Interest Rate Futures

Accrued Interest

- Sometimes bond prices are quoted by deducting accrued interest from the cash price.
- The accrued interest is the amount of the current coupon that is considered to have “accrued” already.
- It is calculated by taking the ratio of the number of days already elapsed in the current coupon period to the total number of days in the coupon period.
- The cash price is called the **dirty price** while the price quoted after deducting accrued interest is called the **clean price**.
- Dirty price is the actual price that is relevant for transaction.
- The definition of accrued interest can be arbitrary. What matters is that accrued interest and quoted price add up to the cash price.

Cash price = Quoted price + Accrued Interest

Day Counts

- Calculation of accrued interest requires determining the number of days already elapsed in the current coupon period and the total number of days in the coupon period.
- Different markets have adopted different day count conventions for these.
- There is no “correct” or “wrong” method.
- The methods are chosen to be reasonable and convenient for traders.
- There are three conventions followed in the U.S.

Treasury Bonds:	Actual/Actual
Corporate Bonds:	30/360
Money Market Instruments:	Actual/360

Example 1: Consider an 8% treasury bond. A coupon of 4% is earned between coupon payment dates. Interest is accrued on an actual/actual basis. When coupons are paid on March 1 and Sept 1, how much interest is earned between March 1 and April 1?

$$Accrued\ interest = 4\% \times \frac{31}{184} = 0.673\%$$

Example 2: Consider an 8% corporate bond. A coupon of 4% is earned between coupon payment dates. Interest is accrued on an 30/360 basis. When coupons are paid on March 1 and Sept 1, how much interest is earned between March 1 and April 1?

$$Accrued\ interest = 4\% \times \frac{30}{180} = 0.667\%$$

Example 3: Consider an 8% T-bill. A coupon of 8% is earned in 360 days. Interest is accrued on an actual/360 basis. How much interest is earned between March 1 and April 1?

$$Interest\ earned = 8\% \times \frac{31}{360} = 0.688\%$$

Price Quotations of U.S. Treasury Bills

The price of T-bills is quoted as a discount rate. The discount is scaled from the life of the bond to 360 days.

$$P = \frac{360}{n}(100 - Y)$$

where *P* is quoted price, *Y* is cash price per $100 of face value and *n* is the number of calendar days to maturity.

Example 4: A T-bill with 60 days to maturity is trading at 98.5% of face value. What is its quoted price?

$$Quoted\ price = \frac{360}{60} \times (100 - 98.50) = 9$$

Price Quotations of U.S. Treasury Bonds

Cash price = Quoted price + Accrued Interest

Treasury Bond Futures

- Treasury bond futures issued by the CME group are one of the most popular long-term interest rate derivatives.
- Interest rate futures need to identify one of the numerous bonds as the underlying asset.
- Instead of creating a separate future for each bond, these futures allow the short-party to deliver any bond with 15 to 25 years of maturity on the first day of the delivery month.
- There are other contracts for treasury securities with maturities longer than 25 years, or for ranges of shorter maturities.

Treasury Bond Futures Quotes

- Treasury bond futures prices are quoted in dollars and thirty-seconds of a dollar for \$100 of face value.
- A quoted price of 127-13 means 127 + 13/32 = \$127.40625 for \$100 of face value.
- For futures on shorter maturities treasuries, prices are quoted in dollars and nearest half or nearest quarter of thirty-seconds of a dollar for \$100 of face value.

Conversion Factors

- While the short party can deliver any of the many eligible bonds, the exchange specifies rules that are used to adjust the price received based upon the bond delivered.
- These rules try to approximately ensure that the price received is greater for more valuable bonds.
- For each eligible bond, the price is determined using a conversion factor calculated based on the exchange's rules.

Cash price received by party with short position =

Most Recent Settlement Price × Conversion factor + Accrued interest

- For example, if the most recent settlement price is 90.00, conversion factor of bond delivered is 1.3800, and the accrued interest on bond is 3.00, then price received for bond is

  $$1.3800 \times 90.00 + 3.00 = \$127.20 \text{ per } \$100 \text{ of principal.}$$
- If the contract is for the delivery of $100,000 face value of bonds, then the short party will receive $127,200 on delivery.

- Conversion factors are used to recognize the differences in prices of bonds rather than general level of prices of bonds.
- If bond prices are generally high or generally low at a point of time, the price quote can be adjusted to reflect that price level.
- However, if two bonds have different prices, a single price quote cannot reflect both prices and that is where conversion factors are useful.
- The conversion factor for a bond is approximately equal to the **value of the bond on the assumption that the yield curve is flat at 6% with semiannual compounding**.
- For this calculation, the maturity of the bond (and the coupon dates) are **rounded down** to the nearest multiple of three months.
- The accrued interest is subtracted from the value to get the conversion factor.
- This adjustment tries to ensure that price received by the short party equals the price of the bond delivered.
- However, the adjustment is only approximate because the interest rates need not be flat at 6% and maturity need not be an exact multiple of three months.
- But these approximations affect all bond prices so the ratio of prices of two bonds is likely to be close to the ratio of their respective conversion factors.

