2.2 — Production Technology

ECON 306 • Microeconomic Analysis • Spring 2023

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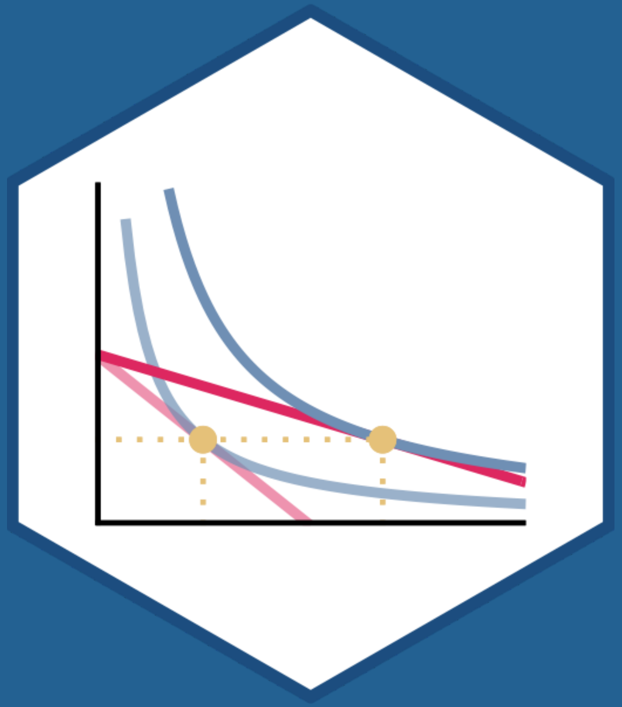
[ryansafner/microS23](https://github.com/ryansafner/microS23)

microS23.classes.ryansafner.com



Outline

Production in the Short Run

The Firm's Problem: Long Run

Isoquants and MRTS

Isocost Lines

The "Runs" of Production

- "Time"-frame usefully divided between short vs. long run analysis

- **Short run**: at least one factor of production is **fixed** (too costly to change)

$$q = f(\bar{k}, l)$$

  - Assume **capital** is fixed (i.e. number of factories, storefronts, etc)
  - Short-run decisions only about using **labor**

The "Runs" of Production

- "Time"-frame usefully divided between short vs. long run analysis

- **Long run**: all factors of production are **variable** (can be changed)

$$q = f(k, l)$$

Production in the Short Run

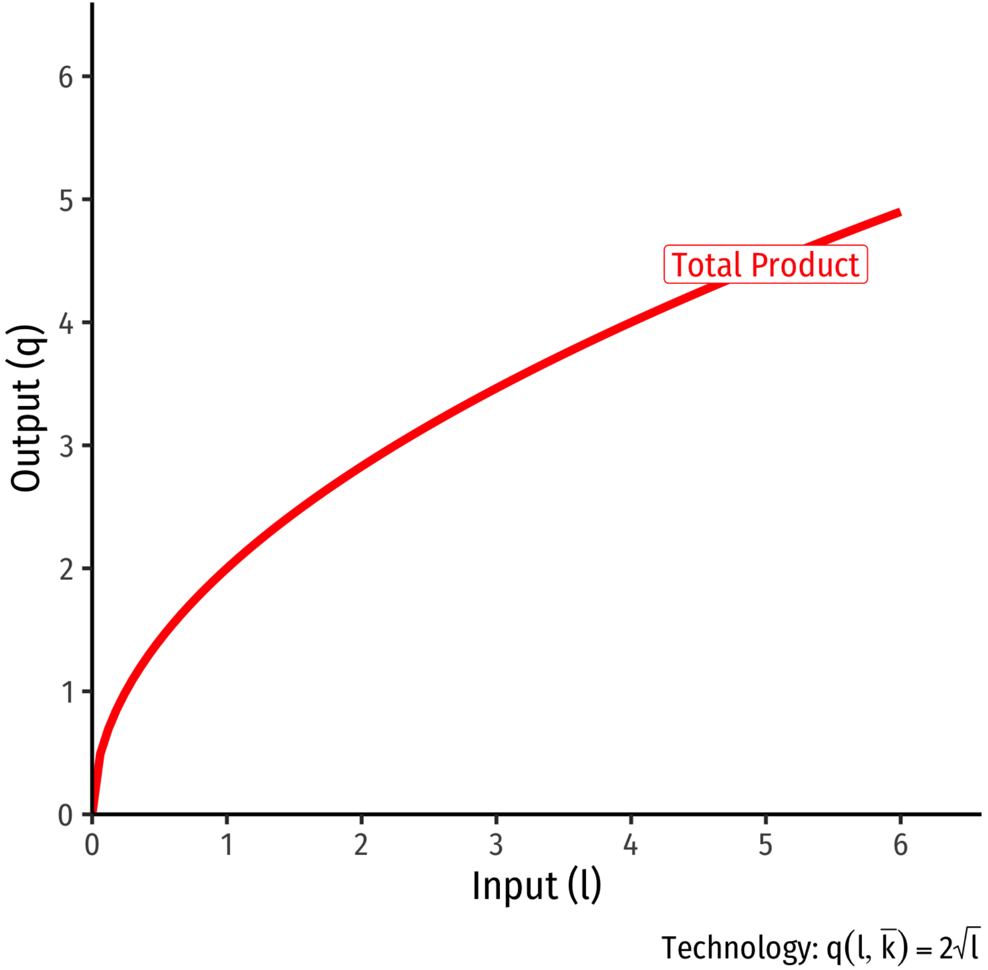
Production in the Short Run: Example

Example: Consider a firm with the production function

$$q = k^{0.5} l^{0.5}$$

- Suppose in the short run, the firm has 4 units of capital.

1. Derive the short run production function.
2. What is the total product (output) that can be made with 4 workers?
3. What is the total product (output) that can be made with 5 workers?

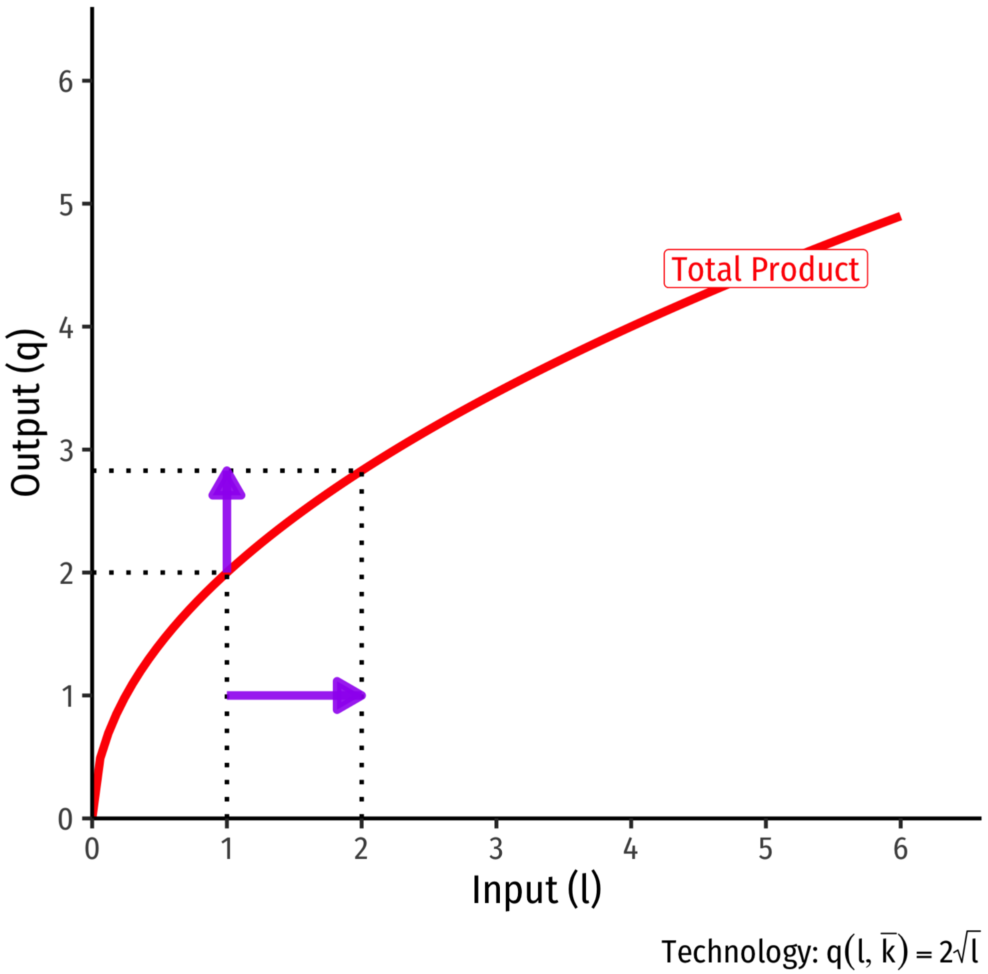


Marginal Products

- The **marginal product** of an input is the *additional* output produced by *one more unit* of that input (*holding all other inputs constant*)

