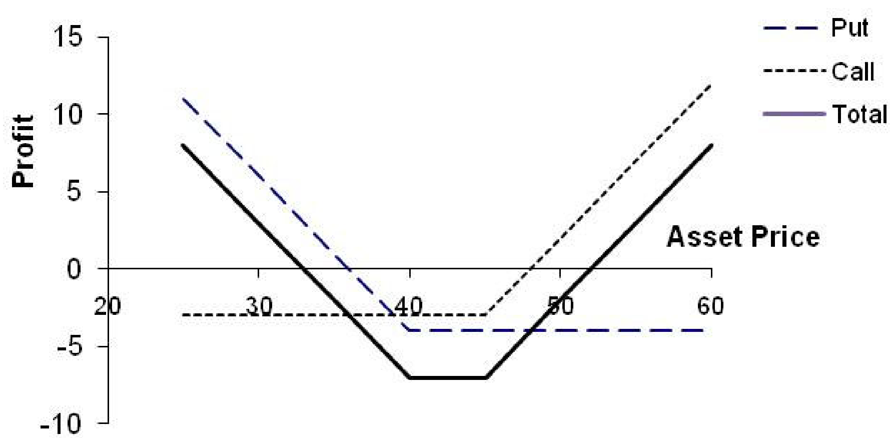
Selected Problems from the Textbook with Solutions

Problem 9.12.
A trader buys a call option with a strike price of $45 and a put option with a strike price of $40. Both options have the same maturity. The call costs $3 and the put costs $4. Draw a diagram showing the variation of the trader's profit with the asset price.

The following figure shows the variation of the trader's position with the asset price. We can divide the alternative asset prices into three ranges:

a) When the asset price less than $40, the put option provides a payoff of $40-S_T$ and the call option provides no payoff. The options cost $7 and so the total profit is $33-S_T$.
b) When the asset price is between $40 and $45, neither option provides a payoff. There is a net loss of $7.
c) When the asset price greater than $45, the call option provides a payoff of S_T-45 and the put option provides no payoff. Taking into account the $7 cost of the options, the total profit is S_T-52.

The trader makes a profit (ignoring the time value of money) if the stock price is less than $33 or greater than $52. This type of trading strategy is known as a strangle.



Problem 9.16.
The treasurer of a corporation is trying to choose between options and forward contracts to hedge the corporation's foreign exchange risk. Discuss the advantages and disadvantages of each.

Forward contracts lock in the exchange rate that will apply to a particular transaction in the future. Options provide insurance that the exchange rate will not be worse than some level. The advantage of a forward contract is that uncertainty is eliminated as far as possible. The disadvantage is that the outcome with hedging can be significantly worse than the outcome with no hedging. This disadvantage is not as marked with options. However, unlike forward contracts, options involve an up-front cost.

Problem 9.19.
What is the effect of an unexpected cash dividend on (a) a call option price and (b) a put option price?

An unexpected cash dividend would reduce the stock price on the ex-dividend date. This stock price reduction would not be anticipated by option holders prior to the dividend announcement. As a result there would be a reduction in the value of a call option and an increase the value of a put option. (Note that the terms of an option are adjusted for cash dividends only in exceptional circumstances.)

Problem 10.11.

A four-month European call option on a dividend-paying stock is currently selling for \$5. The stock price is \$64, the strike price is \$60, and a dividend of \$0.80 is expected in one month. The risk-free interest rate is 12% per annum for all maturities. What opportunities are there for an arbitrageur?

The present value of the strike price is $60e^{-0.12\times 4/12} = \$57.65$. The present value of the dividend is $0.80e^{-0.12\times 1/12} = 0.79$. Because

$$5 < 64 - 57.65 - 0.79$$

the condition in equation (10.8) is violated. An arbitrageur should buy the option and short the stock. This generates 64 – 5 = \$59. The arbitrageur invests \$0.79 of this at 12% for one month to pay the dividend of \$0.80 in one month. The remaining \$58.21 is invested for four months at 12%. Regardless of what happens a profit will materialize.

