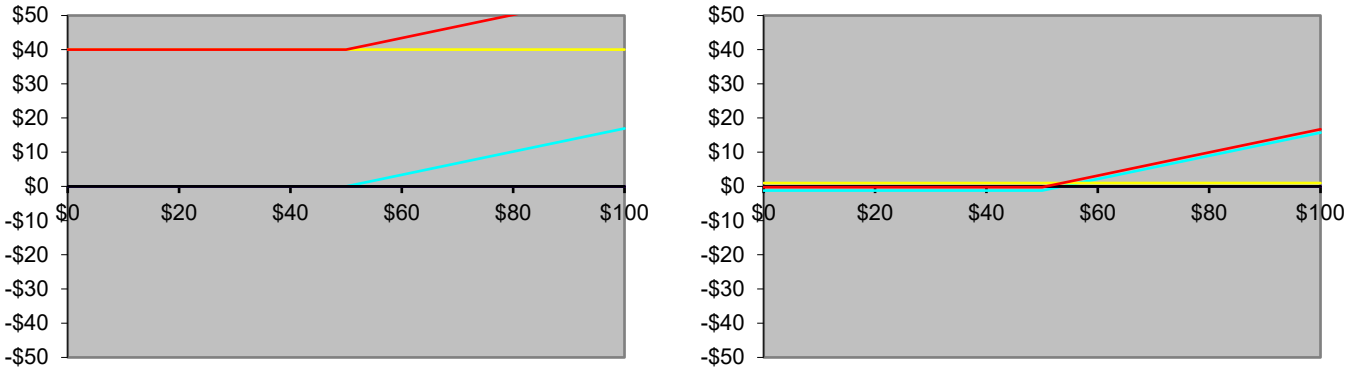
Financial Engineering with Options

There are several trading strategies that use options to meet certain investor needs. We will consider some of these.

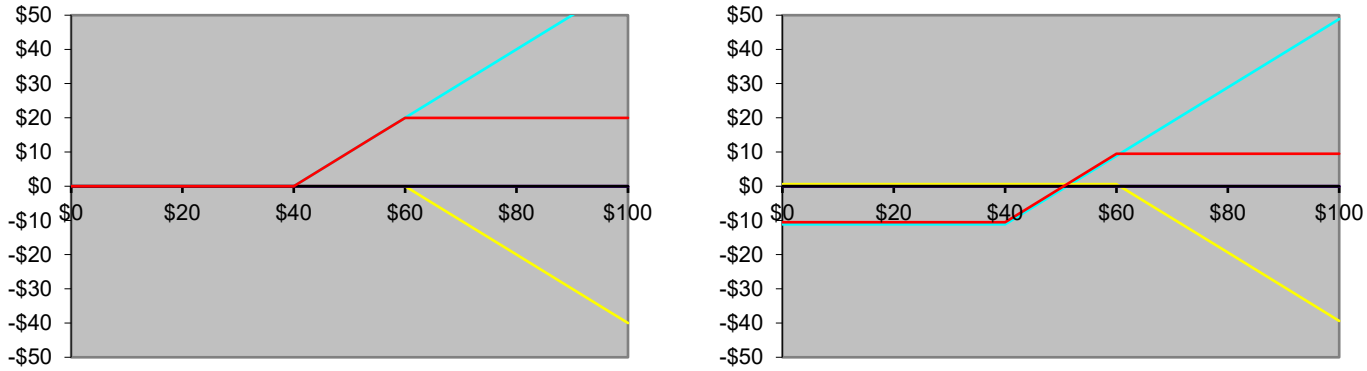
1. **Principal Protected Note**: This is a hybrid of a bond and a stock with some upside of stock but no downside. How is this possible? The note can be constructed using call option on stock rather than the stock itself. But bondholders usually do not want to pay separately for the call option. Instead, the call option is bundled with the bond by increasing the price of the bond to include the price of the call option. An important lesson from the financial crisis of 2007-2008 is that principal protected notes, like some other derivatives, provide protection against market risk but are still vulnerable to **credit risk**. That is, all the protection promised is worthless if the seller of the principal protected notes is unable to meet its promise.



