Derivatives and Risk Management

A derivative is an instrument whose value depends on the values of other more basic underlying variables. Derivatives play a key role in transferring risks in the economy. Examples of derivatives are futures contracts, forward contracts, swaps, and options.

Derivatives are used

- to hedge risks
- to speculate (take a view on the future direction of the market)
- to lock in an arbitrage profit
- to change the nature of a liability
- to change the nature of an investment without incurring the costs of selling one portfolio and buying another

Futures Contracts

A futures contract is an agreement to buy or sell an asset at a certain time in the future for a certain price. By contrast in a **spot contract** there is an agreement to buy or sell the asset immediately.

For example, futures contracts may represent agreement to:

- buy 100 oz. of gold @ US$1100/oz. in December
- sell £62,500 @ 1.5500 US$/£ in March
- sell 1,000 bbl. of oil @ US$40/bbl. in April

Exchanges that trade Futures include

- CME Group
- Intercontinental Exchange
- Euronext
- Eurex
- BM&FBovespa (Sao Paulo, Brazil)
- National Stock Exchange of India
- China Financial futures Exchange

The **futures prices** for a contract is the price at which traders agree to buy or sell at a future time. It is determined by supply and demand in the same way as a spot price

Traditionally futures contracts have been traded using the **open outcry system** where traders physically meet on the floor of the exchange. This has now been largely replaced by **electronic trading** and high frequency (algorithmic) trading has become an increasingly important part of the market

The party that has agreed to buy has a **long position**. The party that has agreed to sell has a **short position**.

Example

In January, an investor enters into a long futures contract to buy 100 oz of gold @ $1,100 per oz in April. In April, the price of gold is $1,175 per oz. What is the investor's profit or loss?

Over-the Counter Markets

These are decentralized markets with bilateral trades. Trades are usually between financial institutions, corporate treasurers, and fund managers. Transactions are much larger than in the exchange-traded market.

Exchange-traded futures and options, by location of exchange

Notional principal, in billions of US dollars

	Open interest			Daily average turnover						
	Dec 2020	Jun 2021	Sep 2021	2019	2020	May 2021	Jun 2021	Jul 2021	Aug 2021	Sep 2021
Futures										
All markets	28,995	34,003	34,042	7,030	5,415	5,155	6,166	5,020	4,357	5,166
Options										
All markets	36,955	53,266	45,487	2,094	1,414	1,357	1,436	1,291	1,067	1,259

Note that the open even though the open interest is comparable for options and futures, the daily turnover is much higher for futures than for options.

Global OTC derivatives market

In billions of US dollars

	Notional amounts outstanding				Gross market value			
	H2 2019	H1 2020	H2 2020	H1 2021	H2 2019	H1 2020	H2 2020	H1 2021
All contracts	**558,513**	**606,821**	**582,055**	**609,996**	**11,598**	**15,481**	**15,783**	**12,617**

The notional outstanding in OTC markets dwarfs that in Exchange-traded markets. Note that gross market values are about 2 percent of notional amount.

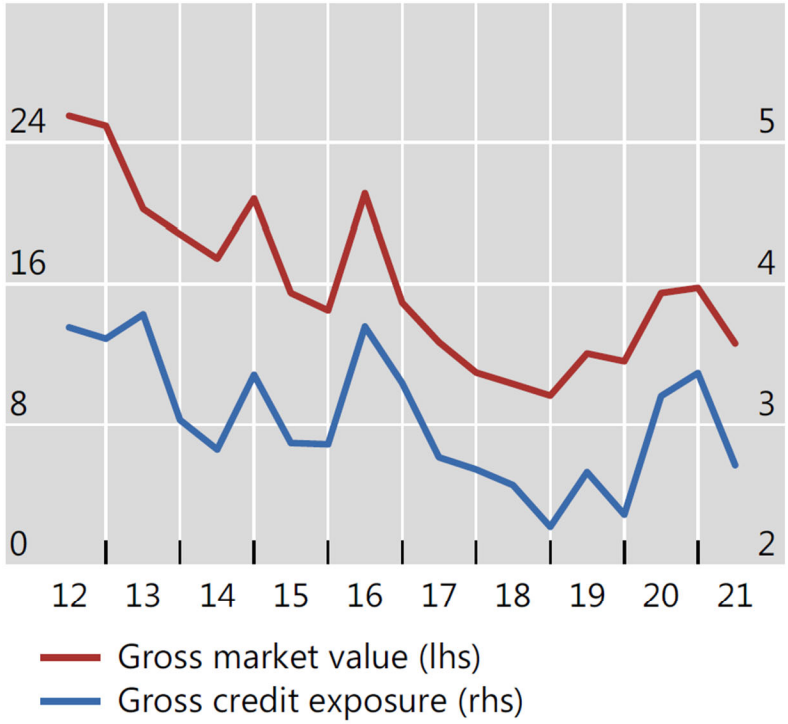
Gross market value of OTC derivatives nears pre-Covid level

