Problem Set I Solutions

QF 430: Introduction to Derivatives
Due Wednesday, September 28

Please submit neatly handwritten or typed answers. You can submit your answers in class or electronically a single pdf file through Canvas. Show your steps or reasoning.

1 Introduction

Problem 1.1. The price of gold is currently \$1,675 per ounce. Forward contracts are available to buy or sell gold at \$1,750 per ounce for delivery in one year. An arbitrageur can borrow or invest money at 4% per annum. What should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

Solution. Since $\$1750 > \$1675 \times 1.04 = \$1742$, forward price is too high given the spot price (or the spot price is too low given the forward price). The arbitrageur should $\boxed{\textbf{borrow money}}$ to $\boxed{\textbf{buy}}$ a certain number of ounces of gold today and $\boxed{\textbf{short forward contracts}}$ on the same number of ounces of gold for delivery in one year. This means that for each ounce of gold purchased today, the arbitrageur borrows \$1,675, enters into short forward contracts to sell at \$1,750 after one year, and repays \$1,675 × 1.04 = \$1,742. The arbitrageur realizes a riskless profit of \$8 next year for each ounce of gold. Note that this profit does not depend on how gold prices move over the next one year. ■

Problem 1.2. The current price of a stock is \$50. Three-month call options with a strike price of \$54 currently sell for \$8. An investor with \$20,000 to invest is considering the following three investment strategies:

(a) Investing all his money in the stock

(b) Doubling the amount to invest by taking a loan of \$20,000 at an interest rate of 1% over three months, investing the resulting \$40,000 in the stock and then repaying \$20,200 on the loan

(c) Investing all his money in the call options

Determine the return of the investor (defined as change in wealth / initial wealth) under each of the three strategies for the following two scenarios: 1) stock price falls to \$40 after three months, 2) stock price rises to \$70 after three months. Compare the risks and returns of the three strategies.

Solution. Consider the three strategies one-by-one:

(a) Investing all his money in the stock

Initial wealth = \$20,000

The investor buys \$20,000/\$50=400 shares.

If the stock price falls to \$40, final wealth = $\$40 \times 400 = \$16,000$.

Return = $\frac{16{,}000-20{,}000}{20{,}000} = -0.20 = \boxed{\textbf{-20\%}}$.

If the stock price rises to \$70, final wealth = $\$70 \times 400 = \$28,000$.

Return = $\frac{28{,}000-20{,}000}{20{,}000} = 0.40 = \boxed{\textbf{40\%}}$.

(b) Doubling the amount to invest by taking a loan of \$20,000 at an interest rate of 1% for three months, investing the resulting \$40,000 in the stock and then repaying \$20,200 on the loan

Initial wealth = \$20,000.

The investor buys \$40,000/\$50=800 shares.

If the stock price falls to \$40, final wealth = $\$40 \times 800 - \$20,200 = \$11,800$.

Return = $\frac{11{,}800-20{,}000}{20{,}000} = -0.41 = \boxed{\textbf{-41\%}}$.

If the stock price rises to \$70, final wealth = $\$70 \times 800 - \$20,200 = \$35,800$. Return = $\frac{35{,}800-20{,}000}{20{,}000} = 0.79 = \boxed{\textbf{79\%}}$.

(c) Investing all his money in the call options

Initial wealth = \$20,000.

The investor buys \$20,000/\$8=2,500 calls.

If the stock price falls to \$40, call options are not exercised (buying stock for \$54 is unprofitable if stock price is \$40). Final wealth = \$0.

Return = $\frac{0-20{,}000}{20{,}000} = -1 = \boxed{\textbf{-100\%}}$.

If the stock price rises to \$70, calls are exercised and final wealth = $(\$70-\$54)\times 2,500 = \$40,000$.

Return = $\frac{40{,}000-20{,}000}{20{,}000} = 1 = \boxed{\textbf{100\%}}$.

As we move from option (a) to (b) to (c), low returns worsen while high returns improve, and $\boxed{\textbf{risk increases}}$. Both borrowing money to invest or investing in call options are examples of levering up to increase risk. In this problem, investing in calls is riskier than investing by borrowing \$1 for every \$1.

