Hedging with Futures

Hedging Reduces Exposure to Risk Factor

- Hedging is used to eliminate exposure to a risk factor. An investor's exposure to a risk is the sensitivity of the investor's wealth to changes in the risk factor.
- Examples of risk factor include price of a commodity, stock price, and interest rate.

$$\begin{aligned} &\textit{Exposure of hedged position to a risk factor} \\ &\quad = \textit{Exposure of unhedged position to the risk factor} \\ &\quad + \textit{Exposure of hedge to the risk factor} \end{aligned}$$

- Typically, the exposure of unhedged position is positive or negative and the investors desires zero exposure to the risk factor.
- This is achieved by choosing a hedge whose exposure to the risk factor is exactly opposite to the exposure of the unhedged position.

Note: We are ignoring daily settlement of futures contracts. Daily settlement can cause differences in timing of the cash flows due to unhedged exposure and the cash flows due to futures. In the following example, we also assume that there is a single price in December. This ignores the possibility that the unhedged exposure and the hedge exposure may be to prices on different dates in December. We will discuss these issues later.

Example 1: An investor's wealth will decrease by $5 million for each $1 increase in (per barrel) price of oil in December. The investor can hedge this risk by entering into a hedge whose value will increase by $5 million for each $1 increase in price of oil in December. Suppose an oil futures contract exists to buy or sell 1 million barrels of oil in December. The oil futures contract value will increase by $1 million for each $1 increase in oil price in December.

$$Unhedged\ exposure\ of\ investor\ to\ oil\ price\ = -\$5\ million\ /\ \$1\ increase\ in\ December\ oil\ price$$

If the investor goes long *k* contracts,

$$\begin{aligned} &Hedged\ exposure\ of\ investor\ to\ oil\ price \\ &\qquad = -\$5\ million\ /\ \$1\ increase\ in\ December\ oil\ price \\ &\qquad + \ k\ \times\ \$1\ million\ /\ \$1\ increase\ in\ December\ oil\ price \end{aligned}$$

To completely hedge exposure to oil price (perfect hedge), the investor should choose *k*=5. That is, the investor should go long 5 futures contracts.

Example 2: A US investor expects to receive 100 million euros in July. All of investor's other cash flows are in US dollars and the investor will convert euros to dollars. The investor has an exposure to the exchange rate between euros and dollars in July.

$$Unhedged\ exposure\ of\ investor\ =\ \$100\ million\ /\ \$1\ increase\ in\ price\ of\ euros\ in\ July$$

For a perfect hedge, the hedge position should have the opposite exposure, that is, it should result in a loss of $100 million for each $1 increase in July euro price.

Suppose there are euro futures contracts such that each contract allows one to buy/sell 10 million euros in July.

If the investor goes long k futures contracts,

$$\begin{aligned} &Hedged\ exposure\ of\ investor \\ &\qquad = \$100\ million/\ \$1\ increase\ in\ price\ of\ euros\ in\ July \\ &\qquad + k \times \$10\ million/\ \$1\ increase\ in\ price\ of\ euros\ in\ July \end{aligned}$$

The investor can achieve perfect hedge by choosing *k*=-10. That is, the investor should short 10 futures contracts. Again, we are ignoring the timing of cash flows and ignoring price variation in July.

Hedging with Long and Short Futures Positions

- Hedging with a short futures position is typically used when the hedger has the underlying asset or expects to get the underlying asset and plans to sell it.
    - The hedger can use the futures contract to lock in a price at which the hedger can deliver the asset.
    - However, it is more common for the hedger to sell in the spot market and close the futures position without delivering.
    - That is, the futures position is used to change the financial exposure, but the costs associated with delivery are avoided.

- Hedging with a long futures position is often used when the hedger plans to buy the underlying asset in the future.
    - By using the futures contract, the hedger can lock-in the purchase price of the asset.
    - However, usually the hedger buys in the spot market and closes the futures position before the delivery period starts to avoid the costs associated with accepting delivery.

Net Impact of Hedging

Example 3: Consider an investor who has an asset at current time t and wants to sell the asset by a future time T. Suppose a futures contract is available on the asset with maturity T. Also assume there are no storage costs or benefits of holding the asset.

Some alternative hedging strategies and their associated payoffs their associated payoffs at maturity are:

Strategy	Payoff at time T
Sell in spot market at T	S_T
Sell in spot market today and invest proceeds	$S_t\ plus\ interest$
Short futures and deliver at T	F_t
Short futures, close and sell in spot market at T	$F_t - F_T + S_T = F_t$

- The first strategy leaves the investor exposed to price risk.
- All other strategies achieve hedging as the payoff is determined in advance.
- The second strategy of selling today may not work if the asset is not available today or if there is any benefit from holding the asset.
- The third and the fourth strategies result in the same payoff but the third strategy may have additional costs associated with delivery in the futures market.

Example 4: Consider an investor who doesn't have an asset at current time t but needs the asset by a future time T. Suppose a futures contract is available on the asset with maturity T. Also assume there are no storage costs or benefits of holding the asset.

Some alternative hedging strategies and their associated payoffs their associated payoffs at maturity are:

Strategy	**Cost at time T**
Buy in spot market at T	S_T
Buy in spot market today by borrowing money	$S_t\ plus\ interest$
Long futures and accept delivery at T	F_t
Long futures, close and buy in spot market at T	$F_t - F_T + S_T = F_t$

- The first strategy leaves the investor exposed to price risk.
- All other strategies achieve hedging as the cost is determined in advance.
- The second strategy of buying today may have an advantage if there is a benefit of holding the asset and a disadvantage if there is a cost of holding the asset.
- The third and the fourth strategies result in the same cost but the third strategy may have additional costs associated with accepting delivery in the futures market.

