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# Learning to Trade Power

by

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# Abstract

Connectionist reinforcement learning methods approximating value-functions offer few convergence guarantees, even in simple systems. Reinforcement learning has been applied previously to agent-based simulation of energy markets using only discrete action and sensor domains. If learning algorithms are to deliver on their potential for application in operational settings, modelling continuous domains is necessary. The contribution of this thesis is to show that policy-gradient reinforcement learning algorithms can be applied to continuous representations of energy trading problems and that their superior use of sensor data results in improved performance over previously applied value-function methods. From this it follows that algorithms which search directly in the policy space will be better suited to decision support applications and automated energy trade.

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# Chapter 1

## Introduction

This thesis describes the application of value-function and policy gradient reinforcement learning algorithms to the electric energy trade problem. This chapter introduces the problem and explains the motivation for the research. The goals of the research are stated along with the principle contributions. Finally, a reading guide is provided that provides an overview of the remaining chapters.

### 1.1 Motivation/Setting the scene

Two trends characterise modern power systems Engineering: Increased liberalisation of the industry through competitive energy trade and increased presence of renewable energy generation on the network. As the number and variety of electricity sources becomes greater, the necessity for automated trade of their power increases. Control algorithms may draw sensor input from data networks and other sources and use some relevant measure of performance, such as profitability, to learn from trading decisions.

Methods including learning classifier systems, genetic algorithms and reinforcement learning have been successfully used to research the characteristics of energy markets in the past (Weidlich & Veit, 2008). This is an alternative to the traditional closed-form equilibrium approaches to game theory research in which behavior emerges from the interactions of many separable, self-serving agents. Typically in these studies, an agent is associated with a portfolio of generating units and/or dispatchable loads. Interaction with the environment involves submission of offers to sell or bids to buy<sup>1</sup> a quantity of power at a specified price in a particular time period. The learning algorithms typically use revenue or earnings as a reward signal and adjust the policy used to select offer/bid price and quantity values.

Research into energy trade using reinforcement learning typically involves discretization of the action domain, often into incremental markups on marginal cost (Weidlich & Veit, 2008). Also, either sensor domains are discretized or state information is disregarded altogether. Despite these

<sup>1</sup>Beware that certain authors may use the term “bid” to refer to an offer to sell when discussing single-sided auctions.

simplifications, authors have been drawn many practical conclusions from this approach[ref].

In the field of robotic control, environment state, action and observation spaces are often continuous or mixed. Traditional reinforcement learning algorithms such as Sarsa and Q-learning can be applied to systems with continuous domains by using connectionist systems for value function approximation(Barto, Sutton, & Anderson, 1983). However, feedback between policy updates and value function changes can result in oscillations or divergence in these algorithms even when applied to simple systems(Peters & Schaal, 2008). In response to this, policy-gradient methods, pioneered by Williams(Williams, 1992), which search directly in the policy space have been developed and applied in many real-life settings(Sutton, Mcallester, Singh, & Mansour, 2000; Peters & Schaal, 2006; Moody & Saffell, 2001; Peshkin & Savova, 2002).

## 1.2 Problem statement/Aims & Objectives

Engineers must strive for complexity in their work. Rarely will a simple solution will perform a function to a higher degree than a more complex one. Certainly, where a function is either performed or not performed, prefer the simpler one, but most often problems can be solved to varying degrees.

The broad aim of the research presented in this thesis is to prove that the above conjecture applies to reinforcement learning algorithms for power trade. Previous research in this field (See Chapter 3 below) has used very simple algorithms in relation to those from the latest advances in artificial intelligence (See Sections 2.2.8 and 2.2.7 below). The goal is to prove that policy gradient methods, using artificial neural networks for policy function approximation, are better suited to learning the complex dynamics of a power system.

## 1.3 Research contributions

This paper compares policy-gradient reinforcement learning algorithms REINFORCE and ENAC with value-function methods Roth-Erev,  $Q(\lambda)$  and Sarsa in their relative ability to trade electricity competitively. Power systems are modelled as balanced three-phase AC networks in the steady-state. Offers/bids for active and reactive power from agent participants are cleared using AC optimal power flow with an auction interface that returns single period revenue and earnings values. Through individual and multi-player experiments the methods are compared in their ability to learn quickly, compete in large systems and exploit characteristics of the power system. It is shown that, in electricity trade, policy-gradient methods:

- converge successfully on an optimal policy,
- are slower to converge than value-function algorithms,
- can learn more complicated characteristics of the power system than value-function algorithms,
- scale better when applied to larger systems and reactive power markets.

Section 2.1 of this paper presents the power system model, the optimal power flow formulation and the auction interface from MATPOWER. Reinforcement learning methods Sarsa,  $Q(\lambda)$ , Roth-Erev, REINFORCE and ENAC are defined in Section 2.2. Section 4 introduces the three experiments used to compare the aforementioned methods. Numerical results from these experiments are reported in Section 5 and an interpretation and critical analysis of them is given in Section 6. Finally, a review of related research is presented in Section 3 and Section 9 provides a conclusion.

## **1.4 Thesis structure/Overview/Reading guide**

This thesis is focussed on the application of standard and advanced reinforcement learning algorithms to a particular problem domain. The reader will require a certain degree of prior knowledge, or must be willing to read much of the referenced material, to fully understand the methodology taken. The intended audience is engineering and economics researchers interested in the application of reinforcement learning algorithms to the problem of trading energy in electric power systems.

