Lecture 18: One-Sample Means With the t-Distribution

Chapter 5.3

Goals for Today

- What do we do when n is small?

Sample Size n

Verifying Normality of Population Distribution

Be cautious when verifying the normality condition for small n. It is important to not only examine the data but also think about where the data come from. For example, ask:

- Would I expect this distribution to be symmetric?
- Am I confident that outliers are rare?

t Distribution

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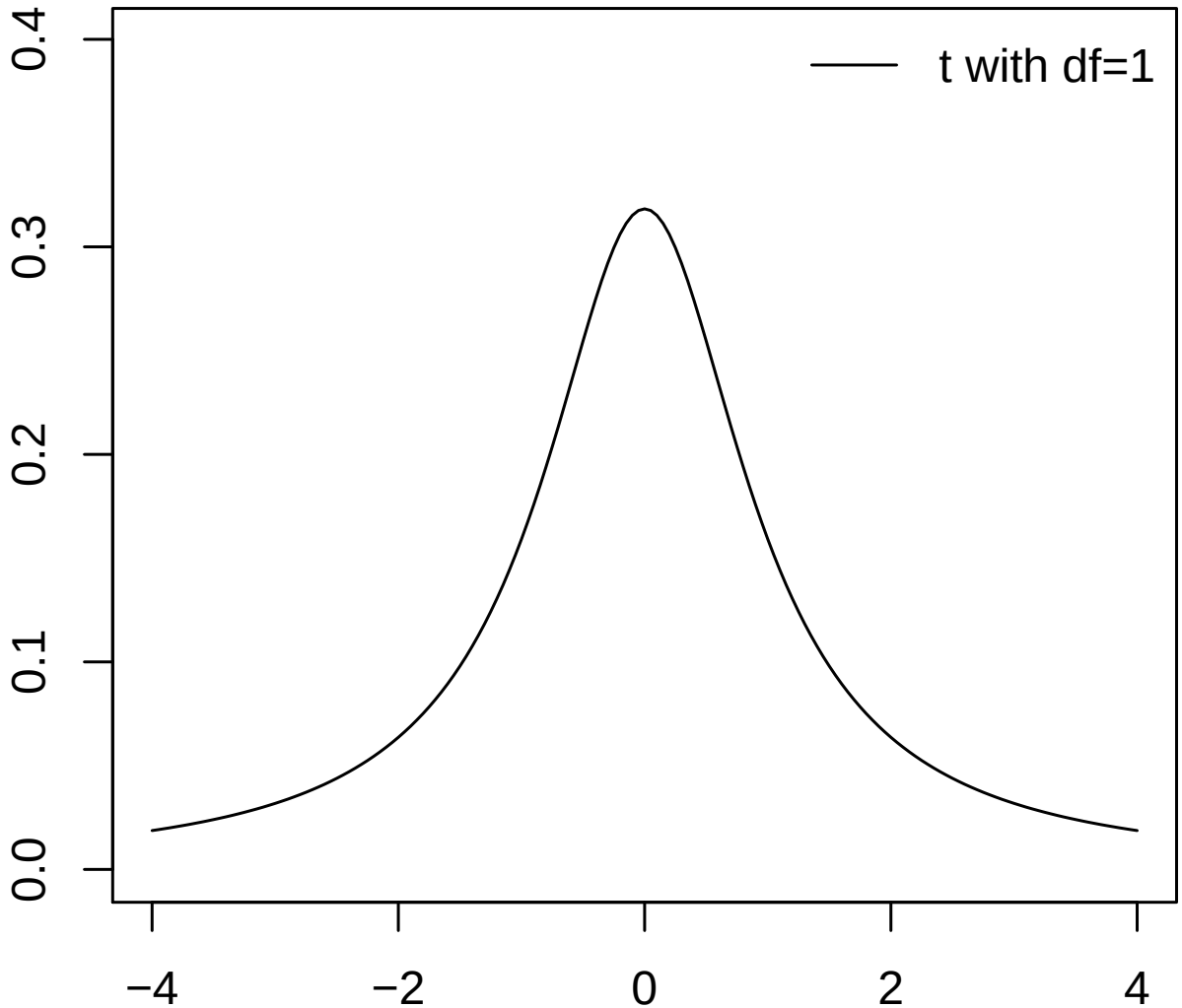
- a t-distribution has only one parameter: the degrees of freedom df.
- It is bell-shaped and centered at 0
- Any t curve is more spread out than a z curve.
  i.e. it has fatter tails
- As the df goes to ∞, the t curve approaches the z curve.

