

# 2.5 – Short Run Profit Maximization

ECON 306 • Microeconomic Analysis • Spring 2023

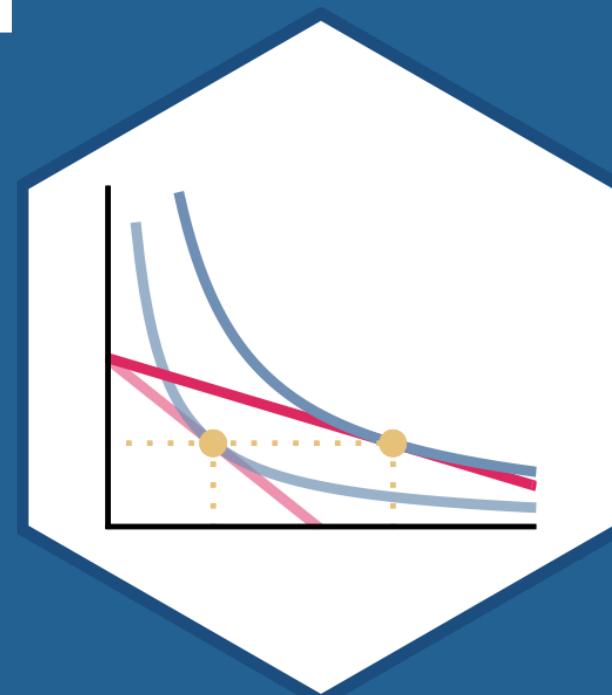
Ryan Safner

Associate Professor of Economics

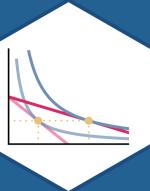
 [safner@hood.edu](mailto:safner@hood.edu)

 [ryansafner/microS23](https://github.com/ryansafner/microS23)

 [microS23.classes.ryansafner.com](https://microS23.classes.ryansafner.com)



# Outline



Revenues

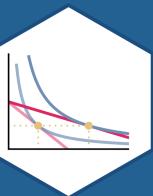
Profits

Comparative Statics

Calculating Profit

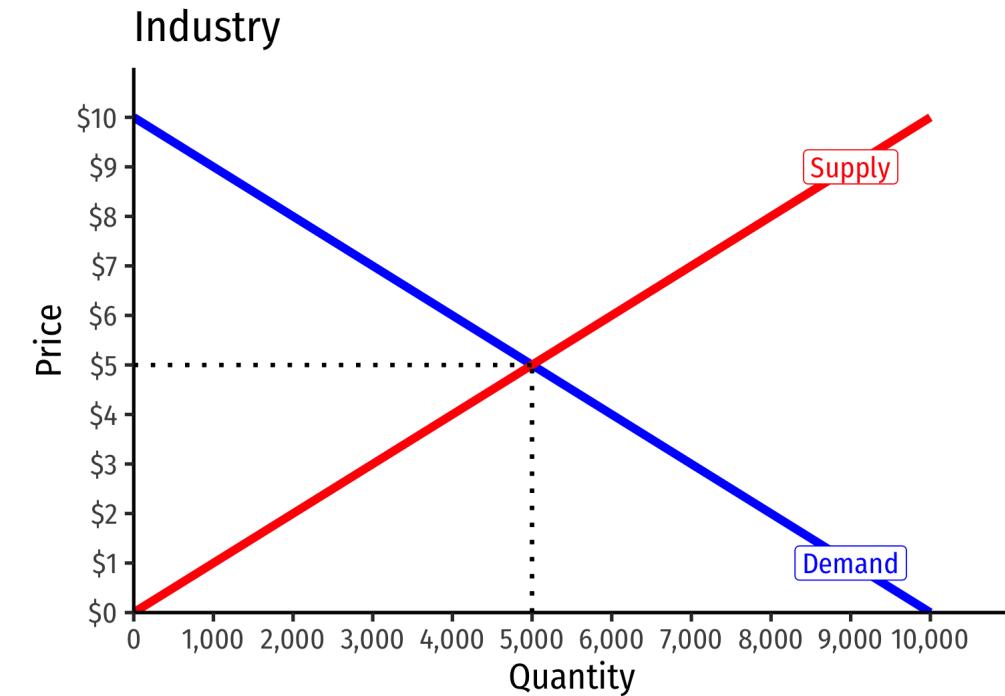
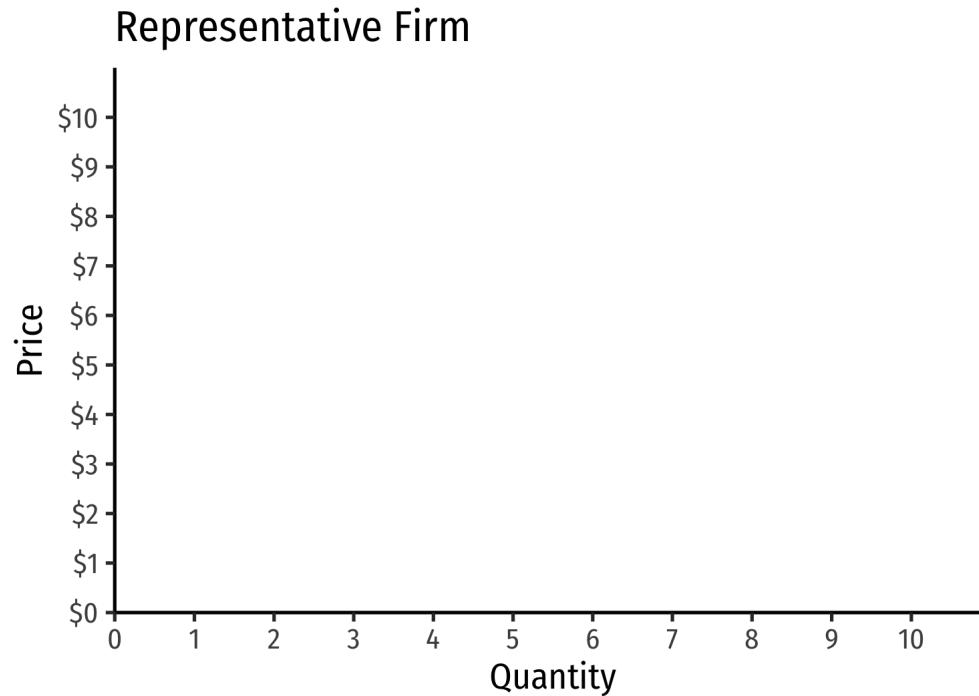
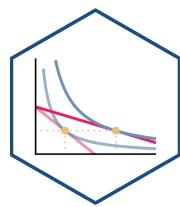
Short-Run Shut-Down Decisions

The Firm's Short-Run Supply Decision

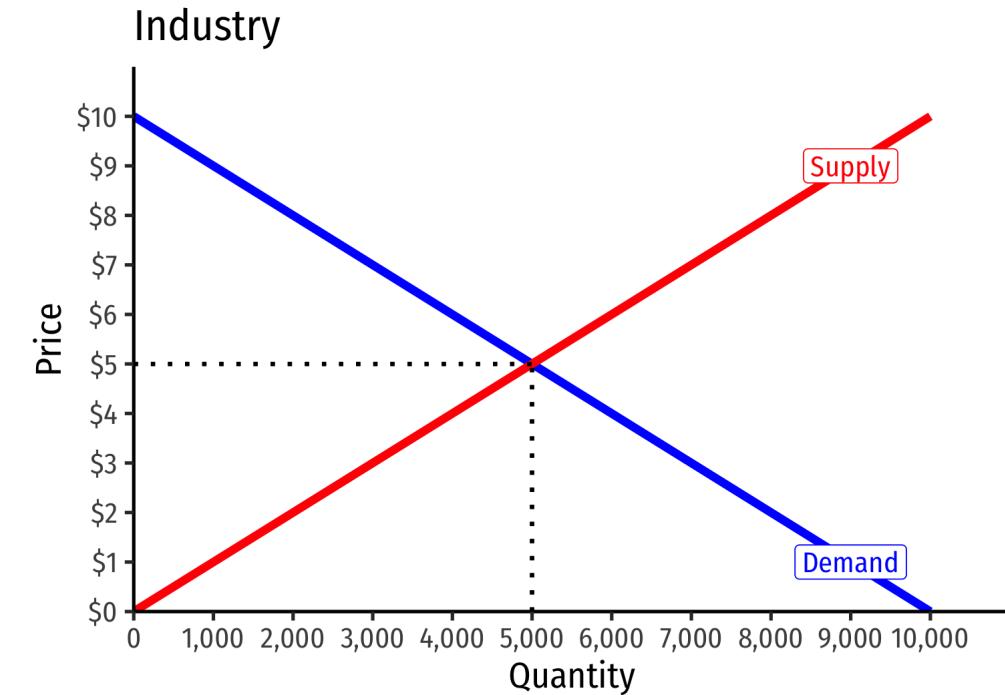
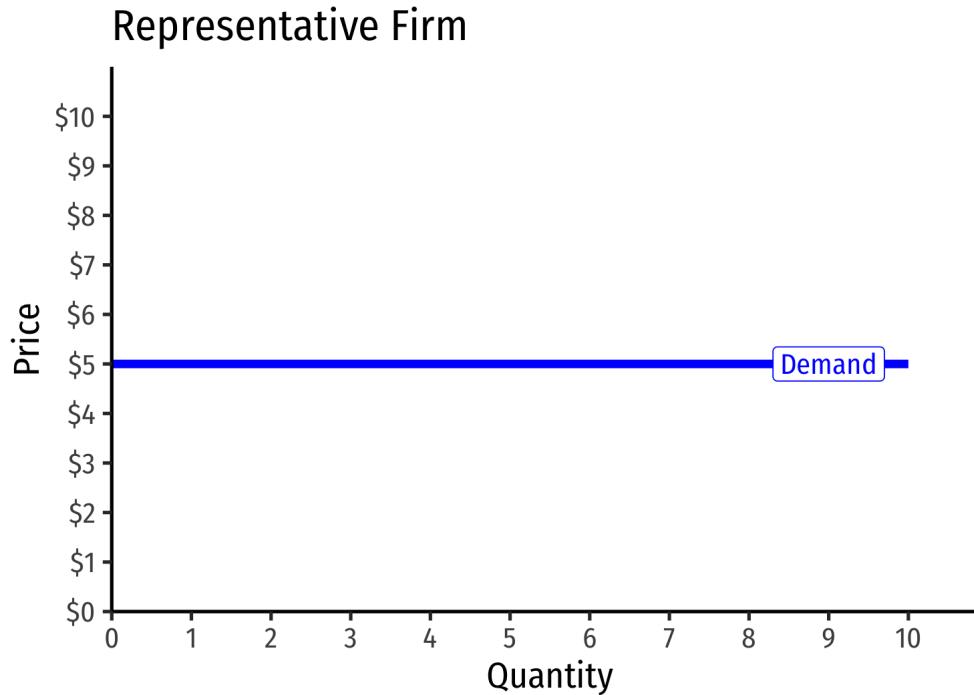
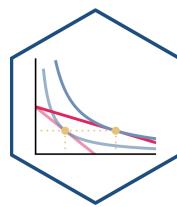


# Revenues

# Revenues for Firms in *Competitive* Industries I

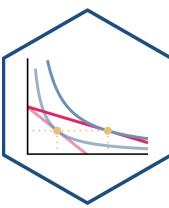


# Revenues for Firms in *Competitive* Industries I

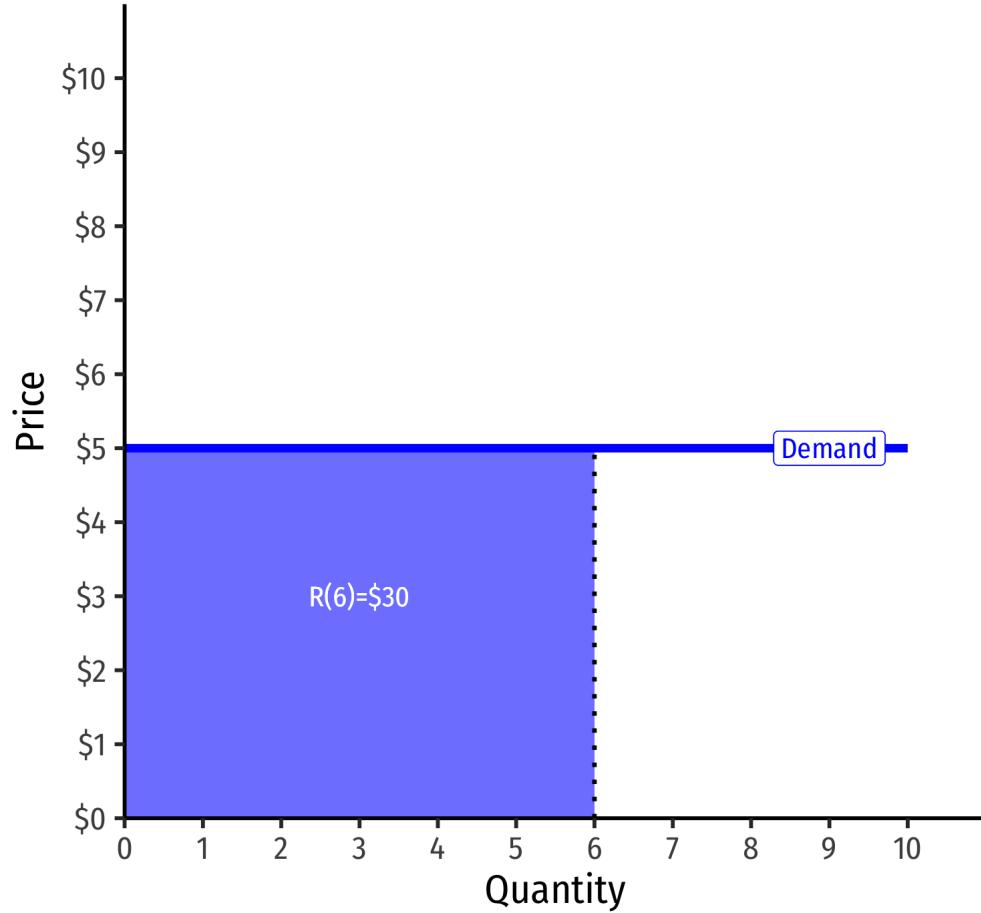


- Demand for a firm's product is **perfectly elastic** at the market price
- Where did the **supply curve** come from? You'll know today

# Revenues for Firms in *Competitive* Industries II

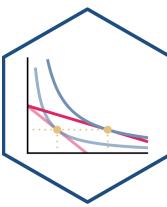


Representative Firm



- **Total Revenue**  $R(q) = pq$

# Average and Marginal Revenues



- **Average Revenue:** revenue per unit of output

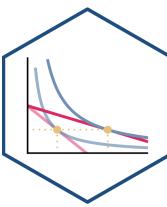
$$AR(q) = \frac{R}{q}$$

- $AR(q)$  is **by definition** equal to the price! (Why?)
- **Marginal Revenue:** change in revenues for each additional unit of output sold:

$$MR(q) = \frac{\Delta R(q)}{\Delta q}$$

- Calculus: first derivative of the revenues function
- **For a competitive firm (only),  $MR(q) = p$ , i.e. the price!**

# Average and Marginal Revenues: Example



**Example:** A firm sells bushels of wheat in a very competitive market. The current market price is \$10/bushel.

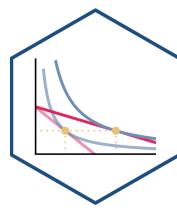
For the 1<sup>st</sup> bushel sold:

- What is the total revenue?
- What is the average revenue?

