Problem Set I

QF 430: Introduction to Derivatives
Due Wednesday, September 28

Please submit neatly handwritten or typed answers. You can submit your answers in class or electronically a single pdf file through Canvas. Show your steps or reasoning.

1 Introduction

Problem 1.1. The price of gold is currently \$1,675 per ounce. Forward contracts are available to buy or sell gold at \$1,750 per ounce for delivery in one year. An arbitrageur can borrow or invest money at 4% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

Problem 1.2. The current price of a stock is \$50. Three-month call options with a strike price of \$54 currently sell for \$8. An investor with \$20,000 to invest is considering the following three investment strategies:

(a) Investing all his money in the stock

(b) Doubling the amount to invest by taking a loan of \$20,000 at an interest rate of 1% over three months, investing the resulting \$40,000 in the stock and then repaying \$20,200 on the loan

(c) Investing all his money in the call options

Determine the return of the investor (defined as change in wealth / initial wealth) under each of the three strategies for the following two scenarios: 1) stock price falls to \$40 after three months, 2) stock price rises to \$70 after three months. Compare the risks and returns of the three strategies.

Problem 1.3. The economic growth next year will be low, medium, or high. There are four assets, A, B, C, and D, whose value (cash flow) in the three states will be as follows:

	Low Growth	Medium Growth	High Growth
A	30	50	80
B	20	40	40
C	30	60	80
D	30	40	50

The prices of the assets A, B, and C, are 49.90, 34.20, and 55.30, respectively.

(a) What are the state prices of the three states (high growth, medium growth, and low growth)?

(b) What is the risk-free rate?

(c) What is the price of asset D?

Problem 1.4. Consider a fixed time T in future. Let S be the price of a stock at time T in \$. For positive integer n, let c_n be the price of a binary call option that will pay \$1 at time T if $S \geq n$ and 0 if $S < n$. Similarly, let p_n be the price of a binary put option that will pay \$1 at time T if $S < n$ and 0 if $S \geq n$. Suppose these prices are available for all positive integral values of n. That is, $c_1, p_1, c_2, p_2, c_3, p_3, \ldots$ are available.

(a) Write an expression for the state price for the state that S lies between n and $n+1$ ($n \leq S < n+1$) using binary call prices. Recall, this is the price of a portfolio created using binary call options that pays \$1 if $n \leq S < n+1$ and 0 otherwise. *Hint*: Buy and/or sell different calls to create a portfolio with the required payoff. The payoff from selling an option is the exact opposite of the payoff from buying the option.

(b) Write an expression for the state price for the state that S lies between n and $n+1$ ($n \leq S < n+1$) using binary put prices.

(c) Equating the two expressions you calculated for the same state price, determine a relation between binary call prices and binary put prices.

2 Futures Markets

Problem 2.1. What is open interest in a derivative contract? Why does the open interest usually decline during the month preceding the delivery month? Suppose 1,000 contracts in a futures market are traded on a given day. Of the 1,000 long positions taken, 400 closed short existing positions, while 600 opened new long positions. Of the 1,000 short positions taken, 700 closed existing long positions and 300 opened new short positions, What is the change in the open interest on this day?

Problem 2.2. A company has a long position in futures contract to buy 5,000 bushels of wheat. The company had entered into the contract when the futures price was 1,094 cents per bushel. The initial margin is \$4,500 and the maintenance margin is \$3,300. The current futures price is 859 cents per bushel and the margin account balance is \$3,700.

(a) Determine the net margin deposits to or margin withdrawals from the margin account so far.

(b) What price change would lead to a margin call?

(c) Following what price change can \$1,000 be withdrawn from the margin account?

Problem 2.3. This problem illustrates the issues faced in setting margin requirements. Use the daily closing prices for crude oil futures contract in dollars per barrel from the given spreadsheet (obtained from Professor Hull's website). You will need to use a spreadsheet or another calculation software for this problem.

(a) Assuming that daily price changes are normally distributed and independent across time, estimate the standard deviation of two-day price changes. To do this, calculate daily price changes, estimate their standard deviation, and then multiply by the square root of two for the standard deviation of two-day price changes. For the purpose of calculating standard deviation, you may assume the mean of the normal distribution to be zero or estimate it using the data.