## Chapter 2

# Background

### 2.1 Optimal Power Flow

Computation of the generator dispatch points is executed using parts of the of the optimal power flow formulation from MATPOWER (R. Zimmerman, Murillo-Sánchez, & Thomas, 2009). In order that the optimal power flow routine could be coupled with agents from the machine learning library PyBrain, the MATLAB<sup>TM</sup> source code from MATPOWER was translated to the Python programming language. With the permission of the MATPOWER developers the resulting package has been released under the terms of version 2 of the GNU General Public License as a project named PYLON (Lincoln, Galloway, & Burt, 2009). Sparse matrix objects from the convex optimisation library CVXOPT were used to allow the implementation to scale well to solving for very large systems.

This section describes parts of the optimal power flow formulation, unit-decommitment algorithm and auction interface from MATPOWER that were used to represent a centralised electricity market. Notable components of the full optimal power flow formulation (available in (R. D. Zimmerman & Murillo-Sánchez, 2007)) that have been ignored are shunt capacitors and inductors, generator P-Q capability curves and dispatchable loads. The power system model is described by defining the bus, branch and generator objects. The power flow equations associated with a network of these components are subsequently defined. The constrained cost variable approach to modelling generator cost functions from (Wang, Murillo-Sanchez, Zimmerman, & Thomas, 2007) is introduced, from which the optimal power flow formulation follows.

The experiments described in Section 4 require an optimal power flow problem to be solved at each step. To accelerate the simulation process for certain experiments the option to use a linearised DC formulation is used, the formulation of which is provided also. The tradeoffs between using DC models over AC have been examined in (Overbye, Cheng, & Sun, 2004) and found reasonable for locational marginal price calculations.

Since the optimal power flow formulations do not facilitate shutting down expensive generators, the unit-decommitment algorithm from MATPOWER is defined. Finally, to provide an interface to agent participants that resembles that of real electricity market, MATPOWER's auction wrapper

for the optimal power flow routine is described.

### 2.1.1 Power System Model

The power system is assumed to be a three-phase AC system operating in the steady-state and under balanced conditions in which it may be represented by a single phase network of busbars connected by branch objects.

#### Branches

Each branch is modelled as a medium length transmission line in series with a transformer at the *from* end. A nominal- $\pi$  model with total series admittance  $y_s = 1/(r_s + jx_s)$  and total shunt capacitance  $b_c$  is used to represent the transmission line. The transformer is assumed to be ideal and both phase-shifting and tap-changing, with the ratio between primary winding voltage  $v_f$  and secondary winding voltage  $N = \tau e^{j\theta_{ph}}$  where  $\tau$  is the tap ratio and  $\theta_{ph}$  is the phase shift angle. From Kirchhoff's current law the current in the series impedance is

$$i_s = \frac{b_c}{2} v_t - i_t \quad (2.1)$$

and from Kirchhoff's voltage law the voltage across the secondary winding of the transformer is

$$\frac{v_f}{N} = v_t + \frac{i_s}{y_s} \quad (2.2)$$

Substituting  $i_s$  from (2.1), gives

$$\frac{v_f}{N} = v_t - \frac{i_t}{y_s} + v_t \frac{b_c}{2y_s} \quad (2.3)$$

and rearranging in terms of  $i_t$ , gives

$$i_t = v_s \left( \frac{-y_s}{\tau e^{j\theta_{ph}}} \right) + v_r \left( y_s + \frac{b_c}{2} \right) \quad (2.4)$$

The current through the secondary winding of the transformer is

$$N^* i_f = i_s + \frac{b_c}{2} \frac{v_f}{N} \quad (2.5)$$

Substituting  $i_s$  from (2.1), gives

$$N^* i_f = \frac{b_c}{2} v_t - i_t + \frac{b_c}{2} \frac{v_f}{N} \quad (2.6)$$

Substituting  $\frac{v_f}{N}$  from (2.3) and rearranging, gives

$$i_s = v_s \left( \frac{1}{\tau^2} \left( y_s + \frac{b_c}{2} \right) \right) + v_r \left( \frac{y_s}{\tau e^{-j\theta}} \right) \quad (2.7)$$

## Generators

Each generator  $i$  is modelled as an apparent power source  $s_g^i = p_g^i + jq_g^i$  at a specific bus  $k$ , where  $p_g^i$  is the active power injection and  $q_g^i$  the reactive power injection, each expressed in per-unit to the system base MVA. Upper and lower limits on  $p_g^i$  are specified by  $p_{max}^i$  and  $p_{min}^i$ , respectively, where  $p_{max}^i > p_{min}^i \geq 0$ . Similarly, upper and lower limits on  $q_g^i$  are specified by  $q_{max}^i$  and  $q_{min}^i$ , respectively, where  $q_{max}^i > q_{min}^i$ .

## Buses and Loads

At each bus  $k$ , constant active power demand is specified by  $p_d^k$  and reactive power demand by  $q_d^k$ . Upper and lower limits on the voltage magnitude at the bus are defined by  $v_m^{k,max}$  and  $v_m^{k,min}$ , respectively. For one bus with an associated generator, designated the *reference* bus, the voltage angle is  $\theta_k^{ref}$  and typically valued zero. Dispatchable loads are modelled as generators with negative  $p_g^i$ , where  $p_{min}^i < p_{max}^i = 0$ .