Hedging Can Lead to Worse Outcomes

- Hedging is used to reduce risk, not increase profitability.
- The risk refers to sensitivity of the investor's value/wealth/objective to a risk factor.
- Hedging reduces the sensitivity in both directions: it not only reduces the impact of adverse outcomes but also the impact of favorable outcomes.
- Hedged value can be higher or lower than the unhedged value.
- The possibility of a decline in value from hedging is the cost paid to buy the possibility of an increase in value from hedging.
- No free lunch: one cannot hedge to get only favorable impact of hedging without the unfavorable impact of hedging.
- To get only gain but no loss, one can buy insurance at a cost.

The New York Times

INTERNATIONAL BUSINESS

Southwest Airlines gains advantage by hedging on long-term oil contracts

By JEFF BAILEY NOV. 28, 2007

The Motley Fool | Latest Stock Picks | Stocks | How to Invest | Retirement | Community | Personal Finance | Ticker or Keyword

Southwest Airlines Gets Burned By Fuel Hedges -- Again

The discount carrier expects its profit margin to decline on a year-over-year basis next quarter -- primarily because of its fuel hedging losses.

Adam Levine-Weinberg (TMFGemHunter)
Jul 25, 2016 at 8:35AM

To Hedge or Not to Hedge

Hedging mitigates risk exposure but does not increase expected profits.

- Risks can be mitigated through hedging or in other ways such as switching jobs, changing investment portfolio, etc.
- Hedge when hedging is more effective than other risk management options.
- One rationale for hedging is that the owners of the organization are risk-averse and hedging by the organization can mitigate the risk exposure of its owners.
- This argument is persuasive for owners of private businesses whose investment in the organization may be a large fraction of their wealth.

- In public firms, shareholders can diversify their investments so the rationale for hedging is not so obvious.
- What is shareholders can't diversify? Suppose a risk factor is systematic (e.g., oil price or interest rates).
- Even in this case, the rationale for hedging is not obvious. Shareholders earn a return premium for bearing systematic risk. If the organization hedges and transfer the systematic risk from shareholders to someone else, it avoids paying the risk premium to shareholders but must pay that risk premium for hedging.

- Hedging can create value if it helps the organization prevent costs that it may incur with adverse outcomes. These costs could include costs of bankruptcy proceedings or the costs of inefficient business actions taken in times of adversity to conserve cash.

How Much To Hedge

- Hedging decisions should be preceded by proper identification of risk exposure.
- The exposure should be based on *aggregate* cash flows rather than a specific investment.
- For example, if one division performs well when oil price increases and another does well when the oil price decreases, it is inefficient to hedge exposure of individual divisions as the two risk exposures offset each other to some extent.

- In competitive industries, hedging decision must also depend on the hedging policies of competitors.
- Suppose most competitors hedge the cost of their inputs.
    - These competitors are immune to changes in input costs so output prices do not change in response to changes in input costs.
    - A firm in this industry that does not hedge input costs will be exposed to changes in input costs without offsetting changes in output prices.
    - This firm should hedge to eliminate exposure to input costs.
- Now suppose most competitors do not hedge.
    - To maintain their profitability, these competitors will pass on the changes in input costs to output prices.
    - That is, input and output prices rise and fall at the same time.
    - If a firm in this industry hedges input costs, when input costs rise, its profits will increase due to increase in output prices.
    - Its profits will fall when input prices decrease.
    - The firm can prevent this unusual risk exposure by abandoning hedging.

Choice of contract

- Choose a delivery month that is as close as possible to, but later than, the end of the life of the hedge.
- When there is no futures contract on the asset being hedged, choose the contract whose futures price is most highly correlated with the asset price.
- Choose the correct number of contracts.

Cross-Hedging

- The exposure to an asset need not be a direct long or short position in an asset. The exposure may arise because the firm's cash flows are expected to change as asset value changes.
- For example, airlines hedge with heating oil futures even though they use jet fuel rather than heating oil. But when heating oil price changes, jet fuel price is also expected to change and impact airline profits.
- In such situations, the cash flows from the hedge are correlated with but do not perfectly offset the cash flows from the unhedged exposure.
- This kind of hedging is called cross-hedging.

Basis Risk

- When an investor hedges a position by taking an opposite position in a derivatives contract, the investor's net exposure depends on the difference between the value S_t of the underlying asset to which the investor is exposed and the value V_t of the derivative contract.
- The difference $S_t - V_t$ is called the basis (b_t) for the hedge.
- Ignoring the timing of cash flows due to daily settlement, the investor's hedged exposure depends on the value of the basis at maturity: $b_T = S_T - V_T$.

- If exposure to an asset is hedged using futures contracts on the same asset, the derivative price V_t equals the futures price F_t which converges to spot price at maturity ($F_T = S_T$) and basis equals zero.
- This results in a perfect hedge:
  - if spot price is higher, futures price will be higher as well
  - if spot price is lower, futures price will be lower as well
  - basis will become zero.

- Even if basis is not zero but is a fixed non-zero value known in advance, hedging is perfect.
  - For example, if it is known that basis at maturity will equal 2, then $V_T = S_T + 2$ so changes in V_T equal the changes in S_T.

