Problem Set V

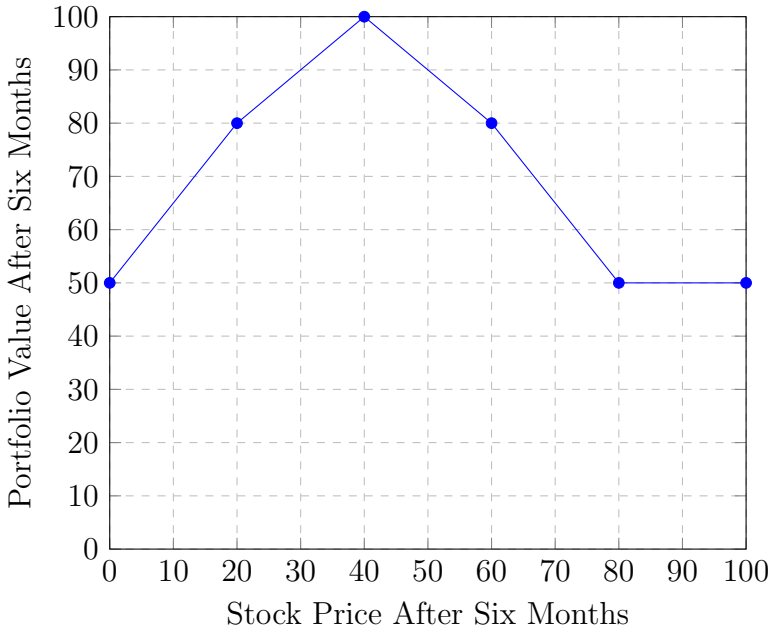
QF 430: Introduction to Derivatives
Due Monday, November 21

Please submit neatly handwritten or typed answers. You can turn in paper submissions in class or submit a single pdf file electronically through Canvas. Show your steps or reasoning. A spreadsheet is provided for Black-Scholes pricing of European options.

1 Financial Engineering with Options

Problem 1.1. You own a stock trading at \$70. The stock does not pay any dividend. You want to reduce your exposure to stock price movement without selling the stock or incurring any cash expense. Use a collar so that the value of your hedged stock after six months will lie within a price range \$$A$ to \$$B$. That is, your position's value will be \$$A$ if stock price after six months is less than \$$A$, \$$B$ if stock price after six months exceeds \$$B$, and equal to the stock price if stock price after six months is between \$$A$ and \$$B$. What collar strategy will provide you a \$20 range (that is, B - A = 20) at zero cost? Ignore commissions or transaction costs. The risk-free rate is 5% per annum with continuous compounding and the volatility of the stock is 40%. Structure the collar using European options. Assume option prices are given by Black-Scholes formulas.

Problem 1.2. A stock is trading at \$50. An investor wants you to design her a portfolio whose value after six months will depend on the price of the stock at that time as shown in the figure below. The portfolio value varies linearly between circular markers and continues linearly to the right of the figure's bounds. How can you structure such a portfolio for her? How much will it cost her? Assume option prices are given by Black-Scholes formulas. Assume that the stock does not pay dividend and its volatility is 40%. The risk-free rate is 5% per annum with continuous compounding. Ignore transaction costs and any other fees. Assume each option is written on one stock and you can buy and sell fractional stocks, bonds, and options. To ensure that your answer is correct, check the value of the portfolio at maturity for a few values of stock price after six months. You don't need to report these checks.



2 Pricing Options with Binomial Trees

Problem 2.1. A stock that does not pay any dividend is currently trading at \$60. After one period, the price will either increase by 20% or decrease by 20%. The risk-free rate is 8% per period (*not continuously compounded*). Consider a European put option on the stock with strike price of \$66. Set up a riskless portfolio and price the option.

Problem 2.2. A stock that does not pay any dividend is currently priced at \$54. After one period, the stock price will increase to \$80 or decrease to \$40. A bond that will pay \$100 after one period is trading at \$96. Find the risk-neutral probabilities of stock price up and down movements. Value a call option with strike \$50. Value an at-the-money European put option. Both options mature after one period.

Problem 2.3. A stock that does not pay dividend is trading at \$100. The stock price will increase by 10% or decrease by 10% in one year. After that, the stock price will increase or decrease by 20% in the second year. The risk-free interest rate is 5% per annum with continuous compounding. Value a two-year American put option with strike price of \$102. Note that risk-neutral probabilities or replicating portfolios may differ across two periods. You must also check for early exercise.

3 Stochastic Processes

Problem 3.1. The quantity X_t follows an Arithmetic Brownian motion with drift 5 and volatility 10. Suppose $X_0 = 100$. What is the probability that X_1 is at least 100? Recall that for an Arithmetic Brownian motion with drift μ and volatility σ, the change in time interval τ is normally distributed with mean $\mu\tau$ and variance $\sigma^2\tau$.

Problem 3.2. A gambler starts with \$40. The gambler has decided to continue placing bets until either her wealth rises to \$100 or her wealth drops to \$20. Each bet is for a dollar and results in a loss of the dollar with 50 percent probability or a doubling to two dollars with 50 percent probability. Explain why the gambler's wealth is a martingale. What is the chance that the gambler ends up with \$100?