### 2.1.2 AC Power Flow Equations

For a network of  $n_b$  buses,  $n_l$  branches and  $n_g$  generators, let  $C_g$  be the  $n_b \times n_g$  bus-generator connection matrix such that the  $(i, j)^{th}$  element of  $C_g$  is 1 if generator  $j$  is connected to bus  $i$ . The  $n_b \times 1$  vector of complex power injections from generators at all buses is

$$S_{g,bus} = C_g \cdot S_g \quad (2.8)$$

where  $S_g = P_g + jQ_g$  is the  $n_g \times 1$  vector with the  $i^{th}$  element equal to  $s_g^i$ .

Combining (2.7) and (2.4), the *from* and *to* end complex current injections for branch  $l$  are

$$\begin{bmatrix} i_f^l \\ i_t^l \end{bmatrix} = \begin{bmatrix} y_{ff}^l & y_{ft}^l \\ y_{tf}^l & y_{tt}^l \end{bmatrix} \begin{bmatrix} v_f^l \\ v_t^l \end{bmatrix} \quad (2.9)$$

where

$$y_{ff}^l = \frac{1}{\tau^2} \left( y_s + \frac{b_c}{2} \right) \quad (2.10)$$

$$y_{ft}^l = \frac{y_s}{\tau e^{-j\theta_{ph}}} \quad (2.11)$$

$$y_{tf}^l = \frac{-y_s}{\tau e^{j\theta_{ph}}} \quad (2.12)$$

$$y_{tt}^l = y_s + \frac{b_c}{2} \quad (2.13)$$

Let  $Y_{ff}$ ,  $Y_{ft}$ ,  $Y_{tf}$  and  $Y_{tt}$  be  $n_l \times 1$  vectors where the  $l$ -th element of each corresponds to  $y_{ff}^l$ ,  $y_{ft}^l$ ,  $y_{tf}^l$  and  $y_{tt}^l$ , respectively. Furthermore, let  $C_f$  and  $C_t$  be the  $n_l \times n_b$  branch-bus connection matrices, where  $C_{f,i,j} = 1$  and  $C_{t,i,k} = 1$  if branch  $i$  connects *from* bus  $j$  *to* bus  $k$ . The  $n_l \times n_b$

branch admittance matrices are

$$Y_f = \mathbf{diag}(Y_{ff})C_f + \mathbf{diag}(Y_{ft})C_t \quad (2.14)$$

$$Y_t = \mathbf{diag}(Y_{tf})C_f + \mathbf{diag}(Y_{tt})C_t \quad (2.15)$$

and relate the complex bus voltages  $V$  to the branch *from* and *to* end current vectors

$$I_f = Y_f V \quad (2.16)$$

$$I_t = Y_t V \quad (2.17)$$

The  $n_b \times n_b$  bus admittance matrix is

$$Y_{bus} = C_f^T Y_f + C_t^T \quad (2.18)$$

and it relates the complex bus voltages to the nodal current injections

$$I_{bus} = Y_{bus} V \quad (2.19)$$

The complex power losses from all branches are expressed as a non-linear function of  $V$

$$\begin{aligned} S_{bus}(V) &= \mathbf{diag}(V) I_{bus}^* \\ &= \mathbf{diag}(V) Y_{bus}^* V^* \end{aligned} \quad (2.20)$$

The complex power injections at the *from* and *to* ends of all branches are also expressed as a non-linear functions of  $V$

$$\begin{aligned} S_f(V) &= \mathbf{diag}(C_f V) I_f^* \\ &= \mathbf{diag}(C_f V) Y_f^* V^* \end{aligned} \quad (2.21)$$

$$\begin{aligned} S_t(V) &= \mathbf{diag}(C_t V) I_t^* \\ &= \mathbf{diag}(C_t V) Y_t^* V^* \end{aligned} \quad (2.22)$$

### 2.1.3 DC Power Flow Equations

The same power system model is used in the formulation of the linearised DC power flow equations, but the following additional assumptions are made:

- The resistance  $r_s$  and shunt capacitance  $b_c$  of all branch can be considered negligible.

$$y_s \approx \frac{1}{jx_s}, b_c \approx 0 \quad (2.23)$$

- Bus voltage magnitudes  $v_{m,i}$ , are all approximately 1 per-unit.

$$v_i \approx 1e^{j\theta_i} \quad (2.24)$$

- The voltage angle difference between bus  $i$  and bus  $j$  is small enough that

$$\sin \theta_{ij} \approx \theta_{ij} \quad (2.25)$$

Applying the assumption that branches are lossless from (2.23), the quadrants of the branch admittance matrix, (2.10), (2.11), (2.12) and (2.13), approximate to

$$y_{ff}^l = \frac{1}{jx_s\tau^2} \quad (2.26)$$

$$y_{ft}^l = \frac{-1}{jx_s\tau e^{-j\theta_{ph}}} \quad (2.27)$$

$$y_{tf}^l = \frac{-1}{jx_s\tau e^{j\theta_{ph}}} \quad (2.28)$$

$$y_{tt}^l = \frac{1}{jx_s} \quad (2.29)$$

Applying the uniform bus voltage magnitude assumption from 2.24 to (2.9), the branch *from* end current approximates to

$$i_f \approx \frac{e^{j\theta_f}}{jx_s\tau^2} - \frac{e^{j\theta_t}}{jx_s\tau e^{-j\theta_{ph}}} \quad (2.30)$$

$$= \frac{1}{jx_s\tau} \left( \frac{1}{\tau} e^{j\theta_f} - e^{j(\theta_t + \theta_{ph})} \right) \quad (2.31)$$