- Hedging is imperfect when there is uncertainty about the value of the basis.
- In this case, the value of the hedged position depends on $b_T = S_T - V_T$ which is uncertain.
- This uncertainty is called basis risk. Some reasons for basis risk are:
  1. The asset underlying the hedge derivative is different from the asset whose risk is being hedged
  2. There is uncertainty about the date of the unhedged exposure – that is the date on which the underlying asset will be bought or sold
  3. The derivative maturity date does not match the date of the unhedged exposure. For example, the hedge needs to be closed before the delivery date.

- Note a change in basis over time is not necessarily basis risk.
  - For example, if basis increases over time but the final value of the basis at maturity is known, there is no basis risk.
  - For some futures contracts, the futures price exceeds the spot price before maturity. In this case, the basis is positive but converges to zero at maturity. A decrease in basis over time is called weakening of the basis.
  - For some other futures contracts, the basis is negative before maturity but converges to zero at maturity. An increase in basis over time is called strengthening of the basis.
  - Changes in basis over time can result in cash inflows or outflows before maturity but do not be themselves indicate basis risk.

Hedging with Long Futures

Let F_t, S_t, and b_t be futures price, spot price, and basis, respectively at time t. Time 0 options to get asset at time T are:

1. Buy now at S_0.
   - No price risk ✓
   - Need cash now ×
   - Asset maybe perishable or there may be storage costs ×
   - Asset may not be available now ×
   - Cost: $\boldsymbol{S_0 e^{rT}}$ (ignoring storage cost or yield)
2. Wait to buy at time T
   - Funds are not tied ✓
   - No transaction costs incurred for risk management ✓
   - Can get what/when is needed ✓
   - Price risk ×
   - Cost: $\boldsymbol{S_T}$
3. Go Long Futures
   - Funds not tied (except margin) ✓
   - Can get asset when needed ✓
   - Price risk is hedged ✓
   - Basis risk remains ×
   - Short-term price movements can lead to liquidity risk ×
   - Cost (cost of buying the asset minus the gain on futures): $\boldsymbol{S_T - (F_T - F_0) = F_0 + b_T}$.

Hedging with Short Futures

Now, consider someone at time 0 who needs to sell an asset at time T. The alternatives are:

1. Sell now at S_0.
   - No price risk ✓
   - Seller may not have cash now ✗
   - May need asset between now and time T ✗
   - Asset may not be available now ✗
   - Price: $\boldsymbol{S_0 e^{rT}}$ (if asset is available for sale)
2. Wait to sell at time T
   - Asset is sold when it is available for sale ✓
   - No transaction costs incurred for risk management ✓
   - Price risk ✗
   - Price: $\boldsymbol{S_T}$
3. Go Short Futures
   - Asset is sold when it is available for sale ✓
   - Price risk is hedged ✓
   - Basis risk remains ✗
   - Short-term price movements can lead to liquidity risk ✗
   - Price (price of asset minus loss on futures): $\boldsymbol{S_T - (F_T - F_0) = F_0 + b_T}$.

Example 5: It is March 1. You will get ¥50M at July end and need to sell them. Yen futures contracts are available for delivery in March, June, September, and December. Each is for ¥12.5M. You short 4 futures contracts. Calculate effective price you obtained in the following cases:

1. F_0=0.78 cents/yen, S_T =0.72 cents/yen, F_T=0.725 cents/yen
2. F_0=0.78 cents/yen, S_T =0.77 cents/yen, F_T=0.77 cents/yen
3. F_0=0.78 cents/yen, S_T =0.82 cents/yen, F_T=0.815 cents/yen

Effective Price:

	1	2	3
Unhedged:	0.72	0.77	0.82
Hedged:	0.775	0.78	0.785

Optimal hedging with no basis risk

- If a hedger hedges too little, the investor will remain exposed to most of the original risk.
- If a hedger hedges too much, the hedge position itself will create new risk exposure for the hedger.
- We are interested in minimizing the risk of the hedged position.

- If there is no basis risk (the asset being hedged matches exactly the asset underlying the futures contract and the maturity of the contract matches the timing of the unhedged risk exposure), then

$$Optimal\ number\ of\ futures\ contracts = -\frac{Unhedged\ exposure\ in\ number\ of\ units\ of\ asset}{Number\ of\ units\ of\ asset\ in\ futures\ contract}$$

- If the unhedged exposure is positive (long), the above formula results in a negative number which indicates a short position in futures contracts.
- If the unhedged exposure is negative (short), the above formula results in a positive number which indicates a long position in futures contract.

Optimal hedging with basis risk

With basis risk, the unhedged risk differs from the risk underlying the futures contract. Consider the notation:

Q_A	Size of unhedged exposure in units of asset A
Q_F	Size of futures contract in units of asset F
P_A	Price of one unit of asset A
P_F	Price of one unit of asset F
σ_A	Standard deviation of P_A
σ_F	Standard deviation of P_F
ρ	Correlation between P_A and P_F

If N futures contracts are used for hedging, the value of the hedged position of the investor is

$$Q_A P_A + N Q_F P_F$$

The optimal value of N minimizes the variance (or the standard deviation) of the above value which equals

$$Var[Q_A P_A + N Q_F P_F] = Q_A^2 \sigma_A^2 + N^2 Q_F^2 \sigma_F^2 + 2\rho N Q_A Q_F \sigma_A \sigma_F$$