If the stock price declines below \$60 in four months, the arbitrageur loses the \$5 spent on the option but gains on the short position. The arbitrageur shorts when the stock price is \$64, has to pay dividends with a present value of \$0.79, and closes out the short position when the stock price is \$60 or less. Because \$57.65 is the present value of \$60, the short position generates at least 64 – 57.65 – 0.79 = \$5.56 in present value terms. The present value of the arbitrageur's gain is therefore at least 5.56 – 5.00 = \$0.56.

If the stock price is above \$60 at the expiration of the option, the option is exercised. The arbitrageur buys the stock for \$60 in four months and closes out the short position. The present value of the \$60 paid for the stock is \$57.65 and as before the dividend has a present value of \$0.79. The gain from the short position and the exercise of the option is therefore exactly 64 – 57.65 − 0.79 = \$5.56. The arbitrageur's gain in present value terms is 5.56 – 5.00 = \$0.56.

Problem 10.14.

The price of a European call that expires in six months and has a strike price of \$30 is \$2. The underlying stock price is \$29, and a dividend of \$0.50 is expected in two months and again in five months. Risk-free interest rates for all maturities are 10%. What is the price of a European put option that expires in six months and has a strike price of \$30?

Using the notation in the chapter, put-call parity, equation (10.10), gives

$$c + Ke^{-rT} + D = p + S_0$$

or

$$p = c + Ke^{-rT} + D - S_0$$

In this case

$$p = 2 + 30e^{-0.1\times 6/12} + (0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12}) - 29 = 2.51$$

In other words the put price is \$2.51.

Problem 10.15.

Explain carefully the arbitrage opportunities in Problem 10.14 if the European put price is \$3.

If the put price is \$3.00, it is too high relative to the call price. An arbitrageur should buy the call, short the put and short the stock. This generates −2 + 3 +29 =\$30 in cash which is invested at 10%. Regardless of what happens a profit with a present value of 3.00 – 2.51 = \$0.49 is locked in.

If the stock price is above \$30 in six months, the call option is exercised, and the put option expires worthless. The call option enables the stock to be bought for \$30, or $30e^{-0.10\times 6/12} = \$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12} = \$0.97$ in present value terms so that there is a profit with a present value of $30 - 28.54 - 0.97 = \$0.49$.

If the stock price is below \$30 in six months, the put option is exercised and the call option expires worthless. The short put option leads to the stock being bought for \$30, or $30e^{-0.10\times 6/12} = \$28.54$ in present value terms. The dividends on the short position cost $0.5e^{-0.1\times 2/12} + 0.5e^{-0.1\times 5/12} = \$0.97$ in present value terms so that there is a profit with a present value of 30 – 28.54 −0.97 = \$0.49.

Problem 11.15.
How can a forward contract on a stock with a particular delivery price and delivery date be created from options?

Suppose that the delivery price is K and the delivery date is T. The forward contract is created by buying a European call and selling a European put when both options have strike price K and exercise date T. This portfolio provides a payoff of $S_T - K$ under all circumstances where S_T is the stock price at time T. Suppose that F_0 is the forward price. If $K = F_0$, the forward contract that is created has zero value. This shows that the price of a call equals the price of a put when the strike price is F_0.

Problem 12.9.

A stock price is currently \$50. It is known that at the end of two months it will be either \$53 or \$48. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a two-month European call option with a strike price of \$49? Use no-arbitrage arguments.

At the end of two months the value of the option will be either \$4 (if the stock price is \$53) or \$0 (if the stock price is \$48). Consider a portfolio consisting of:

$$+\Delta \quad : \quad \text{shares}$$

$$-1 \quad : \quad \text{option}$$

The value of the portfolio is either 48Δ or $53\Delta - 4$ in two months. If

$$48\Delta = 53\Delta - 4$$

i.e.,

$$\Delta = 0.8$$

the value of the portfolio is certain to be 38.4. For this value of Δ the portfolio is therefore riskless. The current value of the portfolio is:

$$0.8 \times 50 - f$$

where f is the value of the option. Since the portfolio must earn the risk-free rate of interest

$$(0.8 \times 50 - f)e^{0.10 \times 2/12} = 38.4$$

i.e.,

$$f = 2.23$$

The value of the option is therefore \$2.23.

