QF 430

Real Options

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What is a simulation?

- Simulate = Imitate
- Simulation
  - Imitation of a system (flight, war, economy, game, etc.)
  - Interesting only if system is complex and faces uncertainty
  - Uncertainty → Multiple scenarios possible
  - Complex → Difficult to use formulae that incorporate all scenarios
    - Coin toss: simulation unnecessary

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Monte Carlo Simulation

- There are some uncertain factors. We are interested in an outcome (or more) which depends on the uncertain factors.
- Multiple scenarios are generated.
- Each scenario consists of a set of values for the uncertain factors. These values are used to compute the value of the outcome in that scenario.
- Important condition: Scenarios must be generated so that they fairly represent uncertainty. That is, different sets of values are generated proportional to the probabilities of those values.
- The values of the outcome calculated in the scenarios are used to determine the average value, the range, or other properties of the outcome.

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Example: Game

- You can play a game by paying $4. You play by rolling two dice. You get as many dollars as the maximum of the two numbers. Should you play?

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Estimate Value of Pi

- The ratio of the area of a circle to the area of a bounding square = $\pi/4$.
- If many points are randomly scattered in the square, then the fraction of squares that lie within the circle = $\pi/4$.
- Simulation: Generate multiple random points.

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Estimate Value of European Call Option

- Call Value = PV of future payoff from option
- Payoff will be zero if call ends up out-of-money and positive if it ends up in-the-money
- Payoff will depend on stock price
- Simulate future stock price
- Assume lognormal probability distribution

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What are real options?

- Real options represent flexibility in decision making.
- Valuing these options means evaluating the extra value due to the flexibility in decision making.
- All real option situations involve
  - Future uncertainty
  - Ability to make future decision based on how uncertainty is resolved
  - If the decision is taken "correctly", one can adapt to situation and cut losses or increase profits.
- Discounted cash flow valuation can be used. However, the complications are:
  - There may be multiple future scenarios
  - Finding "correct" decision for each scenario may not be obvious
  - A probability distribution is not always easy to determine.
  - Discount rate is not always easy to determine

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Decision Trees

- Decision trees are graphical representations of decision making under uncertainty.
- They represent sequence of events starting from the initial (leftmost) node.
- Each path through the tree represents one possible sequence of events.
- Nodes of the tree represent points from which different paths diverge representing different sequences of events.
- Nodes can be
  - Chance nodes: Some uncertainty is resolved
  - Decision nodes: A decision is taken
- Each sequence of events is associated with a payoff.
- Solving a decision tree means
  - Determining the best decision at each decision node
  - Calculating expected payoff
- Usually the difficult part is determining the discount rate

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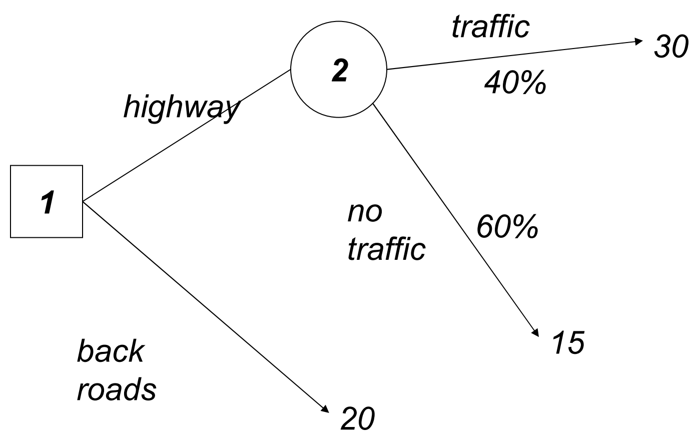
Commuting



