

1.6 – Income & Substitution Effects

ECON 306 • Microeconomic Analysis • Spring 2023

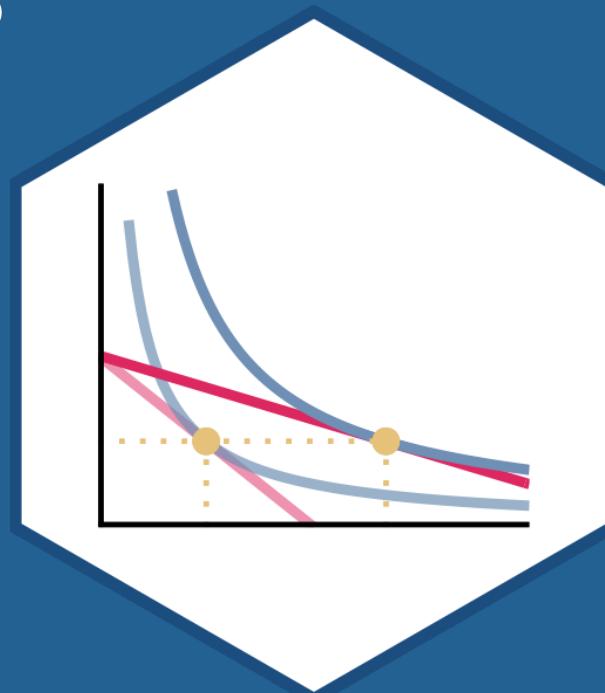
Ryan Safner

Associate Professor of Economics

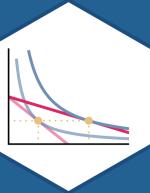
 safner@hood.edu

 [ryansafner/microS23](https://github.com/ryansafner/microS23)

 microS23.classes.ryansafner.com



Outline



The (Own) Price Effect

(Real) Income Effect

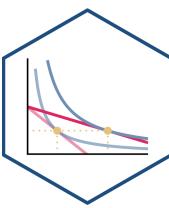
Substitution Effect

Putting the Effects Together

What About Inferior Goods?

On to Demand Curves

A Demand Function (Again)



- A consumer's **demand** (for good x) depends on **current prices & income**:

$$q_x^D = q_x^D(m, p_x, p_y)$$

- **How does demand (for x) change?**

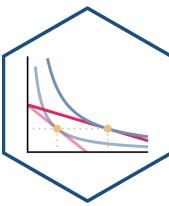
1. **Income effects** $\left(\frac{\Delta q_x^D}{\Delta m} \right)$: how q_x^D changes with changes in income
2. **Cross-price effects** $\left(\frac{\Delta q_x^D}{\Delta p_y} \right)$: how q_x^D changes with changes in prices of *other* goods (e.g. y)
3. **(Own) Price effects** $\left(\frac{\Delta q_x^D}{\Delta p_x} \right)$: how q_x^D changes with changes in price (of x)





The (Own) Price Effect

The (Own) Price Effect



- **Price effect:** change in optimal consumption of a good associated with a change in its price, holding income and other prices constant

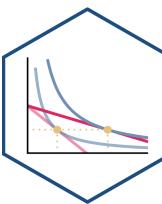
$$\frac{\Delta q_x^D}{\Delta p_x} < 0$$

The law of demand: as the price of a good rises, people will tend to buy less of that good (and vice versa)

- i.e. **the price effect is negative!**



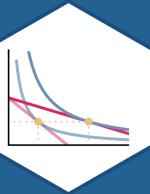
Decomposing the Price Effect



The **price effect** (law of demand) is actually the **net result of two effects**

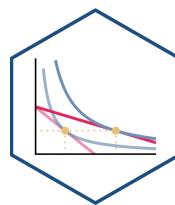
1. **(Real) income effect**: change in consumption due to change in real purchasing power
2. **Substitution effect**: change in consumption due to change in relative prices

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$



(Real) Income Effect

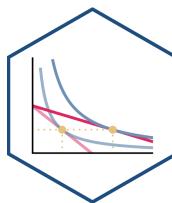
(Real) Income Effect: Demonstration



- Suppose there is only 1 good to consume, x . You have a \$100 income, and the price of x is \$10. You consume 10 units of x
- Suppose the price of x rises to \$20. You now consume 5 units of x .
- This is the **real income effect**



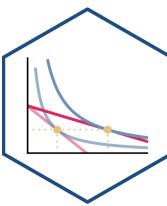
(Real) Income Effect: Demonstration



- **Real income effect:** your consumption mix changes because of the change in the price of x changes your **real income** or **purchasing power** (the amount of goods you can buy)
- Note your ***actual (nominal) income*** (\$100) **never changed!**



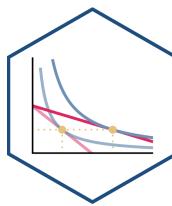
(Real) Income Effect: Size



- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Large-budget items:**
 - e.g. Housing/apartment rent, car prices
 - Price increase/decreases makes you much poorer/wealthier

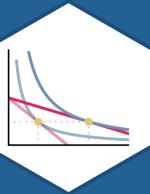


(Real) Income Effect: Size



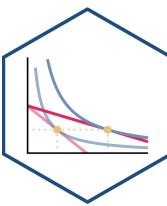
- The *size* of the income effect depends on how large a *portion of your budget* you spend on the good
- **Small-budget items:**
 - e.g. pencils, toothpicks, candy
 - Price changes don't have much of an effect on your wealth or change your behavior much





Substitution Effect

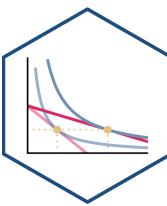
Substitution Effect: Demonstration



- Suppose there are 1000's of goods, none of them a major part of your budget
 - So real income effect is insignificant
- Suppose the price of good x increases
- You would consume *less* of x relative to other goods because x is now *relatively* more expensive
- That's the **substitution effect**

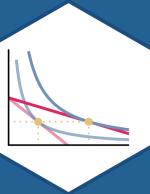


Substitution Effect: Demonstration



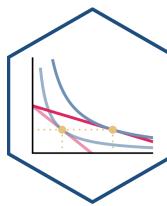
- **Substitution effect:** consumption mix changes because of a change in **relative prices**
- Buy more of the (now) relatively cheaper items
- Buy less of the (now) relatively more expensive item (x)





Putting the Effects Together

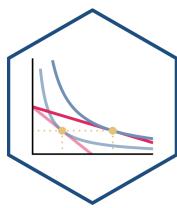
Putting the Effects Together



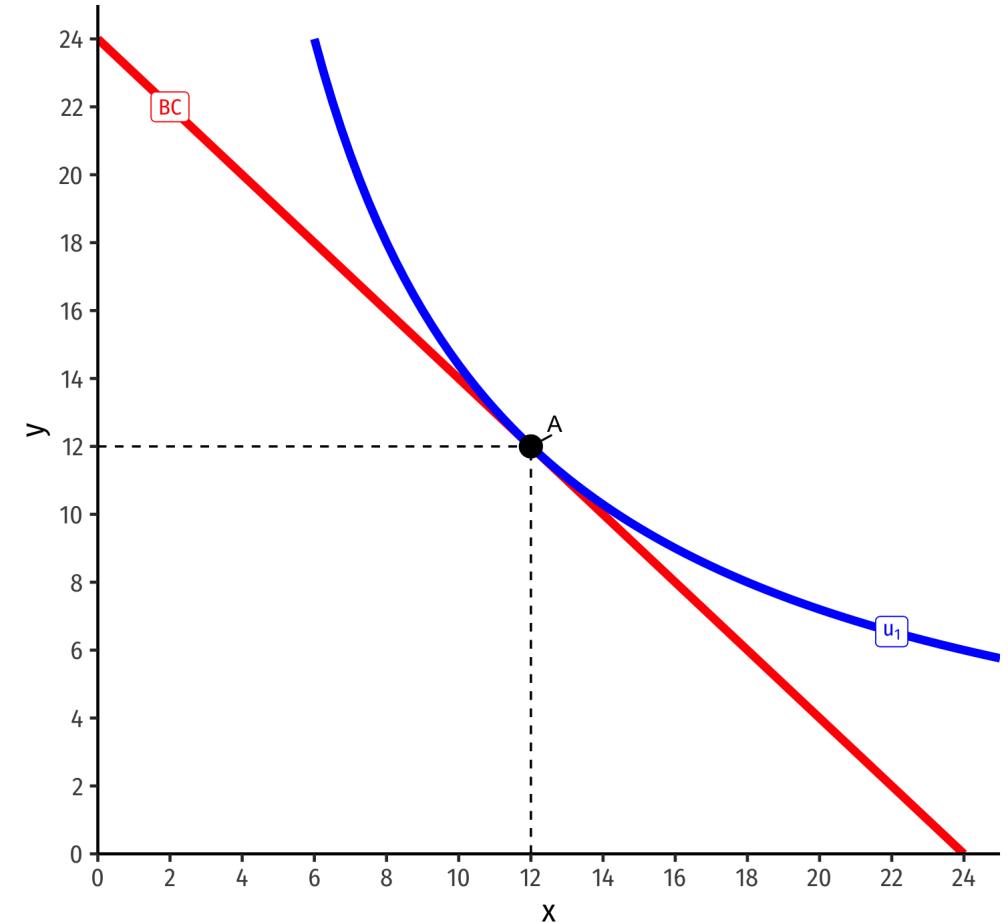
- **Real income effect:** change in consumption due to change in real purchasing power
 - **Could go in different directions:** positive (**normal goods**) or negative (**inferior goods**)
 - Higher price of x means you must buy less x , y , or *both* (depending on your preferences)
- **Substitution effect:** change in consumption due to change in relative prices
 - If x gets more expensive relative to y , consume $\downarrow x$ (and $\uparrow y$)
 - **Always the same direction:** (\downarrow relatively expensive goods, \uparrow relatively cheaper goods)
 - **This is why demand curves slope downwards!**

$$\text{Price Effect} = \text{Real income effect} + \text{Substitution Effect}$$

Real Income and Substitution Effects, Graphically I

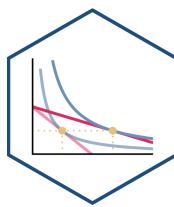


- Original optimal consumption (A)

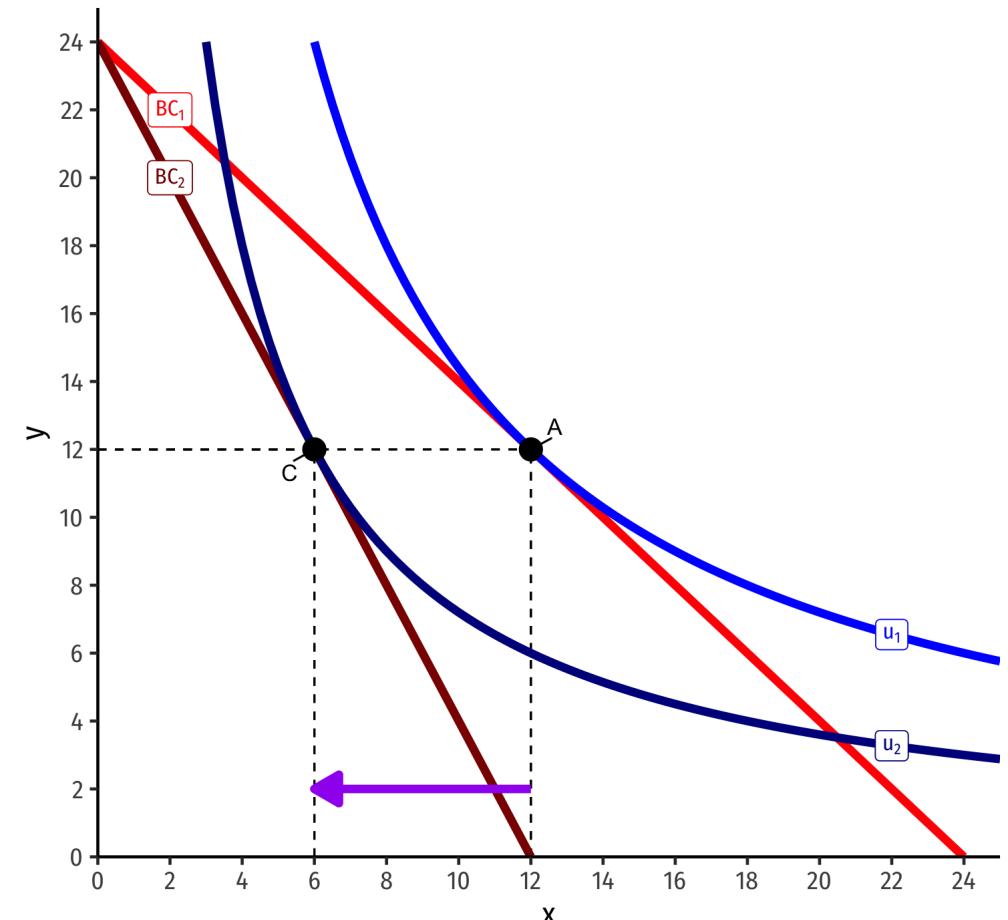


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 24$, $p_y = 1$

Real Income and Substitution Effects, Graphically I

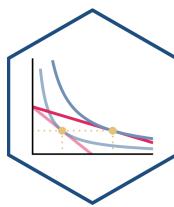


- Original optimal consumption (A)
- **(Total) price effect:** $A \rightarrow C$
- Let's decompose this into the two effects

