University of Strathclyde

Department of Electronic and Electrical Engineering

Learning to Trade Power

by

Richard W. Lincoln

A thesis presented in fulfilment of the
requirements for the degree of

Doctor of Philosophy

2010

Signed: Date: September 4, 2010

Acknowledgements

I wish to thank Professor Jim McDonald for giving me the opportunity to study at The Institute for Energy and Environment and for giving me the freedom to pursue my own research interests. I also wish to thank my supervisors, Professor Graeme Burt and Dr Stuart Galloway, for their guidance and scholarship. I wish to offer very special thanks to my parents, my big brother and my little sister for all of their support throughout my PhD.

This thesis makes extensive use of open source software projects developed by researchers from other institutions. I wish to thank Dr Ray Zimmerman from Cornell University for his work on optimal power flow, researchers from the Dalle Molle Institute for Artificial Intelligence (IDSIA) and the Technical University of Munich for their work on reinforcement learning algorithms and artificial neural networks and Charles Gieseler from Iowa State University for his implementation of the Roth-Erev reinforcement learning method.

Abstract

In Electrical Power Engineering, learning algorithms can be used to model the strategies of electricity market participants. The objective of this work is to establish if *policy gradient* reinforcement learning algorithms can be used to create participant models superior to those using previously applied *value function* based methods.

Supply of electricity involves technology, money, people, natural resources and the environment. All of these aspects are changing and electricity market designs must be suitably researched to ensure that they are fit for purpose. In this thesis electricity markets are modelled as non-linear constrained optimisation problems, which are solved using a primal-dual interior point method. Policy gradient reinforcement learning algorithms are used to adjust the parameters of multi-layer feed-forward artificial neural networks that approximate each market participant's policy for selecting power quantities and prices that are offered in the simulated marketplace.

Traditional reinforcement learning methods that learn a value function have been previously applied in simulated electricity trade, but are largely restricted to discrete representations of a market environment. Policy gradient methods have been proven to offer convergence guarantees in continuous environments and avoid many of the problems that mar value function based methods.

Contents

List of Figures

List of Tables

Chapter 5

Nash Equilibrium Analysis

This chapter presents a simulation that examines a system of agents competing to sell electricity and its convergence to a Nash equilibrium. Value function based and policy gradient reinforcement learning algorithms are compared in their convergence to an optimal policy using a six bus electric power system model.

5.1   Introduction

This thesis presents the first case of policy gradient reinforcement learning methods being applied to simulated electricity trade. As a first step it is necessary to confirm that when using these methods, a system of multiple agents will converge to the same Nash equilibrium[1] that a traditional closed-form simulation would produce.

This is the same approach used by Krause et al. (2006) before performing the study of congestion management techniques that is reviewed in Section 3.2.2. Nash equilibria can be difficult to determine in complex systems so the experiment presented here utilises a model simple enough that it can be determined through exhaustive search.

By observing actions taken and rewards received by each agent over the initial simulation periods it is possible to compare the speed and consistency with which different algorithms converge to an optimal policy. In the following sections the objectives of the simulation are defined, the simulation setup is explained and plots of results, with discussion and critical analysis, are provided.

---

[1]Informally, a Nash equilibrium is a point in a non-cooperative game at which no player is motivated to deviate from its strategy, as it would result in lower gain (Nash, 1950, 1951).

5.2 Aims and Objectives

Some elements of the simulations reported in this chapter are similar to those presented by Krause et al. (2006). One initial aim of this work is to reproduce their findings as a means of validating the approach. The additional objectives are to show:

- That policy gradient methods converge to the same Nash equilibrium as value function based methods and tradtional closed-form simulations,

- Some the characteristics of policy gradient methods and how they differ from value function based methods.

Meeting these objectives aims to provide a basis for using policy gradient methods in more complex simulations, to show that they can *learn to trade power* and to provide guidance for algorithm parameter selection.

5.3 Method of Simulation

Learning methods are compared in this chapter by repeating the same simulation and switching the algorithms used by the agents. An alternative might be to use a combination of methods in the same simulation, but the approach used here is intended to be an extension of the work by Krause et al. (2006).

Each simulation uses a six bus electric power system model adapted from Wood and Wollenberg (1996, pp. 104, 112, 119, 123-124, 549). The model provides a simple environment for electricity trade with a small number of generators and branch flow constraints that slightly increase the complexity of the Nash equilibria. The buses are connected by eleven transmission lines at 230kV. The model contains three generating units with a total capacity of 440MW and loads at three locations, each 70MW in size. The connectivity of the branches and the locations of the generators and loads is shown in Figure **??**. Data for the power system model was taken from a case provided with MATPOWER and is listed in Appendix B.1.

Two sets of quadratic generator operating cost functions, of the form $c(p_i) = ap_i^2 + bp_i + c$ where p_i is the output of generator i, are defined in order to create two different equilibria for investigation. The coefficients a, b and c for cost configuration 1 are listed in Table 5.1. This configuration defines two low cost generators that can not offer a price greater than the marginal cost of the most expensive generator when they apply the maximum possible markup. The set

Table 5.1: Generator cost configuration 1.

Gen	a	b	c
1	0.0	4.0	200.0
2	0.0	3.0	200.0
3	0.0	6.0	200.0

Table 5.2: Generator cost configuration 2.

Gen	a	b	c
1	0.0	5.1	200.0
2	0.0	4.5	200.0
3	0.0	6.0	200.0

of coefficients for cost configuration 2 is listed in Table 5.2. This configuration narrows the cost differences such that offer prices may overlap and may exceed the marginal cost of the most expensive generator.

As in Krause et al. (2006), no load profile is defined. The system load is assumed to be peak in all periods and only one state is defined for methods using look-up tables. Each simulation step is assumed to be one hour in length.

For all generators $P^{min} = 0$ so as to simplify the equilbria and avoid the need for the unit de-commitment algorithm. The maximum capacity for the most expensive generator $P_3^{max} = 220$MW such that it may supply all of the load if dispatched, subject to branch flow limits. This generator is associated with a passive agent that always offers full capacity at marginal cost. For the less expensive generators $P_1^{max} = P_2^{max} = 110$MW. These two generators are each associated with an active learning agent whose activity in the market is restricted to one offer of maximum capacity in each period, at a price representing a markup of between 0 and 30% on marginal cost. Methods restricted to discrete actions may markup in steps of 10%, giving possible markup actions of 0, 10%, 20% and 30%. No capacity withholding is implemented. Discriminatory pricing (pay-as-bid) is used in order to provide a clearer reward signal to agents with low cost generators.