- Like marginal utility

- Similar to marginal utilities, I will give you the marginal product equations

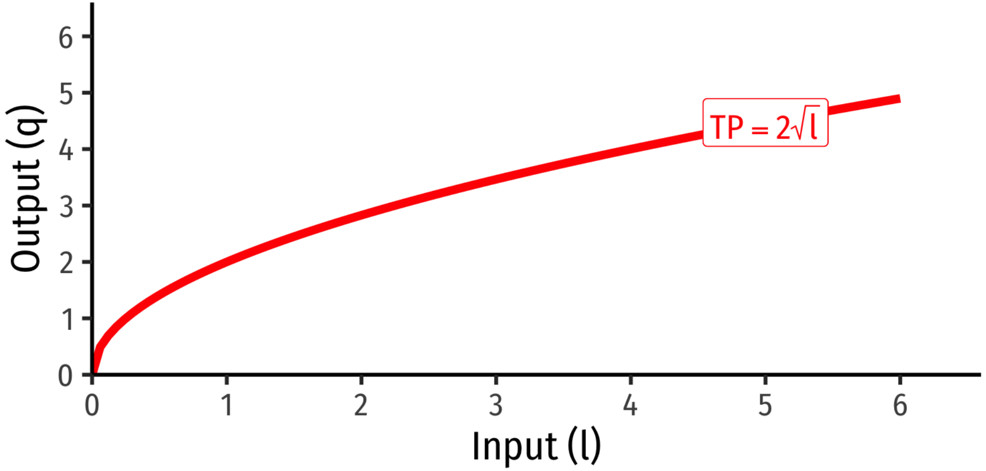


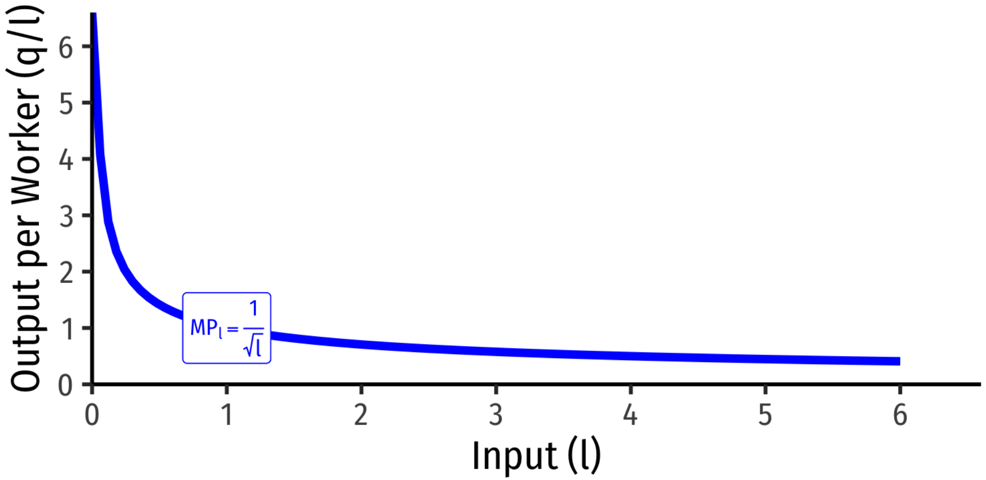
Marginal Product of Labor

- **Marginal product of labor** (MP_l): additional output produced by adding one more unit of labor (holding k constant)

$$MP_l = \frac{\Delta q}{\Delta l}$$

- MP_l is slope of TP at each value of l!
  - Note: via calculus: $\frac{\partial q}{\partial l}$



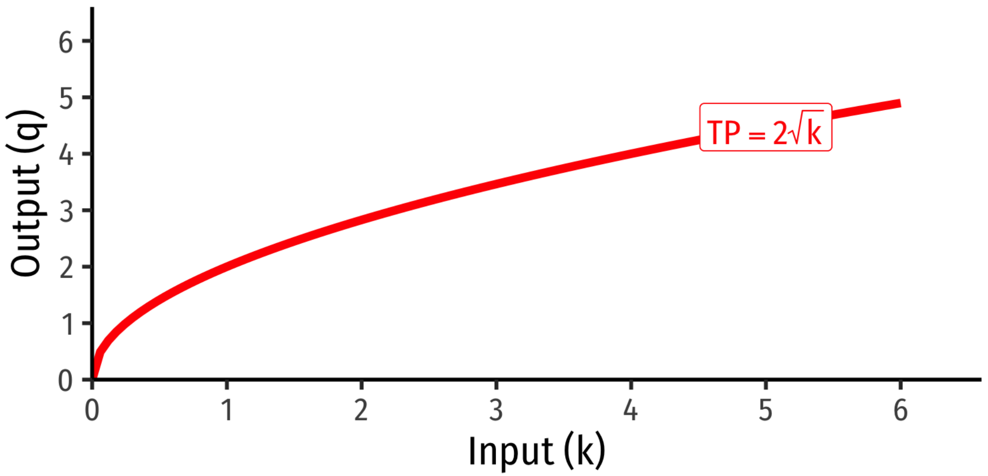


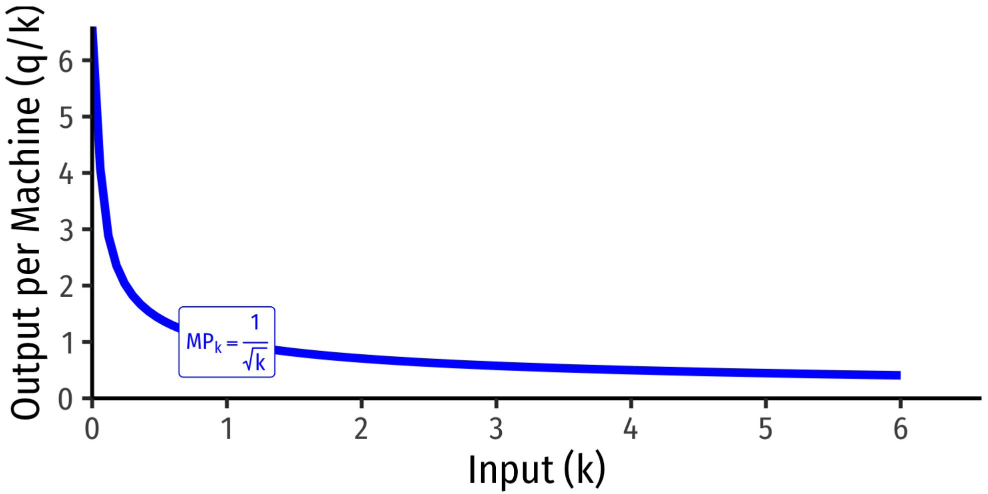
Marginal Product of Capital

- **Marginal product of capital (MP_k)**: additional output produced by adding one more unit of capital (holding l constant)

$$MP_k = \frac{\Delta q}{\Delta k}$$

- MP_k is slope of TP at each value of k!
  - Note: via calculus: $\frac{\partial q}{\partial k}$
- Note we don't consider capital in the short run!

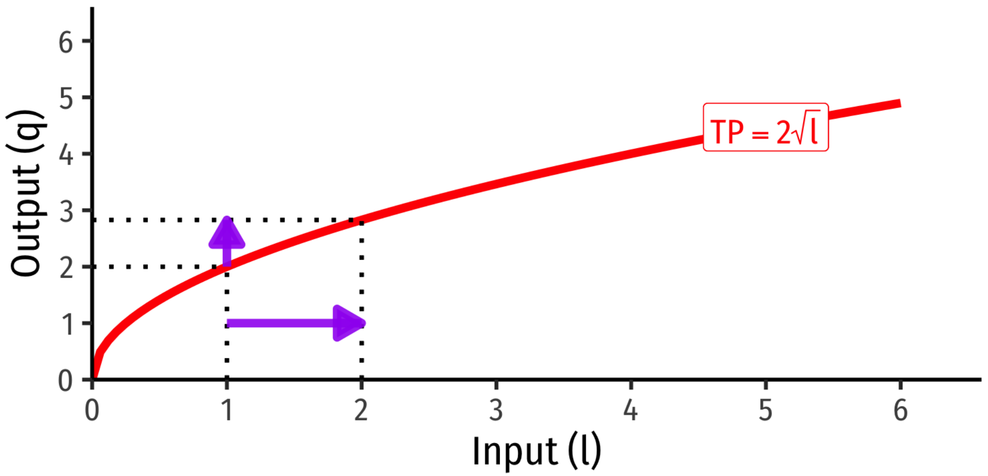


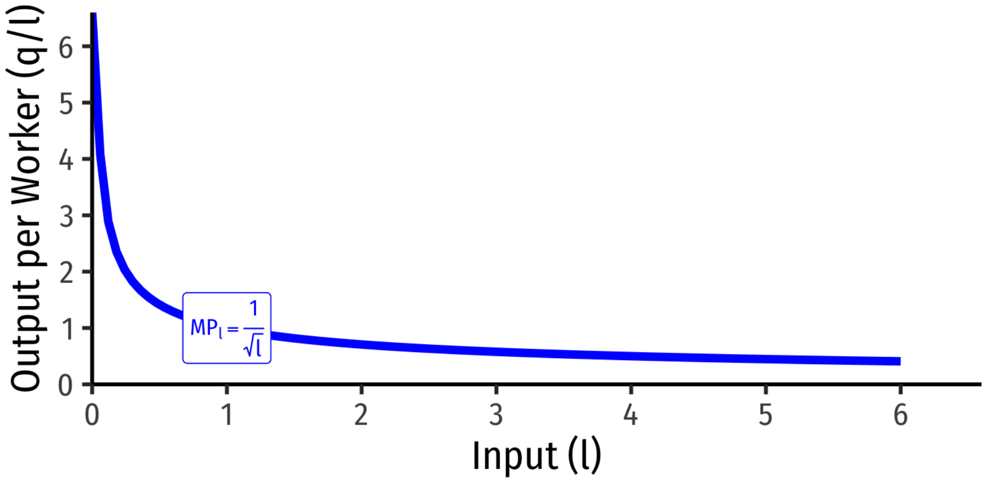


Diminishing Returns

- **Law of Diminishing Returns**: adding more of one factor of production **holding all others constant** will result in successively lower increases in output

- In order to increase output, firm will need to increase *all* factors!





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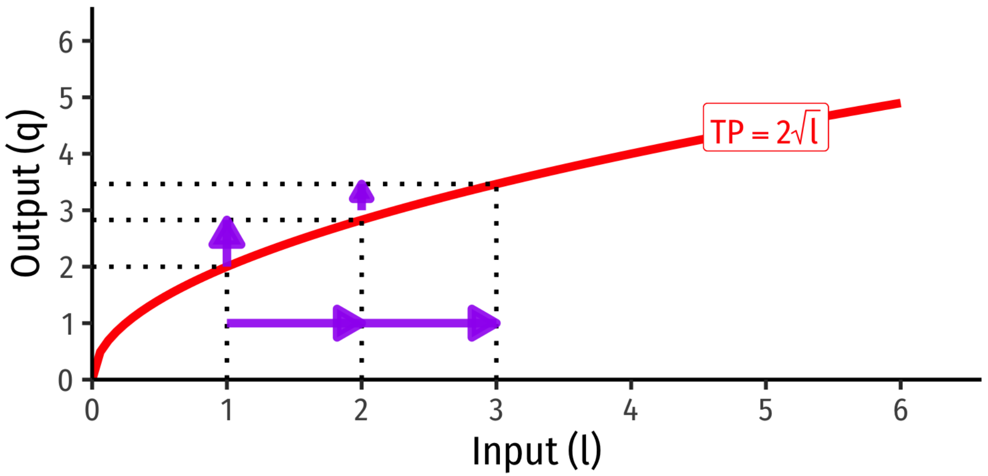
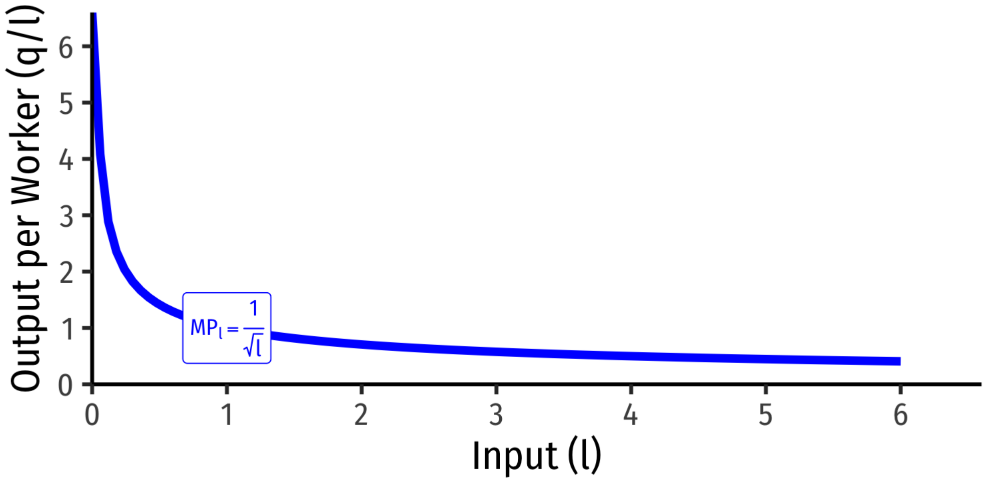