Outstanding OTC derivatives

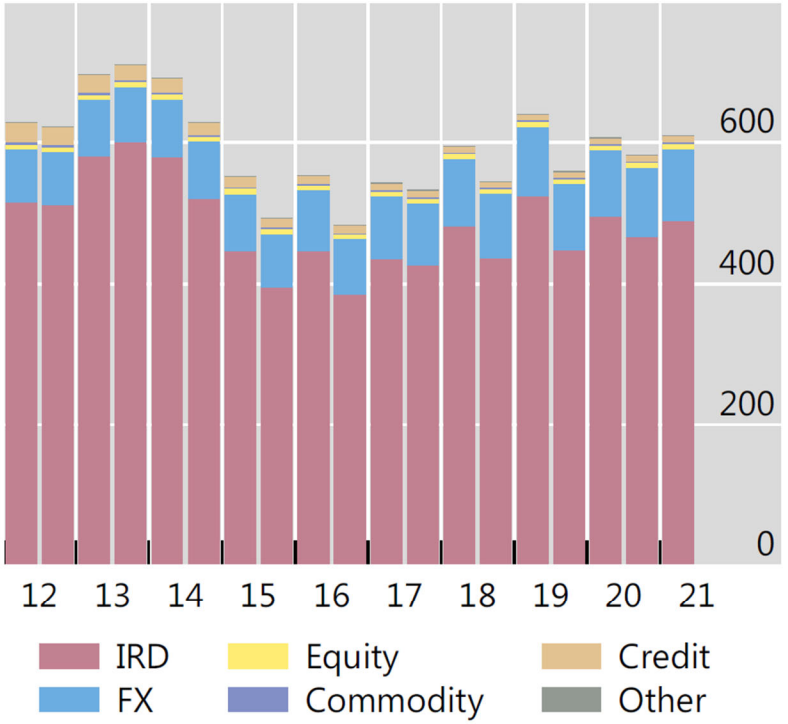
In trillions of US dollars

Graph 1

Gross market value & gross credit exposure



Notional amounts outstanding

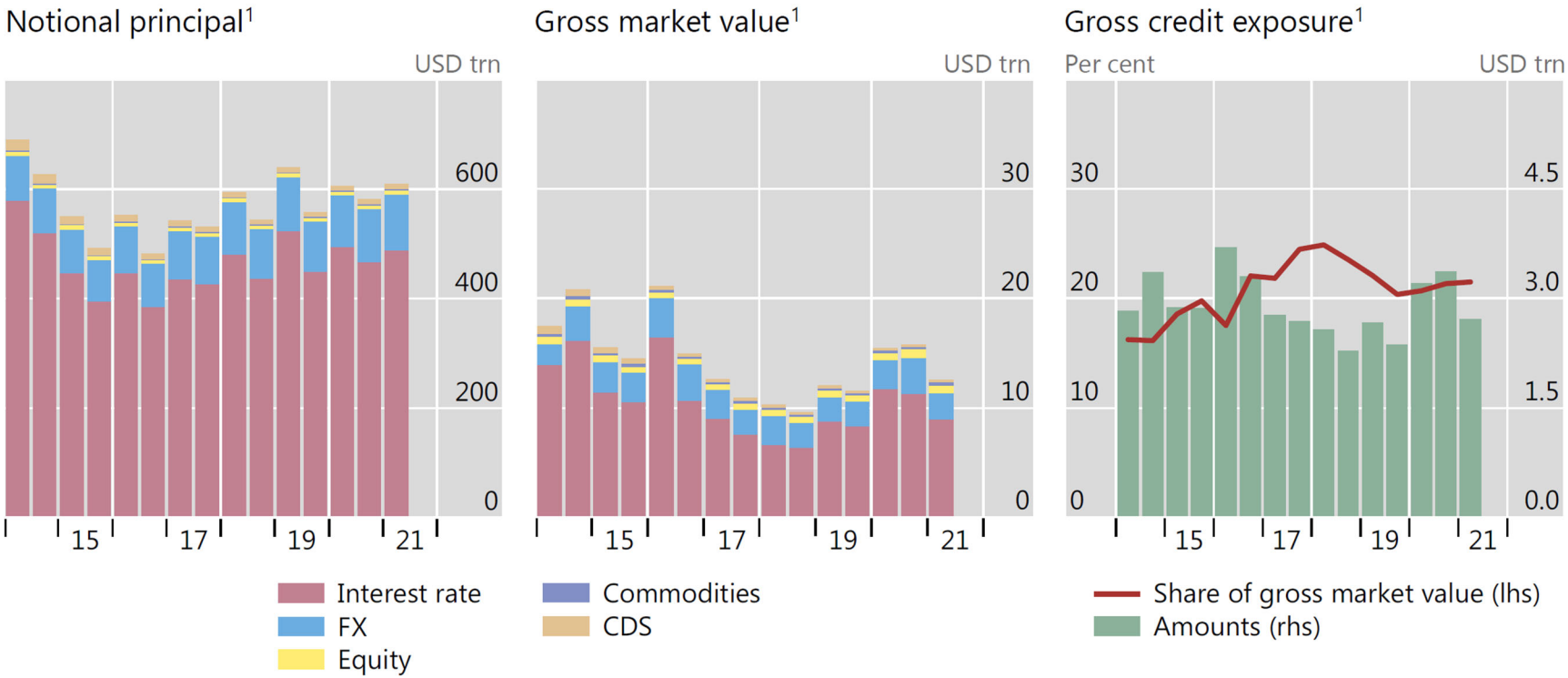


IRD = interest rate derivatives.

Source: BIS OTC derivatives statistics (Tables D5.1 and D5.2).

Global OTC derivatives markets

Graph A.1

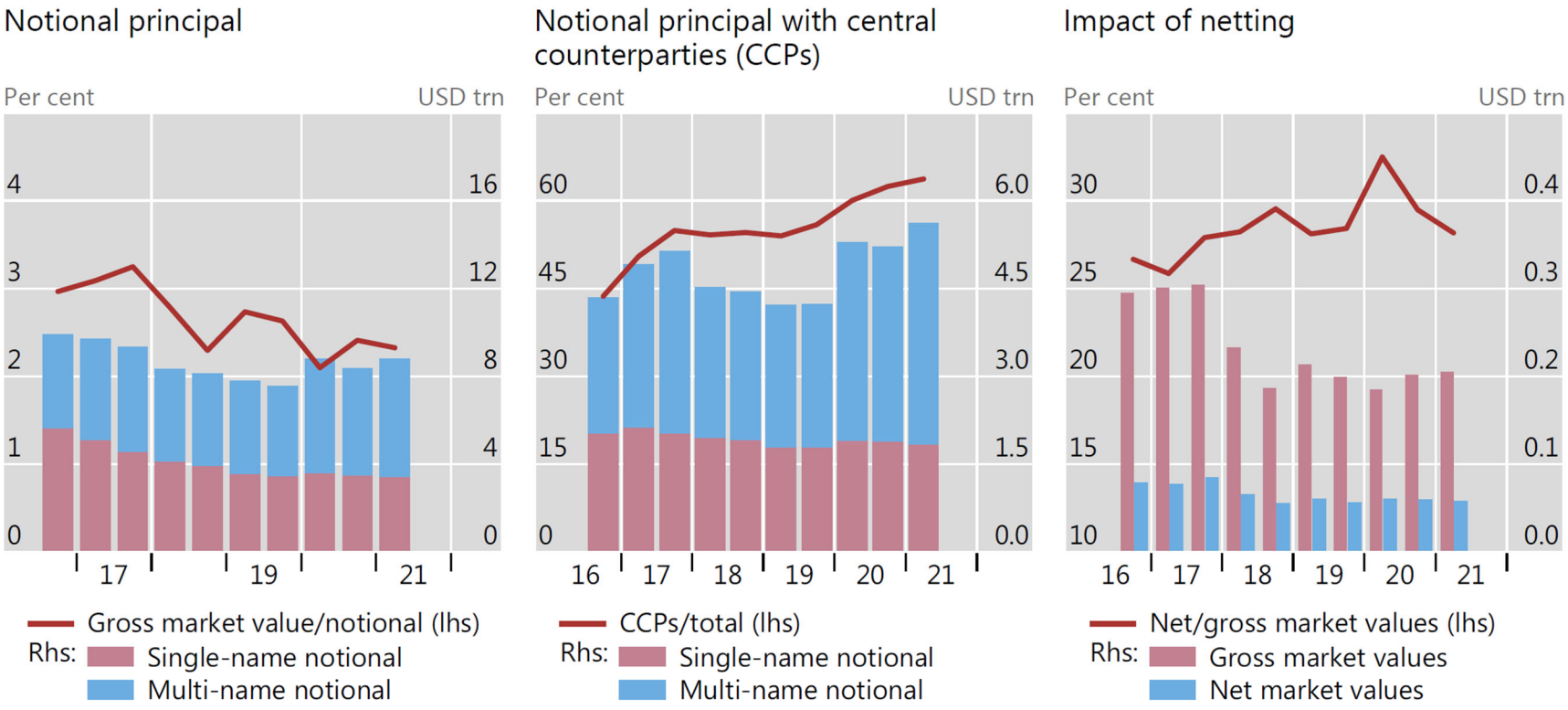


[1] At half-year end (end-June and end-December). Amounts denominated in currencies other than the US dollar are converted to US dollars at the exchange rate prevailing on the reference date.

Source: BIS OTC derivatives statistics (available at www.bis.org/statistics/derstats.htm).