(b) Assume two-day price changes are normal. What maintenance margin should an exchange set for a member with a long position in one contract such that with 99% probability, the decline in contract value over a two-day period will not exceed the maintenance margin? That is, determine maintenance margin such that the decline in the contract value over a two-day period will exceed the maintenance margin with 1% probability. Each contract is on 1,000 barrels of oil. Use the normal distribution with mean and standard deviation you estimated for two-day price changes to find the 1 percentile fall in price over two days. The Excel function norm.inv can be used for this. Scale the price change by contract size to find the corresponding fall in contract value.

(c) In the given data, what percentage of the days does the decline in contract value over a two-day period actually exceed the maintenance margin? What do your results suggest about the appropriateness of the normal distribution assumption?

(d) In practice, margin balance usually exceeds maintenance margin because margin calls restore the margin balance to the initial margin. Assume that the initial margin is set such that the maintenance margin calculated above equals 75% of the initial margin. Track margin balance assuming that (1) the margin balance starts at the initial balance, (2) a margin call is made on any day the margin balance drops below the maintenance margin, and (3) whenever margin balance rises above the initial margin, the excess is withdrawn the same day by the client so that margin balance at the end of the day never exceeds the initial margin.

    Determine the percentage of times that the client has an incentive to default (that is the number of two-day periods over which the margin balance becomes negative if the client does not meet margin calls). Assume that if a margin call is not met one day, margin withdrawal cannot be made the next day. One way to do this is as following. First calculate *true* daily changes in contract value, *true* withdrawals, *true* deposits, and *true* margin balance under assumptions (1)-(3) above, ignoring any defaults. Then calculate the *default* balance on a day assuming *true* margin balance two days ago, no margin deposit that day and the previous day and no margin withdrawal that day. This can also be calculated as: *default* balance on day n equals the *true* balance on day n minus *true* margin deposits on days $n-1$ and n plus *true* margin withdrawal on day n. There is incentive to default if this *default* balance is negative. See the spreadsheet with an example.

3 Hedging with Futures

Problem 3.1. It is now September 2022. A company anticipates that it will purchase 1 million pounds of copper in each of December 2022, June 2023, December 2023, and June

2024. The company has decided to use the futures contracts traded by the CME Group to hedge its risk. Contracts with maturity in January 2023, July 2023, January 2024, and July 2024 are available. One contract is for the delivery of 25,000 pounds of copper. The initial margin is $6,050 per contract and the maintenance margin is $5,500 per contract. The company's policy is to hedge 80% of its exposure. Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the company's needs.

(a) Devise a hedging strategy for the company. That is, specify the company's futures trades for hedging. Do not make the adjustment for daily settlement (tailing the hedge) described in the textbook.

(b) Assume that the market prices (in cents per pound) today and at future dates are as in the following table.

Price / Date	Sep 2022	Dec 2022	Jun 2023	Dec 2023	Jun 2024
Spot	356.2	351.0	344.6	347.9	350.7
Jan 2023 Futures Price	353.8	351.1			
Jul 2023 Futures Price	350.1	347.6	344.4		
Jan 2024 Futures Price		348.0	344.6	348.2	
Jul 2024 Futures Price			345.1	349.1	350.9

What is the impact of the strategy you propose on the price the company pays for copper?

(c) What is the initial margin requirement in September 2022?

(d) Is the company subject to any margin calls? For part, only consider the prices provided, ignoring prices on other days.

Problem 3.2. A fund manager wants to hedge her portfolio against market movements over the next two months. The portfolio is worth $25 million and its CAPM beta is 0.8. The manager plans to use three-month futures contracts on a well-diversified index to hedge its risk. The current level of the index is 3876, one contract is on $50 times the index, the risk-free rate is 4.2% per annum, and the dividend yield on the index is 1.8% per annum. The current 3-month futures price is 3900.

(a) What position should the fund manager take to eliminate all exposure to the market over the next two months?

(b) Calculate the expected gain or loss of the fund manager's hedged position under four cases: the index value in two months is 3,000, 3,500, 4,500, and 5,000. This will consist of the gain or loss on the futures position and gain or loss on the portfolio. In each case, assume that the one-month futures price after two months is 0.4% higher than the index level at that time. Your portfolio value at that time is not provided for these cases. To calculate the expected value of your portfolio after two months in any case, determine the excess return on market as the percentage increase in the index plus the dividend yield over two months minus the risk-free rate over two months. Multiply the excess return on market with the beta of the portfolio to get the expected excess return on the portfolio. Add risk-free rate over two months to get the expected return on the index over two months. For this problem you can ignore compounding so that, for example, 4.2% per annum is equivalent to $4.2\% \times 1/12 = 0.35\%$ per month.