Average Product of Labor (and Capital)

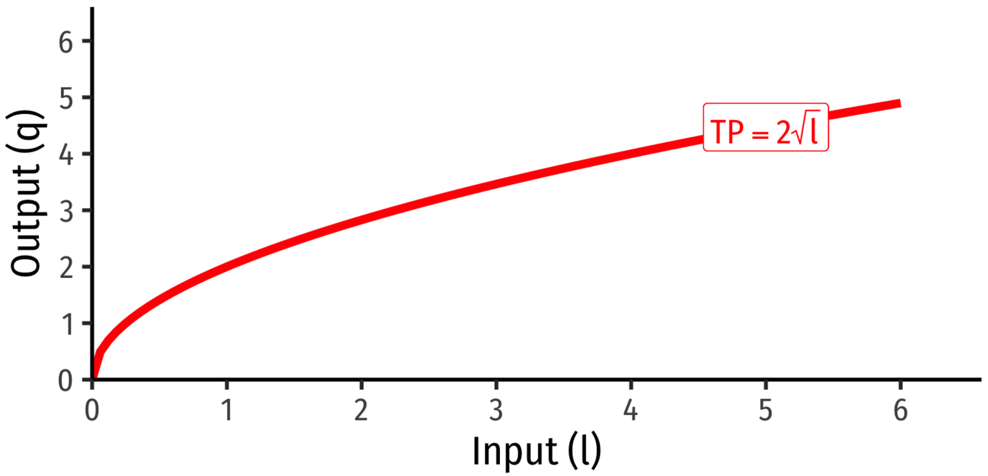
- **Average product of labor** (AP_l): total output per worker

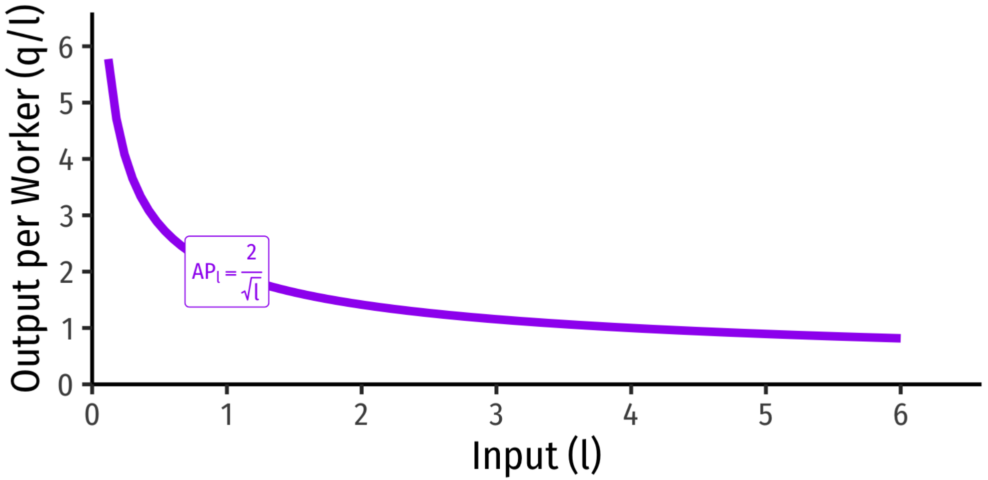
$$AP_l = \frac{q}{l}$$

- A measure of *labor productivity*

- **Average product of capital** (AP_k): total output per unit of capital

$$AP_k = \frac{q}{k}$$





The Firm's Problem: Long Run

The Long Run

- In the long run, *all* factors of production are **variable**

$$q = f(k, l)$$

- Can build more factories, open more storefronts, rent more space, invest in machines, etc.

- So the firm can choose both l *and* k

The Firm's Problem

- Based on what we've discussed, we can fill in a constrained optimization model for the firm
  - But don't write this one down just yet!
- The **firm's problem** is:

1. **Choose:** < inputs and output >
2. **In order to maximize:** < profits >
3. **Subject to:** < technology >

- It's actually much easier to break this into **2 stages**. See today’s class notes page for an example using only one stage.

The Firm's Two Problems

1st Stage: **firm's profit maximization problem**:

1. **Choose:** **< output >**

2. **In order to maximize:** **< profits >**

- We'll cover this later...first we'll explore:

The Firm's Two Problems

1^{st} Stage: **firm's profit maximization problem**:

1. **Choose:** < output >
2. **In order to maximize:** < profits >

- We'll cover this later...first we'll explore:

2^{nd} Stage: **firm's cost minimization problem**:

1. **Choose:** < inputs >
2. **In order to *minimize*:** < cost >
3. **Subject to:** < producing the optimal output >

- Minimizing costs $\iff$ maximizing profits

Long Run Production

Example: $q = \sqrt{lk}$

Capital, k

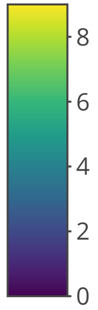
Labor, l

	0	1	2	3	4	5
0	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	1.00	1.41	1.73	2.00	2.24
2	0.00	1.41	2.00	2.45	2.83	3.16
3	0.00	1.73	2.45	3.00	3.16	3.46
4	0.00	2.00	2.83	3.46	4.00	4.47
5	0.00	2.24	3.16	3.87	4.47	5.00

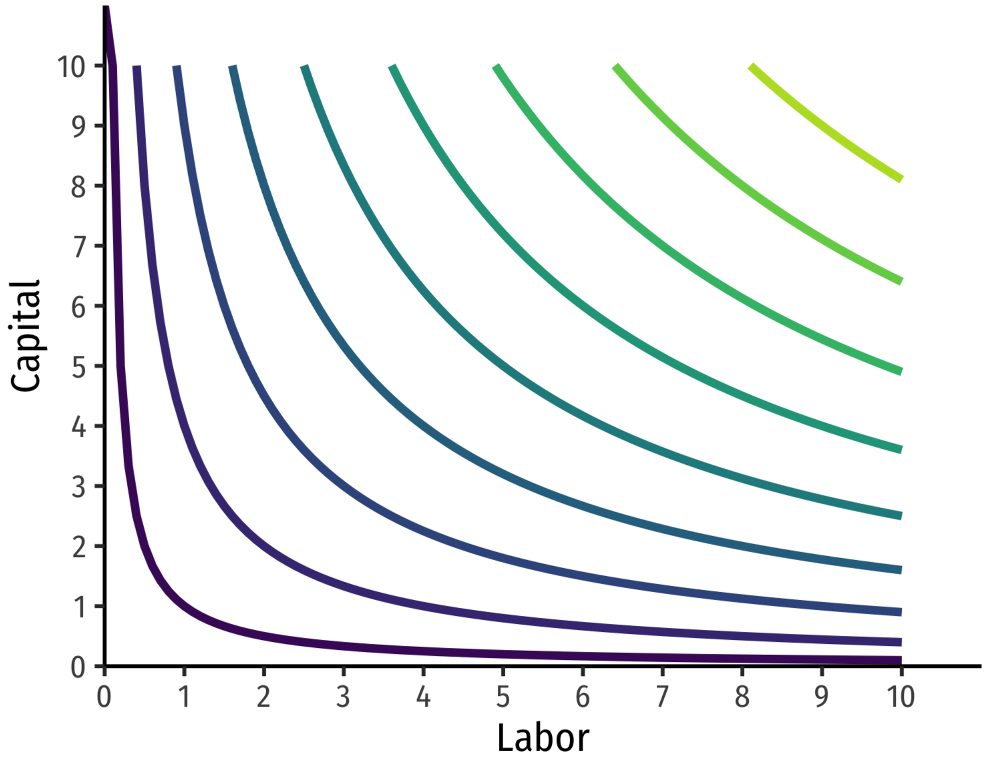
- Many input-combinations yield the same output!
- So how does the firm choose the *optimal* combination?