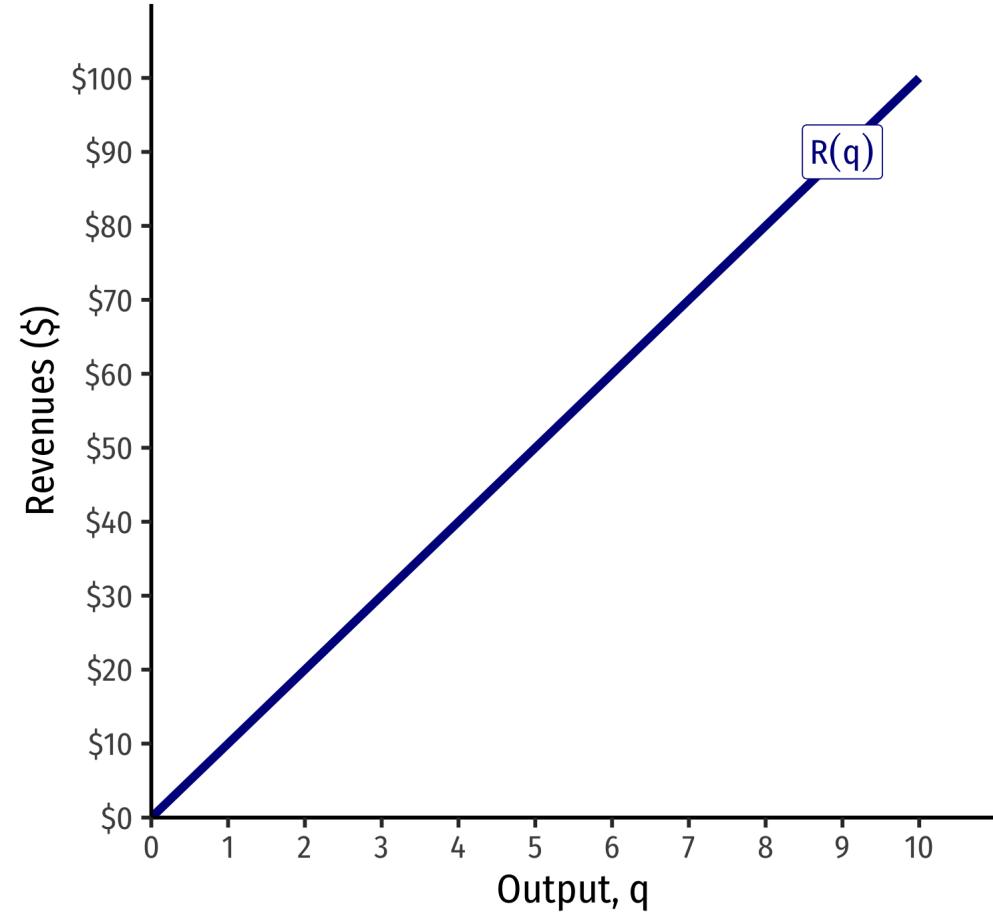
For the 2<sup>nd</sup> bushel sold:

- What is the total revenue?
- What is the average revenue?
- What is the marginal revenue?

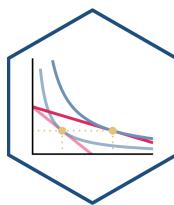
# Total Revenue, Example: Visualized



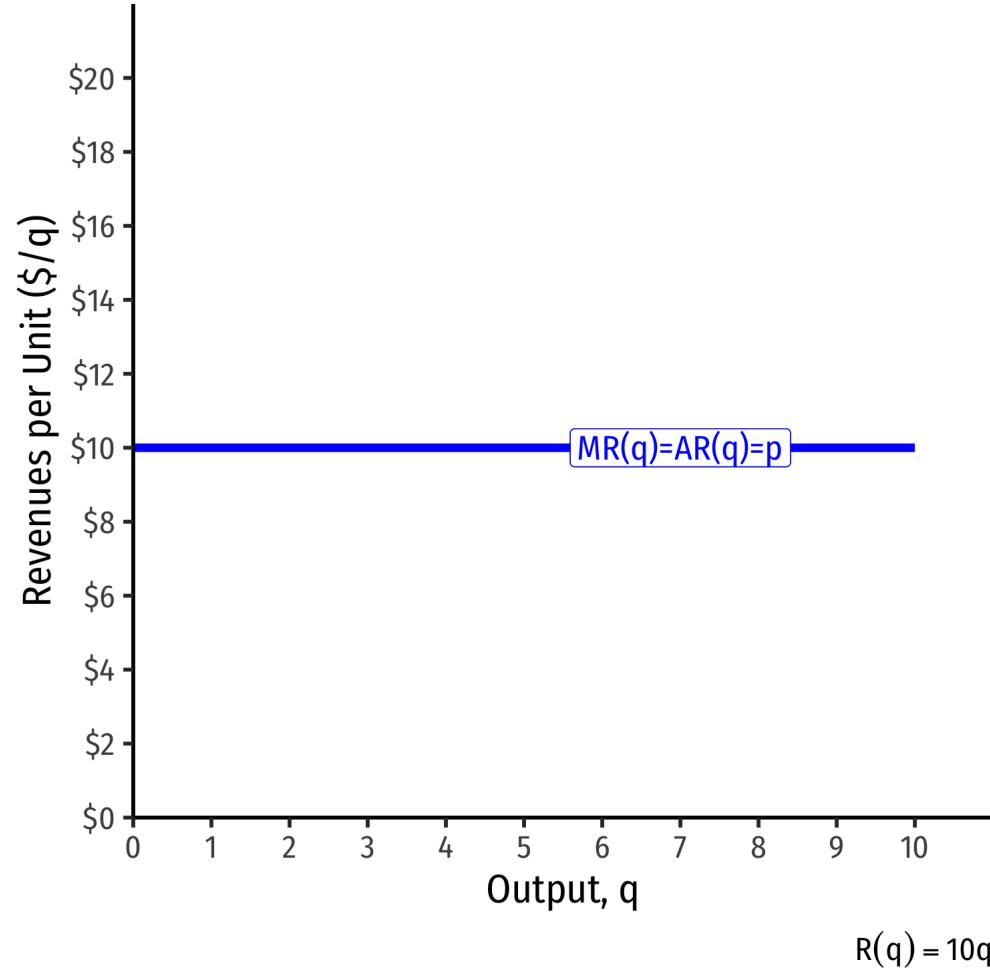
$q$	$R(q)$
0	0
1	10
2	20
3	30
4	40
5	50
6	60
7	70
8	80
9	90
10	100

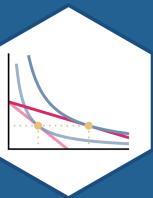


# Average and Marginal Revenue, Example: Visualized



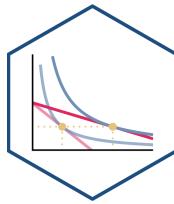
$q$	$R(q)$	$AR(q)$	$MR(q)$
0	0	—	—
1	10	10	10
2	20	10	10
3	30	10	10
4	40	10	10
5	50	10	10
6	60	10	10
7	70	10	10
8	80	10	10
9	90	10	10
10	100	10	10





# Profits

# Recall: The Firm's Two Problems



1<sup>st</sup> Stage: firm's profit maximization problem:

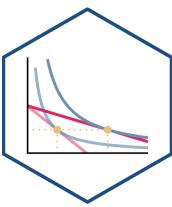
1. Choose: < output >
2. In order to maximize: < profits >

2<sup>nd</sup> Stage: firm's cost minimization problem:

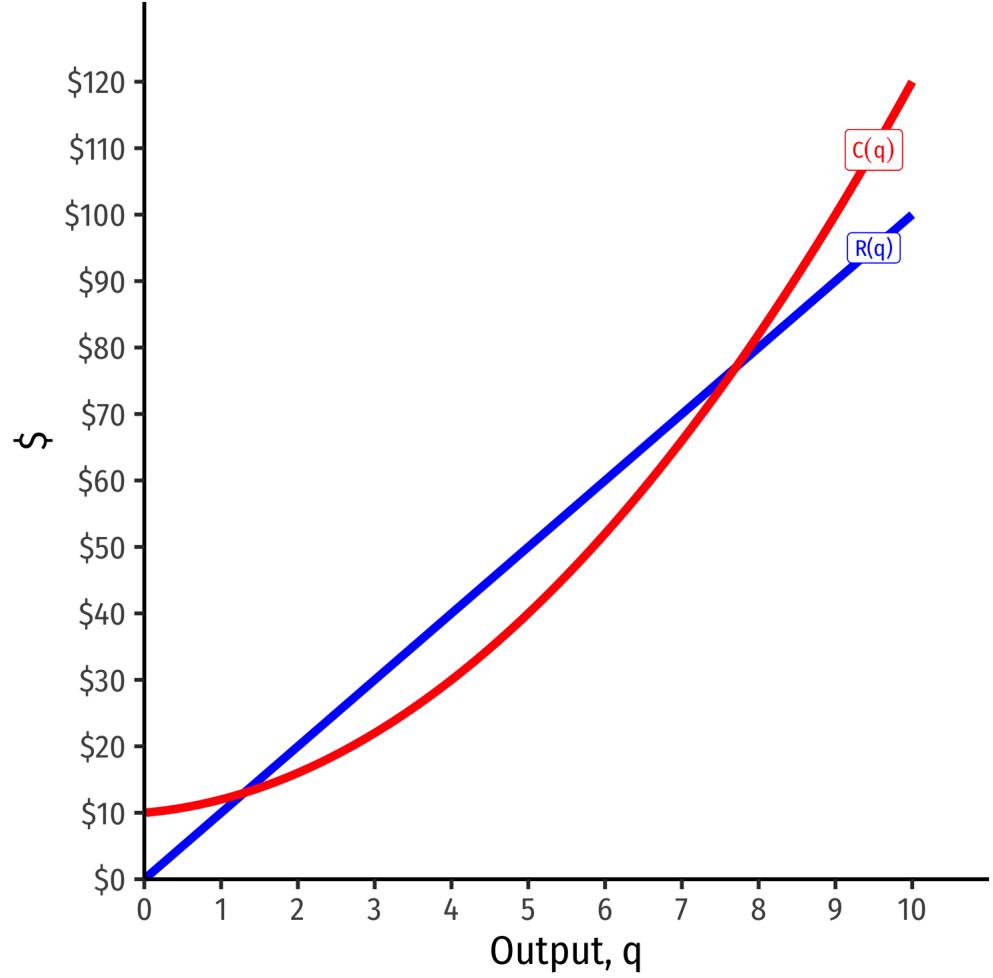
1. Choose: < inputs >
2. In order to minimize: < cost >
3. Subject to: < producing the optimal output >
  - Minimizing costs  $\iff$  maximizing profits



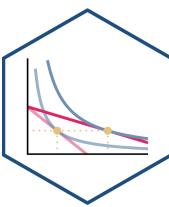
# Visualizing Total Profit As $R(q) - C(q)$



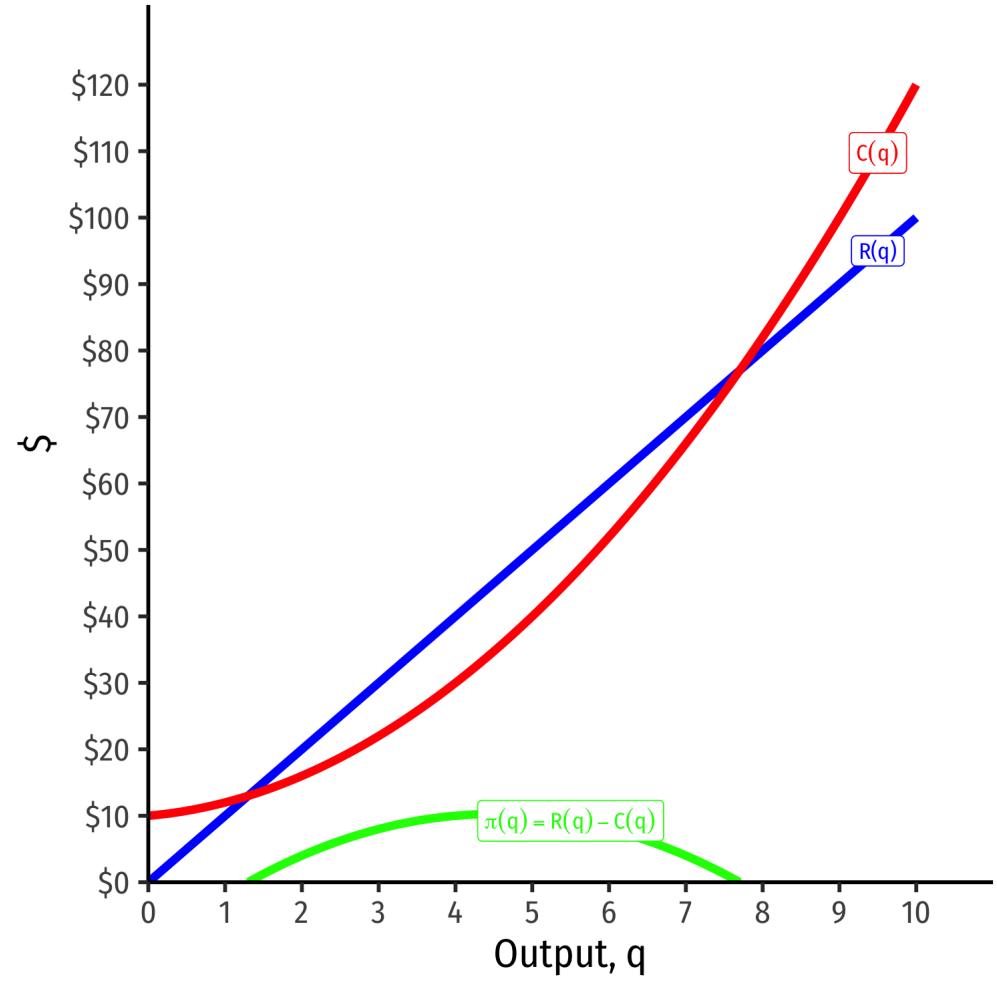
- $\pi(q) = R(q) - C(q)$



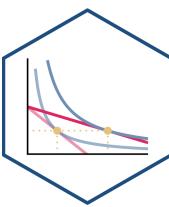
# Visualizing Total Profit As $R(q) - C(q)$



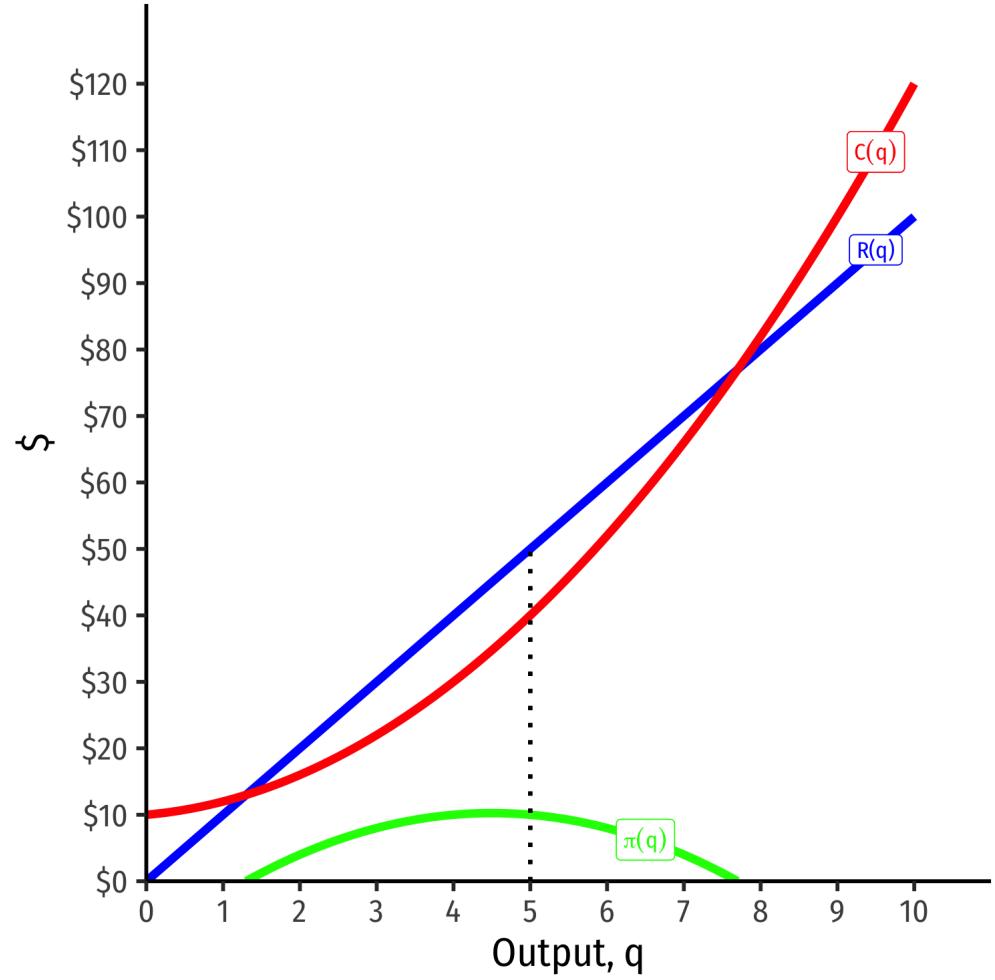
- $\pi(q) = R(q) - C(q)$



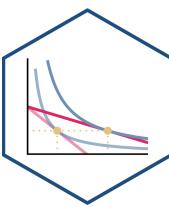
# Visualizing Total Profit As $R(q) - C(q)$



