

# Problem Set II

## QF 430: Introduction to Derivatives

*Due Friday, October 14*

Please submit neatly handwritten or typed answers. You can turn in paper submissions in class or submit electronically a single pdf file through Canvas. Show your steps or reasoning. Do not round too much in intermediate calculation steps. Aim for accuracy of four decimal places in interest rates (0.0337 or 3.37%).

## 1 Interest Rates

**Problem 1.1.** The 6-month, 12-month, 18-month, and 24-month risk-free zero rates are 4%, 4.3%, 4.6%, and 4.9% with semiannual compounding.

- (a) What are the rates with continuous compounding?
- (b) What is the forward rate for the six-month period beginning in 18 months?
- (c) What is two-year par yield?
- (d) What is the value of an FRA where the holder pays LIBOR and receives 6.1% (semi-annually compounded) on \$40 million principal for a six-month period beginning in 18 months? The current forward LIBOR rate for the period is 5.9% (semiannually compounded).

**Problem 1.2.** The following table gives the prices of Treasury bonds:

Bond Principal (\$)	Time to Maturity (years)	Annual Coupon Rate (%)*	Bond Price (\$)
100	0.5	0	97
100	1	0	94
1000	1.5	8	1020
100	2	4	95

\*Half the stated coupon is paid every six months

- (a) Calculate zero rates for maturities of 6 months, 12 months, 18 months, and 24 months.
- (b) What are the forward rates for the periods: 6 months to 12 months, 12 months to 18 months, 18 months to 24 months?
- (c) What are the 6-month, 12-month, 18-month, and 24-month par yields for bonds that provide semiannual coupon payments?
- (d) Estimate the price and yield of a two-year bond providing a semiannual coupon of 5% per annum.

## 2 Forward and Futures Prices

**Problem 2.1.** The risk-free rate of interest is 5% per annum with continuous compounding. A stock is trading at \$49. The stock pays dividend of \$0.60 every quarter. The next dividend will be paid after 2 months. What is the six-month futures price of the stock? Report the price up to one-tenth of a cent.

**Problem 2.2.** The risk-free rate of interest is 5% per annum with continuous compounding, and the dividend yield on a stock index is 6% per annum. The current value of the index is 314. What is the six-month futures price (calculated at least to one-tenth of a cent)?

**Problem 2.3.** The risk-free rate of interest is 5% per annum with continuous compounding, and the fixed dividend yield on a stock index is 7% per annum. The three-month stock index futures price is \$75 and the six-month stock index futures price is \$74. Describe an arbitrage strategy.

**Problem 2.4.** When a known future cash outflow in a foreign currency is hedged by a company using a long forward contract, there is no foreign exchange risk. When it is hedged using long futures contracts, the daily settlement process does leave the company exposed to some risk. Explain the nature of this risk.

Assume that the forward price equals the futures price. In each of the following cases, based on time value of money, explain which company is better off—company A taking a long position in the futures contract or company B taking a long position in the forward contract?

- (a) The value of the foreign currency falls through the life of the contract
- (b) The value of the foreign currency rises through the life of the contract
- (c) The value of the foreign currency first rises and then falls back to its initial value
- (d) The value of the foreign currency first falls and then rises back to its initial value

**Problem 2.5.** Gold price is \$1721 per ounce. The storage costs are \$3.60 per ounce per year payable *quarterly* at the *end* of each quarter. Assuming that interest rate is 5% per annum with continuous compounding for all maturities, calculate the futures price of gold for delivery in six months.

**Problem 2.6.** The current USD/Swiss franc exchange rate is 1.0207 dollars per Swiss franc. The one-year forward exchange rate is 1.0600 dollars per Swiss franc. The one-year USD interest rate is 3% per annum continuously compounded. Estimate the one-year Swiss franc interest rate.

**Problem 2.7.** The six-month interest rates in the United Kingdom and the United States are 4.36% and 4.27% per annum, respectively, with continuous compounding. The spot price of the British pound sterling is \$1.1439. The futures price for a contract deliverable in six months is \$1.1431. What arbitrage opportunities does this create?