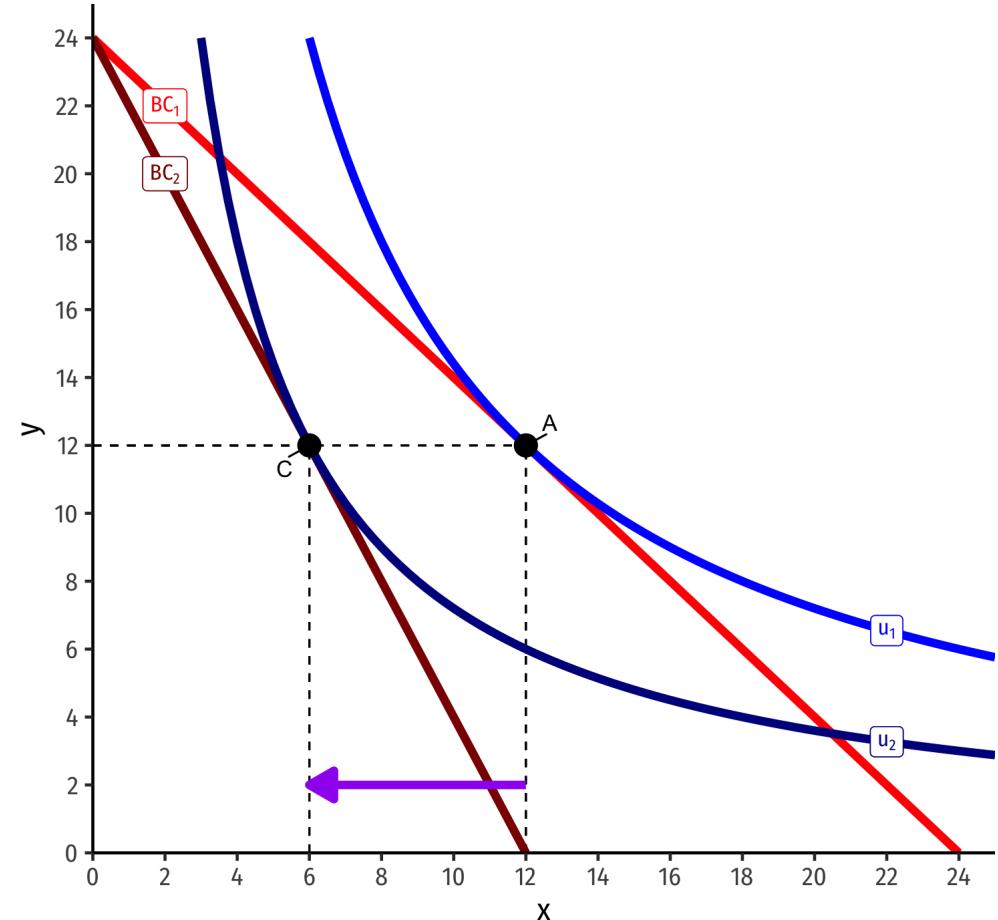


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Real Income and Substitution Effects, Graphically II

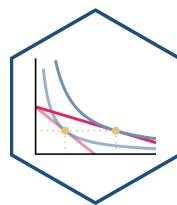


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change

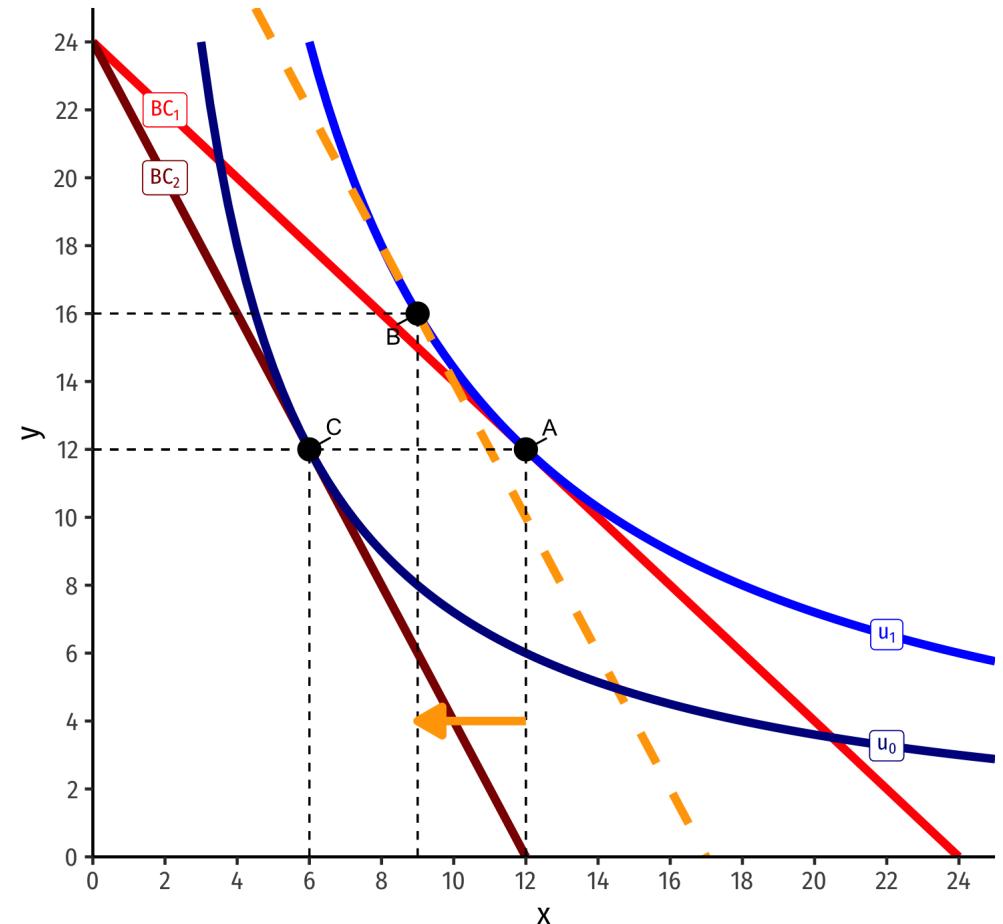


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Real Income and Substitution Effects, Graphically II

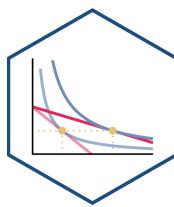


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change
- Graphically: shift *new* budget constraint inwards until tangent with *old* indifference curve
- $A \rightarrow B$ on same I.C. ($\downarrow x, \uparrow y$)
 - Note: Point B *must* be a *different* point on the original curve! Why?

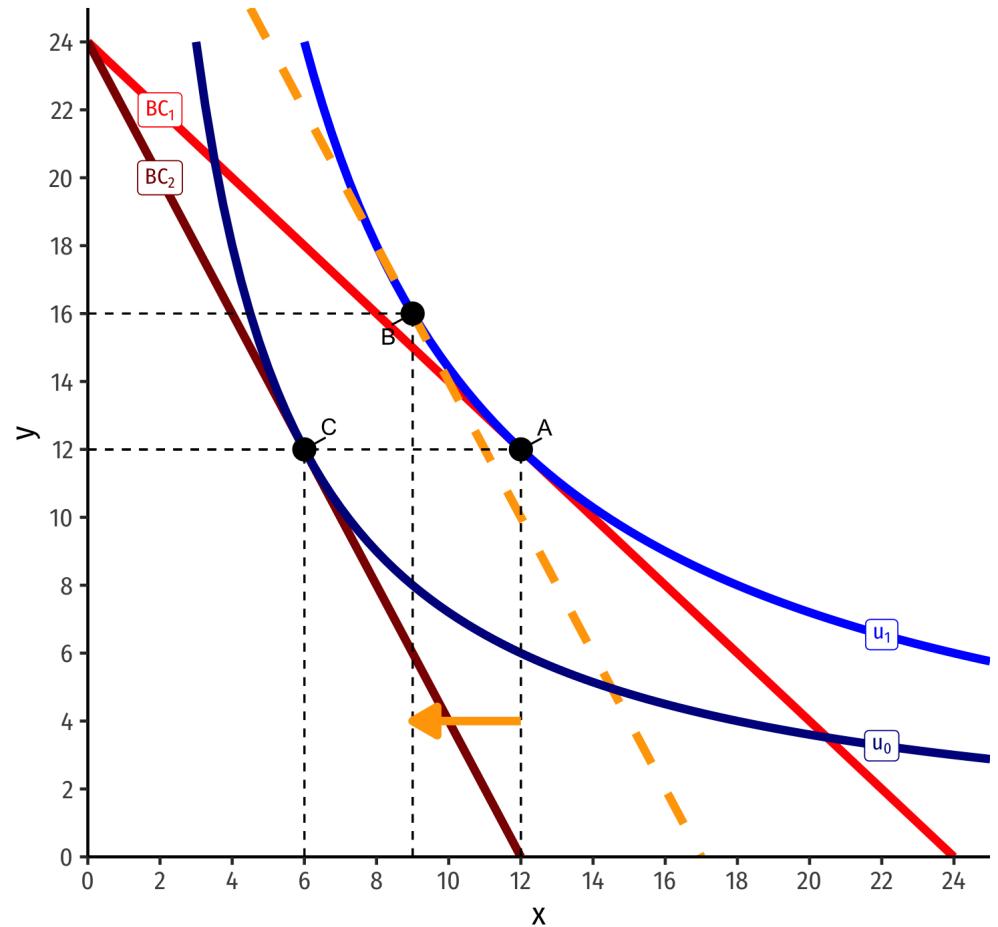


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Real Income and Substitution Effects, Graphically III

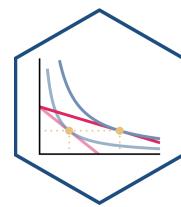


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the price change

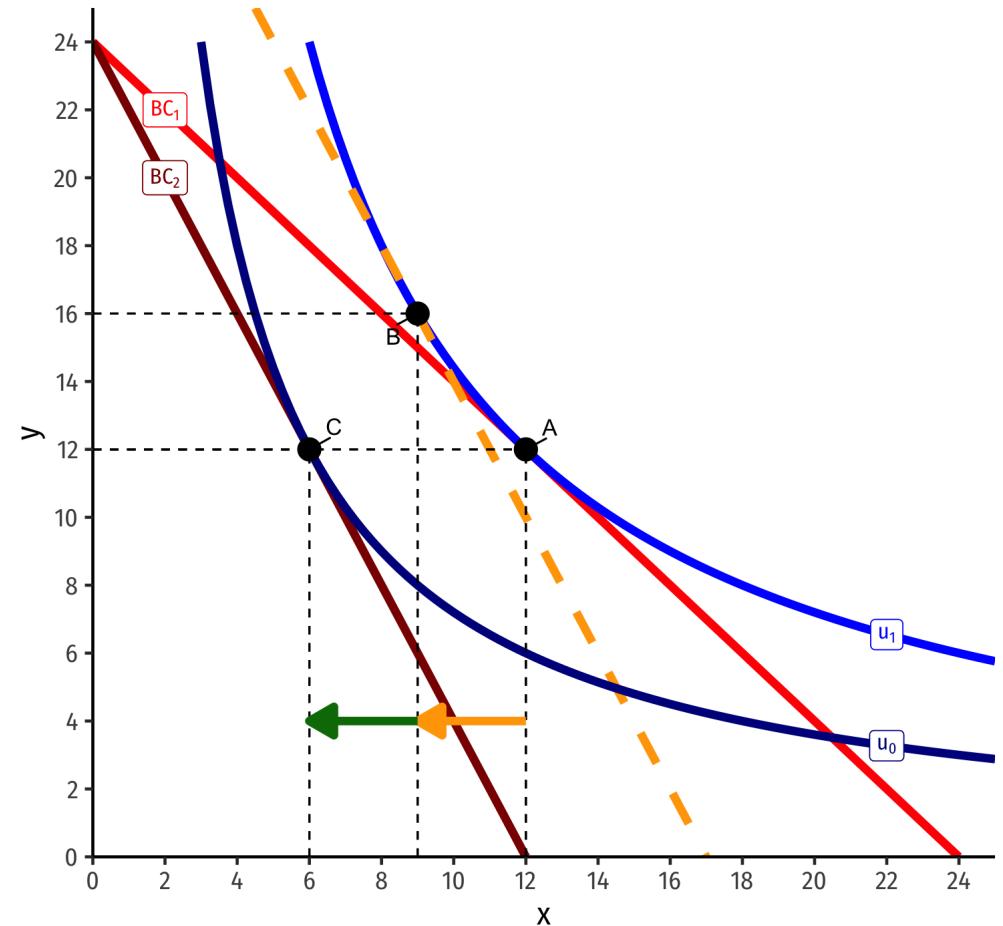


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Real Income and Substitution Effects, Graphically III