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Problem 1.3. The economic growth next year will be low, medium, or high. There are four assets, A, B, C, and D, whose value (cash flow) in the three states will be as follows:

	Low Growth	Medium Growth	High Growth
A	30	50	80
B	20	40	40
C	30	60	80
D	30	40	50

The prices of the assets A, B, and C, are 49.90, 34.20, and 55.30, respectively.

(a) What are the state prices of the three states (high growth, medium growth, and low growth)?

(b) What is the risk-free rate?

(c) What is the price of asset D?

Solution. (a) Let π_L, π_M, and π_H be the state prices of low growth state, medium growth state, and high growth state, respectively. The prices of securities A, B, and C follow the equations:

$$\begin{aligned} 30\pi_L + 50\pi_M + 80\pi_H &= 49.90, \\ 20\pi_L + 40\pi_M + 40\pi_H &= 34.20, \text{ and} \\ 30\pi_L + 60\pi_M + 80\pi_H &= 55.30. \end{aligned}$$

There are many ways of solving these equations. Here is one. Subtract the first equation from the third to eliminate $\pi_L$and π_H and get $10\pi_M = 5.40$ or $\pi_M = \boxed{\mathbf{0.54}}$. Substituting the value of π_M in the first two equations, we get

$$\begin{aligned} 30\pi_L + 80\pi_H &= 22.90, \text{ and} \\ 20\pi_L + 40\pi_H &= 12.60. \end{aligned}$$

Eliminate π_H by multiplying the bottom equation by 2 and then subtracting the top equation. This gives $10\pi_L = 2.30$ or $\pi_L = \boxed{\mathbf{0.23}}$. Substituting π_L in the top equation gives $\pi_H = \boxed{\mathbf{0.20}}$.

(b) Consider a riskless bond that pays \$100 in each state. Its price equals $\$100\pi_L + \$100\pi_M + \$100\pi_H = \$100 \times 0.23 + \$100 \times 0.54 + \$100 \times 0.20 = \$97$. The risk-free rate is the return on this bond: $\$100/\$97 - 1 = \boxed{\mathbf{3.09\%}}$.

(c) Price of asset D is $\$30\pi_L + \$40\pi_M + \$50\pi_H = \$30 \times 0.23 + \$40 \times 0.54 + \$50 \times 0.20 = \boxed{\mathbf{\$38.50}}$.

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Problem 1.4. Consider a fixed time T in future. Let S be the price of a stock at time T in \$. For positive integer n, let c_n be the price of a binary call option that will pay \$1 at time T if $S \geq n$ and 0 if $S < n$. Similarly, let p_n be the price of a binary put option that will pay \$1 at time T if $S < n$ and 0 if $S \geq n$. Suppose these prices are available for all positive integral values of n. That is, $c_1, p_1, c_2, p_2, c_3, p_3, \ldots$ are available.

(a) Write an expression for the state price for the state that S lies between n and $n+1$ ($n \leq S < n+1$) using binary call prices. Recall, this is the price of a portfolio created using binary call options that pays \$1 if $n \leq S < n+1$ and 0 otherwise. *Hint*: Buy and/or sell different calls to create a portfolio with the required payoff. The payoff from selling an option is the exact opposite of the payoff from buying the option.

(b) Write an expression for the state price for the state that S lies between n and $n+1$ ($n \leq S < n+1$) using binary put prices.

(c) Equating the two expressions you calculated for the same state price, determine a relation between binary call prices and binary put prices.

Solution.

(a) The state price for the state $n \leq S < n+1$ is the price of a security or portfolio that pays \$1 if $n \leq S < n+1$ and nothing otherwise. Such a portfolio can be created by buying a binary call that pays \$1 if $S \geq n$ and selling a binary call that pays \$1 if $S \geq n+1$. The binary call bought will pay \$1 if $S \geq n$ while the binary call sold will require a payment of \$ (to the counterparty) if $S \geq n+1$. If $S < n$, there is no cash flow on any of the securities. If $n \leq S < n+1$, we get \$1 from the binary call bought. If $S \geq n+1$, we get \$1 from the call we bought but have to pay it back on the call we sold.