This can also be calculated directly from equations (12.2) and (12.3). $u = 1.06$, $d = 0.96$ so that

$$p = \frac{e^{0.10 \times 2/12} - 0.96}{1.06 - 0.96} = 0.5681$$

and

$$f = e^{-0.10 \times 2/12} \times 0.5681 \times 4 = 2.23$$

Problem 12.13.
A stock price is currently $50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month American put option with a strike price of $51?

A tree describing the behavior of the stock price is shown. The risk-neutral probability of an up move, p, is given by

$$p = \frac{e^{0.05\times3/12} - 0.95}{1.06 - 0.95} = 0.5689$$

We get a payoff of $51 - 50.35 = 0.65$ if the middle final node is reached and a payoff of $51 - 45.125 = 5.875$ if the lowest final node is reached. The value of the option at node B, ignoring early exercise, is

$$(0.5689 \times 0 + 0.4311 \times 0.65)e^{-0.05\times3/12} = 0.277.$$

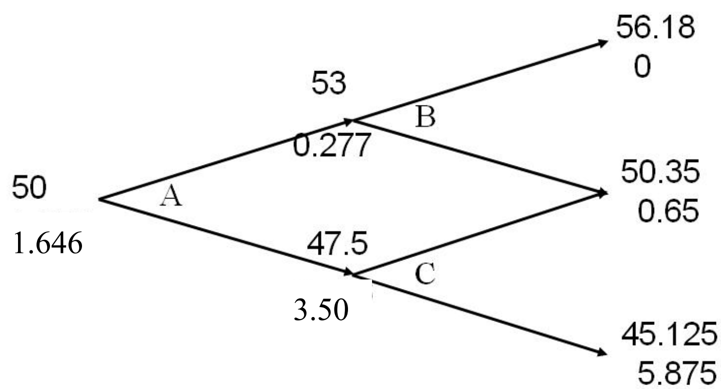
This is the value of the option at node B as option is out-of-money and cannot be exercised early. The value of the option at node C, ignoring early exercise, is

$$(0.5689 \times 0.65 + 0.4311 \times 5.875)e^{-0.05\times3/12} = 2.866.$$

However, early exercise payoff of 3.5 is higher so the option value at C is 3.5. The value of the option at node A, ignoring early exercise, is

$$(0.5689 \times 0.277 + 0.4311 \times 3.5)e^{-0.05\times3/12} = 1.646.$$

Since the payoff from early exercise, $1 is less than this, the value of the put option is $1.646.



Problem 12.14.

A stock price is currently \$25. It is known that at the end of two months it will be either \$23 or \$27. The risk-free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of two months. What is the value of a derivative that pays off S_T^2 at this time?

At the end of two months the value of the derivative will be either 529 (if the stock price is 23) or 729 (if the stock price is 27). Consider a portfolio consisting of:

$$+\Delta \quad : \quad \text{shares}$$

$$-1 \quad : \quad \text{derivative}$$

The value of the portfolio is either $27\Delta - 729$ or $23\Delta - 529$ in two months. If

$$27\Delta - 729 = 23\Delta - 529$$

i.e.,

$$\Delta = 50$$

the value of the portfolio is certain to be 621. For this value of Δ the portfolio is therefore riskless. The current value of the portfolio is:

$$50 \times 25 - f$$

where f is the value of the derivative. Since the portfolio must earn the risk-free rate of interest

$$(50 \times 25 - f)e^{0.10 \times 2/12} = 621$$

i.e.,

$$f = 639.3$$

The value of the option is therefore \$639.3.

This can also be calculated directly from equations (12.2) and (12.3). $u = 1.08$, $d = 0.92$ so that

$$p = \frac{e^{0.10 \times 2/12} - 0.92}{1.08 - 0.92} = 0.6050$$

and

$$f = e^{-0.10 \times 2/12}(0.6050 \times 729 + 0.3950 \times 529) = 639.3$$

Problem 12.16.