The algorithms compared are: Q-learning, ENAC, REINFORCE and the modified Roth-Erev technique (See Section 2.4). Default algorithm parameter values from PyBrain are used and no attempt is made to study parameter sensitivity or variations in function approximator design.

Table 5.3: Agent rewards under cost configuration 1

		G_1							
		0.0%		10.0%		20.0%		30.0%	
		r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
	0.0%	0.0	0.0	40.0	0.0	80.0	0.0	120.0	0.0
G_2	10.0%	0.0	33.0	40.0	33.0	80.0	33.0	120.0	33.0
	20.0%	0.0	66.0	40.0	66.0	80.0	66.0	120.0	66.0
	30.0%	0.0	99.0	40.0	99.0	80.0	99.0	120.0*	99.0*

For the Q-learning algorithm $\alpha = 0.3$, $\gamma = 0.99$ and ϵ-greedy action selection is used with $\epsilon = 0.9$ and $d = 0.98$. For the Roth-Erev technique $\epsilon = 0.55$, $\phi = 0.3$ and Boltzmann action selection is used with $\tau = 100$ and $d = 0.99$.

Both REINFORCE and ENAC use a two-layer neural network with one linear input node, one linear output node, no bias nodes and with the connection weight initialised to zero. This is a typicaly PyBrain configuration taken from the supplied examples. A two-step episode is defined for the policy gradient methods and five episodes are performed per learning step. The exploration parameter σ for these methods is initialised to zero and adjusted manually after each episode such that:

$$\sigma_t = d(\sigma_{t-1} - \sigma_n) + \sigma_n \tag{5.1}$$

where $d = 0.998$ is a decay parameter and $\sigma_n = -0.5$ specifies the value that is converged to asymtotically. In each simulation the learning rate $\gamma = 0.01$ for the policy gradient methods, apart from for ENAC under cost configuration 2 where $\gamma = 0.005$. Both active agents use the same parameter values in each simulation.

As in Krause et al. (2006), the point of Nash equilibrium is established by computing each agent's reward for all possible combinations of discrete markup. The rewards for Agent 1 and Agent 2 under cost configuration 1 are given in Table 5.3. The Nash equilibrium points are marked with a *. The table shows that the optimal policy for each agent is to apply the maximum markup to each offer as their generators are always dispatched. The rewards under cost configuration 2 are given in Table 5.4. This table shows that the optimal point occurs when Agent 2 applies its maximum markup and Agent 1 offers a price just below the marginal cost of the passive agent's generator.

Table 5.4: Agent rewards under cost configuration 2

		G_1							
		0.0%		10.0%		20.0%		30.0%	
		r_1	r_2	r_1	r_2	r_1	r_2	r_1	r_2
	0.0%	0.0	0.0	51.0	0.0	0.0	0.0	0.0	0.0
G_2	10.0%	0.0	49.5	51.0	49.5	0.0	49.5	0.0	49.5
	20.0%	0.0	92.2	51.0	99.0	0.0	99.0	0.0	99.0
	30.0%	0.0	126.8	54.8*	138.4*	0.0	148.5	0.0	148.5

5.4 Simulation Results

Each action taken by an agent and the consequent reward is recorded for each simulation. Values are averaged over the ten simulation runs and standard deviations are calculated using the formula

$$SD = \sqrt{\frac{1}{N-1} \sum_{i=0}^{N} (x_i - \bar{x})^2} \qquad (5.2)$$

where x_i is the action or reward value in simulation i of N simulation runs and $\bar{x}$ is the mean of the values.

Figure ?? shows the average markup on marginal cost and the standard deviation over the ten simulation runs for Agent 1 under price configuration 1, using the four learning methods. The second y-axis in each plot relates to the exploration parameter for each method. Figure ?? shows the same information for Agent 2. Plots of reward are not given as generator prices and the market are configured such that an agent's reward is directly proportional to its action. The plots are vertically aligned and have equal x-axis limits to assist algorithm comparison.

Figures ?? and ?? plot the average markup and reward over ten simulation runs for Agent 1 and Agent 2, respectively, under price configuration 2 for the variant Roth-Erev, Q-learning learning methods. The plots for REINFORCE and ENAC in these figures are for actual values in one simulation run as the number of interactions and variation in values makes the results difficult to observe otherwise.

5.5 Discussion and Critical Analysis

Under cost configuration 1 the agents face a relatively simple control task and receive a clear reward signal that is directly proportional to their markup. The results show that all of the methods consistently converge to the point of Nash equilibrium. The variant Roth-Erev method shows very little variation around the mean once converged, due to the use of Boltzmann exploration with a then low temperature parameter value. The constant variation around the mean that can be seen for Q-learning once converged is due to the use of ϵ-greedy action selection and can be removed if a Boltzmann explorer is selected.

Empirical studies have also shown that the speed of convergence is largely determined by the rate at which the exploration parameter value is reduced. However, the episodic nature of the policy gradient methods requires them to make several interactions per learning step and therefore a larger number of initial exploration steps are required. Policqy gradient methods have also been found to be highly sensitive to the choice of learning rate. High values cause large changes to policy parameters to be made at each step and may cause the algorithm to not converge, but low values cause the algorithm to learn very slowly.

Cost configuration 2 provides a more challenging control problem in which Agent 1 must learn to undercut the passive agent. The results show that the variant Roth-Erev and Q-learning methods both consistently learn their optimal policy and converge to the Nash equilibrium. However, there is space for Agent 1 to markup its offer by slightly more than 10% and still undercut the passive agent, but the methods with discrete actions are not able to exploit this and do not receive the small additional profit.

The results for the policy gradient methods under cost configuration 2 show that they learn to reduce their markup if their offer price starts to exceed that of the passive agent and the reward signal drops. However, a chattering effect below the Nash equilibrium point can be clearly seen for ENAC and the method does not learn to always undercut the other agent. These methods require a much larger number of simulation steps and for the exploration parameter to decay slowly if they are to produce this behaviour. This is due to the need for a lower learning rate that ensures fine policy adjustments can be made and for several interactions to be performed between each learning step.