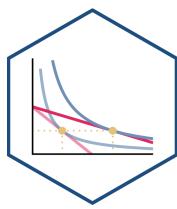


- **(Real) income effect:** change in consumption due to the **change in purchasing power** from the price change
- $B \rightarrow C$ to new budget constraint (can buy less of x and/or y)

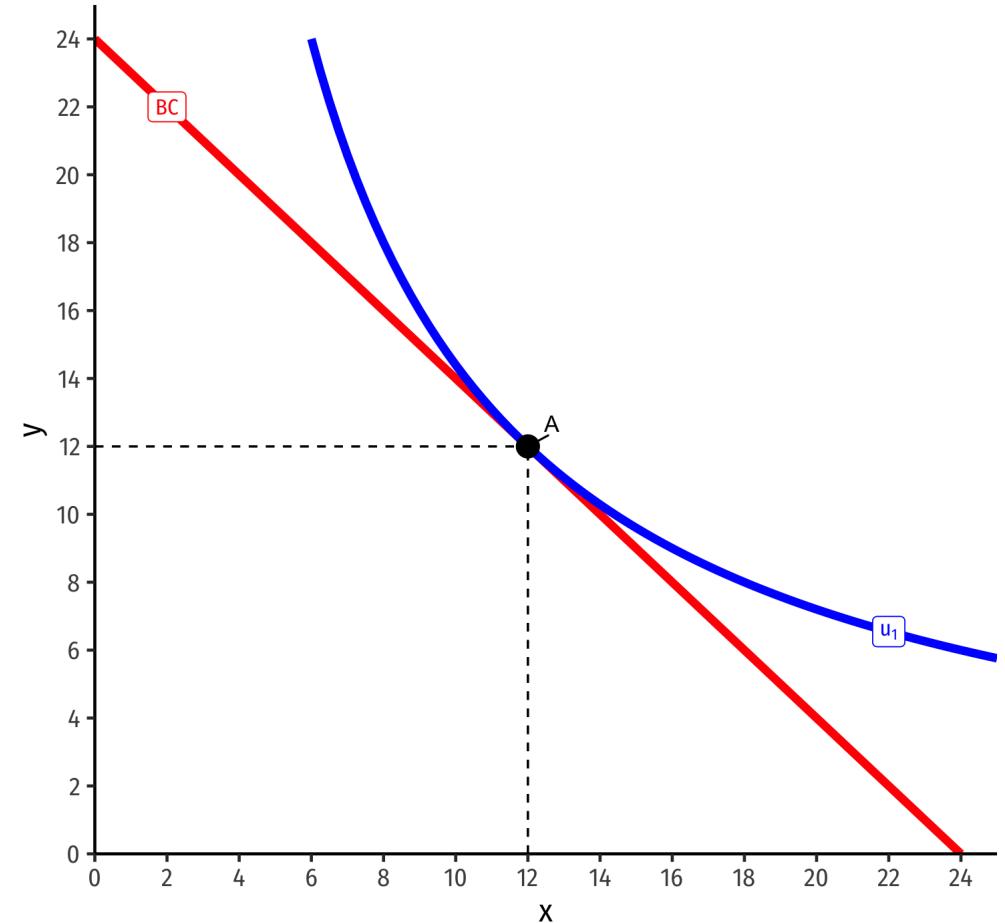


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Real Income and Substitution Effects, Graphically IV

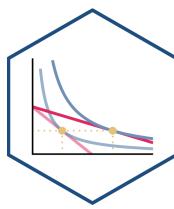


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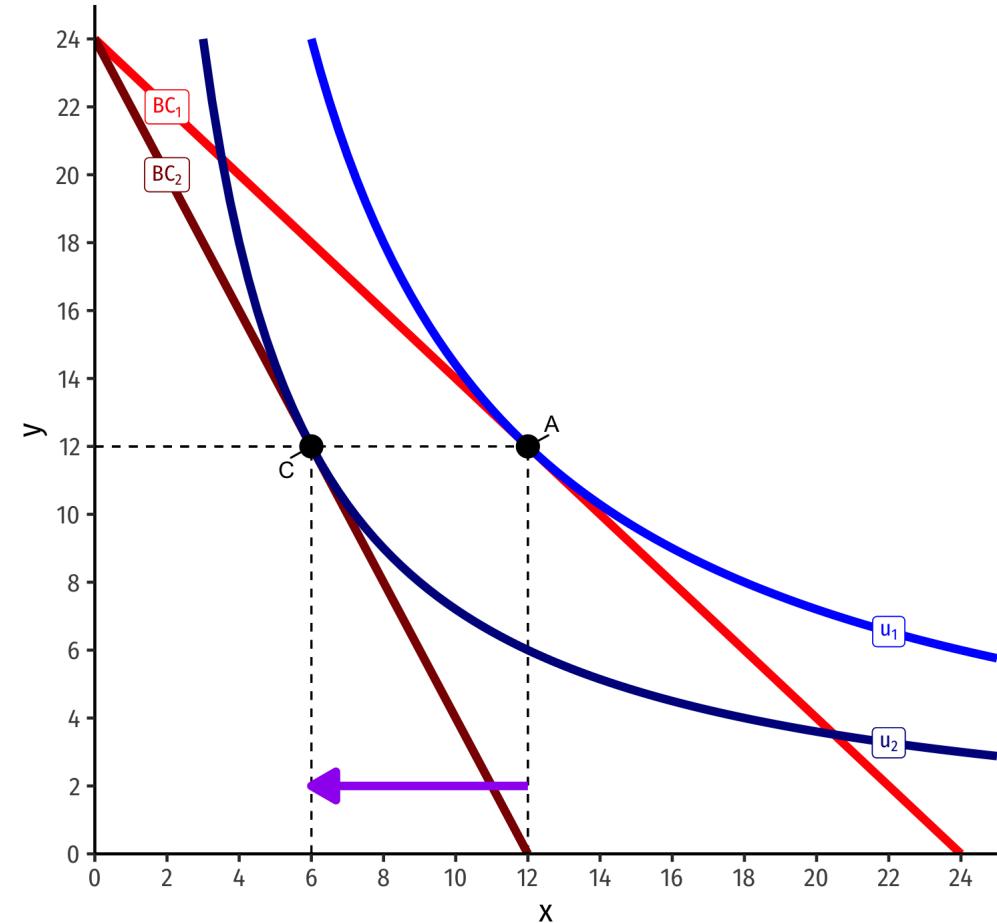


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Real Income and Substitution Effects, Graphically IV

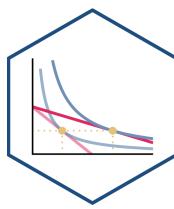


- Original optimal consumption (A)
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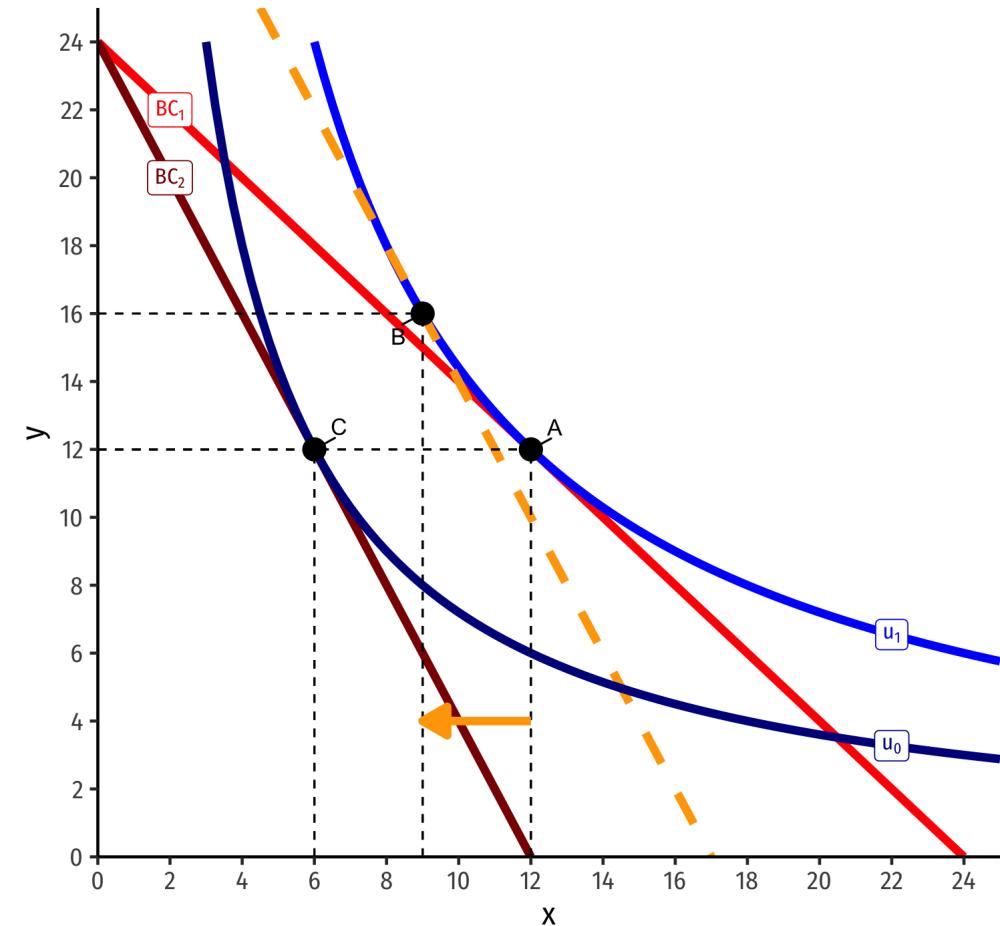


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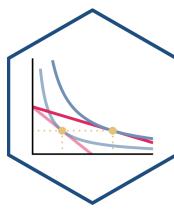


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- **Substitution effect:** $A \rightarrow B$ on same I.C. (\downarrow more expensive x and \uparrow y)

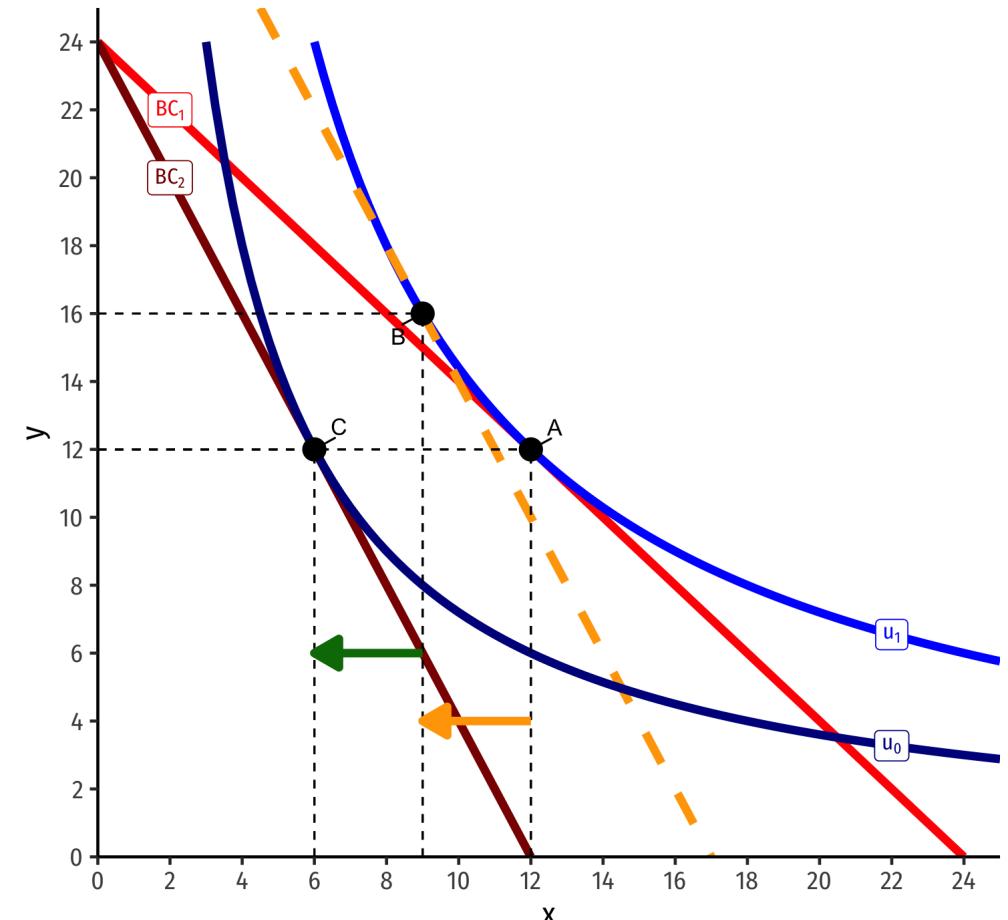


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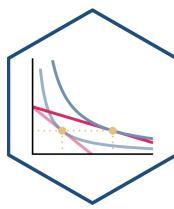


