$\boxed{OCP}$

$$\boxed{\begin{aligned}&\min_{U \in \mathbb{R}^N} U^T U \\ &\text{s.t. } gsim(U) = 0\end{aligned}}$$

$gsim(U) = x_N$

Direct single shooting → sequential approach (only discretized controls in the NLP)

$$\boxed{\begin{aligned}\text{KKT}: \quad & 2U^* + \frac{\partial gsim}{\partial U}(U^*)^T \lambda^* = 0 \\ & gsim(U^*) = 0 \\ & (\mathcal{L}(U,\lambda) = U^T U + gsim(U)\lambda)\end{aligned}}$$

Projection of the objective gradient on constraint surface $= 0$!

Linearize (Newton approach)

$$\begin{bmatrix} 2U_k \\ gsim(U_k) \end{bmatrix} + \begin{bmatrix} H_L & Jsim(U_k)^T \\ Jsim(U_k) & 0 \end{bmatrix} \begin{bmatrix} U_{k+1} - U_k \\ \lambda_{k+1} \end{bmatrix} = 0$$

H_L → $2\mathbb{I}$ (Gauss Newton)
→ B_k (BFGS)

$$KKTRES_k = \left\| \begin{matrix} \nabla_U \mathcal{L}(U_k, \lambda_k) \\ gsim(U_k) \end{matrix} \right\| \quad \text{stopping criterion}$$

$$\boxed{\text{Integrator: } x_{k+1} = \Phi(x_k, u_k), \quad k = 0, \dots, N-1}$$

linearize along the simulated trajectory

$$\delta x_{k+1} = A_k \delta x_k + B_k \delta u_k, \quad k = 0, \dots, N-1$$

A_k → $\frac{\partial \Phi}{\partial x}(x_k, u_k)$

B_k → $\frac{\partial \Phi}{\partial u}(x_k, u_k)$

finite differences

$[x, A_{traj}, B_{traj}] = \text{forwardsweep}(U)$

$$\frac{\partial gsim}{\partial u_k}(U) = (A_{N-1}, A_{N-2}, \dots A_{k+1}) B_k$$

$Jsim = \text{backwardsweep}(A_{traj}, B_{traj})$