(X_2^n, X_2^r) - c_2)\bar{G}\left(\frac{p_2^u(X_2^n, X_2^r)}{1+\alpha}\right)$ is decreasing in Q_1. Therefore, $\Pi(Q_1, 1) - \Pi(Q_1, 0)$ is increasing in Q_1. Hence, $\Pi(\cdot, \cdot)$ is supermodular on the lattice $[0, +\infty) \times \{0, 1\}$ and $Q_1^{u*}(\delta) = \arg\max_{Q_1 \geq 0} \Pi_f^u(Q_1|\delta) \leq \arg\max_{Q_1 \geq 0} \Pi_f(Q_1|\delta) = Q_1^*(\delta)$. Since $m_1^*(\delta) > m_1^u(Q_1^{u*}(\delta)|\delta)$, $\partial_{Q_1}\Pi_f(Q_1^{u*}(\delta)|\delta) > \partial_{Q_1}\Pi_f^u(Q_1^{u*}(\delta)|\delta) = 0$. Since $\Pi_f(\cdot|\delta)$ is concave in Q_1, $Q_1^*(\delta) > Q_1^{u*}(\delta)$. Due to the continuity of $Q_1^*(\cdot)$ and $Q_1^{u*}(\cdot)$ in δ_c, there exists a threshold $\bar{\delta}_q \leq \delta$ such that $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$ for all $\delta > \bar{\delta}_q$.

Part (b). We first show that $m_1^u(Q_1|0)$ is increasing in Q_1. Note that $m_1^u(Q_1|0) = \mu + \delta(\beta_r^u(Q_1) - \beta_n^u(Q_1))$. By Lemma 3, $\beta_r^u(\cdot)$ is increasing whereas $\beta_n^u(\cdot)$ is decreasing in Q_1. Therefore, $m_1^u(Q_1|0)$ is increasing in Q_1.

We then show that there exists a threshold $\bar{Q}_1$ such that $m_1^u(Q_1|0) > m_1^*(0)$ $(m_1^u(Q_1|0) < m_1^*(0))$ if $Q_1 > \bar{Q}_1$ $(Q_1 < \bar{Q}_1)$. Let $\hat{\beta}_r^* = \max_{p \geq 0} \hat{v}_2^r(p) = \lim_{Q_1 \to +\infty} \beta_r^u(Q_1)$. Since $k < 1$, $\hat{\beta}_n^* := v_2^n(\hat{p}_2^{r*}) < \beta_n^*$. It is clear that $\beta_r^* - \hat{\beta}_r^*$ is increasing in r_2, with $\beta_r^* = \hat{\beta}_r^*$ if $r_2 = 0$. Let $\bar{r}_2 > 0$ be the threshold such that $\beta_r^* - \hat{\beta}_r^* = \beta_n^* - \hat{\beta}_n^*$. Hence, $\beta_r^* - \hat{\beta}_r^* < \beta_n^* - \hat{\beta}_n^*$ for all $r_2 < \bar{r}_2$. Moreover, by the monotone convergence theorem, $\lim_{Q_1 \to +\infty} m_1^u(Q_1|0) = \mu + \delta[v_2^r(\hat{p}_2^{r*}) - v_2^n(\hat{p}_2^{r*})] = \mu + \delta[\hat{\beta}_r^* - \hat{\beta}_n^*] > \mu + \delta[\beta_r^* - \beta_n^*] = m_1^*(0)$. Since $m_1^u(Q_1|0)$ is increasing in Q_1, there exists a threshold $\bar{Q}_1$ such that $m_1^u(Q_1|0) > m_1^*(0)$ $(m_1^u(Q_1|0) < m_1^*(0))$ if $Q_1 > \bar{Q}_1$ $(Q_1 < \bar{Q}_1)$.

