Swaps

- A swap is an agreement between two parties to exchange cash flows at specified future times according to certain specified conditions.
- One party agrees to pay one set of cash flows and agrees to receive another set of cash flows.
- Simple example: A person agrees to buy 25,000 pounds of copper after six months at a price of $2.56 per pound.
- Swaps are very popular derivative instruments and are very versatile.
- We will focus on interest rate swaps and currency swaps, the two most popular kinds of swaps.

Interest Rate Swaps

- Most common ("plain vanilla") interest rate swap: one party agrees to pay interest at predetermined fixed rate on a notional principal at regular intervals for a certain time period and in return receive interest at a floating rate.
- The floating rate used for the cash flow exchange at the end of an interval is the market value of that rate at the beginning of the interval.
- We will study interest rate swaps in which the floating interest rate is LIBOR or LIBOR plus a fixed spread.

Example 1: Apple agrees to receive 6-month LIBOR & pay a fixed rate of 3% per annum every 6 months for 3 years on a notional principal of $100 million. Apple enters the swap on March 8, 2022. The following table shows one possible set of LIBOR rates and the resulting cash flows:

		---------Millions of Dollars---------		
Date	LIBOR Rate	FLOATING Cash Flow	FIXED Cash Flow	Net Cash Flow
Mar. 8, 2022	2.2%			
Sept. 8, 2022	2.8%	+1.10	–1.50	–0.40
Mar. 8, 2023	3.3%	+1.40	–1.50	–0.10
Sept. 8, 2023	3.5%	+1.65	–1.50	+0.15
Mar. 8, 2024	3.6%	+1.75	–1.50	+0.25
Sept. 8, 2024	3.9%	+1.80	–1.50	+0.30
Mar. 8, 2025		+1.95	–1.50	+0.45

The first cash flow takes place after six months, at September 8, 2022.

The floating rate used for this exchange is the 6-month rate on March 8, 2022.

There is no exchange of principal amount because both cash flows are based on the same principal amount of $100 million.

The swap can be seen as a series of forward contracts and each forward contract is an exchange of a fixed cash flow for a floating rate cash flow.

Another view of swap is obtained by explicitly adding exchange of principal amounts at the end.

		---------Millions of Dollars---------		
Date	LIBOR Rate	FLOATING Cash Flow	FIXED Cash Flow	Net Cash Flow
Mar. 8, 2022	2.2%			
Sept. 8, 2022	2.8%	+1.10	–1.50	–0.40
Mar. 8, 2023	3.3%	+1.40	–1.50	–0.10
Sept. 8, 2023	3.5%	+1.65	–1.50	+0.15
Mar. 8, 2024	3.6%	+1.75	–1.50	+0.25
Sept. 8, 2024	3.9%	+1.80	–1.50	+0.30
Mar. 8, 2025		+101.95	–101.50	+0.45

Apple receives cash flows of a bond with face value of $100 million and floating interest rate.

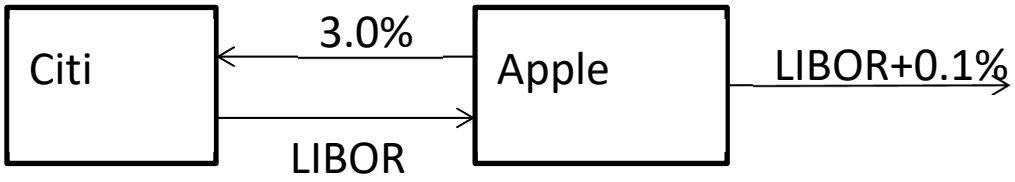
Apple pays cash flows of a bond with face value of $100 million and fixed interest rate.

From Apple's point of view, the swap is a long position in a floating rate bond and a short position in a fixed rate bond.

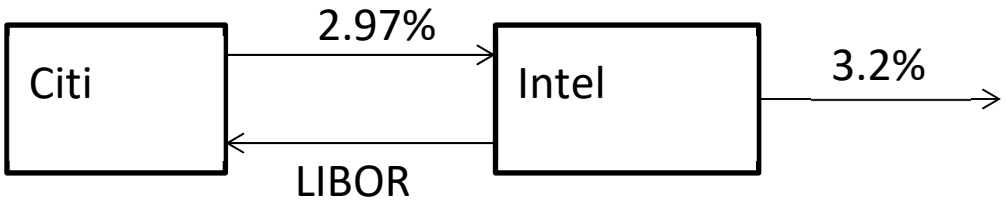
From the counterparty's point of view, the swap is a short position in a floating rate bond and a long position in a fixed rate bond.