t Distribution Examples



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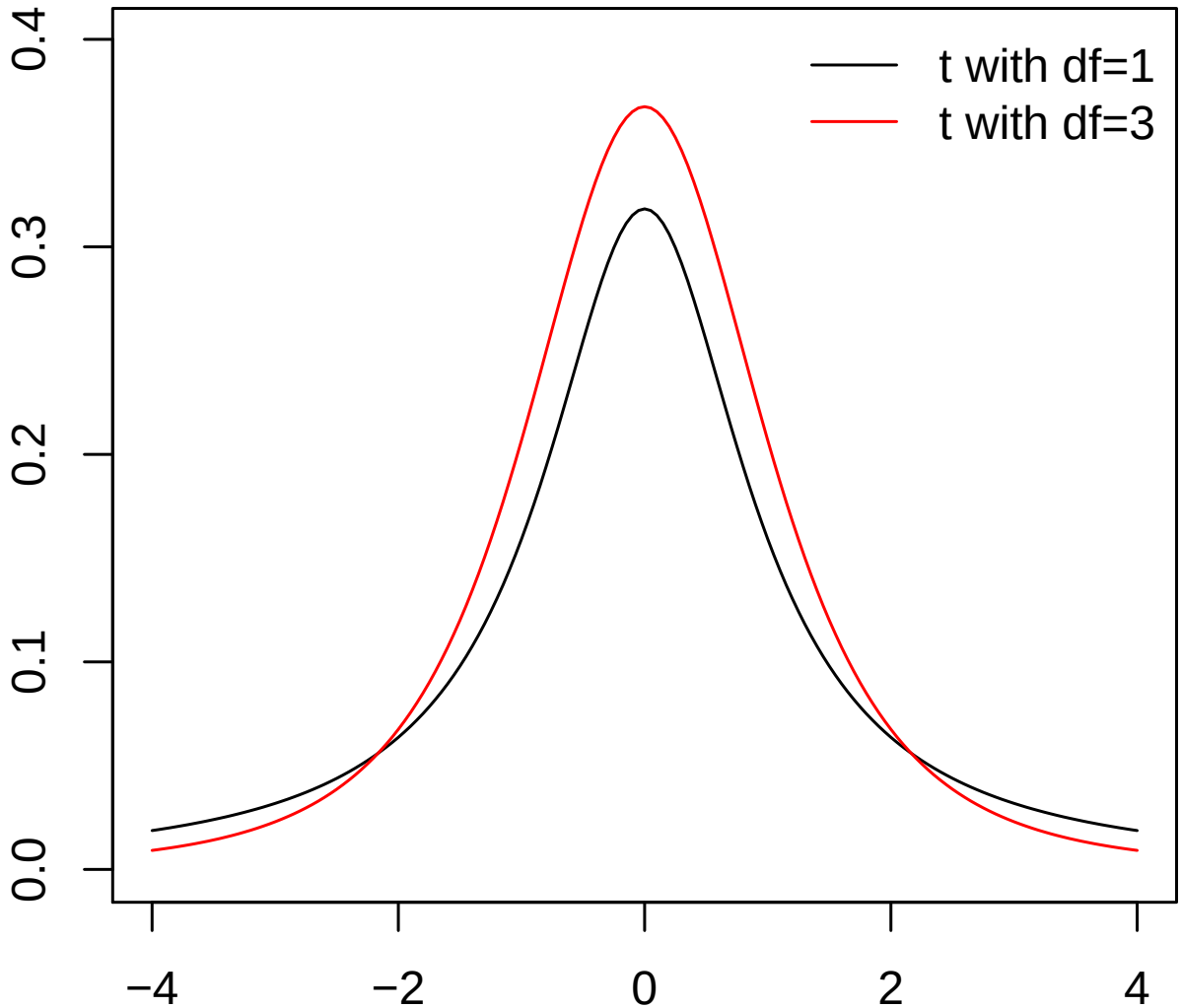