This portfolio costs $c_n - c_{n+1}$. Thus, required state price is $\boxed{c_n - c_{n+1}}$.

(b) Another portfolio that pays \$1 if $n \leq S < n+1$ and nothing otherwise can be created by buying a binary put that pays \$1 if $S < n+1$ and selling a binary put that pays \$1 if $S < n$. This portfolio costs $p_{n+1} - p_n$. Thus, required state price is $\boxed{p_{n+1} - p_n}$.

(c) Equating the two expressions we calculated for the required state price, we get $c_n - c_{n+1} = p_{n+1} - p_n$ or $c_n + p_n = c_{n+1} + p_{n+1}$. It follows that $\boxed{c_1 + p_1 = c_2 + p_2 = \cdots}$

This result can be interpreted as follows. A portfolio consisting of a call that pays \$1 if $S \geq n$ and a put that pays \$1 if $S < n$ always pays \$1 and does not depend on n so the portfolio price equals the price of getting \$1 for sure at time T and does not depend on n.

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2 Futures Markets

Problem 2.1. What is open interest in a derivative contract? Why does the open interest usually decline during the month preceding the delivery month? Suppose 1,000 contracts in a futures market are traded on a given day. Of the 1,000 long positions taken, 400 closed short existing positions, while 600 opened new long positions. Of the 1,000 short positions taken, 700 closed existing long positions and 300 opened new short positions, What is the change in the open interest on this day?

Solution. Open interest is the **number of contracts outstanding** (counting each contract only once, not twice from both long and short positions). Many traders close out their positions just before the delivery month is reached. This is why the open interest declines during the month preceding the delivery month.
The open interest **decreased by 100** on the given day. We can see this as the change in number of short positions: 400 existing short positions were closed and 300 new short positions were created. We can also see this as the change in number of long positions: 700 existing long positions were closed and 600 new long positions were created. ■

Problem 2.2. A company has a long position in futures contract to buy 5,000 bushels of wheat. The company had entered into the contract when the futures price was 1,094 cents per bushel. The initial margin is $4,500 and the maintenance margin is $3,300. The current futures price is 859 cents per bushel and the margin account balance is $3,700.

(a) Determine the net margin deposits to or margin withdrawals from the margin account so far.

(b) What price change would lead to a margin call?

(c) Following what price change can $1,000 be withdrawn from the margin account?

Solution. (a) The company has bought a contract on 5,000 bushels so gain on position = $5,000 \times (\$8.59 - \$10.94) = -\$11,750$ or loss is $11,750. Without any margin deposits or withdrawals, this would have caused margin balance to drop to $4,500 - $11,750 = -$7,250. However, current margin balance is $3,700 so net margin deposits = $3,700 - (-$7,250) = $\boxed{\textbf{\$10,950}}$.

(b) There will be a margin call if the margin balance falls below the maintenance margin of $3,300, that is if the margin balance falls by $3,700-$3,300=$400 or more. This occurs if the price decreases by $400/5,000 bushels = $0.08 = 8 cents per bushel. A price decrease of more than 8 cents per bushel to $\boxed{\textbf{less than 851 cents per bushel}}$ will result in a margin call.

(c) The withdrawal must still leave initial margin of $4,500 so the margin balance before withdrawal must at least be $5,500. This represents an increase of at least $5,500 - $3,700 = $1,800 from the current margin balance. This can occur if the price increases by at least $1,800 / 5,000 bushels = $0.36 = 36 cents per bushel. Thus, $1,000 can be withdrawn from the margin account if the $\boxed{\textbf{price rises to 895 cents per bushel}}$ or more.

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Problem 2.3. This problem illustrates the issues faced in setting margin requirements. Use the daily closing prices for crude oil futures contract in dollars per barrel from the given spreadsheet (obtained from Professor Hull's website). You will need to use a spreadsheet or another calculation software for this problem.

(a) Assuming that daily price changes are normally distributed and independent across time, estimate the standard deviation of two-day price changes. To do this, calculate daily price changes, estimate their standard deviation, and then multiply by the square root of two for the standard deviation of two-day price changes. For the purpose of calculating standard deviation, you may assume the mean of the normal distribution to be zero or estimate it using the data.