5.6   Summary

By observing the state to which a multi-learning-agent system converges, it is possible to verify that learning algorithms produce the same Nash equilibrium that closed-form simulations provide. The results presented in this chapter closely correspond with those from Krause et al. (2006) for Q-learning and show equivalent behaviour for the variant Roth-Erev method. The simulations illustrate how challenging unsupervised learning is in a continuous environment, even for a simple problem. Tasks in which a large reward change can occur for a very small change in policy prove difficult for policy gradient methods to learn and require low learning rates and lengthy periods of exploration. The operation of policy gradient methods with noisy, multi-dimensional state data is not examined in this chapter and deserves investigation.

Chapter 6

System Constraint Exploitation

This chapter explores the exploitation of constraints by learning agents in a dynamic electricity trading environment. Value function based and policy gradient reinforcement learning methods are compared using a modified version of the IEEE Reliability Test System.

6.1 Introduction

Having examined the basic learning characterisitics of four algorithms in Chapter 5, this chapter extends the approach to examine their operation in a complex dynamic environment. It explores the ability of policy gradient methods to operate with multi-dimensional, continuous state and action data in the context of *learning to trade power.*

A reference electric power system model from the IEEE Reliability Test System (RTS) (Application of Probability Methods Subcommittee, 1979) provides a realistic environment for agents to compete with diverse portfolios of generating plant supply dynamic loads. System constraints change as agents adjust their strategies and loads follow a hourly profile that is varied in shape from day-to-day over the course of a simulated year. By observing average profits at each hour of the day, the ability of methods to successfully observe and exploit constraints is examined.

6.2 Aims and Objectives

This experiment aims to compare policy gradient and traditional reinforcement learning methods in a dynamic electricity trading environment. Specifically, the objectives are to determine:

- If the policy gradient methods can achieve greater profitability under dynamic system constraints using a detailed state vector.

- The value of using an AC optimal power flow formulation in agent based electricity market simulation.

Meeting the first objective aims to demonstrate some of the value of using policy gradient methods in electricity market participant modelling and to determine if they warrant further research in this domain.

6.3   Method of Simulation

Learning methods are again compared by repeating simulations of competitive electricity trade switching the algorithms used by the competing agents. Some simplification of the state and action representations for value function based methods is required, but generation portfolios and load profiles are identical for each algorithm test.

The RTS has 24 bus locations that are connected by 32 transmission lines, 4 transformers and 2 underground cables. The transformers tie a 230kV area to a 138kV area. The original model has 32 generators of 9 different types with a total capacity of 3.45GW. To reduce the size of the discrete action space, five 12MW and four 20MW generators are removed. This is deemed reasonable as it reduced the number of generators by 28%, but their combined capacity is only 4.1% of the original total generation capacity and the remainder is more than sufficient to meet demand. To further reduce action space sizes all generators of the same type at the same bus are aggregated into one generating unit. This can be considered to be the representation of each individual power station in the market, rather than each alternator stage. The model has loads at 17 locations and the total demand at system peak is 2.85GW.

Again, generator marginal costs are quadratic functions of output and are defined by the parameters in Table 6.1. Figure **??** shows the cost functions for each of the seven types of generator and illustrates their categorisation by fuel type. Generator cost function coefficients were taken from an RTS model by Georgia Tech Power Systems Control and Automation Laboratory[1] which assumes Coal costs of 1.5 \$/MBtu[2], Oil costs of 5.5 \$/MBtu and Uranium costs of 0.46 \$/MBtu. Data for the modified model is provided in Appendix B.2 and

---

[1] http://pscal.ece.gatech.edu/testsys/
[2] 1 Btu $\approx$ 1055 Joules

Code	c_{down}	a	b	c	Type
U50	0	0.0	0.001	0.001	Hydro
U76	0	0.01414	16.0811	212.308	Coal
U100	0	0.05267	43.6615	781.521	Oil
U155	0	0.00834	12.3883	382.239	Coal
U197	0	0.00717	48.5804	832.758	Oil
U350	0	0.00490	11.8495	665.109	Coal
U400	0	0.00021	4.4231	395.375	Nuclear

Table 6.1: Generator types and cost parameters for the simplified IEEE Reliability Test System.

Agent	U50 Hydro	U76 Coal	U100 Oil	U155 Coal	U197 Oil	U350 Coal	U400 Nuclear	Total (MW)
1		2×		1×			1×	707
2		2×		1×			1×	707
3	6×				3×			891
4			3×	2×		1×		960

Table 6.2: Agent portfolios.

the connectivity of branches and the location of generators and loads is illustrated in Figure **??**.

The generating stock is divided into 4 portfolios (See Table 6.2) that are each endowed to a learning agent. Portfolios were chosen such that each agent has: a mix of base load and peaking plant, approximately the same total generation capacity and generators in different areas of the network. The generator labels in Figure **??** specify the associated agent. The synchronous condenser is associated with a passive agent that always offers 0 MW at 0 \$/MWh (the unit can be dispatched to provide or absorb reactive power).

Markups on marginal cost are restricted to a maximum of 30% and discrete markups of 0, 15% or 30% are defined for value function based methods. Upto 20% of the total capacity of each generator can be withheld and discrete withholds of 0 or 20% are defined. Initially only one offer per generator is required, but this is increased to two in order to explore the effect of increased offer flexibility.

The environment state for all algorithm tests consists of a forecast of the total system demand for the next period. The system demand follows an hourly profile that is adjusted according to the day of the week and the time of year. The profiles are provided by the RTS and are illustrated in Figure **??**. For tests of value function based methods and the Stateful Roth-Erev learning algorithm, the continuous state is divided into 3 discrete states of equal size, that allow

differentiation between low, medium and peak demand.

To investigate the exploration of constraints, AC optimal power flow is used and the state vector for agents using policy gradient methods is optionally adjusted to combine the demand forecast with voltage constraint Lagrangian multipliers for all generator buses and the voltage magnitude at all other buses. Lagrangian multipliers are used for generator buses as generators typically fix the voltage at their associated bus. Branch flows are not included in the state vector as flow limits in the RTS are high and are typically not reached at peak demand. Generator capacity limits are binding in most states of the RTS, but the output of other generators is deemed to be hidden from agents.

The nodal marginal pricing scheme is used and cleared offer prices are determined by the Lagrangian multiplier on the power balance constraint for the bus at which the generator associated with the offer is connected.