- $\pi(q) = R(q) - C(q)$
- Graph: find  $q^*$  to max  $\pi \implies q^*$  where  
max distance between  $R(q)$  and  $C(q)$

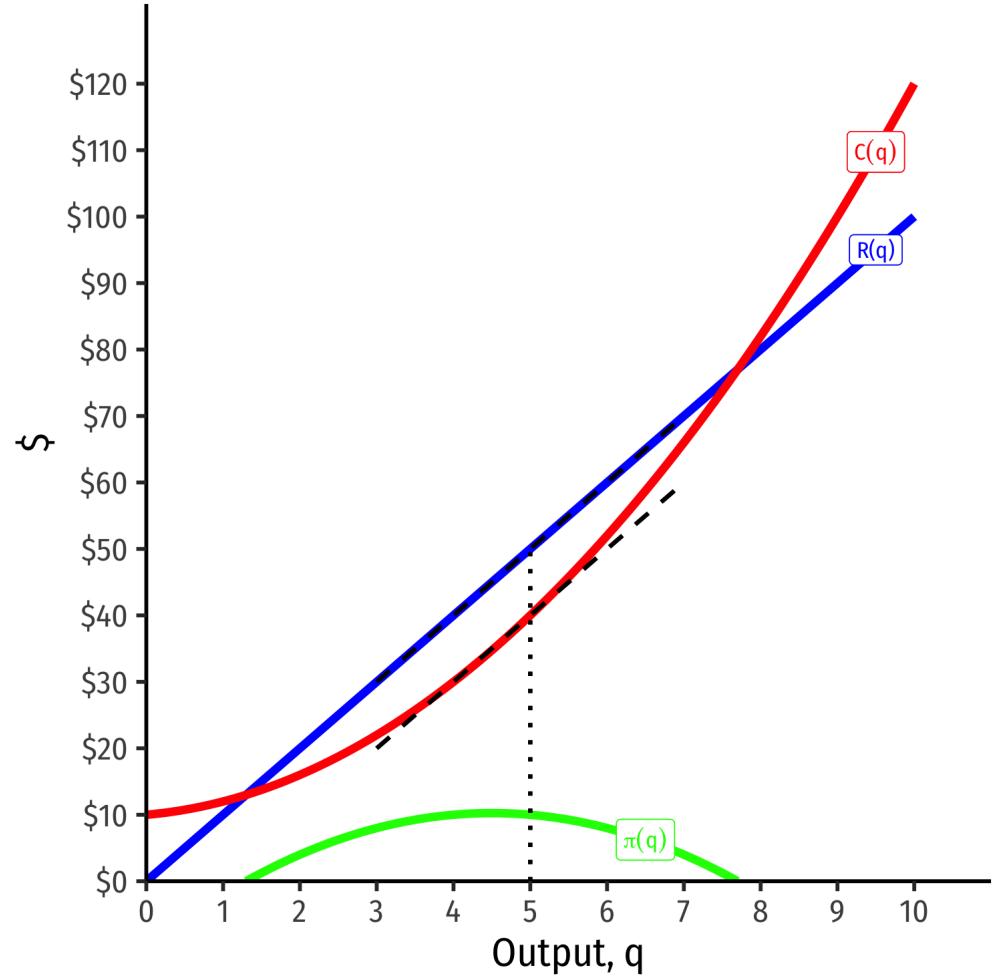


# Visualizing Total Profit As $R(q) - C(q)$

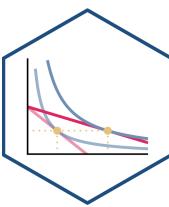


- $\pi(q) = R(q) - C(q)$
- Graph: find  $q^*$  to max  $\pi \implies q^*$  where max distance between  $R(q)$  and  $C(q)$
- Slopes must be equal:

$$MR(q) = MC(q)$$



# Visualizing Total Profit As $R(q) - C(q)$

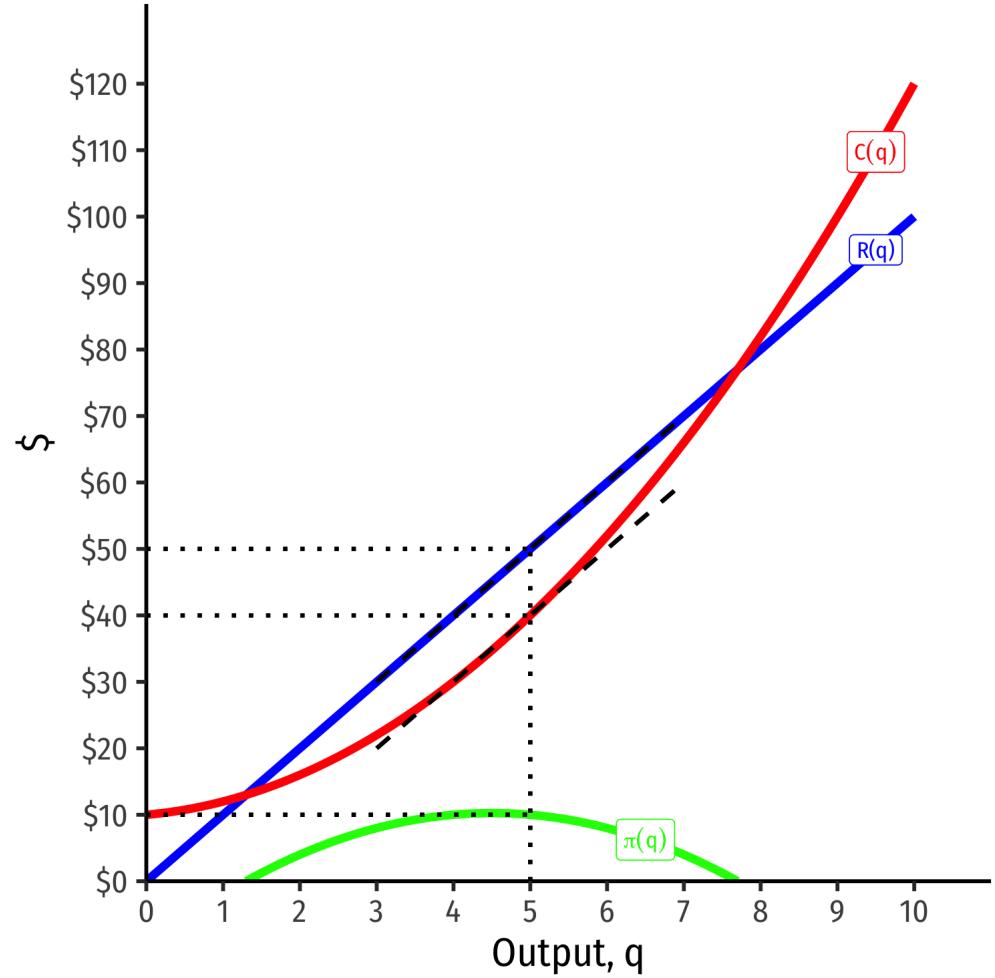


- $\pi(q) = R(q) - C(q)$
- Graph: find  $q^*$  to max  $\pi \implies q^*$  where max distance between  $R(q)$  and  $C(q)$

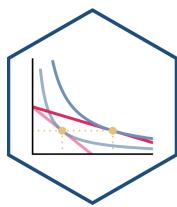
- Slopes must be equal:

$$MR(q) = MC(q)$$

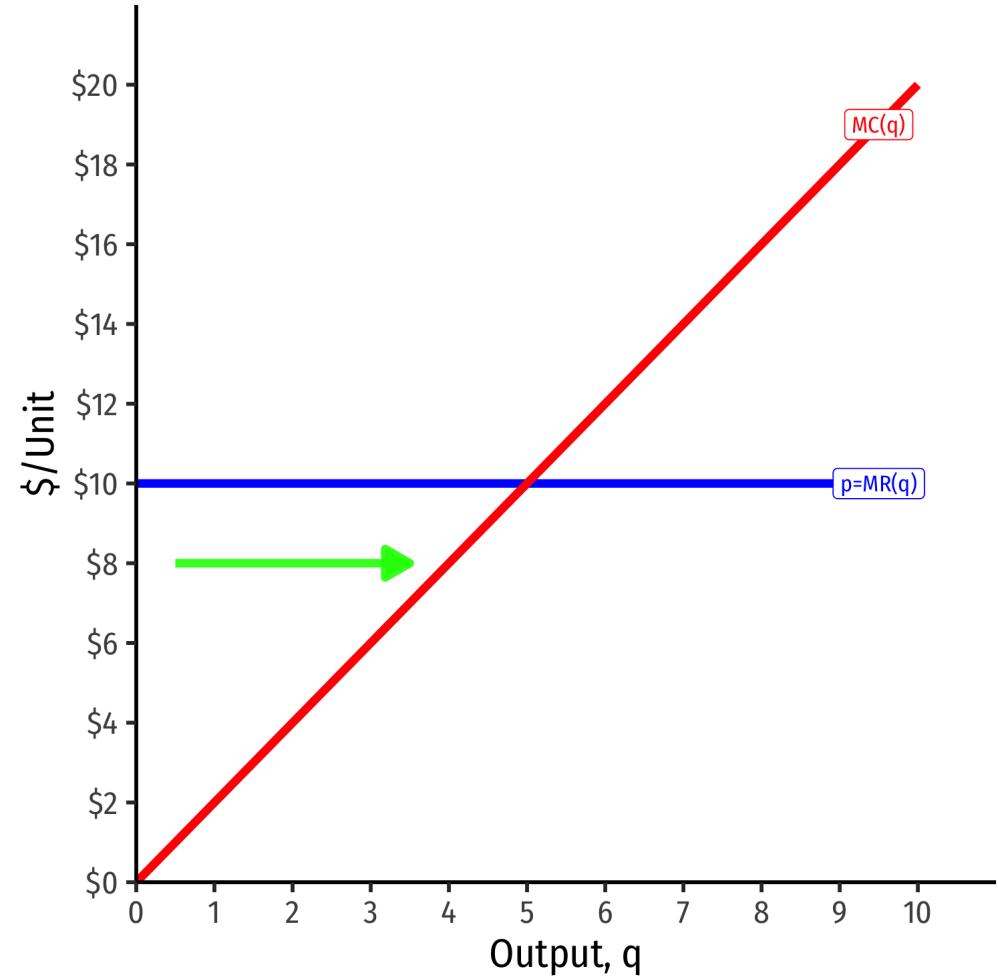
- At  $q^* = 5$ :
  - $R(q) = 50$
  - $C(q) = 40$
  - $\pi(q) = 10$



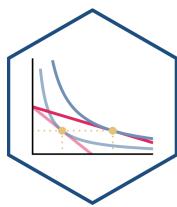
# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



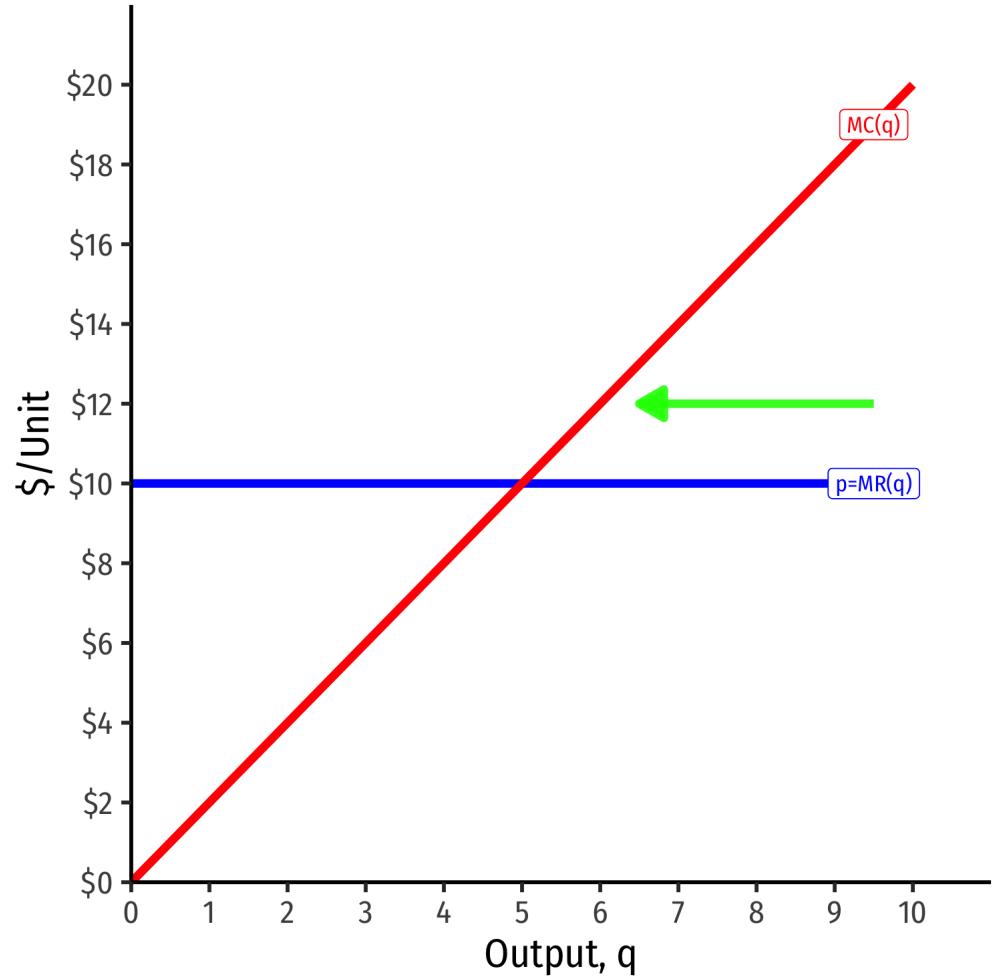
- At low output  $q < q^*$ , can increase  $\pi$  by producing *more*:  $MR(q) > MC(q)$



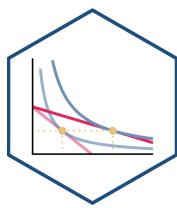
# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



