RWEPA | 程式設計 Python

SciPy 模組 -數值積分, 內插法

Business Data Analytics

- 按訂閱、讚、開啟小鈴鐺
- https://youtube.com/@alan9956
- http://rwepa.blogspot.com/

大綱

- 數值積分
- 內插法
- Q & A

數值積分

(Numerical integration)

scipy.integrate 三大功能

- General integration (quad) 積分- quad
- 數值積分
  - trapezoidal rule 梯形法則 - trapz
  - Simpson‘s rule 辛浦森法則 - simps
- 微分方程 ordinary differential equations - odeint

梯形法則 Trapezoidal rule

- 若 f 在區間 [a , b] 連續，則

$$\int_a^b f(x)dx = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

其中

$$\Delta x = \frac{b-a}{n},\ a = x_0,\ b = x_n,\ n = \text{梯形數目}$$

$$x_1 = x_0 + \Delta x,\ x_2 = x_1 + \Delta x,\ x_3 = x_2 + \Delta x,\ \text{L}$$

梯形法則 - 範例

Use the Trapezoidal Rule to approximate

$$\int_0^{\pi} \sin x\, dx.$$

Compare the results for $n = 4$ and $n = 8$, as shown in Figure 4.44.

Solution When $n = 4$, $\Delta x = \pi/4$, and you obtain

$$\int_0^{\pi} \sin x\, dx \approx \frac{\pi}{8}\left(\sin 0 + 2\sin\frac{\pi}{4} + 2\sin\frac{\pi}{2} + 2\sin\frac{3\pi}{4} + \sin\pi\right)$$

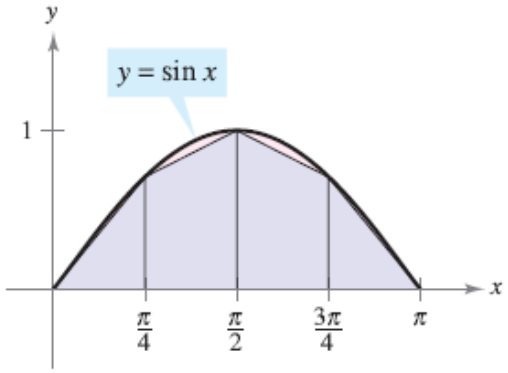
$$= \frac{\pi}{8}\left(0 + \sqrt{2} + 2 + \sqrt{2} + 0\right) = \frac{\pi\left(1 + \sqrt{2}\right)}{4} \approx 1.896.$$

When $n = 8$, $\Delta x = \pi/8$, and you obtain

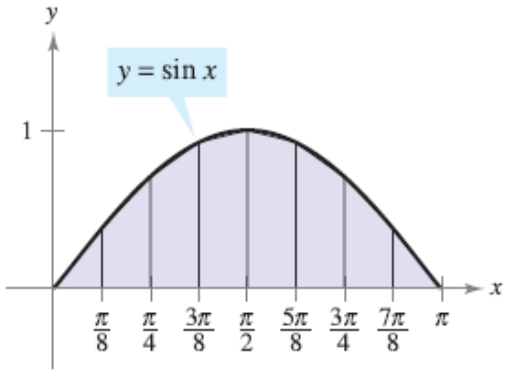
$$\int_0^{\pi} \sin x\, dx \approx \frac{\pi}{16}\left(\sin 0 + 2\sin\frac{\pi}{8} + 2\sin\frac{\pi}{4} + 2\sin\frac{3\pi}{8} + 2\sin\frac{\pi}{2}\right.$$

$$\left. + 2\sin\frac{5\pi}{8} + 2\sin\frac{3\pi}{4} + 2\sin\frac{7\pi}{8} + \sin\pi\right)$$

$$= \frac{\pi}{16}\left(2 + 2\sqrt{2} + 4\sin\frac{\pi}{8} + 4\sin\frac{3\pi}{8}\right) \approx 1.974.$$



Four subintervals



Eight subintervals

辛浦森法則

- 若 f 在區間 連續，則

$$\int_a^b f(x)dx = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\cdots$$
$$+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$$

其中

$$\Delta x = \frac{b-a}{n},\ a = x_0,\ b = x_n,\ n \text{ 必須為偶數}$$

$$x_1 = x_0 + \Delta x, x_2 = x_1 + \Delta x, x_3 = x_2 + \Delta x,\ L$$

辛浦森法則 - 範例

Use Simpson's Rule to approximate

$$\int_0^{\pi} \sin x \, dx.$$

Compare the results for $n = 4$ and $n = 8$.

