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Encrypted Keyword Search Using Path ORAM on MirageOS

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Proforma

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Declaration

I, Rupert Horlick of Homerton College, being a candidate for Part II of the Computer Science Tripos, hereby declare that this dissertation and the work described in it are my own work, unaided except as may be specified below, and that the dissertation does not contain material that has already been used to any substantial extent for a comparable purpose.

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Chapter 1

Introduction

1.1 Motivation

Cloud computing is becoming ubiquitous, with more than an exabyte of data delivered by Cloud Storage Providers (CSPs) in 2013 [1]. For large businesses, a private cloud can be a cost effective way to keep data isolated, but as public cloud services become ever cheaper, even these businesses could be forced to succumb to market pressures and move to the public cloud. With so much important data held by only a few major cloud providers, trust becomes a major issue.

Encryption appears to be the solution to our trust issues; if the providers cannot read the plain-text of our data then surely it is secure? This appears to hold in general, but in the important application of query-based searching, we have a problem: using current methods of homomorphic encryption to perform search over encrypted documents can leak up to 80% of queries [6]. Knowledge about the queries made to a data set, along with the amount of documents returned by each query, could lead to some dangerous inferences. As a motivating example, discovering that a query such as $\langle name,\ disease \rangle$ made to a medical database returned results would allow an adversary to uncover information about a patients medical status, breaching patient confidentiality.

What allowed Islam et al. [6] to infer search queries was knowledge about the documents returned by any specific query. Thus, in order to protect against this kind of attack we need to have some way of preventing the server from knowing which documents it is actually returning in response to any query. Oblivious Random-Access Memory (ORAM) gives us exactly what we want. When using ORAM, not only are two accesses made to exactly the same piece of data computationally indistinguishable to the server, but so too are any two access patterns of the same length.

This project aims to demonstrate that, using a particular ORAM protocol, Stefanov et al.'s Path ORAM [14], it is possible to build a system that allows us to search over a set of encrypted documents without leaking the resulting access pattern, protecting the content of the search queries and therefore the confidentiality of the documents.

1.2 Challenges

The first major challenge that faces any security related project is adequately defining the threat model. In order to be able to reason about and evaluate the security properties of the system, we need to know exactly what we assume an adversary to be capable of. Once we have done this we must show that within these capabilities, the security properties that we desire the system to have remain intact. The threat model will be defined in Section 2.1.

By virtue of being stored in the cloud, we should be able to access our data at any time, from any place, while still maintaining the desired security properties. We also want to be tolerant of network connection errors and client-side crashes. Thus, another challenge is to make the system completely stateless.

In order to make statelessness more efficient, it is necessary to augment ORAM with recursion (Section 2.2), which reduces the amount of transient client-side storage. This enables us to efficiently store the client's state between accesses. Recursion introduces the challenge of choosing how many levels to use. Each extra level of recursion reduces the size of the client's state, but also incurs a time and space overhead. This is explored briefly in Stefanov et al. [14], but we will attempt to have the system automatically choose parameters for the recursion based on the size and block size of the underlying storage used by the system.

Finally in order for the system to perform query-based search over encrypted documents we need to implement a file system, an information retrieval module and an encryption module. These modules will be rudimentary, as the main focus of the project is the implementation and optimisation of the ORAM module, but all of them have inherent design and implementation challenges that will be need to be addressed.

1.3 Related Work

ORAM was first introduced by Goldreich and Ostrovsky [4], motivated by the needed for software protection on disks. The model was then expanded and refined for other settings such as secure processors and cloud computing over a number of years [12]. Stefanov et al. [14] had a large impact on the field, mainly due to the simplicity and elegance of the protocol. Since then, lots of work has been done to optimise ORAM constructions [16, 11], add additional features [8], and build actual systems based on them [13]. An extremely useful thesis by Teeuwen [15], summarised the entire ORAM field, providing insight into the evolution and therefore future of ORAM.

Chapter 2

Preparation

2.1 Defining the Threat Models

Our threat models are defined in terms of a client, a server and an attacker. We have a basic model with passive attackers and an extended model with active attackers.

We assume that the network is under the attacker's control. In the basic model we define the server and the attacker to be passive, and honest, but curious. This means that they will both gather as much information as possible, without deviating from the protocol. Thus the attacker will eavesdrop, but will not prevent messages from arriving at the server, tamper with them in any way or produce their own fake messages. The server will also not tamper with the messages, or with the underlying storage that the protocol is accessing. In this model the attacker is not as interesting as the server, because all communications are encrypted. Thus it is the server, that can see the access patterns of the underlying storage, that we are trying to protect against. The goal of the project is to make sure that this access pattern reveals nothing about the contents of the search queries or the documents in the underlying storage.

We can also extend the model to one where both the attacker and the server become active, freely tampering with messages and introducing new messages, and the server tampers with the underlying storage. In this case we will need to extend our solution with integrity verification.

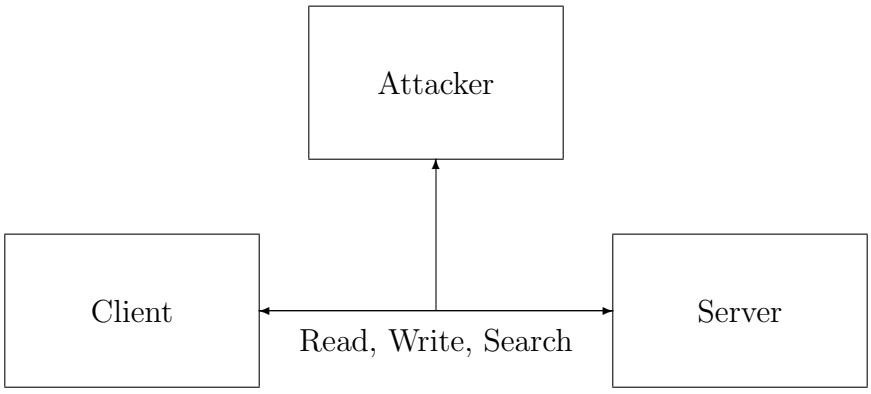


Figure 2.1: The Threat Model: The Attacker and Server are honest, but curious

2.2 Introduction to Path ORAM

Path ORAM is defined in terms of a client and a server, where the client wishes to store data on the server. The data is split into blocks and each block is tagged with a sequence number or address, its position if the data were to be stored sequentially.

On the server blocks are stored in a binary tree of height L, where each node in the tree is a bucket with size Z, containing up to Z blocks. Thus, the tree stores

$$N = Z \cdot (2^{L+1} - 1)$$

blocks in total.

The client requires two local data structures. The stash is a sort of working memory; as blocks are read from the server they are put into the stash and are written back to the server later on. The position map associates with each address a number between 0 and $2^L - 1$, which corresponds to a leaf in the tree. The protocol maintains the invariant that, at anytime, a block with position x in the position map is either in the stash, or in a bucket along the path from the root of the tree to the x^{th} leaf.

This is achieved using Algorithm 1, which is split into four main steps:

Remap Block The current position x of block a is read and then a new position is chosen uniformly at random

Read Path The path to the leaf x is read into the stash. At this point the block for address a is definitely in the stash, if it has ever been written into the ORAM

Write New Data If the operation is a write, then the value in the stash is replaced by the new data^*

Write Path The path to the leaf x is filled with blocks from the stash. A block with address a' can be put in the bucket at level l, if the path to $\mathsf{position}[\mathsf{a}']$ intercepts the path to x at level l. If there are not enough blocks satisfying this criterion, i.e. $min(|S'|, Z) < Z$, then the bucket is filled with dummy blocks

Either it was possible to write a block into the stash along the path to its leaf, or not, in which case it is in the stash, so the invariant holds after each execution of the algorithm.

The security of this operation comes from the fact that positions are assigned uniformly at random. If we consider the sequence of M accesses,

$$\mathbf{p} = (\mathsf{position}_M[\mathsf{a}_M], \mathsf{position}_{M-1}[\mathsf{a}_{M-1}], \ldots, \mathsf{position}_1[\mathsf{a}_1]),$$

any two accesses to the same address will be statistically independent and trivially so will two accesses to different addresses. Thus, by an application of Bayes' rule, we have

$$\Pr(\mathbf{p}) = \prod_{j=1}^{M} \Pr(\mathsf{position}_j[\mathsf{a}_j]) = \left(\frac{1}{2^L}\right)^M$$

Algorithm 1 Read/write data block at address a

```
1: function Access(op, a, data*)

2:     x ← position[a]
3:     position[a] ← UniformRandom(2^L − 1)

4:     for l ∈ {0, 1, ..., L} do
5:         S ← S ∪ ReadBucket(P(x, l))
6:     end for

7:     data ← Read block a from S
8:     if op = write then
9:         S ← (S − {(a, data)}) ∪ {(a, data*)}
10:    end if

11:    for l ∈ {L, L − 1, ..., 0} do
12:        S' ← {(a', data') ∈ S : P(x, l) = P(position[a'], l)}
13:        S' ← Select min(|S'|, Z) blocks from S'
14:        S ← S − S'
15:        WriteBucket(P(x, l), S')
16:    end for

17:    return data

18: end function
```

i.e. the probability of the whole sequence is the probability of each of the individual probabilities, meaning the access pattern is indistinguishable from a random sequence of bit strings.

We might, and for our application will, want to make ORAM stateless, storing all information on the disk, and nothing (except private keys) on the client. We can do this naïvely by simply dumping all of the state to disk after every operation, but in our current construction, the position map takes $O(N)$ storage space on the client, and the stash $O(\log N)$. Adding recursion to ORAM allows us to reduce the size of the overall client storage to $O(\log N)$, by reducing the space required for the position map to $O(1)$, while maintaining $O(\log N)$ for the stash (actually now multiple stashes). We can now write the client-side storage to the server after each call, and read it back again before the next one, without increasing the asymptotic bandwidth overheads.

Recursion works by storing the position map of our original ORAM, now referred to as ORAM_0, in a new ORAM, ORAM_1. Now the client stores the position map of this ORAM, plus the stashes of both. If we can store χ positions in each block of ORAM_1, then the new position map is only $O(N/\chi)$. We can then repeat this process until we have the position map of ORAM_n being of constant size. The number of levels required to achieve this are examined in Stefanov et al. [14] and explored further in Section 4.3.2.