The branch *from* end complex power flow  $s_f = v_f i_f^*$  therefore approximates to

$$s_f \approx e^{j\theta_f} \cdot \frac{j}{x_s\tau} \left( \frac{1}{\tau} e^{-j\theta_f} - e^{j(\theta_t + \theta_{ph})} \right) \quad (2.32)$$

$$= \frac{1}{x_s\tau} \left[ \sin(\theta_f - \theta_t - \theta_{ph}) + j \left( \frac{1}{\tau} - \cos(\theta_f - \theta_t - \theta_{ph}) \right) \right] \quad (2.33)$$

Applying the voltage angle difference assumption from 2.25 yields the approximation

$$p_f \approx \frac{1}{x_s\tau} (\theta_f - \theta_t - \theta_{ph}) \quad (2.34)$$

Let  $B_{ff}$  and  $P_{f,ph}$  be the  $n_l \times 1$  vectors where  $B_{ff_i} = \frac{1}{x_s^i\tau^i}$  and  $P_{f,ph_i} = \frac{-\theta_{ph}^i}{x_s^i\tau^i}$ . If the system  $B$  matrices are

$$B_f = \mathbf{diag}(B_{ff})(C_f - C_t) \quad (2.35)$$

$$B_{bus} = (C_f - C_t)^T B_f \quad (2.36)$$

then the real power bus injections are

$$P_{bus}(\Theta) = B_{bus}\Theta + P_{bus,ph} \quad (2.37)$$



where  $\Theta$  is the  $n_b \times 1$  vector of bus voltage angles and

$$P_{bus,ph} = (C_f - C_t)^\top + P_{f,ph} \quad (2.38)$$

The active power flows at the branch *from* ends are

$$P_f(\Theta) = B_f \Theta + P_{f,ph} \quad (2.39)$$

and  $P_t = -P_f$  since branches are assumed to be lossless.

#### 2.1.4 AC OPF Formulation

Generator active and, optionally, reactive power output costs are defined by convex  $n$ -segment piecewise linear cost functions.

$$c^{(i)}(x) = m_i p + c_i \quad (2.40)$$

for  $p_i \leq p \leq p_{i+1}$ ,  $i = 1, 2, \dots, n$  where  $m_{i+1} \geq m_i$  and  $p_{i+1} > p_i$ . Since these costs are non-differentiable the constrained cost variable approach from (Wang et al., 2007) is used to make the optimisation problem smooth. For each generator  $i$  a helper cost variable  $y_i$  added to the objective function. Inequality constraints

$$y_i \geq m_{i,j}(p - p_j) + c_j, \quad j = 1 \dots n \quad (2.41)$$

require  $y_i$  to lie on the epigraph of  $c^{(i)}(x)$ . The objective of the optimal power flow problem is to minimise the sum of the cost variables for all generators.

$$\min_{\theta, V_m, P_g, Q_g, y} \sum_{i=1}^{n_g} y_i \quad (2.42)$$

Equality constraints enforce the balance between generator complex power injections  $S_g$  and the sum of apparent power demand  $S_d$  and the branch losses expressed in (2.20).

$$S_{bus}(V) + S_d - S_g = 0 \quad (2.43)$$

Branch complex power flow limits  $S_{max}$  are enforced by the inequality constraints

$$|S_f(V)| - S_{max} \leq 0 \quad (2.44)$$

$$|S_f(V)| - S_{max} \leq 0 \quad (2.45)$$

The reference bus voltage angle  $\theta_i$  is fixed by the equality constraint

$$\theta_i^{ref} \leq \theta_i \leq \theta_i^{ref}, \quad i \in \mathcal{I}_{ref} \quad (2.46)$$

Upper and lower limits on the optimisation variables  $V_m$ ,  $P_g$  and  $Q_g$  are enforced by the inequality constraints

$$v_m^{i,min} \leq v_m^i \leq v_m^{i,max}, \quad i = 1 \dots n_b \quad (2.47)$$

$$p_g^{i,min} \leq p_g^i \leq p_g^{i,max}, \quad i = 1 \dots n_g \quad (2.48)$$

$$q_g^{i,min} \leq q_g^i \leq q_g^{i,max}, \quad i = 1 \dots n_g \quad (2.49)$$

### 2.1.5 DC OPF Formulation

Piecewise linear cost functions are also used to define generator active power costs in the DC optimal power flow formulation. Since the power flow equations are linearised, following assumptions (2.23), (2.24) and (2.25), the optimal power flow problem simplifies to a linear program. The voltage magnitude variables  $V_m$  and generator reactive power set-point variable  $Q_g$  are eliminated following assumption (2.25) since branch reactive power flows depend on bus voltage angle differences. The objective function reduces to

$$\min_{\theta, P_g, y} \sum_{i=1}^{n_g} y_i \quad (2.50)$$

Combining the nodal real power injections, expressed as a function of  $\Theta$ , from (2.37) with active power generation  $P_g$  and active demand  $P_d$ , the power balance constraint is

$$B_{bus}\Theta + P_{bus,ph} + P_d - C_g P_g = 0 \quad (2.51)$$

Limits on branch active power flows  $B_f\Theta$  and  $B_t\Theta$  are enforced by inequality constraints

$$B_f\Theta + P_{f,ph} - F_{max} \leq 0 \quad (2.52)$$

$$-B_f\Theta + P_{f,ph} - F_{max} \leq 0 \quad (2.53)$$

The reference bus voltage angle equality constraint from (2.46) and the  $p_g$  limit constraint from (2.48) are applied also.