Solution When $n = 4$, you have

$$\int_0^{\pi} \sin x \, dx \approx \frac{\pi}{12}\left(\sin 0 + 4 \sin \frac{\pi}{4} + 2 \sin \frac{\pi}{2} + 4 \sin \frac{3\pi}{4} + \sin \pi\right) \approx 2.005.$$

When $n = 8$, you have $\int_0^{\pi} \sin x \, dx \approx 2.0003$.

integrate.trapz 梯形法則
integrate.simps 辛浦森法則

```
In [1]: from scipy.integrate import trapz, simps

In [2]: import numpy as np

In [3]: def f1(x):
   ...:     return np.sin(x)

In [4]: x1 = np.linspace(0, np.pi, 5)
   ...: x1
Out[4]: array([0.        , 0.78539816, 1.57079633, 2.35619449, 3.14159265])

In [5]: trapint = trapz(f1(x1), x1)
   ...: print(trapint)
1.8961188979370398

In [6]: sinint = simps(f1(x1), x1)
   ...: print(sinint)
2.0045597549844207
```

1 2

trapz, simps

```
In [7]: x2 = np.linspace(0, np.pi, 9)
   ...: x2
Out[7]:
array([0.        , 0.39269908, 0.78539816, 1.17809725, 1.57079633,
       1.96349541, 2.35619449, 2.74889357, 3.14159265])

In [8]: trapint = trapz(f1(x2), x2)
   ...: print(trapint)
1.974231601945508

In [9]: sinint = simps(f1(x2), x2)
   ...: print(sinint)
2.000269169948387
```

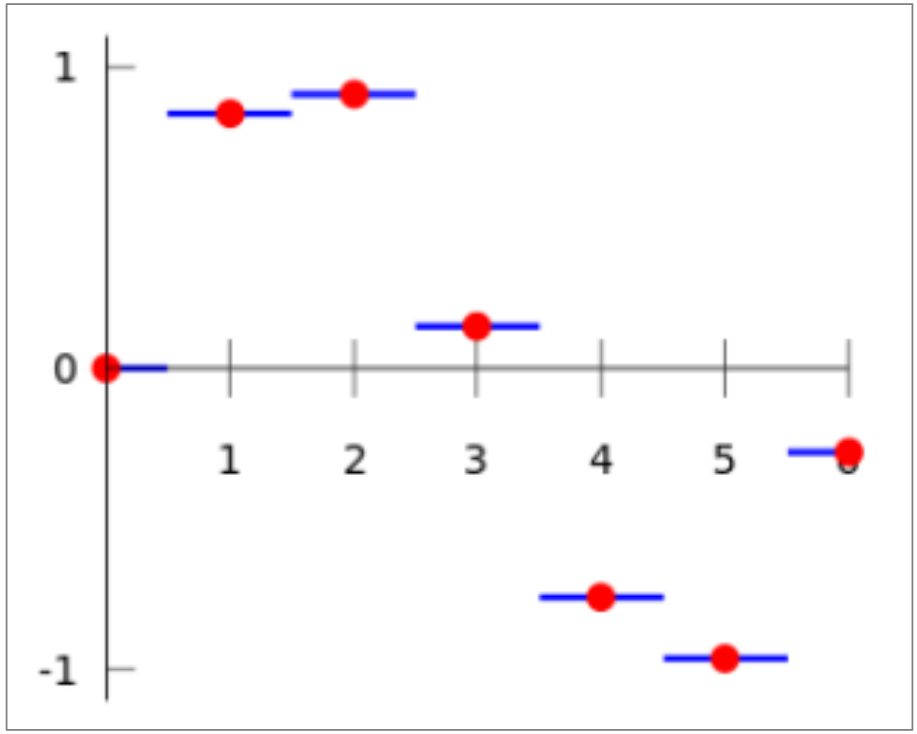
內插法

(Interpolation)