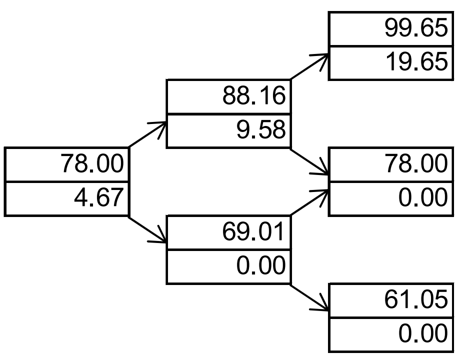
The volatility of a non-dividend-paying stock whose price is \$78, is 30%. The risk-free rate is 3% per annum (continuously compounded) for all maturities. Calculate values for u, d, and p when a two-month time step is used. What is the value of a four-month European call option with a strike price of \$80 given by a two-step binomial tree. Suppose a trader sells 1,000 options (10 contracts). What position in the stock is necessary to hedge the trader's position at the time of the trade?

$$u = e^{0.30\times\sqrt{0.1667}} = 1.1303$$

$$d = 1/u = 0.8847$$

$$p = \frac{e^{0.30\times 2/12} - 0.8847}{1.1303 - 0.8847} = 0.4898$$

The tree is given in Figure S12.3. The value of the option is \$4.67. The initial delta is 9.58/(88.16 – 69.01) which is almost exactly 0.5 so that 500 shares should be purchased.



Problem 13.9.

A stock price has an expected return of 16% and a volatility of 35%. The current price is $38.

a) *What is the probability that a European call option on the stock with an exercise price of $40 and a maturity date in six months will be exercised?*

b) *What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised?*

a) The required probability is the probability of the stock price being above $40 in six months time. Suppose that the stock price in six months is S_T. The probability distribution of ln S_T is

$$\phi\left\{\ln 38+\left(0.16-\frac{0.35^2}{2}\right)0.5,\ 0.35^2\times 0.5\right\}$$

i.e.,

$$\phi(3.687,\ 0.247^2)$$

Since $\ln 40=3.689$, the required probability is

$$1-N\left(\frac{3.689-3.687}{0.247}\right)=1-N(0.008)$$

From normal distribution tables $N(0.008) = 0.5032$ so that the required probability is 0.4968.

b) In this case the required probability is the probability of the stock price being less than $40 in six months. It is

$$1-0.4968=0.5032$$

Problem 13.14.

What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?

In this case, $S_0 = 69$, $K = 70$, $r = 0.05$, $\sigma = 0.35$, and $T = 0.5$.

$$d_1 = \frac{\ln(69/70) + (0.05 + 0.35^2/2) \times 0.5}{0.35\sqrt{0.5}} = 0.1666$$

$$d_2 = d_1 - 0.35\sqrt{0.5} = -0.0809$$

The price of the European put is

$$70e^{-0.05\times 0.5} N(0.0809) - 69N(-0.1666)$$

$$= 70e^{-0.025} \times 0.5323 - 69 \times 0.4338$$

$$= 6.40$$

or \$6.40.

Problem 15.11.

An index currently stands at 696 and has a volatility of 30% per annum. The risk-free rate of interest is 7% per annum and the index provides a dividend yield of 4% per annum. Calculate the value of a three-month European put with an exercise price of 700.

In this case $S_0 = 696$, $K = 700$, $r = 0.07$, $\sigma = 0.3$, $T = 0.25$ and $q = 0.04$. The option can be valued using equation (15.5).

$$d_1 = \frac{\ln(696/700) + (0.07 - 0.04 + 0.09/2) \times 0.25}{0.3\sqrt{0.25}} = 0.0868$$

$$d_2 = d_1 - 0.3\sqrt{0.25} = -0.0632$$

and

$$N(-d_1) = 0.4654, \quad N(-d_2) = 0.5252$$

The value of the put, p, is given by:

$$p = 700e^{-0.07\times 0.25} \times 0.5252 - 696e^{-0.04\times 0.25} \times 0.4654 = 40.6$$

i.e., it is \$40.6.

Problem 16.14.

A futures price is currently 25, its volatility is 30% per annum, and the risk-free interest rate is 10% per annum. What is the value of a nine-month European call on the futures with a strike price of 26?