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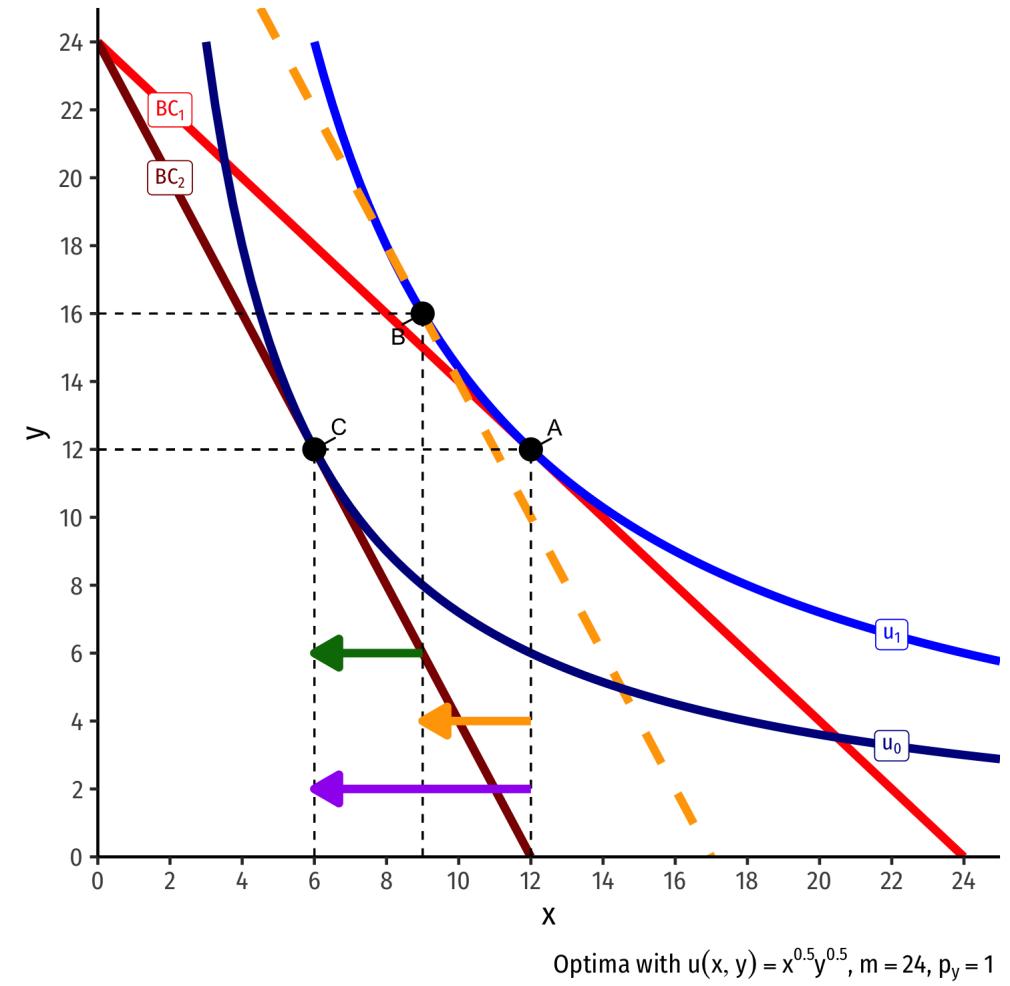


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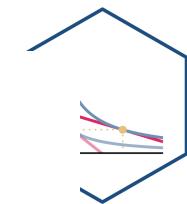


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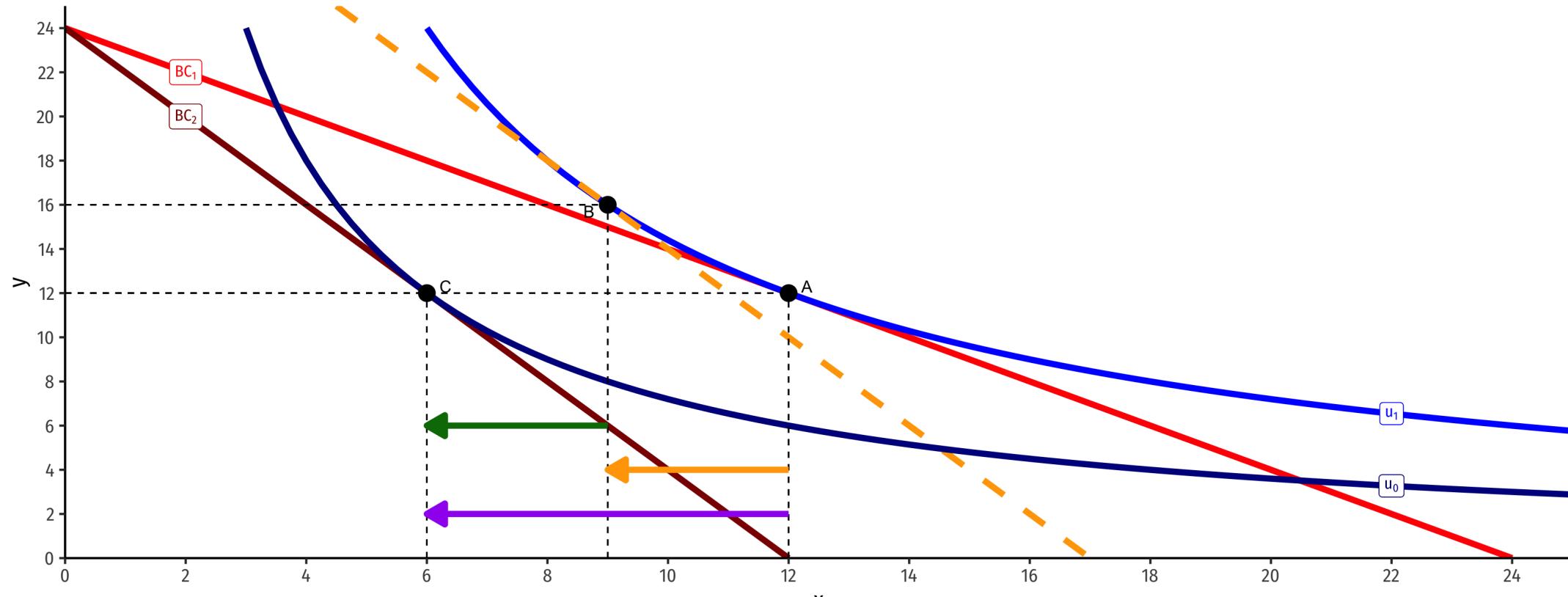


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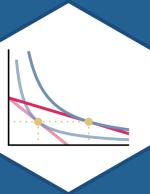
Change in Consumption From an Increase in Price



Normal Good

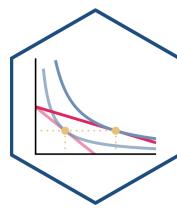


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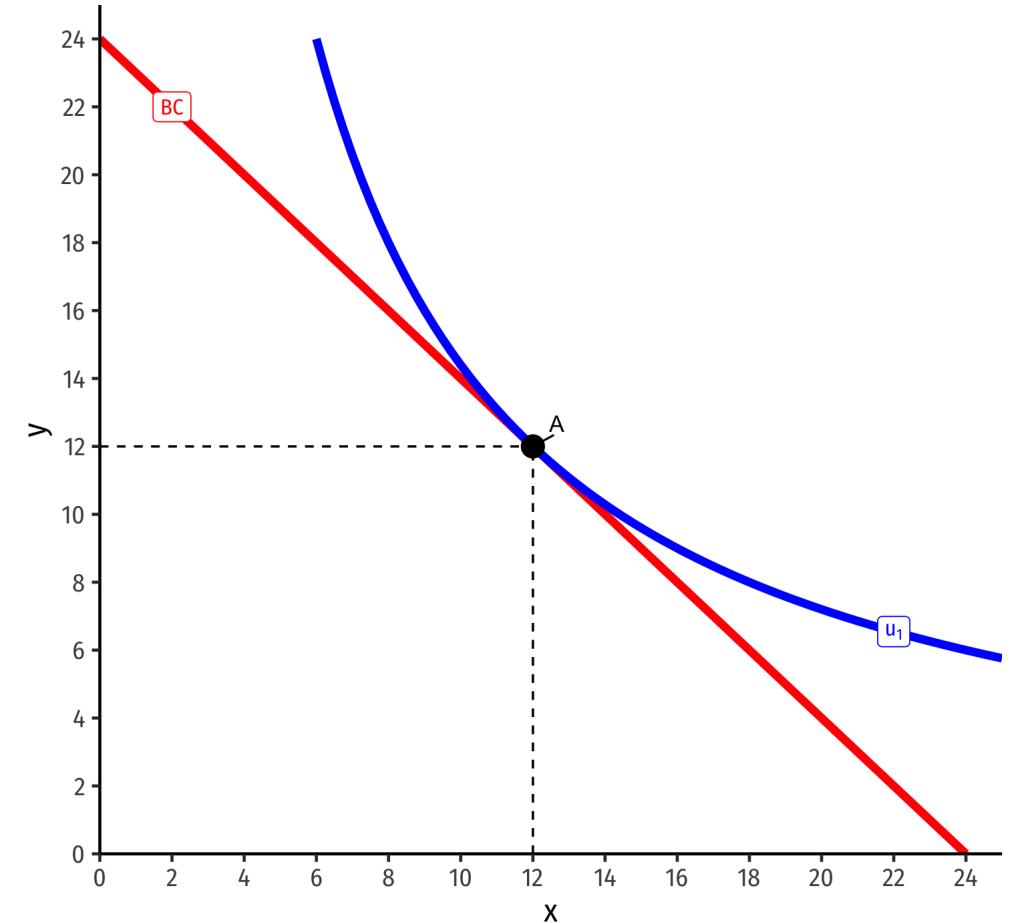


What About Inferior Goods?

Inferior Goods, Graphically I

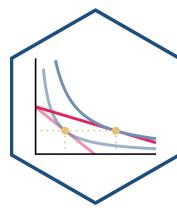


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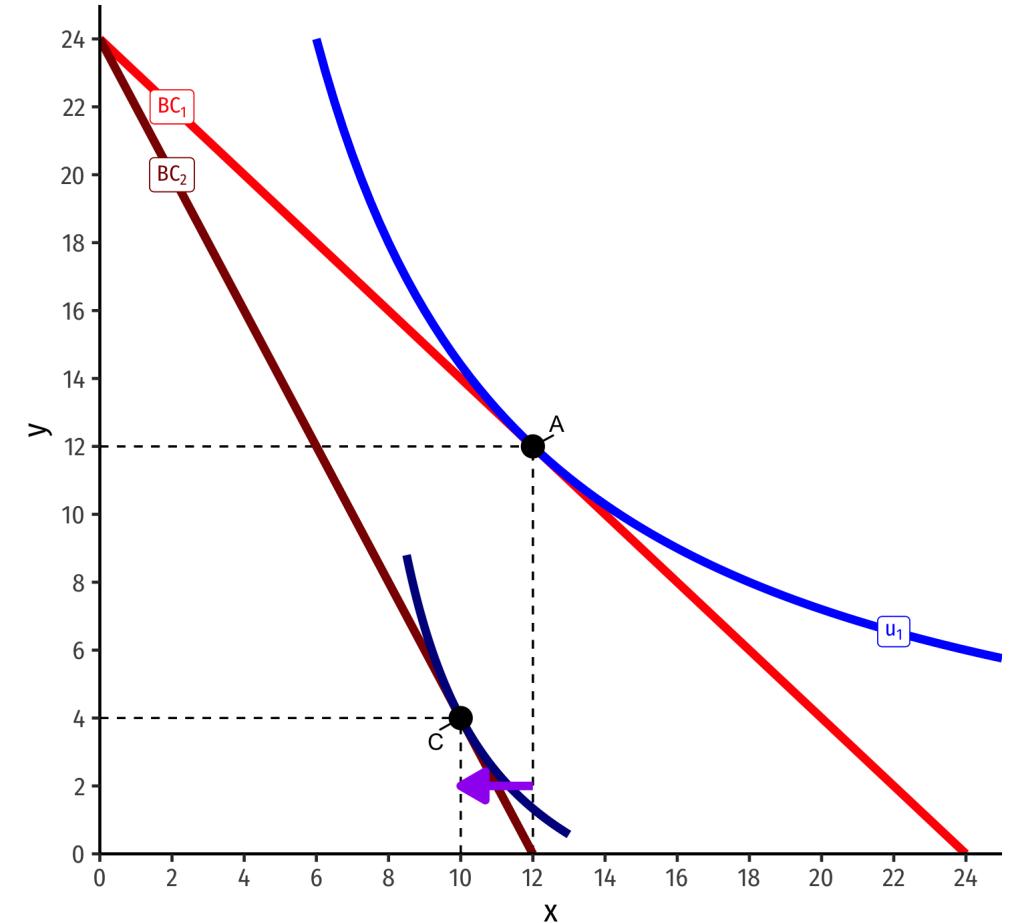