Now we show there exists a $c_q > 0$ such that, if $c_1 < c_q$, $Q_1^*(0) < Q_1^{u*}(0)$. It is clear that $Q_1^{u*}(0) \uparrow \bar{X}$ and $Q_1^*(0) \uparrow \bar{X}$ as $c_1 \downarrow 0$, where $\bar{X}$ is the upper bound of the support of X ($\bar{X}$ may take the value of $+\infty$). Hence, there exists a threshold $c_q > 0$ (dependent on r_2) such that if $c_1 < c_q$, $Q_1^{u*}(0) > \bar{Q}_1$ and $Q_1^*(0) > \bar{Q}_1$. Let $\hat{\pi}_2(Q_1) := \delta\mathbb{E}[v_2^n(p_2^u(X_2^n, X_2^r))X]$, where $X_2^n = (X - Q_1)^+$ and $X_2^r = X \wedge Q_1$. It's clear that $\hat{\pi}_2(\cdot)$ is differentiable and, by the chain rule $\hat{\pi}_2'(Q_1) = \delta\mathbb{E}[\partial_p v_2^n(p_2^u(X_2^n, X_2^r))(\partial_{X_2^n} p_2^u(X_2^n, X_2^r) + \partial_{X_2^r} p_2^u(X_2^n, X_2^r))1_{\{X \geq Q_1\}}X]$. As $Q_1 \to \bar{X}$, for any realization of $X \leq \bar{X}$, $\partial_{X_2^n} p_2^u(X_2^n, X_2^r)$ and $\partial_{X_2^r} p_2^u(X_2^n, X_2^r)$ converges to 0. Hence, by the dominated convergence theorem, there exits a threshold $\hat{Q} \in [\bar{Q}_1, \bar{X})$, such that $\hat{\pi}_2'(Q_1) \in [-\epsilon\mathbb{P}(X \geq Q_1), 0]$ for all $Q_1 \geq \hat{Q}$, where $\epsilon := (\tilde{m}_1^u(\hat{Q}) - \tilde{m}_1^*)/2 > 0$. Let $\bar{c}_1(r_2) \in (0, \tilde{c}(r_2)]$ be the threshold such that, if $c_1 < \bar{c}_1(r_1)$, we have $Q_1^{u*}, Q_1^* > \hat{Q} \geq \bar{Q}_1$. Therefore, $\partial_{Q_1}\Pi_f(Q_1^{u*}(0)|0) = (m_1^*(0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) - (c_1 - r_1) < (m_1^u(Q_1^{u*}(0)|0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) - \epsilon\mathbb{P}(X \geq Q_1^{u*}(0)) - (c_1 - r_1) \leq (m_1^u(Q_1^{u*}(0)|0) - r_1)\mathbb{P}(X \geq Q_1^{u*}(0)) + \hat{\pi}_2'(Q_1^{u*}(0)) - (c_1 - r_1) \leq \partial_{Q_1}\Pi_f^u(Q_1^{u*}(0)|0) = 0$, where the first inequality follows from $m_1^u(Q_1^{u*}(0)|0) - m_1^*(0) \geq (m_1^u(\hat{Q}|0) - m_1^*(0)) = 2\epsilon > \epsilon$, the second from $\hat{\pi}_2'(Q_1^{u*}(0)) \in [-\epsilon\mathbb{P}(X \geq Q_1^{u*}(0)), 0]$, and the last from the monotonicity that $m_1^u(\cdot|0)$ is increasing in Q_1. Because $\Pi_f(\cdot|0)$ is concave in Q_1, $Q_1^*(0) = \arg\max_{Q_1} \Pi_f(Q_1|0) < Q_1^{u*}(0)$ follows immediately. Since $Q_1^*(\delta_c)$ and $Q_1^{u*}(\delta_c)$ are continuous in δ_c, there exists a threshold $\underline{\delta}_q$ such that $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$ for all $\delta_c \in [0, \underline{\delta}_q)$.

