Problem Set III

QF 430: Introduction to Derivatives
Due Monday, October 24

Please submit neatly handwritten or typed answers. You can turn in paper submissions in class or submit electronically a single pdf file through Canvas. Show your steps or reasoning. Do not round too much in intermediate calculation steps. Aim for accuracy of four decimal places in interest rates (0.0337 or 3.37%).

1 Interest Rate Derivatives

Problem 1.1. The Wall Street Journal's quoted ask price on October 13, 2022 for a 2.875% U.S. government treasury note maturing on May 15, 2032 is 91.1240. This quote means that the ask quote is \$91 and 12.5 32nds for every \$100 of face value. Assume that the note pays semiannual coupons. Treasury bond transactions settle the next business day so this is the quote for making the payment and getting the bond on October 14, 2022.

(a) What is the cash price? You can optionally verify your answer to within a cent using the reported (semiannually compounded) yield of 3.963%. See Treasury Bond Pricing Example on Canvas.

(b) How does your answer change if the bond is a corporate bond?

Reference: Slides Examples 1-2

Problem 1.2. The price of a 90-day Treasury bill is quoted as 4.40. What continuously compounded return (on an actual/365 basis) does an investor earn on the Treasury bill for the 90-day period?

Reference: Slides Example 4

Problem 1.3. Download the treasury conversion factors spreadsheet from CME group website at https://www.cmegroup.com/trading/interest-rates/treasury-conversion-factors.html. Open the tab with conversion factors and scroll down to the table titled "U.S. TREASURY BOND FUTURES CONTRACTS." Each row provides conversion factors for a specific bond. Use the *second* row from the table (we will ignore the first row because it has only one conversion factor). Find the conversion factors for the futures contract with the two earliest delivery months. Fill in the bond information, the delivery months, and the conversion factors below. Next, perform and show your conversion factor calculations to verify both conversion factor values.

Coupon	
Maturity Date	
CUSIP	
Delivery Month 1	
Conversion Factor 1	
Delivery Month 2	
Conversion Factor 2	

Reference: Slides Example 6

Problem 1.4. Suppose that the Treasury bond futures price is 102-11. Which of the following four bonds is cheapest to deliver?

Bond	Quoted Price	Conversion Factor
1	91-19	0.8913
2	97-28	0.9535
3	107-10	1.0441
4	116-18	1.1349

Reference: Slides Example 6

Problem 1.5. It is October 14, 2022. The cheapest-to-deliver bond in an April 2023 Treasury bond futures contract is a 3% coupon bond, and delivery is expected to be made on April 30, 2023. Coupon payments on the bond are made on March 1 and September 1 each year. The rate of interest with continuous compounding is 4% per annum for all maturities. The conversion factor for the bond is 0.6403. The current quoted bond price is \$85.31. Calculate the quoted futures price for the contract.

Reference: Slides Example 7

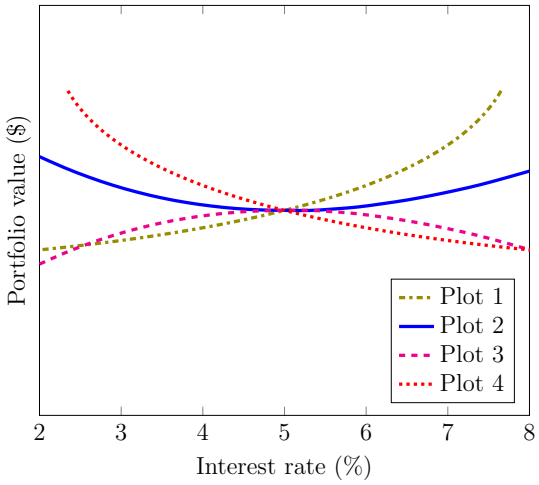
Problem 1.6. Suppose the yield curve is flat at 5% per annum continuously compounded. Moreover, yield curve can only have parallel shifts. That is, interest rates can change but interest rate stays same for all maturities.