The right-hand side of the above expression can be written as

$$(1-\rho^2) Q_A^2 \sigma_A^2 + (N Q_F \sigma_F + \rho Q_A \sigma_A)^2$$

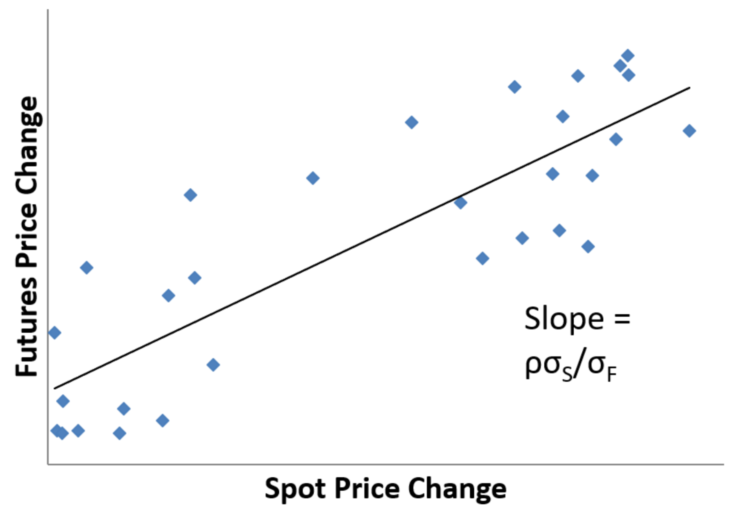
In the above expression, the first term does not depend on N. The second term is minimized to zero at optimal value of N given by

$$N^* = -\rho \frac{\sigma_A}{\sigma_F} \frac{Q_A}{Q_F}$$

(1)

Observations:

1. Use more contracts (absolute value) for larger or more volatile unhedged position.
2. Use fewer contracts (absolute value) with larger or more volatile futures contract.
3. Use more contracts (absolute value) if prices of unhedged position and futures contract are more correlated.
4. The quantity $\rho\frac{\sigma_A}{\sigma_F}$ is the expected change in the unhedged position's price for a unit change in the futures price. It can be estimated as the coefficient in a regression of the changes in unhedged position's price on the changes in the futures price. It is the slope of the best-fit line in a graph of historical data where the vertical axis represents the changes in the price of the unhedged position and the horizontal axis represents the changes in futures price. The price changes should be for the same duration as the duration of the exposure.



Example 6: It is January 4, 2021. Suppose Southwest Airlines wants to hedge its jet fuel exposure using NYMEX heating oil futures. The quarterly jet fuel consumption of Southwest in a typical year is about 500 million gallons (see https://www.transtats.bts.gov/fuel.asp) but was about 300 million gallons in 2020. Assume Southwest wants to hedge price of 400 million gallons of jet fuel at March-end using heating oil futures with delivery month of April 2021. Southwest plans to close the futures position at the end of March. Heating oil futures contract is for 42,000 gallons of heating oil (https://www.cmegroup.com/trading/energy/refined-products/heating-oil_contract_specifications.html). The current spot price of jet fuel is $1.312 per gallon while the current futures price is $1.464 per gallon.

- Here, Q_A = -400 million gallons of jet fuel (because Southwest needs to buy jet fuel), Q_F = 42,000 gallons of heating oil, P_A = spot price / gallon of jet fuel, and P_F = futures price / gallon of heating oil.
- We estimate σ_A, σ_F, and ρ using historical data.
- We use previous 40 quarters of data (2011-2020).
- For each quarter, we need the change in jet fuel spot price (downloaded from https://www.eia.gov/dnav/pet/hist/eer_epjk_pf4_rgc_dpgD.htm) and the change in heating oil futures price (downloaded from https://www.eia.gov/dnav/pet/pet_pri_fut_s1_d.htm).[1]

[11] Futures prices are given as four series representing the four earliest delivery months: Contract 1, Contract 2, Contract 3, and Contract 4. The January-March price change is calculated as the price change for the April delivery contract using Contract 3 price at the beginning of January and Contract 1 price at the end of March.