Cheapest-to-Deliver Bond

- The short party should deliver the bond where its profit is maximized. The cost of buying a bond is

  Quoted bond price + accrued interest.

- The price received on delivery is

  Most recent settlement price × conversion factor + accrued interest

- The short party maximized the difference. The cheapest-to-deliver bond is the one with the least value of

  Quoted bond price - most recent settlement price × conversion factor

Example 6: It is February 25, 2020. The futures price for the February 2020 bond futures contract is 128-27.

a. Calculate the conversion factor for a bond maturing on September 1, 2038, paying a coupon of 8%.
b. Calculate the conversion factor for a bond maturing on December 1, 2043, paying coupon of 5%.
c. Suppose that the quoted prices of the bonds in (a) and (b) are 157.00 and 113.00, respectively. Which bond is cheaper to deliver?
d. Assuming that the cheapest to deliver bond is actually delivered on February 25, 2020 what is the cash price received for the bond?

a. On the first day of the delivery month the bond has 18 years and 7 months to maturity. The value of the bond assuming it lasts 18.5 years and all rates are 6% per annum with semiannual compounding is

$$\frac{4}{0.03}\left(1-\frac{1}{1.03^{37}}\right)+\frac{100}{1.03^{37}}=122.17$$

The conversion factor is therefore 1.2217.

b. On the first day of the delivery month the bond has 23 years and 10 months to maturity. The value of the bond assuming it lasts 23.75 years and all rates are 6% per annum with semiannual compounding is

$$\frac{1}{\sqrt{1.03}}\left(2.5+\frac{2.5}{0.03}\left(1-\frac{1}{1.03^{47}}\right)+\frac{100}{1.03^{47}}\right)=88.67$$

Subtracting the accrued interest of 1.25, this becomes 87.42. The conversion factor is therefore 0.8742.

c. For the first bond, the quoted futures price times the conversion factor is

$$128\frac{27}{32} \times 1.2217 = 128.84375 \times 1.2217 = 157.4084.$$

This is 0.4084 more than the quoted bond price. For the second bond, the quoted futures price times the conversion factor is

$$128\frac{27}{32} \times 0.8742 = 128.84375 \times 0.8742 = 112.6352.$$

This is 0.3648 less than the quoted bond price. The first bond is therefore the cheapest to deliver.

d. The price received for the bond is 157.4084 plus accrued interest. There are 177 days between September 1, 2019 and February 25, 2020. There are 182 days between September 1, 2019 and March 1, 2020. The accrued interest is therefore

$$4 \times \frac{177}{182} = 3.8901$$

The cash price received for the bond is therefore$157.4084 + 3.8901 = 161.2985.$

Treasury Bond Futures Price

Since a bond is an investment asset with a known income, the futures price is calculated as

$$F_0 = (S_0 - I)e^{rT}$$

where S_0 is the spot price of the bond (assuming we know the cheapest-to-deliver bond), I is the present value of income from the bond during the life of the futures contract, T is the time until the futures contract matures, and r is the risk-free rate.

Example 7: It is February 27, 2020. The cheapest-to-deliver bond in a December 2020 Treasury bond futures contract is a 4% coupon bond, and delivery is expected to be made on December 31, 2020. Coupon payments on the bond are made on March 1 and September 1 each year. The risk-free rate of interest with continuous compounding is 2% per annum for all maturities. The conversion factor for the bond is 0.7833. The current quoted bond price is $130.63. Calculate the quoted futures price for the contract.

The cash bond price S_0 is currently

$$130.63 + \frac{179}{182} \times 2 = 132.5970.$$

A coupon of 2 will be received after 3 days. The present value of the coupon on the bond is $2e^{-0.02\times 3/365} = 2.0000$. Another coupon of 2 will be received after 187 days. The present value of the coupon on the bond is $2e^{-0.02\times 187/365} = 1.9796$. Thus, income from the bond before futures maturity is $I = 2 = 1.9796 = 3.9796$. The futures contract lasts 308 days. The cash futures price if it were written on the 4% bond would therefore be

$$(132.5970 - 3.9796)e^{0.02\times 308/365} = 130.8068$$

At delivery there are 121 days of accrued interest. The quoted futures if the contract were written on the 4% bond would therefore be

$$130.8068 - \frac{121}{181} \times 2 = 129.4698.$$

The quoted price should therefore be

$$\frac{129.4698}{0.7833} = 165.2876.$$

Eurodollar Futures

- A Eurodollar is a dollar deposited in a bank outside the United States.
- Eurodollar futures are futures on the 3-month Eurodollar deposit rate (same as 3-month LIBOR rate).
- One contract is on the rate earned on $1 million.
- A change of one basis point or 0.01 in a Eurodollar futures quote corresponds to a contract price change of $25.
- A Eurodollar futures contract is settled in cash.
- When it expires (on the third Wednesday of the delivery month) the final settlement price is 100 minus the actual three-month LIBOR rate.