(b) Assume two-day price changes are normal. What maintenance margin should an exchange set for a member with a long position in one contract such that with 99% probability, the decline in contract value over a two-day period will not exceed the maintenance margin? That is, determine maintenance margin such that the decline in the contract value over a two-day period will exceed the maintenance margin with

1% probability. Each contract is on 1,000 barrels of oil. Use the normal distribution with mean and standard deviation you estimated for two-day price changes to find the 1 percentile fall in price over two days. The Excel function norm.inv can be used for this. Scale the price change by contract size to find the corresponding fall in contract value.

(c) In the given data, what percentage of the days does the decline in contract value over a two-day period actually exceed the maintenance margin? What do your results suggest about the appropriateness of the normal distribution assumption?

(d) In practice, margin balance usually exceeds maintenance margin because margin calls restore the margin balance to the initial margin. Assume that the initial margin is set such that the maintenance margin calculated above equals 75% of the initial margin. Track margin balance assuming that (1) the margin balance starts at the initial balance, (2) a margin call is made on any day the margin balance drops below the maintenance margin, and (3) whenever margin balance rises above the initial margin, the excess is withdrawn the same day by the client so that margin balance at the end of the day never exceeds the initial margin.

Determine the percentage of times that the client has an incentive to default (that is the number of two-day periods over which the margin balance becomes negative if the client does not meet margin calls). Assume that if a margin call is not met one day, margin withdrawal cannot be made the next day. One way to do this is as following. First calculate *true* daily changes in contract value, *true* withdrawals, *true* deposits, and *true* margin balance under assumptions (1)-(3) above, ignoring any defaults. Then calculate the *default* balance on a day assuming *true* margin balance two days ago, no margin deposit that day and the previous day and no margin withdrawal that day. This can also be calculated as: *default* balance on day n equals the *true* balance on day n minus *true* margin deposits on days $n-1$ and n plus *true* margin withdrawal on day n. There is incentive to default if this *default* balance is negative. See the spreadsheet with an example.

Solution.

(a) If the mean is estimated from data, we get mean of one-day price change to be 0.01659 and standard deviation to be 1.5777. For the two-day price changes, mean is $2 \times 0.01659 = 0.03318$ and standard deviation is $\sqrt{2} \times 1.5777 = \boxed{\mathbf{2.2312}}$. If the mean is assumed to be zero, variance is calculated as sum of squares of all price changes divided by number of price changes minus one. Standard deviation is square root of variance. Multiply by $\sqrt{2}$ to get two-day price change standard deviation of $\boxed{\mathbf{2.2313}}$.

(b) The 1-percentile two-day price change is calculated from normal distribution using the mean and standard deviation of the two-day price change. This is a negative number (showing decline in value). Make it positive and multiply by the contract size of 1,000 to get the maintenance margin.

If the mean is estimated from data,

$$\begin{aligned}\text{Maintenance margin} &= -1000 \times (\text{mean} + \text{SD} \times \Phi^{-1}(0.01)) \\ &= -1000 \times (0.03318 + 2.2312 \times (-2.3263)) \\ &= \boxed{\mathbf{\$5157.40}}.\end{aligned}$$

where Φ is the normal cumulative distribution function.

If the mean is assumed to be zero,

$$\begin{aligned}\text{Maintenance margin} &= -1000 \times \text{SD} \times \Phi^{-1}(0.01) \\ &= -1000 \times 2.2313 \times (-2.3263)) \\ &= \boxed{\mathbf{\$5190.87}}.\end{aligned}$$

(c) The decline in contract value over a two-day period exceeds the maintenance margin 24 times which is $\boxed{\textbf{2.31\%}}$ of the days in the data. This is more than the expected 1% if the data were truly normal and prices changes were independent. This suggests that the distribution may have $\boxed{\textbf{fatter tails}}$ than the normal distribution and price changes across days may $\boxed{\textbf{not be independent}}$.