### 2.1.6 Unit Decommitment

In the OPF formulation above (See section 2.1) the solver must attempt to dispatch generators within their minimum and maximum power limits. Expensive generators can not be completely shutdown even if doing so would result in a lower total system cost. To achieve this an implementation of the *unit decommitment* algorithm (See Algorithm 1, below) from MATPOWER was used (R. D. Zimmerman & Murillo-Sánchez, 2007, p. 20). The algorithm finds the least cost dispatch by solving repeated OPF problems with different combinations of generating units at their minimum active power limit deactivated. The lowest cost solution is returned when no further improvement can be made and no candidate generators remain.

---

**Algorithm 1** Unit decommitment

---

```
1: initialise  $N \leftarrow 0$ 
2: solve initial OPF
3:  $L_{tot} \leftarrow$  total load capacity
4: while total min gen. capacity  $> L_{tot}$  do
5:    $N \leftarrow N + 1$ 
6: end while
7: repeat
8:   for  $c$  in candidates do
9:     solve OPF
10:  end for
11: until done = True
```

---

### 2.1.7 Auction Interface

Solving the optimisation problem defined above (See section 2.1) is intended to represent the function of a pool market operator. To present agents participating in this market with a simpler interface, more representative of a pool market an implementation of the “smart market” auction clearing mechanism from MATPOWER was used (R. D. Zimmerman & Murillo-Sánchez, 2007, p. 31). Using this interface the OPF problem is formulated from a list of offers to sell and bids to buy power.

An offer/bid specifies a quantity of power in MW and a price for that power in \$/MWh, to be traded over a particular period of time. The market accepts sets of offers and bids and uses the solution of the unit decommitment algorithm to return sets of *cleared* offer and bids. The cleared offers/bids can then be used to produce dispatch orders from which values of revenue and earnings/losses may be determined.

The market features the ability to set maximum offer price limits and minimum bids price limits. The process of clearing the market begins by withholding offers/bids outwith these limits, along with those specifying non-positive quantities. Valid offers/bids for each generator are then sorted into non-decreasing/increasing order and used to form new piecewise-linear cost functions and adjust the generator’s active power limits.

The dispatch points and nodal prices from solving a unit decommitment OPF with the newly configured generators as input are used in the auction clearing mechanism to determine the proportion of each offer/bid block that should be cleared and the associated price for each.

Different pricing options arise from the fact that a gap in which any price is acceptable to all participants may exist between the last accepted offer price and the last accepted bid price (See Figure X). This allows, for example, the prevention of bids setting the price, even when they are marginal, by selecting the *last accepted offer* auction type.

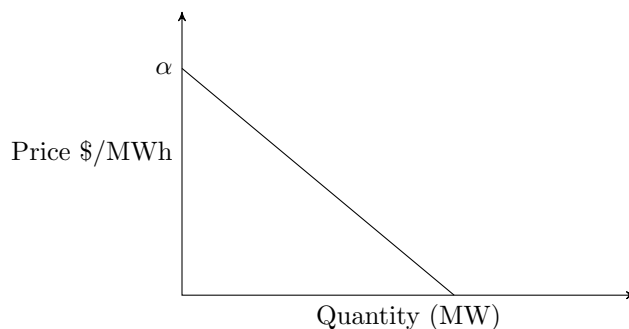


Figure 2.1: Acceptable price range

## 2.2 Reinforcement Learning

This section describes agent's policies that represent a store of experience and the learning algorithms that its modify parameters. Together these components form models of individual behavior which are used to determine the actions to be performed in the agent's environment and to learn from received rewards.

For a comprehensive introduction to reinforcement learning with evaluations of algorithm designs through mathematical analysis and computational experiments the interested reader is directed to the seminal work by Barto and Sutton

### 2.2.1 Introduction

The problem of learning how best to interact with an environment so as to maximise some long-term reward is one that arises in many aspect of life. Reinforcement learning is a term that is typically applied to understanding, automating and solving this problem through computational approaches. Unlike with the majority of Machine Learning techniques, the algorithms are not instructed as to which actions to take, but must learn to maximise the long-term reward through trial-and-error.

Reinforcement learning starts with an interactive, goal-seeking individual and an associated environment. The individuals require the ability to sense aspects of their environment, perform actions that influence the state of their environment and be assigned rewards as a response to their chosen action. An agent is said to follow a particular *policy* when mapping the perceived state of its environment to an action choice.

Value-based methods attempt to find the optimal policy by approximating a *value-function* which returns the total reward an agent can expect to accumulate, given an initial state and following the current policy thereafter.

Policy-gradient methods are an alternative to this which represent a policy using a learned function approximator with its own parameters The function approximator is updated according to the gradient of expected reward with respect to these parameters.

## 2.2.2 Basic Roth-Erev

The Roth-Erev reinforcement learning algorithm uses a stateless policy to select actions from a discrete domain (Roth et al., 1995; Erev & Roth, 1998). The dataset stored by each agent,  $j$ , contains an array of length  $K$ , where  $K$  is the number of feasible actions  $k$ . Each value in the array represents the propensity for selection of the associated action in all states of the environment. Following interaction  $t$  in which agent  $j$  performed on the environment action  $k'$ , for arbitrary positive  $t$ , a reward,  $r_{jk'}(t)$ , is calculated. The propensity for agent  $j$  to select action  $k$  for interaction  $t + 1$  is

$$q_{jk}(t+1) = \begin{cases} (1 - \phi)q_{jk}(t) + r_{jk'}(t)(1 - \epsilon), & k = k' \\ (1 - \phi)q_{jk}(t) + r_{jk'}(t)(\frac{\epsilon}{K-1}), & k \neq k' \end{cases} \quad (2.54)$$

where  $\phi$  and  $\epsilon$  denote *recency* and *experimentation* parameters, respectively. The recency (forgetting) parameter degrades the propensity for all actions and prevents the value from going unbounded. It is intended to represent the tendency for players to forget older action choices and to prioritise more recent experience. The experimentation parameter prevents the probability of choosing an action from going to zero and thus encourages exploration of the action space.