Principal protected note payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

2. **Bull Spread**: This is created by going long in a call with a lower strike price (K_1) and going short in a call with a higher strike price (K_2). Alternatively, it can be created by going long in a put with a lower strike price and going short in a put with a higher strike price. A bull spread allows investor to bet on stock price increase while limiting the investor's upside as well as downside.

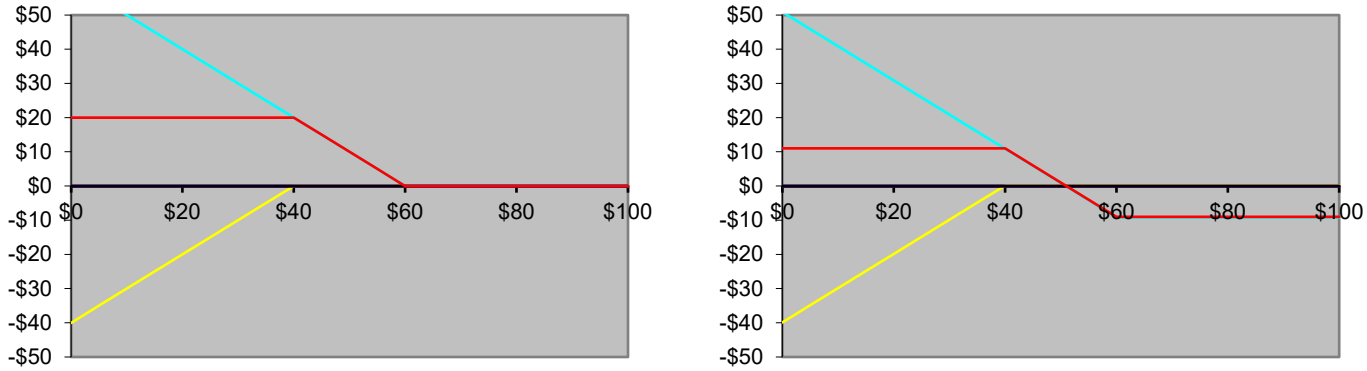
$$Value\ of\ bull\ spread = C(K_1) - C(K_2) = P(K_1) - P(K_2)$$



Bull spread payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

3. **Bear Spread**: This is created by going short in a call with a lower strike price (K_1) and going long in a call with a higher strike price (K_2). Alternatively, it can be created by going short in a put with a lower strike price and going long in a put with a higher strike price. A bear spread allows investor to bet on stock price decline while limiting the investor's upside as well as downside.

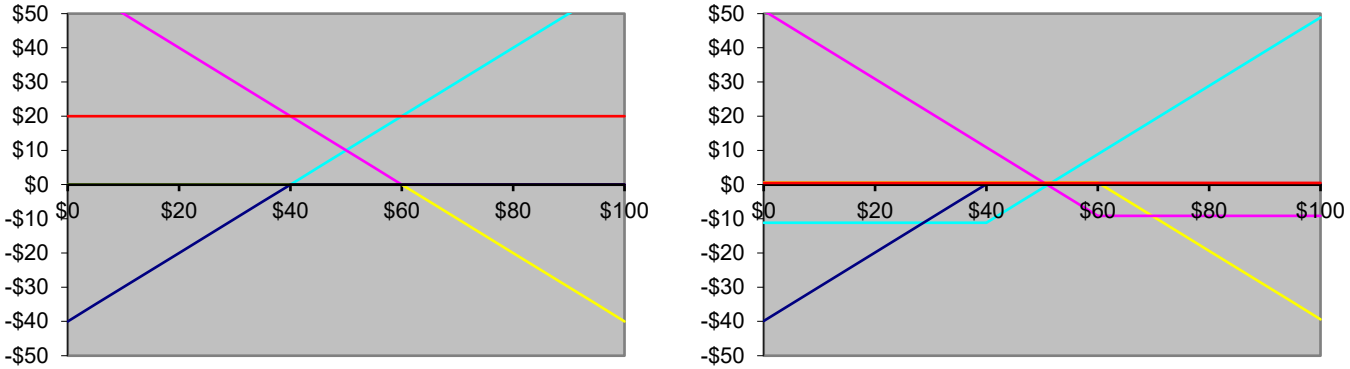
$$Value\ of\ bear\ spread = C(K_2) - C(K_1) = P(K_2) - P(K_1)$$