Credit default swaps[1]

Graph A.6

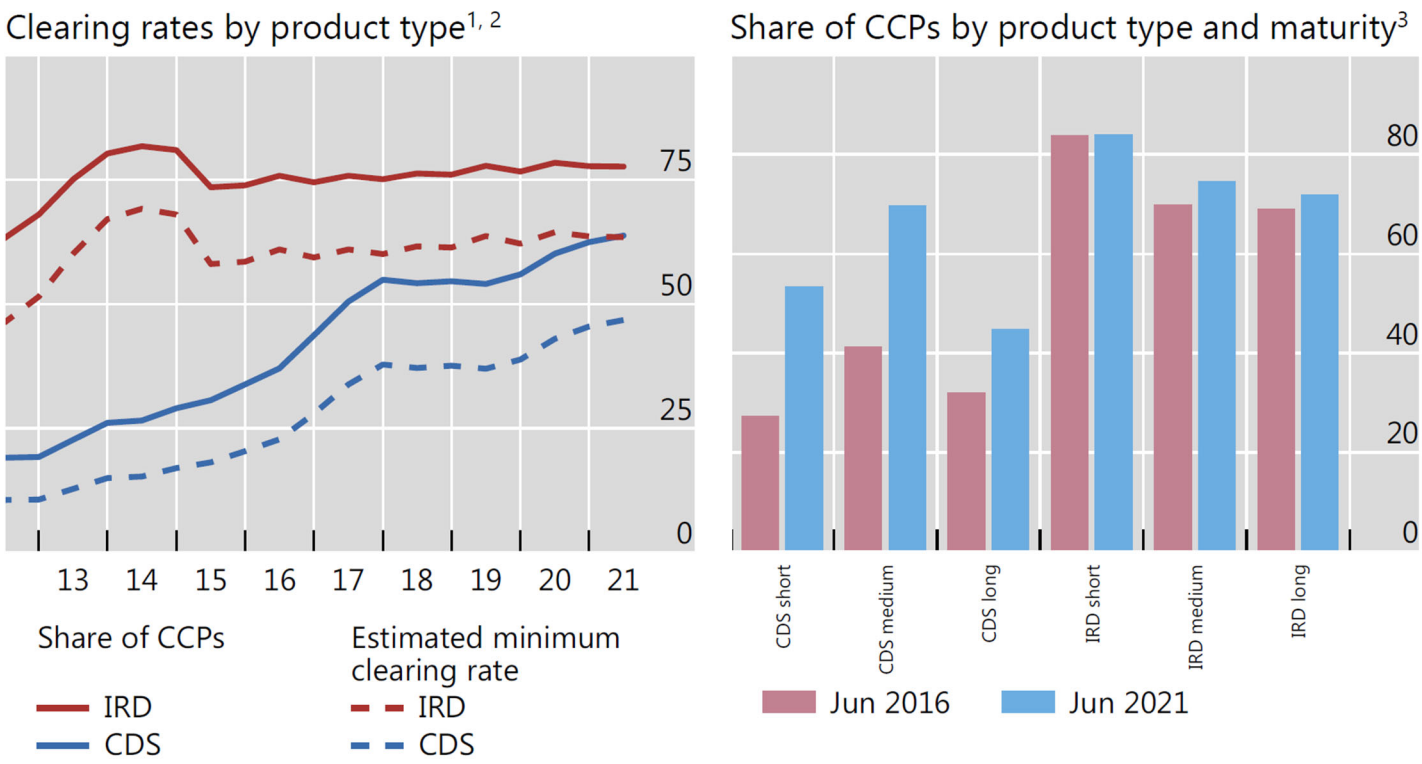


[1] At half-year end (end-June and end-December). Amounts denominated in currencies other than the US dollar are converted to US dollars at the exchange rate prevailing on the reference date.

Source: BIS OTC derivatives statistics (available at www.bis.org/statistics/derstats.htm).

Central clearing of interest rate and credit derivatives

Shares, in per cent — Graph 3



[1] Percentage of notional amounts outstanding of OTC interest rate derivatives (IRD) and credit default swaps (CDS) cleared by central counterparties (CCP). The estimated minimum clearing rate is the proportion of trades that are cleared, calculated as (CCP / 2) / (1 – (CCP / 2)), where CCP represents the share of notional amounts outstanding that dealers report against CCPs. [2] The CCP share is halved to adjust for the potential double-counting of inter-dealer trades novated to CCPs. [3] Short refers to maturity below one year, medium refers to maturity between one and five years, and long refers to maturity above five years.

Source: BIS OTC derivatives statistics (Tables D5.1 and D10.1).

The Lehman Bankruptcy

- Lehman filed for bankruptcy on September 15, 2008.
- Biggest bankruptcy in US history.
- Lehman was an active participant in the OTC derivatives markets and got into financial difficulties because it took high risks and found it was unable to roll over its short-term funding.
- It had hundreds of thousands of OTC derivatives transactions outstanding with about 8,000 counterparties.
- Unwinding these transactions has been challenging for both the Lehman liquidators and their counterparties.

New Regulations for OTC Market

The OTC market is becoming more like the exchange-traded market. New regulations introduced since the crisis mean that

- Standard OTC products traded between financial institutions must be traded on swap execution facilities
- A central clearing party must be used as an intermediary for standard products when they are traded between financial institutions
- Trades must be reported to a central registry

Systemic Risk

New regulations were introduced because of concerns about systemic risk. OTC transactions between financial institutions lead to systemic risk because a default by one large financial institution can lead to losses by other financial institutions.

Forward Contracts

- Forward contracts are similar to futures except that they trade in the over-the-counter market.
- These contracts are popular on currencies and interest rates
- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero).
- The forward price may be different for contracts of different maturities

Example: Foreign Exchange Quotes for USD/GBP exchange rate on May 13, 2015

	Bid	Offer
Spot	1.5746	1.5750
1-month forward	1.5742	1.5747
3-month forward	1.5736	1.5742
6-month forward	1.5730	1.5736

On May 13, 2015 the treasurer of a corporation might enter into a long forward contract to sell £100 million in six months at an exchange rate of 1.5730. This obligates the corporation to pay £1 million and receive $157.30 million on December 13, 2015.

Options

- A **call option** is an option to buy a certain asset by a certain date for a certain price (the strike price).
- A **put option** is an option to sell a certain asset by a certain date for a certain price (the strike price).
- An **American option** can be exercised at any time during its life. A **European option** can be exercised only at maturity

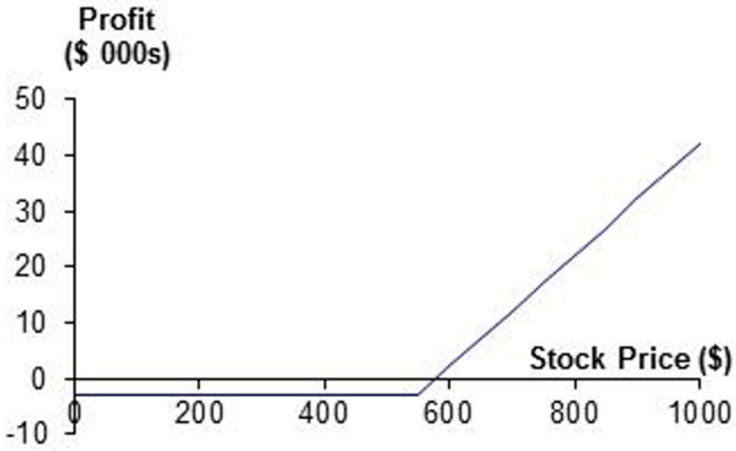
Google Call Option Prices (May 13, 2015 Stock Price: bid 532.20, offer 532.34)

Strike Price ($)	June Bid	June Offer	Sept Bid	Sept Offer	Dec Bid	Dec Offer
475	57.90	61.80	66.00	68.90	73.50	76.50
500	34.80	37.10	45.90	47.90	54.90	56.60
525	16.70	17.30	30.40	31.30	40.20	41.10
550	5.60	6.20	18.60	19.40	28.10	29.00
575	1.55	1.80	10.50	11.30	18.80	20.20