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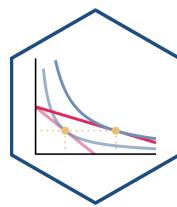


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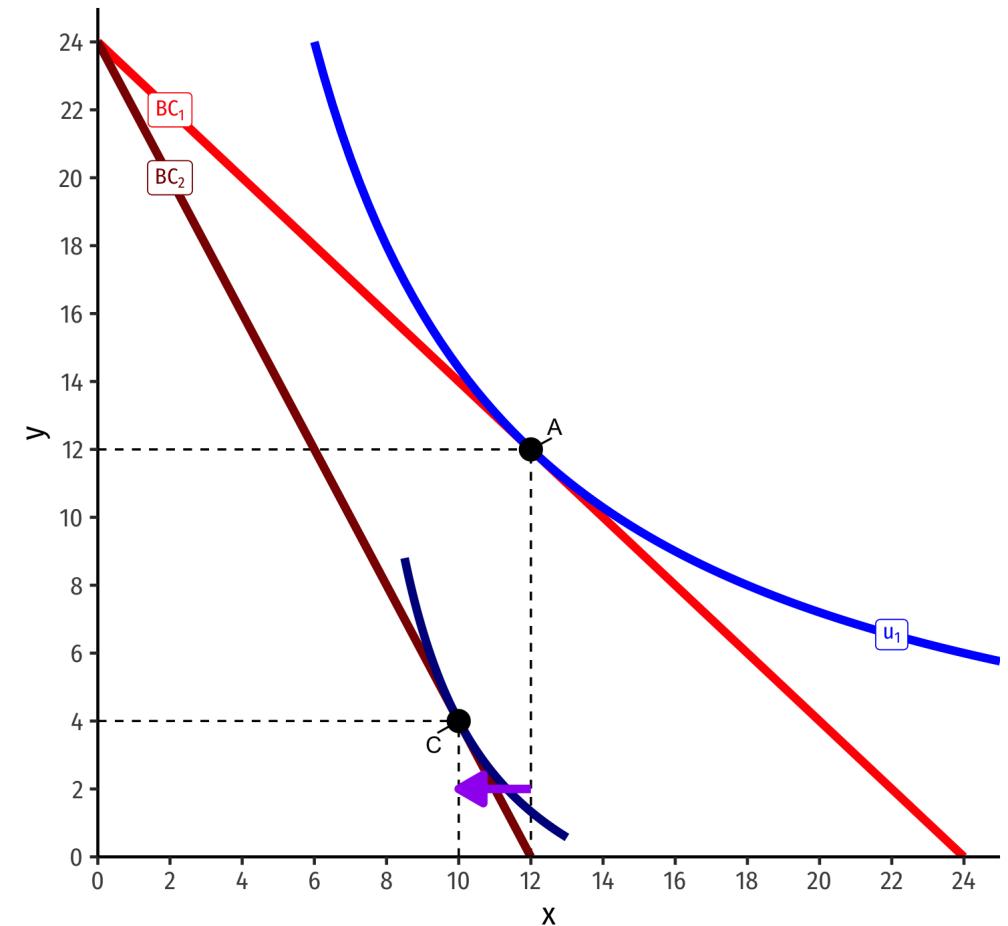


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Inferior Goods, Graphically II

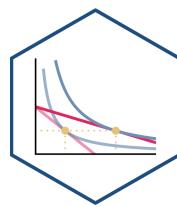


- **Substitution effect:** what you would choose under the **new exchange rate** to **remain indifferent** as before the change

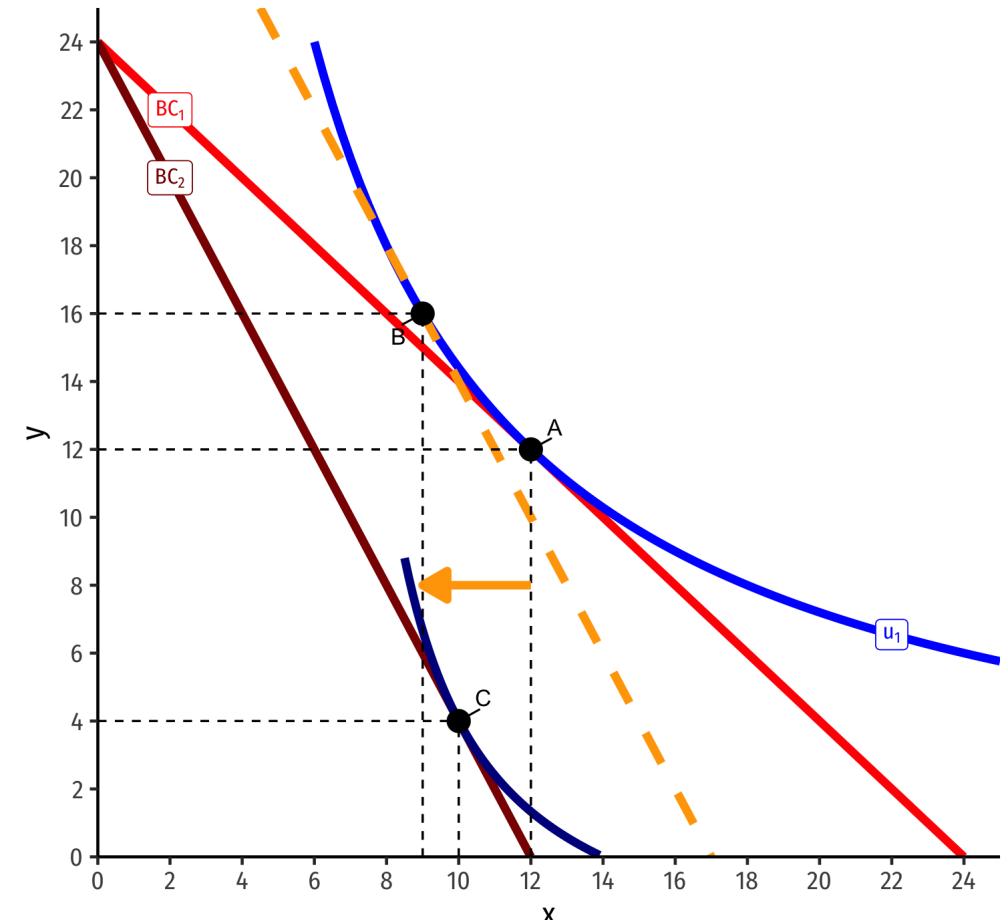


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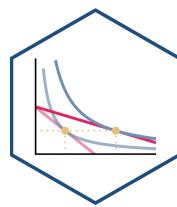


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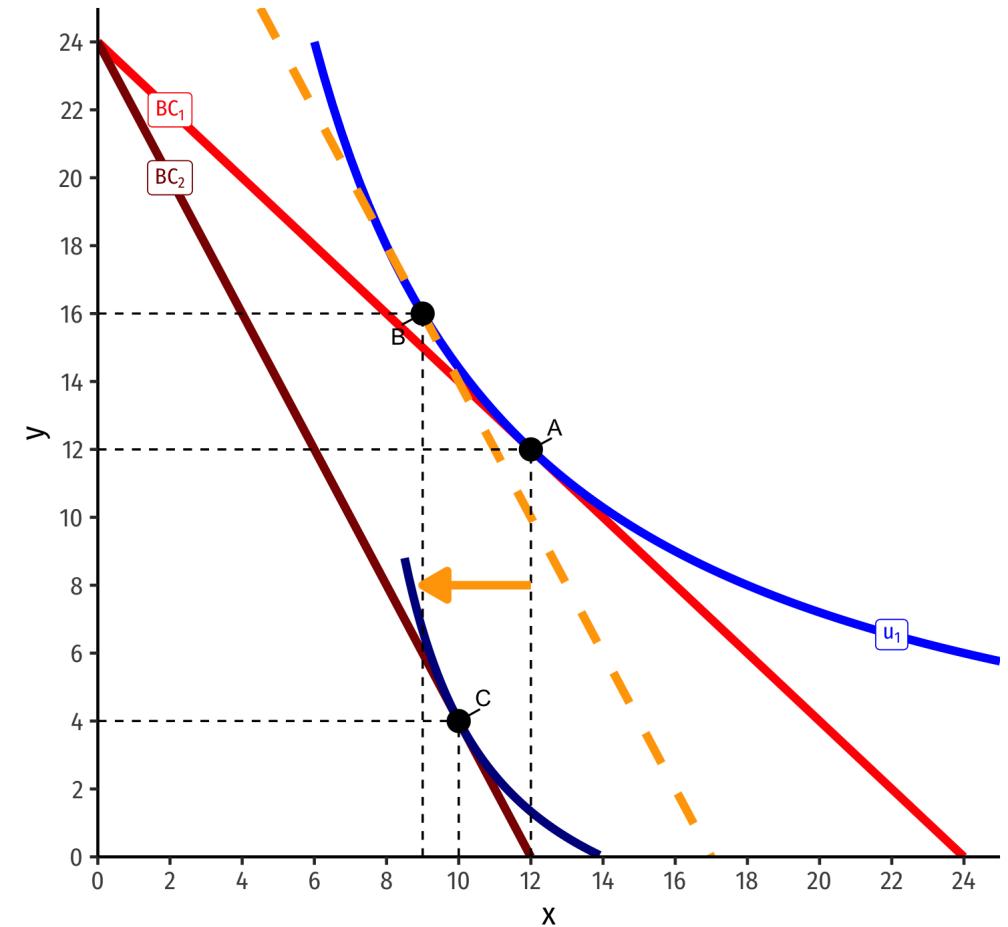


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Inferior Goods, Graphically III

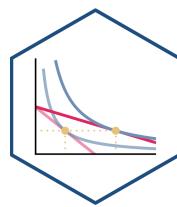


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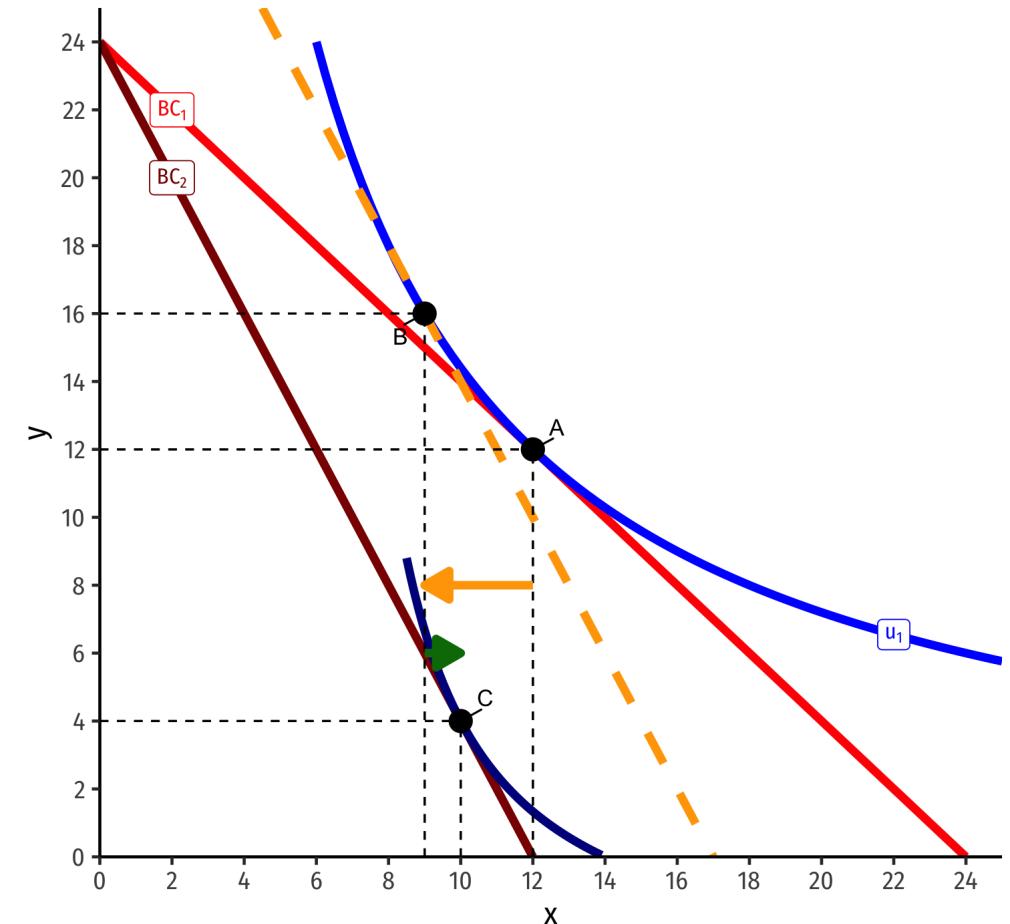