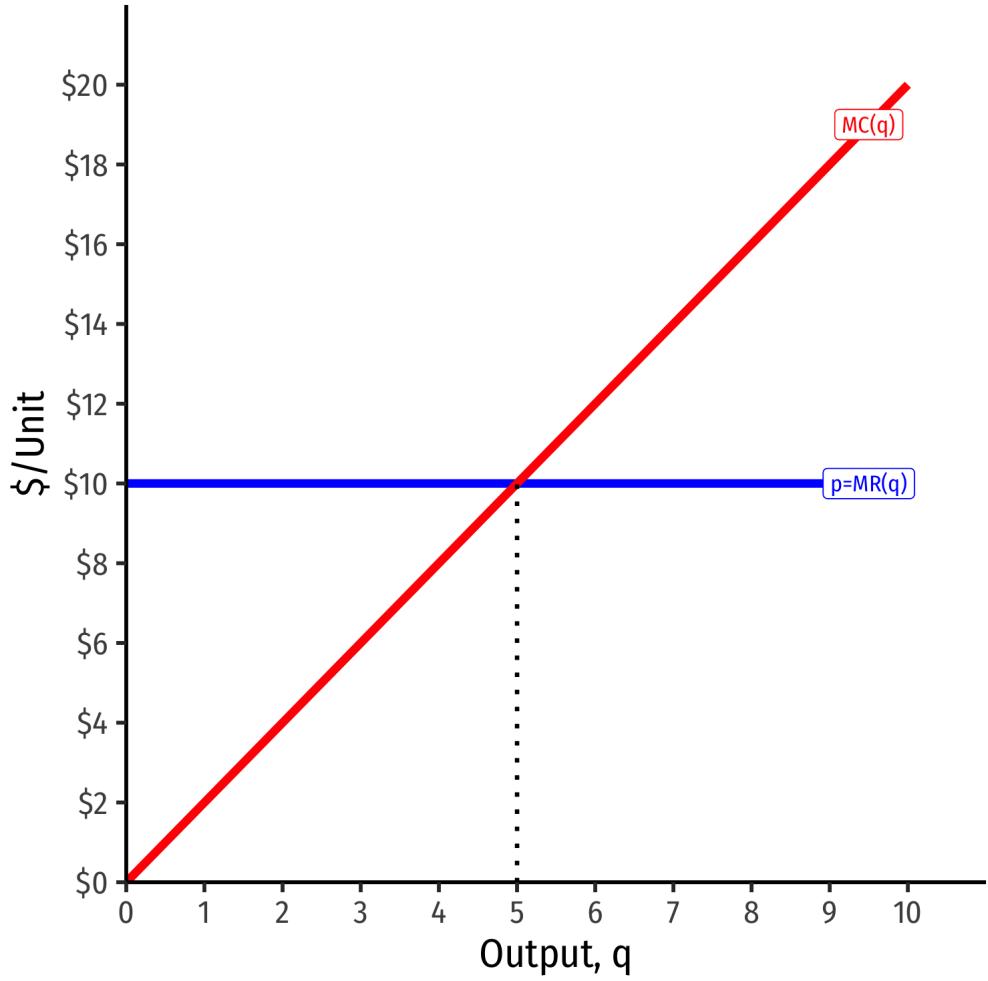
- At high output  $q > q^*$ , can increase  $\pi$  by producing less:  $MR(q) < MC(q)$

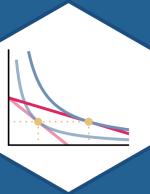


# Visualizing Profit Per Unit As $MR(q)$ and $MC(q)$



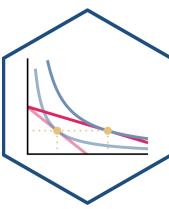
- $\pi$  is *maximized* where  
 $MR(q) = MC(q)$



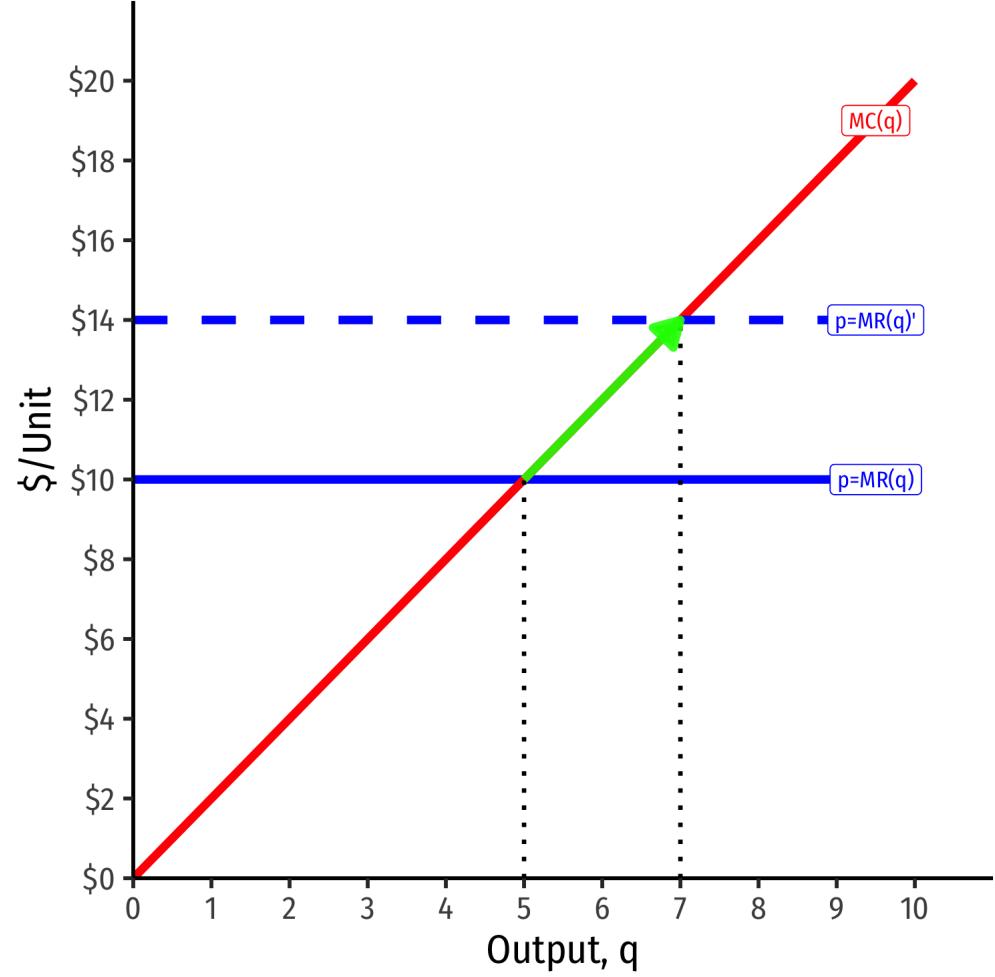


# Comparative Statics

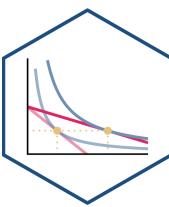
# If Market Price Changes I



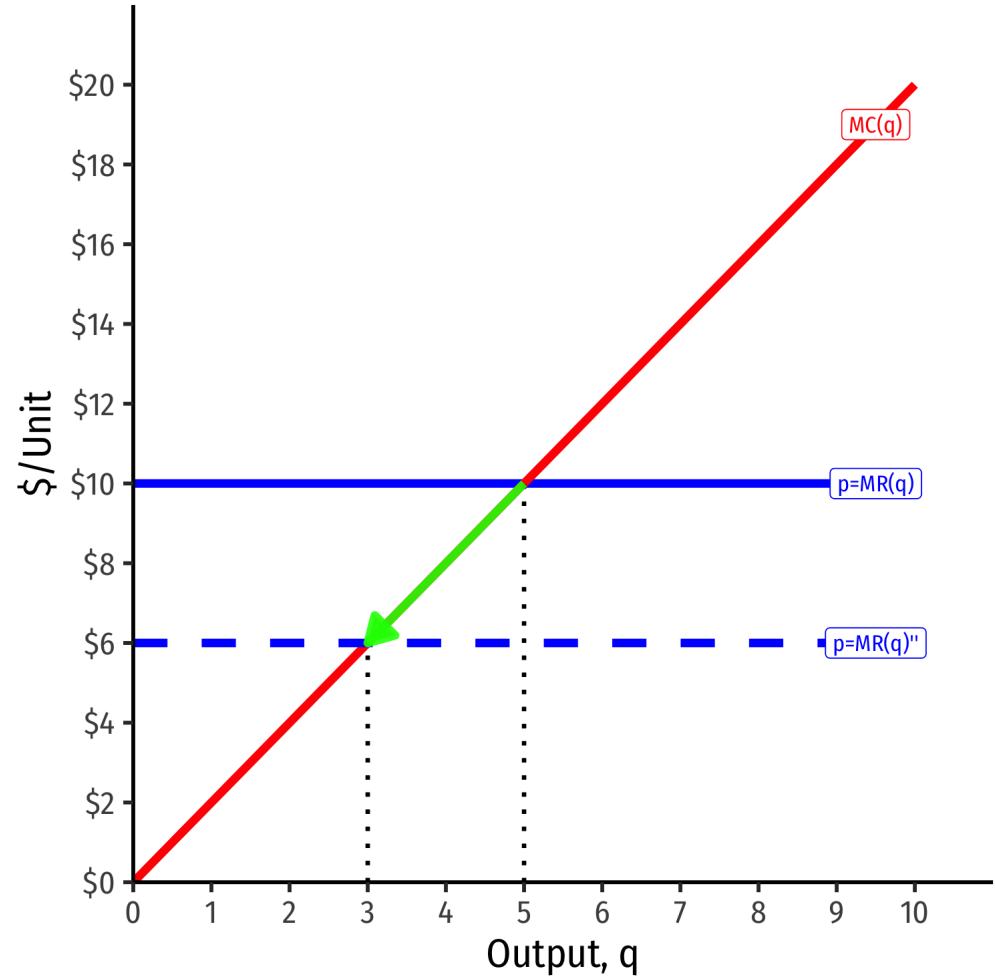
- Suppose the market price **increases**
- Firm (always setting  $MR=MC$ ) will respond by **producing more**



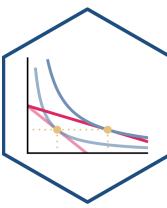
# If Market Price Changes II



- Suppose the market price **decreases**
- Firm (always setting  $MR=MC$ ) will respond by **producing less**



# The Firm's Supply Curve

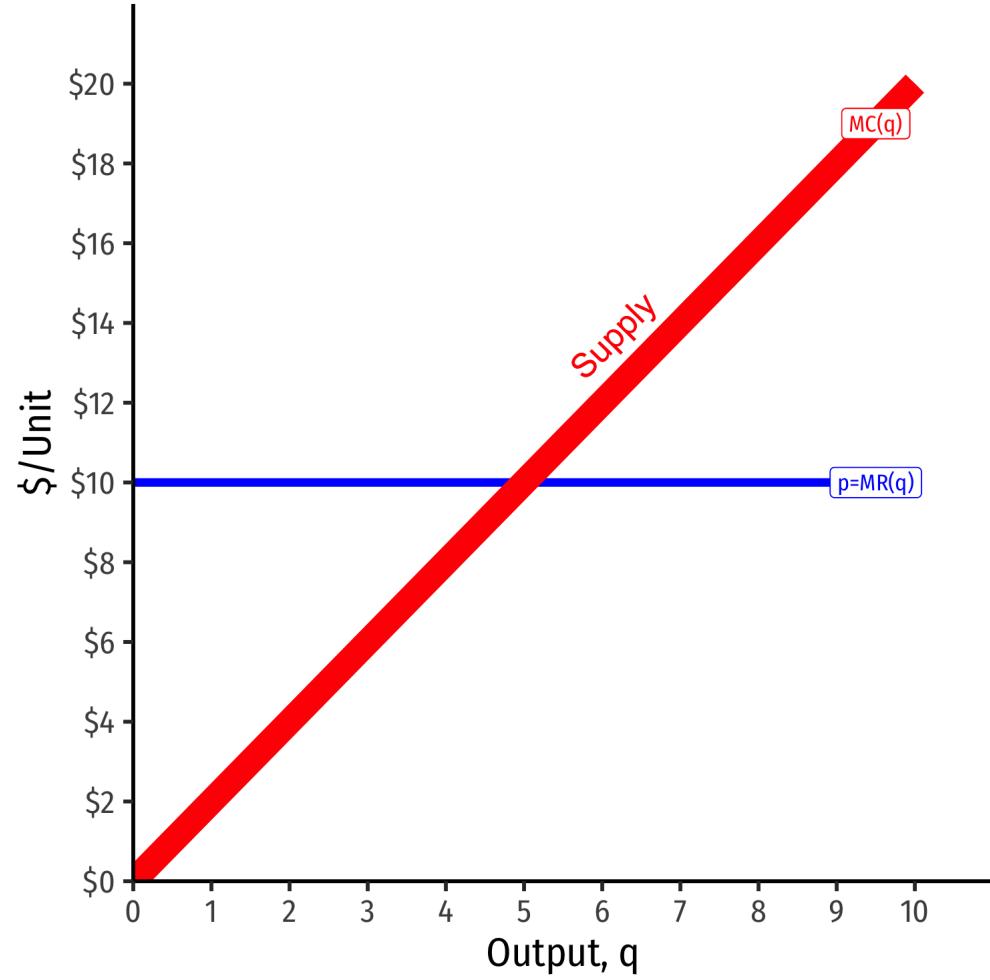


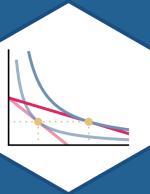
- The firm's marginal cost curve is its supply curve<sup>†</sup>

$$p = MC(q)$$

- How it will supply the optimal amount of output in response to the market price
- Firm always sets its price equal to its marginal cost

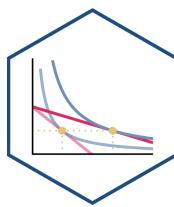
<sup>†</sup> Mostly...there is an important **exception** we will see shortly!