Part (c). By Theorem 2, $Q_1^*(\delta_c)$ is strictly increasing in δ_c if $\sigma_r^* > \sigma_n^*$, whereas $Q_1^{u*}(\delta_c)$ is strictly decreasing in δ_c. Therefore, $\underline{\delta}_q = \bar{\delta}_q$ if $\sigma_r^* > \sigma_n^*$. *Q.E.D.*

Proof of Theorem 6: Part(a). A straightforward algebraic manipulation yields $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$, where $I_e(Q_1) := \kappa_1 Q_1 + \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right) - \delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{1+\alpha}\right) - \delta\iota_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X]$. If $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$, it is easy to check that $I_e'(Q_1^*(\delta_c)) > \left[\kappa_1 - \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right) + \delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{P}(X \geq$

$Q_1^*(\delta_c)) > 0$ where the first inequality follows from $\mathbb{P}(X \geq Q_1^*(\delta_c)) < 1$, whereas the second inequality follows from the assumptions that $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$ and $\kappa_2 > \iota_2$. Thus, by Theorem 2(a), if $\sigma_r^* > \sigma_n^*$, $Q_1^*(\delta_c)$ is strictly increasing in δ_c, so is $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$; if $\sigma_r^* < \sigma_n^*$, $Q_1^*(\delta_c)$ is strictly decreasing in δ_c, so is $I_e^*(\delta_c) = I_e(Q_1^*(\delta_c))$. Furthermore, by Theorem 2, $\sigma_r^* > \sigma_n^*$ if and only if $k \in (\underline{k}, \bar{k})$, and $\sigma_r^* < \sigma_n^*$ if and only if $k < \underline{k}$ or $k > \bar{k}$.

Part (b). As shown in part(a), $I_e^*(\delta_c)$ is strictly increasing in $Q_1^*(\delta_c)$ and, by the proof of Theorem 3(c), $Q_1^*(\delta_c)$ is increasing in k when k is small, and decreasing in k when k is big. Therefore, with the same argument as Theorem 3, we know $I_e^*(\delta_c)$ is quasiconcave in k, and thus there exists a threshold K_e, such that $I_e^*(\delta_c)$ is increasing (resp. decreasing) in k for $k \leq K_e$ (resp. $k \geq K_e$). *Q.E.D.*

Proof of Theorem 4: Part (a). Since $\delta_c > \bar{\bar{\delta}}_q$, Theorem 5(a) implies that $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$. Now we compute $I_e^{u*}(\delta_c)$. Given the market size (X_2^n, X_2^r), the equilibrium total second-period production quantity, $Q_2^u(X_2^n, X_2^r)$, is given by $Q_2^u(X_2^n, X_2^r) = X_2^n\bar{G}\left(\frac{p_2^u(X_2^n,X_2^r)}{1+\alpha}\right) + X_2^r\bar{G}\left(\frac{p_2^u(X_2^n,X_2^r)}{k+\alpha}\right)$. Therefore, following the same argument as in the proof of Theorem 6, we have $I_e^{u*}(\delta_c) = \mathbb{E}\{\kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2 Q_2^u(X_2^{n*}, X_2^{r*})\} = \kappa_1 Q_1^{u*}(\delta_c) + \mathbb{E}\left[\left(\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{k+\alpha}\right) - \delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)\right)(X \wedge Q_1^{u*}(\delta_c))\right] + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)X\right]$, where $X_2^{n*} = (X - Q_1^{u*}(\delta_c))^+$ and $X_2^{r*} = X \wedge Q_1^{u*}(\delta_c)$. For any δ_c, $I_e^*(\delta_c)$ is strictly linearly decreasing in ι_2. Thus, let $\bar{\iota}_e := \max\{\iota_2 : I_e^*(\delta_c) \geq I_e^{u*}(\delta_c)\}$. We have $I_e^*(\delta_c) > I_2^{u*}(\delta_c)$, if $\iota_2 < \bar{\iota}_e$. In particular, if $\iota_2 = 0$, $Q_1^*(\delta_c) > Q_1^{u*}(\delta_c)$, $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$, and $\kappa_1 \geq \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)$ imply that $\kappa_1 Q_1^*(\delta_c) - \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1^*(\delta_c)) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X] > \kappa_1 Q_1^{u*}(\delta_c) - \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)\right](X \wedge Q_1^{u*}(\delta_c)) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)X\right]$, and $\delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^*(\delta_c)) > \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{k+\alpha}\right)(X \wedge Q_1^{u*}(\delta_c))\right]$. Thus, for $\iota_2 = 0$, we follow the same argument as the proof of Theorem 6 to establish that $I_e^*(\delta_c) = \kappa_1 Q_1^*(\delta_c) - \left[\delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\right]\mathbb{E}(X \wedge Q_1^*(\delta_c)) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\mathbb{E}[X] + \delta\kappa_2\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^*(\delta_c)) > \kappa_1 Q_1^{u*}(\delta_c) - \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)\right](X \wedge Q_1^{u*}(\delta_c)) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)X\right] + \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{k+\alpha}\right)\mathbb{E}(X \wedge Q_1^{u*}(\delta_c))\right] = I_e^{u*}(\delta_c)$, i.e., $I_e^*(\delta_c) > I_e^{u*}(\delta_c)$ for $\iota_2 = 0$. Therefore, $\bar{\iota}_e > 0$.