2.3 Introduction to Inverted Indexes

The inverted index is the single most important data structure in Information Retrieval. It allows us to perform a large amount of work in advance in order to give performance that is linear in the number of documents involved in a query, much better than a linear scan of all documents that is $O(NK)$, for N documents with an average of K words. It consists of two parts: the dictionary and the postings. The dictionary is a list, usually stored as a hash table, of all of the terms that appear in a set of documents. Then for each term, we have a postings list, a list of all the documents that contain a term. Collectively these postings lists are referred to as the postings.

To look up a single keyword in a hash-based inverted index we hash the keyword and return the relevant postings list, if it exists. We can add simple Boolean operations such as disjunction, which takes the union of two postings lists, and conjunction, which takes the intersection.

In this project we will restrict ourselves to conjunctive queries, where a query is a space-separated list of keywords, and we take the conjunction of the postings lists for all of these keywords. We do this because we are focusing on the correctness and efficiency of search using the ORAM implementation, rather than creating an advanced IR system.

Constructing an inverted index can be done in three simple steps. We first split the text into tokens, units of the document that are separated by spaces. We then perform some linguistic preprocessing on these tokens, for example stemming, which removes suffixes of words in order to get them into a normalised form. Finally, assuming that we have assigned each document an ID, we add the document ID to the postings list of each token.

2.4 Introduction to MirageOS

MirageOS is a unikernel operating system. The idea of unikernels is to provide the minimum amount of functionality required of an application by pulling together a number of libraries, and compiling down to an executable. Compared to the traditional approach of running a full operating system such as Ubuntu, with databases, web servers, and the like built on top, the unikernel provides significant speed and memory improvements.

Building the implementation of ORAM on Mirage allows us to have a consistent interface between client machines and cloud providers. We can run the ORAM software on a trusted cloud instance and connect through it from any client machine, to any cloud provider that we like. We also do not need to have the instance running at all times. Mirage is so lightweight, that we could spin up an instance every time we need to perform some actions. This saves us money because, using cloud services like AWS, we only pay for the compute time we actually use. This is the main motivation behind implementing statelessness in the project.

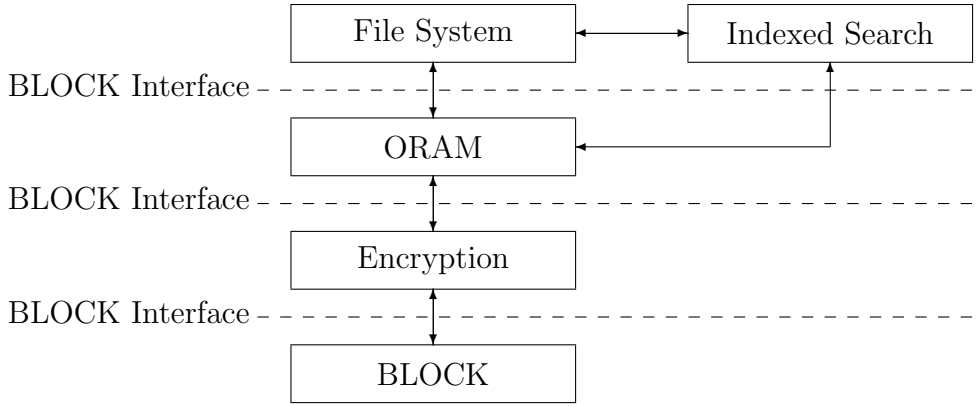


Figure 2.2: The Application Stack: We can use any underlying BLOCK implementation and we can add/remove ORAM, Encryption or Search modules as we please

2.5 System Architecture

We have discussed a couple of the major building blocks of the implementation, but how does it all fit together? We start with the architecture that we already have, a client and a server, communicating using the server's cloud storage API. Now what we want to do is insert our trusted ORAM instance in between the client and the server. In order to do this with the least inconvenience to the user, we need to define an API that unifies those of the cloud providers, providing all of the same facilities that the user already has access to. Then we need to have ORAM actually interface with the cloud providers. We can do this by creating a module for each provider that translates their individual API to satisfy MirageOS's BLOCK interface. Actually writing these modules will not be part of this project.

Now all of the important stuff can happen in between the two interfaces. Figure 2.2 shows the overall structure of the ORAM instance. From the bottom up, we have the underlying BLOCK module, which will be the bindings for some cloud provider, or a local block device for the purposes of evaluating this project. Then there is an encryption layer, that will take care of all cryptographic operations in one module, exposing another BLOCK interface. On top of this we have the ORAM layer, which will implement Algorithm 1, along with marshalling data into and out of the BLOCK interface on both sides of the operation. We can build a file system at the top layer that takes any BLOCK implementation, which will allow us to perform tests of the overhead introduced by different combinations of modules. Then we will add our search module along side the file system, which will construct and store the inverted index that we described in Section 2.3, as well as exposing search operations to the client. Combining all of these modules will give a complete system, capable of performing secure search over encrypted documents.

2.6 Requirements Analysis

High Priority Basic ORAM implementation

Medium Priority File System

Medium Priority Search Module

Low Priority Encryption

Low Priority Further Optimisations including Recursion and Statelessness

Low Priority Integrity Verification (Extension)

ORAM forms the basis for the whole project and as such is given high priority. Once the basic ORAM is completed, adding a file system and search module gives us a complete system that we can evaluate, so these two modules are of medium priority. Encryption and further optimisations will round off the project, but are not necessary in order to get something that we are able to evaluate, so are given low priority. Finally, integrity verification is left as an extension, and therefore has low priority.

2.7 Choice of Tools

2.7.1 OCaml

Having decided to build ORAM on top of MirageOS, OCaml was really the only choice of programming language, because Mirage applications and libraries are all built in OCaml. However, OCaml was also part of the reason for choosing to build on Mirage.

OCaml has an extremely powerful module system, making it easy to parameterise the ORAM implementation over module signatures for Block devices or other system components. This not only encourages more generalised programming, but also allows powerful techniques like recursive modules to be exploited. For instance, to implement recursive ORAM, the position map of the main ORAM is another ORAM, so we can simply parameterise over a position map signature and pass the ORAM implementation to itself as the position map.

OCaml's static typing system is also indispensable for ensuring correctness of programs and increasing productivity.

2.7.2 Libraries

The libraries used for building ORAM are listed in table 2.1.

2.7.3 Development Environment

Good tooling is essential to productivity when building large projects. For OCaml, one of the most important tools is OASIS, which automatically generates Makefiles for a project, based on a specification file. Using the OCaml compilers directly quickly gets infeasible, but OASIS takes care of everything for you. Emacs also proved to be an incredibly useful tool, because there are a number of OCaml specific plugins that provide everything from code indenting and syntax highlighting, to autocompletion and type inspection. It is also

Library	*Version*	*Purpose*	*License*
Mirage	2.6.1	System Component Interface Definitions, Application Configuration Framework	ISC
Jane Street's Core	113.00.00	Data Structures, Algorithms	Apache-2.0
LWT	2.5.1	Threading	LGPL-2.1
Cstruct	1.8.0	Data Structure	ISC
Alcotest	0.4.6	Unit Testing	ISC
Mirage Block CCM	1.0.0	Encryption	ISC
Stemmer	0.2	Linguistic Processing	GNUv2

Table 2.1: Libraries used by Mirage ORAM

possible to run a shell from within Emacs itself, where it is possible to interpret, compile, and run the code, which gives a big boost to productivity.

Tool	*Version*	*Purpose*	*License*
Mac OSX	10.11.2	Operating System	Proprietary
Emacs	24.5	Text Editor	GPL
OPAM	1.2.2	Package Manager	GPLv3
OASIS	0.4.5	Build Tool	LGPL-2.1

Table 2.2: Tools used in the development of Mirage ORAM

2.8 Software Engineering Techniques

I employed two major techniques to ensure my code was well thought through and well built, while remaining productive.

First of all a Test Driven Development approach allowed me to fail fast [5]. I wrote unit tests for each new piece of code that was written, until I was sure that it was working correctly. This meant that when it came to combining small modules into a larger system, things worked together as expected more often than not.

Secondly, I wrote interface files before writing the actual implementation. This meant that I had to think about the design of each module thoroughly before writing any code and also meant that I could make adjustments to other modules in order to fit with the new design.

Combining these techniques with documentation and structuring of both the source code and the source repository, led to a manageable and feasible development workflow.

2.9 Summary

In this chapter I have discussed the work that was undertaken before development began. This included a rigorous definition of the threat model, a brief introduction to the major

algorithms, data structures and libraries, an overview of preliminary architectural designs, and a discussion of the techniques and tools used in the development process.

The next chapter will show how all of this information was used to achieve the aims of the project.

Chapter 3

Implementation

This chapter describes the process that took the designs and algorithms of the previous chapter and turned them into a functioning system. An overview of the implemented system is given in Figure 3.1. The structure of the text will follow the structure of this diagram, working up from the bottom. We thus start with a discussion of encryption in Section 3.1, then we have a longer discussion of ORAM in Section 3.2, because this is the main focus of the project. Following that we discuss the file system and the search module in Sections 3.3 and 3.4 respectively.

Looking at Figure 3.1, we can summarise the major challenges of the implementation. At the highest level, the plumbing together of all the parts of the system required careful API design and some intricate usage of the powerful OCaml module system. Choosing a library to use for the encyption module was an important decision and I actually ended up filing a pull request to fix an important bug. The ORAM module presented many challenges, not least of which was simply understanding the Path ORAM protocol and translating its terse pseudocode into a fully functioning program. Adding recursion to this implementation required an even deeper understanding of the module system, including first class modules. The implementation of a minimal, but complete, inode-based file system, required appreciation of the choices made by many file system designer before me, and the realisation of a B-Tree library in OCaml, which did not exist before. Finally the search module leveraged many concepts from the field of information retrieval, including algorithms and data structures, but also an appreciation of the trade-offs that must be made between the space used and the precision of the indexing process.