Example 8: Suppose you buy (take a long position in) a contract on November 1. The contract expires on December 21. The prices are as shown.

Date	Quote
Nov 1	97.12
Nov 2	97.23
Nov 3	96.98
……..	……..
Dec 21	97.42

If on Nov. 1 you know that you will have $1 million to invest on for three months on Dec 21, the contract locks in a rate of 100 - 97.12 = 2.88%.

How much do you gain or lose a) on the first day, b) on the second day, c) over the whole time until expiration?

On the first day, the price increased from 97.12 to 97.23. That is, an increase of 11 basis points. Each basis point results in a gain of $25 so the first day gain is $25 × 11 = $275. What this means is that on November 1, you locked in a rate of 2.88% on the $1 million 3-month deposit you will make on December 21. On November 2, this rate is 100 – 97.23 = 2.77%. So your locked-in interest rate exceeds the market rate by 11 basis points. That will result in additional interest of $1 million × 0.11% × 3/12 = $275.

On the second day, the price falls by 25 basis points, resulting in a loss of $625 on the long futures position.

Over the entire period until expiration, the price rise equals 97.42 = 97.12 = 0.30 or 30 basis points. The cumulative gain is $25 × 30 = $750.

If Q is the quoted price of a Eurodollar futures contract, the value of one contract is

$$10,000[100-0.25(100-Q)]$$

- Verify that this corresponds to the $25 per basis point rule.

Forward vs. Futures Interest Rates

- Eurodollar futures contracts last as long as 10 years.
- For Eurodollar futures lasting beyond two years we cannot assume that the forward rate equals the futures rate.
- There are two reasons for this:
  1. Futures is settled daily where forward is settled once
  2. Futures is settled at the beginning of the underlying three-month period; FRA is settled at the end of the underlying three- month period
- Both factors cause gains or losses to be realized earlier in futures than in forwards.
- This can cause their values to be slightly different.

Convexity Adjustment

A “convexity adjustment” often made is

$$Forward\ Rate\ =\ Futures\ Rate\ -\ 0.5\sigma^2 T_1 T_2$$

where T_1 is the start of period covered by the forward/futures rate, T_2 is the end of period covered by the forward/futures rate (90 days later that T_1), σ is the standard deviation of the change in the short rate per year.

Duration

- The duration of a bond that provides cash flow c_i at time t_i and has a continuously-compounded yield y is

$$D = \frac{\sum_{i=1}^{n} t_i c_i e^{-y t_i}}{B} = \sum_{i=1}^{n} t_i \frac{c_i e^{-y t_i}}{B}.$$

- It is the weighted average of the times when bond's cash flows are paid.
- The weights are the proportions of the bond's value arising from those cash flows.
- The duration is a measure of the sensitivity of the bond price to small parallel shifts in interest rates.

$$\frac{\Delta B}{B} = -D \Delta y.$$

- Notice that this is only an approximation and works better for small changes interest rates.
- When interest rates are expressed with m compounding periods per year, the corresponding measure is the modified duration defined as

$$D^* = \frac{D}{1 + y/m}$$

so that

$$\frac{\Delta B}{B} = -D^* \Delta y.$$

Duration Matching

- Duration matching involves hedging against interest rate risk by matching the durations of assets and liabilities.
- It provides protection against small parallel shifts in the zero curve.

Duration-Based Hedging

- Suppose we want to hedge a portfolio against parallel movements in interest rates.
- Let P be the value of the portfolio and D_P be its duration.
- Suppose we use futures contract for hedging.
- Let V_F be the value of one futures contract and D_F be the duration of the futures contract.
- Then, the number of contracts required to hedge is

$$N^* = -\frac{PD_P}{V_F D_F}.$$

Example 9: Suppose we want to hedge a $10 million portfolio against interest rate changes in the next 3 months. Duration of the portfolio in 3 months will be 6.8 years. The 3-month T-bond futures price is 93-02 so that contract price is $93,062.50. The duration of cheapest to deliver bond in 3 months is 9.2 years.

Number of contracts for a 3-month hedge is

$$-\frac{10{,}000{,}000 \times 6.8}{93{,}062.50 \times 9.2} = -79.42$$