(d) Start by keeping track of the margin balance assuming no defaults. For each day, keep track of starting and ending margin balances, contract value changes, deposits, and withdrawals. The initial margin is $\boxed{\textbf{\$6,876.53}}$ or $\boxed{\textbf{\$6,921.16}}$ based on the assumption made about the mean. The margin balance at the end of first day is the initial margin. On any subsequent day, starting balance is the ending balance from the previous day. Calculate the end-of-trading day balance as the starting balance plus the change in contract value due to price change. If this balance is below the maintenance margin, there is a deposit equal to the initial margin minus the end-of-trading-day balance. If the end-of-trading-day balance exceeds the initial margin, there is a withdrawal equal to the end-of-trading-day balance minus the initial margin. The ending balance equals end-of-trading-day balance plus deposits minus withdrawals. The following figure shows such calculations for a few days.

Starting Margin Balance	Change in Contract Value	Balance after Change in Value	Deposit if Margin Call	Withdrawal if excess	Ending Margin Balance	Margin Balance without Deposits in Two Days and without Withdrawal Today
					6921.16	
6921.16	-730	6191.16	0	0	6191.16	
6191.16	-1110	5081.16	1840	0	6921.16	5081.16
6921.16	870	7791.16	0	870	6921.16	5951.16
6921.16	880	7801.16	0	880	6921.16	7801.16
6921.16	1070	7991.16	0	1070	6921.16	7991.16
6921.16	-390	6531.16	0	0	6531.16	6531.16
6531.16	530	7061.16	0	140	6921.16	7061.16
6921.16	1170	8091.16	0	1170	6921.16	8091.16
6921.16	-1020	5901.16	0	0	5901.16	5901.16
5901.16	630	6531.16	0	0	6531.16	6531.16
6531.16	450	6981.16	0	60	6921.16	6981.16
6921.16	990	7911.16	0	990	6921.16	7911.16
6921.16	30	6951.16	0	30	6921.16	6951.16
6921.16	-1390	5531.16	0	0	5531.16	5531.16
5531.16	-1270	4261.16	2660	0	6921.16	4261.16
6921.16	-1040	5881.16	0	0	5881.16	3221.16

There is an incentive to default on day n if the margin balance becomes negative when the investor doesn't make deposit in response to margin calls on days $n-1$ and n. Of course, if the investor didn't meet a margin call on day $n-1$, the exchange will not let the investor withdraw any amount on day n. So calculate a *default* balance on day n as the *true* day n balance minus any *true* deposits on days $n-1$ and n plus any *true* withdrawals on day n. Alternatively, calculate the *default* balance on day n as the *true* balance on day $n-2$ plus contract value changes on days $n-1$ and n minus any *true* withdrawal on day $n-1$. There is incentive to default when this *default* balance is negative. This happens 9 times, that is $\boxed{\textbf{0.87\%}}$ of the days.

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3 Hedging with Futures

Problem 3.1. It is now September 2022. A company anticipates that it will purchase 1 million pounds of copper in each of December 2022, June 2023, December 2023, and June 2024. The company has decided to use the futures contracts traded by the CME Group to hedge its risk. Contracts with maturity in January 2023, July 2023, January 2024, and July 2024 are available. One contract is for the delivery of 25,000 pounds of copper. The initial margin is \$6,050 per contract and the maintenance margin is \$5,500 per contract. The company's policy is to hedge 80% of its exposure. Contracts with maturities up to 13 months into the future are considered to have sufficient liquidity to meet the company's needs.

(a) Devise a hedging strategy for the company. That is, specify the company's futures trades for hedging. Do not make the adjustment for daily settlement (tailing the hedge) described in the textbook.

(b) Assume that the market prices (in cents per pound) today and at future dates are as in the following table.

Price / Date	Sep 2022	Dec 2022	Jun 2023	Dec 2023	Jun 2024
Spot	356.2	351.0	344.6	347.9	350.7
Jan 2023 Futures Price	353.8	351.1			
Jul 2023 Futures Price	350.1	347.6	344.4		
Jan 2024 Futures Price		348.0	344.6	348.2	
Jul 2024 Futures Price			345.1	349.1	350.9

What is the impact of the strategy you propose on the price the company pays for copper?

(c) What is the initial margin requirement in September 2022?

(d) Is the company subject to any margin calls? For part, only consider the prices provided, ignoring prices on other days.