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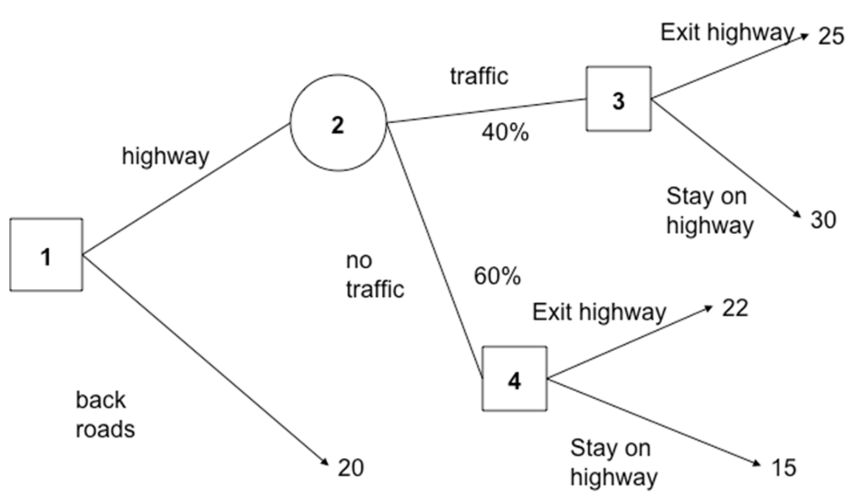
Commuting Options, with exit



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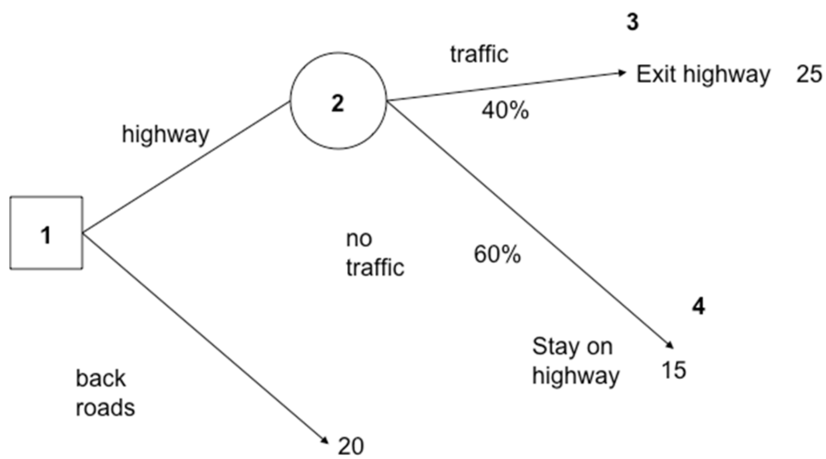
Commuting Options, pruned



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Types of Real Options

- Delay
- Expand
- Extend
- Abandon
- Shrink
- Switch
- It is not always clear what name should be used for an option.
- Usually, it is based on what you consider as the default choice and how does option let you change things.
- For example, if you consider small size of the business as default choice, then you have the option to expand if the business is successful.
- On the other hand, if you consider the large size as the default choice , then you have the option to shrink if the business is unsuccessful.

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Example: Fuelco Project A

Fuelco is considering a development project using its patented fuel-cell technology. If Fuelco pays $200M to start the project, then they are permitted to bid for a government contract. The objective probability of winning the contract is 50 percent, and there is no beta risk for the government's decision. If Fuelco's bid is accepted (one year later), then they can choose to finish the project by accepting the contract (cost = $300M), when they will earn an NPV (as of one year from now) of $600M (not including the $300M cost of finishing the project). If they do not receive the contract, then they can still finish the development project (cost = $300M), but they could only receive $200M for the project by selling it to some nongovernmental buyer (not including the $300M cost of finishing the project). The risk-free rate is zero.

<u>Problems</u>

a) Draw the tree for Fuelco's problem under the assumption that it starts the project.

b) Compute the NPV for the project. Should Fuelco start the project?

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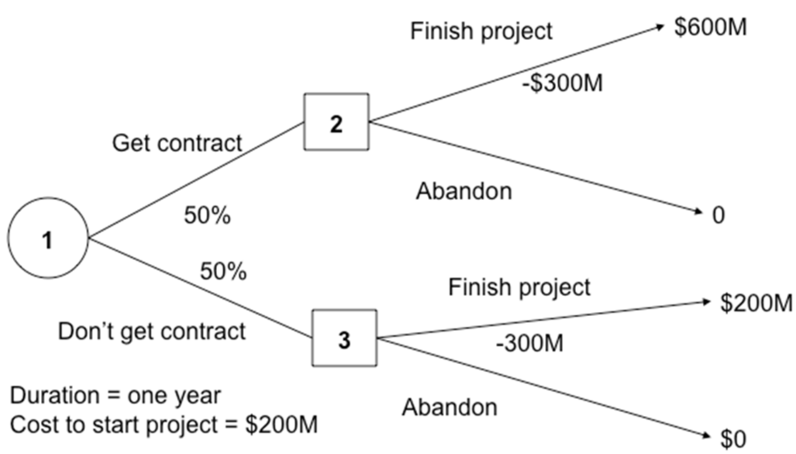
Fuelco's Decision Tree (Project A)



