HOMEWORK 1 FOR MODERN OPTIMIZATION METHODS FALL SEMESTER 2020

1. **Profit Maximization Problem for Fertilizer Company.** *(15 points)* A fertilizer company purchases nitrates, phosphates, potash and an inert chalk base at a cost of \$1500, \$500, \$1000 and \$100 per ton, respectively, and produces four fertilizers A, B, C, and D. The production cost, selling price, and composition of the four fertilizers are given below.

	Production	Selling	Percentage Composition			
Fertilizer	Cost	Price	Nitrates	Phosphates	Potash	Chalk
A	100	350	5	10	5	80
B	150	550	5	15	10	70
C	200	450	10	20	10	60
D	250	700	15	5	15	65

   During any week, no more than 1000 tons of nitrate, 2000 tons of phosphates, 1500 tons of potash will be available. The company is reqired to supply a minimum of 5000 tons of fertilizer A and 4000 tons of fertilizer D per week to its customers, but it is otherwise free to produce the fertilizers in any quantities it pleases. Formulate the problem of finding the quantity of each fertilizer to be produced by the company to maximize the profit. YOU ARE NOT REQUIRED TO SOLVE THIS OPTIMIZATION PROBLEM.

2. **Study Time Allocation Problem for a Student.** *(15 points)* A graduate student is enrolled in a 4-unit mathematics course and a 3-unit statistics course. Since the student has to work for 20 hours a week at a convenient store, he can spend a maximum of 40 hours a week to study outside the class. It is known from students who took the courses previously that the numerical grade (g) in each course is related to te study time spent outside the class as $g_m = t_m/6$ and $g_s = t_s/5$, where g indicates the numerical grade ($g = 4$ for A, $g = 3$ for B, $g = 2$ for C, $g = 1$ for D and $g = 0$ for F), t represents the time spent in hours per week to study outside the class, and the subscripts m and s denote the mathematics and statistics courses. The student enjoys statistics more than mathematics and hence would like to spend at least 75 minutes to study for statistics for every 60 minutes he spends to study mathematics. Also, as far as possible, the student does not want to spend more time on any course beyond the time required to earn a grade of A. The student wishes to maximize his grade point P, given by $P = 4g_m + 3g_s$ by suitably distributing his study time. Identify the data structure, objective function and all constraints in this optimization problem. YOU ARE NOT REQUIRED TO SOLVE THIS OPTIMIZATION PROBLEM.

3. **Programming of the Traditional Methods** *(70 points)* In this problem, you are going to write programs for four traditional methods applied to a search of maximum values of a 4D matrix. Please follow the guideline below:

   (a) Create a search domain. First, you need to create by yourself a $10 \times 10 \times 10 \times 10$ dataset X filled with 10000 randomly generated numbers ranging from 0 to 1. Depending on the programming language you use, find a way to save this data. You need to submit this data together with your homework, so that I can double check if your programs work correctly.

(b) To make things simple, your search space and your solution space are the same in this problem. In your program, define (i, j, k, l) as your search space, and the value $X[i, j, k, l]$ is your objective function value.

(c) Write the program of Exhaustive Enumeration. Consider if it is necessary to define the lower or upper bounds y in this program. Write down your optimal candidate solution and its objective function value.

(d) Define gb as the global best value you have obtained in (c). In addition, for the rest of three algorithms, we define the termination criterion as $pb \geq 0.95gb$ or the number of iterations reaches 100.

(e) Write the program of Random Sampling for the search of maximum values that terminates when the termination criterion reaches. Plot the objective function value of pb over the 100 iterations.

(f) Define the mutation of a candidate solution as it randomly chooses a new position within 1 step of each axis. For example, if the solution is now at $(2, 3, 4, 5)$, the mutation of this solution will choose one position from 16 choices of combinations among $i = 1, 3$, $j = 2, 4$, $k = 3, 5$ and $l = 4, 6$.

(g) Write the program of Hill Climbing for the search of maximum values that uses the mutation in (f) and terminates when the termination criterion reaches. Plot the objective function value of pb over the 100 iterations.

(h) Write the program of Random Walk for the search of maximum values that uses the mutation in (f) and terminates when the termination criterion reaches. Plot the objective function value of pb over the 100 iterations.

(i) Summary: please summarize and comment on the results of Random Sampling, Hill Climbing and Random Walks in terms of solution quality and runtime.

4. **More on Study Time Allocation Problems for a Student.** *(Bonus 10 points)* Linear programming is a useful and deterministic optimization method. It can be used to solve the problem like Question 2. Formally speaking, a linear program is to find X which minimize a linear function $f(X)$ subject to several linear equality constraints $\sum_i a_{ij}x_i = b_j$ and some boundary constraints $x_i \geq 0$. Given the statement of optimization problem in Question 2, use the linear programming method to find the optimal way to distribute the study time so that his grade can be maximized.