Typical parameter values are used for each of the algorithms. Again, no attempt to study parameter sensistivity is made. Learning rates are set low and exploration parameters decay slowly due to the length and complexity of the simulation. For Q-learning $\alpha = 0.2$, $\gamma = 0.99$ and ϵ-greedy action selection is used with $\epsilon = 0.9$ and $d = 0.999$. For Roth-Erev learning $\epsilon = 0.55$, $\phi = 0.3$ and Boltzmann action selection is used with $\tau = 100$ and $d = 0.999$.

Again a typical PyBrain two-layer neural network configuration with linear input and output nodes, no bias nodes and randomised initial connection weights are used for policy function approximation. The initial exploration rate $\sigma = 0$ for both policy gradient methods and decays according to Equation (5.1) with $d = 0.995$ and $\sigma_n = -0.5$. Constant learning rates are used in each simulation with $\gamma = 0.01$ for REINFORCE and $\gamma = 0.005$ for ENAC.

6.4   Simulation Results

Each agent's rewards are recorded for a simulated year of 364 trading episodes, each consisting of 24 interactions. To compare algorithms, the average reward for each hour of the day is calculated for each agent and plotted. Only results for Agent 1 and Agent 4 are given as Agents 1 and Agent 2 have identical portfolios and most of Agent 3's portfolio consists of hydro-electric plant with zero cost. The method of applying percentage markups on marginal cost has not effect for generators with zero cost and almost identical results are found for all algorithms.

Figure **??** compares the modified Roth-Erev method with the Stateful Roth-Erev method. The plots show average rewards for Agent 1 and Agent 4 when

using Q-learning and the two Roth-Erev variants.

Figure **??** and Figure **??** compare policy gradient methods under two state vector configurations. Figure **??** concerns Agent 1 and shows the average reward received for a state vector consisting solely of a demand forecast and for a combined demand forecast and bus voltage profile state vector. Figure **??** shows average rewards for Agent 4 under the same configurations.

Figure **??** shows average rewards for agents 1 and 4 from a repeat of the bus voltage profile state simulation, but with two offers required per generator. Due to time constraints and limited simulation resources only results for Q-learning and ENAC are given.

6.5 Discussion and Critical Analysis

Agents with a discrete environment have 216 possible actions to choose from in each state when required to submit one offer per generator. Figure **??** shows that, using Q-learning, agents are able to learn an effective policy that yields increased profits under two different portfolios. The importance of utilising environment state data in a dynamic electricity setting is illustrated by the differences in average reward received by the modified Roth-Erev method and the Stateful Roth-Erev method. The optimal action for an agent depends upon the current system load and the stateless Roth-Erev formulation is unable to interpret this. The Stateful Roth-Erev method can be seen to achieve approximately the same performance as Q-learning.

Including bus voltage constraint data in the state for a discrete environment would result in a state space of impractical size, but including it in a continuous environment was straight-forward. The results show that ENAC achieves greater profits when presented with a combined demand forecast and bus voltage state vector. REINFORCE performs less well than ENAC, but also shows improvement over the pure demand forecast case. ENAC achieves equivalent, but not greater performance than Q-learning in all periods of the trading day when using the voltage data. ENAC is not able to use the additional state information to any further advantage, but does learn a profitable policy.

Simply changing the number of offers that are required to be submitted for each generator from 1 to 2, increases the number of discrete action possibilities in each state to 46,656. Figure **??** shows that Q-learning is still able to achieve a similar level of reward as in the one offer case. The profitability for both methods is degraded, but ENAC receives significantly lower average reward when the agent

is required to produce an action vector of twice the size and is not able to use the increased flexibility in its offer structure to any advantage.

With state and action spaces on this scale, computing updates to an agent's look-up table or neural network adds considerably to the computational expense of the simulation. Researchers wishing to apply these methods in larger problems must be willing to investigate program optimisation and parallel or distributed processing. Studies not requiring this level of complexity are seemingly best using a state-value function based method, such as Q-learning or the Stateful Roth-Erev formulation.

The lack of involvement in the market from the hydro-electric power plant largely negates the participation of Agent 3 and exposes one significant shortcomming of the approach. This could be overcome by allowing markups in dollars, rather than as a percentage of marginal cost.

Generation portfolios were configured such that agents would receive a mix of low-cost base-load plant and expensive peak-supply plant. However, the cost differences between fuel types are such that an offer of power from a coal or nuclear power station can not exceed in price that from a unit with a more expensive fuel type. Greater competition and more complex equilibria could introduced to the simulation if fuel cost differences were not as large or maximum markups on price were greater. In Rastegar, Guerci, and Cincotti (2009), for example, a 300% markup limit is set.

The dynamics of this simulation could also be greatly increased by introducing demand-side participation. It could allow agents to directly influence the state of the environment, through the demand forecast sensor. It would also give agents more options for competition, increasing the complexity of their optimal policy and posing a greater challenge to the learning algorithms.

6.6 Summary

In this chapter policy gradient reinforcement learning algorithms have been applied in a complex dynamic electricity trading simulation and assessed in their ability to exploit constraints in the system. They were found to be a valid technique for *learning to trade power*, but were outperformed by Q-learning in most configurations of environment state and action space. This includes a simulation with action spaces that were expected to be too large for Q-learning to explore, but to be of no significant challenge to policy gradient methods.

The addition of bus voltage sensor data in the state vector of agents operating

policy gradient methods was found to improve their performance. However, no evidence was found to suggest that they could use this information to increase their profitability above that of agents operating the Q-learning or Stateful Roth-Erev method. Indeed, it is beleived that this can be considered a general finding in reinforcement learning research, that despite great effort and the devlopment of many new algorithms, few surpass the original temporal difference methods from Sutton and Barto (1998).

Shortcommings in the price markup methodology and competition levels have been identified and possible solutions proposed. The implications of increased computational expense for further development of this work have also been noted. AC optimal power flow adds enormously to simulation times when analysing an entire year of hourly trading interactions. The addition of bus voltage data was found to improve performance of the policy gradient methods, but it has not been show if the same could not be achieved by perhaps using bus voltage angles from a DC optimal power flow formulation.

Chapter 7

Conclusions and Further Work

This final chapter summarises the conclusions that can be drawn from the results presented in this thesis and gives some ideas for further development of the contributions that have been made.

7.1   Summary Conclusions

This thesis has introduced the use of policy gradient reinforcement learning algorithms in modelling strategies of electricity market participants. Over the last two decades, competitive markets have become an essential component of electricity supply industries in many large countries. They will play an important role in the future as the world population continues to grow and finite primary energy fuel resources become increasingly scarce. Electric energy trade requires a unique market design, but radical architecture changes can not be experimented with on real systems.