Part (b). Since $\delta_c < \underline{\delta}_q$, Theorem 5(b) implies that $Q_1^*(\delta_c) < Q_1^{u*}(\delta_c)$. Lemma 2 implies that $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$. Hence, $\kappa_1 Q_1^*(\delta_c) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\{\mathbb{E}[X] - \mathbb{E}(X - Q_1^*(\delta_c))^+\} < \kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)X - (X \wedge Q_1^{u*}(\delta_c))^+\right]$. Let $\tilde{\iota}_e := (\bar{G}(\frac{p_2^{r*}}{k+\alpha}) - \bar{G}(\frac{p_2^{n*}}{k+\alpha}))\kappa_2/\bar{G}(\frac{p_2^{r*}}{k+\alpha}) < \kappa_2$. If $\iota_2 > \tilde{\iota}_e$, since $Q_1^{u*}(\delta_c) > Q_1^*(\delta_c)$ and $p_2^{r*} < p_2^u(\cdot,\cdot) < p_2^{n*}$, $\mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{k+\alpha}\right)(Q_1^{u*}(\delta_c) \wedge X)\right] > \left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(Q_1^{u*}(\delta_c) \wedge X) > \left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right]\mathbb{E}(Q_1^*(\delta_c) \wedge X)$. Putting everything together, if $\iota_2 > \tilde{\iota}_e$, we have that $I_e^{u*}(\delta_c) = \kappa_1 Q_1^{u*}(\delta_c) + \delta\kappa_2\mathbb{E}\left[\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{1+\alpha}\right)X - (X \wedge Q_1^{u*}(\delta_c))^+\right] + \mathbb{E}\left[\delta\kappa_2\bar{G}\left(\frac{p_2^u(X_2^{n*},X_2^{r*})}{k+\alpha}\right)\right](Q_1^{u*}(\delta_c) \wedge X) > \kappa_1 Q_1^*(\delta_c) + \delta\kappa_2\bar{G}\left(\frac{p_2^{n*}}{1+\alpha}\right)\{\mathbb{E}[X] - \mathbb{E}(X - Q_1^*(\delta_c))^+\} + \mathbb{E}\{\left[\delta(\kappa_2 - \iota_2)\bar{G}\left(\frac{p_2^{r*}}{k+\alpha}\right)\right](Q_1^*(\delta_c) \wedge X)\} = I_e^*(\delta_c)$. This shows part (b). *Q.E.D.*