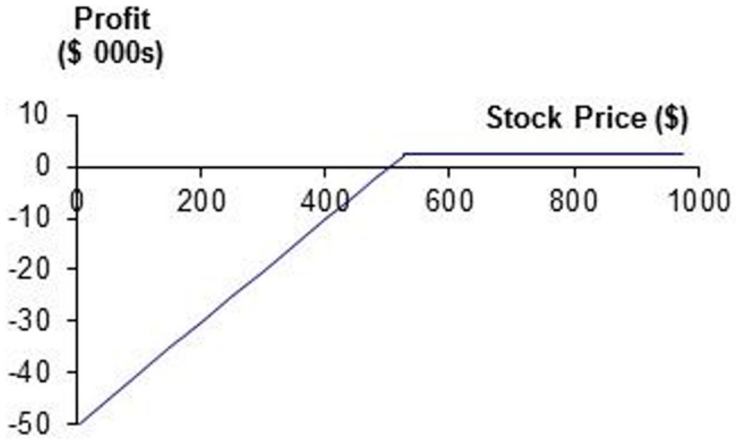
Google Put Option Prices (June 25, 2015 Stock Price: bid 532.20, offer 532.34)

Strike Price ($)	June Bid	June Offer	Sept Bid	Sept Offer	Dec Bid	Dec Offer
475	0.95	1.05	5.50	9.20	12.50	15.20
500	2.95	3.30	13.00	13.80	21.30	22.10
525	9.40	9.90	22.40	23.20	31.30	32.00
550	22.90	24.40	35.20	36.40	44.10	45.00
575	42.70	45.80	51.90	53.50	59.70	61.00

Net profit from purchasing a contract consisting of 100 December call options with a strike price of $550 for $29 per option



Net profit from selling a contract consisting of 100 September put options with a strike price of $525 for $22.40 per option



Exchanges on which options trade include

- Chicago Board Options Exchange
- International Securities Exchange
- NYSE Euronext
- Eurex (Europe)

Options vs Futures/Forwards

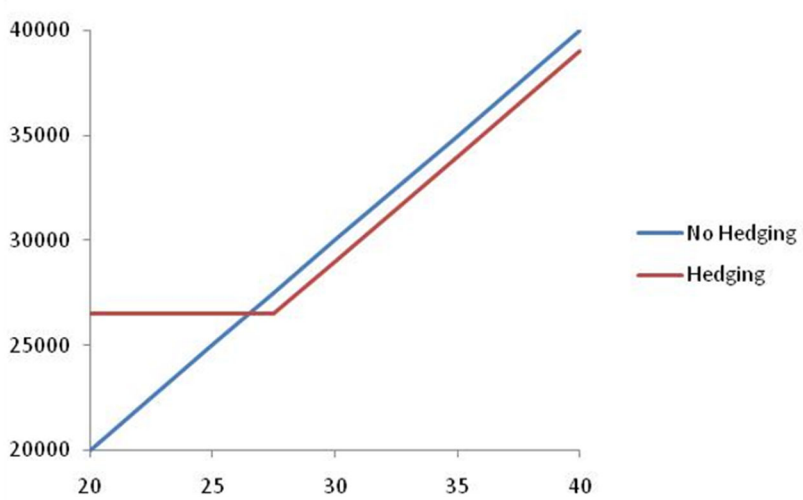
A futures/forward contract gives the holder the obligation to buy or sell at a certain price. An option gives the holder the right to buy or sell at a certain price.

Uses of Derivatives

Derivatives can be used for hedging, speculation, and arbitrage. When a trader has a mandate to use derivatives for hedging or arbitrage, but then switches to speculation, large losses can result. There are numerous examples of such mishaps.

Examples:

1. A US company will pay £10 million for imports from Britain in 3 months and decides to hedge using a long position in a forward contract.
2. An investor owns 1,000 shares currently worth $28 per share. A two-month put with a strike price of $27.50 costs $1. The investor decides to hedge by buying 10 contracts. Investor's value of shares with and without hedging is:



3. An investor with $2,000 to invest feels that a stock price will increase over the next 2 months. The current stock price is $20 and the price of a 2-month call option with a strike of $22.50 is $1.

4. A stock price is quoted as £100 in London and $152 in New York. The current exchange rate is 1.5500. What is the arbitrage opportunity?

Example: Gold Arbitrage Opportunity

Suppose that the spot price of gold is US$1,100 per ounce. The quoted 1-year futures price of gold is US$1,200. The 1-year US$ interest rate is 2% per annum. Assume that there are no income or storage costs for gold. Is there an arbitrage opportunity?

Now suppose that the spot price of gold is US$1,100. The quoted 1-year futures price of gold is US$1,050. The 1-year US$ interest rate is 2% per annum. Again, assume that there are no income or storage costs for gold. Is there an arbitrage opportunity?

What futures price will prevent any arbitrage?

If the spot price of gold is S and the futures price is for a contract deliverable in T years is F, then

$$F = S\,(1+r\,)^{T}$$

where r is the 1-year (domestic currency) risk-free rate of interest.

In our examples, S=1100, T=1, and r=0.02 so that F = 1100(1+0.02) = 1122.

Example: Oil Arbitrage Opportunity

Suppose that the spot price of oil is US$40. The quoted 1-year futures price of oil is US$50. The 1-year US$ interest rate is 2% per annum. The storage costs of oil are 1% per annum. Is there an arbitrage opportunity?

Now suppose that the spot price of oil is US$40, the quoted 1-year futures price of oil is US$35, the 1-year US$ interest rate is 2% per annum, and the storage costs of oil are 1% per annum. Is there an arbitrage opportunity?

Pricing: Derivatives Versus Underlying Securities

- Derivative securities are usually priced by invoking absence of arbitrage arguments.
  - During a short time interval, the change in the price of derivative security is proportional to the change in price of the underlying security.
  - So a derivative security is a combination of the underlying security and risk-free bond.
- The price of the derivative security can then be calculated from the price of the underlying security and the risk-free rate.

- However, how do we determine the price of the underlying security?

- Non-derivative securities are priced so that investors earn an expected return that equals the required return.
- The investor pays the price for the security and later receives the cash flows from the security.
- The required return compensates the investor for the time delay in receiving cash flows and for the risk of the cash flows.
- This compensation depends on impatience and risk-aversion of investors.

Recipe for pricing a security from scratch

1. What cash flows are expected from the security?
2. What is the time delay in getting cash flows in step 1?
3. What is the risk of cash flows in step 1?
4. How much return should the investor earn as compensation for the time delay in step 2?
5. How much return should the investor earn as compensation for the risk in step 3?
6. Required return = compensate for time delay (step 4) + compensation for risk (step 5).
7. Price = present value of the expected cash flows (step 1) at the required return (step 6).

- Step 2 depends on how important different kinds of risks are to investors.
- Capital Asset Pricing Model (CAPM) assumes that investors diversify so they only care about non-diversifiable risk measured by beta.
- However, not everyone believes in CAPM and other measures of risk are possible. For example, bond rating is often used as a risk measure for bonds.
- The extra return investors require for delay in cash flows is taken to be the risk-free rate.
- The extra return investors require for risk (for example, additional return for a stock beta of 1.2 or for a bond rating of A-) is reverse-engineered from observed prices of other securities.