Computational simulation is a well established technique for evaluating market design concepts and agent-based simulation is an approach that allows large complex systems to be modelled. There are many examples of learning algorithms being used to model electricity market participants in the literature, but policy gradient methods have not been previously applied. These methods use function approximation techniques to operate in environments with state and actions spaces that are continuous, discrete or mixed and have been successfully applied in robot control and other problems.

To examine the properties of policy gradient methods and compare their performance with previously applied value function based methods, a modular simulation framework has been defined and implemented. The framework uses a power exchange auction market model with nodal marginal pricing to provide an

environment in which agents can *learn to trade power* competitively.

The framework has first been used in a simulation that compares the convergence to Nash equilibria of four different learning algorithms. The simulation reproduced the findings of Krause et al. (2006) and presented similar results for policy gradient methods. They were found: to exhibit very different characteristics to value function based methods, to require a larger number of interactions before learning an optimal policy and to require low learning rate and exploration rate decay parameters for complex equilibria to be approached.

In a second simulation the framework was used to compare the same algorithms in a complex dynamic electricity trading environment. A reference electric power system model, designed for reliability analysis, was used to provide a realisitic environment of a reasonable size. The algorithms were compared in their ability to observe and exploit constraints in the system as loads followed an hourly profile. It was not found that policy gradient methods could use additional bus voltage data to achieve greater performance than a action-value function based method or a stateful variant of the Roth-Erev technique. However, they were shown to learn effective strategies under noisy dynamic conditions and to imporove performance using complex continuous state representations.

In conclusion, policy gradient methods are a valid option for modelling the strategies of electricity market participants. They can use profit feedback from an electricity market model to adjust the parameters of a policy function approximator in the direction of increased reward. Function approximators allow market participants to be modelled using agents that accept complex power system state data and produce offers for diverse portfilios of generators with price markups applied and capacity withheld. This thesis has compared a selection of techniques for participant modelling and provides guidance for algorithm and parameter selection.

7.2   Further Work

This final section highlights some of the shortcommings of the methodology presented in this thesis and explains how the models could be further developed. It introduces some alternative learning algorithms that might also be used to simulate electricity market participant behaviour. It explains is how a model formulated using data from National Grid Ltd. could be used in practical simulations of the UK electricity market and describes some other possibilities for using AC optimal power flow in agent-based electric power market simulation.

7.2.1 Parameter Sensitivity and Delayed Reward

The simulations presented in this thesis use typical algorithm parameters that are either the default values from PyBrain or inspired by the literature. No study of parameter sensitivity is performed. Alternative function approximation and back-propagation techniques could also be investigated. Parameter sensitivity analysis is typically conducted by the algorithm developers using standard benchmark problems (such as mazes or pole balancing problems) that are familar to researchers in Artificial Intelligence and allow results to be compared. The shortage of published results and lack of standardised electricity trading models might limit the benefits of using this problem for general parameter sensitivity analysis.

The reward signals received by agents in all of the simulations presented in this thesis result directly from the agent's previous action. In reality, a market settlement process would introduce delays to payments for electricity production. Time did not permit value function based methods with eligibility traces (See Section 2.4.1) to be compared with policy gradient methods, but the ability to learn under delayed reward is a fundamental part of both reinforcement learning and market trade and deserves investigation in this context.

7.2.2 Alternative Learning Algorithms

This thesis has concentrated on traditional value function based methods, the Roth-Erev technique and two policy gradient reinforcement learning methods. However, there are other learning algorithms that have been published recently that could also be used in electric power trade simulations.

Riedmiller (2005) presented Neuro-Fitted Q-Iteration (NFQ) algorithms that attempt to overcome many of the problems experienced when implementing Q-learning methods with value function approximation using neural networks. They store all transition experiences and perform off-line updates using supervised learning techniques such as RProp (Riedmiller & Braun, 1993). The method has been shown to be robust against parameterization and to learn quickly in standard benchmark tests and real-world applications (Kietzmann & Riedmiller, 2009).

The GQ(λ) algorithm by Maei and Sutton (2010) is another extension of Q-learning for operation in continuous environments. Convergence guarantees have been shown and the scaling properties suggest the method is suitable for large-scale reinforcement learning applications. A software implementation of GQ(λ)

has been developed by the authors and recently made available as open source.

Four new Natural Actor-Critic algorithms have been presented by Bhatnagar, Sutton, Ghavamzadeh, and Lee (2009). Like ENAC, they use function approximation techniques and are suitable for large-scale applications of reinforcement learning. Three of the algorithms are extensions to ENAC, but are fully incremental: the gradient computation is never reset while the policy is updated at every simulation step. The authors state a need to assess the ultimate utility of these algorithms through application in real-world problems.

This thesis provides a framework that would allow implementations of these algorithms to be assessed and used to research aspects of electricity markets.

7.2.3 UK Transmission System

Some of the more ambitious agent-based electricity market simulations have used stylised models of national transmission systems (Rastegar et al., 2009; Weidlich & Veit, 2006). This work has often been motivated by recent or expected changes to the arrangements in the associated regions. In the UK, nine large power stations are due to be decommissioned by 2016 in accordance with EU Large Combustion Plant Directive (National Electricity Transmission System Operator, 2007). Coupled with obligations, made in the Climate Change Act 2008, to cut greenhouse gas emissions by 80% of 1990 levels by 2050, coming years are likely to see major changes in the way the UK power system is operated. Examination of the situation could be enhanced by advanced participant behavioural models and accurate electric power system simulations such as those presented in this thesis.

Figure ?? illustrates a model of the UK transmission system that has been formulated from data provided by the National Electricity Transmission System Operator (2010). This model has been converted into PSS/E raw file format and is included with the code developed for this thesis (See Appendix A.11). It is currently too computationally expensive to be solved repeatedly in an agent-based simulation, but optimisation efforts might allow for it to be used in studies pertinent to the UK energy industry.

7.2.4 AC Optimal Power Flow

This thesis presents the first application of AC optimal power flow in electricity market simulation using reinforcement learning agents. AC optimal power flow formulations are more difficult to implement and more computationally expensive

to solve than their linearised DC counterparts. The additional time and effort required for their use does not always add sufficient value to simulations. However, the option to use AC formulations does provide certain opportunities for further work.

