3.3 SECTION EXERCISES

VERBAL

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2. If a function f is increasing on (a, b) and decreasing on (b, c), then what can be said about the local extremum of f on (a, c)?

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ALGEBRAIC

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13. $j(x) = 3x^3$ on $[1, 1 + h]$

14. $r(t) = 4t^3$ on $[2, 2 + h]$

15. $\dfrac{f(x + h) - f(x)}{h}$ given $f(x) = 2x^2 - 3x$ on $[x, x + h]$

GRAPHICAL

For the following exercises, consider the graph of f shown in **Figure 15**.

16. Estimate the average rate of change from $x = 1$ to $x = 4$.

17. Estimate the average rate of change from $x = 2$ to $x = 5$.

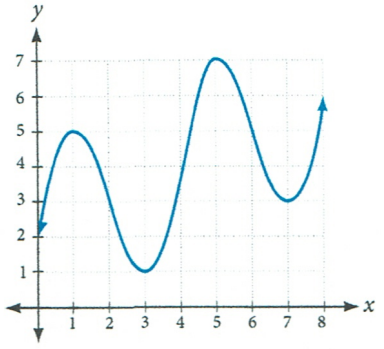
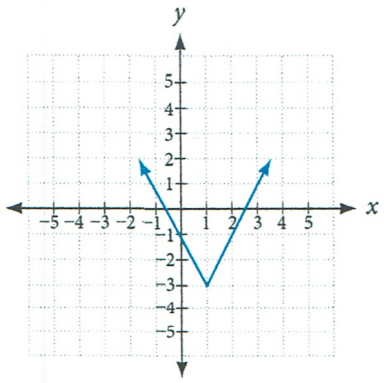


Figure 15

For the following exercises, use the graph of each function to estimate the intervals on which the function is increasing or decreasing.

18.



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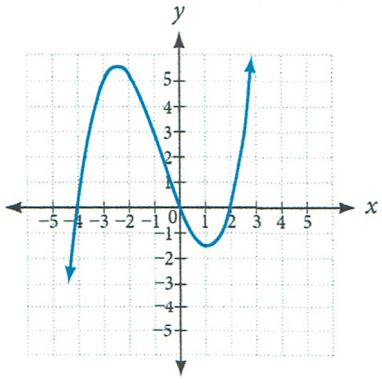

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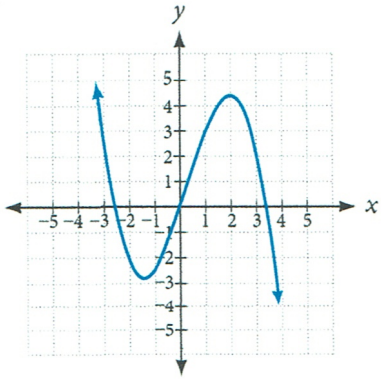

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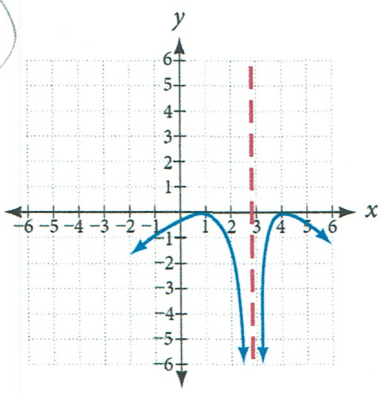

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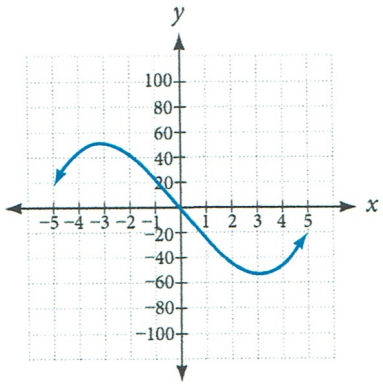

Figure 16

22. Estimate the intervals where the function is increasing or decreasing.

23. Estimate the point(s) at which the graph of f has a local maximum or a local minimum.

For the following exercises, consider the graph in **Figure 17.**

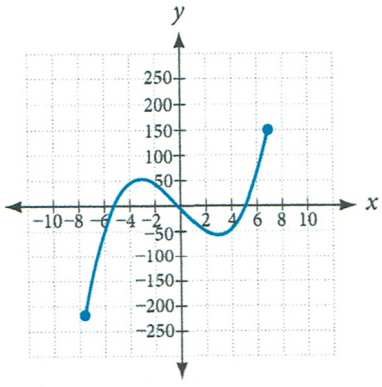

Figure 17

24. If the complete graph of the function is shown, estimate the intervals where the function is increasing or decreasing.