Uses of Interest Rate Swaps

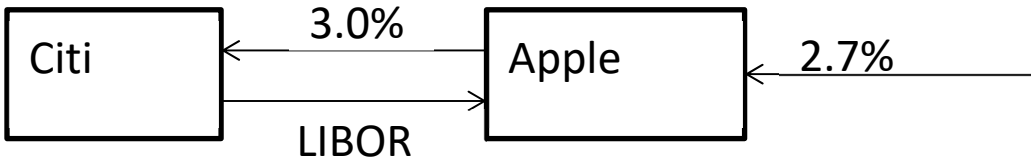
- Interest rate swaps are typically used to converting a liability or an investment from a fixed rate to a floating rate or from a floating rate to a fixed rate.
- Here are some examples of swaps in which Citibank provides swaps that allow Apple and Intel to transform their investments and liabilities. In each case, determine the net cash flow of Apple or Intel.



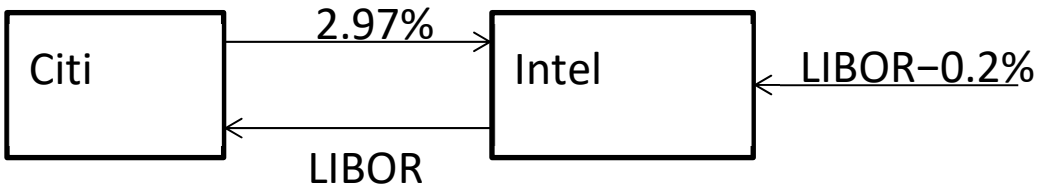
Apple transforms liability from floating to fixed. Net rate paid = 3.1%



Intel transforms liability from fixed to floating. Net rate paid = LIBOR + 0.23%



Apple transforms assets from fixed to floating. Net rate earned = LIBOR - 0.3%



Intel transforms liability from floating to fixed. Net rate earned = 2.77%

Role of Financial Intermediaries

- Suppose Apple wants to transform its floating rate liability to a fixed rate liability and Intel wants to change its fixed rate liability to a floating rate liability.
- They can enter into a swap with each other.
- However, it will be costly for them to find a counterparty and negotiate.
- Typically, they will each approach a financial intermediary and the financial intermediary will enter into a separate swap with each other.
- The financial intermediary sets terms that nets it a positive spread amount.
- Apple and Intel get terms which are slightly worse than what they would if they directly negotiated with each other.
- But they don't have to worry about each other's credit risk and they avoid search costs.

Market Makers

- The financial intermediary cannot always get two counterparties that are interested in two offsetting swaps.
- In this case, the financial intermediary enters into swap with a single counterparty and manages its risk through derivative transactions such as forward agreements, bonds, or futures.
- The financial intermediary act as market maker by willing to act as counterparty in swaps.
- It posts fixed rates that it is willing to pay or receive in exchange for floating rate in swaps. These fixed rates are called **swap rates**.
- The difference between the rate that the market maker is willing to pay (bid) and the rate that the market maker is willing to receive (offer) is the spread that the market maker earns for its services.
- The swap rate is the average of these two rates and can be considered the fair rate at which the swap has zero value to each counterparty.
- The swap rate is that rate at which the fixed-rate bond and the floating-rate bond exchanged in the swap have the same market value.

Day Count Convention

- We have assumed that the cash flows are determined by multiplying the notional principal with the interest rate and the swap interval.
- In practice, a day count convention is specified for fixed and floating payments. For example, LIBOR is likely to be actual/360 in the U.S. because LIBOR is a money market rate. We will ignore these details.

Confirmations

- Confirmations specify the terms of a transaction.
- The International Swaps and Derivatives Association (ISDA) has developed Master Agreements that can be used to cover all agreements between two counterparties.
- CCPs are used for most standard swaps.