The inclusion of reactive power costs in the objective function of an AC optimal power flow problem provides an opportunity to run auctions for voltage support in parallel with those for active power. These could be open to agents associated with reactive compensation equipment such as that commonly needed for wind farm developments. Traditionally, reactive power markets have been mostly of academic interest, but as the UK makes greater use of on and off-shore wind power, the topic could become of increasing interest and importance.

Bus voltages are not all assumed to be 1 per-unit in AC optimal power flow problems, but are part of the vector of optimisation variables. Adjusting phase shift angles, θ_{ph}, can offer a degree of control over the direction of power flows. The control of transformer tap ratios, τ, and phase shift angles by learning agents could become a topic of interest in congestion management research.

7.2.5 Multi-Market Simulation

Finally, the global economy is a holistic system of systems and the analysis of markets independently must be of limited value. Recent agent-based electricity market studies have investigated the interaction between electricity, gas and emission allowance markets (Kienzle, Krause, Egli, Geidl, & Andersson, 2007; J. Wang, Koritarov, & Kim, 2009).

Data for the UK gas transmission network provided by the National Electricity Transmission System Operator (2010) is of limited detail, compared to that for the electricity transmission system, but suitable models could be used to study the the relationships between UK gas and electricity markets. As in Kienzle et al. (2007), actions in the gas market would constrain the generators options to sell power in subsequent electricity auctions. Add to this the option to trade in emissions allowance markets and agents would be presented with large state and action spaces and would require suitably advanced learning methods.

Bibliography

31.04, W. (1983). Electric power transmission at voltages of 1000 kv and above plans for future ac and dc transmission -data on technical feasibility and on general design. Electra. (ELT_091_3)

Alam, M. S., Bala, B. K., Huo, A. M. Z., & Matin, M. A. (1991). A model for the quality of life as a function of electrical energy consumption. Energy, 16(4), 739-745.

Aleksandrov, V., Sysoyev, V., & Shemeneva, V. (1968). Stochastic optimization. Engineering Cybernetics, 5, 11-16.

Amerongen, R. van. (1989, May). A general-purpose version of the fast decoupled load flow. Power Systems, IEEE Transactions on, 4(2), 760-770.

Application of Probability Methods Subcommittee. (1979, November). IEEE reliability test system. Power Apparatus and Systems, IEEE Transactions on, PAS-98(6), 2047-2054.

Auer, P., Cesa-Bianchi, N., Freund, Y., & Schapire, R. E. (2003). The nonstochastic multiarmed bandit problem. SIAM Journal of Computing, 32(1), 48-77.

Baird, L. (1995). Residual algorithms: Reinforcement learning with function approximation. In Proceedings of the Twelfth International Conference on Machine Learning (p. 30-37). Morgan Kaufmann.

Bellman, R. E. (1961). Adaptive control processes – A guided tour. Princeton, New Jersey, U.S.A.: Princeton University Press.

Bertsekas, D. P., & Tsitsiklis, J. N. (1996). Neuro-dynamic programming. Belmont, MA: Athena Scientific.

Bhatnagar, S., Sutton, R. S., Ghavamzadeh, M., & Lee, M. (2009). Natural actor-critic algorithms. Automatica, 45(11), 2471–2482.

Bishop, C. M. (1996). Neural networks for pattern recognition (1st ed.). Oxford University Press, USA. Paperback.

Bower, J., & Bunn, D. (2001, March). Experimental analysis of the efficiency of uniform-price versus discriminatory auctions in the england and wales

electricity market. Journal of Economic Dynamics and Control, 25(3-4), 561-592.

Bower, J., Bunn, D. W., & Wattendrup, C. (2001). A model-based analysis of strategic consolidation in the german electricity industry. Energy Policy, 29(12), 987-1005.

Boyd, S., & Vandenberghe, L. (2004). Convex optimization. Cambridge University Press. Hardcover.

Bunn, D., & Martoccia, M. (2005). Unilateral and collusive market power in the electricity pool of England and Wales. Energy Economics.

Bunn, D. W., & Oliveira, F. S. (2003). Evaluating individual market power in electricity markets via agent-based simulation. Annals of Operations Research, 57-77.

Carpentier, J. (1962, August). Contribution à l'étude du Dispatching Economique. Bulletin de la Society Francaise Electriciens, 3(8), 431-447.

Cole, S. (2010, February 4). MatDyn [Computer software manual]. Katholieke Universiteit Leuven.

Department of Energy and Climate Change. (2009). Digest of United Kingdom Energy Statistics 2009. In (chap. 5). National Statistics – Crown.

Erev, I., & Roth, A. E. (1998). Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. The American Economic Review, 88(4), 848-881.

Ernst, D., Minoia, A., & Ilic, M. (2004, June). Market dynamics driven by the decision-making of both power producers and transmission owners. In Power Engineering Society General Meeting, 2004. IEEE (p. 255-260).

Fausett, L. (Ed.). (1994). Fundamentals of neural networks: architectures, algorithms, and applications. Upper Saddle River, NJ, USA: Prentice-Hall, Inc.

Gieseler, C. (2005). A Java reinforcement learning module for the Repast toolkit: Facilitating study and implementation with reinforcement learning in social science multi-agent simulations. Unpublished master's thesis, Department of Computer Science, Iowa State University.

Glimn, A. F., & Stagg, G. W. (1957, April). Automatic calculation of load flows. Power Apparatus and Systems, Part III. Transactions of the American Institute of Electrical Engineers, 76(3), 817-825.

Glynn, P. W. (1987). Likelilood ratio gradient estimation: an overview. In Wsc '87: Proceedings of the 19th conference on winter simulation (p. 366-375). New York, NY, USA: ACM.

Goldfarb, D., & Idnani, A. (1983). A numerically stable dual method for solving strictly convex quadratic programs. Mathematical Programming, 27, 1-33.

Gordon, G. (1995). Stable function approximation in dynamic programming. In Proceedings of the Twelfth International Conference on Machine Learning (p. 261-268). Morgan Kaufmann.

Grainger, J., & Stevenson, W. (1994). Power system analysis. New York: McGraw-Hill.

Guo, M., Liu, Y., & Malec, J. (2004, October). A new Q-learning algorithm based on the metropolis criterion. Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 34(5), 2140-2143.

ICF Consulting. (2003, August). The economic cost of the blackout: An issue paper on the northeastern blackout. (Unpublished)

IEEE Working Group. (1973, November). Common format for exchange of solved load flow data. Power Apparatus and Systems, IEEE Transactions on, 92(6), 1916-1925.