Proof of Theorem 7: We first derive $S_c^*(\delta_c)$ and $S_c^{u*}(\delta_c)$. Let $\mathfrak{a}_1^*(\delta_c)$ and $\mathfrak{a}_1^{u*}(\delta_c)$ be the in-stock probability in the base model and the NTR model, respectively. The expected surplus of a customer with discount factor δ_c in the base model is given by: $\mathfrak{a}_1^*(\delta_c)(\mu - p_1^*(\delta_c) + \delta\sigma_r^*) + (1 - \mathfrak{a}_1^*(\delta_c))\delta\sigma_n^* = \mathfrak{a}_1^*(\delta_c)(\mu - \mu - \delta_c(\sigma_r^* - \sigma_n^*) + \delta\sigma_r^*) + (1 - \mathfrak{a}_1^*(\delta_c))\delta\sigma_n^* = \mathfrak{a}_1^*(\delta_c)(\delta - \delta_c)(\sigma_r^* - \sigma_n^*) + \delta\sigma_n^*$, where the first equality follows from $p_1^*(\delta_c) = \mu + \delta_c(\sigma_r^* - \sigma_n^*)$. Therefore, the equilibrium total customer surplus is given by

$S_c^*(\delta_c) = \mathbb{E}[(\mathfrak{a}_1^*(\delta_c)(\delta - \delta_c)(\sigma_r^* - \sigma_n^*) + \delta\sigma_n^*)X]$. Analogously, the expected surplus of a customer with discount factor δ_c in the NTR model is given by: $\mathfrak{a}_u^*(\delta_c)(\mu - p_1^{u*}(\delta_c) + \delta\sigma_r^u(Q_1^{u*}(\delta_c))) + (1 - \mathfrak{a}_u^*(\delta_c))\delta\sigma_n^u(Q_1^{u*}(\delta_c)) = \mathfrak{a}_u^*(\delta_c)(\delta - \delta_c)(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))) + \delta\sigma_n^u(Q_1^{u*}(\delta_c))$. Therefore, the equilibrium total customer surplus is given by $S_c^{u*}(\delta_c) = \mathbb{E}[(\mathfrak{a}_u^*(\delta_c)(\delta - \delta_c)(\sigma_r^u(Q_1^{u*}(\delta_c)) - \sigma_n^u(Q_1^{u*}(\delta_c))) + \delta\sigma_n^u(Q_1^{u*}(\delta_c)))X]$.

Next, we show that $S_c^*(\delta_c) < S_c^{u*}(\delta_c)$ for $\delta_c > \bar{\delta}_s$. Note that, when $\delta_c = \delta$, $S_c^*(\delta_c) = \delta\mathbb{E}[\sigma_n^* X]$ and $S_c^{u*}(\delta_c) = \delta\mathbb{E}[\sigma_n^u(Q_1^{u*}(\delta_c))X]$. By Lemma 2(c), $\sigma_n^* < \sigma_n^u(Q_1^{u*}(\delta_c))$. Hence, it follows immediately that $S_c^{u*}(\delta_c) = \delta\mathbb{E}[\sigma_n^u(Q_1^{u*}(\delta_c))X] > \delta\mathbb{E}[\sigma_n^* X] = S_c^*(\delta_c)$ for $\delta_c = \delta$. Since $S_c^*(\delta_c)$ and $S_c^{u*}(\delta_c)$ are continuous in δ_c, there exists a threshold $\tilde{\delta}_s < \delta$ such that $S_c^{u*}(\delta_c) > S_c^*(\delta_c)$ for $\delta_c \in (\tilde{\delta}_s, \delta]$. *Q.E.D.*