內插法

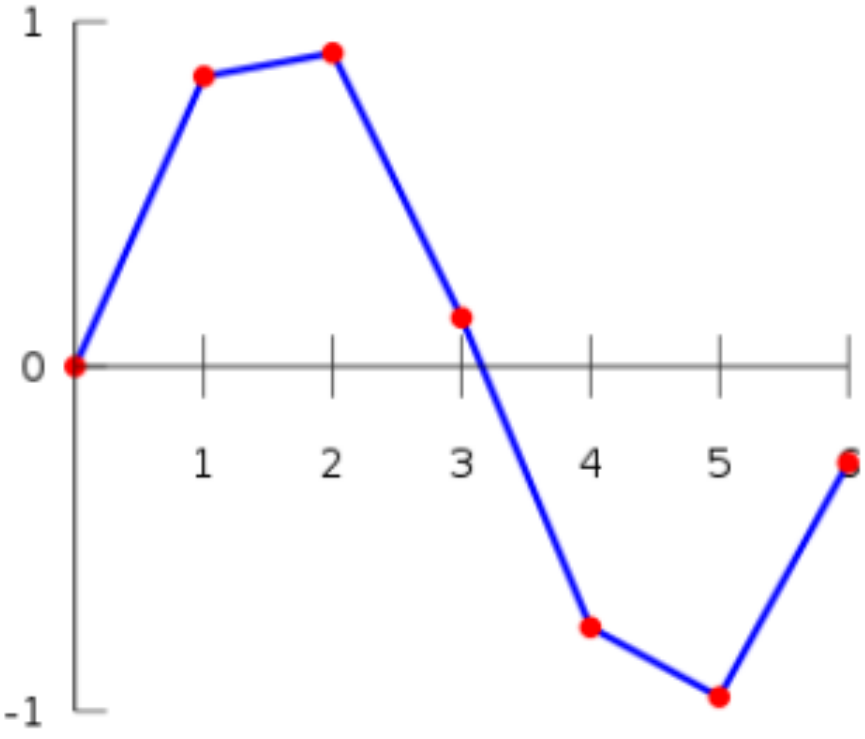
- 在數值分析的數學領域，內插法是一種估計，在一組已知數據點範圍內構造新數據點的方法。
- 求解科學和工程的問題時，通常有許多數據點藉由採樣、實驗等方法獲得，這些數據可能代表了有限個數值函數。而根據這些數據，我們希望得到一個連續的函數（也就是曲線）；或者更密集的離散方程式與已知數據互相吻合，這個過程叫做擬合（fitting）。
- 常用內插法技術
  - Nearest-neighbor interpolation　近鄰插值法
  - Linear interpolation　線性插值法
  - Polynomial interpolation　多項式插值法
  - Spline interpolation　樣條插值法