Historical Return

- If you invest P_t in an investment at time t and then get back P_{t+1} from the investment at time t+1.
- The quantity P_{t+1} is the future cash flow from the investment.
- For a stock investment, this may be in the form of dividends and stock price.
- For bonds, it may be bond price and interest.
- For a derivative, it may be the value of the derivative.
- The return from t to t+1 is calculated as

$$R = \frac{P_{t+1} - P_t}{P_t} = \frac{P_{t+1}}{P_t} - 1$$

Expected and Required Return

- The expected return of an investment is the expectation of what the historical return will be.
- It depends on beliefs about how value of investment will change over time.
- The actual return may vary from expected return because of uncertainty. This introduces risk in investing.
- Investors prefer high expected return and low risk.
- They compare expected return and risk across investments to determine the return they require from an investment of a given risk.
- They then use this required return to discount future cash flows and price the investment.
- Discounted cash flow pricing:

$$P_t = \frac{E_t[P_{t+1}]}{1+R}$$

Discounted Cash Flow Valuation

The two main tasks in discounted cash flow valuation are the estimation of cash flows and the estimation of the discount rate.

Example 1: A stock pays annual dividends. The next dividend will be paid after one year and is expected to be $2. Dividends are expected to grow at 5%. The discount rate for the stock is 15%. What is stock price?

$$Price = \frac{2}{1.15} + \frac{2 \times 1.05}{1.15^2} + \frac{2 \times 1.05^2}{1.15^3} + \cdots = \frac{2}{0.15 - 0.05} = \$20$$

Example 2: A bond pays a coupon of $5 every year. The bond will mature after 6 years. Investors require an yield of 8% from the bond. What is bond price?

$$Price = \frac{5}{1.08} + \frac{5}{1.08^2} + \frac{5}{1.08^3} + \frac{5}{1.08^4} + \frac{5}{1.08^5} + \frac{105}{1.08^6} = \frac{5}{0.08}\left(1 - \frac{1}{1.08^6}\right) + \frac{100}{1.08^6} = \$86.13$$

- The strength of discounted cash flow valuation is that any kind of business, asset, or security can be valued if we can determine discount rate and cash flows.
- Specifically, even a new business or security unlike any other can also be valued.
- The weakness of discounted cash flow valuation is that estimation of cash flows and discount rate is hard, and it is difficult to be confident that the estimation is correct.

Multiples/Comparables Method

- In multiples method, neither cash flows, nor discount rate are estimated.
- Instead, one or more peers are identified whose cash flows are similar to the cash flows of the security or business being valued except for a difference in size.
- The cash flows of the firm being valued are assumed to be a multiple of the peer's cash flows.
- When valuing firms using multiples method, peers are usually other firms in the same industry.
- The discount rate for the security or business being valued is assumed to be the same as the discount rate for peer.
- The price of the security being valued equals the price of the peer scaled by the multiple.
- Thus, the valuation task reduces to observing the price of the peer and identifying the multiple, usually as the ratio of some characteristic such as size of assets, earnings, etc.

- By eliminating the need for estimating cash flows and discount rate, multiples method is not sensitive to errors in these estimations.
- And by using market prices of peers as an input, multiples method can allow valuation to be consistent with changes in market's sentiment or risk tolerance.
- However, multiples method is susceptible to other faulty assumptions, such as poor choice of peers or multiples.

Example 3: Company A and Company B are in same industry and their cash flows are expected to be similar except that Company A's size and cash flows are double that of Company B. Company B has 10 million shares, each priced at $40. Company A has 50 million shares. What should be Company A's share price?

Value of Company A = Multiple × Value of Company B = 2 × 10 million shares × $40 / share = $800 million

Share price of Company A = $800 million / 50 million shares = $16 / share.

- **Derivative Pricing**
- Can discounted cash flow valuation and multiples method be used to value derivatives? Yes.
- However, since the cash flows of derivatives depend on the cash flows of the underlying security, first the cash flows of the underlying security and the discount rate for the underlying security must be determined and then both the cash flows and the discount rate must be adjusted for the derivative.
- A lot of this work is duplication of the work required in pricing the underlying security.
- One can avoid duplication of this effort if two conditions are met
    - the price of the underlying security is available and
    - there is a way to calculate derivative price based on the price of the underlying asset.
- The first condition is usually met.
- Thanks to the analytical work in derivative pricing, we now have formulas and numerical methods to calculate derivative price based on the price of the underlying security.
- Thus, in practice,
    - prices of underlying securities are calculated based on first principles, taking cash flows and discount rate into account
    - prices of derivatives are calculated using models based on absence-of-arbitrage arguments, taking price of underlying security as an input

Risk Management

Uncertainty

- We use the term uncertainty when we are not sure how a situation may evolve.
- For example, what will be the outcome of an election, how will the stock market perform over the next year, will I be able to get a good job, will the winter be mild, how will my favorite sports team perform, what will be the revenue growth of our firm this year?
- To model uncertainty, start by specifying the events of interest.
- Think of each **event** as one possible outcome of an uncertain experiment.
- Alternative names for events are **states** or **scenarios**.
- Events are **mutually exclusive** if two events cannot occur simultaneously.
- Events are **exhaustive** if at least one of the events must occur.

- For example, in case of US Presidential election, the two possible events are a win by Democrats and a win by Republicans.
- In case of stock market performance over the next one year, you may define events as
  1. S&P500 will decline,
  2. S&P500 will remain unchanged or rise by less than 10%, and
  3. S&P500 will rise by 10% or more.
- Or if you wanted to consider more possibilities, you could define events as
  1. S&P500 will decline by more than 10%,
  2. S&P500 will decline but no more than 10%
  3. S&P500 will remain unchanged or rise by less than 10%,
  4. S&P500 will rise by at least 10% but less than 20%, and
  5. S&P500 will rise by 20% or more.

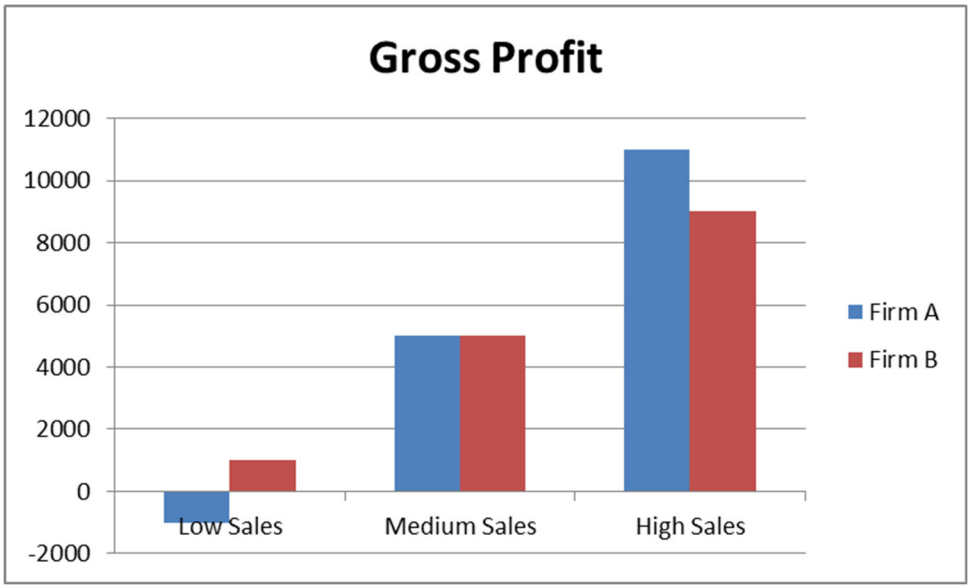
Risk

- Risk is defined with respect to some **objective**.
- An uncertainty becomes a risk for an individual (or corporation) if it impacts the individual's (or corporation's) objective.
- The objective could be profit or wealth for an investor.
- For a corporation, some possible objectives are revenue, growth, margins, profit, earnings per share, or share price.
- If the value of the objective is the same across all events, there is no risk.
- If the objective value differs across events, there is risk.
- The risk is greater if the objective varies more across the events.