Proof of Theorem 8: Part (a). It follows from the same argument as the proof of Theorem 1(a) that, $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$. Let $W_s(Q_1|\delta_c)$ denote the expected total social welfare with first-period production quantity Q_1 and customer discount factor δ_c. To compute $W_s(Q_1|\delta_c)$, Since $w_2(X_2^n, X_2^r) = \sigma_n^{s*}X_2^n + \sigma_r^{s*}X_2^r$, we have $W_s(Q_1|\delta_c) = p_1^{s*}(\delta_c)\mathbb{E}(X \wedge Q_1) + (\mu - p_1^{s*}(\delta_c))\mathbb{E}(X \wedge Q_1) - (c_1 + \kappa_1)Q_1 + s\mathbb{E}(Q_1 - X)^+ + \delta\mathbb{E}\{w_2(X - (X \wedge Q_1), X \wedge Q_1)\} = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s + \kappa_1)Q_1 + \delta\sigma_n^{s*}\mathbb{E}(X)$. Therefore, $Q_1^{s*}(\delta_c)$ is the solution to a newsvendor problem with marginal revenue $m_1^{s*} - s$, marginal cost $c_1 + \kappa_1 - s$, and demand distribution $F(\cdot)$. Hence, $Q_1^{s*}(\delta_c) = \bar{F}^{-1}(\frac{c_1+\kappa_1-s}{m_1^{s*}-s})$, and the equilibrium social welfare is $W_s^*(\delta_c) = W_s(Q_1^{s*}(\delta_c)|\delta_c) = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 + \kappa_1 - s)Q_1^{s*}(\delta_c) + \delta\sigma_n^{s*}\mathbb{E}(X)$.

Part (b). It follows immediately from part (a) that $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$ is strictly increasing (resp. decreasing) in δ_c if and only if $\sigma_r^{s*} > \sigma_n^{s*}$ (resp. $\sigma_r^{s*} < \sigma_n^{s*}$). Note that, by part (a), $\sigma_r^{s*} = \mathbb{E}[(k + \alpha)V_2 - c_2 - \kappa_2 + e_2]^+$, where $e_2 := r_2 + \iota_2$. The same argument as the proof of Theorem 2(a) implies that if σ_r^{s*} is increasing in k at $k = k_0$, then σ_r^{s*} is increasing in k for all $k \le k_0$. Hence, σ_r^{s*} is quasiconcave in k. Let $\underline{k}_s := \arg\max_k \sigma_r^{s*} > \sigma_n^{s*}$ and $\bar{k}_s := \arg\max_k \sigma_r^{s*} > \sigma_n^{s*}$. The quasiconcavity of σ_r^{s*} in k suggests that $\sigma_r^{s*} > \sigma_n^{s*}$ if and only if $k \in (\underline{k}_s, \bar{k}_s)$, and $\sigma_r^{s*} < \sigma_n^{s*}$ if and only if $k < \underline{k}_s$ or $k > \bar{k}_s$. Since m_1^{s*} is independent of δ_c, $Q_1^{s*}(\delta_c)$ is independent of δ_c as well. As a result, $W_s^*(\delta_c) = (m_1^{s*} - s)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1 + \kappa_1 - s)Q_1^{s*}(\delta_c) + \delta\sigma_n^{s*}\mathbb{E}(X)$ is independent of δ_c.

Part (c). The same argument as the proof of Theorem 3(c) demonstrates that $\frac{c_1+\kappa_1-s}{m_1^{s*}-s}$ is quasiconvex in k. Let $K_s := \arg\min_k \left[\frac{c_1+\kappa_1-s}{m_1^{s*}-s}\right]$. We have Q_1^{s*} is increasing in k for $k \le K_s$ and decreasing in k otherwise. Since c_1 is convexly decreasing in k, for any realization of X and production quantity $Q_1 = Q_1^{s*} = \bar{F}^{-1}\left(\frac{c_1+\kappa_1-s}{m_1^{s*}-s}\right)$, $(m_1^{s*} - s)(Q_1 \wedge X) - (c_1 + \kappa_1 - s)Q_1$ is increasing in k for $k \le K_s$ and decreasing in k otherwise. Therefore, $W_s^* = \mathbb{E}[(m_1^{s*} - s)(Q_1^{s*} \wedge X) - (c_1 + \kappa_1 - s)Q_1^{s*} + \delta\sigma_n^{s*}X]$ is also increasing in k if $k \le K_s$ and decreasing in k otherwise. *Q.E.D.*