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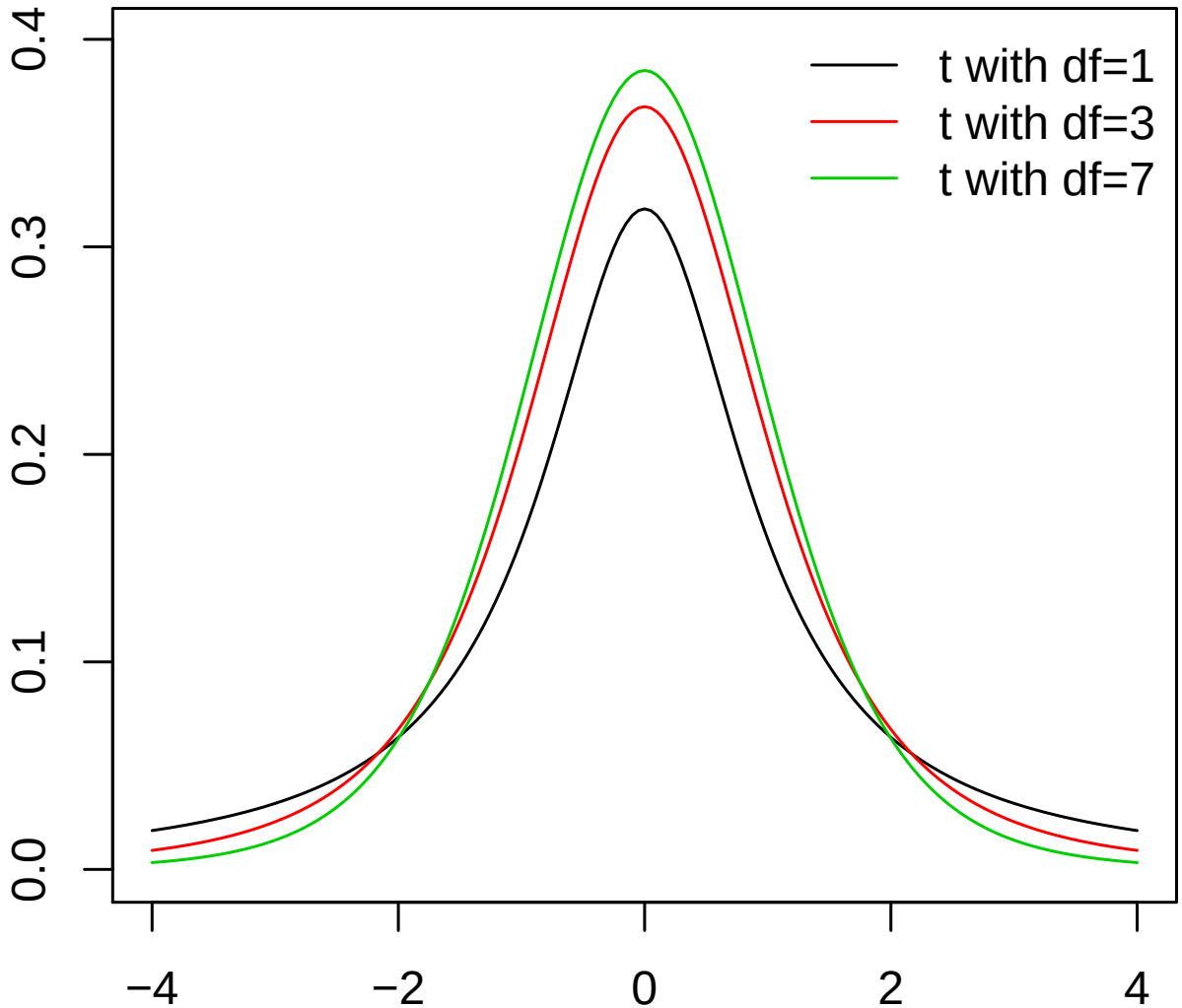