3.1 Encryption

The security of our entire system is dependent on the security of the underlying encryption layer. Without encryption the ORAM construction is completely superfluous, thus we want to use tested and trusted cryptographic libraries, rather than rolling our own. Luckily for us, there is an existing implementation of an encrypted block device satisfying Mirage's `BLOCK` interface. It provides a functor, a module parameterised in another module. This functor takes an implementation of a block device, satisfying the `BLOCK` interface, and returns a new module, also satisfying the same interface. This is very similar

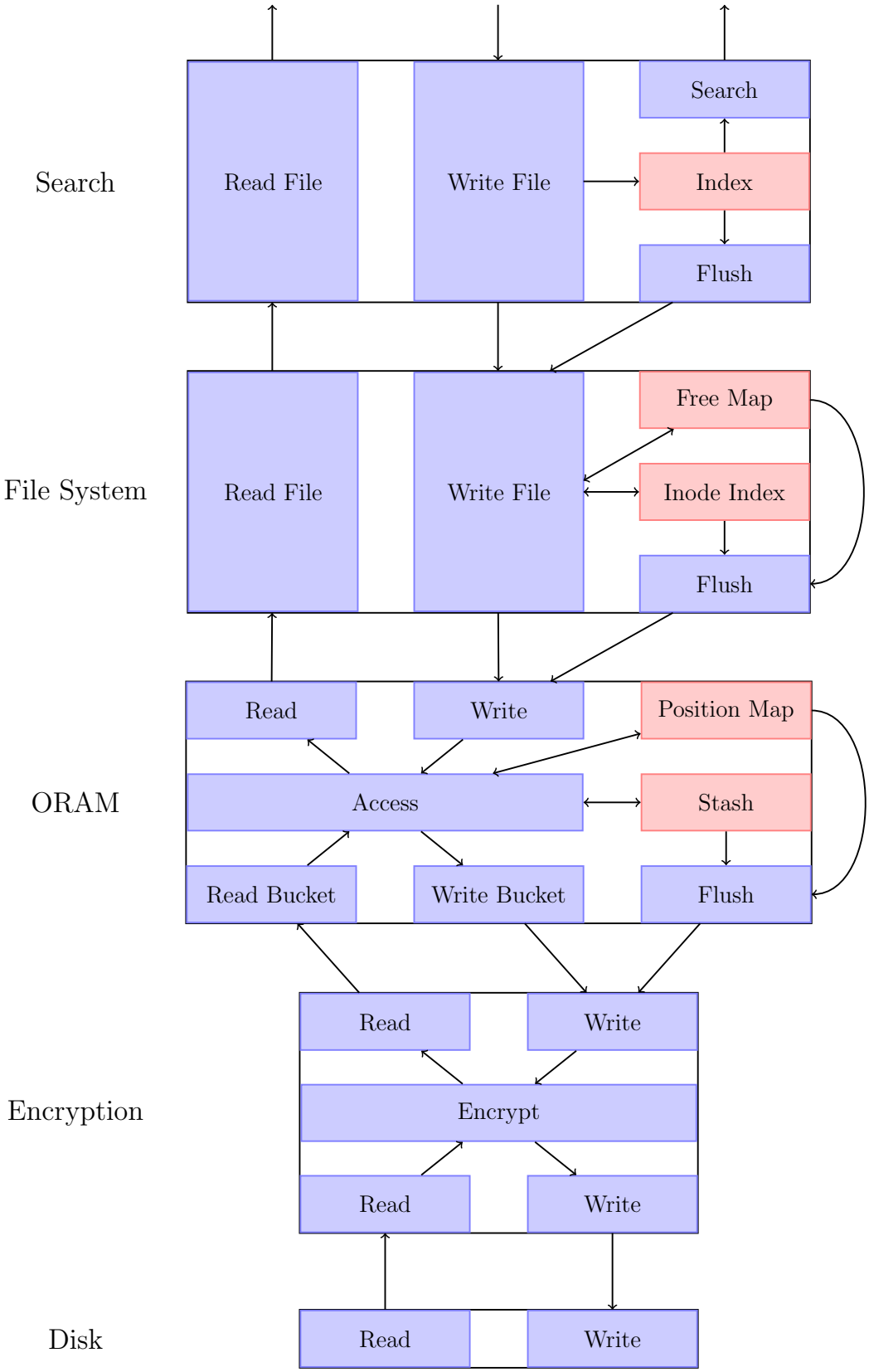


Figure 3.1: An overview of the system, showing the flow of data. Black boxes are modules, blue are functions, and red are data structures.

Listing 1 Mirage `BLOCK` module signature

```
module type BLOCK = sig

  type page_aligned_buffer = Cstruct.t

  type +'a io = 'a Lwt.t

  type t

  type error = [
    | `Unknown of string (** an undiagnosed error *)
    | `Unimplemented     (** operation not yet implemented in the code *)
    | `Is_read_only      (** you cannot write to a read/only instance *)
    | `Disconnected      (** the device has been previously disconnected *)
  ]

  type id

  val disconnect: t -> unit io

  type info = {
    read_write: bool;    (** True if we can write, false if read/only *)
    sector_size: int;    (** Octets per sector *)
    size_sectors: int64; (** Total sectors per device *)
  }

  val get_info: t -> info io

  val read: t -> int64 -> page_aligned_buffer list -> [ `Error of error | `Ok of unit ] io

  val write: t -> int64 -> page_aligned_buffer list -> [ `Error of error | `Ok of unit ] io

end
```

to the way that ORAM is implemented, and allows us to chain together the application of this functor with the ones used in ORAM to get an encrypted block device. To create the encrypted block device and later connect to it, we need to provide a key. Key management is a very important problem, but it is left out of the scope of this project. For our purposes we use a constant, known key throughout the evaluation stage of the project.

3.2 Path ORAM

The structure of Path ORAM is described abstractly in Section 2.2, in terms of the core data structures and the access algorithm. In this section we discuss how those data structures were realised and the design decisions involved in the implementation.

3.2.1 Inherent Constraints

By virtue of writing an implementation that satisfies an existing module signature, there are a number of constraints put on the design of our system.

The first major constraint is the use of the Cstruct library, introduced in Section 2.7.2. The underlying block device requires a buffer of type `Cstruct.t` to read from/write to,

Security Parameter (λ)	Bucket Size (Z)		
	4	5	6
	Max Stash Size		
80	89	63	53
128	147	105	89
256	303	218	186

Table 3.1: Empirical results for maximum persistent stash size from Stefanov et al. [14]

and in order to satisfy the `BLOCK` module signature, ORAM needs to return data as a `Cstruct.t`. Thus, to avoid unnecessary marshalling of data, we will pass our data around in this form.

Another constraint is the usage of `int64` as the type of addresses in `BLOCK`'s, `read` and `write` operations. Again, to avoid unnecessary (and potentially unsafe) work converting between types, we will use `int64`s wherever possible.

3.2.2 Stash

The stash stores blocks of data temporarily before they are written back into ORAM. It needs to support operations of insertion, lookup based on address, and removal. For this job I chose an `int64`-keyed hash table from Jane Street's Core library, with `Cstruct.t` values. This was put into its own module, abstracting away the underlying type, meaning that we could swap implementations of the stash module without breaking the core code of the ORAM module.

The hash table gives us constant time for the operations of insertion, lookup and removal, making it ideal for our needs. The hash table implementation takes an initial size as a parameter, and expands when necessary. This would add a large overhead, because we would have to copy the entire contents of the stash. However, as shown in Stefanov et al. [14], we require exactly $Z \cdot (L + 1)$ blocks of transient storage, for a tree of height L and bucket size Z, and on top of that, we require a constant amount of space for persistent stash storage. Table 3.1 shows the maximum stash size required depending on the security parameter, λ, and the bucket size Z. A stash with security parameter λ has probability $2^{-\lambda}$ of exceeding this stash size.

In order to achieve statelessness, we need to be able to write the stash to disk, but we want our security parameter to be high enough that we have long term security. We must make a trade-off between security and speed, which we can do by keeping the security parameter high and reducing the size of a block. As we have just seen there is a constant maximum number of blocks in the stash, so reducing the block size will have a direct affect on the time taken for each access.

3.2.3 Position Map

The position map associates a leaf position with each block of the data. As mentioned in Section 3.2.1, we are constrained to using `int64` block addresses, thus we require a

Algorithm 2 Calculate the dimensions of a 3D array given total desired size

Require: size > 0

function PosMapDims(size)
  $(x, y, z) \leftarrow$ SplitIndices(size)
  $x \leftarrow x + 1$
  $y \leftarrow y + 1$
  if $x > 1$ **then**
    $y \leftarrow$ 0x3FFFFFFF
    $z \leftarrow$ 0x3FFFFFFF
  else if $y > 1$ **then**
    $z \leftarrow$ 0x3FFFFFFF
  end if
  return (x, y, z)
end function

position map that is indexed by 64-bit integers. OCaml provides us with a Bigarray module, but the size of these arrays is specified using the OCaml `int` type. This type actually uses only 63 bits on a 64-bit machine and 31 on a 32-bit machine. Both of these types are also signed, so we need to represent a type that can range up to $2^{64} - 1$, using a type that can only go up to $2^{30} - 1$ on a 32-bit machine.

To accommodate this, we can use 3-dimensional arrays. We take an `int64` value and split its bits into a 4-bit value and 2 30-bit values. The 4-bit value consists of the 4 most significant bits, and will therefore have a value of 0 unless we store more than 2^{60} blocks. The 30-bit values are guaranteed to be converted into non-negative `int`s, which we can then use to address two dimensions of the array.

Now the only remaining problem is creating the position map. Algorithm 2 shows how we translate from a desired `int64` size to the dimensions of a three dimensional array. After splitting the `int64` as described above, we must add one to the first two dimensions, because we always want them to be at least of size 1. If a higher dimension is greater than 1, then all lower dimensions become their maximum value, in this case $2^{30} - 1$. We can now create an array using these values that is guaranteed to be at least the size that we require on both 32-bit and 64-bit machines.