Comparative Advantage Agreement

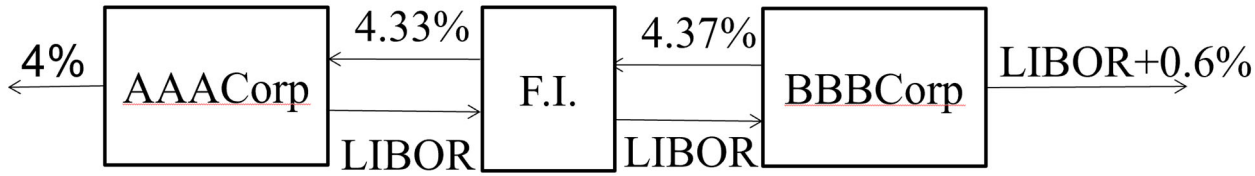
- How do swaps create value for both counterparties?
- One party has an advantage in fixed-rate market but wants to borrow in floating-rate market while the other party has the opposite situation.
- Swaps help both achieve their desired outcome.
- In general, the relative advantage of one party over the other is greater in one market than in the other market.
- Why does one party have a greater relative advantage in one market than in the other market? If there is an economic rationale for that, how do swaps overcome that rationale?

Example 2: Consider AAACorp which has a higher creditworthiness than BBBCorp. AAACorp wants to borrow floating while BBBCorp wants to borrow fixed. The rates at which they can borrow are the following:

	Fixed	Floating
AAACorp	4.00	6-month LIBOR – 0.1%
BBBCorp	5.20	6-month LIBOR + 0.6%

- AAACorp has a relative advantage of 1.2% in the fixed rate market and a relative advantage of 0.7% in the floating rate market.
- It makes sense for AAACorp to borrow fixed and BBBCorp to borrow floating and then enter into a swap.

- Or they can enter into a swap with a financial intermediary:



- Each party is better off compared to borrowing without swap. How is this possible?
- The reason is that the 4.0% and 5.2% rates available to AAACorp and BBBCorp in fixed rate markets are 5-year rates.
- The LIBOR−0.1% and LIBOR+0.6% rates available in the floating rate market are on the other hand six-month rates.
- **These rates are not guaranteed in the long run and may diverge for AAACorp and BBBCorp as their creditworthiness changes**.
- BBBCorp's fixed rate obtained as a result of the swap depends on the spread above LIBOR it borrows at in the future and this spread may increase.

Valuation of Interest Rate Swap

- Initially interest rate swaps are worth close to zero.
- At later times they can be valued as a portfolio of forward rate agreements (FRAs).
- The procedure is to
  1. Calculate LIBOR forward rates
  2. Calculate the swap cash flows that will occur if LIBOR forward rates are realized
  3. Discount these swap cash flows at OIS rates

Two Roles of Interest Rates

- Interest rates are used to determine cash flows of an interest rate swap and also to discount the cash flows.
- The cash flows substituted for the floating payment of the swap are based on LIBOR forward rates.
- In the past, discounting was done using LIBOR zero rates.
- Now, OIS rates are used to discount cash flows so valuation of swap requires LIBOR forward as well as OIS rates.

Example 3: Suppose that some time ago a financial institution agreed to receive 6-month. LIBOR and pay 3% per annum (with semiannual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The LIBOR rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 2.8%, 3.2%, and 3.4%, respectively. The 6-month LIBOR rate at the last payment date was 2.9% (with semiannual compounding).

Approach 1: Swap = Exchange of Bonds

- The fixed rate bond is valued by discounting all cash flows.
- The floating rate bond is valued at par right after the first cash flow exchange because it will pay LIBOR for all future cash flows and the discount rate is also LIBOR.
- The discount factors for 0.25 years, 0.75 years, and 1.25 years are calculated as $e^{-0.028\times0.25}$, $e^{-0.032\times0.75}$, and $e^{-0.034\times1.25}$, respectively.

Time	B_{fix} *cash flow*	B_{fl} *cash flow*	*Discount factor*	*Present value* B_{fix} *cash flow*	*Present value* B_{fl} *cash flow*
0.25	1.5	101.4500	0.9930	1.4895	100.7423
0.75	1.5		0.9763	1.4644	
1.25	101.5		0.9584	97.2766	
Total:				100.2306	100.7423

- Value of swap to the financial institution = 100.7423 – 100.2306 = $0.5117 million.