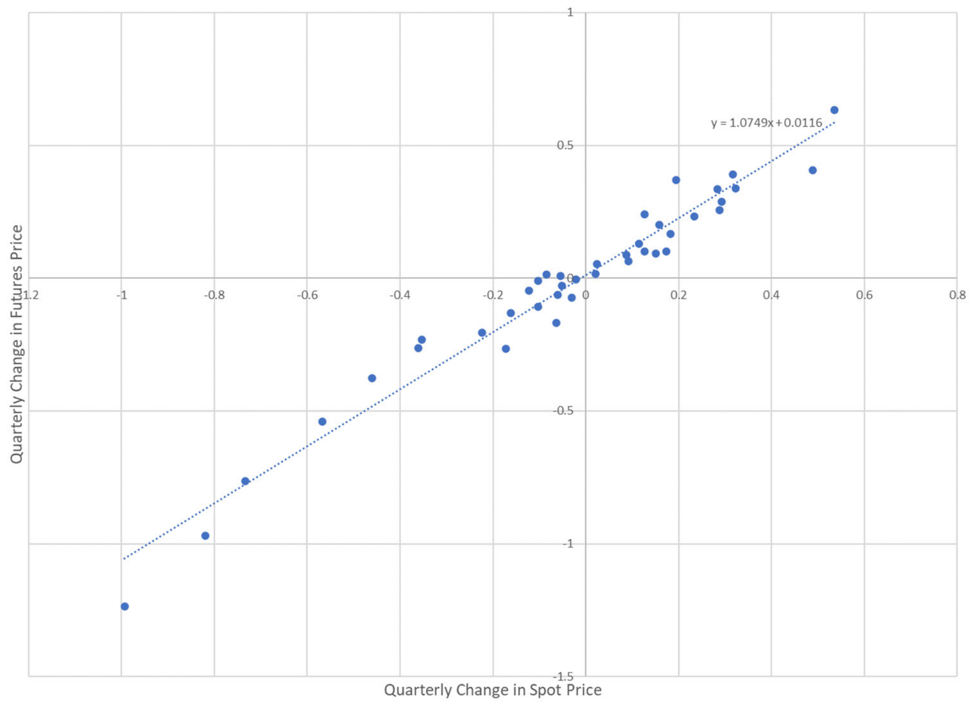
Quarter	Initial Spot	Final Spot	Initial Futures	Final Futures	Change in Spot Price	Change in Futures Price
2011Q1	2.528	3.161	2.554	3.09	0.633	0.536
2011Q2	3.202	2.996	3.157	2.933	-0.206	-0.224
2011Q3	2.976	2.844	2.956	2.795	-0.132	-0.161
2011Q4	2.815	2.917	2.744	2.918	0.102	0.174
2012Q1	3.04	3.242	3.009	3.168	0.202	0.159
2012Q2	3.315	2.774	3.263	2.696	-0.541	-0.567
2012Q3	2.749	3.156	2.681	3.169	0.407	0.488
2012Q4	3.151	2.984	3.108	3.045	-0.167	-0.063
2013Q1	2.993	2.982	3.017	2.915	-0.011	-0.102
2013Q2	3.002	2.737	3.052	2.88	-0.265	-0.172
2013Q3	2.754	2.842	2.883	2.971	0.088	0.088
2013Q4	2.773	3.014	2.95	3.077	0.241	0.127
2014Q1	2.935	2.861	2.962	2.932	-0.074	-0.03
2014Q2	2.809	2.874	2.879	2.971	0.065	0.092
2014Q3	2.858	2.627	3	2.647	-0.231	-0.353
2014Q4	2.636	1.666	2.666	1.847	-0.97	-0.819
2015Q1	1.559	1.567	1.772	1.718	0.008	-0.054
2015Q2	1.598	1.727	1.772	1.887	0.129	0.115
2015Q3	1.685	1.422	1.873	1.513	-0.263	-0.36
2015Q4	1.403	1.026	1.562	1.101	-0.377	-0.461
2016Q1	1.039	1.056	1.164	1.185	0.017	0.021
2016Q2	1.005	1.396	1.169	1.485	0.391	0.316
2016Q3	1.424	1.418	1.549	1.528	-0.006	-0.021
2016Q4	1.451	1.552	1.578	1.704	0.101	0.126
2017Q1	1.528	1.481	1.697	1.574	-0.047	-0.123
2017Q2	1.471	1.364	1.579	1.476	-0.107	-0.103
2017Q3	1.351	1.686	1.528	1.812	0.335	0.284
2017Q4	1.647	1.904	1.764	2.052	0.257	0.288
2018Q1	1.903	1.957	2.004	2.028	0.054	0.024
2018Q2	1.916	2.148	1.976	2.209	0.232	0.233

2018Q3	2.1	2.267	2.17	2.352	0.167	0.182
2018Q4	2.327	1.564	2.414	1.681	-0.763	-0.733
2019Q1	1.607	1.896	1.681	1.973	0.289	0.292
2019Q2	1.916	1.888	1.997	1.945	-0.028	-0.052
2019Q3	1.915	1.852	1.967	1.906	-0.063	-0.061
2019Q4	1.85	1.942	1.877	2.028	0.092	0.151
2020Q1	1.934	0.697	2.005	1.012	-1.237	-0.993
2020Q2	0.65	1.02	0.984	1.178	0.37	0.194
2020Q3	1.025	1.038	1.229	1.145	0.013	-0.084
2020Q4	1.003	1.34	1.153	1.476	0.337	0.323

The standard deviation and correlations are:

Standard deviation of Spot Price	0.36665
Standard deviation of Futures Price	0.33384
Correlation	0.97872

A scatterplot follows.