(a) How much should you invest in a 5-year zero coupon bond such that an increase in (continuously compounded) interest rate of 0.01% results in a loss of 10 cents? How much does this investment pay after 5 years?

(b) After the investment you calculated in the 5-year bond, how much of a 10-year zero coupon bond should you short to immunize yourself against small changes in interest rate? That is, such that the gain in your short position on an interest rate increase of 0.01% is 10 cents. How much do you need to repay after 10 years on the short position?

(c) Duration matching does not protect against large interest rate changes because bond prices are convex in interest rates (prices of longer-term bonds are more convex). Which of the following plots shows how your portfolio value changes with interest rates? Why?

Reference: Duration



Problem 1.7. A portfolio manager wants to hedge a bond portfolio against interest rate risk over the next three months. The manager plans to hedge using a Treasury bond futures contract maturing in four months. The portfolio is worth $38 million and will have a duration of 11.2 years in four months. The futures price is 113, and each futures contract is on $100,000 of bonds. The bond that is expected to be cheapest to deliver will have a duration of 17.5 years at the maturity of the futures contract.

(a) What position in futures contracts is required?

(b) Suppose that all rates increase over three months, but longer-term rates increase less than shorter-term rates. What is the effect of this on the performance of the hedge?

Reference: Slides Example 9

2 Swaps

Problem 2.1. A U.S. company wants to borrow Australian dollars (AUD) at a fixed rate of interest. An Australian company wants to borrow U.S. dollars at a fixed rate of interest. They have been quoted the following interest rates (per annum):

	US Dollars	AUD
US company	7.8%	8.4%
Australian company	8.2%	8.0%

Design a swap that will net a bank, acting as intermediary, 10 basis points per annum and that will produce a gain of 35 basis points per annum for each of the two companies.

Reference: Slides/Book - Currency Swaps Comparative Advantage

Problem 2.2. A financial institution has agreed to pay three-month LIBOR and to receive 7% per annum in return in an interest rate swap. The notional principal is $50 million and payments are exchanged every three months. The swap has a remaining life of 11 months. Three-month forward LIBOR for all maturities is currently 6.6% per annum. The

three-month LIBOR rate two months ago was 6.8% per annum. OIS rates for all maturities are currently 6% with continuous compounding. All other rates are compounded quarterly. What is the value of the swap?

Reference: Slides Example 3

Problem 2.3. Consider fixed-for-floating swaps with *annual exchanges* of cash flows and one-year LIBOR as the floating rate. The two-year swap rate and the three-year swap rate are 4% per annum and 4.1% per annum, respectively, both annually compounded. The risk-free zero interest rates are 3% for one year, 3.1% for two years, and 3.2% for three years, each continuously compounded. What is the forward LIBOR rate for the period between 2 years and 3 years? (Hint: Both 2-year and 3-year swaps have value zero so the differences in their cash flows also have value zero. You can assume the principal amount to be \$100 million but it does not matter.)

Reference: Bootstrapping - Book Example 7.2, Slides Example 4

Problem 2.4. A company wants a floating-for-fixed swap where it receives semiannual payments at 5.2% per annum with semiannual compounding on a principal of \$20 million for three years. The three-year swap rate with semiannual cash flows is 5.0% per annum with semiannual compounding. The OIS zero curve is flat at 4% per annum with continuous compounding. How much should a derivatives dealer charge the company?

Reference: Valuation of Swaps

Problem 2.5. Suppose that the term structure of risk-free interest rates is flat in the United States and Britain. The USD interest rate is 5.1% per annum continuously compounded and the British rate is 4.8% per annum continuously compounded. The current value of a pound (GBP) is 1.12 USD. Under the terms of a swap agreement, a financial institution pays 6% per annum in USD and receives 6% per annum in GBP. The principals in the two currencies are 12 million USD and 10 million GBP. Payments are exchanged every year, with one exchange having just taken place. The swap will last two more years. What is the value of the swap to the financial institution?

Reference: Slides Example 6