1.近鄰插值法



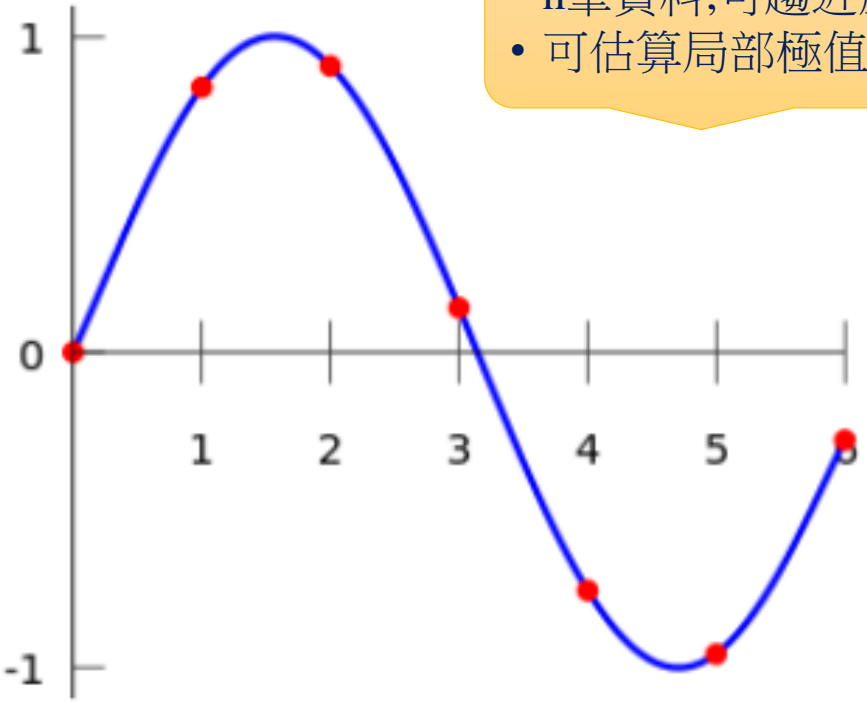
紅色:原始資料

2.線性插值法



3.多項式插值法

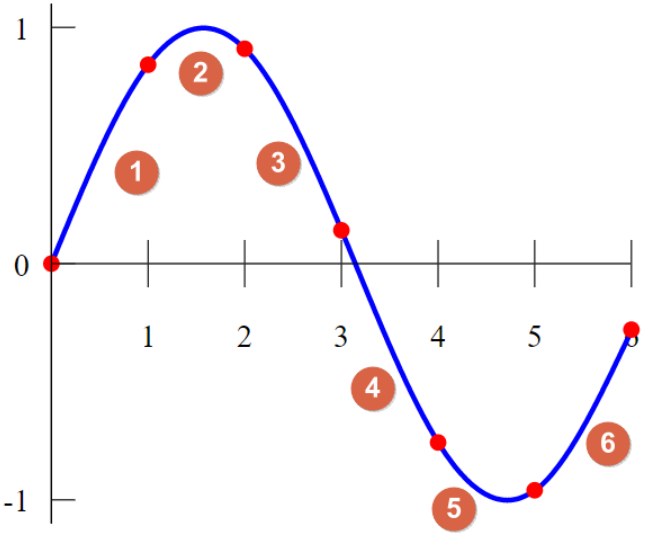
- n筆資料,可趨近於(n-1)階多項式