3.2.4 Creating ORAM

We want ORAM to be able to replace any existing block device in any Mirage program. In order to do this we need access to the methods of the underlying block device as well as the block device itself. Thus, we need to build an ORAM functor, that takes a module satisfying the `BLOCK` interface, shown in Listing 1, and gives us a new module satisfying the same interface.

In order to get access to this underlying device, we need a `create` method, which

takes a block device as input and returns something of type `Oram.Make(B).t`, shown in Listing 2. This type contains the ORAM parameters such as `bucketSize` and `blockSize`, structural information such as the `height` of the ORAM and the `numLeaves`, and pointers to the stash, position map, and underlying block device.

The following parameters are passed as input to the `create` method, along with the block device:

`size` The desired size of the ORAM in blocks

`blockSize` The desired size of a single block in bytes

`bucketSize` The number of blocks in a bucket

Using these, the `create` method can calculate new structural information. The `BLOCK` interface defines the size of the block device in sectors, using the variable `size_sectors`, and defines the size of a sector using `sector_size`. We will continue to refer to data blocks in ORAM as blocks, but to satisy `BLOCK`, we will need to expose a value for `sector_size`. First we need to calculate the number of sectors required for a block as

$$\texttt{sectorsPerBlock} = \frac{\texttt{blockSize} - 1}{\texttt{sector\_size}} + 1,$$

which rounds up the number of sectors so we can always fit the desired block size. Now, the `sector_size` that ORAM uses is the size of the part of the block that stores data. Thus, we must subtract the size of the address, 8 bytes, giving

$$\texttt{sector\_size} = \texttt{sector\_size} \times \texttt{sectorsPerBlock} - 8,$$

Now we can calculate the height of the ORAM, but we need to consider two cases. If the desired size of the ORAM is specified, then we simply calculate the height as

$$L = \left\lfloor \log_2 \left(\frac{N}{Z} + 1 \right) \right\rfloor - 1.$$

This comes from rearranging the equation for the size of the binary tree in buckets, $2^{L+1} - 1$, introducing the floor operator so that the resulting binary tree is less than or equal to the desired size, so that it will definitely fit on the block device. If the size is unspecified, we assume that we should fill as much of the device as possible, so we calculate

$$N = \frac{\texttt{size\_sectors}}{\texttt{sectorsPerBlock}}$$

and then perform the same calculation as above with this new value. Finally we calculate, `numLeaves` and a new value for `size_sectors` from L using the equations we have already seen.

This is all of the structural information we require, so finally we must create instances of the client-side data structures and we must initialise the ORAM space. To do the former we call the creation functions of the associated data structures. For the latter, we loop through the block device, writing dummy blocks to every location. Dummy blocks have address -1, and are ignored by the access protocol. Now we can return everything that we have as an instance of `ORAM.Make(B).t`.

Listing 2 The type of an ORAM device `ORAM.Make(B).t`

```
type info = {
    read_write: bool;
    sector_size: int;
    size_sectors: int64;
  } [@@ deriving bin_io]

type structuralInfo = {
    height : int;
    numLeaves : int64;
    sectorsPerBlock : int;
  } [@@ deriving bin_io]

type core = {
    info : info;
    structuralInfo : structuralInfo;
    bucketSize : int64;
    offset : int64;
    desiredBlockSize : int;
    stash : Stash.t;
  } [@@ deriving bin_io]

type t = {
    info : info;
    structuralInfo : structuralInfo;
    bucketSize : int64;
    offset : int64;
    desiredBlockSize : int;
    stash : Stash.t;
    positionMap : PositionMap.t;
    blockDevice : BlockDevice.t;
  }
```

3.2.5 Accessing ORAM

Now that we have the client-side data structures and our ORAM module, we can almost implement Algorithm 1. Before we can though, we need the surrounding plumbing. The `BLOCK` interface functions `read` and `write` input/output data as a list of `Cstruct.t`s, so they must split this into chunks before calling `access` on each one. On the other side of access, we need the subroutines ReadBucket and WriteBucket, which access the underlying block storage.

It is the second two of these plumbing functions that have a much more interesting role. They are actually responsible for maintaining the logical binary tree structure. There are no physical pointers, but instead the structure is built purely through calculating the appropriate address of a bucket. The bucket on the path to leaf x at level l is calculated using Algorithm 3.

This is most easily explained using Figure 3.2. Here we can see the nodes labelled in order of their position in memory. The leaves are also labelled, but the binary representations of these labels are actually more important. The binary representation, read from left to right, tells us how to reach the leaf. To get to leaf 5, with binary representation 101, we go right, left, right. When we go left, we double the label on the node and add one, and when we go right, we add two. Multiplying this node label by the block size and the bucket size gives us the physical address of the node. So for a node at level l, we only

Algorithm 3 Calculating the address of the bucket at level l on the path to leaf x

```
function BucketAddress(x,l)
    address ← 0
    for i = 0; i < l; i++ do
        if x >> (i + height − l) && 1 = 1 then
            address ← (2 × address) + (bucketSize × sectorsPerBlock × 2)
        else
            address ← (2 × address) + (bucketSize × sectorsPerBlock)
        end if
    end for
    return address
end function
```

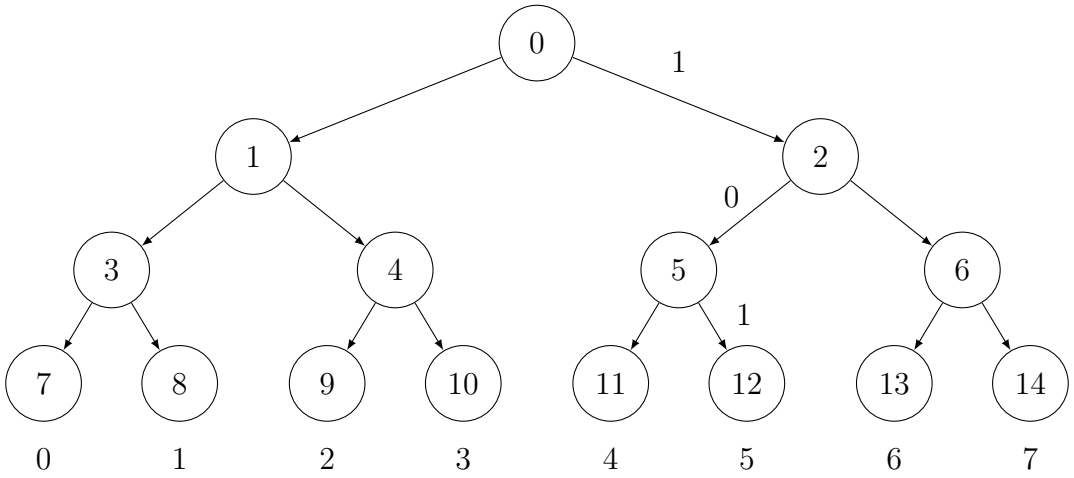


Figure 3.2: Visualisation of Algorithm 3

compare the first l bits, following the procedure above.

With this plumbing in place, we can input/output data from the outside and the underlying block device. Now we can implement Algorithm 1. Remapping the blocks requires a call to a pseudo-random function, reading in the path adds the contents of each bucket to the stash, and writing new data looks up a block in the stash and gives it a new data value. The final part is lines 11-16. This is where we decide which blocks should be put into the path.

The naive implementation of this, and that suggested by Algorithm 1, loops through the stash and finds blocks such that the bucket at level l on the path to leaf $\mathsf{position}[\mathsf{a}']$ is the same as the bucket at level l on the path to leaf x. There are two small optimisations we can make here. The first is to perform the position lookup only once, attaching positions to the blocks in a temporary data structure, avoiding repeated work at each level. The second avoids calculating the bucket addresses entirely. We can do this by noting that in order for the paths to two leaves to intersect at level l, the leaves must have the same first l bits. Thus, we can perform a right bit shift on both x and $\mathsf{position}[\mathsf{a}']$ of $\texttt{height} - l$ bits, and check for equality.

We now have a functioning ORAM functor, that can be used to augment a block

device in any Mirage program. In Sections 3.2.6 and 3.2.7 we look at optimisations and extensions to this ORAM construction, before moving onto to the further aims of the project.

3.2.6 Recursion

The essence of recursive ORAM, is that the position map of one ORAM is another ORAM. Thus, we can extend our original ORAM functor, parameterising it in the implementation of the position map. To do this we use a single interface, that is satisfied by both the in memory position map and the ORAM module. We can apply this new ORAM functor with the in memory position map module to get our original construction. However, we can now take this further, applying the functor again with the result of the first application, and this can be done to an arbitrary depth. This is an exhibition of the power of OCaml's module system.

We can use the method above to build the recursive ORAM module by hand, applying the functor $n + 1$ times for n levels of recursion. However, it is better to have a more automatic solution, where the recursive module is built automatically based on the size of the data ORAM, ORAM_0. Our addresses are 64-bit integers, so they take 8 bytes of storage each. If ORAM_0 has size N in blocks, and the size of each block is B bytes, then one block in ORAM_1 being used as the position map for ORAM_0 can store $\chi = B/8$ addresses. The number of blocks required for ORAM_1 will therefore be N/χ. After $\log N/\log \chi$ recursions, we will have a constant sized position map.

So, to create a recursive ORAM, we take in the size, N, and the block size, B, and automatically apply the ORAM functor recursively $\log N/\log \chi$ times. Calling the `create` function of the resulting module will create ORAM instances with the correct number of levels of recursion.

3.2.7 Statelessness

In order to achieve statelessness for ORAM, we need to store a few different things. We need to store type information, the stash, and the position map. We will first talk about how to layout all of this information on disk, before moving on to talk about how we actually get it there.

Once we have initialised ORAM and begun to use it, we do not want to move it around on disk, because we would have to copy the entire thing. We do however need to store the information necessary to discover ORAMs existence in a well-known location, in order to allow us to reconnect to it. This leads us to storing a superblock in the first sector of the block device. This block will contain a pointer to the location of the rest of the state, along with its length. This way, we can store ORAM starting at the second location on disk, and then append all of the state at the end of the ORAM section, meaning we will never have to move ORAM once we have initialised it.

