

Lecture 18: One-Sample Means With the *t*-Distribution

Chapter 5.3

Goals for Today

- ▶ What do we do when n is small?

Sample Size n

Verifying Normality of Population Distribution

When verifying normality for small n , it is important to not only examine the data but also think about where the data come from. For example, ask:

- ▶ Would I expect this distribution to be symmetric?
- ▶ Am I confident that outliers are rare?

t Distribution

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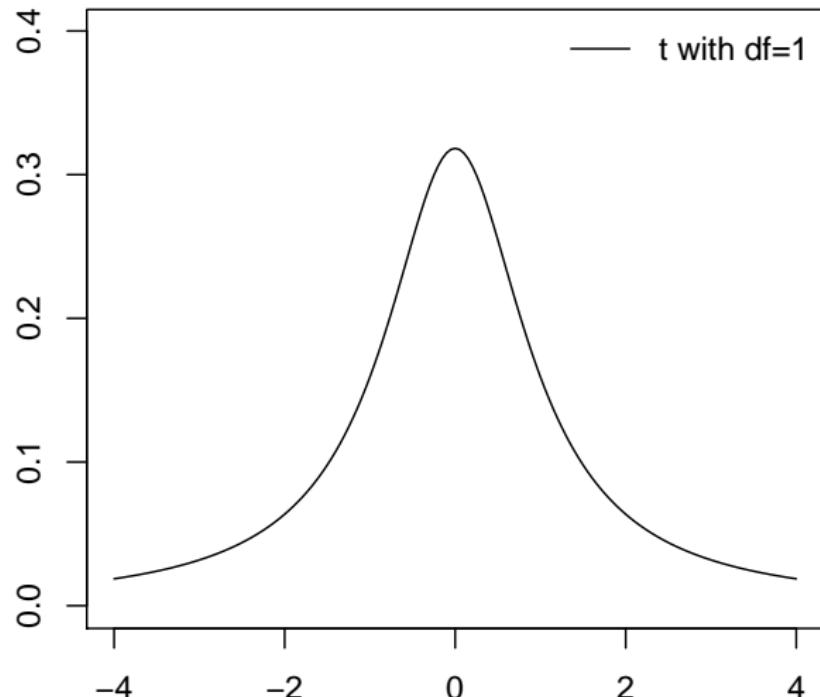
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i.e. it has fatter tails

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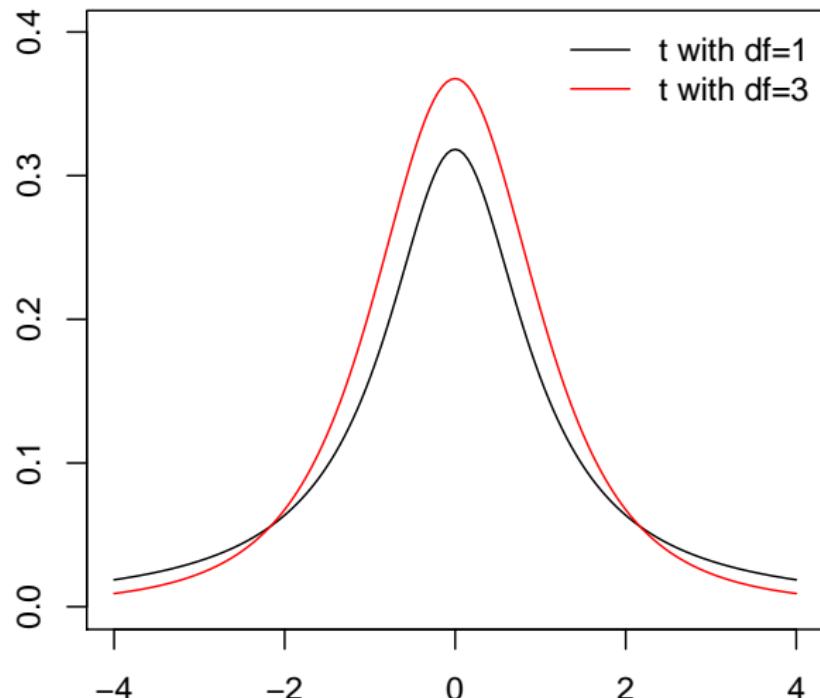
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- ▶ As the df goes to ∞ , the *t* curve approaches the *z* curve.