Erev and Roth proposed that actions be selected according to a discrete probability distribution function where action  $k$  is selected for interaction  $t + 1$  with probability:

$$p_{jk}(t+1) = \frac{q_{jk}(t+1)}{\sum_{l=0}^K q_{jl}(t+1)} \quad (2.55)$$

Since  $\sum_{l=0}^K q_{jl}(t+1)$  increases with  $t$ , a reward  $r_{jk}(t)$  for performing action  $k$  will have a greater effect on the probability  $p_{jk}(t+1)$  during early interactions while  $t$  is small. This is intended to represent Psychology's *Power Law of Practice* in which it is qualitatively stated that, with practice, learning occurs at a decaying exponential rate and that a learning curve will eventually flatten out.

This algorithm may alternatively use a form of the *softmax* method (Sutton & Barto, 1998) using the Gibbs, or Boltzmann, distribution to select action  $k$  for the  $t + 1$ th interaction with probability

$$p_{jk}(t+1) = \frac{e^{q_{jk}(t+1)/\tau}}{\sum_{l=0}^K e^{q_{jl}(t+1)/\tau}} \quad (2.56)$$

where  $\tau$  is the *temperature* parameter. This parameter may be decreased in value over the course of an experiment since high values give all actions similar probability and encourage exploration of the action space, while low values promote exploitation of past experience.

## 2.2.3 Variant Roth-Erev

Two shortcomings of the basic Roth-Erev algorithm (§2.2.2) have been identified and a variant formulation proposed (Nicolaisen, Petrov, & Tesfatsion, 2002). The problems are that the values by which propensities are updated can be zero or very small for certain combinations of the experimentation parameter  $\epsilon$  and the total number of feasible actions  $K$ . Also, all propensity values are

decreased by the same amount when the reward,  $r_{jk'}(t)$  is zero. Under the variant algorithm the propensity of agent  $j$  to select action  $k$  for interaction  $t + 1$  becomes:

$$q_{jk}(t+1) = \begin{cases} (1 - \phi)q_{ik}(t) + r_{jk'}(t)(1 - \epsilon), & k = k' \\ (1 - \phi)q_{ik}(t) + q_{jk}(t)(\frac{\epsilon}{K-1}), & k \neq k' \end{cases} \quad (2.57)$$

As with the basic Roth-Erev algorithm, the propensity for the action that the reward is associated with is adjusted by the experimentation parameter. All other action propensities are adjusted by a small proportion of their current value.

#### 2.2.4 SARSA

The SARSA algorithm is an on-policy Temporal Difference control method, similar to Q-learning. The action-value update for agent  $j$  is defined by

$$Q_j(s_{jt}, a_{jt}) + \alpha[r_{jt+1} + \gamma Q_j(s_{jt+1}, a_{jt+1}) - Q_j(s_{jt}, a_{jt})]. \quad (2.58)$$

While the Q-learning algorithm updates action-values using a greedy policy, which is a different policy to that being followed, SARSA uses the discounted future reward of the next state-action observation following the original policy.

#### 2.2.5 Q-Learning

The formulation of the Q-learning algorithm used is that of the original off-policy Temporal Difference algorithm developed by Watkins(Watkins, 1989). The action-value function,  $Q(s, a)$ , returns values from a  $M \times N$  matrix where  $M$  and  $N$  are arbitrary positive numbers equal to the total number of feasible states and actions, respectively. Each value represents the *quality* of taking a particular action,  $a$ , in state  $s$ . Actions are selected using either the  $\epsilon$ -greedy or softmax (See section 2.2.2) methods. The  $\epsilon$ -greedy method either selects the action (or one of the actions) with the highest estimated value or it selects an action at random, uniformly, independently of the estimated values with, typically small, probability  $\epsilon$ .

Agent  $j$  will observe a reward,  $r_{jt}$ , and a new state,  $s_{jt+1}$ , after taking action  $a_{jt}$  at step  $t$  when in state  $s_{jt}$ . The state-action value,  $Q_j(s_{jt}, a_{jt})$ , is updated according to the maximum value of available actions in state  $s_{t+1}$  and becomes

$$Q_j(s_{jt}, a_{jt}) + \alpha[r_{jt+1} + \gamma \max_a Q_j(s_{jt+1}, a_{jt}) - Q_j(s_{jt}, a_{jt})] \quad (2.59)$$

where  $\alpha$  and  $\gamma$  are the learning rate,  $0 \leq \alpha \leq 1$ , and discount factor,  $0 \leq \gamma \leq 1$ , respectively. The learning rate determines the extent to which new rewards will override the effect of older rewards. The discount factor allows the balance between maximising immediate rewards and future rewards to be set.

### 2.2.6 $Q(\lambda)$

With the Q-learning formulation, described in equation 2.59, only the quality associated with the previous state,  $s_{jt}$ , is updated. However, the preceding states can also, in general, be said to be associated with the reward  $r_{jt+1}$ . Eligibility traces are a mechanism for facilitating this effect and in algorithms such as  $Q(\lambda)$ , the  $\lambda$  refers to it. The eligibility trace for a state  $e(s)$  represents how eligible the state  $s$  is to receive credit or blame for the error. The term “trace” refers to fact that only recently visited states become eligible. The eligibility value for the current state is increased, while for all other states it is attenuated by a factor  $\lambda$ .