Now we actually need to store the state, which means we need some method of serialising it, ready to be written to disk. We could implement this manually, but for the majority of the information we can take advantage of an existing serialisation library,

Jane Street's Bin_prot. This is a binary protocol, that allows you to annotate a type with `[@@ deriving bin_io]` and will then generate functions to read and write instances of the type into buffers. We are able to use this for all of the type information for ORAM, as well as for the stash. This leads us to splitting the ORAM type into a core, that can use Bin_prot, and an extended type, that includes the position map and the underlying block device. This structure is shown in Listing 2.

The position map is more difficult to serialise, because under recursive ORAM, it might actually be another ORAM. We obviously don't want to write the entire of the position map ORAM onto the disk a second time, so we need a custom serialisation function for the position map that will store only meta-data for ORAM position maps, and will store the entire position map in the base case. We want to store all state right at the end of the block device, after all recursive instances, so this function actually collects the data from all the levels of recursion together into one buffer.

Now that we can write all of the major parts into the buffer, we simply collect everything together and dump it at the end of the block device. After this is done, we are able to disconnect from ORAM safely. Reconnecting to ORAM is now a case of checking for the presence of the superblock, reading in the location and length of the state, reading the actual state, and then calling the connect function on the position map. The connect function allows each recursive ORAM instance to have its own reference to the underlying block device, but does nothing in the case of the in memory position map.

3.3 File System

In order to perform search over documents, we need some way of actually storing those documents. This section describes the design and implementation of a basic file system that satisfies the requirements of the project.

3.3.1 General Design

The most common way of building a file system on top of a block device is through the use of inodes. An inode contains meta-information about a file along with pointers to the actual data blocks. For the purposes of this project, an inode will simply be one sector of the block device, containing the length of the file, followed by the list of pointers. In a system with more complex needs the inode would contain more information, such as modification/access timestamps, file permissions, etc., but we simply want to be able to read and write documents.

We need to be able to access the inode for a particular file quickly, so we should store its location in an index. We could perform lookup based on the actual filename, but the names have variable lengths, therefore, because we want to store the index, it is better to lookup based on the hash of the filename. Section 3.3.2 describes the implementation of the Inode Index.

We also need to allocate space on the block device for inode index blocks, for inode blocks and for data blocks. So we need a map that tells us which blocks on the device

are free and allows us to update it as new blocks are needed. Section 3.3.3 describes the implementation of the Free Map.

We could now build a working file system, but we want it to be stateless. In order to do that, we need to store enough information to be able to reconstruct its in-memory representation. All we actually need to store is the root address of the Inode Index and the length of the Free Map. These two alone allow us to locate the data structures on disk when reconnecting to the block device. We store these two pieces of information at address 0 in what we call the Superblock.

3.3.2 The Inode Index

We need a data structure that associates keys, in the form of file hashes, with values, in the form of pointers to inodes. We want to support operations of insertion, lookup and deletion efficiently, but we also want to store the data structure on disk. This leads us naturally to an implementation using B-Trees.[1]

B-Trees are a generalisation of self-balancing binary search trees, where each node can have more than one child. If a node has n children, then it stores $n - 1$ keys. It is guaranteed that

$$\forall k \in child_m, j \in child_{m+1}.k < key_m < j,$$

that is, a key is greater than all the keys to its left and less than all the keys to its right.

B-Trees are an efficient on-disk data structure, because we can use the whole of a disk sector for one node. This gives us an extremely high branching factor, reducing the depth of the tree and therefore the number of disk sectors that we need to access in any single operation. On creation of the file system, we calculate the branching factor of the tree in order to fill as much of each sector as possible with useful information.

3.3.3 The Free Map

In order to allocate space efficiently, we can simply use an array of bits the size of the block device. We again want to have an on-disk data structure, or at least one that can easily be flushed to disk regularly. It was therefore beneficial to write my own bit array based on `Cstruct`s, rather than using a library implementation. This gives us the ability to write the whole structure directly onto the disk using the block device methods, without any cumbersome translation.

The `Cstruct` library performs data access in bytes. This leads us to Algorithm 4 for getting and setting individual bits. To get the n^{th} bit, we must get the $\frac{n}{8}^{th}$ byte and extract it from there. To do this, we calculate the index of the bit in the byte, shift a 1 to that position, and perform an and, masking that bit. Setting is a similar operation, but seeks to preserve the surrounding bits. To set a 1, we calculate the index of the bit in the byte, shift a 1 to that position, and perform an or, preserving all other bits. Setting a 0 is slightly trickier. We want to perform an and with a bit string that is 0 at the desired position and 1 everywhere else, but shifting fills empty bits with 0s. We can however use

[1] The algorithms for B-Tree operations were adapted from Cormen et al. [2]

Algorithm 4 Getting and setting individual bits in a byte array

```
function GetBit(index)
    byte ← byteArray[index]
    shift ← 7 − index mod 8
    return byte >> shift && 1
end function

function SetBit(index, boolean)
    byte ← byteArray[index]
    shift ← 7 − index mod 8
    if boolean then
        byte ← byte || 1 << shift
    else
        byte ← ¬(¬byte || 1 << shift)
    end if
    byteArray[index] ← byte
end function
```

De Morgan's Law

$$a \;\&\&\; b = \neg(\neg a \;||\; \neg b)$$

to convert this to an operation involving a bit string that has a 1 at the desired position and 0s everywhere else.

3.4 Search Module

So we have our documents and we can access them without revealing which ones we are accessing. Now the final piece of the puzzle is actually performing search. We discuss building an Inverted Index, the data structure that will allow us to search efficiently, in Section 3.4.1. We then discuss the front-end of the whole application, the search API, in Section 3.4.2.

3.4.1 Inverted Index

The basics of Inverted Indexes are discussed in Section 2.3. As stated there, the Index consists of two main structures, the dictionary and the postings. For the dictionary, we will do the usual thing, which is to implement it as a hash table. This provides us with $O(1)$ lookup and insertion, which are the main operations we will be performing. The postings are more flexible. In our implementation we want to store the file names, because they are not actually stored in the filesystem itself. We also want to perform intersection of postings lists, allowing us to perform phrase search. Thus, we will use a Hash Set, a data structure built on top of a hash table, that stores a set of keys, and has the added benefit of keeping them unique for us.

Having decided on our data structure, we need to actually index files. As usual in this project, the files will be `Cstruct`s. So we have a number of steps to perform in order to process a file. We first convert the file to a string and immediately strip it of characters we don't need, pretty much all punctuation. At this point we have a sequence of alphanumeric character strings, separated by spaces and newlines, so we can perform a split on these characters to get a list of words.

We could perform indexing now, adding the name of the current file to the postings list of every word in our list, but there are a couple of things that we want to do first. We really don't want to store separate words for 'run', 'ran', 'runs', and so on, so we will perform some linguistic preprocessing, stemming the words using Porter's stemming algorithm. I used a small open-source library implementation of this algorithm. This technique not only reduces the size of the index, but also arguably improves search, because now queries for 'run' can automatically returns documents containing any morphological derivation.

Finally, we remove duplicates from our list of words, reducing insertion overhead, and put the entries into the index.

For our purposes we will only implement simple conjunctive phrase search, meaning we look for documents containing all of the words in a space separated query string. Now search is as simple as performing the same preprocessing on the query string, performing lookup on each of the queries in turn and taking the big intersection of the resulting list of hash sets. In order to make this intersection operation efficient, we sort the list by order of hash set size. Then we can filter the smaller hash set by checking for membership in the larger hash set. This means we perform one constant time lookup for each member of the small set, rather than the large, giving a large performance boost, as this smaller set is monotonically decreasing.

3.4.2 Keyword Search API

The final step in an end-to-end system is the API. We need to wrap file system access, indexing and search all into one module that provides a single point of entry for an encrypted search system.

We have three main operations that are the most important to support. Writing files, reading files, and searching over files. We will not be concerned with deleting files at the moment, but it is left as an extension if time permits.

Writing files is the most important step, because this is where the search module does the indexing. On write, we first write through to the first system, then index the file and finally flush the index to disk to make sure it persists. Reading files is simply a pass through to the file system and search makes calls to inverted index.

3.5 Summary

In this chapter I have discussed the implementation of all of the components of my system, including the important design decisions and trade-offs that were made. We have seen how to implement recursive Path ORAM in a way that allows it to plug into existing

MirageOS programs, and we have seen how to build a file system and search operations on top of it.

We now move on to the important task of evaluation, where we make sure that I have implemented ORAM in a way that ensures functionality, performance, and security, and have therefore achieved the aims of the project.

Chapter 4

Evaluation

This chapter explains the methodology used to ensure the correctness of the ORAM implementation, as well as analysing its performance and security properties. Section 4.1 gives a summary of the results and compares them with expectations. Section 4.2 discusses functional testing through the use of Unit Testing and Randomised Testing. Section 4.3 then goes on to discuss performance, and then finally we analyse the security of ORAM in Section 4.4.

4.1 Overall Results

Overall, ORAM performed as expected in terms of functionality, performance, and security. It operated correctly, writing files and reading the same data back out. It continued to do so with the additions of statelessness, recursion, and encryption, so all parts of the system were functional separately and as a whole. In terms of performance, my implementation agreed with the theoretical bounds given in Stefanov et al. [14], which state $O(\log N)$ time overhead. Finally, statistical analysis showed that ORAM did indeed have a statistically random access pattern, ensuring the security of the implementation.

4.2 Unit Tests

Unit testing was used throughout the development process, which allowed me to be sure that individual components were functioning correctly, before I combined them into larger more complicated systems. This section describes the most important test cases that were examined for each module and discusses the use of randomised testing to cover a larger range of input values. Code coverage testing was also used to make sure that the tests were visiting all parts of the system.

4.2.1 Stash

There three main cases to test for the stash:

- Values not expected to be in the stash are not found

- Values that have been added to the stash are found
- Adding dummy blocks to the stash has no effect

During the development process, I simply hand-coded a small number of example cases, but in order to test more extensively, I coded the above three cases as properties for randomised testing. This generates a number of test cases and verifies that the properties hold in every case and allows us to cover far more cases than hand-coded tests alone.