Approach 2: Swap = Series of Forward Rate Agreements

- Value each exchange of cash flows separately and then discount them.
- Assume that all unknown floating rates equal the forward rates for the corresponding time periods.
- The forward rates can be calculated as follows. For the period 0.25 years to 0.75 years, the forward rate is

$$f = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = \frac{3.2\% \times 0.75 - 2.8\% \times 0.25}{0.75 - 0.25} = 3.4\%.$$

- The floating cash flow after 0.75 years is then calculated as

$$\$100\, million \times (e^{0.034 \times 0.5} - 1) = \$1.7145\, million.$$

- Similarly, the forward rate and the cash flow after 1.25 years can be calculated. The net cash flow at each rate is then discounted using zero rates.

Time	*Fixed cash flow*	*Floating cash flow*	*Net cash flow*	*Discount factor*	*Present value of net cash flow*
0.25	−1.5000	+1.4500	−0.0050	0.9930	−0.0497
0.75	−1.5000	+1.7145	+0.2145	0.9763	+0.2094
1.25	−1.5000	+1.8672	+0.3672	0.9584	+0.3519
Total:					+0.5117

Note: In Example 2, LIBOR forward rates were used to calculate cash flows and LIBOR zero rates were used to discount. If OIS rates are available, OIS rates should be used for discounting.

Bootstrapping LIBOR forward rates

- We have seen how to value swap if LIBOR forward rates are known.
- However, this process can also be carried out in reverse.
- If swap rates are available at different maturities, a swap with a fixed interest rate equal to the corresponding swap rate is valued at zero.
- That is, the present value of the known fixed cash flows equals the present value of the floating cash flows.
- This helps us infer the market's estimate of the floating cash flows which reflect LIBOR forward rates.

Example 4: The 6, 12, 18, and 24 month OIS rates are 3.8%, 4.3%, 4.6%, and 4.75% respectively with continuous compounding. The 6-month LIBOR rate is 4% (semiannually compounded). Suppose forward LIBOR rates for 6-12 and 12-18 months have already been calculated as 5% and 5.5%, respectively (semiannually compounded). The two year swap rate is 5%. Calculate the LIBOR forward rate *F* for the 18-24 month period.

A 2-year swap where 5% is paid and LIBOR is received on $100 is worth zero. The values of the first three exchanges are

$$0.5 \times (0.04 - 0.05) \times 100e^{-0.038\times 0.5} = -0.4906,$$

$$0.5 \times (0.05 - 0.05) \times 100e^{-0.043\times 1} = 0,$$

$$0.5 \times (0.055 - 0.05) \times 100e^{-0.046\times 1.5} = 0.2333.$$

The value of the fourth payment must be +0.2573 so that the total value is zero.

$$0.5 \times (F - 0.05) \times 100e^{-0.0475\times 2} = 0.2573.$$

$$F = 0.05566 \; or \; 5.566\% \; pacc$$

Swap Rates

- What is the difference between swap rates and LIBOR rates?
- The LIBOR is the rate of interest at which AA-rated banks borrow for periods up to 12 months from other banks.
- The swap rate is the average of
    - the fixed rate that a swap market maker is prepared to pay in exchange for receiving LIBOR (its bid rate) and
    - the fixed rate that it is prepared to receive in return for paying LIBOR (its offer rate).
- Since the floating cash flows of a swap are tied to different forward LIBOR rates while the fixed cash flows are tied to the swap rate, the **swap rate is related to a series of forward LIBOR rates**.

- Neither LIBOR rates, nor swap rates are risk-free rates.
- However, **swap rates are closer to risk-free rates than LIBOR rates** in normal market conditions.
- A financial institution can earn the 5-year swap rate on a certain principal by lending the principal to one AA borrower for the first 6 months, to another AA borrower for the next 6 months and so on, and entering into a swap to exchange the LIBOR income for the 5-year swap rate.
- In this case, the bank is always lending to a AA borrower for each 6-month period. This risk is lower than the risk of 5-year LIBOR where a 5-year loan is made to a AA borrower because the borrower may not continue to be to AA over the five years.
- That is why the 5-year swap rate is less than the 5-year AA borrowing rate.