Mapping Input-Combination Choices Graphically

3-D Production Function



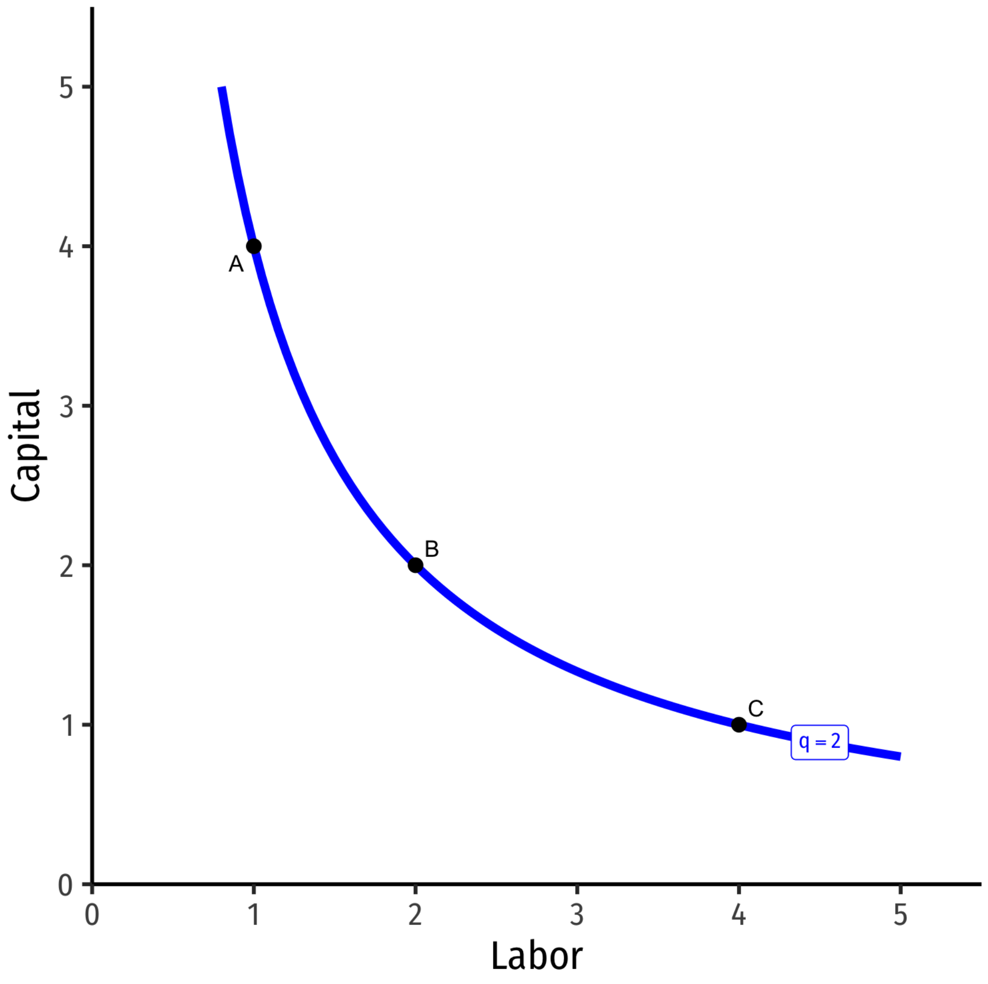
2-D Isoquant Contours



Isoquants and MRTS

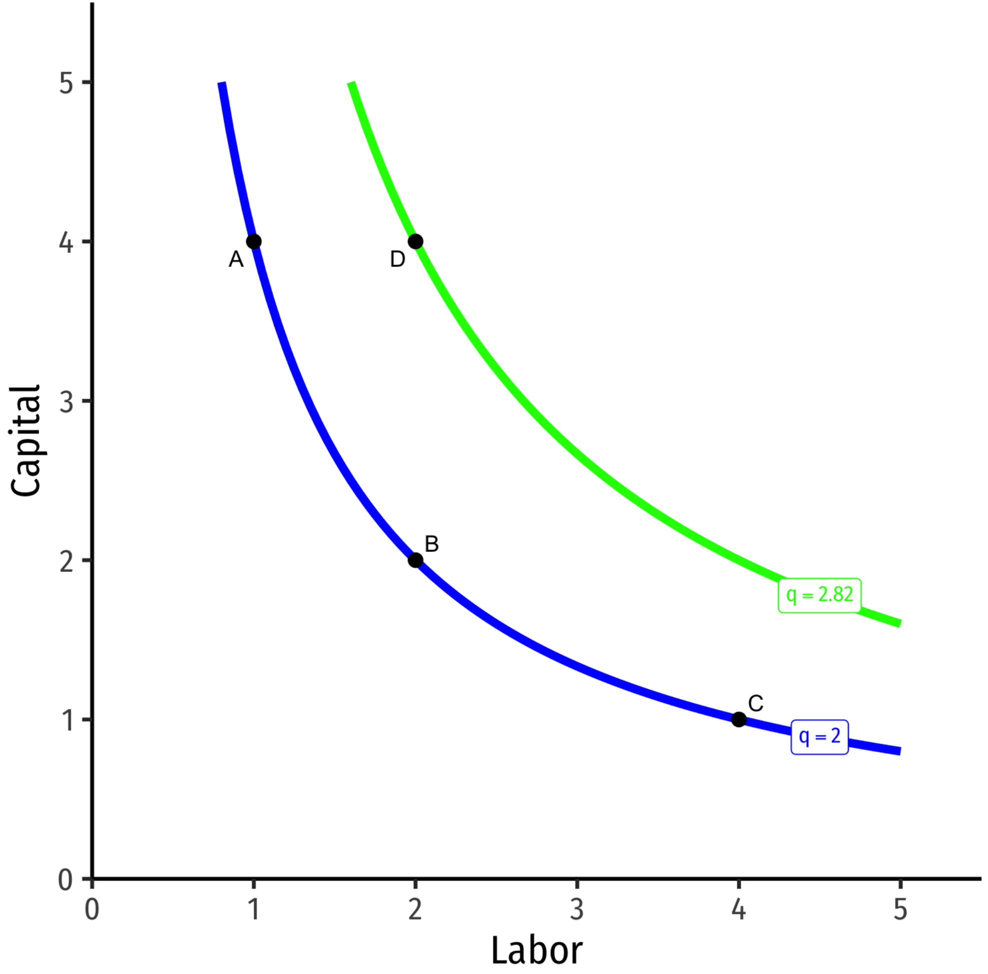
Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q



Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q
- Combinations *above* curve yield **more output**; on a **higher curve**
  - $D > A = B = C$

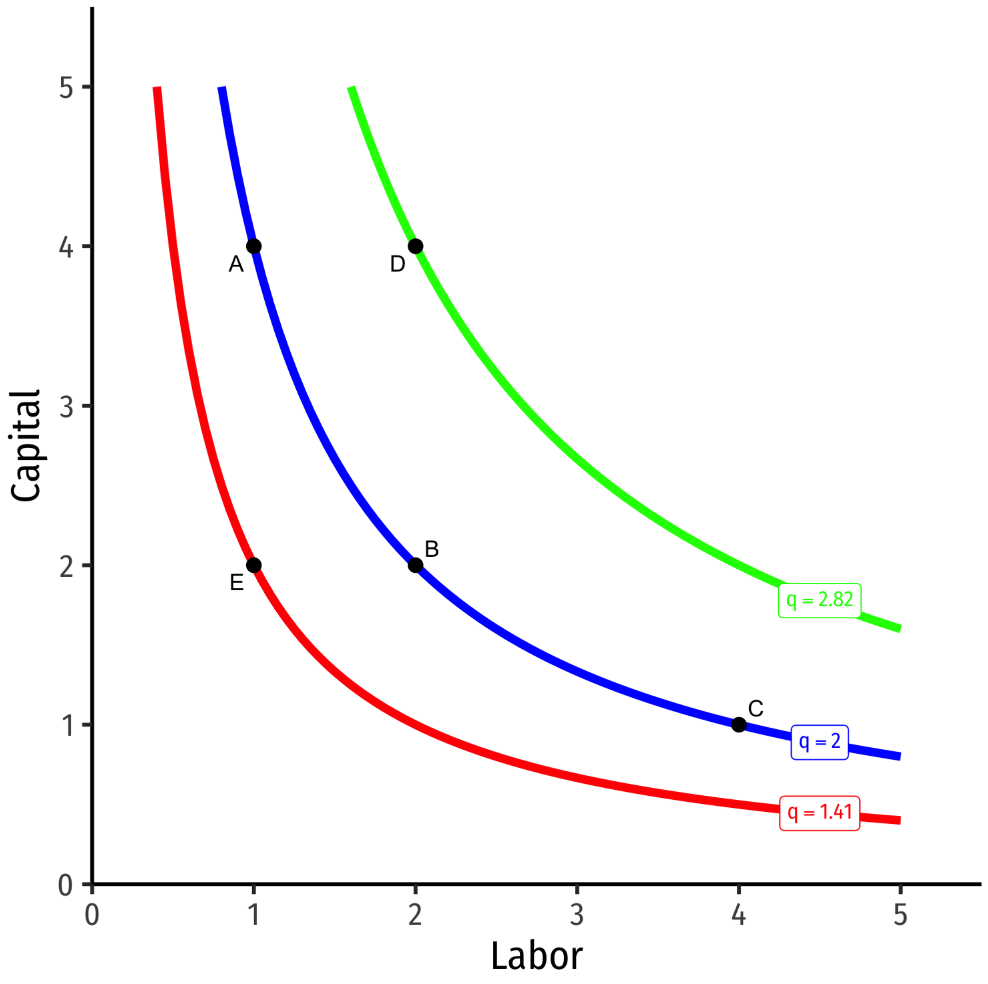


Isoquant Curves

- We can draw an **isoquant** indicating all combinations of l and k that yield the same q

- Combinations *above* curve yield **more output**; on a **higher curve**
  - $D > A = B = C$

- Combinations *below* the curve yield **less output**; on a **lower curve**
  - $E < A = B = C$



Marginal Rate of *Technical* Substitution I

- If your firm uses fewer workers, how much more capital would it need to produce the same amount?