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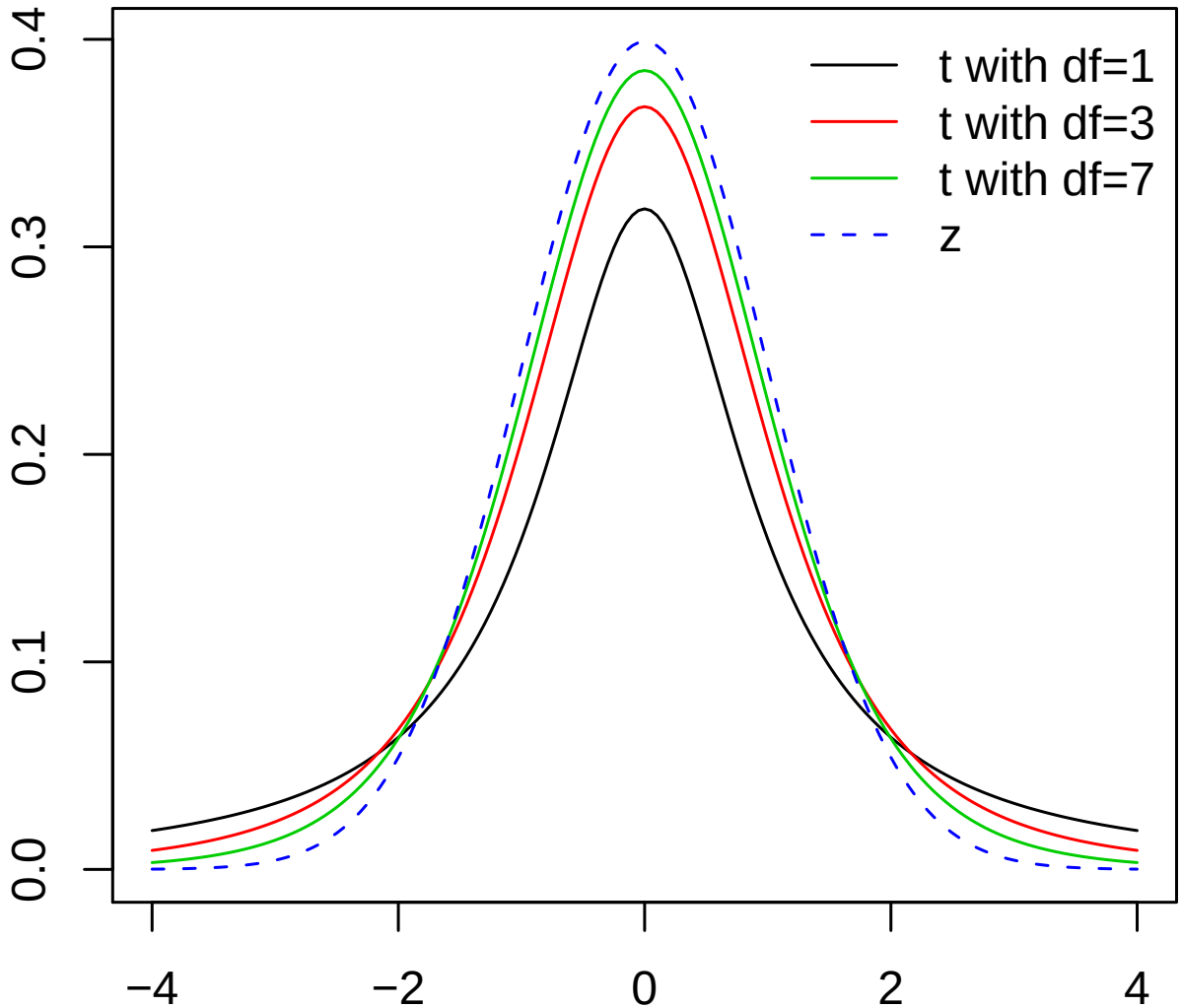