Currency Swaps

- In a currency swap, cash flows in one currency are exchanged with cash flows in another currency.
- The cash flows may be fixed in both currencies, floating in both currencies, or fixed in one currency and floating in the other currency.
- We will consider **fixed-for-fixed swaps**. The discount rate for each currency's cash flow is the risk-free rate for that currency.
- Currency swaps differ from interest rate swaps in one other respect.
- In a currency swap, principal amounts in the two currencies are exchanged at the beginning of the swap and then at the maturity of the swap.

Example 5: Consider a five-year agreement by BP to pay 3% on a US dollar principal of $15,000,000 and receive 4% on a sterling principal of £10,000,000. The cash flows of the swap will be

Date	Dollar Cash Flows (millions)	Sterling Cash Flows (millions)
March 1, 2022	+15.00	-10.00
March 1, 2023	-0.45	+0.40
March 1, 2024	-0.45	+0.40
March 1, 2025	-0.45	+0.40
March 1, 2026	-0.45	+0.40
March 1, 2027	-15.45	+10.40

Typical Use of a Currency Swap

Currency swaps can be used to

1. convert a liability in one currency to a liability in another currency
2. convert an investment in one currency to an investment in another currency

Comparative Advantage Agreement

- We argued that the comparative advantage argument for interest rate swaps is weak.
- Comparative advantage arguments for currency swaps are more realistic.
- One party may have a greater comparative advantage in structuring their assets or liabilities in one currency while the other party may have a greater comparative advantage in structuring their assets or liabilities in a another currency.
- Differential tax laws across countries and dependent on the location of different companies can lead to such comparative advantage situations.
- The comparative advantage can be seen by comparing the borrowing costs after adjusting for the differential impact of taxes:

	USD	AUD
General Electric	5.0%	7.6%
Quantas	7.0%	8.0%

Valuation of Currency Swaps

Fixed for fixed currency swaps can be valued either using forward rates or as the difference between two bonds.

Example 6: All Japanese interest rates are 1.5% per annum (cont. comp.). All USD interest rates are 2.5% per annum (cont. comp.). A swap is structured so that 3% is received in yen and 4% is paid in dollars. Payments are made annually. The principals are $10 million and 1,200 million yen. The swap will last for 3 more years. The current exchange rate is 110 yen per dollar.

Approach 1: Valuation in Terms of Forward Rates

Time	Dollar Cash Flow	Yen Cash Flow	Forward Rate	Dollar Value of Yen Cash Flow	Net Cash Flow	Present Value
1	-0.4	+36	0.009182	0.3306	-0.0694	-0.0677
2	-0.4	+36	0.009275	0.3339	-0.0661	-0.0629
3	-10.4	+1236	0.009368	11.5786	+1.1786	+1.0934
Total						+0.9629

Approach 2: Valuation in Terms of Bonds

Time	Cash Flows ($ millions)	PV ($ millions)	Cash Flows (millions of Yen)	PV (millions of Yen)
1	0.4	0.3901	36	35.46
2	0.4	03805	36	34.94
3	10.4	9.6485	1,236	1,181.61
Total		10.4191		1,252.01

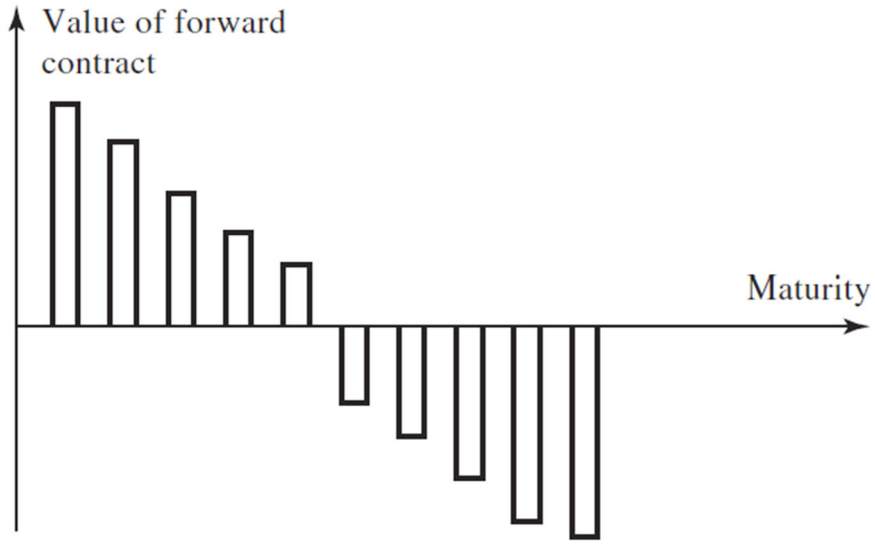
Value = 1,252.01/110−10.4191 = +0.9629 millions of dollars