4.2.2 Position Map

We need to test the two main aspect of the position map. Firstly, the translation of 64 bit addresses into 3 regular integers, and then its actually operation as a data structure.

The code performing the translation is essentially a mathematical definition in itself, so any property definition that we might use for randomised testing would probably just be the original code. Thus, in this instance we simply check a few hand chosen random cases, along with the important edge cases. The edge cases we want to check are the maximum, and minimum values, along with values either side of a change in the higher indices, that is values with output (x, y, max_int) and $(x, y + 1, 0)$, and similarly for the higher index.

In terms of operation, we want to check that on adding a value to the position map, we read back the same value, and that trying to add a value at an address outside of the allowed range results in an error. Both of these cases can be coded up as properties for randomised testing.

4.2.3 ORAM

The ORAM implementation is particularly amenable to randomised testing, because we tend to define inverse functions for most of its functionality. This makes it very easy to write properties for randomised testing of the form $f(f^{-1}(x)) = x$. This allows us to check that each stage of ORAM is operating correctly, from writing individual blocks, through writing entire paths, to writing entire files.

For other functions that were not so easy to code properties for, regular unit testing was used. These include reconnecting to ORAM to test statelessness, and performing simple calculations, such as computing the height for ORAM.

4.2.4 Free Map

The Free Map allocates blocks in the block device to be used by different parts of the file system. We need to make sure that on creation, it allocates the correct number of initial blocks, and then subsequently always allocates the correct number of blocks, if they are available, and only ever allocates blocks that are actually free.

During development, I used only a handful of test cases, covering some of the most important edge cases. This included creating the map, and checking that nothing was allocated to begin with except the first n, where n is passed into the creation function.

Then, I checked that allocating and deallocating a few different sequences of blocks led to the expected results. Finally, I checked that if there were not enough free blocks, that an error was returned.

After main development had finished, I added some randomised testing to this module to cover a wider range of inputs.

4.2.5 B-Trees

It is not possible to test the B-Tree library directly, because it only gives us a functor to create a B-Tree. Thus the B-Tree was tested using the Inode Index, the canonical use case of the library in this system. In order to test the B-Tree properly, it was necessary to use randomised testing to create a reasonable access pattern that was guaranteed to cause splitting of the root node.

4.2.6 Inodes

The inodes needed to be tested to make sure they could add and delete pointers, while maintaining a correct count. This is again amenable to randomised testing, although unit tests were also used throughout the development to test specific cases that would be expected to cause exceptions.

4.2.7 File System

In order to test the file system I created a large number of random files and used randomised testing to ensure that under any access pattern the correct files were always read back out once they had been written in. This also included testing to ensure that reasonable exceptions were produced when the file system became full, or a file did not exist in the system.

4.2.8 Search Module

The search module was tested for correctness, i.e. if a file containing a word had been put into the system, then it would appear in the search results. This was difficult to randomise, so I manually constructed a number of test scenarios.

4.3 Performance Testing

4.3.1 Parameter Optimisation

Before performing the main experiments, I decided to optimise the parameters of ORAM, in order to allow the experiments to run faster. The main parameter in question is the block size, which has been shown in Ousterhout et al. [9] to dramatically affect the speed of IO operations. I discovered that this was indeed the case with ORAM, as can be seen in Figure 4.1. It appeared that increasing the block size has an unbounded increase on

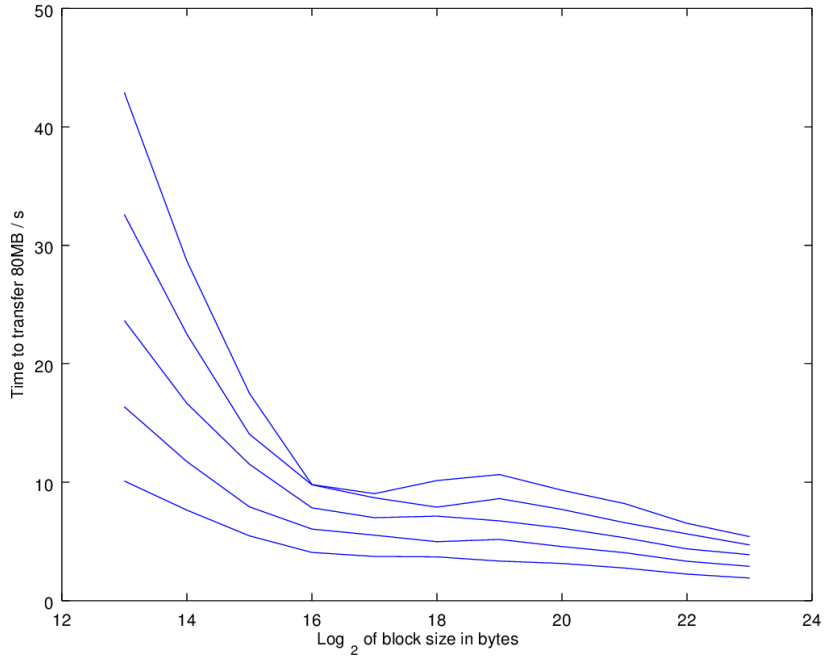


Figure 4.1: Plot of the time taken to transfer 80MB of data at varying block sizes and sizes of ORAM. Each line represents one ORAM size, so we can see that as block size increases, the time decreases.

performance, but I settled with a block size of 1MB to trade-off between speed and the possibility to specify the size the block device.

4.3.2 Comparison with Literature

In order to effectively test the overhead due to ORAM, two things needed to be done very carefully. The first was isolating and removing major sources of uncertainty. When I first ran the experiments, I attempted to run them on my local machine. In this environment, other processes interfered with the ORAM process, making the results unreliable. I secured a remote testing machine in order to run the experiments in complete isolation. Here, at first, they were running on an NFS mounted drive, meaning that network latency and protocol overheads were affecting the results. After moving the experiments to local disk, it was clear that ORAM overheads were finally dominating the results. Now, secondly, I needed to initialise the ORAMs properly. Using a fresh ORAM, the stash is empty and most of the blocks are dummy blocks that get disregarded. It is necessary to run an initialisation sequence in order to remove the effects of these on the results. We use the worst case sequence, writing each block in turn, then reading each block in turn, to ensure that all blocks get used multiple times, leaving the ORAM in a state that it would likely be in after extended use. Only once this steady state has been reached can we reliably test the performance.

For the actual experiment, rather than using the worst case sequence, we use a random sequence of block accesses. We perform 10 runs of 1000 iterations each, oscillating between

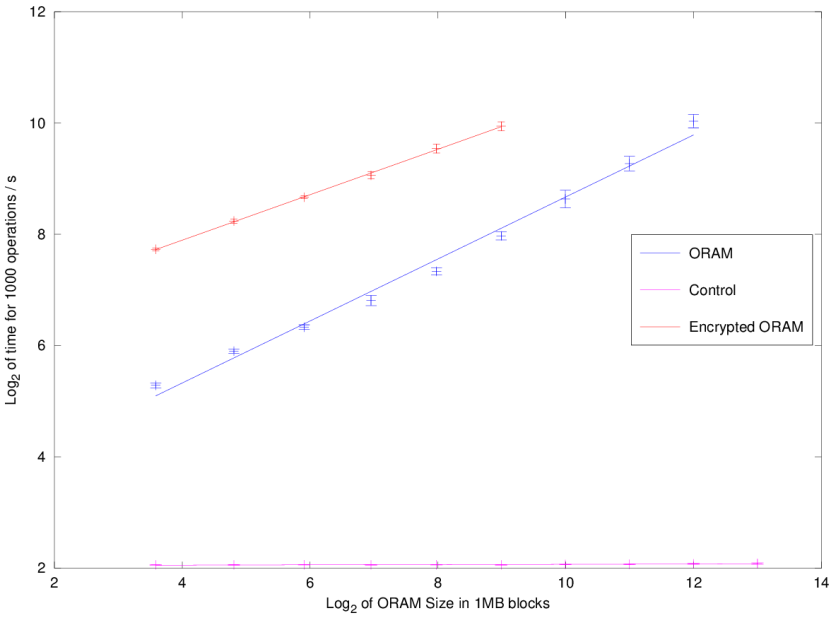


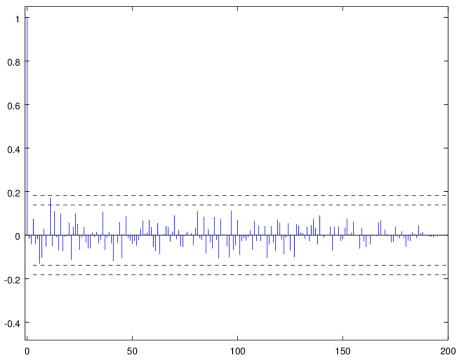
Figure 4.2: The relationship between size of an ORAM in blocks and the time taken for 1000 operations, plotted for ORAM, encrypted ORAM, and a control block device with no ORAM. We take logs of both axes, because block size was increased in powers of two and we expect a log relationship.

reads and writes in order to balance the different overheads. We are trying to see how the performance changes as the block size increases, so we use ORAMs of a range of sizes, from 12 blocks (a tree of depth 1) to 4092 blocks (a tree of depth 9). The log of the time taken for each ORAM to perform 1000 iterations is plotted against the logarithm of the size of the ORAM in Figure 4.2. We expect this to be a straight line, because we expect logarithmic overheads from ORAM, and we increase the block sizes roughly in powers of two. We can see that ORAM does indeed show a logarithm overhead compared to the control experiment that performed the same sequence of accesses to a block device without ORAM. I performed the same experiment adding encryption to ORAM and we can see that the overhead was still logarithmic, but with a larger constant.

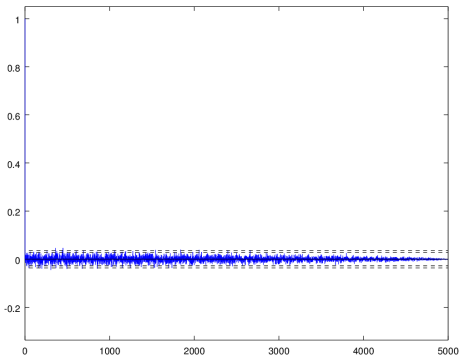
Although it has not been discussed here, bandwidth is another important measure when examining the performance of ORAM, especially when it is being used in the cloud. With more time, a detailed study of the bandwidth used with and without recursion would have been carried out, which would have given further insight into the trade-off that recursion presents us with.