25. If the complete graph of the function is shown, estimate the absolute maximum and absolute minimum.

NUMERIC

26. **Table 3** gives the annual sales (in millions of dollars) of a product from 1998 to 2006. What was the average rate of change of annual sales (**a**) between 2001 and 2002, and (**b**) between 2001 and 2004?

Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Sales (millions of dollars)	201	219	233	243	249	251	249	243	233

Table 3

27. **Table 4** gives the population of a town (in thousands) from 2000 to 2008. What was the average rate of change of population (**a**) between 2002 and 2004, and (**b**) between 2002 and 2006?

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008
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Table 4

For the following exercises, find the average rate of change of each function on the interval specified.

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31. $g(x) = 3x^3 - 1$ on $[-3, 3]$

32. $y = \frac{1}{x}$ on $[1, 3]$

33. $p(t) = \frac{(t^2 - 4)(t + 1)}{t^2 + 3}$ on $[-3, 1]$

34. $k(t) = 6t^2 + \frac{4}{t^3}$ on $[-1, 3]$

TECHNOLOGY

For the following exercises, use a graphing utility to estimate the local extrema of each function and to estimate the intervals on which the function is increasing and decreasing.

35. $f(x) = x^4 - 4x^3 + 5$

36. $h(x) = x^5 + 5x^4 + 10x^3 + 10x^2 - 1$

37. $g(t) = t\sqrt{t + 3}$

38. $k(t) = 3t^{\frac{2}{3}} - t$

39. $m(x) = x^4 + 2x^3 - 12x^2 - 10x + 4$

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EXTENSION

41. The graph of the function f is shown in **Figure 18**.

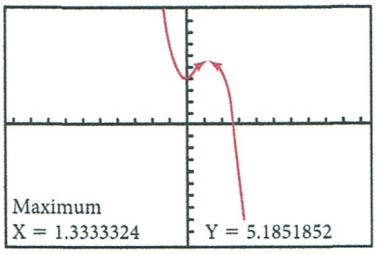

Figure 18

Based on the calculator screen shot, the point (1.333, 5.185) is which of the following?

a. a relative (local) maximum of the function
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42. Let $f(x) = \frac{1}{x}$. Find a number c such that the average rate of change of the function f on the interval $(1, c)$ is $-\frac{1}{4}$

43. Let $f(x) = \frac{1}{x}$. Find the number b such that the average rate of change of f on the interval $(2, b)$ is $-\frac{1}{10}$.

REAL-WORLD APPLICATIONS

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46. Near the surface of the moon, the distance that an object falls is a function of time. It is given by $d(t) = 2.6667t^2$, where t is in seconds and $d(t)$ is in feet. If an object is dropped from a certain height, find the average velocity of the object from $t = 1$ to $t = 2$.

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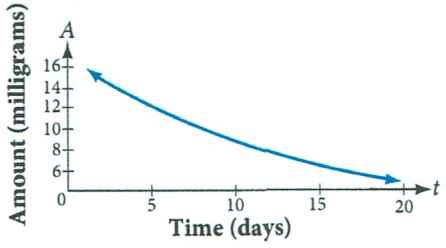

Figure 19

Use the graph to estimate the average decay rate from $t = 5$ to $t = 15$.