Other Currency Swaps

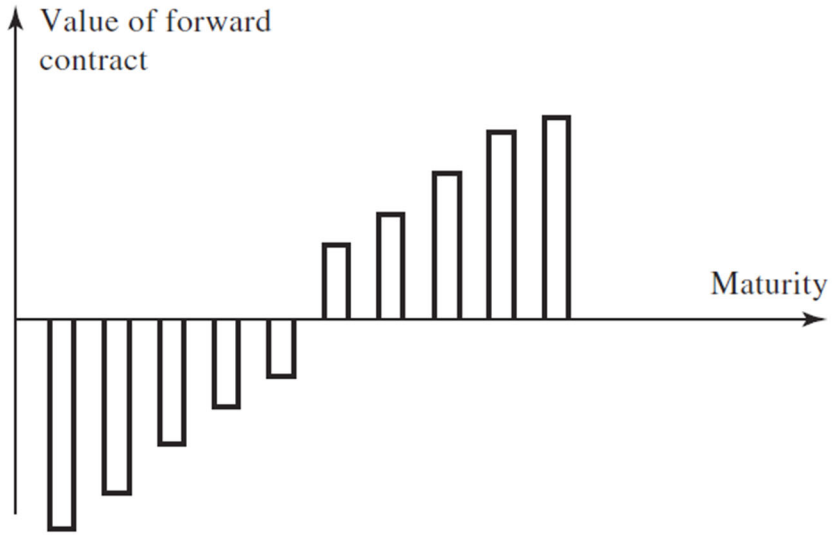
- A fixed-for-floating currency swap is equivalent to a fixed-for-fixed currency swap plus a fixed for floating interest rate swap.
- A floating-for-floating currency swap is equivalent to a fixed-for-fixed currency swap plus two floating interest rate swaps.

Swaps and Forwards

- A swap is a convenient way of packaging forward contracts.
- When a swap is initiated the swap has zero value, but typically some forwards have a positive value and some have a negative value.
- For interest rate swaps, the value of forwards is related to the term structure of interest rates.
- If the term structure of interest rates is upward-sloping, from the perspective of the party receiving fixed rate, earlier forward contracts will have positive value and later forward contracts will have negative value:



- If the term structure of interest rates is downward-sloping, from the perspective of the party receiving fixed rate, earlier forward contracts will have negative value and later forward contracts will have positive value:



Credit Risk

- Credit risk is the risk of default by a counterparty.
- A counterparty is less likely to default on one swap if its losses on one position with a financial institution are offset by gains on another position with the same financial institution.
- That is why when derivatives transactions with a counterparty are cleared bilaterally, they are **netted**.
- Another measure that reduces credit risk is posting of **collateral**.
- There is (credit risk) exposure if the net value of outstanding transactions is greater than the collateral posted.
- In the previous two figures for pattern in value of forward contracts, which one poses more credit risk?

Credit Default Swaps

- A credit default swap (CDS) is a swap that allows investors to hedge credit risk.
- A CDS is like an insurance contract that pays off if a particular company or country, called the **reference entity**, defaults.
- The buyer of credit protection pays an insurance premium, known as the **CDS spread**, to the seller of protection for the life of the contract or until the reference entity defaults.

- Suppose the **notional principal** of a CDS is $100 million, maturity is 5 years, and the CDS spread is 150 basis points.
- Then, the protection buyer pays $1.5 million each year as insurance premium to the protection seller.
- If the reference entity does not default, the protection buyer does not receive anything in return.
- If the reference entity defaults, the protection seller buys the bonds issued by the reference entity for their face value and the spread payments stop.
- The face value of bonds bought equals the notional principal of the CDS.
- Thus, if CDS buyer owned a portfolio of bonds issued by the reference entity with a principal of $100 million, in the event of a default by the reference entity, CDS with notional principal of $100 million will bring the value of the portfolio back up to $100 million.