Solution. (a) To hedge the $\boxed{\textbf{December 2022 purchase}}$, the company should take a $\boxed{\textbf{long}}$ $\boxed{\textbf{position in January 2023 contracts}}$ for the delivery of 800,000 pounds of copper. The total number of contracts required is $800,000/25,000 = \boxed{\textbf{32}}$. Similarly, a $\boxed{\textbf{long}}$ position in $\boxed{\textbf{32 July 2023 contracts}}$ is required to hedge the $\boxed{\textbf{June 2023 purchase}}$. For the $\boxed{\textbf{December 2023 purchase}}$, the company could take a $\boxed{\textbf{long}}$ position in $\boxed{\textbf{32 July 2023 contracts}}$ and $\boxed{\textbf{roll}}$ them into $\boxed{\textbf{January 2024 contracts}}$ during $\boxed{\textbf{June 2023}}$. (As an alternative, the company could hedge the December 2023 purchase by taking a long position in 32 January 2023 contracts and rolling them into January 2024 contracts in December 2022.) For the $\boxed{\textbf{June 2024 purchase}}$, the company could take a $\boxed{\textbf{long}}$ position in $\boxed{\textbf{32 July 2023 contracts}}$ and $\boxed{\textbf{roll}}$ them into $\boxed{\textbf{July 2024 contracts}}$ during $\boxed{\textbf{June 2023}}$. The strategy is therefore as follows

Sep 2022:	Enter into a long position in $\boxed{\textbf{32 January 2023}}$ contracts
	Enter into long position in $\boxed{\textbf{96 July 2023}}$ contracts
December 2022:	$\boxed{\textbf{Close out 32}}$ January 2023 contracts
June 2023:	$\boxed{\textbf{Close out 96}}$ July 2023 contracts
	Enter into long position in $\boxed{\textbf{32 January 2024}}$ contracts
	Enter into long position in $\boxed{\textbf{32 July 2024}}$ contracts
December 2023:	$\boxed{\textbf{Close}}$ out 32 January 2024 contracts
June 2024:	$\boxed{\textbf{Close}}$ out 32 July 2024 contracts

(b) For copper in December 2022, the company pays

$$351 - 0.8(351.1 - 353.8) = \boxed{\textbf{353.16}} \text{ cents per pound.}$$

For copper in June 2023, the company pays

$$344.6 - 0.8(344.4 - 350.1) = \boxed{\textbf{349.16}} \text{ cents per pound.}$$

For the December 2023 purchase, it loses $350.1 - 344.4 = 5.7$ cents per pound on the July 2023 futures and gains $348.2 - 344.6 = 3.6$ cents per pound on the January 2024 futures. The net price paid is

$$\begin{aligned}&347.9 - 0.8(344.4 - 350.1) - 0.8(348.2 - 344.6)\\&= 347.9 + 0.8 \times 5.7 - 0.8 \times 3.6 = \boxed{\mathbf{349.78}} \text{ cents per pound.}\end{aligned}$$

For the June 2024 purchase, it loses $350.1 - 344.4 = 5.7$ cents per pound on the July 2023 futures and gains $350.9 - 345.1 = 5.8$ cents per pound on the July 2024 futures. The net price paid is

$$\begin{aligned}&350.7 - 0.8(344.4 - 350.1) - 0.8(350.9 - 345.1)\\&= 350.7 + 0.8 \times 5.7 - 0.8 \times 5.8 = \boxed{\mathbf{350.62}} \text{ cents per pound.}\end{aligned}$$

The hedging strategy ends up keeping the price paid in the range 349.16 cents per pound to 353.16 cents per pound.

(c) In February 2022, the initial margin requirement on the 128 contracts is $128 \times \$2,700 = \boxed{\mathbf{\$345{,}600}}$.

(d) There is a margin call when the futures price drops by more than $(\$6,050 - \$5,500)/25000 = \$0.022$, or 2.2 cents per pound. This happens to the **January 2023 contract between September 2022 and December 2022** and to the **July 2023 contract between September 2022 and December 2022** and also **between December 2022 and June 2023**. (Under the plan above the January 2024 contract is not held between December 2022 and June 2023, but if it were there would be a margin call during this period.)