4.4 Statistical Analysis

In order to test the effectiveness of ORAM, we need to show that the access pattern observed by the block device is actually statistically random. Of course there is structure to this randomness, because on each access a whole path is read and written, but the actual path that gets written should be random. Thus, we should perform a test for randomness on a sequence of path indices.



(a) Autocorrelation of a 200 iteration access pattern



(b) Autocorrelation of a 5,000 iteration access pattern

Figure 4.3: Two autocorrelation plots, with the autocorrelation coefficient on the y-axis and time lag on the x-axis. The dashed black lines represent confidence bands of 95% and 99%.

In order to do this we will use two common techniques: autocorrelation plotting and runs testing.

Autocorrelation plotting plots the correlation of a sequence with itself at various time lags [3]. For a random sequence the noise cancels itself out, so we want to see a plot of values very close to zero, apart from at lag 0, where the correlation will be exactly the power of the signal. I plotted this for two access patterns, one of length 200 so that the results would be easily visible, and one of length 5,000 to show longer term effects. For both of these patterns, the underlying access pattern, the one that would be seen without ORAM, was simply a succession of reads and writes to the same location. These plots are shown in Figure 4.3a and Figure 4.3b respectively.

The dashed black lines represent confidence bands of 95% and 99%, and in order to conclude that a sequence is random, we need almost all of the autocorrelation coefficients to lie within these bands. This is indeed the case, so ORAM has successfully taken a heavily non-random access pattern and turned it into a random one.

Runs testing attempts to detect non-random beheviour in a signal by counting the number of runs, sequences of values that are all above or below the median value. Too few runs in a sequence suggests a trend and too many runs suggests cyclic behaviour. For a large sample, we can approximate the distribution of runs using a normally distributed random variable, and compare this with the distribution that we measure. To generate the distribution, we take 1000 sample access patterns, each of length 180. We cut each pattern into 18 equally sized segments of size 10, take the mean of each segment, and count the number of runs in the sequence of means. There can be between 2 and 18 runs in each sequence, but we would expect 90% of the sequences to have between 7 and 14 runs, the 0.05% tail cut-offs for this distribution [7]. Figure 4.4 plots this distribution, along with the tail cut-offs, and 92.2% of samples fall within the bounds. Thus, we can conclude that the samples were generated from a random process.

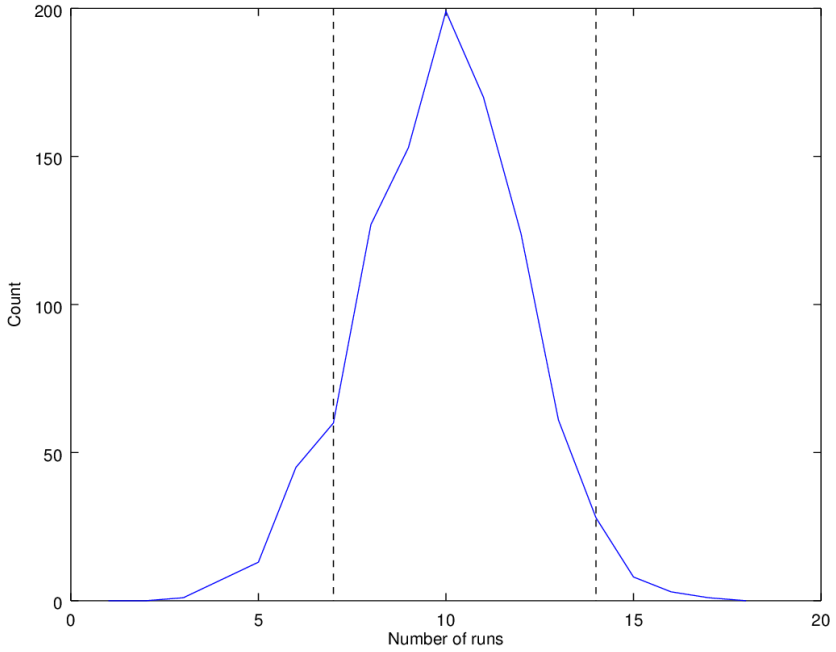


Figure 4.4: The distribution of the number of runs in 1000 access patterns of length 180. The dashed black lines represent 0.05% tail cut-offs, and 92.2% of values fall within these bounds.

Chapter 5

Conclusions

5.1 Results

I successfully implemented the Path ORAM protocol and showed that my implementation agreed with the theoretical overhead of $O(\log N)$ and that it is a statistically secure implementation.

5.2 Lessons Learnt

This project has taught me many important lessons. The one that has had the most impact is time management. I got stuck into the project straight away and did as much work on it as I could during the Michaelmas term and vacation, which meant that hiccoughs later down the line had a large buffer and didn't cause severe consequences. Related to this I learnt that predicting the time that each part of a project will take is very difficult and it is therefore important to be both conservative and realistic about the number of things that will go wrong. Finally, I learnt that you should seek advice as soon as there are signs of trouble, rather than waiting for a scheduled meeting to brings something up. The advice of experienced supervisors can turn the seemingly catastrophic into a minor disruption.

5.3 Future Work

There is still much work to be done in the field of ORAM. One of the main drawbacks of current schemes is that the size of the ORAM must be determined in advance and the overhead associated with expanding it is enormous. I would like to investigate the possibility of creating a resizable version of my implementation, however this was unfortunately not in the scope of this project.

There are also many optimisations to be performed on this implementation. Along with many interesting optimisations mentioned in the literature, there is of course plenty of room for analysis and optimisation of this specific implementation using benchmarking to identify code hot-spots. This would of course only reduce the constants involved in the performance, rather than providing an asymptotic improvement.

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Appendix A

Project Proposal

Computer Science Project Proposal

Encrypted Keyword Search Using
Path ORAM on MirageOS

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23 October 2015

Project Supervisors: Dr N. Sultana & Dr R. M. Mortier

Director of Studies: Dr B. Roman

Project Overseers: Dr M. G. Kuhn & Prof P. M. Sewell

Introduction and Description of the Work

As the cost of large-scale cloud storage decreases and the rate of data production grows, more and more sensitive data is being stored in the cloud. We, of course, want to encrypt our data, to ward off prying eyes, but this comes at a cost. We can no longer selectively retrieve parts of the data at will. We need some method of searching over encrypted data to find the parts we are interested in.

So let us say that Alice has a set of documents that she wants to store on an untrusted server, run by Bob. We'll first assume that Bob is "honest, but curious", that is, he will attempt to gather all knowledge that he can without deviating from the protocol. Alice wants to store her documents encrypted, but also wants to search over them without Bob being able to learn either the keywords she is searching for, or the results of any query, the documents that contain the keyword. In order to enable efficient search over the documents, Alice stores an encrypted index on the server along with the documents.

There are a number of schemes in the literature that use symmetric encryption techniques to build a searchable encryption scheme. They rely on the use of a trapdoor generating function, that allows Bob to search over the encrypted index and respond to Alice with the matching line from the encrypted index. Then Alice requests the relevant documents from Bob. Bob has a complete view of the communications channel, but does not have access to the trapdoor generating function. He simply sees a query in the form of trapdoor and then a number of requests for specific documents.

The problem is that these all leak the access pattern, so Bob knows which documents matched any query, even if he doesn't know what they matched. It turns out that this pattern of access can leak large amounts of information. In a study [6] on an encrypted email repository, up to 80% of plaintext search queries could be inferred from the access pattern alone! So clearly this a leak worth plugging, but how can we do it?

One solution to our problem is to use Oblivious Random Access Memory (ORAM), a cryptographic primitive that hides data access patterns. In our case, we move the searching and object retrieval functionality back to the client. That is, we turn Bob's server into a block device and we attempt to maintain the property that any two sequences of accesses of the form $(operation, address, data)$, that are the same length, have computationally indistinguishable physical access patterns. Bob should have no way of learning what *address* we are really accessing, and therefore will never know which documents matched a given search query.

A trivial ORAM algorithm operates by scanning over the whole ORAM and reading/updating only the relevant block, but this has $O(N)$ bandwidth cost, where N is the number of blocks, which is highly impractical for large-scale storage. Luckily, much better algorithms have been proposed. We choose to focus on Path ORAM [14], because it has only $O(\log N)$ bandwidth cost in the worst case if $B = \Omega(\log^2 N)$, as well as being incredibly simple conceptually.

Now let's assume that Bob has become malicious, and is modifying our encrypted data. In order to combat this, we can provide integrity verification by treating the ORAM as a Merkle tree, but with data in every node. The details of this scheme are outlined below after Path ORAM has been described further.

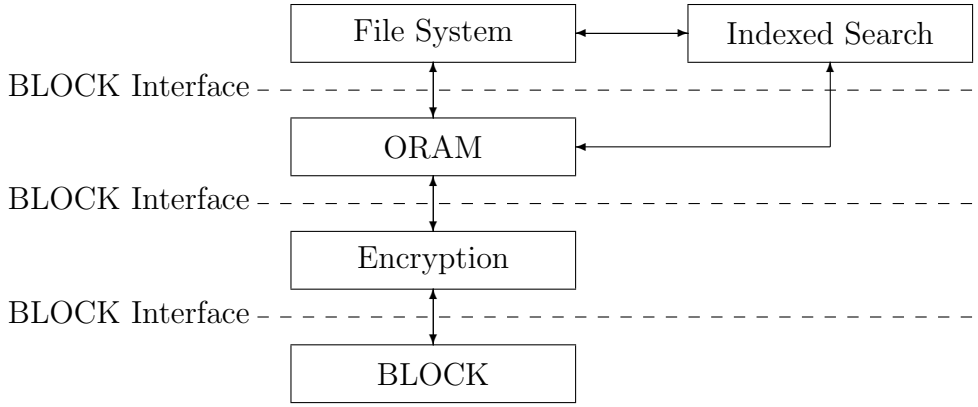

Figure A.1: The Application Stack: We can use any underlying BLOCK implementation and we can add/remove ORAM, Encryption or Search modules as we please

So the project is a searchable encrypted object store, with integrity verification. It will provide a simple, name-value pair API, that allows more complex filesystems to be built on top of it. A block diagram of the system is shown in Figure A.1.