Example 1: Firms A and B are concerned about the risk of their gross profit next year. Firm A has fixed cost of $10,000 and variable cost of $2 per unit. Firm B has fixed cost of $5,000 and variable cost of $3 per unit. The revenue per unit is $5. There is uncertainty about sales. Each firm expects its sales next year to be 3,000 units, 5000 units, or 7000 units.

Event		Sales = 3000 units	Sales = 5000 units	Sales = 7000 units
Firm A	Revenue	15000	25000	35000
	Fixed Costs	10000	10000	10000
	Variable Costs	6000	10000	14000
	Gross Profit	-1000	5000	11000
Firm B	Revenue	15000	25000	35000
	Fixed Costs	5000	5000	5000
	Variable Costs	9000	15000	21000
	Gross Profit	1000	5000	9000

The uncertainty about sales affects gross profit for both firms but the risk is greater for Firm A because it has higher fixed costs that do not adjust as the level of sales changes.


Gross Profit
12000
10000
8000
6000
4000
2000
0
-2000
Low Sales
Medium Sales
High Sales
Firm A
Firm B

Risk Management with Real Decisions

- **Real decisions** impact the business operations of the firm.
- A change in production technology, introduction of a new product, abandoning a poorly performing product, acquiring another firm, opening a new plant, moving operations to a different country, changing the production level, outsourcing information technology needs, etc. are all examples of real decisions.

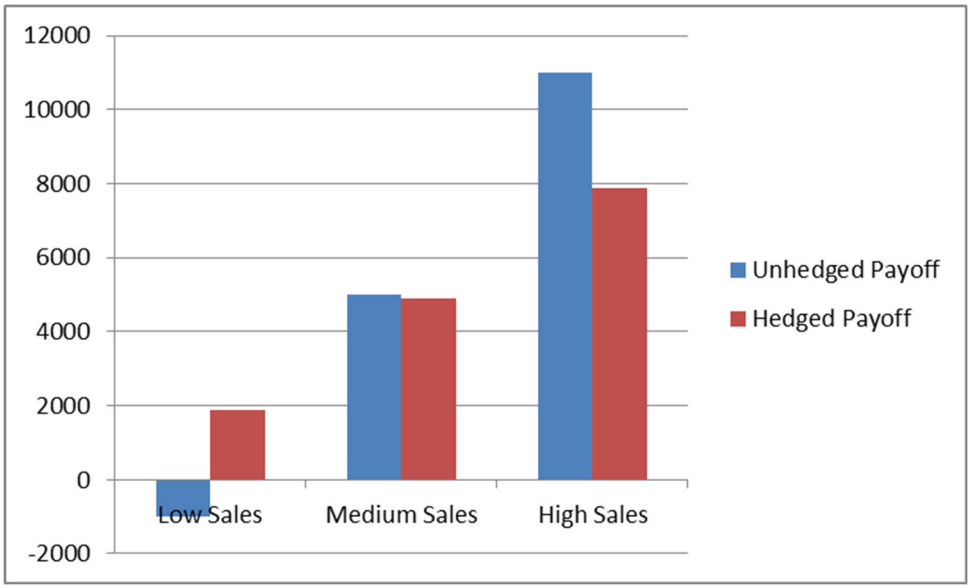
Risk Management through Financial Decisions

- **Financial decisions** are results of financial contracts or trades that do not impact the business operations of the firm but can impact the financial performance of the firm.
- Examples of financial decisions include using derivatives such as futures or options to hedge a risk, buying insurance, investing in stocks of other firms, etc.
- Financial decisions affect the firm's performance across events. **Hedging** is the use of financial transactions by a firm to lower its risk.
- However, financial transactions can also be used to increase risk. The latter is called **speculation**.

Example 2: Firm A from Example 1 enters into a financial contract to hedge its gross profit next year. If the gross profit turns out to be less than $5000, the firm will receive half of the difference between $5000 and its gross profit. If the gross profit turns out to be more than $5000, the firm will pay half of the difference between its gross profit and $5000. The firm has to pay now $100 to enter into the contract. The risk-free rate is 5% per year.

Event	Sales = 3000 units	Sales = 5000 units	Sales = 7000 units
Revenue	15000	25000	35000
Fixed Costs	10000	10000	10000
Variable Costs	6000	10000	14000
Unhedged Gross Profit	-1000	5000	11000
Payoff from hedging	3000	0	-3000
Payoff from cost of hedging	-105	-105	-105
Hedged payoff	1895	4895	7895

Notice that the $100 cost of hedging today is equivalent to a cost of $105 next year because the risk-free rate is 5% per year. Hedging reduces the risk of Firm A's payoffs by increasing low unhedged payoffs and decreasing high unhedged payoffs.

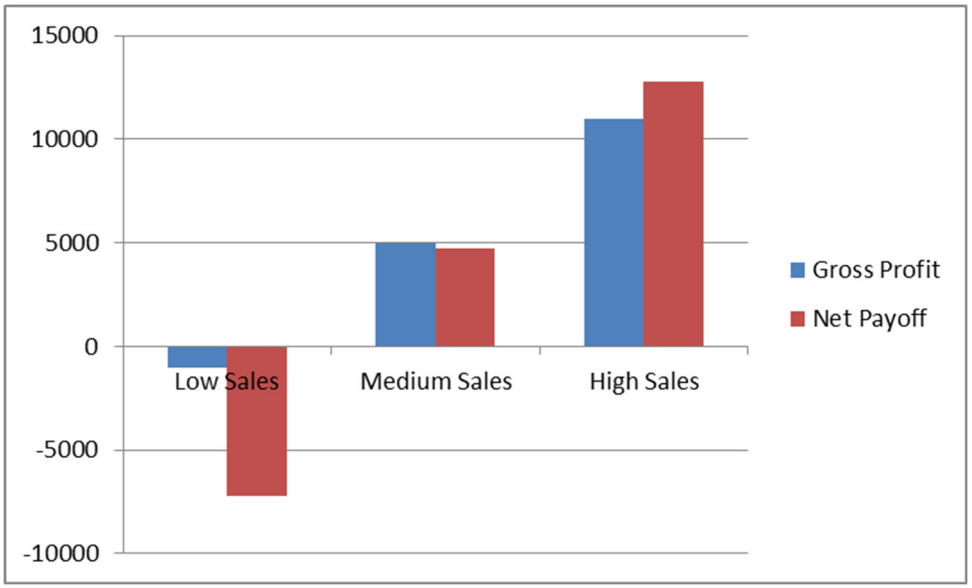


Hedging can be used to completely eliminate risk. However, often firms only partially reduce risk.