Kallrath, J., Pardalos, P., Rebennack, S., & Scheidt, M. (2009). Optimization in the energy industry. Springer.

Kienzle, F., Krause, T., Egli, K., Geidl, M., & Andersson, G. (2007, September). Analysis of strategic behaviour in combined electricity and gas markets using agent-based computational economics. In 1st European workshop on energy market modelling using agent-based computational economics (p. 121-141). Karlsruhe, Germany.

Kietzmann, T. C., & Riedmiller, M. (2009). The neuro slot car racer: Reinforcement learning in a real world setting. Machine Learning and Applications, Fourth International Conference on, 0, 311-316.

Kirschen, D. S., & Strbac, G. (2004). Fundamentals of power system economics. Chichester: John Wiley & Sons.

Krause, T., & Andersson, G. (2006). Evaluating congestion management schemes in liberalized electricity markets using an agent-based simulator. In Power Engineering Society General Meeting, 2006. IEEE.

Krause, T., Andersson, G., Ernst, D., Beck, E., Cherkaoui, R., & Germond, A. (2004). Nash Equilibria and Reinforcement Learning for Active Decision Maker Modelling in Power Markets. In Proceedings of 6th IAEE European Conference 2004, modelling in energy economics and policy.

Krause, T., Beck, E. V., Cherkaoui, R., Germond, A., Andersson, G., & Ernst, D. (2006). A comparison of Nash equilibria analysis and agent-based modelling for power markets. International Journal of Electrical Power & Energy

Systems, 28(9), 599-607.

Li, H., & Tesfatsion, L. (2009a, July). The ames wholesale power market test bed: A computational laboratory for research, teaching, and training. In IEEE Proceedings, Power and Energy Society General Meeting. Alberta, Canada.

Li, H., & Tesfatsion, L. (2009b, March). Capacity withholding in restructured wholesale power markets: An agent-based test bed study. In Power systems conference and exposition, 2009 (p. 1-11).

Lincoln, R., Galloway, S., & Burt, G. (2007, May 23-25). Unit commitment and system stability under increased penetration of distributed generation. In Proceedings of the 4th International Conference on the European Energy Market, 2007. EEM 2007. Cracow, Poland.

Lincoln, R., Galloway, S., & Burt, G. (2009, May). Open source, agent-based energy market simulation with Python. In Proceedings of the 6th International Conference on the European Energy Market, 2009. EEM 2009. (p. 1-5).

Lincoln, R., Galloway, S., Burt, G., & McDonald, J. (2006, 6-8). Agent-based simulation of short-term energy markets for highly distributed power systems. In Proceedings of the 41st international universities power engineering conference, 2006. UPEC '06. (Vol. 1, p. 198-202).

Maei, H. R., & Sutton, R. S. (2010). Gq(λ): A general gradient algorithm for temporal-difference prediction learning with eligibility traces. In In proceedings of the third conference on artificial general intelligence. Lugano, Switzerland.

McCulloch, W., & Pitts, W. (1943, December 21). A logical calculus of the ideas immanent in nervous activity. Bulletin of Mathematical Biology, 5(4), 115-133.

Micola, A. R., Banal-Estañol, A., & Bunn, D. W. (2008, August). Incentives and coordination in vertically related energy markets. Journal of Economic Behavior & Organization, 67(2), 381-393.

Micola, A. R., & Bunn, D. W. (2008). Crossholdings, concentration and information in capacity-constrained sealed bid-offer auctions. Journal of Economic Behavior & Organization, 66(3-4), 748-766.

Minkel, J. R. (2008, August 13). The 2003 northeast blackout–five years later. Scientific American.

Momoh, J., Adapa, R., & El-Hawary, M. (1999, Feb). A review of selected optimal power flow literature to 1993. I. Nonlinear and quadratic programming approaches. Power Systems, IEEE Transactions on, 14(1), 96-104.

74

Momoh, J., El-Hawary, M., & Adapa, R. (1999, Feb). A review of selected optimal power flow literature to 1993. II. Newton, linear programming and interior point methods. Power Systems, IEEE Transactions on, 14(1), 105-111.

Moody, J., & Saffell, M. (2001, July). Learning to trade via direct reinforcement. IEEE Transactions on Neural Networks, 12(4), 875-889.

Moody, J., Wu, L., Liao, Y., & Saffell, M. (1998). Performance functions and reinforcement learning for trading systems and protfolios. Journal of Forecasting, 17, 441-470.

Naghibi-Sistani, M., Akbarzadeh-Tootoonchi, M., Javidi-D.B., M., & Rajabi-Mashhadi, H. (2006, November). Q-adjusted annealing for Q-learning of bid selection in market-based multisource power systems. Generation, Transmission and Distribution, IEE Proceedings, 153(6), 653-660.

Nash, J. F. (1950, January). Equilibrium points in n-person games. Proceedings of the National Academy of Sciences of the United States of America, 36(1), 48-49.

Nash, J. F. (1951, September). Non-cooperative games. The Annals of Mathematics, 54(2), 286-295. Available from `http://dx.doi.org/10.2307/1969529`

National Electricity Transmission System Operator. (2007, September). Large combustion plant directive (Tech. Rep.). National Grid Electricity Transmission plc. (GCRP 07/32)

National Electricity Transmission System Operator. (2010, May). 2010 National Electricity Transmission System Seven Year Statement (Tech. Rep.). National Grid Electricity Transmission plc.

Nicolaisen, J., Petrov, V., & Tesfatsion, L. (2002, August). Market power and efficiency in a computational electricity market with discriminatory double-auction pricing. Evolutionary Computation, IEEE Transactions on, 5(5), 504-523.

Nicolaisen, J., Smith, M., Petrov, V., & Tesfatsion, L. (2000). Concentration and capacity effects on electricity market power. In Evolutionary Computation. Proceedings of the 2000 Congress on (Vol. 2, p. 1041-1047).

Overbye, T., Cheng, X., & Sun, Y. (2004, Jan.). A comparison of the AC and DC power flow models for LMP calculations. In System sciences, 2004. Proceedings of the 37th annual Hawaii international conference on (p. 9-).

Peshkin, L., & Savova, V. (2002). Reinforcement learning for adaptive routing. In Neural Networks, 2002. IJCNN 2002. Proceedings of the 2002 International Joint Conference on (Vol. 2, p. 1825-1830).