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Example: Fuelco Project B

In addition to Project A, Fuelco is also considering a separate investment in fuel-cell technology designed to replace oil-based energy for some types of engines . By investing $100M today to start the project, Fuelco would maintain the option to finish the project with a further investment (= $200M) in one year. If oil prices are at least $60 per barrel in one year (objective probability = 50%), then on completion of the project, Fuelco would have an NPV (as of one year from now) of $1000M (not including the $200M cost of finishing the project). If oil prices are less than $60 a barrel in one year (objective probability = 50%), then the project would not be economical for most applications and would have an NPV (one year from now) of $300M (not including the $200M cost of finishing the project). If Fuelco decides not to finish the project, then they can sell the technology to a competitor for $200M, regardless of the price of oil. The beta for the project is unknown, but we do have some information about oil prices: the market price of a European binary call option (payoff = $1) on oil with a strike price of $60 per barrel, and an expiration of 1 year is 25 cents.

Problems

a) Draw the tree for Fuelco's problem under the assumption that it starts the project.

b) Compute the NPV for the project. Should Fuelco start the project?

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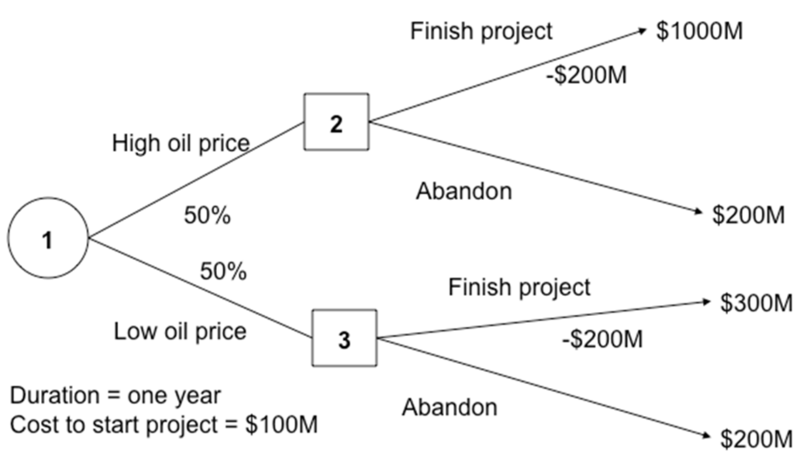
Fuelco's Decision Tree (Project B)



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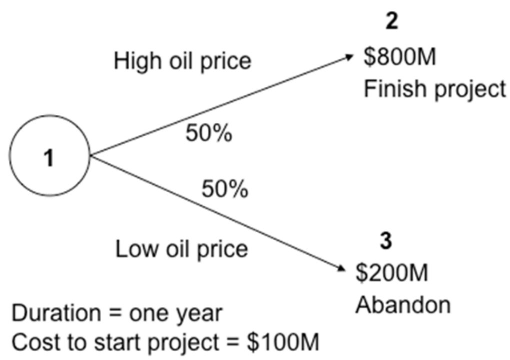
Fuelco's Decision Tree (Project B, pruned)



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Risk-neutral probabilities

- If everyone in the world was risk neutral, what probabilities would have to go into the tree?
- These probabilities are a fiction. They are make-believe. They are fantasy. They are not real.

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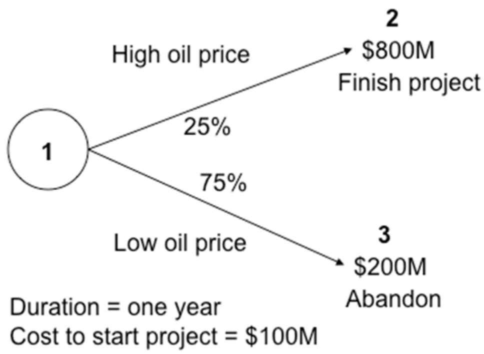
Fuelco's Decision Tree (Project B, pruned) *Risk-Neutral World*