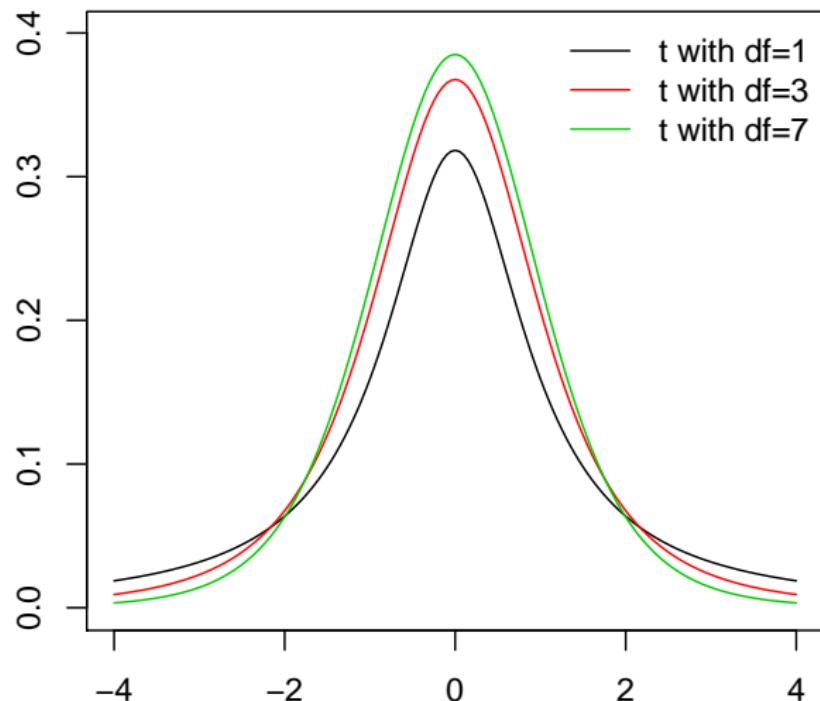
t Distribution Examples



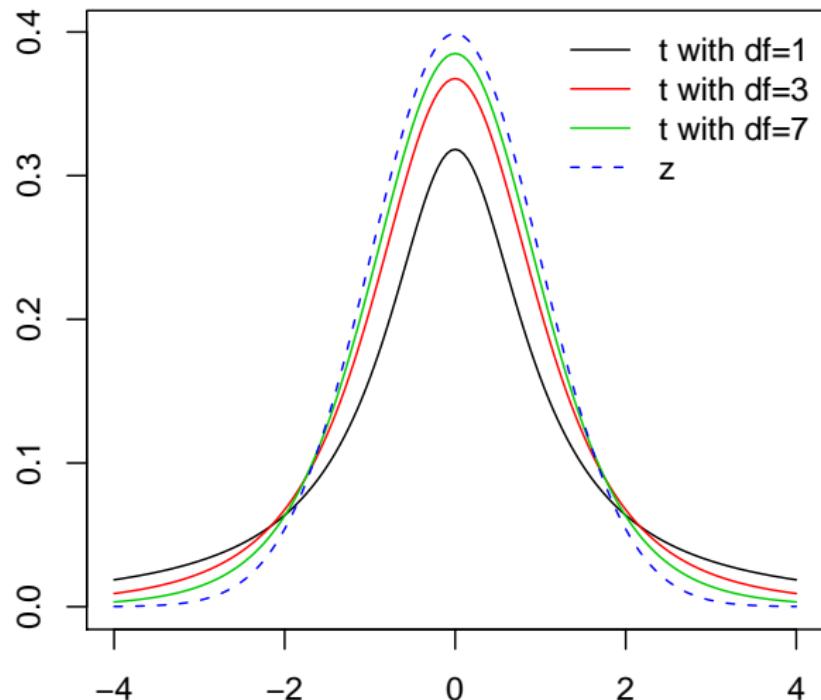
t Distribution Examples



t Distribution Examples



t Distribution Examples



Conditions for Using t Distribution

t-Tables

If $n = 11$, we use $df = 11 - 1 = 10$ and do a look up on the *t*-table on page 410:

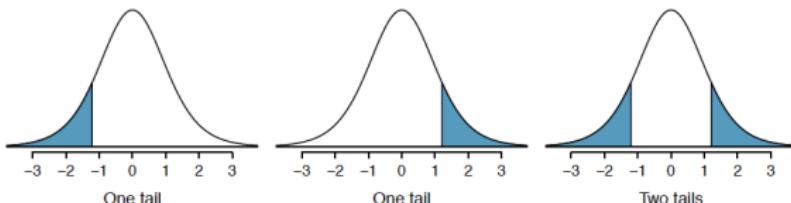


Figure B.1: Three *t* distributions.

	one tail	0.100	0.050	0.025	0.010	0.005
	two tails	0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11

Confidence Intervals

t-Test Example

Example 5.19 on page 252: A random sample of 25 New Yorkers were asked how much sleep they get per night. Does the data below provide strong evidence that New Yorkers sleep less than 8 hours a night on average? Set $\alpha = 0.10$

n	\bar{x}	s	min	max
25	7.73	0.77	6.17	9.78

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Decision: Since the *p*-value < $\alpha = 0.10$, we reject H_0 that NY'ers sleep 8 hours a night at the $\alpha = 0.10$ significance level in favor of the hypothesis they sleep more.

History of t Distribution

The t distribution was derived by William Sealy Gosset in 1908, a chemist/statistician at the Guinness Brewery in Dublin, Ireland.



History of t Distribution

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The t -test's complete name is the (Student's) t-test.

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In fact if you go to the Guinness Brewery at St James's Gate in Dublin, Ireland...



History of t Distribution