Marginal Rate of *Technical* Substitution I

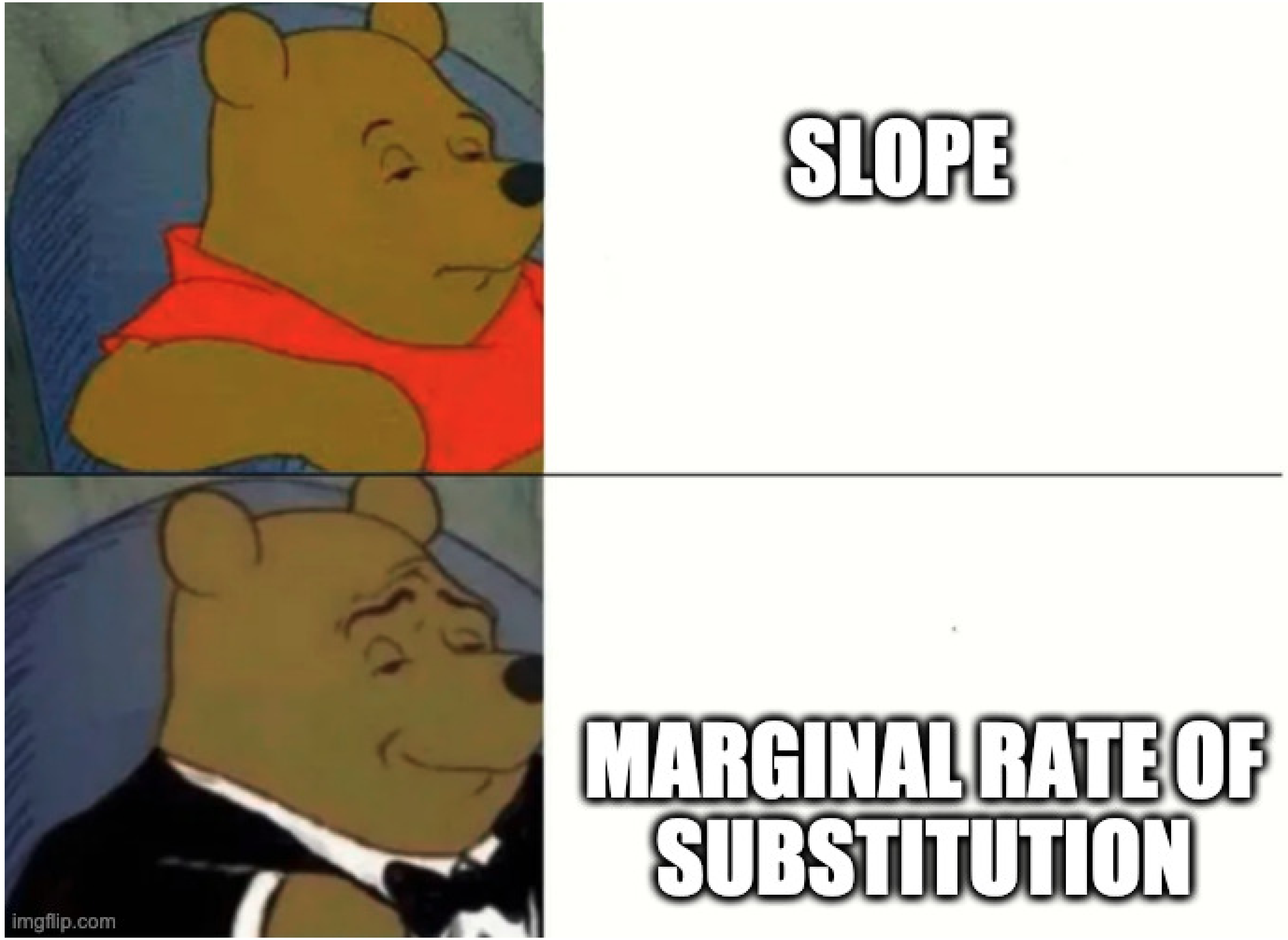
- If your firm uses fewer workers, how much more capital would it need to produce the same amount?

- **Marginal Rate of Technical Substitution (MRTS)**: rate at which firm trades off one input for another to *yield same output*

- Firm's **relative value** of using l in production based on its tech:

> "We could give up (MRTS) units of k to use 1 more unit of l to produce the same output."

Marginal Rate of *Technical* Substitution II

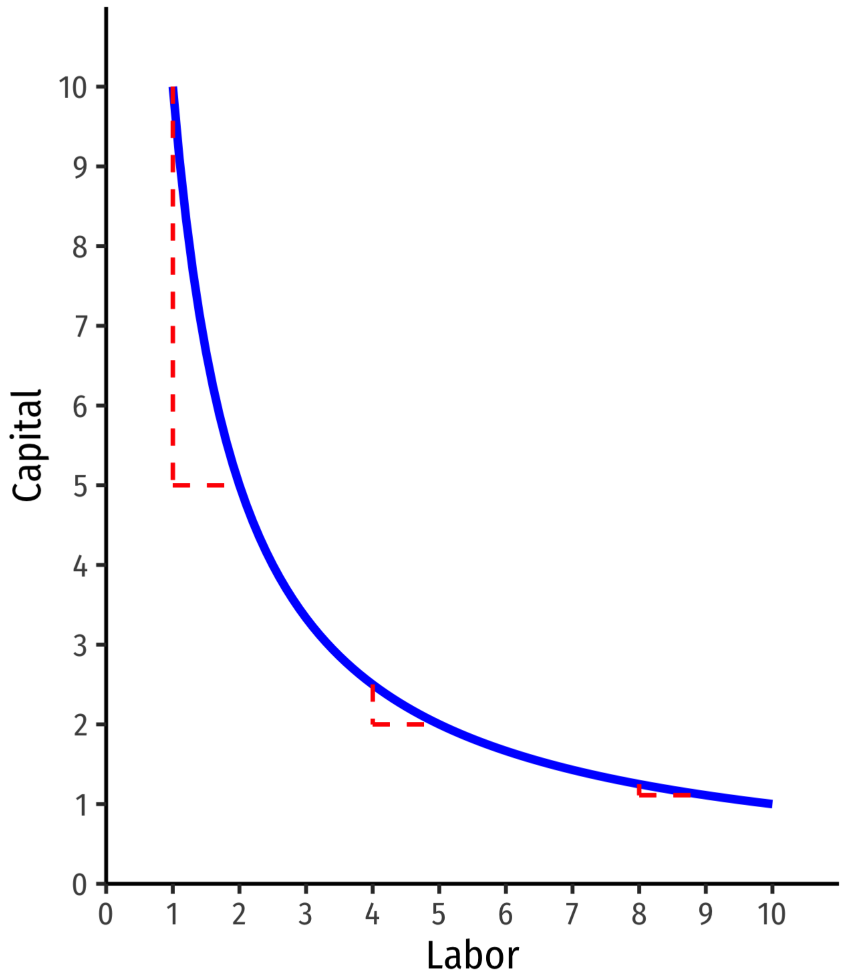


Marginal Rate of *Technical* Substitution II

- MRTS is the slope of the isoquant

$$MRTS_{l,k} = -\frac{\Delta k}{\Delta l} = \frac{rise}{run}$$

- Amount of k given up for 1 more l
- Note: slope (MRTS) changes along the curve!
- **Law of diminishing returns**!



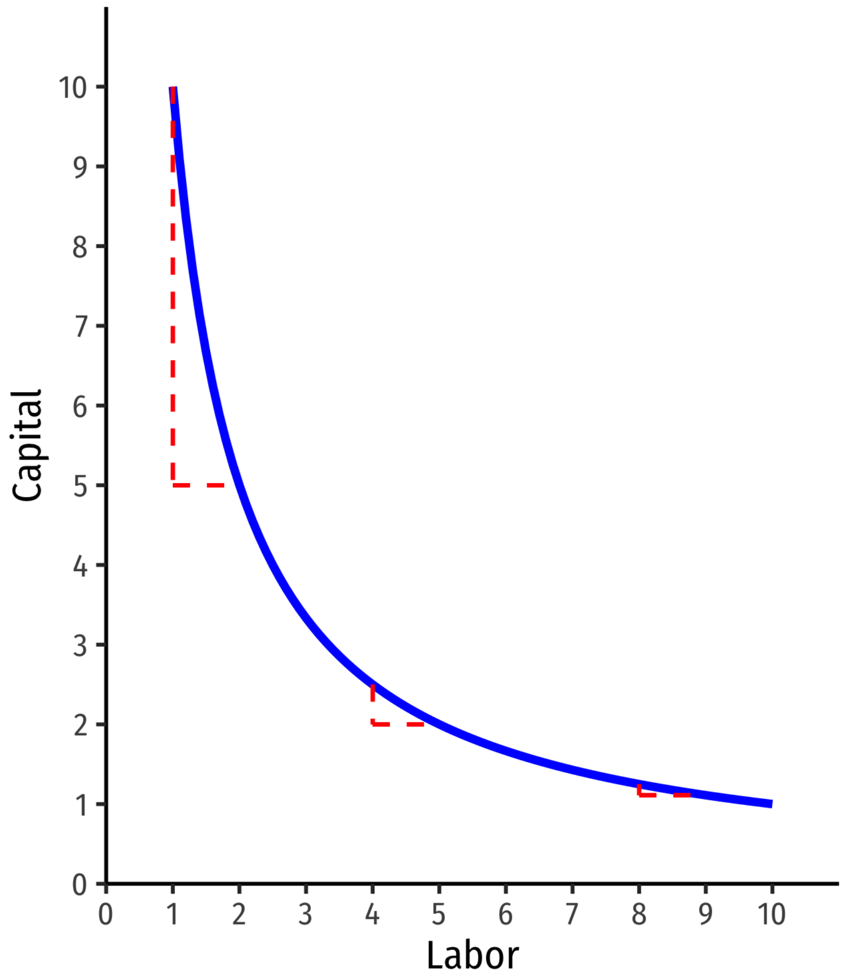
MRTS and Marginal Products

- Relationship between MP and $MRTS$:

$$\underbrace{\frac{\Delta k}{\Delta l}}_{MRTS} = -\frac{MP_l}{MP_k}$$

- See proof in [today's class notes]()

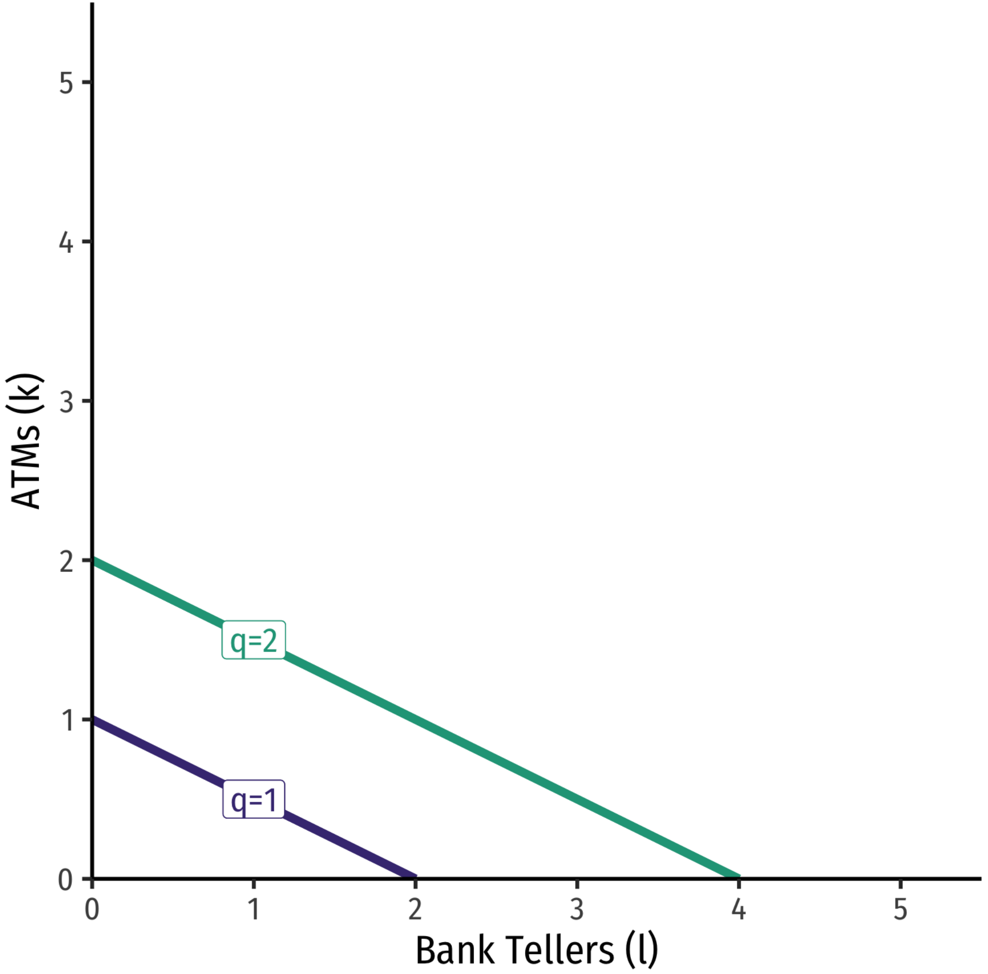
- Sound familiar? 🧐



Special Case I: Perfect Substitutes

Example: Consider Bank Tellers (l) and ATMs (k)