The off-policy nature of Q-learning requires special care to be taken when implementing eligibility traces. While the algorithm may learn a greedy policy, in which the action with the maximum value would always be taken, typically a policy with some degree of exploration will be followed when choosing actions. If an exploratory (pseudo-random) step is taken the preceding states can no longer be considered eligible for credit or blame. Setting  $\lambda$  to 0 for non-greedy actions removes much of the benefit of using eligibility traces if exploratory actions are frequent. A solution to this has been developed, but requires a very complex implementation (Peng & Williams, 1996). A naïve approach can be taken, where the effect of exploratory actions is ignored, but the results of this are unexplored.

### 2.2.7 REINFORCE

The previously defined learning methods typically rely upon discretisation of the sensor and action spaces so the associated values may be stored in tables. The memory requirements for this restrict the application of these methods to only small environments. Many environments, particularly from real applications, exhibit continuous sensor and/or action spaces and require generalisation techniques to be employed to provide a more compact policy representation.

REINFORCE is an associative reinforcement learning algorithm that determines a policy by modifying the parameters of a policy function approximator, rather than approximating a value function (Williams, 1992). Commonly, feedforward artificial neural networks are used to represent the policy, where the input is a representation of the state and the output is action selection probabilities. In learning, a *policy gradient* approach is taken where the weights of the network are adjusted in the direction of the gradient of expected reinforcement.

Defining the network, let  $\mathbf{x}^i$  denote the vector of inputs to the  $i$ th unit and  $y_i$  denote output of the unit. In the input layer of the network the elements  $x_j$  of  $\mathbf{x}^i$  are normalised sensor values from the environment and in the output layer, or in any hidden layers, they are outputs from the  $j$  unit in the preceding layer. Let  $\mathbf{w}^i$  denote the vector of the weights,  $w_{ij}$ , on the connections to the  $i$ th unit. The output of the  $i$ th unit is dependant on the vector of inputs,  $\mathbf{x}^i$ , and the associated weights,  $\mathbf{w}^i$ .

For each interaction of the agent with the environment, each parameter  $w_{ij}$  is incremented by

$$\Delta w_{ij} = \alpha_{ij}(r - b_{ij}) \frac{\partial \ln \rho_i}{\partial w_{ij}} \quad (2.60)$$

where  $\alpha_{ij}$  is the *learning factor*,  $b_{ij}$  is the *reinforcement baseline* and  $\rho_i$  is the performance of

the policy (e.g., the average reward per interaction).

### **2.2.8 ENAC**

ToDo: Episodic Natural Actor Critic(Peters & Schaal, 2008).



## Chapter 3

# Related Work

Relative to the traditional closed-form equilibrium approaches, agent-based simulation of (electricity) markets is a new field of research. For comprehensive reviews and surveys of the many different techniques that have been applied in recent years the interested reader is directed to (Weidlich & Veit, 2008; Tesfatsion & Judd, 2006; Visudhiphan, 2003). This section will focus on reviewing literature from the field in which reinforcement learning techniques were applied in combination with explicit power system models. A short review is also provided of some more general applications of reinforcement learning with connectionist systems and policy-gradient methods.

### 3.1 Learning algorithm comparison

### 3.2 Q-learning

Krause et al. have published agent-based energy market research in which Q-learning methods were applied while considering physical system properties. In a comparison between Nash equilibrium analysis and agent-based simulation, the suitability of bottom-up modelling for the assessment of market evolution was assessed (Krause et al., 2004). This is built upon in subsequent publications which evaluate the influence on market power and social welfare of three congestion management schemes and which analyse strategic behavior in combined gas and electricity markets (Krause & Andersson, 2006; Kienzle, Krause, Egli, Geidl, & Andersson, 2007). Power Transmission Distribution Factors (PTDF) are used in place of explicit power flow equations in determining line flows. The action domain of generating agents is limited to 0%, 5% and 10% markups on true marginal costs. The implementation of the Q-learning method used does not differentiate between environment states when selecting actions. This is a modification to the traditional formulation that still results in convincing conclusions, as with the popular Roth-Erev method.

There are similar applications of Q-learning in which states *are* defined, but none model the AC transmission system. A common approach is to use categorised market price from the previous period as state information (Bakirtzis & Tellidou, 2006; Xiong, Okuma, & Fujita, 2004).

### 3.3 Roth-Erev

The AMES (Agent-based Modeling of Electricity Systems) power market test bed is a software package that models core features of the Wholesale Power Market Platform (WPMP) – a market design proposed by the US Federal Energy Regulatory Commission (FERC) in April 2003 for common adoption in regions of the US(Sun & Tesfatsion, 2007a). The design features:

- a centralised structure managed by an independent market operator,
- parallel day-ahead and real-time markets and
- locational marginal pricing.

Learning agents may represent load serving entities or generating companies and learn using implementations of the Roth-Erev method (See sections 2.2.2 and 2.2.3, above) built on the Repast agent simulation toolkit(Gieseler, 2005). The permissive license under which the source code for these algorithms has been released allowed direct translation of them for use in this study. Agents learn from the solutions of hourly bid/offer based DC-OPF problems formulated as quadratic programs and solved using QuadProgJ(Sun & Tesfatsion, 2007b).