Proof of Theorem 9: If $s_2^*(\delta_c)$ is the solution to $p_s^{n*} = \arg\max_{p_2^n \ge 0}(p_2^n + s_2 - c_2)\bar{G}\left(\frac{p_2^n}{1+\alpha}\right)$, it is clear that the subsidy/tax scheme with $s_2 = s_2^*(\delta_c)$ can induce the equilibrium price p_s^{n*} for new customers. We now show that $s_2^*(\delta_c)$ exists. Since $v_2^n(p_2^n)$ is quasiconcave in p_2^n for any s_2, the first-order condition $\partial_{p_2^n} v_2^n(p_2^n) = 0$ guarantees the optimal price for new customers. Moreover, $\partial_{p_2^n} v_2^n(p_s^{n*}) = \bar{G}\left(\frac{p_s^{n*}}{1+\alpha}\right) - \frac{p_s^{n*}+s_2-c_2}{1+\alpha} g\left(\frac{p_s^{n*}}{1+\alpha}\right)$, which is strictly decreasing in s_2. Hence, there exists a unique $s_2^*(\delta_c)$, such that $\partial_{p_2^n} v_2^n(p_s^{n*}) = 0$, thus inducing the socially optimal equilibrium price p_s^{n*} for new customers.

If $s_r^*(\delta_c)$ is the solution to $p_s^{r*} = \arg\max_{p_2^r \geq 0}(p_2^r + s_2^*(\delta_c) + s_r - c_2 + r_2)\bar{G}\left(\frac{p_2^r}{k+\alpha}\right)$, the subsidy/tax scheme with $s_r = s_r^*(\delta_c)$ can induce the equilibrium trade-in price p_s^{r*} for repeat customers. We now show that $s_r^*(\delta_c)$ exists. Since $v_2^r(p_2^r)$ is quasiconcave in p_2^r for any (s_2, s_r), the first-order condition $\partial_{p_2^r} v_2^r(p_2^r) = 0$ guarantees the optimal price for new customers. Moreover, if $s_2 = s_2^*(\delta_c)$, $\partial_{p_2^r} v_2^r(p_s^{r*}) = \bar{G}\left(\frac{p_s^{r*}}{k+\alpha}\right) - \frac{p_s^{r*}+s_2^*(\delta_c)+s_r-c_2+r_2}{k+\alpha} g\left(\frac{p_s^{r*}}{k+\alpha}\right)$, which is strictly decreasing in s_r. Hence, there exists a unique $s_r^*(\delta_c)$, such that $\partial_{p_2^r} v_2^r(p_s^{r*}) = 0$, thus inducing the socially optimal equilibrium trade-in price for repeat customers p_s^{r*}.

Given the subsidy/tax scheme $(s_1, s_2^*(\delta_c), s_r^*(\delta_c))$, as shown above, the firm adopts the same second-period pricing strategy as the social welfare maximizing one: (p_s^{n*}, p_s^{r*}). Hence, the first-period price should also be the same as the one that is socially optimal: $p_1^{s*}(\delta_c) = \mu + \delta_c(\sigma_r^{s*} - \sigma_n^{s*})$. Thus, the expected profit of the firm in period 1 is $\Pi_f^s(Q_1|\delta_c) = (m_1^s(s_1|\delta_c) - s)\mathbb{E}(X \wedge Q_1) - (c_1 - s)Q_1 + \delta(p_s^{n*} + s_2^*(\delta_c) - c_2)\bar{G}\left(\frac{p_s^{n*}}{1+\alpha}\right)\mathbb{E}(X)$, where $m_1^s(s_1|\delta_c) = p_1^{s*}(\delta_c) + \delta[(\kappa_2 + s_2^*(\delta_c) + s_r^*(\delta_c) - \iota_2)\bar{G}(\frac{p_s^{r*}}{k+\alpha}) - (\kappa_2 + s_2^*(\delta_c))\bar{G}(\frac{p_s^{n*}}{1+\alpha})] + s_1$. Thus, $\Pi_f^s(Q_1|\delta_c)$ has a unique optimizer $\bar{F}^{-1}(\frac{c_1-s}{m_1^s(s_1|\delta_c)-s})$. Moreover, as shown in Theorem 8, $Q_1^{s*}(\delta_c) = \bar{F}^{-1}(\frac{c_1+\kappa_1-s}{m_1^{s*}-s})$. Therefore, if $s_1^*(\delta_c)$ is the unique solution to $\frac{c_1-s}{m_1^s(s_1|\delta_c)-s} = \frac{c_1+\kappa_1-s}{m_1^{s*}-s}$, the optimal production quantity with the linear subsidy/tax scheme $s_g^*(\delta_c) = (s_1^*(\delta_c), s_2^*(\delta_c), s_r^*(\delta_c))$ is $Q_1^{s*}(\delta_c)$, which is the socially optimal first-period production quantity. *Q.E.D.*