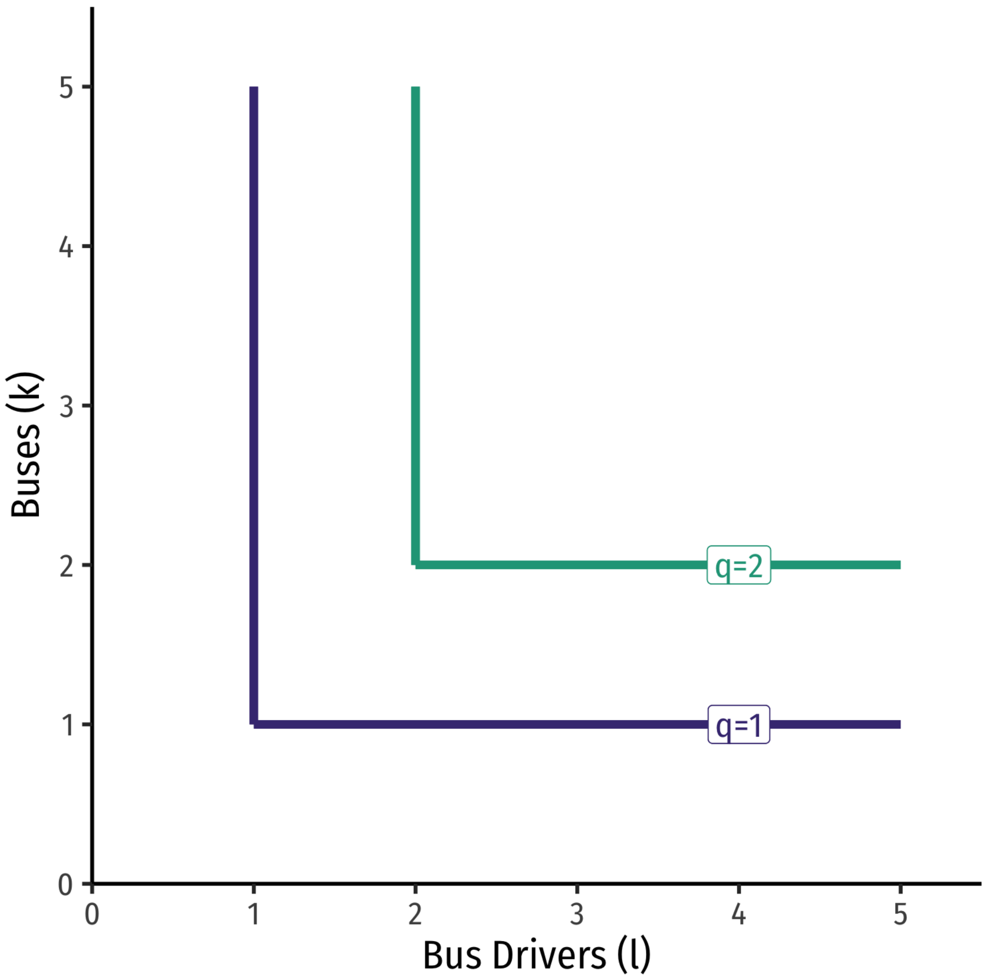
- Suppose 1 ATM can do the work of 2 bank tellers
- **Perfect substitutes**: inputs that can be substituted at same fixed rate and yield same output
- $MRTS_{l,k} = -0.5$ (a constant!)



Special Case II: Perfect Complements

Example: Consider buses (k) and bus drivers (l)

- Must combine together in fixed proportions (1:1)
- **Perfect complements**: inputs must be used together in same fixed proportion to produce output
- $MRTS_{l,k}$: ?

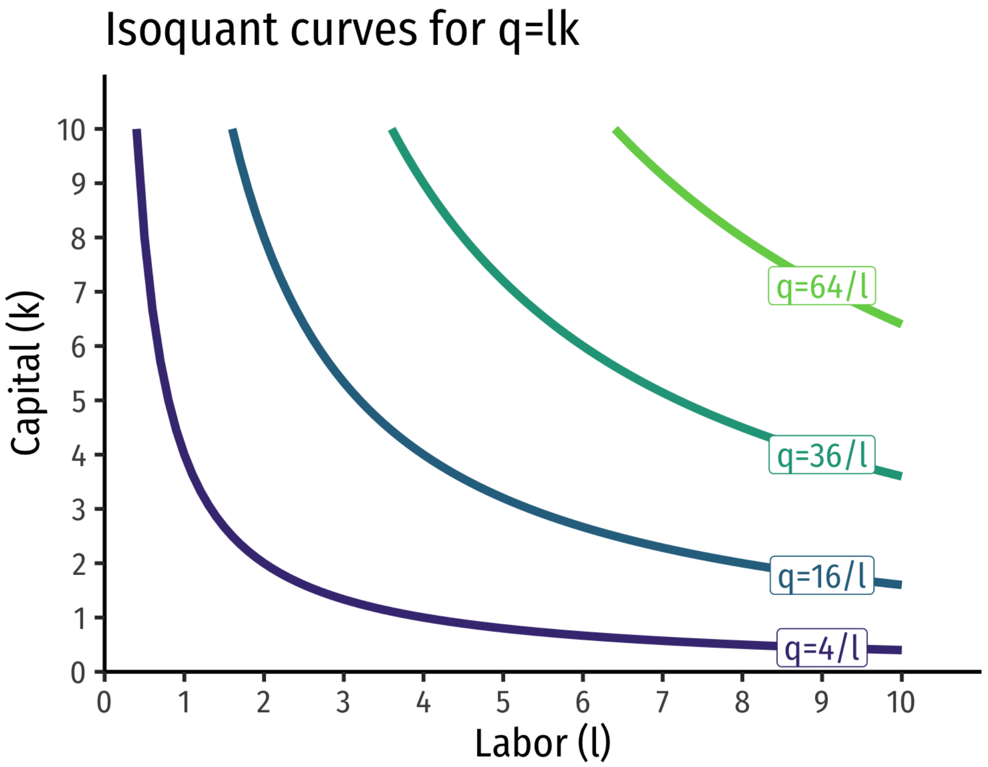


Common Case: Cobb-Douglas Production Functions

- Again: very common functional form in economics is **Cobb-Douglas**

$$q = A\,k^a l^b$$

- Where $a, b > 0$
  - often $a + b = 1$
- A is total factor productivity



Practice

Example: Suppose a firm has the following production function:

$$q = 2lk$$

Where its marginal products are:

$$MP_l = 2k$$
$$MP_k = 2l$$

1. Put l on the horizontal axis and k on the vertical axis. Write an equation for $MRTS_{l,k}$.

2. Would input combinations of $(1, 4)$ and $(2, 2)$ be on the same isoquant?

3. Sketch a graph of the isoquant from part 2.

Isocost Lines

Isocost Lines

- If your firm can choose among *many* input combinations to produce q, which combinations are optimal?

- Those combination that are **cheapest**

- Denote prices of each input as:

  - w: price of labor (wage)
  - r: price of capital

- Let C be **total cost** of using inputs (l, k) at market prices (w, r) to produce q units of output:

$$C(w, r, q) = wl + rk$$

The Isocost Line, Graphically

$$wl + rk = C$$

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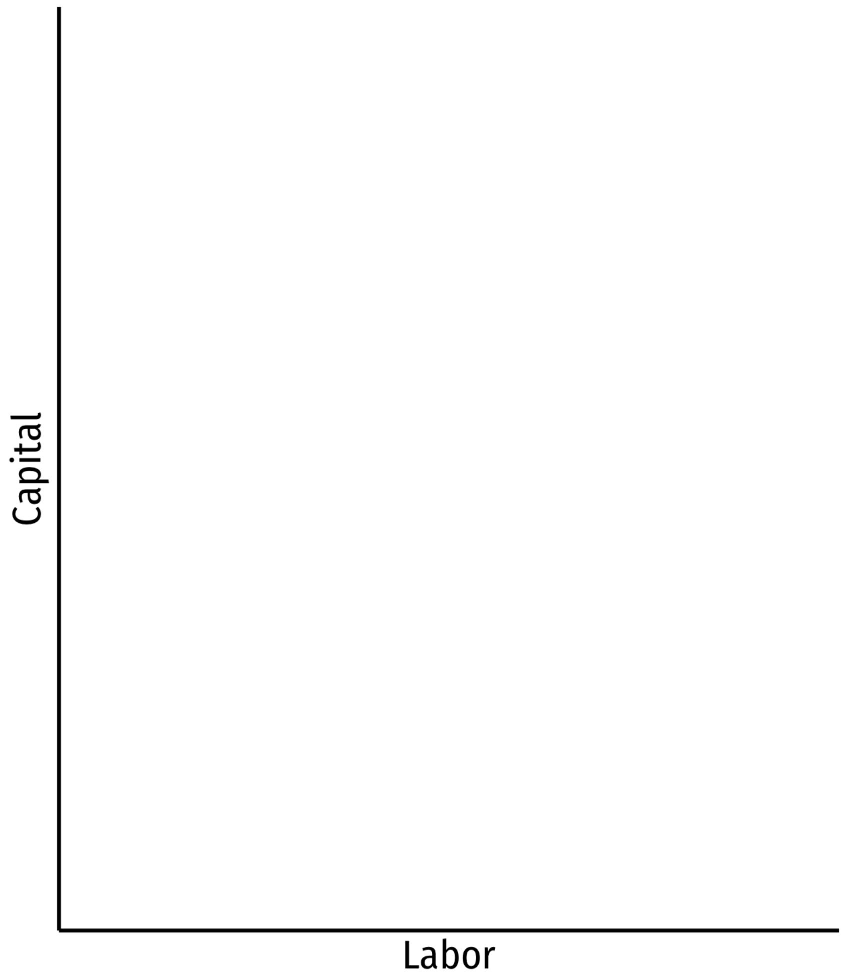
$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$