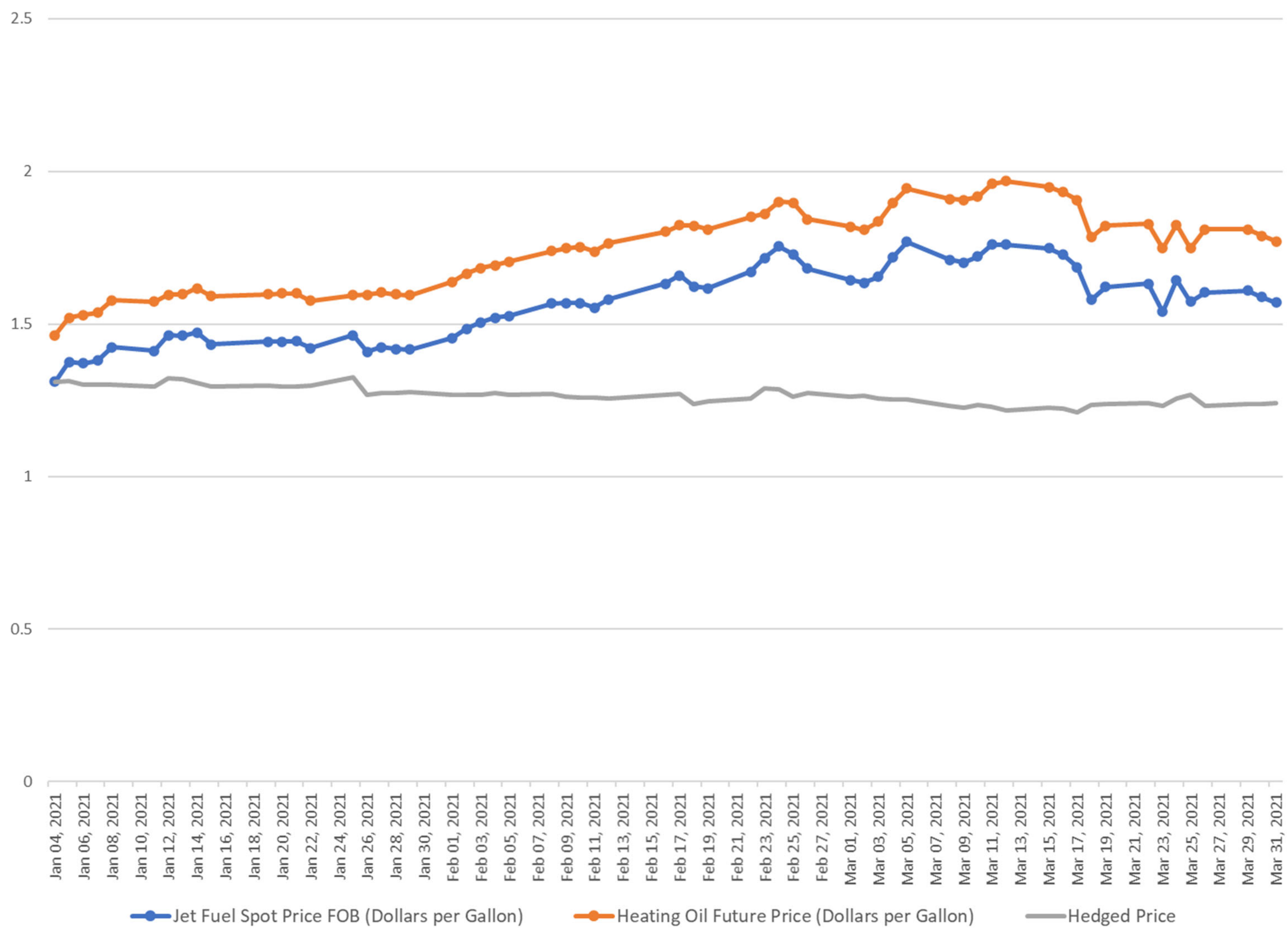


- The number of futures contracts needed for optimal hedging is

$$N^* = -\rho \frac{\sigma_A}{\sigma_F}\frac{Q_A}{Q_F} = -0.97872 \times \frac{\$0.36665 \ / \ gallon\ of\ jet\ fuel}{\$0.33384 \ / \ gallon\ of\ heating\ oil} \times \frac{-400{,}000{,}000\ gallons\ of\ jet\ fuel}{42{,}000\ gallons\ of\ heating\ oil}$$
$$= 10{,}237$$

- Let us evaluate the performance of this hedge using actual prices in March 2021.
- The spot and futures price at the end of March 2021 were \$1.571 per gallon of jet fuel and \$1.771 per gallon of heating oil, respectively.
- Gain on futures position = $(\$1.771 - \$1.464) \times 42{,}000 \times 10{,}237 = \$131.996\ million$
- Effective price paid for jet fuel = $\frac{\left(\frac{\$1.571}{gallon} \times 400\ million\ gallons - \$131.996\ million\right)}{400\ million\ gallons} = \$1.241\ per\ gallon$

The following chart shows the movement of spot and futures prices moved between January and March and the effective hedged price.



In Example 6, we calculated volatility and correlations using historical price changes. This method assumes that the future price changes will have the same distribution as past price changes. An alternative assumption commonly seen in financial data is that the distribution of percentage price changes, rather than absolute price changes, is stable over time.

Example 7: We reconsider the problem in Example 6 by calculating volatilities and correlations using percentage price changes. The percent price change is calculated as the logarithm of the ratio of final price to initial price. The price change of interest is the price change in $ per $ of initial price, rather than per gallon. Therefore, the unit of measurement is dollars rather than gallons.

- The size of exposure must now be measured in dollars rather than gallons.
- Here, Q_A = -400 million gallons of jet fuel × \$1.312 / gallon of jet fuel = -\$524.8 million, Q_F = 42,000 gallons of heating oil × \$1.464 / gallon of heating oil = \$61,488, P_A = spot price of jet fuel currently worth \$1, and P_F = futures price of heating oil currently worth \$1.
- We estimate σ_A, σ_F, and ρ using historical data:

Quarter	Initial Spot	Final Spot	Initial Futures	Final Futures	% Change in Spot Price	% Change in Futures Price
2011Q1	2.528	3.161	2.554	3.09	0.2235	0.1905
2011Q2	3.202	2.996	3.157	2.933	-0.0665	-0.0736
2011Q3	2.976	2.844	2.956	2.795	-0.0454	-0.0560
2011Q4	2.815	2.917	2.744	2.918	0.0356	0.0615
2012Q1	3.04	3.242	3.009	3.168	0.0643	0.0515
2012Q2	3.315	2.774	3.263	2.696	-0.1782	-0.1909
2012Q3	2.749	3.156	2.681	3.169	0.1381	0.1672
2012Q4	3.151	2.984	3.108	3.045	-0.0545	-0.0205
2013Q1	2.993	2.982	3.017	2.915	-0.0037	-0.0344
2013Q2	3.002	2.737	3.052	2.88	-0.0924	-0.0580