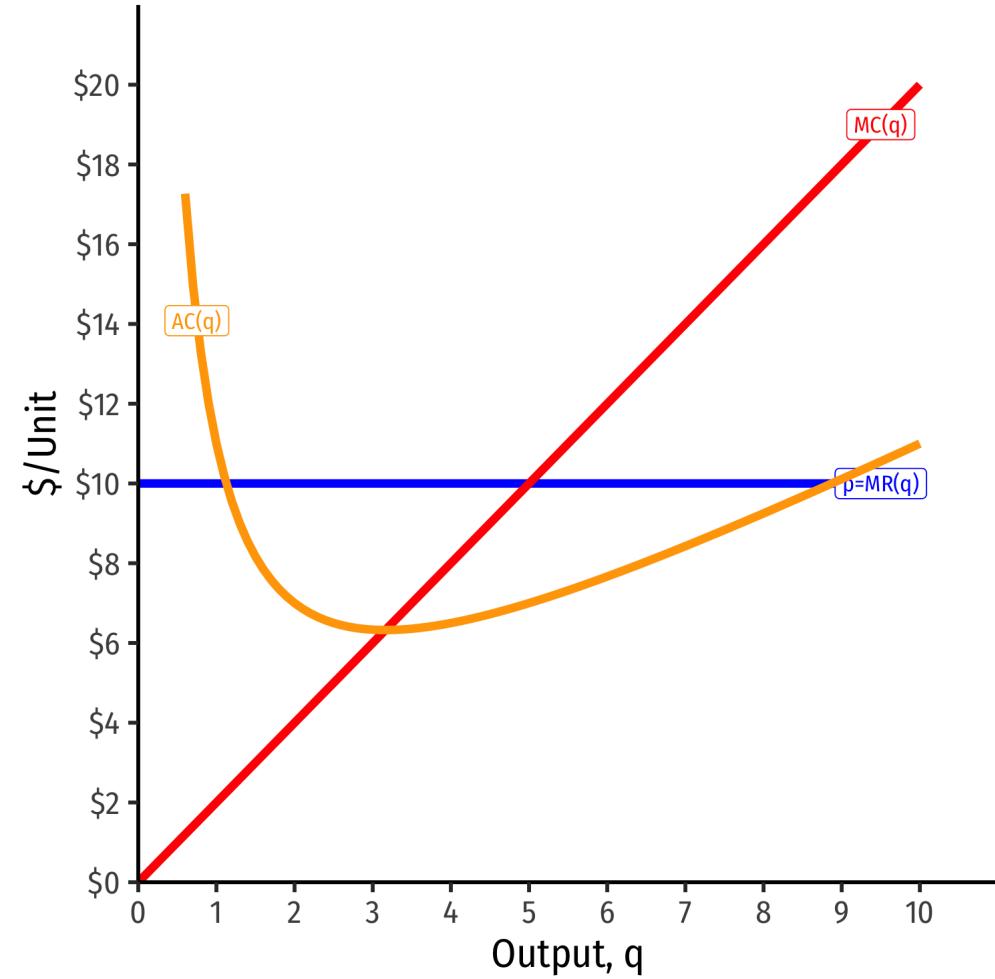
# Calculating Profit

# Calculating (Average) Profit as AR(q)-AC(q)

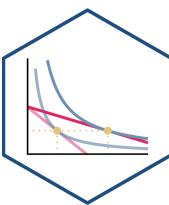


- Profit is

$$\pi(q) = R(q) - C(q)$$



# Calculating (Average) Profit as AR(q)-AC(q)

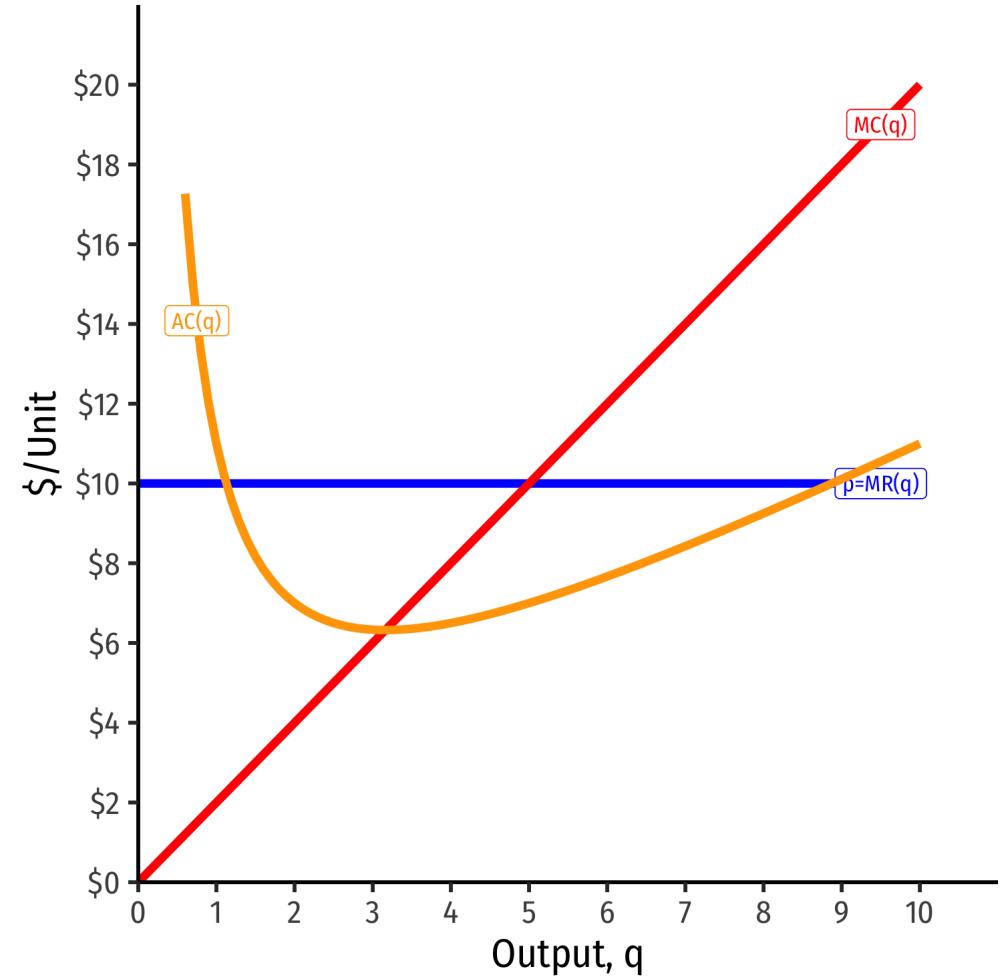


- Profit is

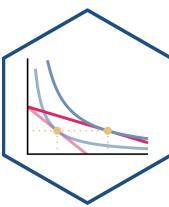
$$\pi(q) = R(q) - C(q)$$

- Profit per unit can be calculated as:

$$\begin{aligned}\frac{\pi(q)}{q} &= AR(q) - AC(q) \\ &= p - AC(q)\end{aligned}$$



# Calculating (Average) Profit as AR(q)-AC(q)



- Profit is

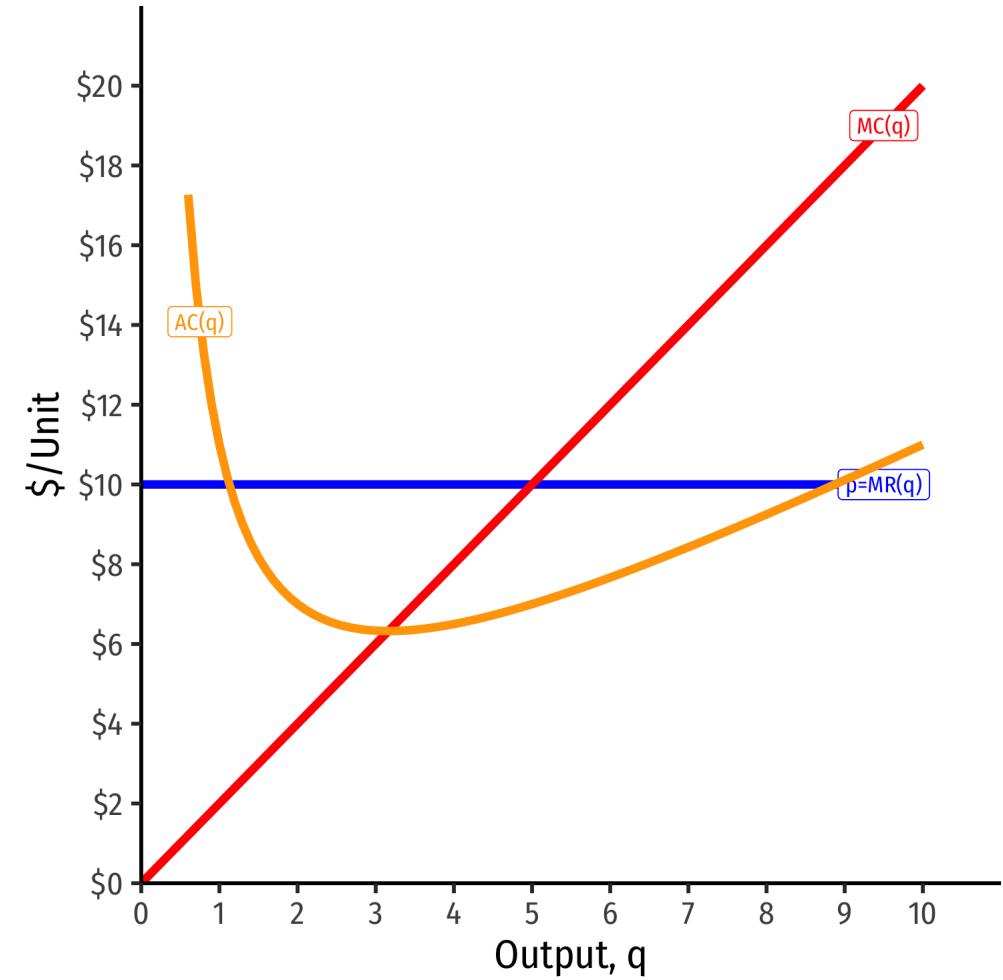
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- Profit per unit can be calculated as:

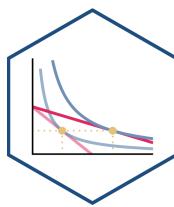
$$\begin{aligned}\frac{\pi(q)}{q} &= AR(q) - AC(q) \\ &= p - AC(q)\end{aligned}$$

- Multiply by  $q$  to get total profit:

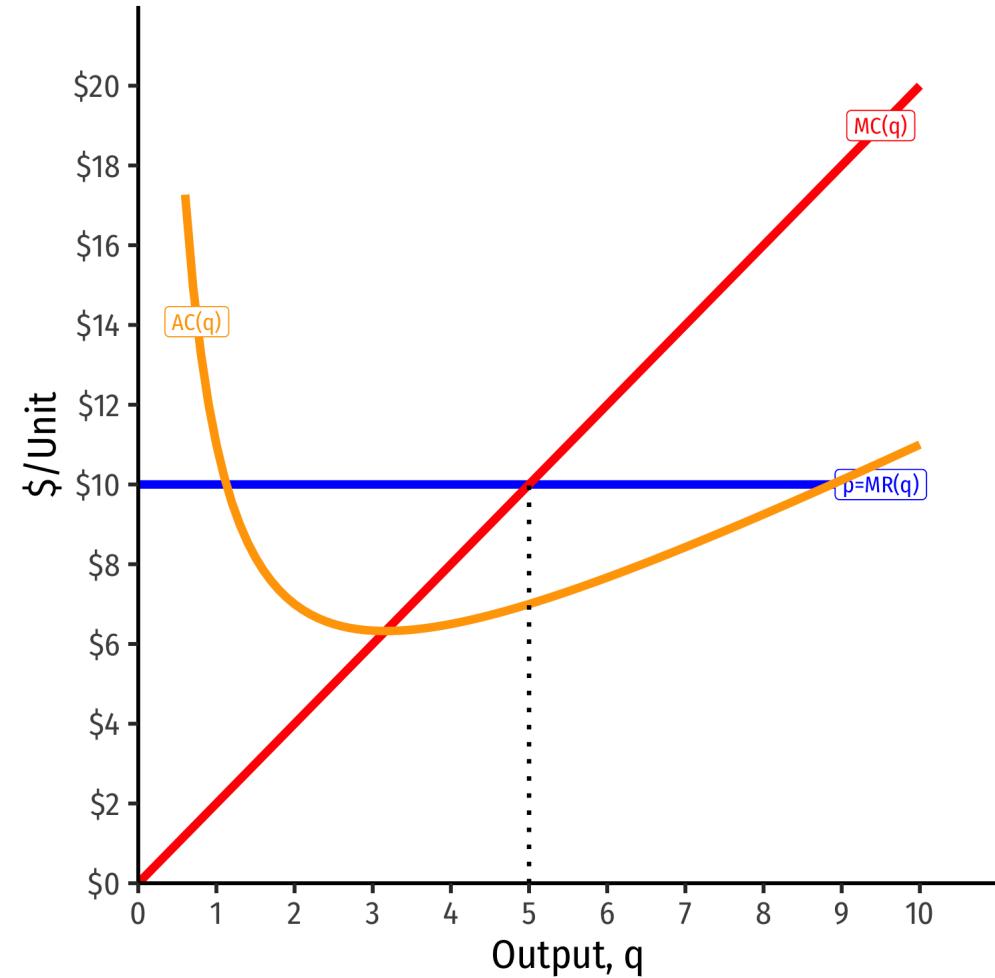
$$\pi(q) = q [p - AC(q)]$$



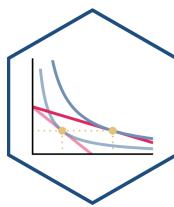
# Calculating (Average) Profit as AR(q)-AC(q)



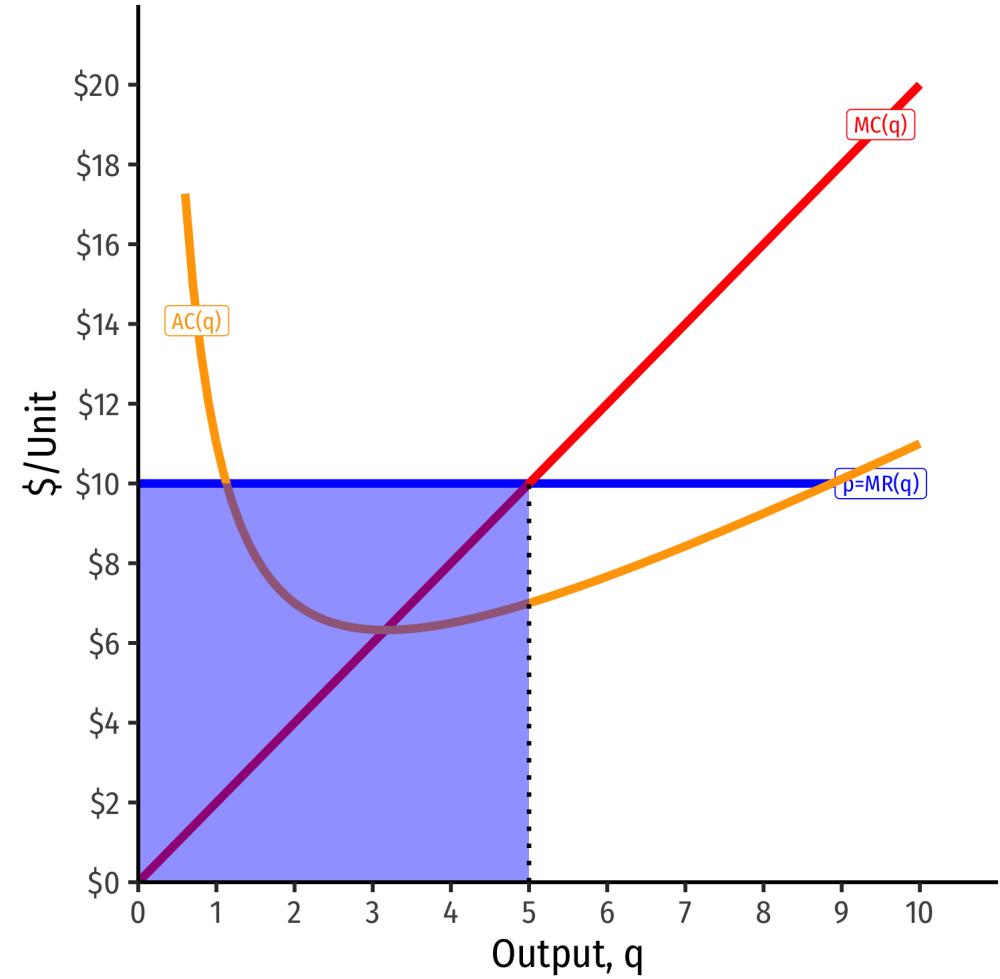
- At market price of  $p^* = \$10$
- At  $q^* = 5$  (per unit):
- At  $q^* = 5$  (totals):