Conditions for Using t Distribution

t-Tables

If $n = 11$, we use $df = 11 - 1 = 10$ and do a look up on the t-table on page 410:

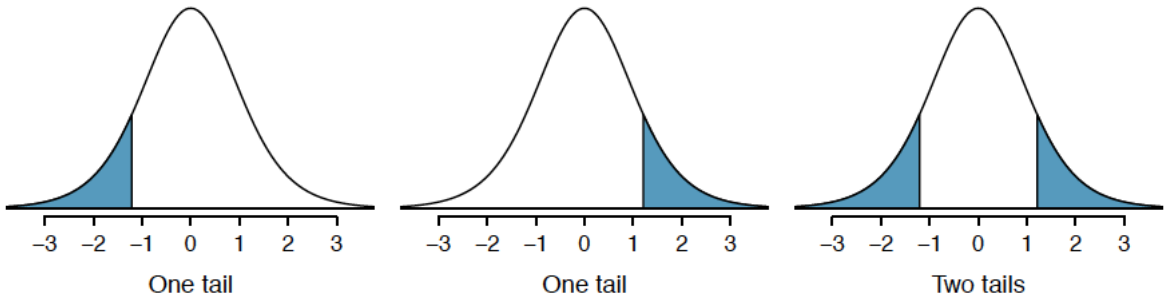

Figure B.1: Three t distributions.

one tail		0.100	0.050	0.025	0.010	0.005
two tails		0.200	0.100	0.050	0.020	0.010
df	1	3.08	6.31	12.71	31.82	63.66
	2	1.89	2.92	4.30	6.96	9.92
	3	1.64	2.35	3.18	4.54	5.84
	4	1.53	2.13	2.78	3.75	4.60
	5	1.48	2.02	2.57	3.36	4.03
	6	1.44	1.94	2.45	3.14	3.71
	7	1.41	1.89	2.36	3.00	3.50
	8	1.40	1.86	2.31	2.90	3.36
	9	1.38	1.83	2.26	2.82	3.25
	10	1.37	1.81	2.23	2.76	3.17
	11	1.36	1.80	2.20	2.72	3.11

Confidence Intervals

t-Test Example

Example 5.19 on page 252: A random sample of 25 New Yorkers were asked how much sleep they get per night. Does the data below provide strong evidence that New Yorkers sleep less than 8 hours a night on average? Set $\alpha = 0.05$

n	$\overline{x}$	s	min	max
25	7.73	0.77	6.17	9.78

t-Test

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p-Value: we use the t distribution i.e. the t-table on page 410:

one-tail	0.100	0.050	0.025	0.010	0.005
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df $= 24$	1.32	1.71	2.06	2.49	2.80

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- by symmetry -1.75 is in $[-2.06, -1.71]$
- the one-sided p-value is in between $[0.025, 0.05]$

Decision: Since the p-value $< \alpha = 0.05$, we reject H_0 that NY'ers sleep 8 hours a night at the $\alpha = 0.05$ significance level in favor of the hypothesis they sleep more.

History of t Distribution

The t distribution was derived by William Sealy Gosset in 1908, a chemist/statistician at the Guinness Brewery in Dublin, Ireland.

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The t-test's complete name is the (Student's) t-test.

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In fact if you go to the Guinness Brewery at St James's Gate in Dublin, Ireland...

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