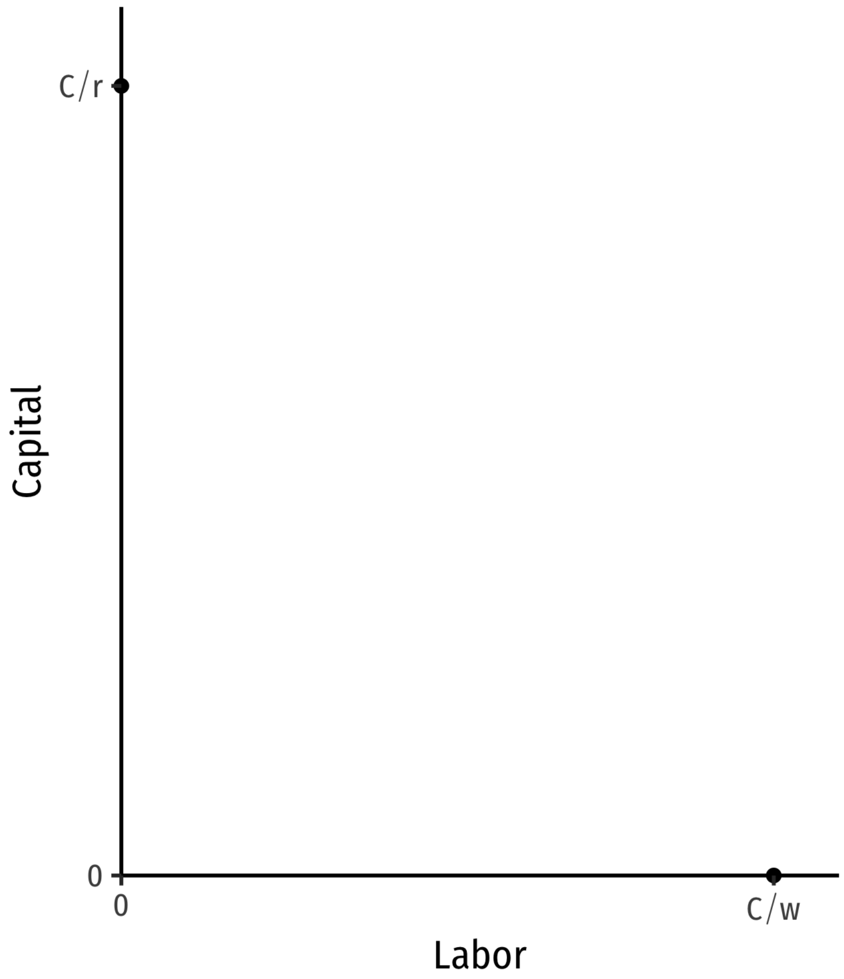
The Isocost Line, Graphically

$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$

- Vertical-intercept: $\frac{C}{r}$
- Horizontal-intercept: $\frac{C}{w}$



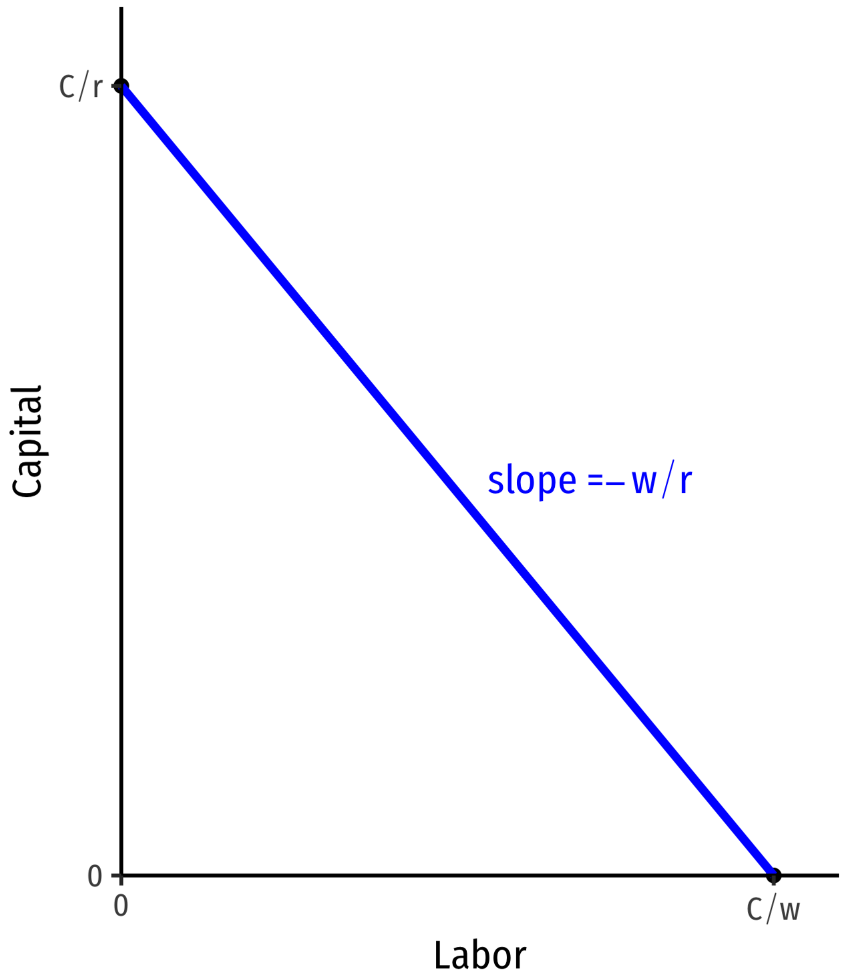
The Isocost Line, Graphically

$$wl + rk = C$$

- Solve for k to graph

$$k = \frac{C}{r} - \frac{w}{r}l$$

- Vertical-intercept: $\frac{C}{r}$
- Horizontal-intercept: $\frac{C}{w}$
- slope: $-\frac{w}{r}$



The Isocost Line: Example

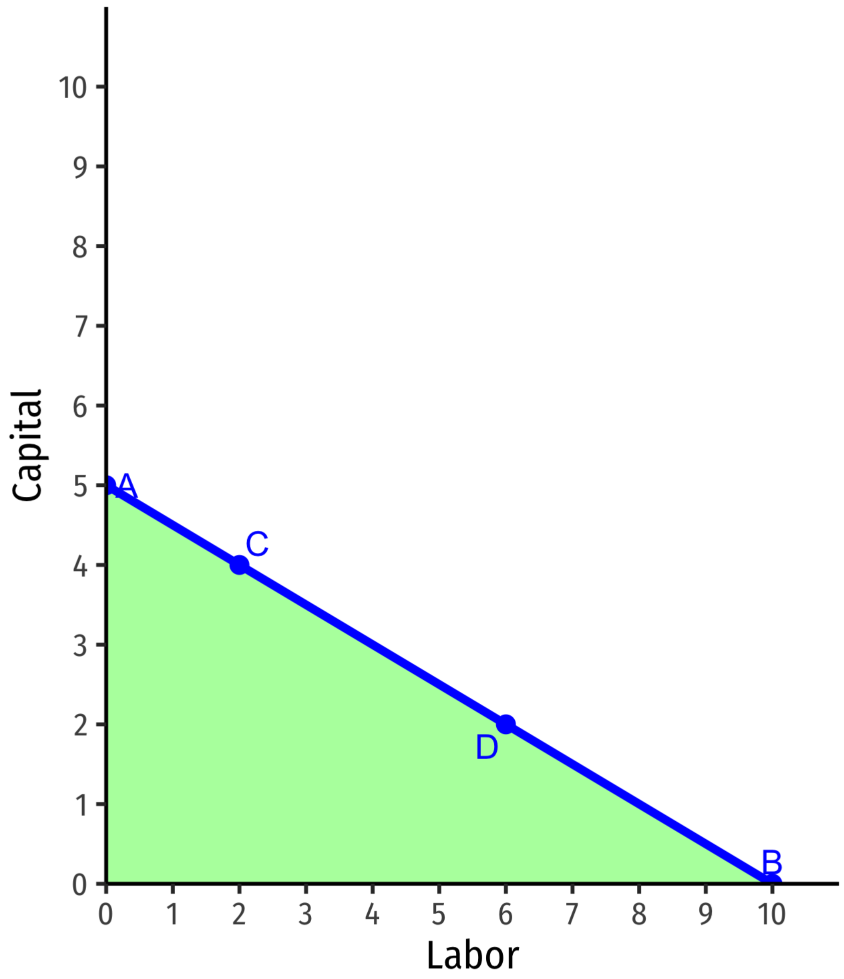
Example: Suppose your firm has a purchasing budget of \$50. Market wages are \$5/worker-hour and the mark rental rate of capital is \$10/machine-hour. Let l be on the horizontal axis and k be on the vertical axis.

1. Write an equation for the isocost line (in graphable form).

2. Graph the isocost line.

Interpreting the Isocost Line

- Points on the line are same total cost
  - A: $\$5(0l) + \$10(5k) = \$50$
  - B: $\$5(10l) + \$10(0k) = \$50$
  - C: $\$5(2l) + \$10(4k) = \$50$
  - D: $\$5(6l) + \$10(2k) = \$50$