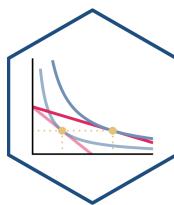
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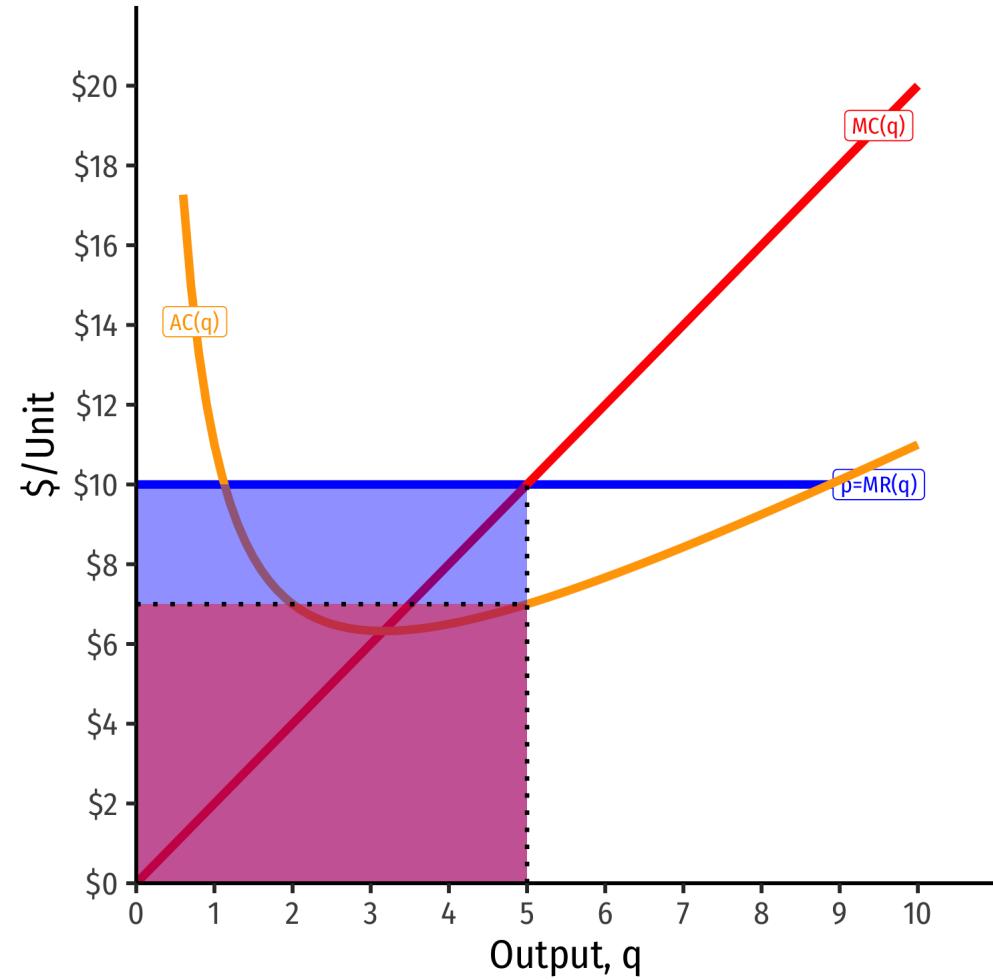
- At market price of  $p^* = \$10$
- At  $q^* = 5$  (per unit):
  - $AR(5) = \$10/\text{unit}$
- At  $q^* = 5$  (totals):
  - $R(5) = \$50$



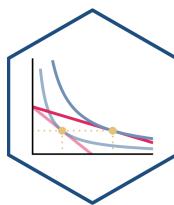
# Calculating (Average) Profit as AR(q)-AC(q)



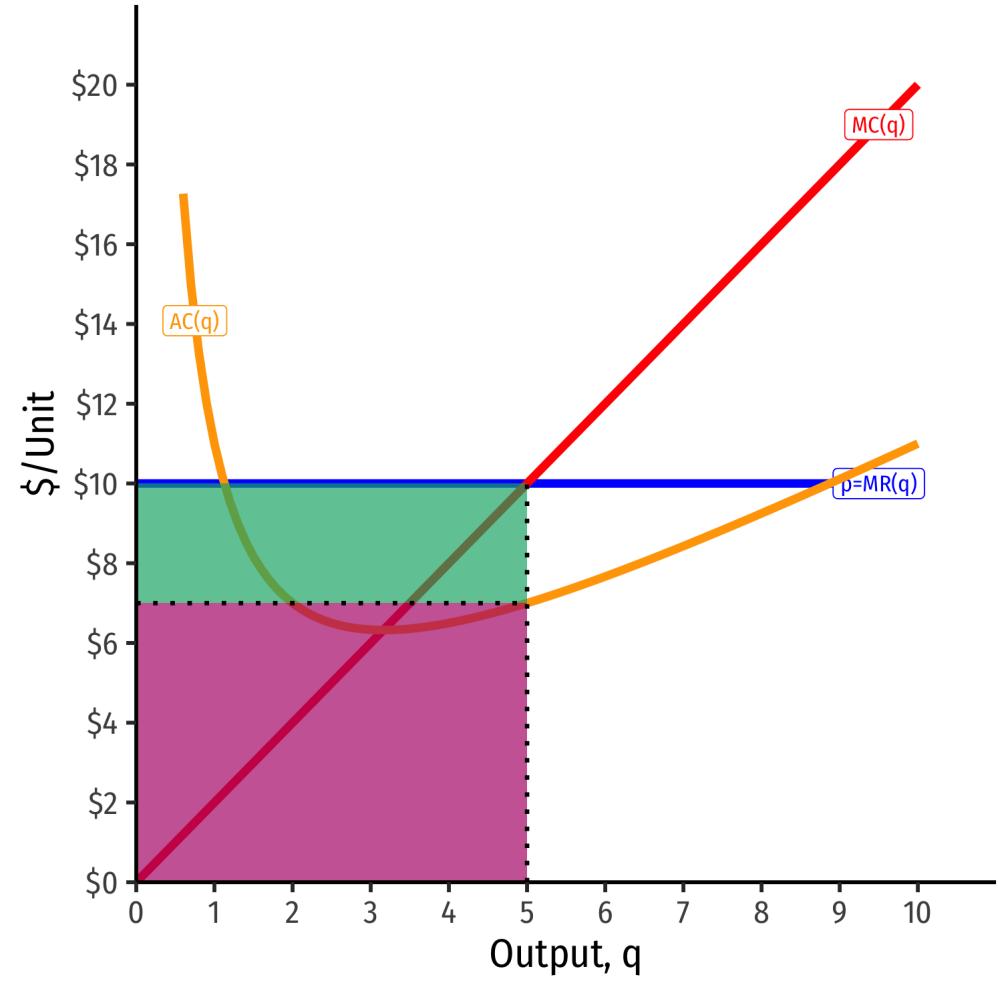
- At market price of  $p^* = \$10$
- At  $q^* = 5$  (per unit):
  - $AR(5) = \$10/\text{unit}$
  - $AC(5) = \$7/\text{unit}$
- At  $q^* = 5$  (totals):
  - $R(5) = \$50$
  - $C(5) = \$35$



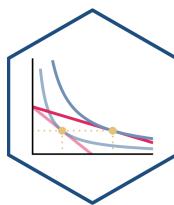
# Calculating (Average) Profit as AR(q)-AC(q)



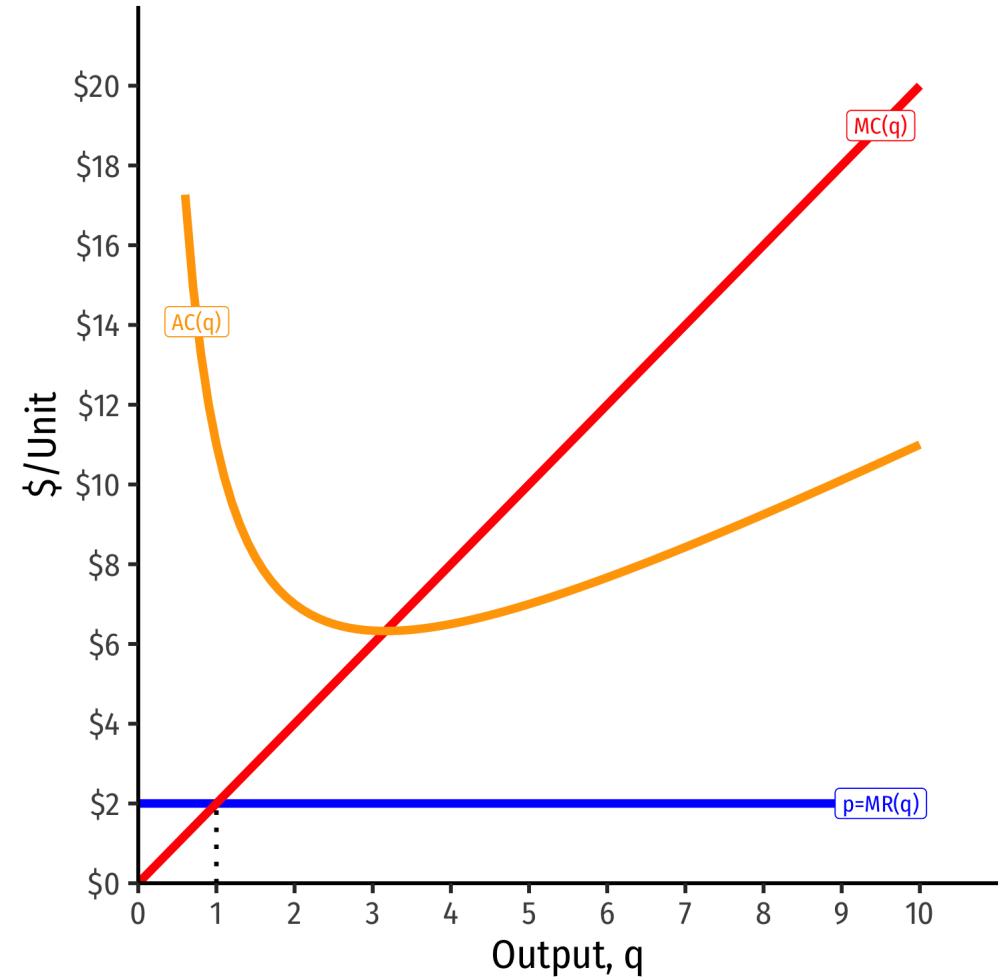
- At market price of  $p^* = \$10$
- At  $q^* = 5$  (per unit):
  - $AR(5) = \$10/\text{unit}$
  - $AC(5) = \$7/\text{unit}$
  - $A\pi(5) = \$3/\text{unit}$
- At  $q^* = 5$  (totals):
  - $R(5) = \$50$
  - $C(5) = \$35$
  - $\pi = \$15$



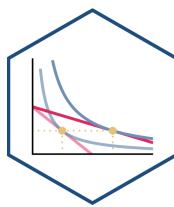
# Calculating (Average) Profit as AR(q)-AC(q)



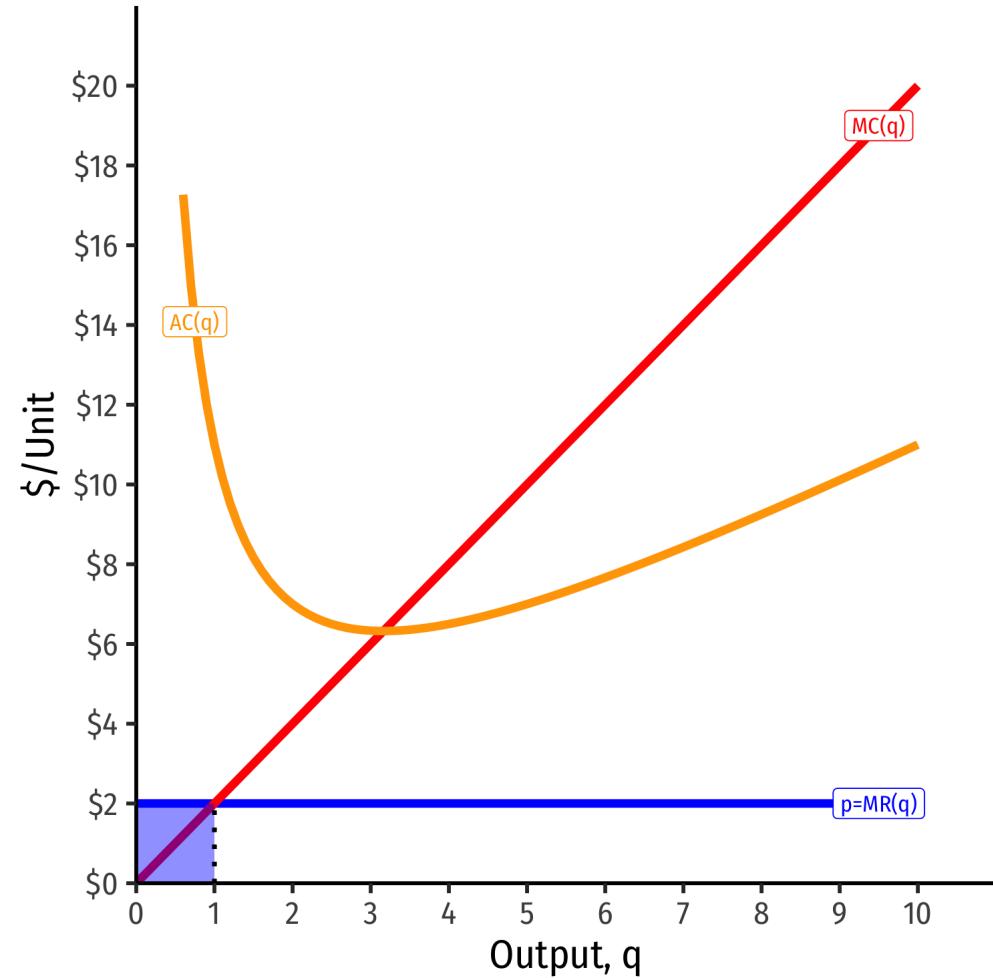
- At market price of  $p^* = \$2$
- At  $q^* = 1$  (per unit):
- At  $q^* = 1$  (totals):



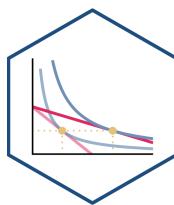
# Calculating (Average) Profit as AR(q)-AC(q)



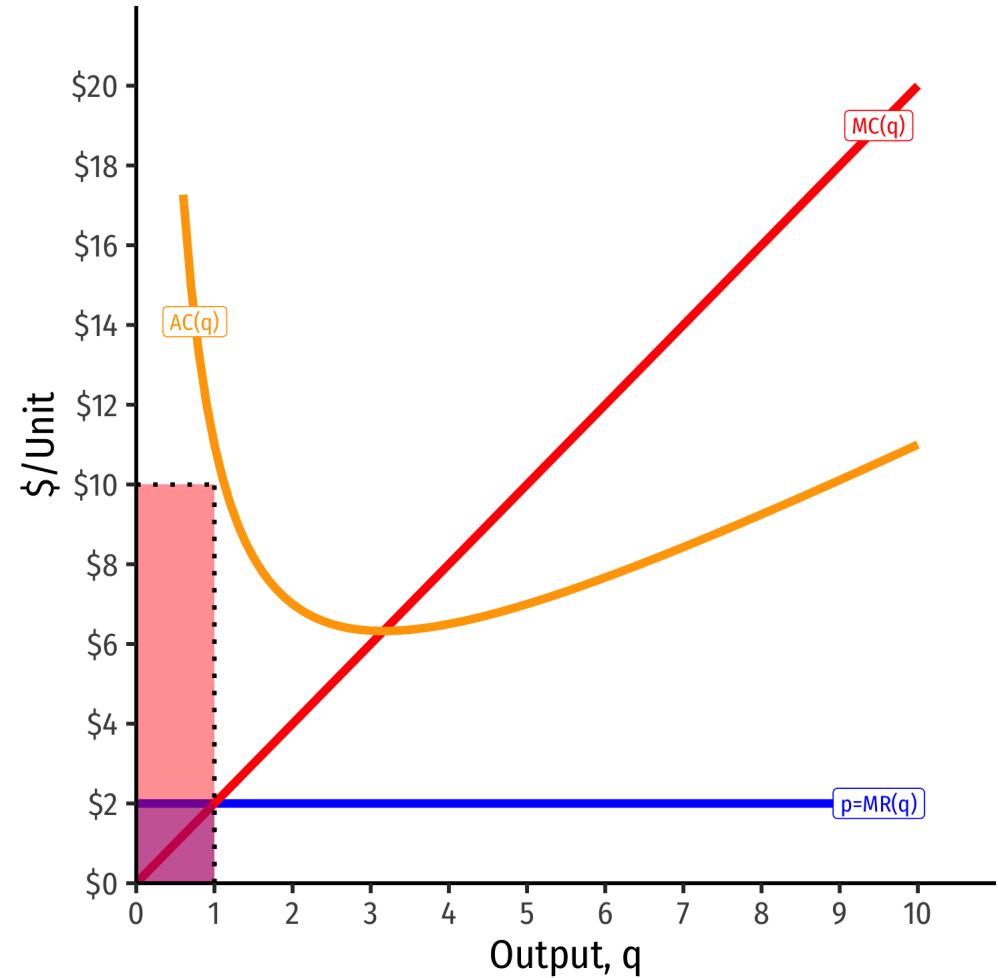
- At market price of  $p^* = \$2$
- At  $q^* = 1$  (per unit):
  - $AR(1) = \$2/\text{unit}$
- At  $q^* = 1$  (totals):
  - $R(1) = \$2$