2013Q3	2.754	2.842	2.883	2.971	0.0315	0.0301
2013Q4	2.773	3.014	2.95	3.077	0.0833	0.0421
2014Q1	2.935	2.861	2.962	2.932	-0.0255	-0.0102
2014Q2	2.809	2.874	2.879	2.971	0.0229	0.0315
2014Q3	2.858	2.627	3	2.647	-0.0843	-0.1252
2014Q4	2.636	1.666	2.666	1.847	-0.4588	-0.3670
2015Q1	1.559	1.567	1.772	1.718	0.0051	-0.0309
2015Q2	1.598	1.727	1.772	1.887	0.0776	0.0629
2015Q3	1.685	1.422	1.873	1.513	-0.1697	-0.2134
2015Q4	1.403	1.026	1.562	1.101	-0.3129	-0.3497
2016Q1	1.039	1.056	1.164	1.185	0.0162	0.0179
2016Q2	1.005	1.396	1.169	1.485	0.3286	0.2393
2016Q3	1.424	1.418	1.549	1.528	-0.0042	-0.0136
2016Q4	1.451	1.552	1.578	1.704	0.0673	0.0768
2017Q1	1.528	1.481	1.697	1.574	-0.0312	-0.0752
2017Q2	1.471	1.364	1.579	1.476	-0.0755	-0.0675
2017Q3	1.351	1.686	1.528	1.812	0.2215	0.1705
2017Q4	1.647	1.904	1.764	2.052	0.1450	0.1512
2018Q1	1.903	1.957	2.004	2.028	0.0280	0.0119
2018Q2	1.916	2.148	1.976	2.209	0.1143	0.1115
2018Q3	2.1	2.267	2.17	2.352	0.0765	0.0805
2018Q4	2.327	1.564	2.414	1.681	-0.3973	-0.3619
2019Q1	1.607	1.896	1.681	1.973	0.1654	0.1602
2019Q2	1.916	1.888	1.997	1.945	-0.0147	-0.0264
2019Q3	1.915	1.852	1.967	1.906	-0.0335	-0.0315
2019Q4	1.85	1.942	1.877	2.028	0.0485	0.0774
2020Q1	1.934	0.697	2.005	1.012	-1.0206	-0.6837
2020Q2	0.65	1.02	0.984	1.178	0.4506	0.1799
2020Q3	1.025	1.038	1.229	1.145	0.0126	-0.0708
2020Q4	1.003	1.34	1.153	1.476	0.2897	0.2470

The standard deviation and correlations are:

Standard deviation of Spot % Change	0.23783
Standard deviation of Futures % Change	0.18330
Correlation	0.96582

- The number of futures contracts needed for optimal hedging is
- $N^* = -\rho \frac{\sigma_A}{\sigma_F} \frac{Q_A}{Q_F} = -0.96582 \times \frac{\$0.23783 \; / \; \$1}{\$0.18330 \; / \; \$1} \times \frac{-\$524{,}800{,}000}{\$61{,}488} = 10{,}696$
- The optimal number of contracts is slightly different from but close to the value we calculated in Example 6.
- The difference can be large if the current ratio of prices is far from the historical average.
- In such cases, it is preferable to use the methodology of Example 7.
- The disadvantage of this approach is that the optimal number of contracts depends on current prices and changes over time as prices change.
- In practice, this can be ignored as long as price changes are small.

Stock Index Futures

- A stock index tracks changes in the value of a hypothetical portfolio of stocks.
- Usually, the stock index tracks only the capital gains/losses of the portfolio, not dividends paid by stocks in the portfolio.
- Stock index futures allow investors to hedge or speculate on performance of stock indices.
- Some popular stock index futures contracts are based on Dow Jones Industrial Average, S&P 500, and NASDAQ-100.

Hedging an Equity Portfolio

- When stock index futures are used to hedge an equity portfolio, the volatilities and correlation of the equity portfolio and the futures contract are calculated using percentage changes.
- To calculate the number of contracts for optimal hedging in equation (1),
  - the portfolio size Q_A and futures size Q_F are measured in dollars
  - σ_A and $\sigma_F$are the standard deviations of returns of the portfolio and the stock index, respectively, and
  - ρ is the correlation between the return on the portfolio and the return on the stock index.
- The quantity $\rho \frac{\sigma_A}{\sigma_F}$ is called the beta of the equity portfolio when the stock index represents the entire stock market.
- Representing this quantity by the symbol β, the optimal number of contracts for hedging the equity portfolio is given by

$$N^* = -\beta \frac{Q_A}{Q_F} \tag{2}$$

- The optimal number of contracts is proportional to the portfolio beta because beta measures the risk in the equity portfolio that depends on the market movements (represented by the stock index).
- Movements of the equity portfolio that are unrelated to the market movements are unaffected by this hedging.
- Hedging a portfolio with stock index futures can allow an investor to place bets on individual stocks and avoid the risk of market movements.
- It can also be used to insulate a portfolio temporarily from market movements.

Changing the Beta of a Portfolio

- If an investor wants to partially hedge the market risk of an equity portfolio, the investor can reduce the effective beta of the portfolio from unhedged beta β to hedged beta β^*.
- The number of futures contracts needed for this hedging is given by

$$N^* = -(\beta - \beta^*) \frac{Q_A}{Q_F} \tag{3}$$

Example 8: In February, an investor holds a portfolio valued at $12 million and with a beta of 1.5. The investor wants to reduce the market risk of the portfolio by half using E-mini S&P 500 Futures. The most liquid contract is for the delivery month of March and the futures price is 3303.00. The futures contract is written on 50 times the S&P 500 index.