Bear spread payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

4. **Box Spread**: This consists of a bull spread created using calls and a bear spread created using puts with the same exercise prices K_1 and K_2. The payoff from the box spread is fixed at $K_2 - K_1$.

$$Value\ of\ box\ spread = C(K_1) - C(K_2) - P(K_1) + P(K_2)$$
$$= \{C(K_1) - P(K_1)\} - \{C(K_2) - P(K_2)\}$$
$$= \{S_0 - PV\ of\ K_1\} - \{S_0 - PV\ of\ K_2\}$$
$$= PV\ of\ (K_2 - K_1)$$



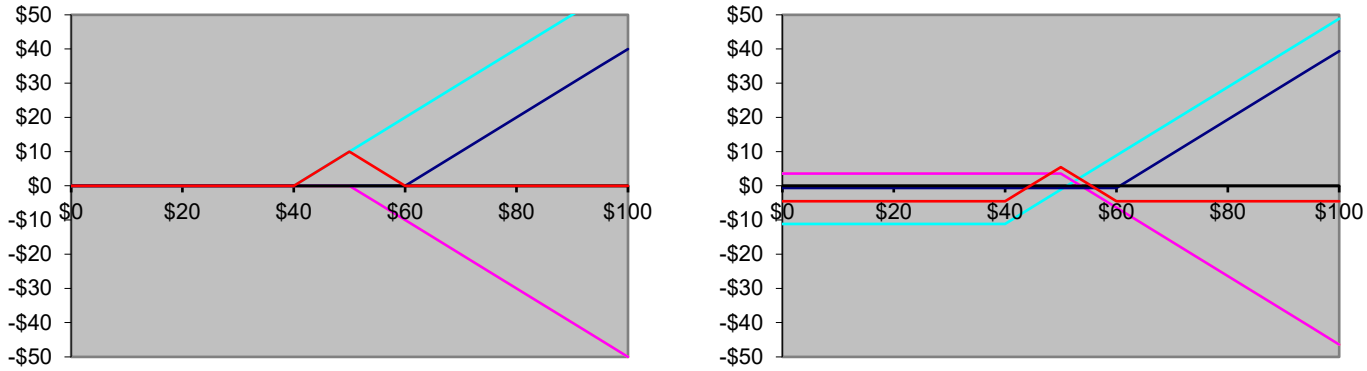
Box spread payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

5. **Butterfly Spread**: This can be used by an investor who expects volatility to be low in the future. It can be created by going long in an option with a low strike price (K_1) and another option with a high strike price (K_3) and simultaneously taking short position in two options with a middle strike price (K_2). All of the options should be calls or all should be puts.