- 可估算局部極值



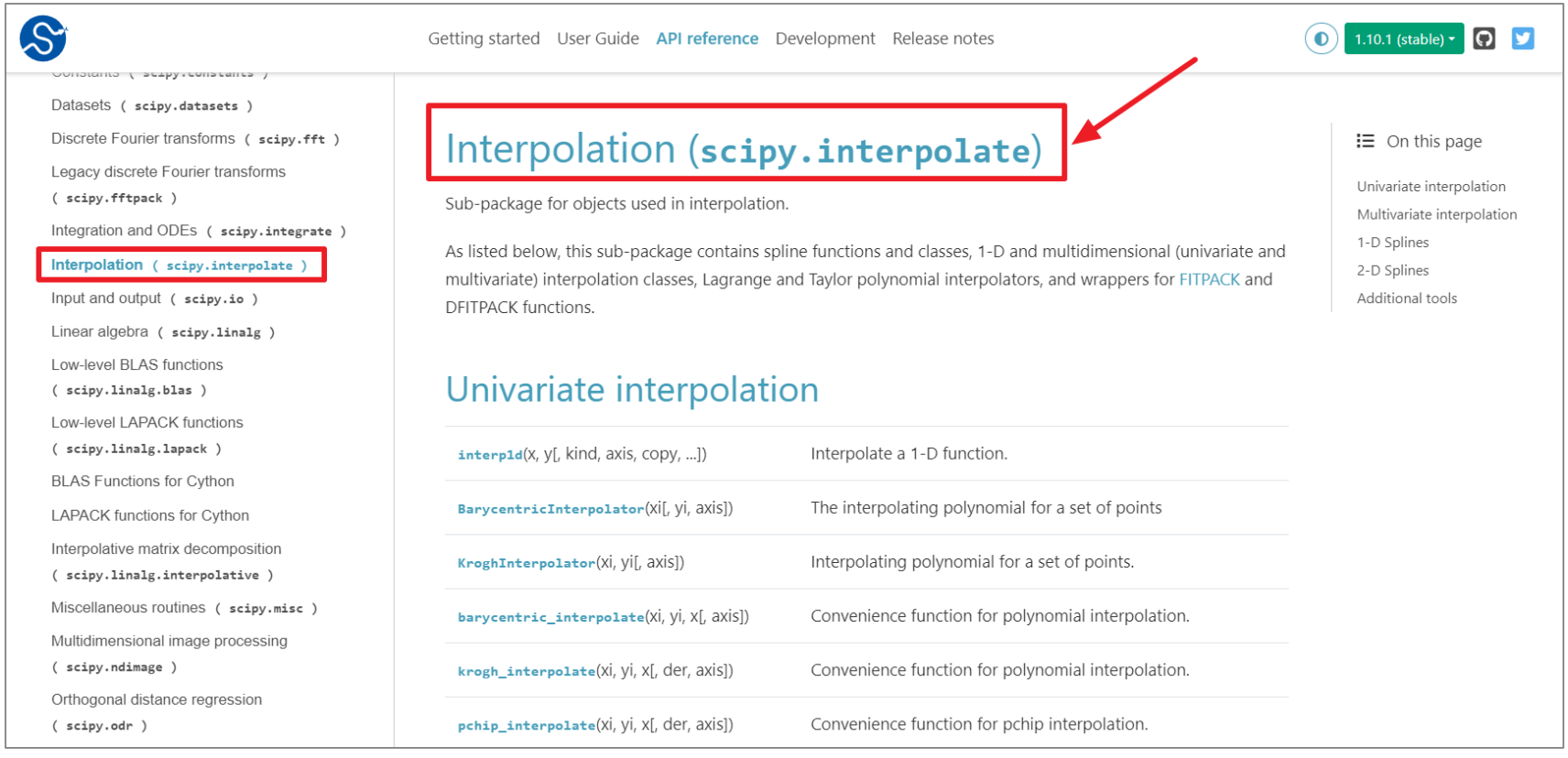
$$f(x) = -0.0001521x^6 - 0.003130x^5 + 0.07321x^4 - 0.3577x^3 + 0.2255x^2 + 0.9038x.$$