Starting Point

MirageOS is a framework, that pulls together a number of libraries and syntax extensions, to provide a lightweight unikernel operating system, that is designed to run on the Xen hypervisor. A unikernel operating system is a single-address space machine image, customised to provide the minimum set of features to run an application. It provides a command line tool for generating the main file, that links together implementations of various parts of the system, and passes them to the unikernel. There are a number of module signatures that define the operation of devices, such as CONSOLE for consoles, ETHIF for ethernet, and most importantly for us BLOCK, for block devices.

I have chosen to use Mirage for a number of reasons. Firstly, it is lightweight and designed to be run in the cloud, meaning that simple cloud services can be built on top of it that fully leverage the ORAM. Secondly, it is written in OCaml, meaning that I can take full advantage of static typing and a rich module system.

This will allow me to write my implementation of ORAM and Encryption as a pair of functors that take an implementation of Mirage's BLOCK interface and create new BLOCK implementations, augmented with new features. This means that we can add and remove ORAM and Encryption as we like and the Object Store remains agnostic. This is shown in Figure A.1. It also means that we could use any underlying implementation of the BLOCK interface and that it would plug seamlessly into existing programs. There are currently two implementations of the BLOCK interface, one for Unix and one for Xen, and I would like to support both. This abstraction also allows for the use of cloud storage, implemented as a mapping between the BLOCK interface and a cloud provider's RESTful API.

Other OCaml libraries that will be of most use to me include nocrypto, which provides

a wide variety of cryptographic tools, Jane Street's Core library, which standardises and optimises many of OCaml's core modules, and LWT, a lightweight cooperative threading library that is used throughout Mirage.

Substance and Structure of the Project

Substance

The main focus of this project is the implementation and evaluation of the Path ORAM protocol. Encrypted search is our target domain and as such will be an integral part of the project, but it is the performance and security properties that Path ORAM provides that we are really interested in.

The Path ORAM protocol has three main components: a binary tree, a stash and a position map. The binary tree is the main storage space. Every node in the tree is a bucket, which can contain up to Z blocks. The tree has height L, where the tree of height 0 consists only of the root node, and the leaves are at level L. The stash is temporary client-side storage, consisting only of a set of blocks waiting to be put back into the tree. The position map associates, with each block ID, an integer between 0 and $2^L - 1$. The invariant that the Path ORAM algorithm maintains is that if the position of a block x is p, then x is either in some bucket along the path from the root node to the p^{th} leaf, or in the stash. On every access to the tree, a whole path is read into the stash, the accessed block is assigned a new random position and then as many blocks as possible are written back into the same path. The assignment to a random position means that, in any two access to the same block, the paths that are read are statistically independent.

We can extend the basic path ORAM algorithm with recursion. That is, calling the data ORAM $ORAM_0$, we store the position map of $ORAM_0$ in a smaller ORAM, $ORAM_1$, and the position map for this in an even smaller ORAM, $ORAM_2$. We can do this until we have a sufficiently small position map on the client. Supposing that we store χ leaf addresses in each PosMap ORAM, the position for a data block with address a_0 is at $a_1 = a_0/\chi$ in $ORAM_1$, and in general $a_n = a_0/\chi^n$ for the address in $ORAM_n$.

There is actually an issue with using Path ORAM in the context of MirageOS and cloud storage. If the Mirage instance crashes, then we lose the client-side state. With no position map, the ORAM becomes useless. To remedy this, we would have to read the entire contents out in one go and then reinsert it, resulting in a large overhead. There are two solutions to this problem: store the client-side state in persistent storage on the client, or upload the state to the server after every access. The second option is preferable, because it separates the ORAM implementation from the client machine. The client-side state is actually $O(\log N)$, so we should be able to store it on the server without increasing our complexity bounds.

As mentioned above, we can add integrity verification to Path ORAM by treating it as a Merkle tree. Each node will store a hash of the form $H = (b_1|| \cdots ||b_n||h_1||h_2)$, where b_n is the n^{th} block stored in the node, and h_1 and h_2 are the hashes of the left and right children. We always read and write the whole path at a time, so for the read or write of any single node we only have to read or write two hashes. For instance, on write, we

calculate the hash of the leaf node, which is then available for calculating the next level hash. So we only have to read the hash of the sibling of the leaf. This pattern is the same all the way up to the root of the tree.

In order to perform searches over our data, we will store along with it an encrypted inverted index. This is a data structure that, for any keyword, list the documents that contain it. The search module will build the index from the object store and then store the index using the object store. It will provide a search function that, given a keyword, will perform a simple scan over the inverted index and return the identifiers of documents that match it.

Evaluation

We need to test for functionality. Does the ORAM successfully write data and read it back out? Does the search function return documents correctly? This will consist of fairly trivial tests, writing objects to and from the block device and searching over them. A range of different types of documents will be used, including randomly (pre-)generated ones and entirely non-random ones, from sources such as Project Gutenberg.

We then want to evaluate performance. What is the overhead when we add the ORAM functor? How do recursion and statelessness further affect this? Does this correspond to the theoretical values from the literature? This will use tests similar to the above, but specifically focusing on time and space efficiency. Using the plain Object Store as a baseline, I will add in encryption and ORAM, separately and in combination to try and isolate the effects of each individual module.

Finally we want to test the security properties of the project. Is there any statistical correlation between access patterns? Do we provide adequate integrity verification? What, if anything, can be inferred about the search queries? Apart from integrity verification, which we can test by simply corrupting the ORAM and making sure that this is detected, the security comes down to the statistical independence of access patterns. If we can show, using statistical methods, that there is no correlation between two sequences of accesses with identical length, then we have security for not just storage, but for search as well, because this is protected by ORAM's security.

Structure

The project breaks down into the following sub-projects:

1. Familiarising myself with OCaml, MirageOS and related libraries
2. Implementing the basic Path ORAM functor and testing that it works in place of existing BLOCK device implementations
3. Implementing the Object Store, testing this and further testing ORAM using it
4. Adding recursion and statelessness to ORAM
5. Implementing and testing the search module

6. Adding the encryption layer
7. Creation of a suite of tests and experiments to evaluate the performance and security properties of each individual component and of the system as a whole
8. Writing the dissertation

Success Criteria for the Main Result

1. To demonstrate, through well chosen examples, that I have implemented a functionally correct Path ORAM functor with search capabilities
2. To demonstrate, through well chosen examples, that the implementation has the expected security properties, i.e. keeps access patterns hidden

Possible Extensions

There are a number of ways that this project could be extended. By the nature of the modular design, we can perform optimisations at any layer of the system. There have been a large number of optimisations to Path ORAM proposed in the literature, so if I achieve the goals of my main project, including evaluation, ahead of schedule I will examine these and potentially implement some of them.

In particular for Path ORAM, recursion does add an overhead, but we can reduce this overhead by exploiting locality. Assuming that programs will access adjacent data blocks, we can cache PosMap blocks in a PosMap Lookaside Buffer, so that if all χ data blocks that are referenced in a PosMap block are accessed in turn we only need to do the recursion once. Doing this naïvely, however, breaks security, because we are revealing information through the cache hit pattern. To avoid this we use Unified ORAM, which combines all of the recursive ORAMs into a single logical tree. We then use the address space to separate the levels of recursion, so addresses 1 to N are for data blocks, $N + 1$ to $N + (N/\chi)$ for $ORAM_1$ and so on. Now all accesses occur in the same tree, and the security of Path ORAM keeps the cache miss pattern hidden.

Another optimisation compresses the PosMaps, reducing the number of levels of recursion required to achieve the desired client side storage, resulting in an asymptotic bandwidth complexity decrease for ORAM with small block size.

The last two optimisations were originally designed and tested in a secure processor setting [10], so their application to the cloud storage setting is novel.

Another area that could be addressed is the limitation of Path ORAM (and other tree based ORAMs) to fixed-sized trees. We either need to know our storage requirements before setting up the ORAM, potentially wasting resources, or resize them in the naïve way as storage requirements increase. Resizability has been implemented for the ORAM construction of *Shi et al.* [12] in [8], but exploring the possibility of making Path ORAM resizable was left as an open research topic.

Timetable: Work plan and Milestones

Planned starting date is 16/10/2015.

1. **16/10/15 – 26/10/15** Familiarise myself with relevant Mirage libraries. Implement basic Path ORAM functor.
2. **27/10/15 – 09/11/15** Implement basic test harness. Start implementation of object store.
3. **10/11/15 – 23/11/15** Finish object store and use it to build more complex tests of the ORAM.
4. **24/11/15 – 04/12/15** Add recursion and statelessness to the ORAM.
5. **05/12/15 – 18/12/15** Write up implementation section of the dissertation for all parts completed so far.
6. **18/12/15 – 31/12/15** Write the search module and design tests for it.
7. **01/01/16 – 08/01/16** Write implementation section for the search module.
8. **09/01/16 – 29/01/16** Evaluate the project in its current state, achieving an acceptably complete project. Write the progress report.
9. **30/01/16 – 08/02/16** Write up the evaluation of the project so far.
10. **09/02/16 – 21/02/16** Incorporate encryption model and perform further evaluation using this.
11. **22/02/16 – 06/03/16** Submit first draft to supervisors for feedback and modify based on feedback.
12. **07/03/16 – 11/03/16** Perform further evaluation and refinement as necessary
13. **12/03/16 – 25/03/16** Write final draft of dissertation and then leave it until submission time, in order to focus on revision.
14. **01/05/16 – 13/05/16** Reread and make any final edits and then submit.

Resources Required

- My own laptop for implementation and testing
- My own external hard disk for backups
- GitHub for version control and backup storage
- MirageOS libraries as a basis for the project