$$Value\ of\ butterfly\ spread = C(K_1) - 2C(K_2) + C(K_3)$$

or

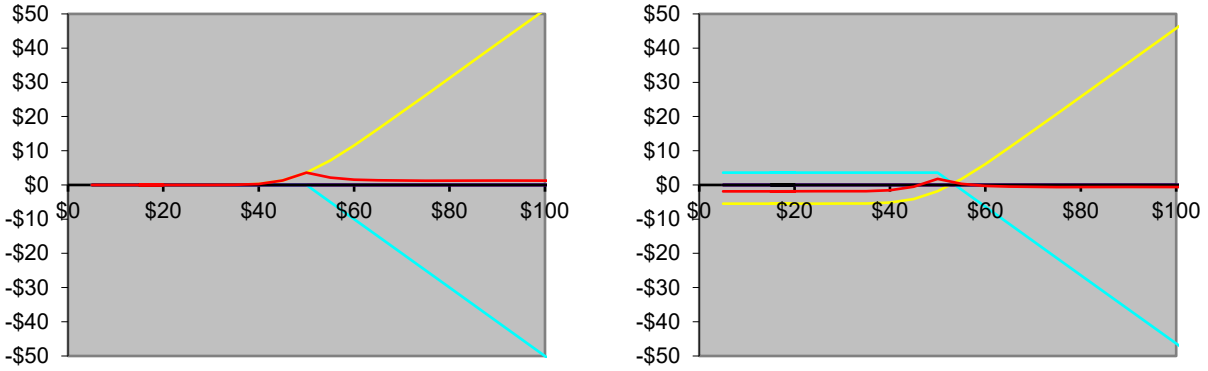
$$Value\ of\ butterfly\ spread = P(K_1) - 2P(K_2) + P(K_3)$$



Butterfly spread payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

6. **Calendar Spread**: A calendar spread can also be used to bet on low volatility. It is created by being long in a longer maturity (T_2) call and being short in a shorter maturity (T_1) call with the same strike price.

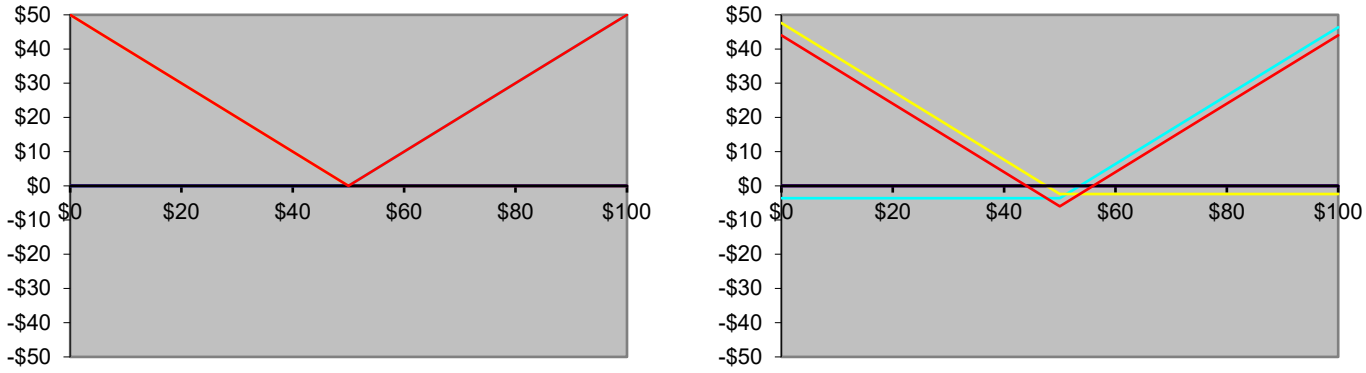
$$Value\ of\ calendar\ spread = C_{T_2}(K) - C_{T_1}(K)$$



Calendar spread payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

7. **Straddle**: It is a combination of a long position in a call and a long position in a put, both with same strike and maturity. A straddle allows an investor to bet on high volatility.