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Inferior Goods, Graphically III

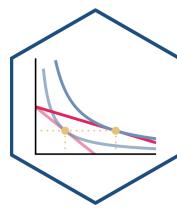


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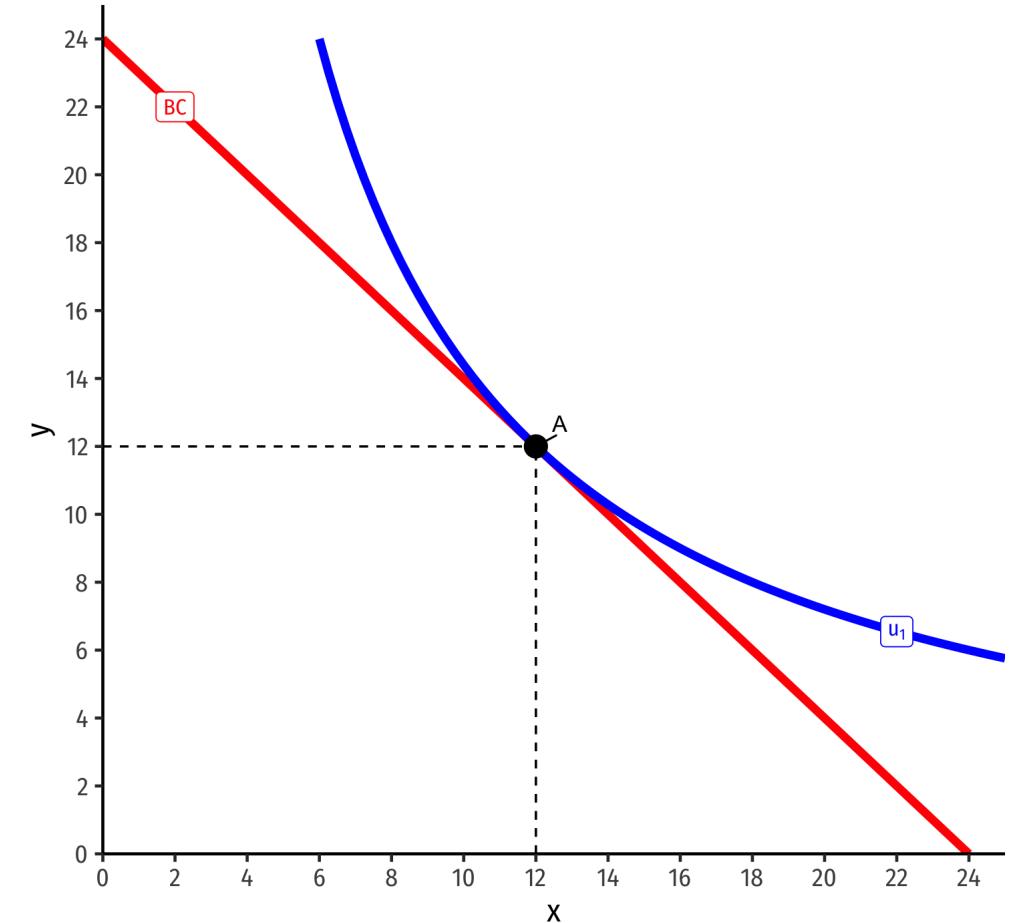


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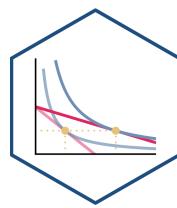
Inferior Goods, Graphically IV



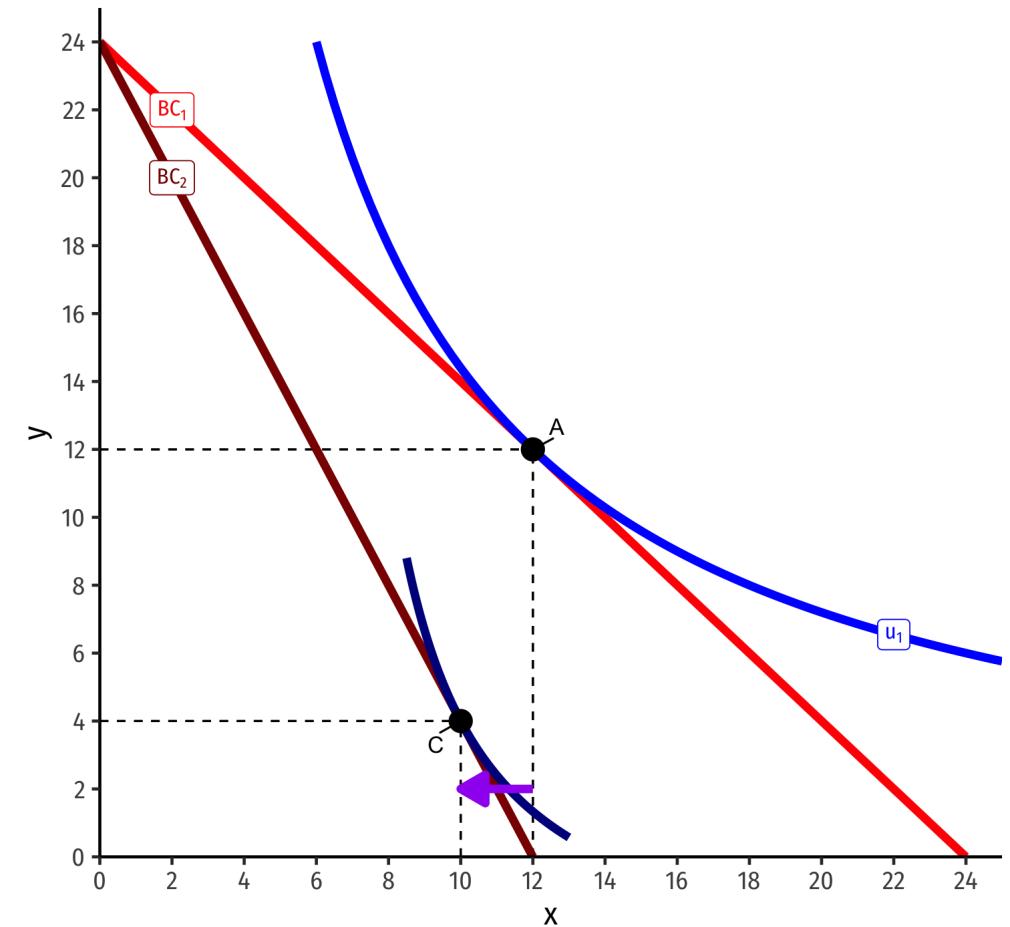
- Original optimal consumption (A)



Inferior Goods, Graphically IV

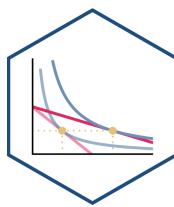


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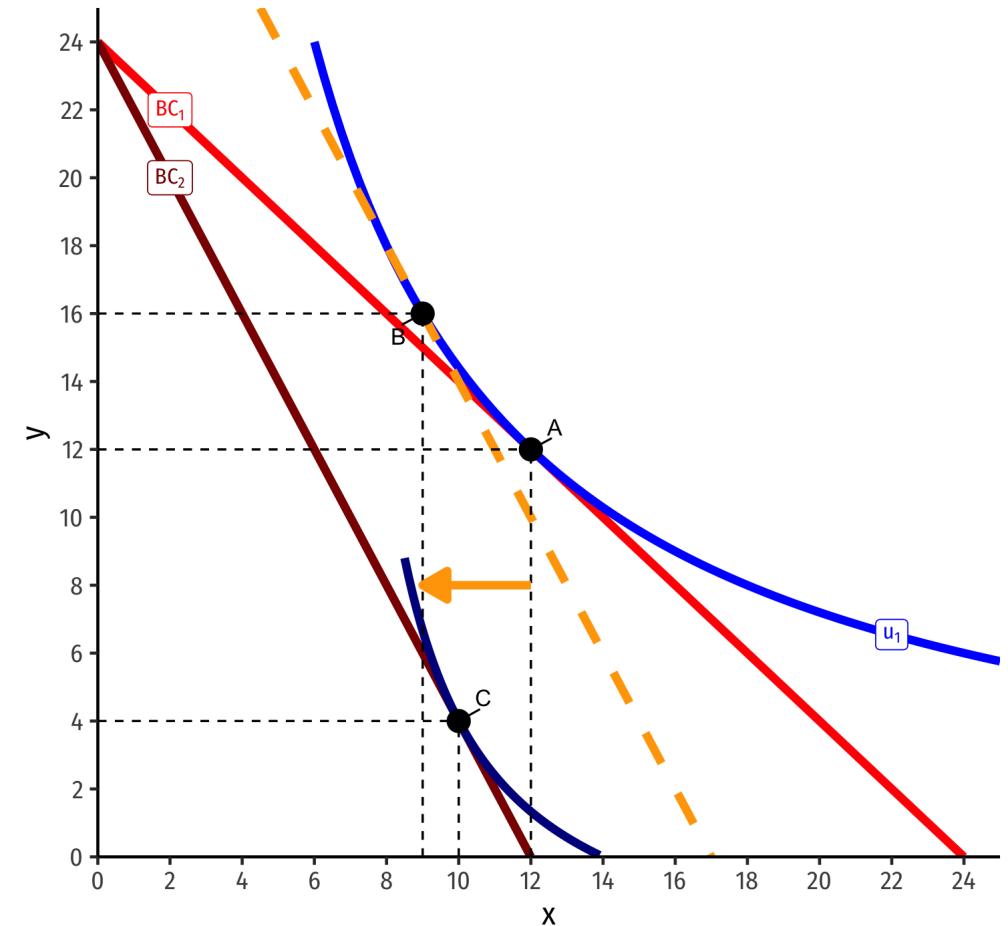


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Inferior Goods, Graphically IV

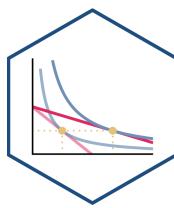


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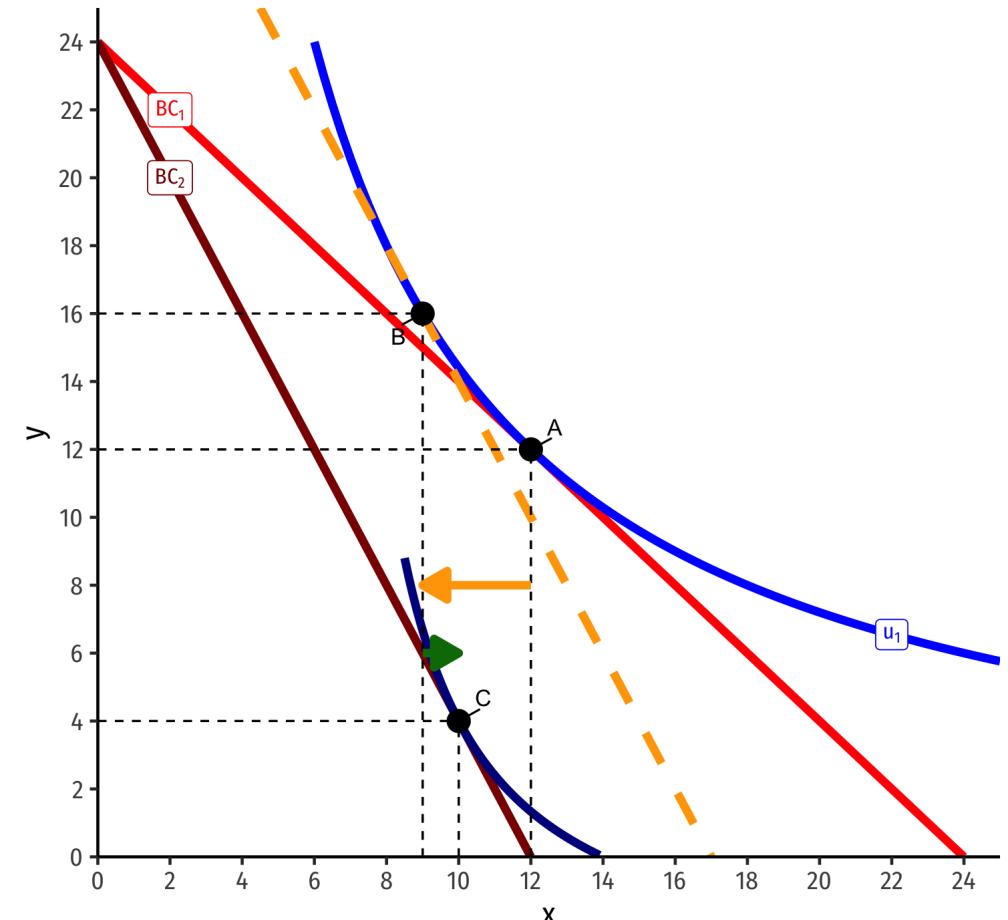


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Inferior Goods, Graphically IV