Peters, J. (2010). Policy gradient methods. (Available online: www.scholarpedia.org)

Peters, J., & Schaal, S. (2006, October). Policy gradient methods for robotics. In Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on (p. 2219-2225).

Peters, J., & Schaal, S. (2008). Natural actor-critic. Neurocomputing, 71(7-9), 1180-1190.

Rastegar, M. A., Guerci, E., & Cincotti, S. (2009, May). Agent-based model of the Italian wholesale electricity market. In Energy Market, 2009. 6th International Conference on the European (p. 1-7).

Riedmiller, M. (2005). Neural fitted Q iteration - first experiences with a data efficient neural reinforcement learning method. In In 16th European conference on machine learning (pp. 317–328). Springer.

Riedmiller, M., & Braun, H. (1993). A direct adaptive method for faster backpropagation learning: the rprop algorithm.

Robbins, H. (1952). Some aspects of the sequential design of experiments. Bulletin American Mathematical Society, 58(5), 527-535.

Roth, A. E., Erev, I., Fudenberg, D., Kagel, J., Emilie, J., & Xing, R. X. (1995). Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term. Games and Economic Behavior, 8(1), 164-212.

Schaul, T., Bayer, J., Wierstra, D., Sun, Y., Felder, M., Sehnke, F., et al. (2010). PyBrain. Journal of Machine Learning Research, 11, 743-746.

Schweppe, F., Caramanis, M., Tabors, R., & Bohn, R. (1988). Spot pricing of electricity. Dordrecht: Kluwer Academic Publishers Group.

Sharpe, W. F. (1966, January). Mutual fund performance. Journal of Business, 119-138.

Sharpe, W. F. (1994). The Sharpe ratio. The Journal of Portfolio Management, 49-58.

Stott, B., & Alsac, O. (1974, May). Fast decoupled load flow. Power Apparatus and Systems, IEEE Transactions on, 93(3), 859-869.

Sun, J., & Tesfatsion, L. (2007a). Dynamic testing of wholesale power market designs: An open-source agent-based framework. Computational Economics, 30(3), 291-327.

Sun, J., & Tesfatsion, L. (2007b, June). Open-source software for power industry research, teaching, and training: A DC-OPF illustration. In Power Engineering Society General Meeting, 2007. IEEE (p. 1-6).

Sutton, R. S., & Barto, A. G. (1998). Reinforcement learning: An introduction. MIT Press. Gebundene Ausgabe.

Sutton, R. S., McAllester, D., Singh, S., & Mansour, Y. (2000). Policy gradient methods for reinforcement learning with function approximation. In Advances in neural information processing systems (Vol. 12, p. 1057-1063).

Tellidou, A., & Bakirtzis, A. (2007, Novemeber). Agent-based analysis of capacity withholding and tacit collusion in electricity markets. Power Systems, IEEE Transactions on, 22(4), 1735-1742.

Tesauro, G. (1994). TD-Gammon, a self-teaching backgammon program, achieves master-level play. Neural Computation, 6(2), 215-219.

Tesfatsion, L., & Judd, K. L. (2006). Handbook of computational economics, volume 2: Agent-based computational economics (handbook of computational economics). Amsterdam, The Netherlands: North-Holland Publishing Co.

The International Energy Agency. (2010, Septemeber). Key world energy statistics 2010. Paris.

Tinney, W., & Hart, C. (1967, Novemeber). Power flow solution by Newton's method. Power Apparatus and Systems, IEEE Transactions on, 86(11), 1449-1460.

Tsitsiklis, J. N., & Roy, B. V. (1994). Feature-based methods for large scale dynamic programming. In Machine learning (p. 59-94).

United Nations. (2003, December 9). World population in 2300. In Proceedings of the United Nations, Expert Meeting on World Population in 2300.

U.S.-Canada Power System Outage Task Force. (2004, April). Final report on the august 14, 2003 blackout in the united states and canada: Causes and recommendations (Tech. Rep.). North American Electric Reliability Corporation.

Veit, D., Weidlich, A., Yao, J., & Oren, S. (2006). Simulating the dynamics in two-settlement electricity markets via an agent-based approach. International Journal of Management Science and Engineering Management, 1(2), 83-97.

Vengerov, D. (2008). A gradient-based reinforcement learning approach to dynamic pricing in partially-observable environments. Future Generation Computer Systems, 24(7), 687-693.

Visudhiphan, P. (2003). An agent-based approach to modeling electricity spot markets. Unpublished doctoral dissertation, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology.

Visudhiphan, P., & Ilic, M. (1999, February). Dynamic games-based modeling

of electricity markets. In Power Engineering Society 1999 Winter Meeting, IEEE (Vol. 1, p. 274-281).

Wang, H., Murillo-Sanchez, C., Zimmerman, R., & Thomas, R. (2007, Aug.). On computational issues of market-based optimal power flow. Power Systems, IEEE Transactions on, 22(3), 1185-1193.

Wang, J., Koritarov, V., & Kim, J.-H. (2009, July). An agent-based approach to modeling interactions between emission market and electricity market. In Power Energy Society General Meeting, 2009. PES 2009. IEEE (p. 1-8).

Weidlich, A., & Veit, D. (2006, July 7-10). Bidding in interrelated day-ahead electricity markets - insights from an agent-based simulation model. In Proceedings of the 29th IAEE International Conference.

Weidlich, A., & Veit, D. (2008, July). A critical survey of agent-based wholesale electricity market models. Energy Economics, 30(4), 1728-1759.

Williams, R. J. (1992). Simple statistical gradient-following algorithms for connectionist reinforcement learning. In Machine learning (p. 229-256).

Wood, A. J., & Wollenberg, B. F. (1996). Power Generation Operation and Control (second ed.). New York: Wiley, New York.

Yao, J., Adler, I., & Oren, S. S. (2008). Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network. Operations Research, 56(1), 34-47.

Yao, J., Oren, S. S., & Adler, I. (2007). Two-settlement electricity markets with price caps and cournot generation firms. European Journal of Operational Research, 181(3), 1279-1296.

Zimmerman, R. (2010, March 19). MATPOWER 4.0b2 User's Manual [Computer software manual]. School of Electrical Engineering, Cornell University, Ithaca, NY 14853.

Zimmerman, R., Murillo-Sánchez, C., & Thomas, R. J. (2009, July). MATPOWER's extensible optimal power flow architecture. In IEEE PES General Meeting. Calgary, Alberta, Canada.