Properties of Option Prices

Option prices depend on

1. Spot price S_0
2. Exercise price K
3. Maturity T
4. Volatility of the return on the underlying asset σ
5. Risk-free rate r
6. Dividends D

The direction of impact of these variables on call and put option prices is described in the following table. A '+' means that the option price moves in the same direction as the variable and a '-' means that the option price moves in an opposite direction to that of the variable. European and American call option prices are denoted by c and C, respectively. European and American put option prices are denoted by p and P, respectively.

Variable	Effect on c	Effect on C	Effect on p	Effect on P
S_0	+	+	-	-
K	-	-	+	+
T	?	+	?	+
σ	+	+	+	+
r	+	+	-	-
D	-	-	+	+

Bounds on Prices for Options on Non-Dividend-Paying Stock

- European call

$$max(S_0 - PV\ of\ K, 0) \leq c \leq S_0$$

- European put

$$max(PV\ of\ K - S_0, 0) \leq p \leq PV\ of\ K$$

- American call

$$max(S_0 - K, S_0 - PV\ of\ K, 0) \leq C \leq S_0$$

- American put

$$max(K - S_0, PV\ of\ K - S_0, 0) \leq P \leq K$$

Example 1: An American put option is written on a non-dividend-paying stock. The stock price is $50, the exercise price is $66, expiration is after one year, and the risk-free rate is 10% per year.

a. What is the no-arbitrage price range for the put option?
b. Suggest an arbitrage strategy if the put price is $9.
c. Suggest an arbitrage strategy if the put price is $70.

Bounds on Prices for Options on Dividend-Paying Stock

If the dividends to be paid on the stock during the life of the option are known and have present value D, then

- European call

$$max(S_0 - D - PV\ of\ K, 0) \leq c \leq S_0$$

- European put

$$max(PV\ of\ K + D - S_0, 0) \leq p \leq PV\ of\ K$$

- American call

$$max(S_0 - K, S_0 - D - PV\ of\ K, 0) \leq C \leq S_0$$

- American put

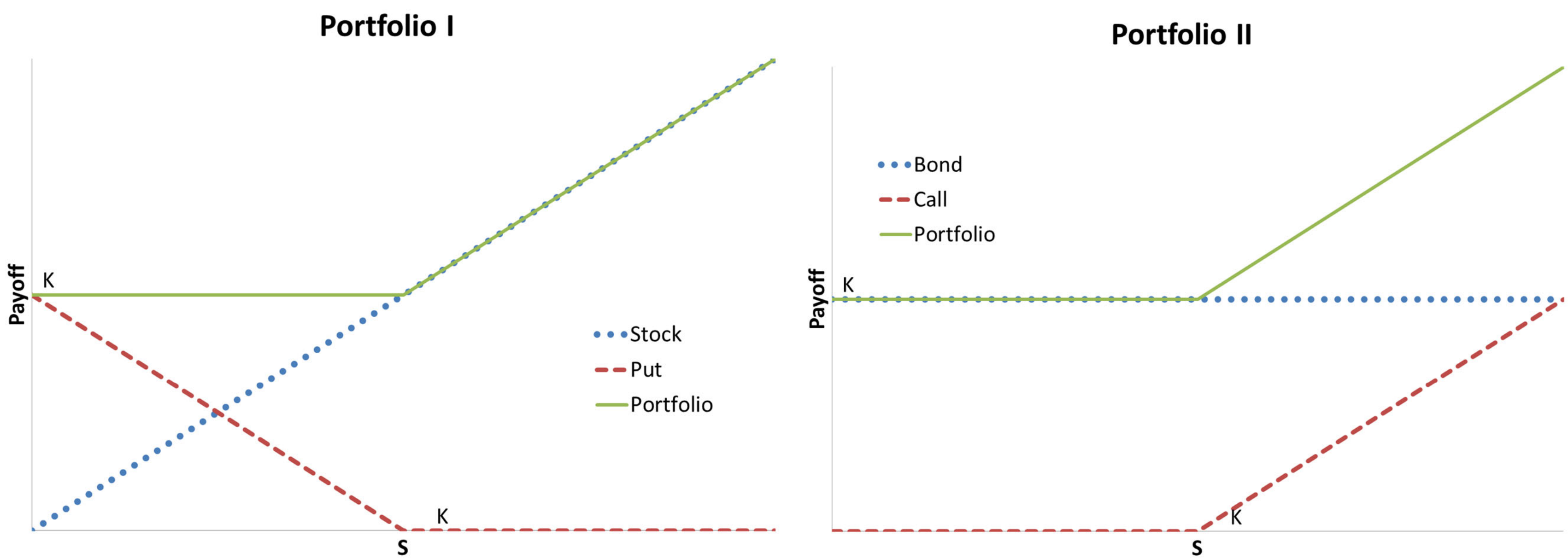
$$max(K - S_0, PV\ of\ K + D - S_0, 0) \leq P \leq K$$

Put Call Parity

Consider two portfolios:

I. A non-dividend-paying stock + a European put on the stock with strike K and maturity T
II. A bond with face value K and maturity T + a European call on the stock with strike K and maturity T

The value of each portfolio after time T depends on the stock price at that time as follows:



The two portfolios will have same value regardless of the stock price at maturity. Then, the two portfolios should cost the same. That is,

$$S_0 + p = PV\ of\ K + c$$

$$S_0 = c - p + PV\ of\ K$$

This equation represents put-call parity. The left-hand-side of the equation is the price of the **real stock**. The right-hand-side is the price of a **synthetic stock** created from a long call, a short put, and a long bond. The real stock and the synthetic stock will have the same value at maturity so they must cost the same today. If this equation is violated, investors can buy the cheaper of the two and simultaneously sell the costlier of the two to earn a positive profit without any future risk. Put-call parity holds for European options on non-dividend-paying stocks.

Put Call Parity with Dividends

When the underlying stock pays dividends, put-call parity is modified as follows:

$$S_0 = c - p + PV\ of\ K + D$$

where D is the present value of dividends on the underlying stock before option maturity.

Example 2: A non-dividend-paying stock is trading at \$50. A European call option with strike price of \$50 and 6 months to maturity is trading at \$5. Determine the price of the corresponding put option. A bond with face value of \$100 with 6 months to maturity is trading at \$96.

$$S_0 = c - p + PV\ of\ K$$

Substituting $S_0 =$, $c =$, and $PV\ of\ K = \quad \times \frac{}{}$,

$$= \quad - \quad + \quad \times \frac{}{}$$

so $p = 3$.

Put Call Parity with American Options

With American options, put-call parity does not hold as equality but results in inequalities.

$$S_0 - D - K = C - P \leq S_0 - PV\ of\ K$$

Early Exercise

American calls on non-dividend-paying stocks should not be exercised early (before maturity). Why?

1. There is a possibility that the stock price declines a lot after early exercise. The resulting losses could have been reduced by not exercising the option.
2. It is better to pay exercise price later rather than earlier.

Since American call options on non-dividend-paying stocks should not be exercised early, they offer no advantage over European call options and must have the same value. That is, $C = c$ for non-dividend-paying stock.