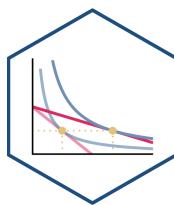
# Calculating (Average) Profit as AR(q)-AC(q)



- At market price of  $p^* = \$2$
- At  $q^* = 1$  (per unit):
  - $AR(1) = \$2/\text{unit}$
  - $AC(1) = \$10/\text{unit}$
- At  $q^* = 1$  (totals):
  - $R(1) = \$2$
  - $C(1) = \$10$



# Calculating (Average) Profit as AR(q)-AC(q)



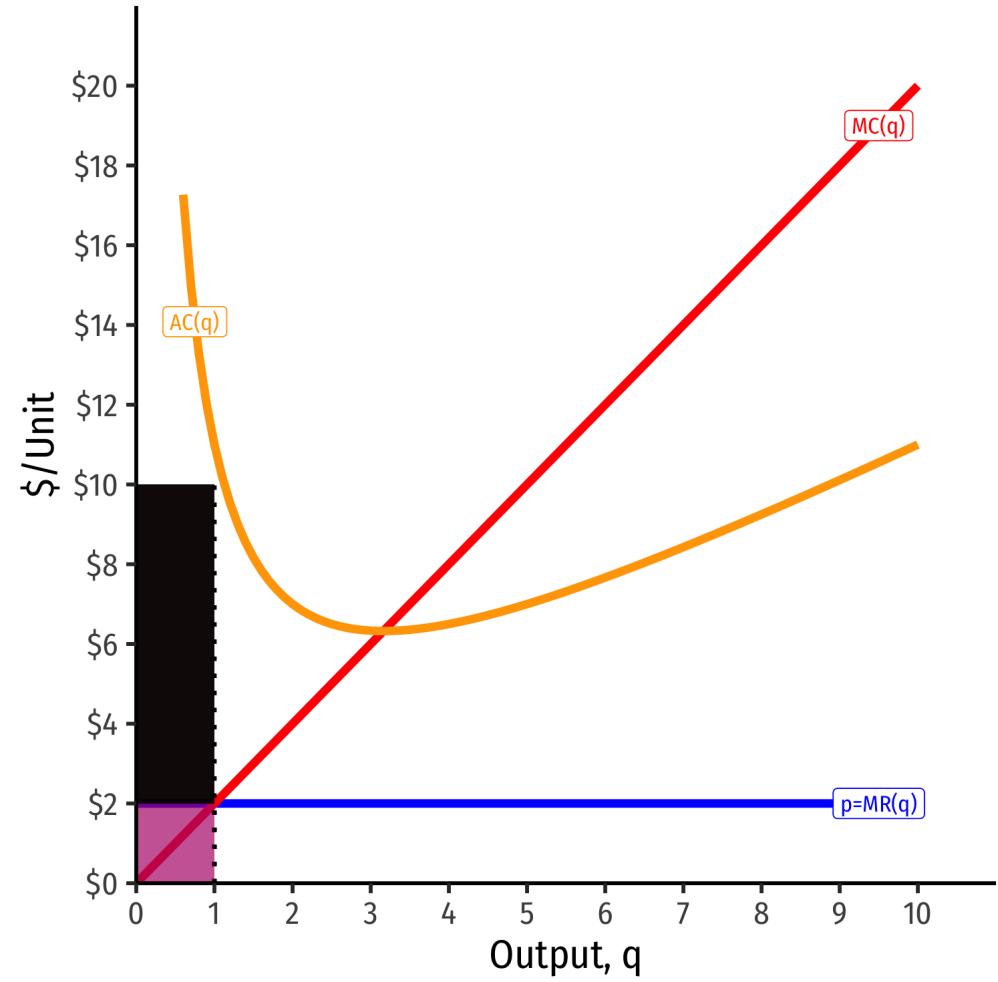
- At market price of  $p^* = \$2$

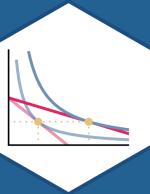
- At  $q^* = 1$  (per unit):

- $AR(1) = \$2/\text{unit}$
- $AC(1) = \$10/\text{unit}$
- $A\pi(1) = -\$8/\text{unit}$

- At  $q^* = 1$  (totals):

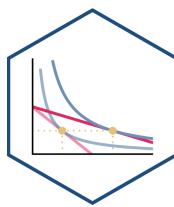
- $R(1) = \$2$
- $C(1) = \$10$
- $\pi(1) = -\$8$





# Short-Run Shut-Down Decisions

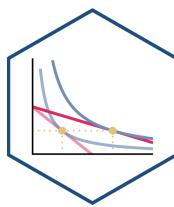
# Short-Run Shut-Down Decisions



- What if a firm's profits at  $q^*$  are **negative** (i.e. it earns **losses**)?
- **Should it produce at all?**



# Short-Run Shut-Down Decisions

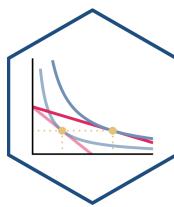


- Suppose firm chooses to produce **nothing** ( $q = 0$ ):
- If it has **fixed costs** ( $f > 0$ ), its profits are:

$$\pi(q) = pq - C(q)$$



# Short-Run Shut-Down Decisions



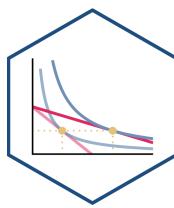
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# Short-Run Shut-Down Decisions



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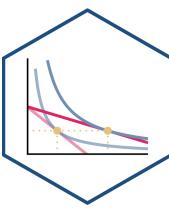
$$\pi(q) = pq - f - VC(q)$$

$$\pi(0) = -f$$

i.e. it (still) pays its fixed costs



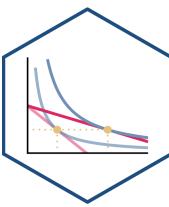
# Short-Run Shut-Down Decisions



- A firm should choose to produce **no output** ( $q = 0$ ) only when:

$\pi$  from producing  $<$   $\pi$  from not producing

# Short-Run Shut-Down Decisions

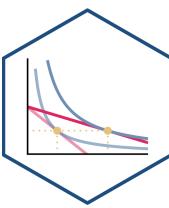


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# Short-Run Shut-Down Decisions



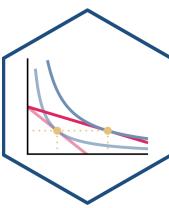
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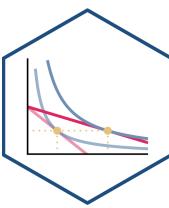
$\pi$  from producing  $<$   $\pi$  from not producing

$$\pi(q) < -f$$

$$pq - VC(q) - f < -f$$

$$pq - VC(q) < 0$$

# Short-Run Shut-Down Decisions



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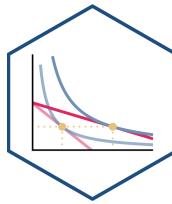
$$\pi(q) < -f$$

$$pq - VC(q) - f < -f$$

$$pq - VC(q) < 0$$

$$pq < VC(q)$$

# Short-Run Shut-Down Decisions



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$\pi$  from producing  $<$   $\pi$  from not producing

$$\pi(q) < -f$$

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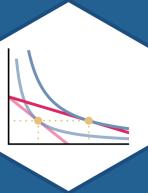
$$pq - VC(q) < 0$$

$$pq < VC(q)$$

$$p < AVC(q)$$

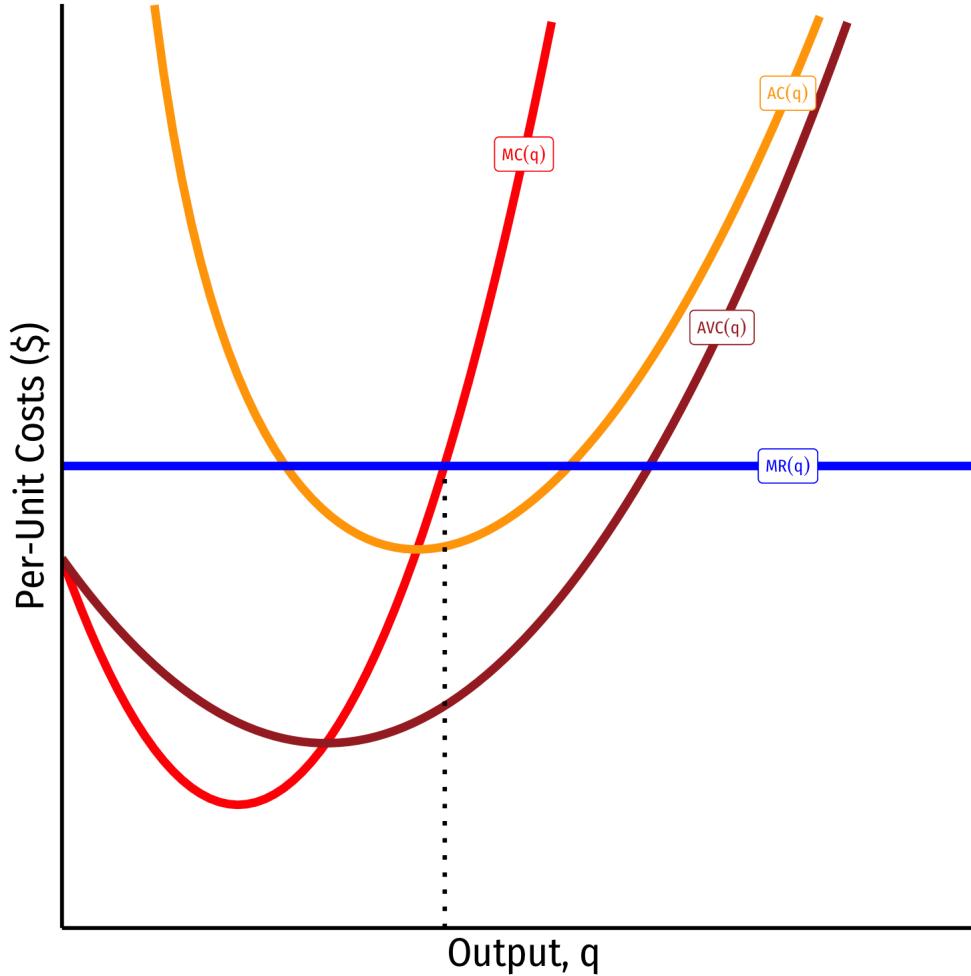
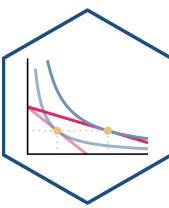
- **Shut down price:** firm will shut down production *in the short run* when  $p < AVC(q)$



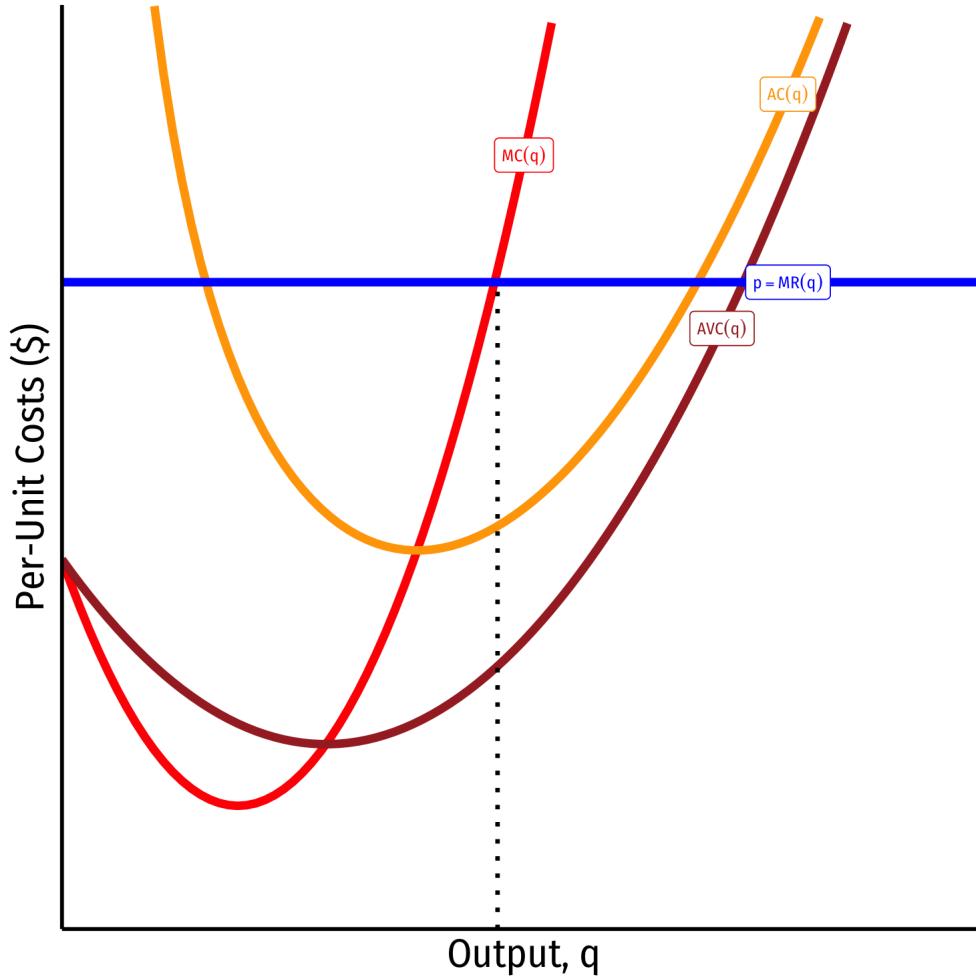
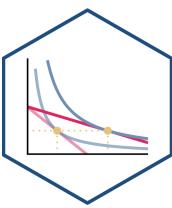