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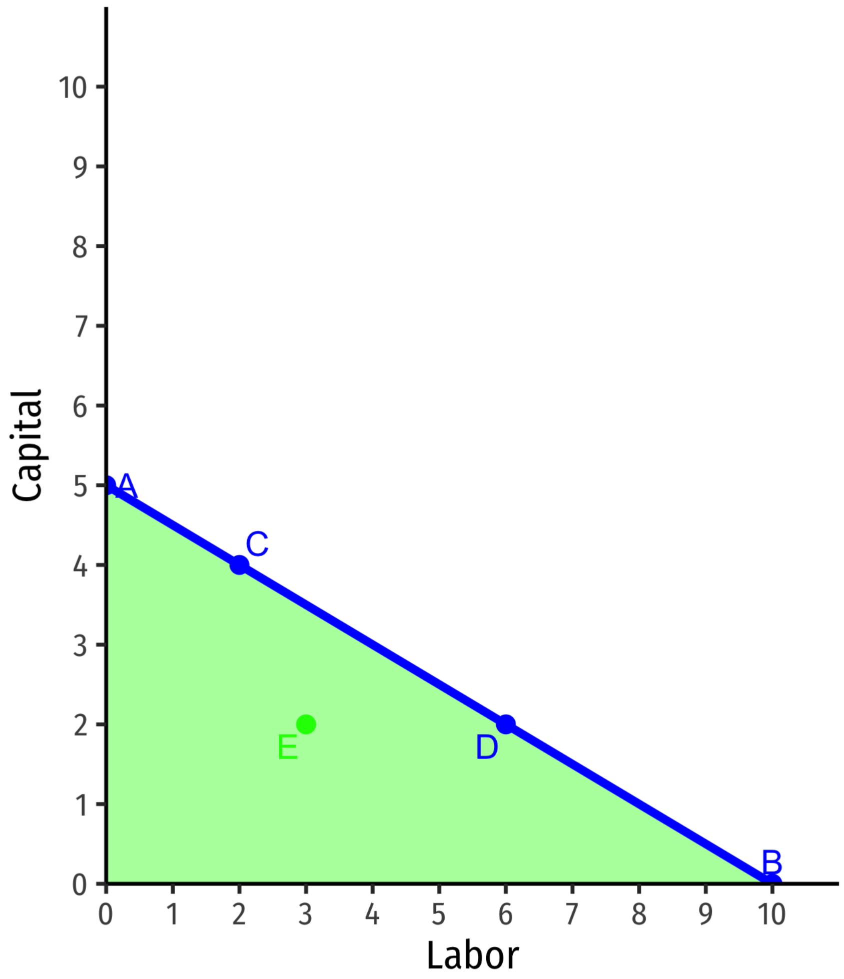
    - D: $\$5(6l) + \$10(2k) = \$50$

- Points beneath the line are cheaper (but may produce less)
    - E: $\$5(3l) + \$10(2k) = \$35$



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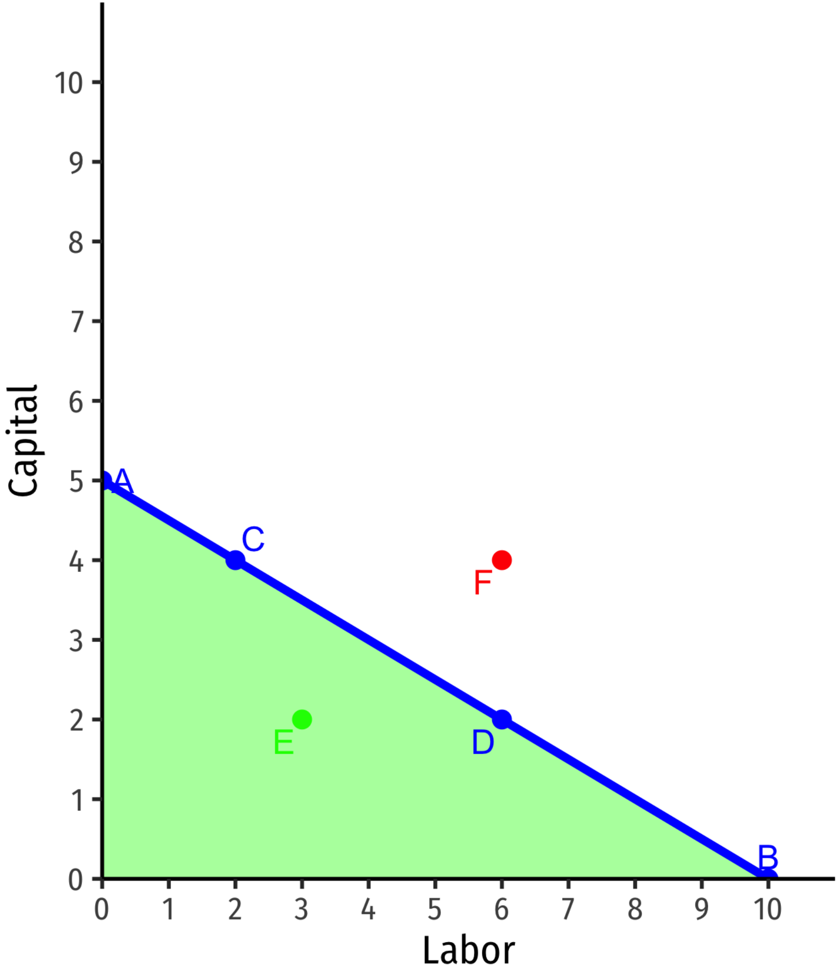
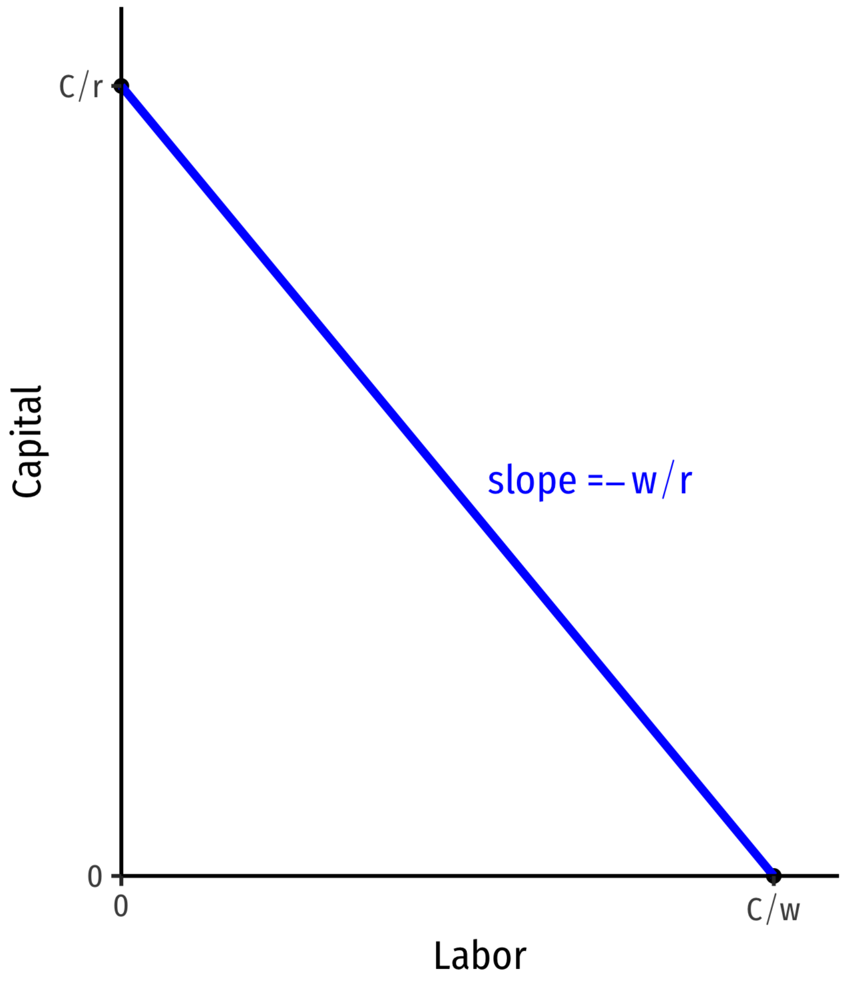
- Points beneath the line are cheaper (but may produce less)
  - E: $\$5(3l) + \$10(2k) = \$35$
- Points above the line are more expensive (and may produce more)
  - F: $\$5(6l) + \$10(4k) = \$70$



Interpretting the Slope

- **Slope**: **tradeoff** between l and k at market prices
    - Market **"exchange rate"** between l and k
- ***Relative* price** of l or the **opportunity cost** of l:

> Hiring 1 more unit of l requires giving up $\left(\frac{w}{r}\right)$ units of k

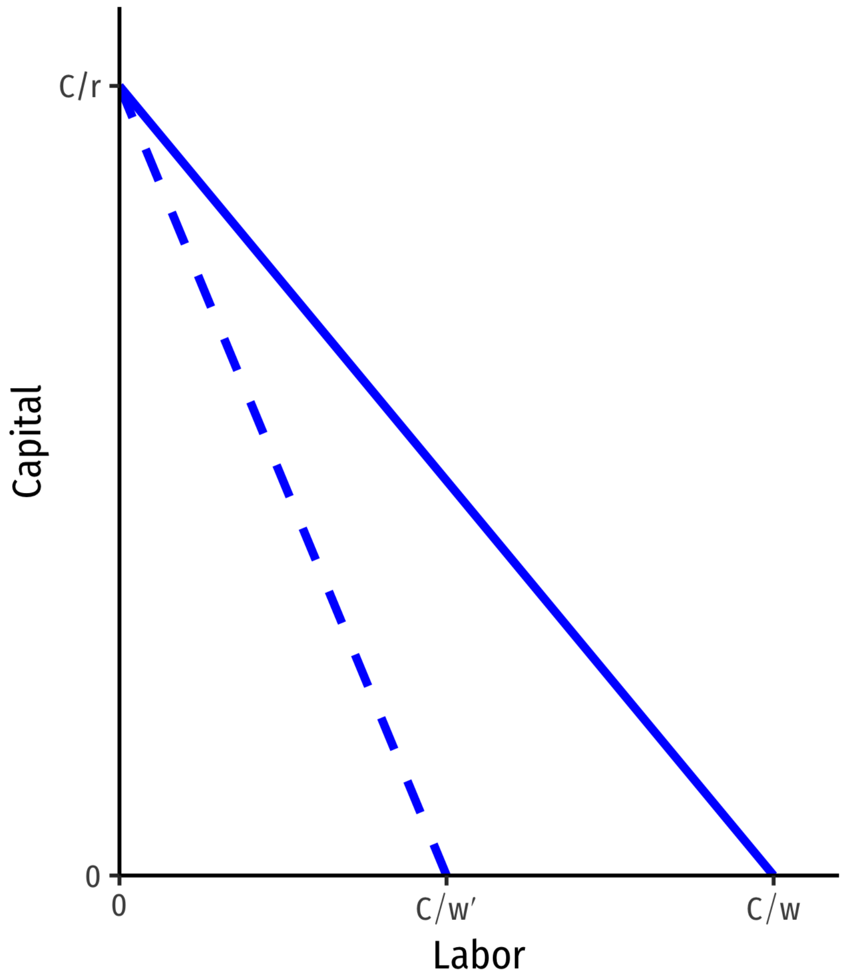


Changes in Relative Factor Prices I

- Changes in relative factor prices: *rotate* the line

Example: An increase in the price of l

- Slope changes: $-\frac{w'}{r}$



Changes in Relative Factor Prices II

- Changes in **relative factor prices**: *rotate* the line

Example: An increase in the price of k

- Slope changes: $-\frac{w}{r'}$