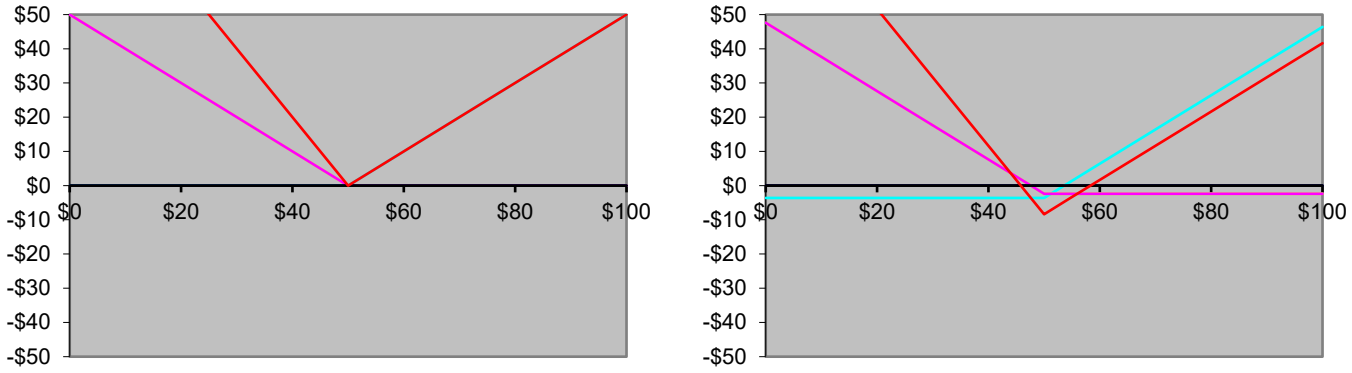
$$Value\ of\ straddle = C(K) + P(K)$$



Straddle payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

8. **Strip**: It is a combination of a long position in one call and a long position in two puts with the same strike and maturity.

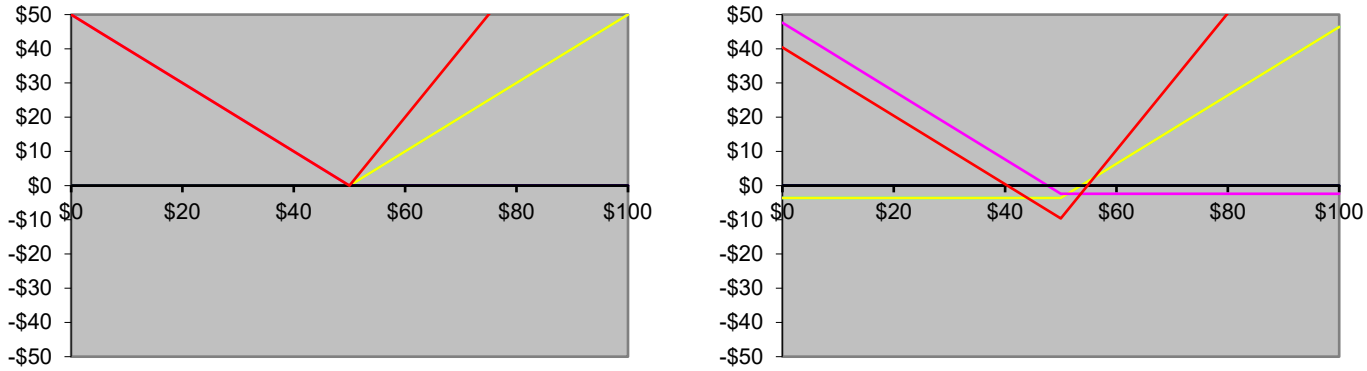
$$Value\ of\ strip = C(K) + 2P(K)$$



Strip payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

9. **Strap**: It is a combination of a long position in two calls and a long position in one put with the same strike and maturity.