# The Firm's Short Run Supply Decision

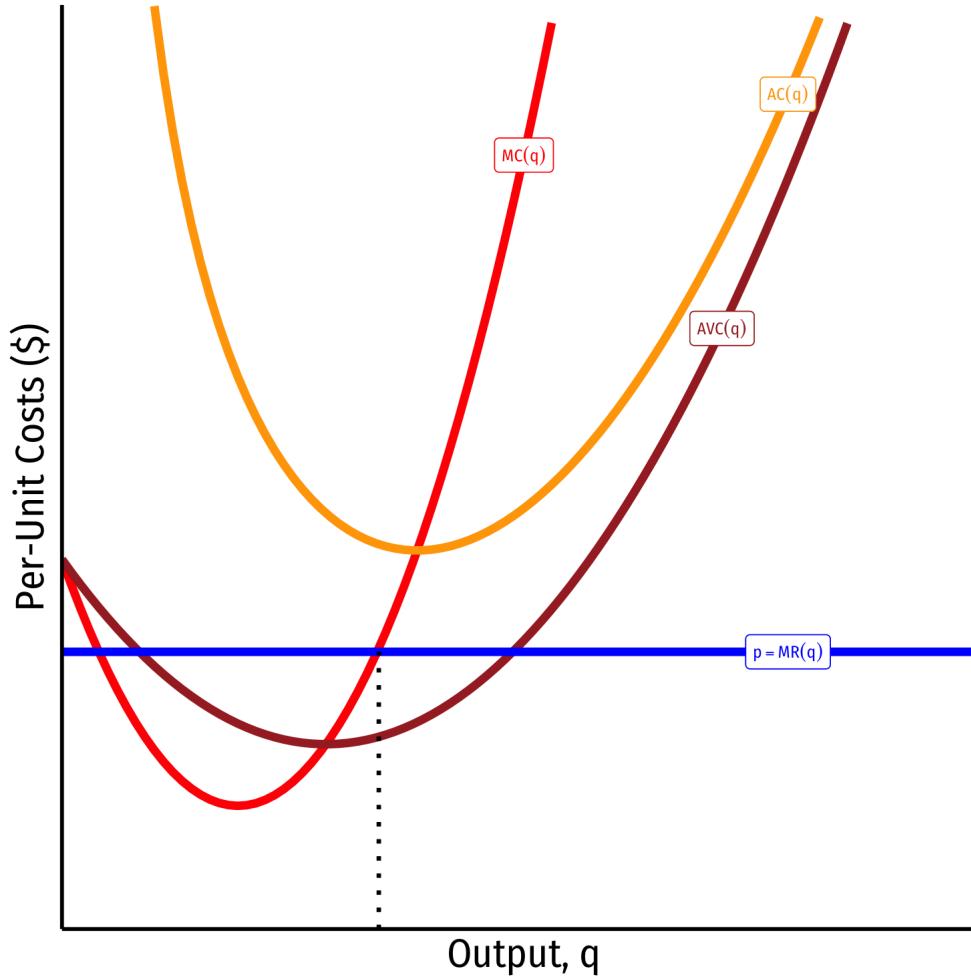
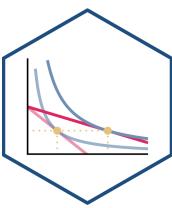
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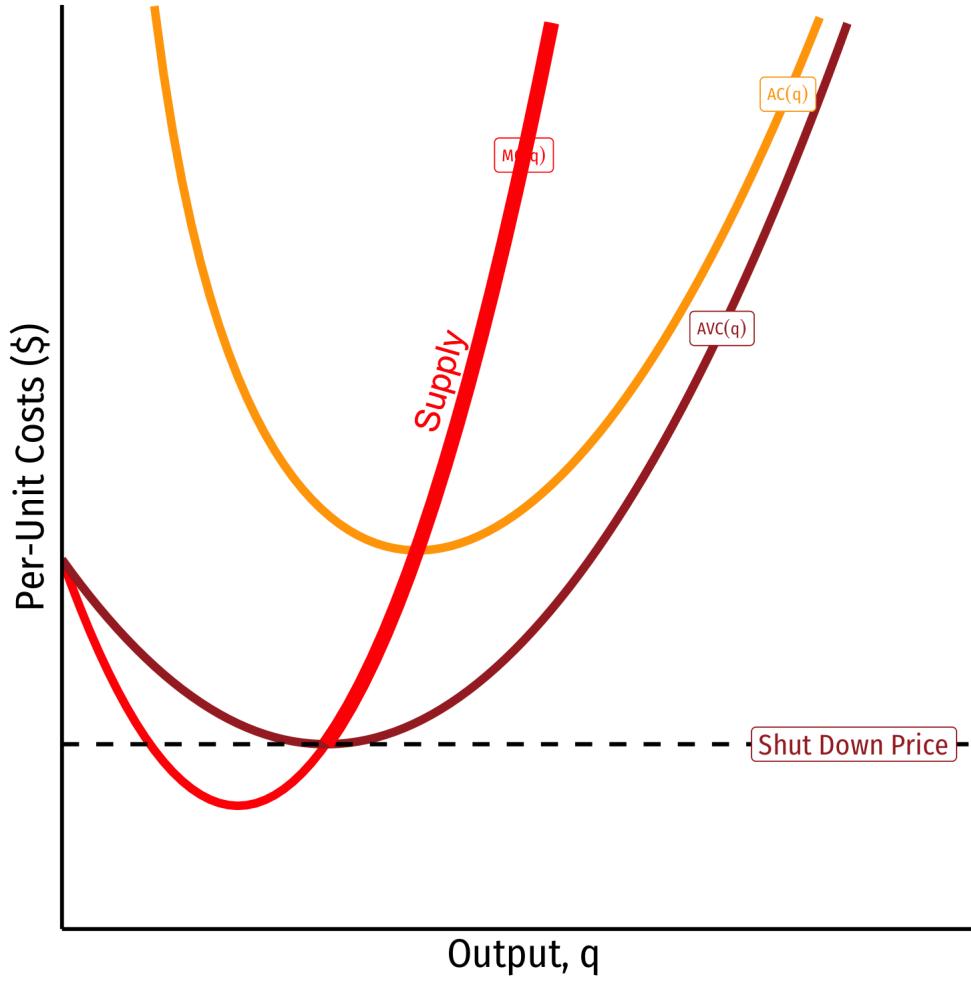
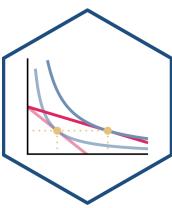
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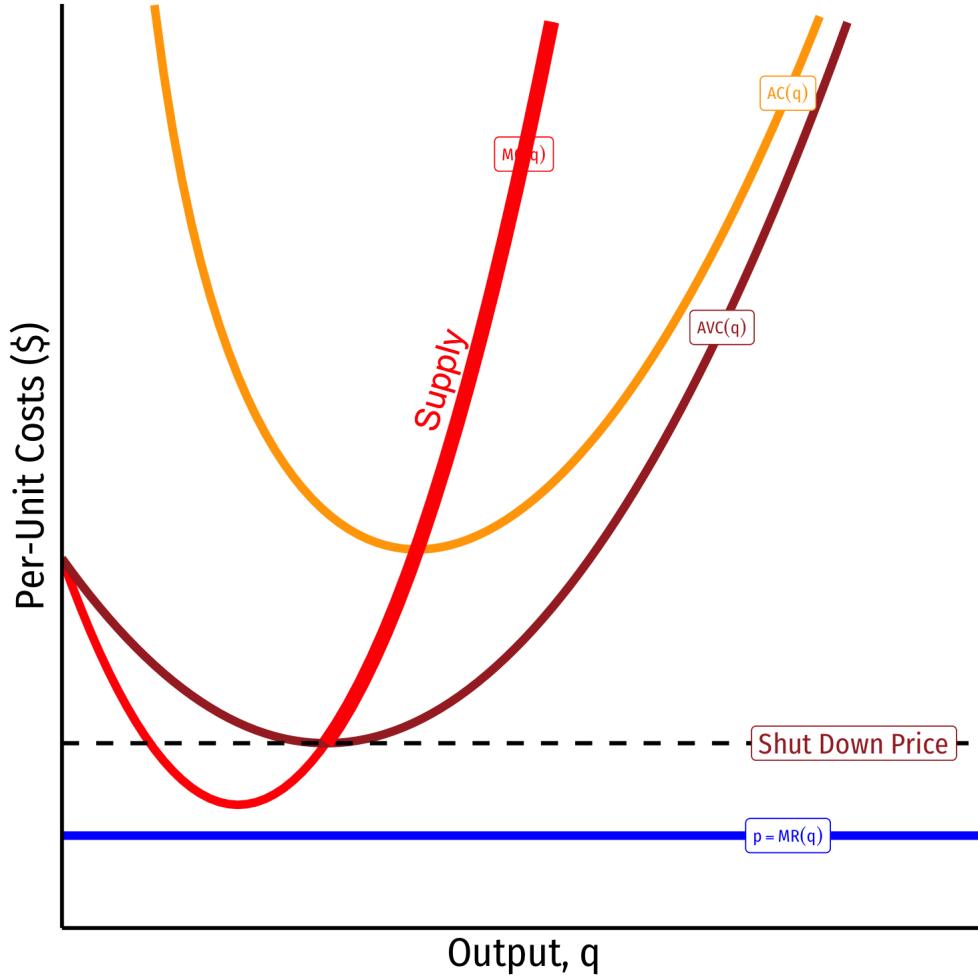
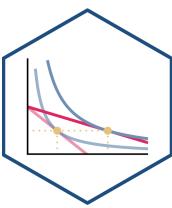
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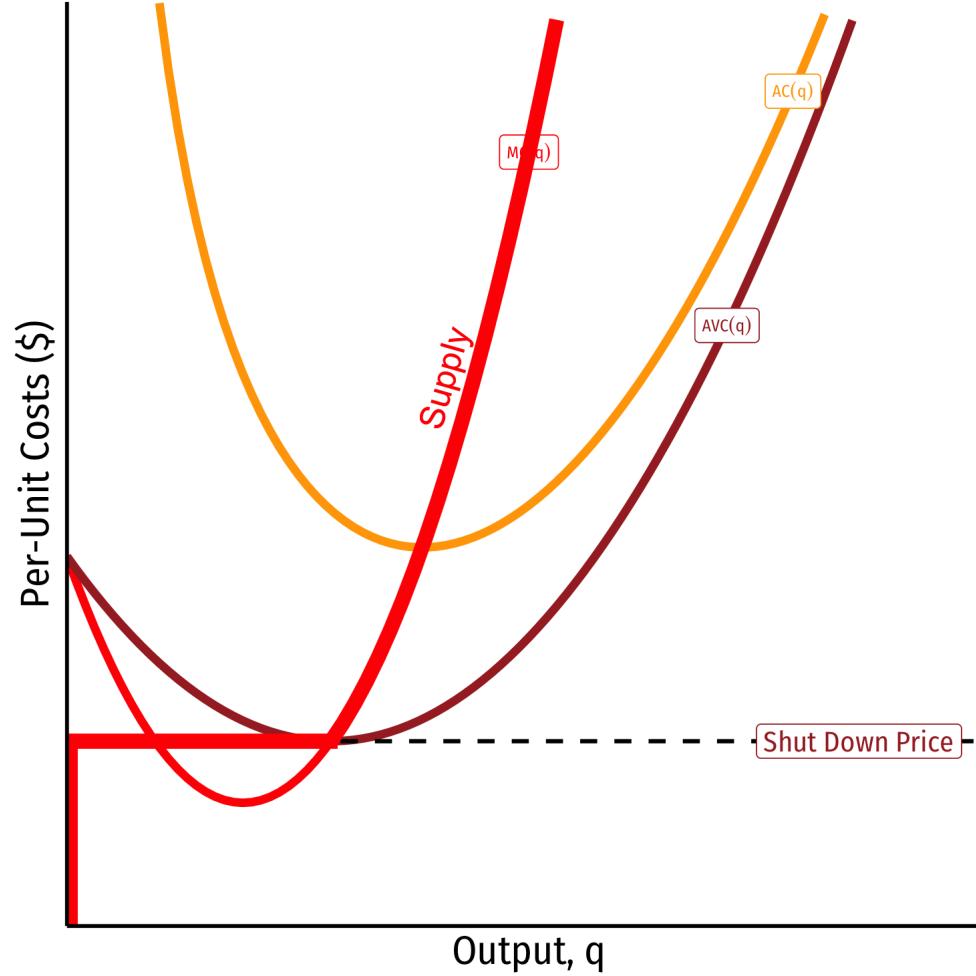
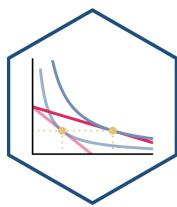
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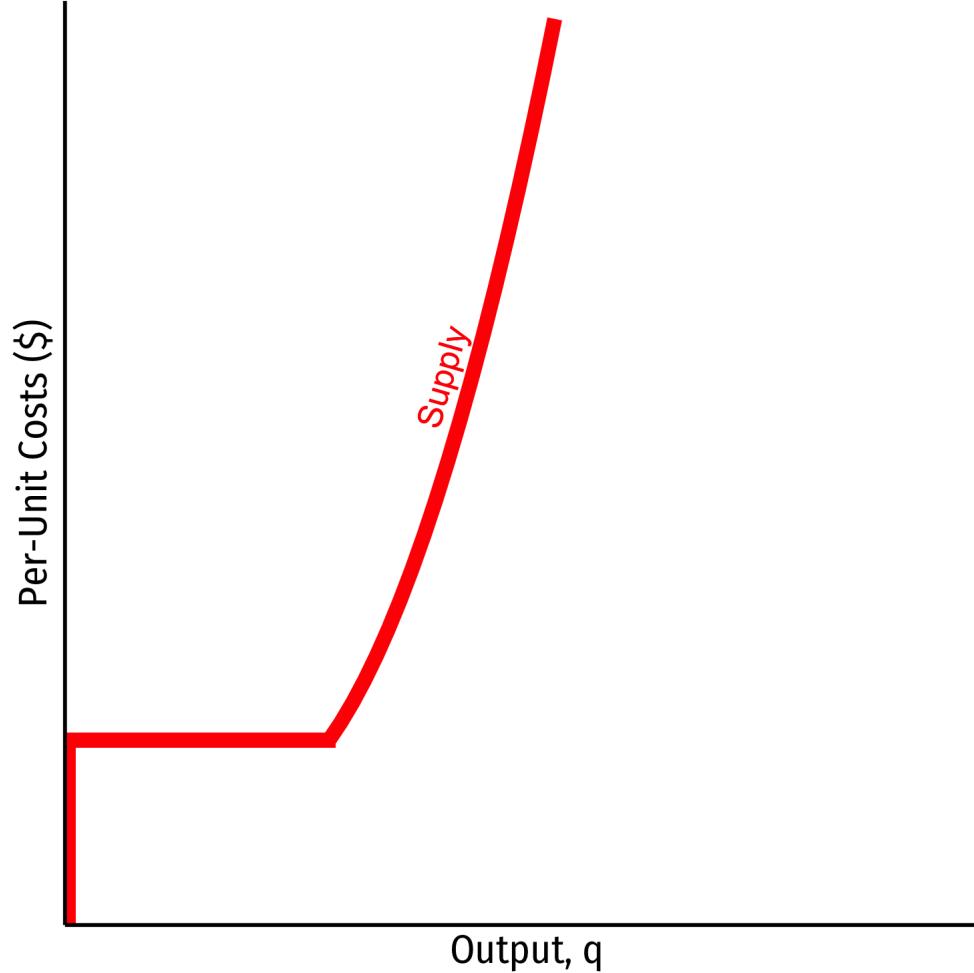
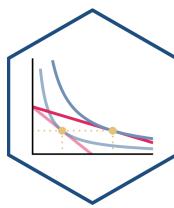
# The Firm's Short Run Supply Decision



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

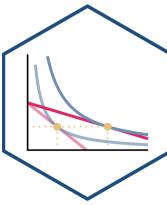
# The Firm's Short Run Supply Decision



Firm's short run supply curve:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

# Summary:



**1. Choose  $q^*$  such that  $MR(q) = MC(q)$**

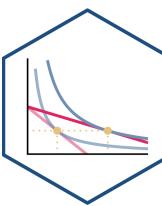
**2. Profit**  $\pi = q[p - AC(q)]$

**3. Shut down if  $p < AVC(q)$**

Firm's short run (inverse) supply:

$$\begin{cases} p = MC(q) & \text{if } p \geq AVC \\ q = 0 & \text{If } p < AVC \end{cases}$$

# Choosing the Profit-Maximizing Output $q^*$ : Example



**Example:** Daniel's Midland Archers has the following cost structure for producing archery bows:

$$C(q) = 2q^2 + 3q + 50$$

$$MC(q) = 4q + 3$$

Suppose the market is very competitive and the current market price is \$15.

1. How many bows should the firm produce?
2. How much profit will it earn per day?
3. At what price would the firm break even?