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Example: Fuelco Project C

Fuelco is considering a consumer application for their patented fuel-cell technology. They have already completed several R&D projects with this technology, so they have eliminated the technical risk for this new project. To begin producing and marketing to the consumer market would require a new investment of $200M, to be paid in one year. The value of Project C depends on consumer demand. If demand is "high" (50 percent chance), then the value of the project would be $600M (one year from now). If demand is "low" (50 percent chance), then the value of the project would be $200M. If Fuelco chooses not to undertake the project, then they can still sell some of the related patents to another firm. If demand is "high" (50 percent chance), then the salvage value of these patents would be $300M (one year from now). If demand is "low" (50 percent chance), then the salvage value of these patents would be $100M. Selling the patents has no effect on any of Fuelco's other projects. We will use the CAPM to estimate expected returns in this problem, where the expected market premium is 7 percent, and the risk-free rate is 5 percent.

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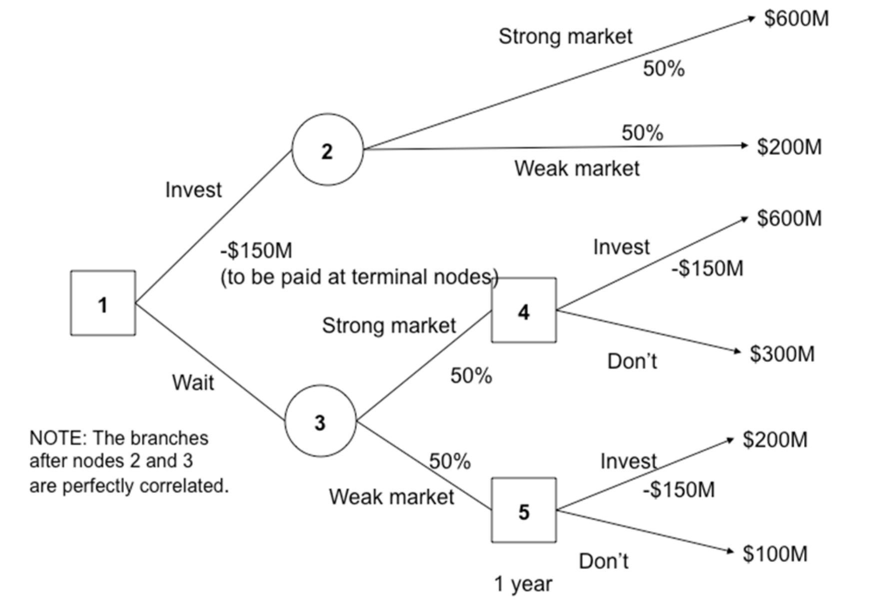
Fuelco's Decision Tree, Project C



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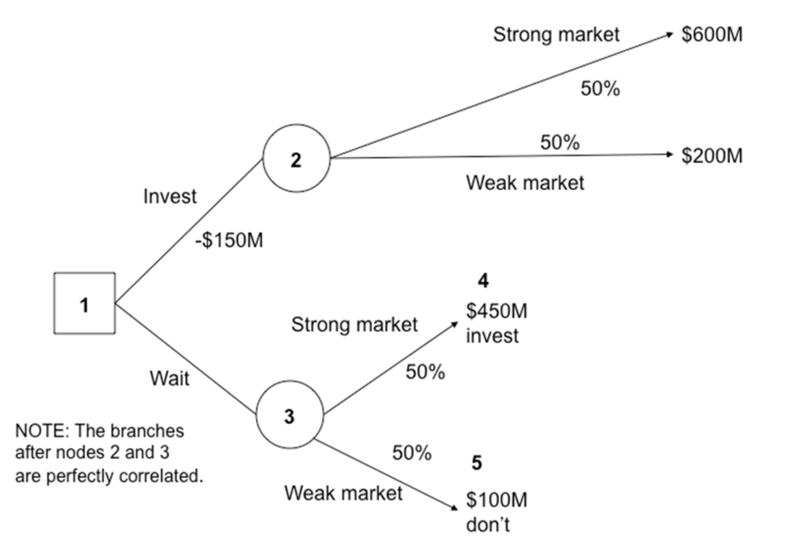
Fuelco's Decision Tree, pruned