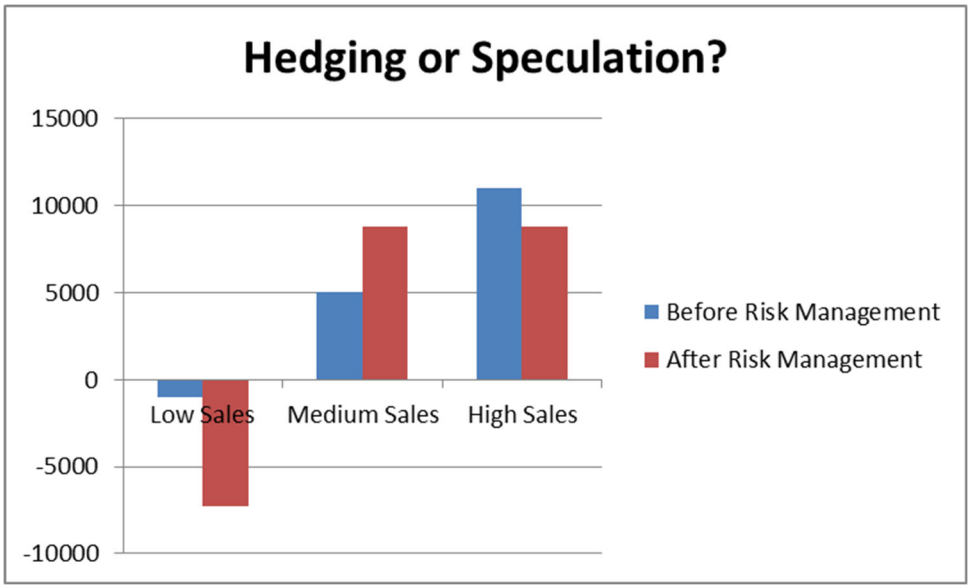
Example 3: Firm A from Example 1 buys a financial contract that promises to pay one-fifth of the firm's revenue next year in exchange for an upfront cost of $5,000. The risk-free rate is 5% per year.

Event	Sales = 3000 units	Sales = 5000 units	Sales = 7000 units
Revenue	15000	25000	35000
Fixed Costs	10000	10000	10000
Variable Costs	6000	10000	14000
Unhedged Gross Profit	-1000	5000	11000
Payoff from contract	3000	5000	7000
Payoff from cost of contract	-5250	-5250	-5250
Net payoff	-7250	4750	12750

Notice that the $5,000 cost of hedging today is equivalent to a cost of $5,250 next year because the risk-free rate is 5% per year. In this case, the financial contract increases the risk of Firm A's payoffs by decreasing low unhedged payoffs and increasing high unhedged payoffs. This is an example of speculation rather than risk management or hedging.



Examples 2 and 3 show that risk management can be used to decrease risk (hedging) or increase risk (speculation). Sometimes such a clear distinction is not possible and a risk management strategy may be lowering some risks while simultaneously speculating on some other risks. The following chart shows an example.


Hedging or Speculation?
15000
10000
5000
0
-5000
-10000
Low Sales
Medium Sales
High Sales
Before Risk Management
After Risk Management

Risk Management as Transfer of Cash Flows across Events

- As Example 2 illustrates, risk management results in a reduction of cash flows in some events and an increase in cash flows in some other events.
- A risk management strategy based on financial transactions cannot result in increase of cash flows in all events once the cost of risk management is taken into account.
- The benefit of risk management is the increase in cash flows under some events.
- The cost is the decrease in cash flows under some other events.
- So risk management results in a transfer of cash flows from some events to some other events.
- Two questions arise:
  1. Is the extra cash flow in some events worth the reduction in cash flows in some other events? The answer to that question determines whether risk management is valuable or not. The following discussion of state prices addresses this question.
  2. If risk management is valuable, why does the counterparty provide risk management as its cash flows are exactly opposite? The following discussion of market imperfections addresses this.

Arbitrage Opportunities Do Not Exist in Perfect Markets

- An **arbitrage opportunity** is a situation in which it is possible to make a profit without any risk of loss and without making any investment.
- Following are some examples of arbitrage opportunities:
    - You can buy apples from a store for $2 and sell to another store for $3 with no transaction costs.
    - You can borrow at 5% and lend at 6%
    - Stock X is trading at $50 in one stock exchange and at $51 in another stock exchange and there are no trading costs
- In a perfect market, prices should be such that there is no arbitrage opportunity. This is the **absence of arbitrage principle**.
- This principle implies that two trading strategies (or portfolios) with same payoffs must cost the same.

State Prices

- In a perfect market with no frictions, security prices are determined by **state prices**.
- The state price of a state is the price investors pay to get 1 unit of cash flow in that state (event) and nothing in other states (events).
- If a stock pays 60 in recession, 90 in normal event, and 120 in boom, its price equals 60 times the state price of recession plus 90 times the state price of normal event plus 120 times the state price of boom.
- If a bond pays 100 regardless of whether the economy is in recession, normal, or boom, its price must equal 100 times the state price of recession plus 100 times the state price of normal event plus 100 times the state price of boom.
- The price of a security that pays one unit of cash flow in recession and nothing in other events is the state price of recession.
- Such securities that pay one unit of cash flow in one event and zero in other events are called **Arrow-Debreu securities** based on work by Nobel Prize winning economists Kenneth Arrow and Gerard Debreu.
- The price of an Arrow-Debreu security is the state price of the event in which the security pays.
- Any other security with more complicated cash flows can be created as a combination of Arrow-Debreu securities.
- Suppose you bet $2 on the horse Silver Blaze in a horse race. You are promised $5 if Silver Blaze wins and nothing otherwise. What is the state price of the Silver Blaze winning the race?

Example 4: Consider a bond and a stock. The bond will yield $100 regardless of how the market performs. The stock will result in $60 in recession, $90 in normal event, and $120 in boom. The current price of stock is $78 and the current price of bond is $95. Suppose the state price of recession is 0.35, the state price of normal event is 0.5, and the state price of boom is 0.1. Verify stock and bond prices. Determine the price of a derivative security that pays 100 in normal event or recession but nothing in boom.

Stock price = $60 \times 0.35 + 90 \times 0.5 + 120 \times 0.1 = 78$

Bond price = $100 \times 0.35 + 100 \times 0.5 + 100 \times 0.1 = 95$

Derivative price = $100 \times 0.35 + 100 \times 0.5 + 0 \times 0.1 = 85$

What do state prices depend on?

- One factor is the **probability of the event**.
- For example, consider two bets.
- The first bet pays you one unit of cash flow if S&P500 stock index increases the next day.
- The second bet pays you one unit of cash flow if S&P500 stock index decreases by at least 10% the next day.
- Since the first event is more likely than the second, you would be willing to pay more for the first bet than for the second.
- In general, state prices are higher for events which are considered more likely.

- The second factor that determines state prices is the **investors' preference (need) for cash flow in that event**.
- Investors may be in greater need of additional cash in some events than in other events.
- For example, recessions result in lower wealth for most investors so investors value additional cash more than they would in a boom where they have more wealth.
- So even if recession and boom are equally likely to occur, the state price of recession is higher than the state price of boom.

Reasons for trade

- The cash flows from a financial transaction are mirror images for the two parties in the transaction.
- If both parties evaluate the transaction using same state prices, then if one party finds the transaction valuable, the counterparty will not find it valuable.
- So why do financial transactions take place? They occur
    - either because the two parties do not agree on state prices
    - or because the two parties have different circumstances and their valuation of the transaction depends on state prices and their special circumstances.

Example 5: An airline will need $100 million barrels of oil next year. The oil price next year will be high ($110/barrel), medium ($100/barrel), or low ($90/barrel). If the oil price is high, the airline will experience financial distress. An option that will pay $100 if the oil price next year is $110 (and nothing otherwise) is trading for $37. An option that will pay $100 if the oil price next year is $100 is trading for $32. An option that will pay $100 if the oil price next year is $90 is trading for $27.

a. What are the state prices for the three events representing different levels of oil price next year?

   The option that pays $100 if oil price is high is trading for $37. So the price of getting $100 in high oil price event is $37. The price of getting 1 unit in high oil price event is $37/$100 = 0.37. Thus, the state price of the high oil price event state, $\pi_H = 0.37$. Similarly, the state prices for the event of medium oil price and low oil price are $\pi_M = 0.32$ and $\pi_L = 0.27$, respectively.

b. What is the risk-free rate?