In this case, $F_0 = 25$, $K = 26$, $\sigma = 0.3$, $r = 0.1$, $T = 0.75$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma\sqrt{T}} = -0.0211$$

$$d_2 = \frac{\ln(F_0 / K) - \sigma^2 T / 2}{\sigma\sqrt{T}} = -0.2809$$

$$c = e^{-0.075}[25N(-0.0211) - 26N(-0.2809)]$$

$$= e^{-0.075}[25 \times 0.4916 - 26 \times 0.3894] = 2.01$$

Problem 17.9.
The Black–Scholes–Merton price of an out-of-the-money call option with an exercise price of \$40 is \$4. A trader who has written the option plans to use a stop-loss strategy. The trader's plan is to buy at \$40.10 and to sell at \$39.90. Estimate the expected number of times the stock will be bought or sold.

The strategy costs the trader 0.10 each time the stock is bought or sold. The total expected cost of the strategy, in present value terms, must be \$4. This means that the expected number of times the stock will be bought or sold is approximately 40. The expected number of times it will be bought is approximately 20 and the expected number of times it will be sold is also approximately 20. The buy and sell transactions can take place at any time during the life of the option. The above numbers are therefore only approximately correct because of the effects of discounting. Also the estimate is of the number of times the stock is bought or sold in the risk-neutral world, not the real world.

Problem 17.15.
A financial institution has just sold 1,000 seven-month European call options on the Japanese yen. Suppose that the spot exchange rate is 0.80 cent per yen, the exercise price is 0.81 cent per yen, the risk-free interest rate in the United States is 8% per annum, the risk-free interest rate in Japan is 5% per annum, and the volatility of the yen is 15% per annum. Calculate the delta, gamma, vega, theta, and rho of the financial institution's position. Interpret each number.

In this case $S_0 = 0.80$, $K = 0.81$, $r = 0.08$, $r_f = 0.05$, $\sigma = 0.15$, $T = 0.5833$

$$d_1 = \frac{\ln(0.80/0.81) + \left(0.08 - 0.05 + 0.15^2/2\right) \times 0.5833}{0.15\sqrt{0.5833}} = 0.1016$$

$$d_2 = d_1 - 0.15\sqrt{0.5833} = -0.0130$$

$$N(d_1) = 0.5405; \quad N(d_2) = 0.4998$$

The delta of one call option is $e^{-r_f T}N(d_1) = e^{-0.05\times 0.5833} \times 0.5405 = 0.5250$.

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2} = \frac{1}{\sqrt{2\pi}} e^{-0.00516} = 0.3969$$

so that the gamma of one call option is

$$\frac{N'(d_1)e^{-r_f T}}{S_0 \sigma \sqrt{T}} = \frac{0.3969 \times 0.9713}{0.80 \times 0.15 \times \sqrt{0.5833}} = 4.206$$

The vega of one call option is

$$S_0\sqrt{T}N'(d_1)e^{-r_f T} = 0.80\sqrt{0.5833} \times 0.3969 \times 0.9713 = 0.2355$$

The theta of one call option is

$$-\frac{S_0 N'(d_1)\sigma e^{-r_f T}}{2\sqrt{T}} + r_f S_0 N(d_1)e^{-r_f T} - rKe^{-rT}N(d_2)$$
$$= -\frac{0.8 \times 0.3969 \times 0.15 \times 0.9713}{2\sqrt{0.5833}}$$
$$+0.05 \times 0.8 \times 0.5405 \times 0.9713 - 0.08 \times 0.81 \times 0.9544 \times 0.4948$$
$$= -0.0399$$

The rho of one call option is

$$KTe^{-rT}N(d_2)$$
$$= 0.81 \times 0.5833 \times 0.9544 \times 0.4948$$
$$= 0.2231$$

Delta can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the value of an option to buy one yen increases by 0.525 times that amount. Gamma can be interpreted as meaning that, when the spot price increases by a small amount (measured in cents), the delta increases by 4.206 times that amount. Vega can be interpreted as meaning that, when the volatility (measured in decimal form) increases by a small amount, the option's value increases by 0.2355 times that amount. When volatility increases by 1% (= 0.01) the option price increases by 0.002355. Theta can be interpreted as meaning that, when a small amount of time (measured in years) passes, the option's value decreases by 0.0399 times that amount. In particular when one calendar day passes it decreases by $0.0399/365 = 0.000109$. Finally, rho can be interpreted as meaning that, when the interest rate (measured in decimal form) increases by a small amount the option's value increases by 0.2231 times that amount. When the interest rate increases by 1% (= 0.01), the options value increases by 0.002231.