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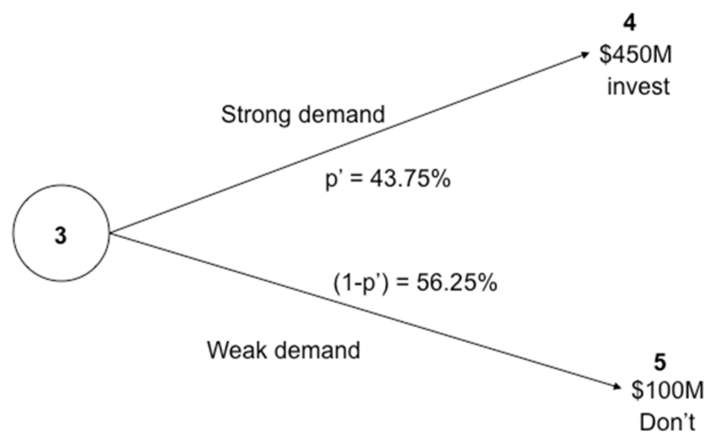
Fuelco's event tree, after node 3 *Risk-Neutral World, β = 1*



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Property Development Example

- A builder is developing a shopping complex Pinewood Mall that will be ready in two years. You are interested in buying the complex and then leasing it out to different businesses. The amount you can earn from leasing and the amount of maintenance costs are uncertain at this time and will be known after two years. You have data for some other complexes which appeared similar to Pinewood Mall at the time of development. Their cash flows are:

Complex	I	II	III	IV	V
PV of Leasing Revenues ($M)	250	275	300	350	400
PV of Future Maintenance Costs ($M)	125	150	125	125	150

- You can buy at the market price of Pinewood Mall after two years. However, you don't want to pay a very high price so you are considering buying an agreement in which you pay an upfront non-refundable fee now and in turn, you get the right (but not the obligation) to buy the complex after two years and then get all future leasing revenues. You will have to pay $150 million plus all future maintenance costs to the builder. What is the maximum upfront fee that you should pay for this agreement? Investors require a return of 10% per annum on such properties. Assume the continuously compounded risk-free rate is r=5%.

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Property Development Example

- Exercise right (flexibility) bought?
  - Right to buy the complex.
- Call or put?
  - Right to leasing cash flows (uncertain) by paying $150 million (fixed) and maintenance costs (uncertain).Treat it as call option in two ways.
  - Pay $150 million to get leasing cash flows minus maintenance costs, or
  - Ignoring uncertainty of maintenance costs by using the average maintenance cost of (125 + 150 + 125 + 125 + 15)/5 = 135 million plus $150 million = $285 million as what you will pay to get leasing revenues. Let us use the second approach.
- Underlying asset?
  - The leasing revenues.
- Spot price S?
  - Leasing revenues. Based on comparables, PV after two years = (250 + 275 + 300 + 350 + 400)/5=$315 million. Current value S= $315 million /⟦1.1⟧^2 = $260.33 million.
- Exercise price X?
  - $285 million
- Maturity T?
  - T= 2 years.

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Property Development Example

- Volatility?
  - Volatility of lognormal returns on the underlying asset, which is the value of leasing revenues. We need volatility of leasing revenue returns over time. Leasing revenues today are worth $283.6 million. Based on comparables, we know that in two years, they could be any of the five values from $250 million to $400 million. Assuming each one is equally likely, we calculate log returns and their volatility.

Complex	I	II	III	IV	V
Current Value S_0	260.33	260.33	260.33	260.33	260.33
PV of Leasing Revenues S_2	250	275	300	350	400
Two-year log return ($\ln(S_2/S_0)$)	-4.05%	5.48%	14.18%	29.60%	42.95%

  - Standard deviation of log returns over two years = 18.8%
  - Annual standard deviation of log returns, $\sigma=(18.8\%)/(\sqrt{2})=13.29\%$.
- Using Black-Scholes formula for price of European call,
  - c = $20.66 million
- The maximum upfront fee you should pay is $20.66 million.

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Does Market Ignore Real Options?

- A DCF analysis that considers expected cash flows and the appropriate discount rate for those cash flows
  - Ignores real options
  - Underestimates value
- This is true even if it recognizes that cash flows are risky and uses the correct discount rate.
- Why do people use models that ignore real options?
  - They may be making mistakes.
  - They may be aware of real options but incorporating their value indirectly by
    - Choosing higher expected cash flows
    - Choosing lower discount rate

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