   A risk-free security will provide the same cash flow regardless of the oil price. Consider a risk-free security that pays $100 in all states. Such a security can be created by buying the three options each of which pays $100 when oil price reaches a certain level. So the price of

the risk-free security equals the price of the three options, $37 + $32 + $27 = $96. Risk-free rate = return on this risk-free security = $100/$96 – 1 = 4.17% with annual compounding.

The airline's current market value incorporates a discount of $48 million due to the possibility of financial distress from high oil price next year. This discount can be avoided if the airline doesn't have to pay the high oil price next year. The airline is considering the following two risk management possibilities: insurance or hedging.

- Insurance: If the oil price exceeds $100, the insurer will reimburse the airline for the difference between oil price and $100/barrel. The airline must pay the insurer a fixed fee now. The present value of transaction costs of providing insurance is $6 million for the insurer.

c. If the airline buys insurance, what will be its future cash flows from insurance?

   The insurance will pay ($110/barrel - $100/barrel) × 100M barrel = $1B if the oil price reaches $110/barrel. The insurance will pay 0 if the oil price is $100/barrel or $90/barrel.

d. What is the maximum fee the airline should be willing to pay for insurance?

The maximum fee the airline should pay = value of cash flows from insurance based on market prices + additional value (positive or negative) of insurance to airline due to market imperfections.

Value of cash flows from insurance based on market prices = $\pi_H \times \$1B + \pi_M \times 0 + \pi_L \times 0 = 0.37 \times \$1B = \$370M$.

Additional value of insurance to airline due to market imperfections = increase in firm value due to avoidance of financial distress = $48M.

The airline should pay a maximum fee of $370M + $48M = $418M.

e. What is the minimum fee that the insurer will accept for insurance?

The insurer's cost of providing insurance = value of cash flows from insurance based on market prices + additional costs (positive or negative) due to market imperfections. We already calculated the value of cash flows from insurance based on market prices = $370M.

Additional costs due to market imperfections = transaction costs of providing insurance = $6M.

So the insurer's cost of providing insurance = $370M + $6M = $376M. This is the minimum fee that the insurer should accept.

f. If the airline buys insurance by paying $390 million, what is the NPV of this transaction to the insurer? What is the impact on the airline's share price? The airline has 10 million shares outstanding.

NPV to insurer = fee – costs = $390M - $376M = $14M
NPV to firm = value of insurance – fee = $418M - $390M = $28M
Buying insurance increases the value of each share by $28M/10M shares = $2.8/share.
The total NPV of transaction = $14M + $28M = $42M. Note that the NPV arises only due to market imperfections. Insurance allows airline to avoid financial distress costs of $48M but results in transaction costs of $6M so total NPV = $48M - $6M = $42M.

- Hedging: The airline will enter into a financial contract with an oil company to fix the price it will pay for oil next year. There will be no initial cash payment. If the oil price is higher than the fixed price, the oil company will compensate the airline with the difference. If the oil price is lower than the fixed price, the airline will pay the difference to the oil company. The oil company's present value of transaction costs of providing hedging is also $6 million.

g. If the airline hedges its exposure at a fixed oil price of $*P* /barrel, what will be its future cash flows from hedging?

If the oil price is \$110/barrel, the airline will receive (\$110/barrel - \$*P*/barrel) × 100M barrels = (110 – *P*) × \$100M. Similarly, if the oil price is \$100/barrel, the airline will receive (100 – *P*) × \$100M and if the oil price is \$90/barrel, the airline will receive (90 – *P*) × \$100M. Note that some of these cash flows may be negative.

h. What is the highest fixed oil price at which the airline will agree to hedge its exposure?

The value of hedging to airline = value of cash flows from hedging at market prices + additional value (positive or negative) from hedging to airline due to market imperfections. Suppose the fixed price is *P*.

Value of cash flows from hedging at market prices = $\pi_H \times (110 - P) \times \$100M + \pi_M \times (100 - P) \times \$100M + \pi_L \times (90 - P) \times \$100M = 0.37 \times (110 - P) \times \$100M + 0.32 \times (100 - P) \times \$100M + 0.27 \times (90 - P) \times \$100M = (97 - 0.96P) \times \$100M$.

Additional value (positive or negative) from hedging to airline due to market imperfections = increase in firm value due to avoidance of financial distress = \$48M.

Total value of hedging to airline = $(97 - 0.96P) \times \$100M + \$48M$.

At the highest value of *P* that the airline will agree to, hedging's value to airline will be exactly 0. So we solve $(97 - 0.96P) \times \$100M + \$48M = 0$ to get $P = 101.5417$.

i. What is the lowest fixed oil price at which the oil company will agree to hedge airline's exposure?

The cost of hedging to the oil company = value to the airline of cash flows from hedging at market prices + additional costs (positive or negative) due to market imperfections. We already calculated that if the fixed price is P, value to the airline of cash flows from hedging at market prices = $(97 - 0.96P) \times \$100M$.
Additional costs due to market imperfections = transaction costs of providing hedging = \$6M.
So the oil company's cost of providing hedging = $(97 - 0.96P) \times \$100M + \$6M$.
At the lowest value of P that the oil company will agree to, hedging's cost to the oil company will be exactly 0. So we solve $(97 - 0.96P) \times \$100M + \$6M = 0$ to get $P = 101.1042$.

j. If the airline hedges its exposure at a fixed oil price of $101.25/barrel, what is the NPV of this transaction to the oil company? What is the impact of the transaction on the airline's share price?

We already calculated that the cost of hedging to the oil company = $(97 – 0.96P) \times \$100M + \$6M$. Substituting $P = 101.25$, cost = $(97 – 0.96 \times 101.25) \times \$100M + \$6M = -\$14M$. So NPV to oil company = $14M.
We also calculated earlier that the total value of hedging to airline = $(97 – 0.96P) \times \$100M + \$48M$. Substituting $P = 101.25$, the total value of hedging to airline = $(97 – 0.96 \times 101.25) \times \$100M + \$48M = \$28M$.
Hedging increases the value of each share by $28M/10M shares = $2.8/share.
The total NPV of transaction = $14M + $28M = $42M. Note that the NPV arises only due to market imperfections. Hedging allows airline to avoid financial distress costs of $48M but results in transaction costs of $6M do total NPV = $48M - $6M = $42M.

k. If there was no possibility of financial distress and therefore, no financial distress related discount for the airline, is insurance or hedging feasible for the airline? In other words, can there be terms which result in positive NPV for both the airline and the insurer (or the oil company)?

Costs of Risk Management

- Risk management results in administrative costs incurred by the financial institution providing risk management.
- These include costs of preparing financial statements, legal costs, and overhead costs of sales and advertising.
- Some other costs arise from the suspicion of the counterparty about the motive for risk management.
- A firm requesting insurance or hedging knows more about its risk than the counterparty does.
- This information asymmetry puts the counterparty at a disadvantage which charges a premium to make up for this possible disadvantage.
- A counterparty may also be worried about moral hazard.
- Moral hazard refers to the possibility of manipulation by the firm requesting risk management.
- For example, if a firm insures its earnings, managers may not work hard since any shortfall in earnings from not working hard will be covered by insurance.[1]
- The counterparty realizes this possibility of moral hazard and charges a premium.

[1] Moral hazard is common in other situations. Patients with health insurance are not as cost conscious about their medical bills as patients without health insurance. Employees who are about to switch jobs may be less concerned in their current job performance as it is less likely to influence their future. Fund managers are likely to make riskier investments with borrowed money than they would have with their own money.