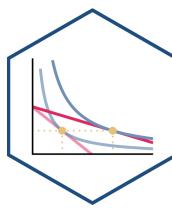


- Original optimal consumption (A)
- Price of x rises, new optimal consumption at (C)
- **Substitution effect:** $A \rightarrow B$ on same I.C. (\downarrow more expensive x and $\uparrow y$)
- **(Real) income effect:** $B \rightarrow C$ to new budget constraint (can buy less x and/or y)

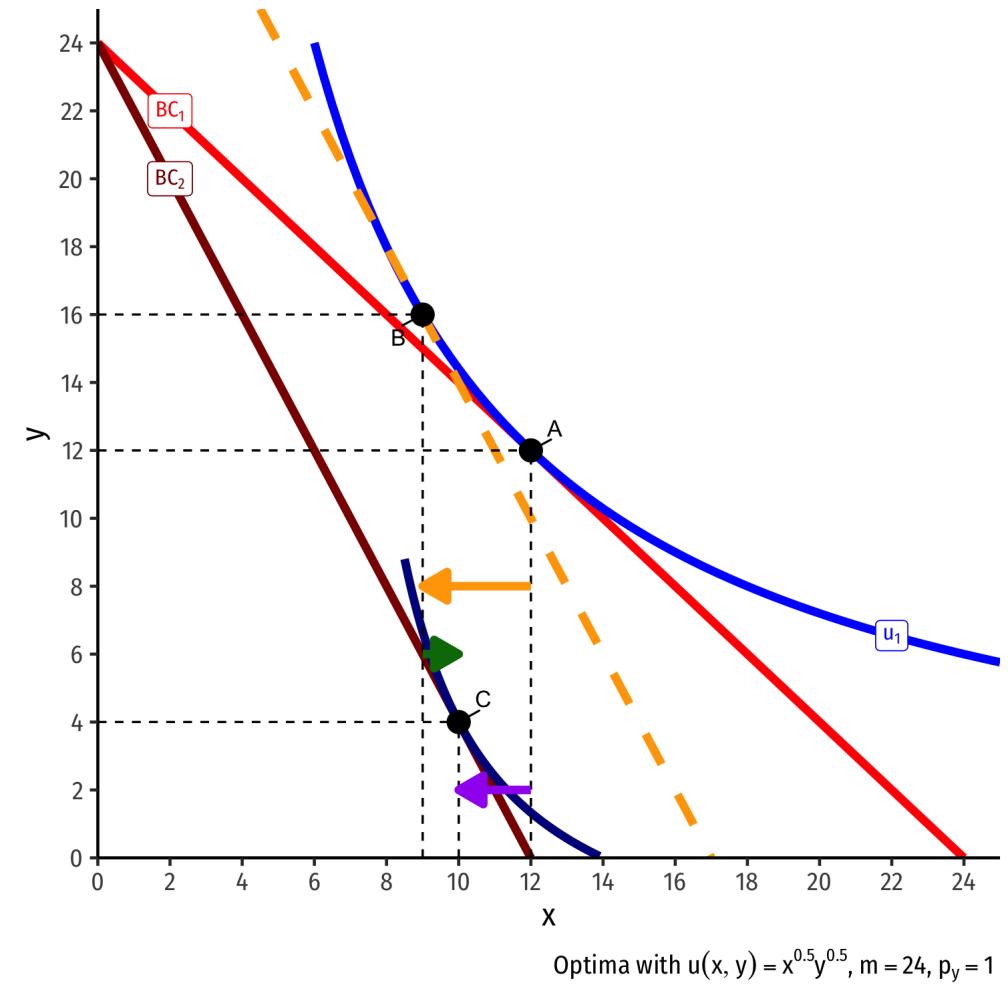


Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 24$, $p_y = 1$

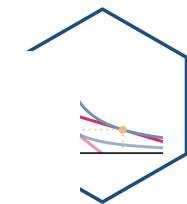
Inferior Goods, Graphically IV



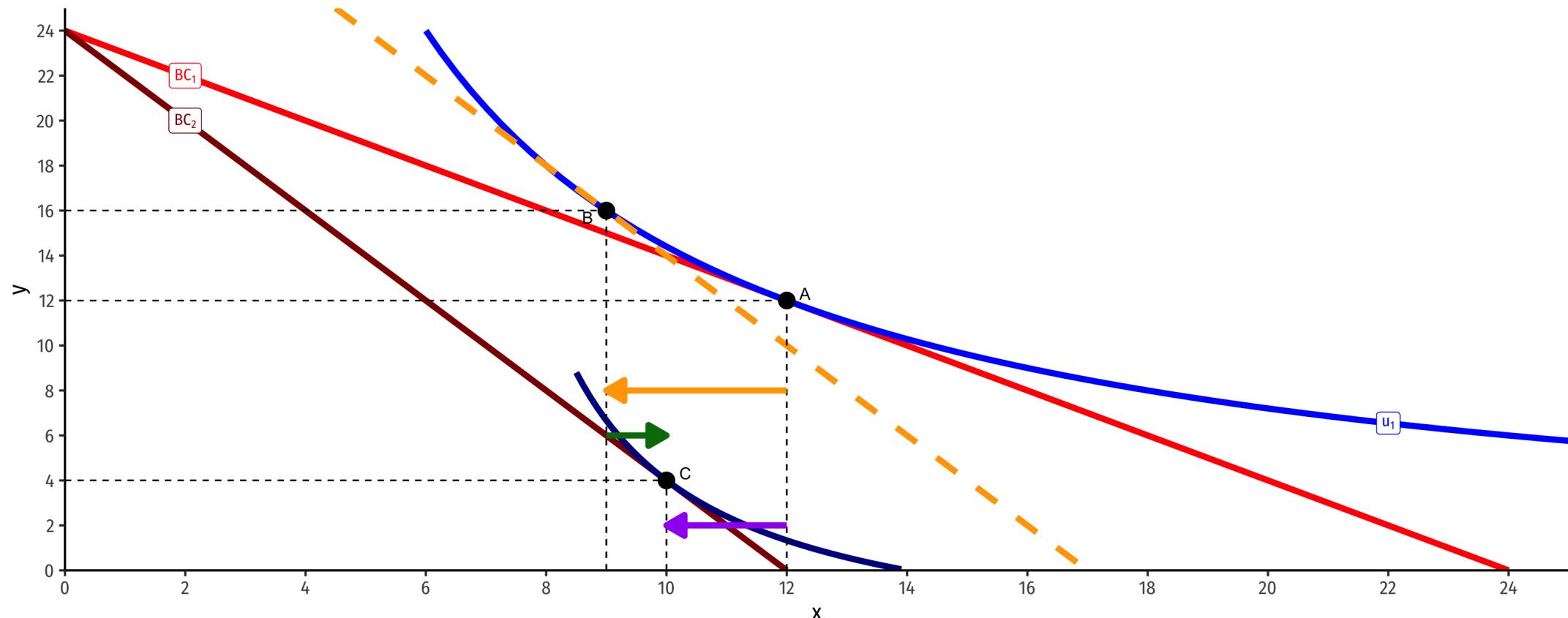
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- **(Total) price effect:** $A \rightarrow C$



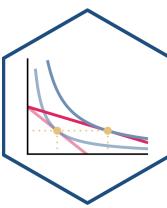
Change in Consumption From an Increase in Price



Inferior Good

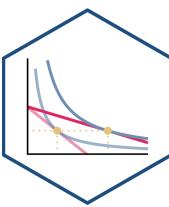


Violating the Law of Demand



Example: What would it take to violate the law of demand?

A Giffen Good



- **Giffen good:** theoretical good that violates law of demand

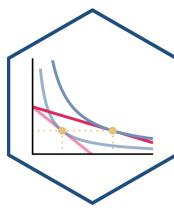
(negative) real income effect > substitution effect

1. Few substitutes (small substitution effect)
2. An inferior good (negative real income effect)
3. A large portion of income spent on it (large real income effect)

- Price increase (decrease) causes person to buy *more* (less)

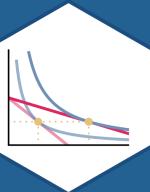


Recap: Real Income and Substitution Effects



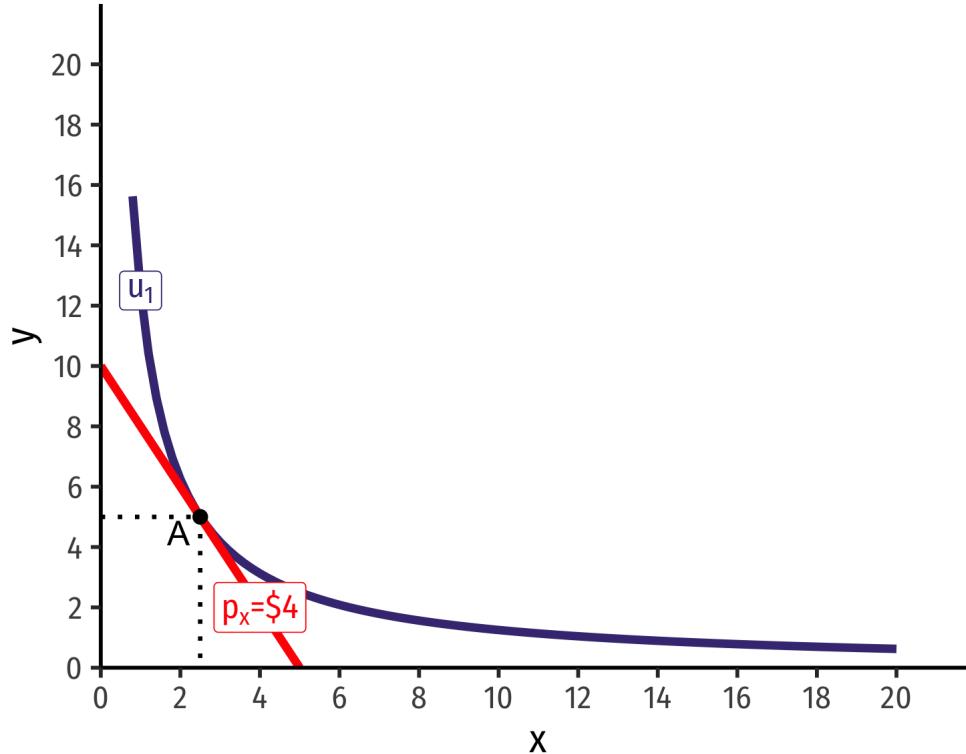
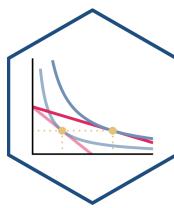
Price Effect = **Real income effect** + **Substitution Effect**

- **Substitution effect**: is always in the direction of the cheaper good
- **Real Income effect**: can be positive (normal) or negative (inferior)
- **Law of Demand**/Demand curves slope downwards (**Price effect**) mostly because of the substitution effect
 - Even (inferior) goods with negative real income effects overpowered by substitution effect
- Theoretical **Giffen good** exception: **negative R.I.E. > S.E.**