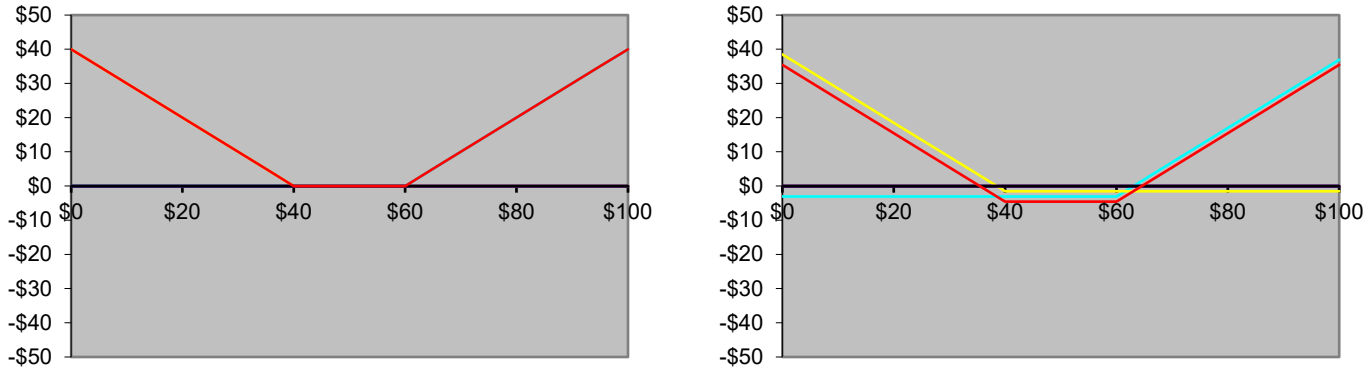
$$Value\ of\ strip = 2C(K) + P(K)$$



Strap payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

10. **Strangle**: It is a combination of a long position in one call and a long position in one put with the maturity but with call strike greater than put strike.

$$Value\ of\ strangle = P(K_1) + C(K_2)$$



Strangle payoff without and with accounting for initial cash flow (horizontal axis is stock price at expiration, vertical axis is portfolio cash flow)

You should try to remember names of some of these strategies as some of these terms are commonly used in the industry. However, from the point of understanding derivatives, it is more important that you can predict the payoff structure for a given portfolio of derivatives. Given a desired payoff structure, a financial engineer should be able to come up with a portfolio that can achieve the desired payoff. Here are some observations that can help you in these tasks (the price of the underlying at option expiration is on the horizontal x-axis and the payoff of the derivatives or the portfolio is on the vertical y-axis):

1. A (riskless zero coupon) bond represents a horizontal line at face value of the bond. Its slope is zero. Each additional bond in a portfolio raises portfolio value plot up by the face value of the bond.
2. A stock represents a 45˚ line passing through original. Its slope is 1. The addition of n stocks to a portfolio leaves the portfolio value at y-axis unchanged but increases the slope of the portfolio value plot by n.
3. A long call option represents a horizontal line that starts at the origin and moves right and at exercise price, it turns upwards at 45˚. The slope is zero on the left of the exercise price and 1 on the right of the exercise price. The addition of n long calls to a portfolio leaves the portfolio value to the left of exercise price unchanged but increases the slope of the portfolio value plot to the right of the exercise price by n.
4. A short call option represents a horizontal line that starts at the origin and moves right and at exercise price, it turns downwards at 45˚. The slope is zero on the left of the exercise price and -1 on the right of the exercise price. The addition of n short calls to a portfolio leaves the portfolio value to the left of exercise price unchanged but decreases the slope of the portfolio value plot to the right of the exercise price by n.
5. A long put option represents a horizontal line with value zero to the right of the exercise price and a line that rises at 45˚ towards the left of the exercise price. The slope is -1 on the left of the exercise price and zero on the right of the exercise price. The addition of n long puts to a portfolio leaves the portfolio value to the right of exercise price unchanged but decreases the slope of the portfolio value plot to the left of the exercise price by n.

6. A short put option represents a horizontal line with value zero to the right of the exercise price and a line that falls at 45˚ towards the left of the exercise price. The slope is 1 on the left of the exercise price and zero on the right of the exercise price. The addition of n short puts to a portfolio leaves the portfolio value to the right of exercise price unchanged but increases the slope of the portfolio value plot to the left of the exercise price by n.