3.3 SECTION EXERCISES

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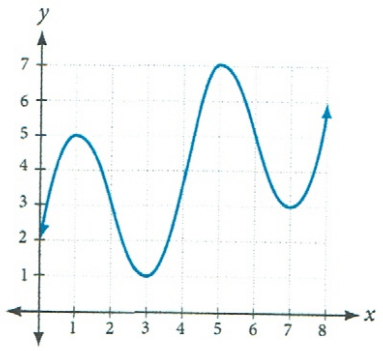

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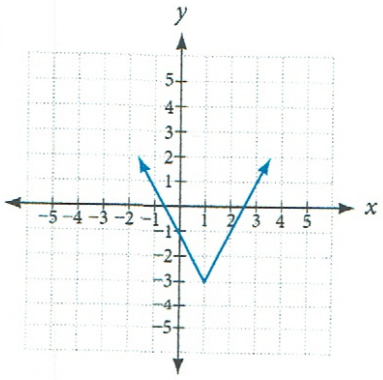

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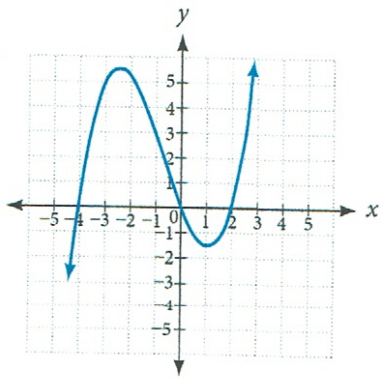

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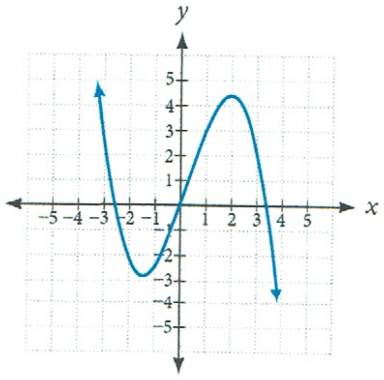

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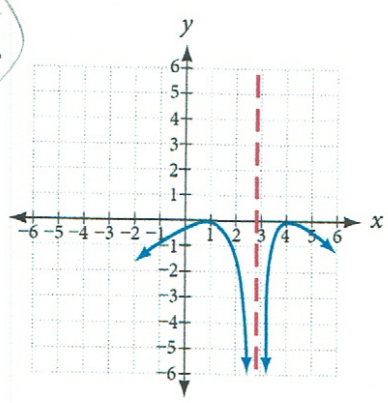

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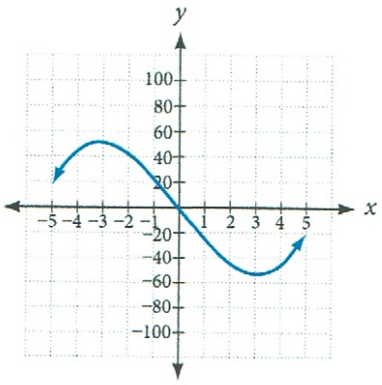

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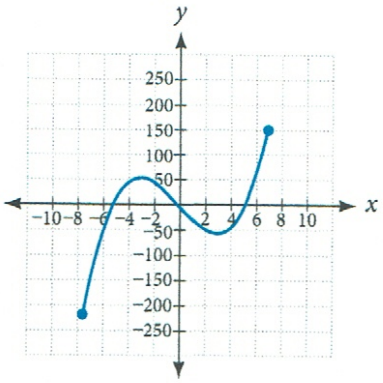

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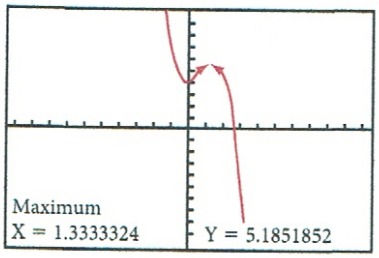

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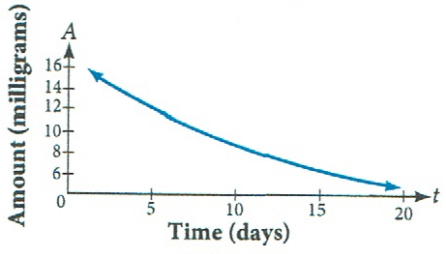

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