The ability of generator agents to learn particular supply offers has been demonstrated along with the plotting and data handling capabilities of AMES using a 5-bus network model(Li & Tesfatsion, 2009). The same network has been used to investigate strategic capacity withholding in FERC wholesale power market design(Hongyan & Tesfatsion, 2009). Generator agents report linear marginal cost functions to the market operator and supply functions are formed through linear interpolation between the prices at minimum and maximum production limits. Load serving agents submit combinations of fixed demand bids and price-sensitive bid functions, the ratio between which is varied between 0.0 and 1.0 to test physical and economic capacity withholding potential. Comparing results from a benchmark case (in which true production costs are reported, but higher than marginal cost functions may be reported) and cases in which reported production limits may be less than the true values, the authors find, that with sufficient capacity reserve, no evidence to suggest potential for inducing higher net earnings through capacity withholding in the WPMP.

# Chapter 4

## Methodology

Societies reliance on secure energy supplies and the high volumes of electricity typically consumed render it impractical to experiment with radically new approaches to energy trade on real systems. This section explains the approach taken modelling real systems in software such that they may be simulated computationally. The method by which the physical power systems, that deliver electricity to consumers, were modeled is given, as well as for the mechanisms that facilitate trade and participants that utilise these mechanisms.

### 4.1 Electricity network model

High voltage transmission and distribution networks are the mechanisms by which traded electric energy is delivered to consumers. Limits to line/cable power flows, outages and reactive power availability can impose constraints on particular trades. As such, certain technical characteristics of the networks are fundamental to energy market operation and must be duly modeled.

#### 4.1.1 Power Flow

The problem to be solved is finding the steady-state operating point of the network when given levels of generation and load are present. The primary constraints in a power system are the branch flow limits and the voltage limits at each bus. The system must be operated such that these constraints are not violated.

#### 4.1.2 Common Information Model

Many tools exist for steady-state analysis of balanced three-phase AC networks and most are centred around bespoke models that describe the power system data. Several attempts have been made in the past to standardise the format in which power system data is stored [CDF, UKGDS, ODF] and latest and most popular is the Common Information Model.

The Common Information Model (CIM) is an abstract ontological model that describes the elements of national electric power systems and the associations between them. CIM is an evolving

international standard approved by the International Electrotechnical Commission (IEC).

Unlike many tool specific models the CIM does not simplify the power system into a graph of buses connected by branches. Instead it describes each of the components in the system and the electrical connectivity between them. Conventional numerical techniques for steady-state analysis of AC power systems require a simplified bus-branch model such that when the voltage angle and magnitude at each bus is determined the power flows on each branch may be calculated.

#### **4.1.3 Energy market model**

Mechanisms for facilitating competitive trade between electricity producers and consumers differ greatly in the specifics of their implementations in countries throughout the world. However, fundamentally they either provide a centralised pool through which all electricity is bought and sold or they permit producers and suppliers to trade directly.

The UK transmission network is frequently congested[1]. The thermal limits of transmission lines between particular areas are often reached. The balancing mechanism is the financial instrument used by the system operator to resolve constraint issues and energy imbalances. Should the market not be suitably effective in this function the system operator may choose to contract outwith the balancing mechanism. By way of incentive to match demand and avoid congestion, imbalance charges are imposed on responsible participants. There is some evidence to suggest that centralised resolution by a system operator and socialisation of the incurred costs leads to inefficient despatch of generators[Neuhoff].

There are a number of alternative approaches to congestion resolution.

#### **4.1.4 Transmission capacity rights**

One approach is to issue contracts for transmission capacity rights or equivalent financial rights. The maximum available transmission capacity being auctioned for certain periods of time and firm contracts made entitling owners to full compensation upon curtailment or withdrawal.

The states of Pennsylvania, New Jersey and Maryland (PJM) operate a non-compulsory power pool with nodal market-clearing prices based on competitive bids. This is complemented by daily and monthly capacity markets plus the monthly auction of Financial Transmission Rights to provide a hedging mechanism against future congestion charges.

#### **4.1.5 Transmission charging**

Impose delivery charges which increase as network constraints are approached.

#### **4.1.6 Extended bids/offers**

Request extended bids and offers which include costs associated with the adjustment of participant's desired position.

## 4.2 Market participant model

Without competition between market participants there is no driver for individuals to improve efficiency and reduce costs paid by the consumers. Traders are typically responsible for this, but it is not feasible to use humans for this project. In a highly distributed power system, a very large number of items of plant may be supplying the demand and, depending on the levels of aggregation, this could require many traders to be used. Also, this project requires that experiments be repeated numerous times under a variety of scenarios.

### 4.2.1 Software agents

Participants are modeled in software also. The nature of a highly distributed power system dictates that a very large number of entities may be interacting in the marketplace. Economic studies regularly integrate participant logic into the same optimisation problem as the market. However, this does not scale to large numbers of individual participants. Separating participant logic into individual software agents allows their action selection procedures to be processed in simultaneously. The definition of an agent in this context emerges from the machine learning technique employed to implement the competitive decision making process.

### 4.2.2 Reinforcement learning

While there is a wealth of data available on past energy market activity involving conventional transmission connected plant, there exists no such resource for trade performed in highly distributed power systems. Consequently, reactive machine learning techniques that use new data to influence the decision making policy are used.

Reinforcement learning is a sub-area of machine learning and can be applied to a wide variety of problems. To allow the same learning algorithms developed for traditional, academic reinforcement learning problems (chess, backgammon, lift scheduling etc.) to be applied to models of energy markets (and vice versa) a modular machine learning library is used.

## Chapter 5

# Results

## Chapter 6

# Discussion

## Chapter 7

# Critical analysis



## Chapter 8

### Future work

## Chapter 9

# Summary conclusions

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