4.樣條插值法

$$f(x) = \begin{cases} -0.1522x^3 + 0.9937x, & \text{if } x \in [0,1], \\ -0.01258x^3 - 0.4189x^2 + 1.4126x - 0.1396, & \text{if } x \in [1,2], \\ 0.1403x^3 - 1.3359x^2 + 3.2467x - 1.3623, & \text{if } x \in [2,3], \\ 0.1579x^3 - 1.4945x^2 + 3.7225x - 1.8381, & \text{if } x \in [3,4], \\ 0.05375x^3 - 0.2450x^2 - 1.2756x + 4.8259, & \text{if } x \in [4,5], \\ -0.1871x^3 + 3.3673x^2 - 19.3370x + 34.9282, & \text{if } x \in [5,6]. \end{cases}$$

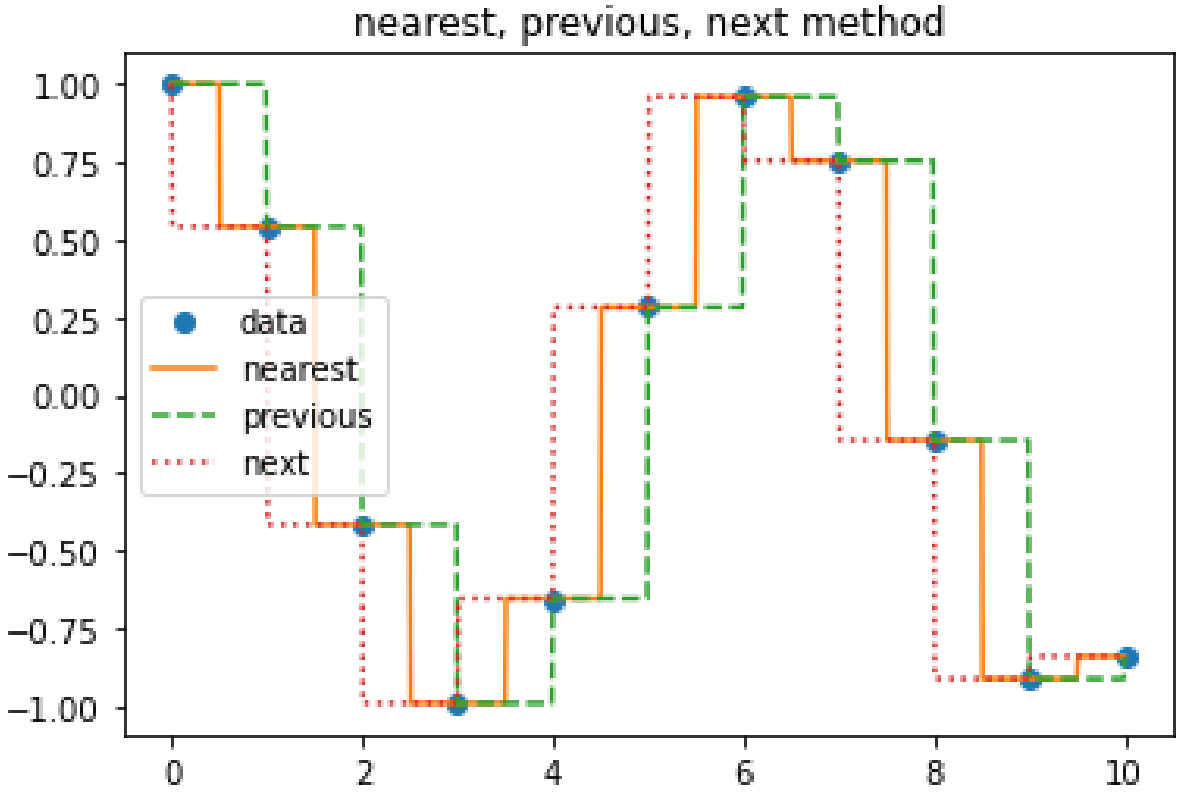


Interpolation (scipy.interpolate)

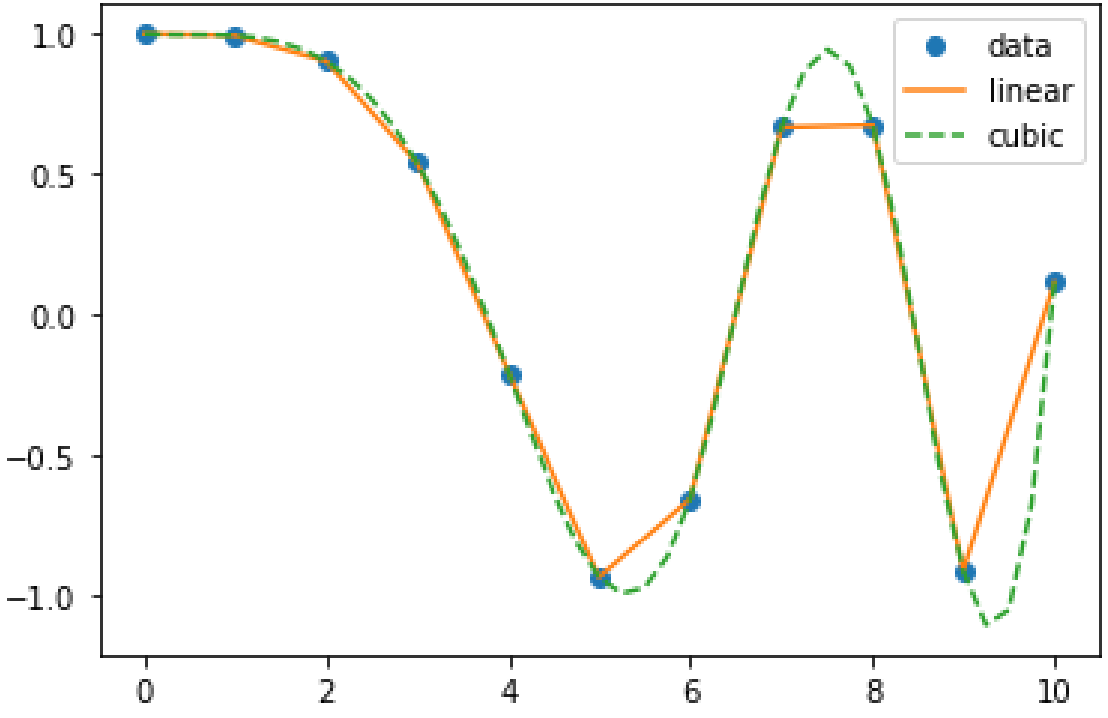


https://docs.scipy.org/doc/scipy/reference/interpolate.html

近鄰插值法


nearest, previous, next method
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
2
4
6
8
10
data
nearest
previous
next

interp1d – cubic (續)



謝謝您的聆聽

Q & A

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