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Problem 3.2. A fund manager wants to hedge her portfolio against market movements over the next two months. The portfolio is worth \$25 million and its CAPM beta is 0.8. The manager plans to use three-month futures contracts on a well-diversified index to hedge its risk. The current level of the index is 3876, one contract is on \$50 times the index, the risk-free rate is 4.2% per annum, and the dividend yield on the index is 1.8% per annum. The current 3-month futures price is 3900.

(a) What position should the fund manager take to eliminate all exposure to the market over the next two months?

(b) Calculate the expected gain or loss of the fund manager's hedged position under four cases: the index value in two months is 3,000, 3,500, 4,500, and 5,000. This will consist of the gain or loss on the futures position and gain or loss on the portfolio. In each case, assume that the one-month futures price after two months is 0.4% higher than the index level at that time. Your portfolio value at that time is not provided for these cases. To calculate the expected value of your portfolio after two months in any case, determine the excess return on market as the percentage increase in the index plus the dividend yield over two months minus the risk-free rate over two months. Multiply the excess return on market with the beta of the portfolio to get the expected excess return on the portfolio. Add risk-free rate over two months to get the expected return on the index over two months. For this problem you can ignore compounding so that, for example, 4.2% per annum is equivalent to 4.2% $\times$ 1/12 = 0.35% per month.

Solution. (a) The number of contracts the fund manager should short is

$$0.8 \times \frac{25,000,000}{3900 \times 50} = 102.56$$

Rounding to the nearest whole number, $\boxed{\textbf{103 contracts should be shorted}}$.

(b) The following table shows the impact of the strategy. To illustrate the calculations in the table, consider the first column for the case in which the index in two months is 3,000. The excess return on the market is calculated as

$$\begin{aligned}\text{excess return on market} &= \text{percentage increase in index} + \text{dividend yield} - \text{risk-free return} \\ &= \frac{3000-3876}{3876} + 1.8\% \times \frac{2}{12} - 4.2\% \times \frac{2}{12} \\ &= -22.601\% + 0.3\% - 0.7\% = -23.001\%.\end{aligned}$$

From the capital asset pricing model,

$$\begin{aligned}\text{excess return on portfolio} &= \text{portfolio beta} \times \text{excess return on market} \\ &= 0.8 \times (-23.001\%) = -18.4005\%.\end{aligned}$$

The expected return on the portfolio is calculated as,

$$\begin{aligned}\text{expected return on portfolio} &= \text{excess return on portfolio} + \text{risk-free return} \\ &= -18.4005\% + 4.2\% \times \frac{2}{12} = -17.7005\%.\end{aligned}$$

The expected gain on portfolio is $-17.70\% \times 25,000,000 = -\$4,425,125$.

If the index in two months is 3,000, the futures price is $\$3,000 \times 1.004 = \$3,012$. The gain on the short futures position is therefore,

$$(3,900 - 3,012) \times 50 \times 103 = \$4,573,200.$$

The net gain on hedged position is $\$4,573,200 - \$4,425,125 = \boxed{\textbf{\$148,075}}$.

Index now	3876	3876	3876	3876
Index in two months	3000	3500	4500	5000
Return on index (%)	-22.6006	-9.7007	16.0991	28.999
Dividend yield (%)	0.3	0.3	0.3	0.3
Risk-free return (%)	0.7	0.7	0.7	0.7
Excess return on index (%)	-23.0006	-10.1007	15.6991	28.599
Portfolio beta	0.8	0.8	0.8	0.8
Excess return on portfolio (%)	-18.4005	-8.0806	12.5593	22.8792
Expected return on portfolio (%)	-17.7005	-7.3806	13.2593	23.5792
Portfolio size	25000000	25000000	25000000	25000000
Expected gain on portfolio	-4425125	-1845150	3314825	5894800
Futures price now	3900	3900	3900	3900
Futures price in two months	3012	3514	4518	5020
Change in Futures price	-888	-386	618	1120
Contract size	50	50	50	50
Number of contracts	-103	-103	-103	-103
Gain on Futures	4573200	1987900	-3182700	-5768000
Hedged Gain on portfolio	148075	142750	132125	126800

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