Problem 17.22.

A bank's position in options on the dollar–euro exchange rate has a delta of 30,000 and a gamma of $-80,000$*. Explain how these numbers can be interpreted. The exchange rate (dollars per euro) is 0.90. What position would you take to make the position delta neutral? After a short period of time, the exchange rate moves to 0.93. Estimate the new delta. What additional trade is necessary to keep the position delta neutral? Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange-rate movement?*

The delta indicates that when the value of the euro exchange rate increases by \$0.01, the value of the bank's position increases by $0.01 \times 30{,}000$ = \$300. The gamma indicates that when the euro exchange rate increases by \$0.01 the delta of the portfolio decreases by $0.01 \times 80,000 = 800$. For delta neutrality 30,000 euros should be shorted. When the exchange rate moves up to 0.93, we expect the delta of the portfolio to decrease by $(0.93 - 0.90) \times 80,000 = 2,400$ so that it becomes 27,600. To maintain delta neutrality, it is therefore necessary for the bank to unwind its short position 2,400 euros so that a net 27,600 have been shorted. As shown in the text (see Figure 17.8), when a portfolio is delta neutral and has a negative gamma, a loss is experienced when there is a large movement in the underlying asset price. We can conclude that the bank is likely to have lost money.

Problem 23.8.

Suppose that the risk-free zero curve is flat at 7% per annum with continuous compounding and that defaults can occur half way through each year in a new five-year credit default swap. Suppose that the recovery rate is 30% and hazard rate is 3%. Estimate the credit default swap spread? Assume payments are made annually.

The table giving unconditional default probabilities, is

Time (years)	*Probability of surviving to year end*	*Default Probability during year*
1	0.9704	0.0296
2	0.9418	0.0287
3	0.9139	0.0278
4	0.8869	0.0270
5	0.8607	0.0262

The table giving the present value of the expected regular payments (payment rate is s per year), is

Time (yrs)	*Probability of survival*	*Expected Payment*	*Discount Factor*	*PV of Expected Payment*
1	0.9704	$0.9704s$	0.9324	$0.9048s$
2	0.9418	$0.9418s$	0.8694	$0.8187s$
3	0.9139	$0.9139s$	0.8106	$0.7408s$
4	0.8869	$0.8869s$	0.7558	$0.6703s$
5	0.8607	$0.8607s$	0.7047	$0.6065s$
Total				$3.7412s$

The table giving the present value of the expected payoffs (notional principal =$1), is

Time (yrs)	*Probability of default*	*Recovery Rate*	*Expected Payoff*	*Discount Factor*	*PV of Expected Payment*
0.5	0.0296	0.3	0.0207	0.9656	0.0200
1.5	0.0287	0.3	0.0201	0.9003	0.0181
2.5	0.0278	0.3	0.0195	0.8395	0.0164
3.5	0.0270	0.3	0.0189	0.7827	0.0148
4.5	0.0262	0.3	0.0183	0.7298	0.0134
Total					0.0826

The following table gives the present value of accrual payments.

Time (yrs)	*Probability of default*	*Expected Accrual Payment*	*Discount Factor*	*PV of Expected Accrual Payment*

0.5	0.0296	0.0148*s*	0.9656	0.0143s
1.5	0.0287	0.0143*s*	0.9003	0.0129s
2.5	0.0278	0.0139*s*	0.8395	0.0117s
3.5	0.0270	0.0135*s*	0.7827	0.0106s
4.5	0.0262	0.0131*s*	0.7298	0.0096s
Total				0.0590s

The credit default swap spread s is given by:

$$3.7412s+0.0590s = 0.0826$$

It is 0.0217 or 217 basis points.