The number of futures contracts needed for hedging is

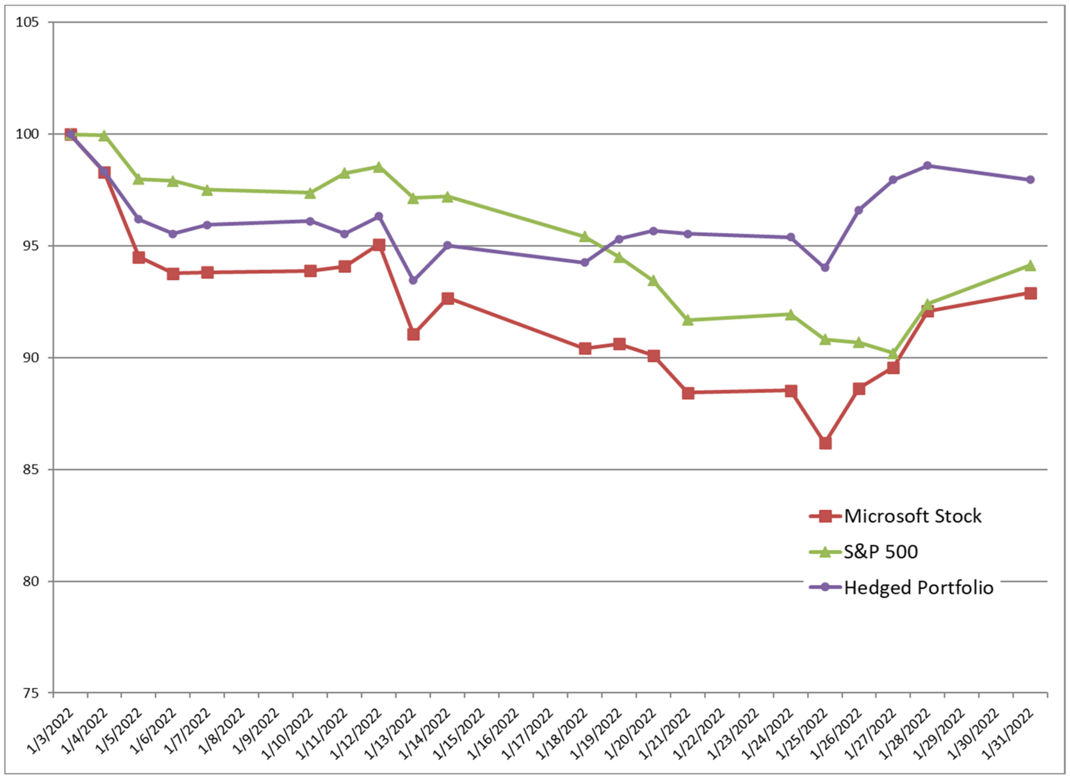
$$N^* = -(\beta - \beta^*)\frac{Q_A}{Q_F} = -(1.5 - 0.75)\frac{12{,}000{,}000}{50 \times 3303.00} = -54.5$$

The investor should short 54.5 futures contracts.

Example 9: Suppose on January 2, 2022, you wanted to hedge your holding of 20,000 Microsoft shares against the risk of aggregate stock price movements. You chose to short E-mini S&P500 futures March 2022 contract. Microsoft share price was \$334.75 and S&P 500 Futures price was 4786. Microsoft stock beta was 0.86 according to Yahoo! Finance.

The optimal hedge ratio is 0.86 and the number of futures that must be shorted is $0.86 \times (20{,}000 \times \$334.75)/(50 \times 4786) = 24.06$ rounded to 24.

The following chart shows how this hedge would have performed.



Reasons for Hedging Equity Returns

- An investor may want to be out of the market for a while.
- Hedging avoids the costs of selling and repurchasing the portfolio.
- Hedging can also be used to remove market risk from a portfolio.
- For example, you feel that the stocks in your portfolio will outperform the market in both good and bad times. But you have no particular opinion about how the market will move. In that case, you may not want to take on the market risk associated with the portfolio. Hedging to make your portfolio zero-beta ensures that the return you earn is the risk-free return plus the excess return of your portfolio over the market.

Stack and Roll

- When futures contracts with multiple delivery months are available, hedgers need to decide which contract to use for hedging.
- It is preferable to use a contract with delivery month that exactly coincides with the timing of the investors' risk exposure or the earliest delivery month following that.
- Using a contract with an earlier delivery month can leave the investor unhedged for some time and may also expose the investor to the risk of delivery.
- However, the transaction costs may be high for futures contracts with later delivery months.
- Some hedgers hedge using more liquid futures contracts by first stacking contracts with earlier delivery months and roll over their position.
- To roll over their position, they close the position in the contract with earlier delivery month and take a new position in the contract with the next delivery month.
- The implicit assumption here is that the prices of the futures contracts with different delivery months will move in a similar way.
- However, sometimes, short-term price and long-term price may deviate and stack-and-roll strategy may result in mismatch in the timing of the cash flows between the hedging strategy and the underlying risk.

Liquidity Issues

- In any hedging situation there is a danger that losses will be realized on the hedge while the gains on the underlying exposure are unrealized.
- This can create liquidity problems.
- One example is Metallgesellschaft which sold long term fixed-price contracts on heating oil and gasoline and hedged using stack and roll.
- The price of oil fell leading to margin calls on futures position.
- While the company expected to gain on its underlying exposure in the long-term, it was unable to absorb the short-term cash problems.