Proof of Theorem 10: Part (a). Under the optimal subsidy/tax policy $s_g^*(\delta_c)$, the firm's profit is $\Pi_f^{s*}(\delta_c) = (p_1^{s*}(\delta_c) + s_1^*(\delta_c) - s)\mathbb{E}(Q_1^{s*}(\delta_c) \wedge X) - (c_1 - s)Q_1^{s*}(\delta_c) + \delta\mathbb{E}(X \wedge Q_1^{s*}(\delta_c))(k+\alpha)J\left(\frac{c_2+\kappa_2-e_2}{k+\alpha}\right) + \delta\mathbb{E}(X - Q_1^{s*}(\delta_c))^+(1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right) = \left(\frac{c_1-s}{c_1+\kappa_1-s}(m_1^{s*}-s)\right)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1-s)Q_1^{s*}(\delta_c) + \delta\mathbb{E}[X](1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$, where we plug in $s_1^*(\delta_c) = \frac{c_1-s}{c_1+\kappa_1-s}(m_1^{s*}-s) + s - \mu - \delta_c(\sigma_s^{r*} - \sigma_s^{n*}) + \delta(k+\alpha)J\left(\frac{c_2+\kappa_2-e_2}{k+\alpha}\right) - \delta(1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$, with $J(x) := \bar{G}(x)/h(x)$. It follows immediately from its formula expression that $\Pi_f^{s*}(\delta_c)$ is independent of δ_c.

Part (b). It's clear that $\delta\mathbb{E}[X](1+\alpha)J\left(\frac{c_2+\kappa_2}{1+\alpha}\right)$ is independent of the depreciation factor k, whereas $\left(\frac{c_1-s}{c_1+\kappa_1-s}(m_1^{s*}-s)\right)\mathbb{E}(X \wedge Q_1^{s*}(\delta_c)) - (c_1-s)Q_1^{s*}(\delta_c)$ is a constant proportion of the optimal social welfare that is influenced by the production decision (i.e., $(m_1^{s*}-s)\mathbb{E}[X \wedge Q_1^{s*}(\delta_c)] - (c_1 - s + \kappa_1)Q_1^{s*}(\delta_c)$). By Theorem 8(c), $(m_1^{s*}-s)\mathbb{E}[X \wedge Q_1^{s*}(\delta_c)] - (c_1 - s + \kappa_1)Q_1^{s*}(\delta_c)$ is increasing in k for $k \leq K_s$ and decreasing in k for $k \geq K_s$, i.e., W_s^* is maximized at $k = K_s$. Therefore, the firm's profit under the subsidy/tax scheme $s_g^*(\delta_c)$, $\Pi_f^{s*}(\delta_c)$, is maximized at $k = K_s$ as well. In other words, if the firm has the flexibility to control the depreciation factor k (equivalently, the remanufacturing efficiency), it will set the socially optimal one K_s under the optimal government policy $s_g^*(\delta_c)$. *Q.E.D.*