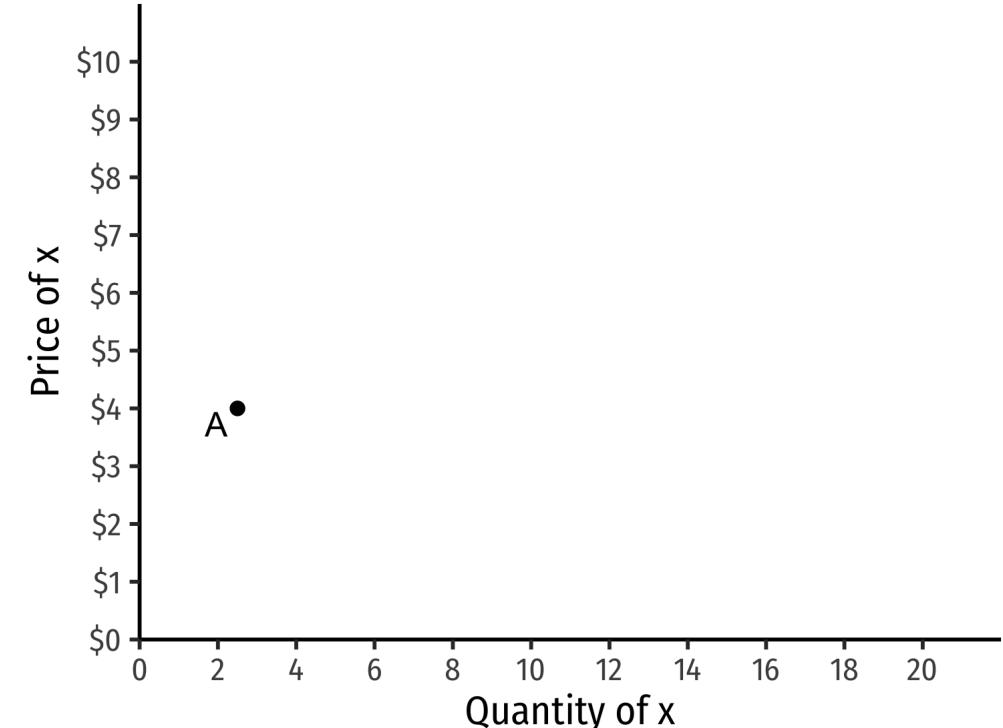


On To Demand Curves

Deriving a Demand Curve Graphically



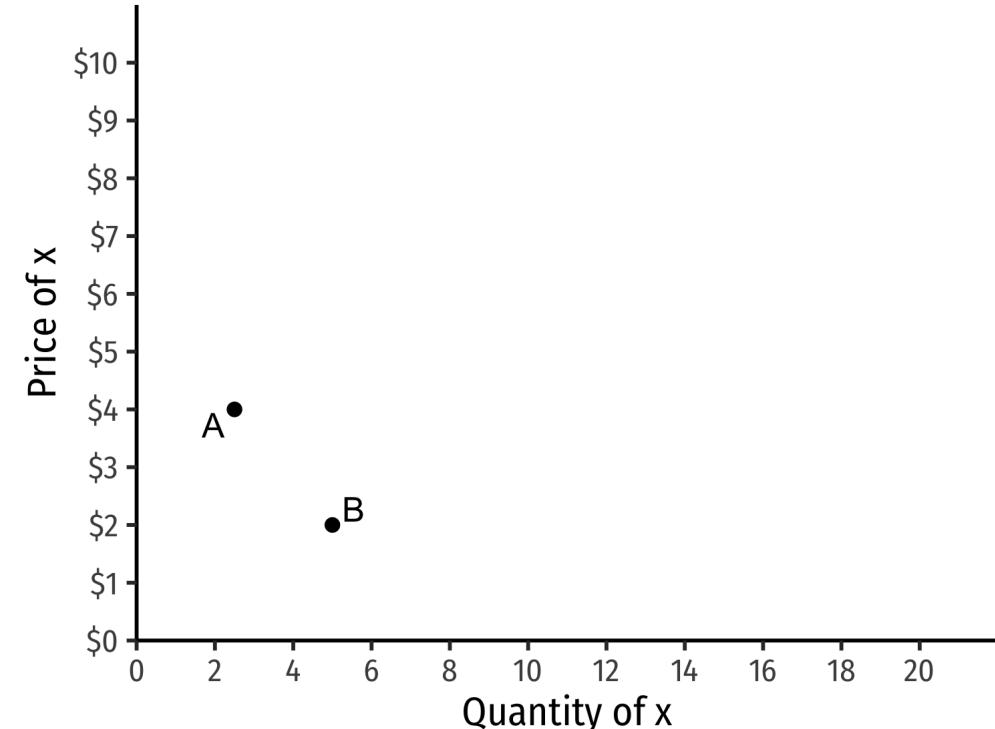
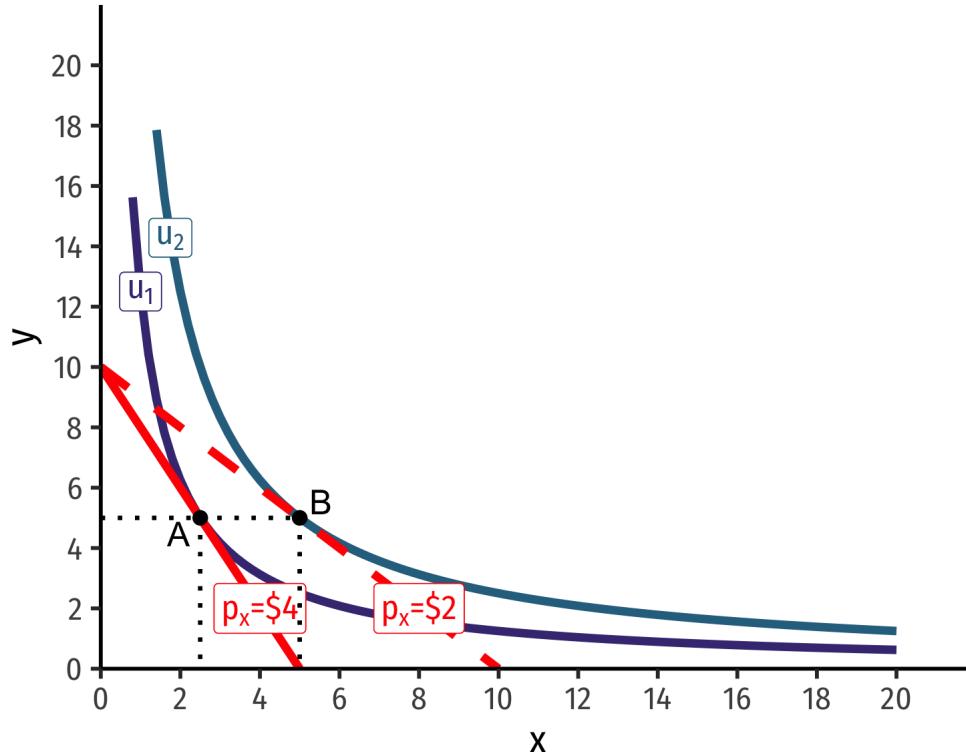
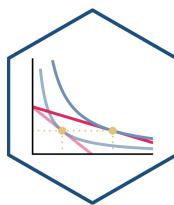
Optima with $u(x, y) = x^{0.5}y^{0.5}$, $m = 20$, $p_y = 2$



Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x

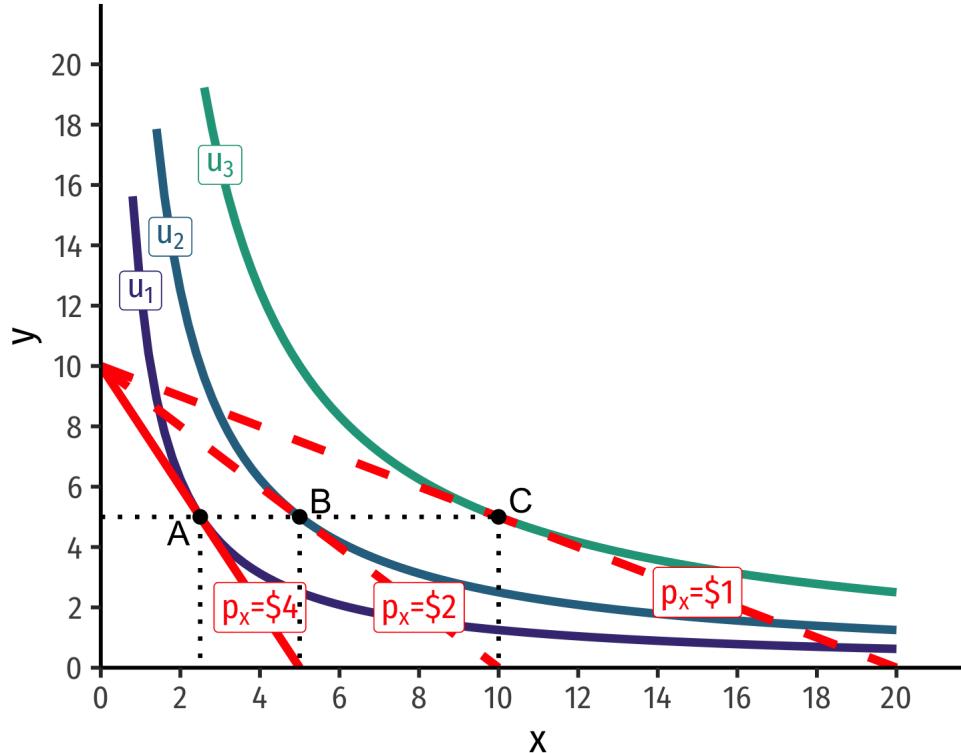
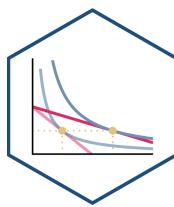
Deriving a Demand Curve Graphically



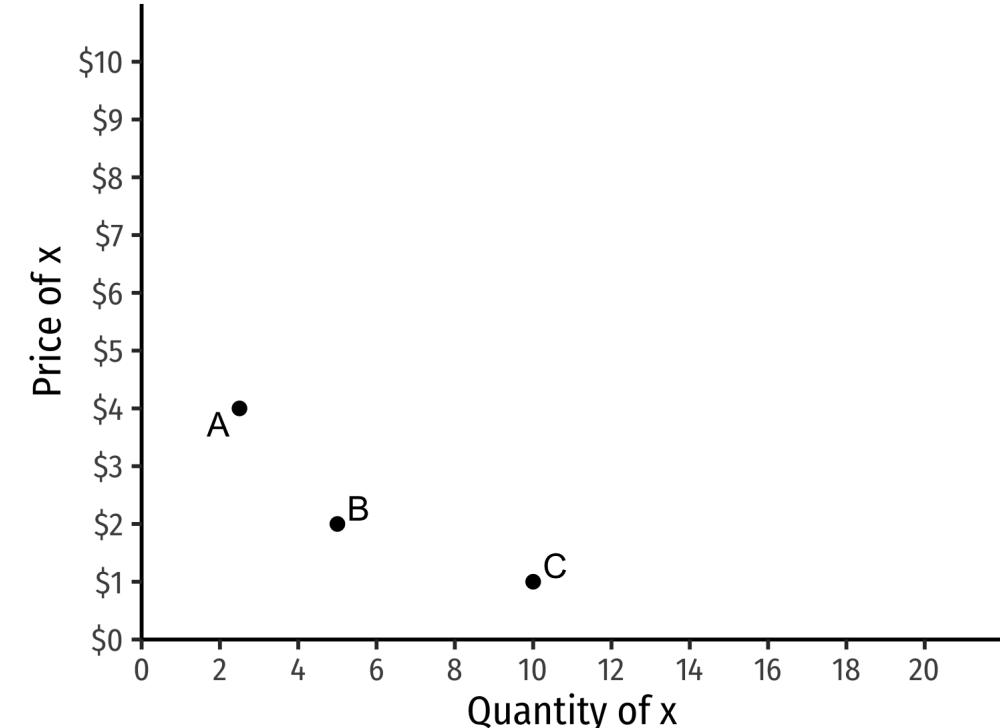
Demand function: $\frac{m}{2p}$; Inverse Demand function: $p = \frac{m}{2q}$

- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x ; at $p_x = 2$, consumer buys 5 x

Deriving a Demand Curve Graphically



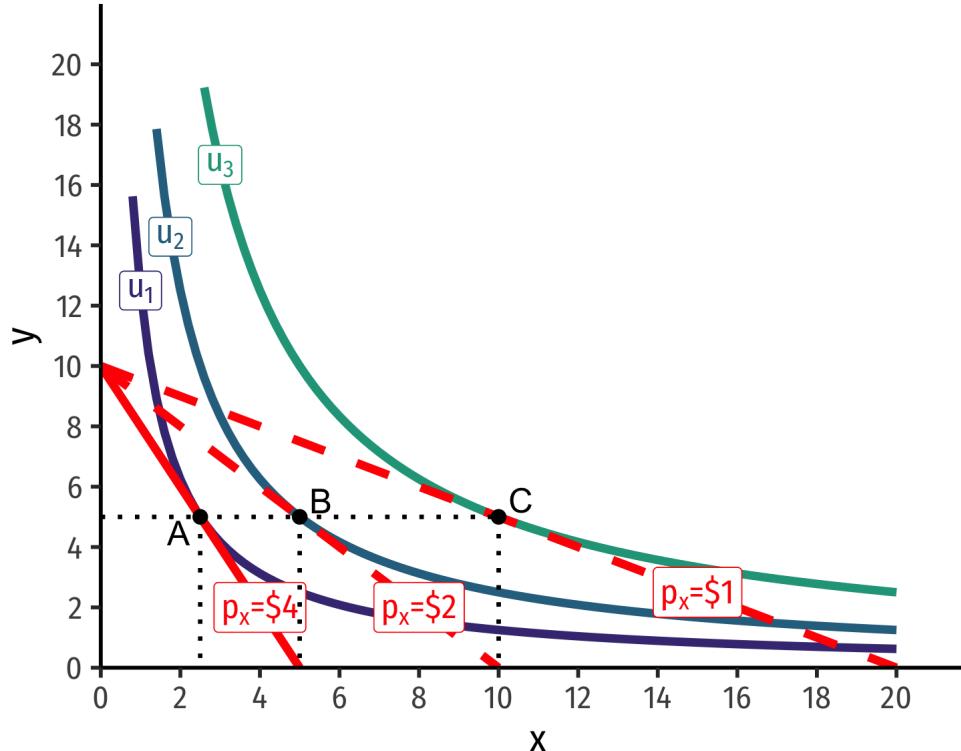
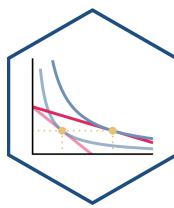
$$\text{Optima with } u(x, y) = x^{0.5}y^{0.5}, m = 20, p_y = 2$$



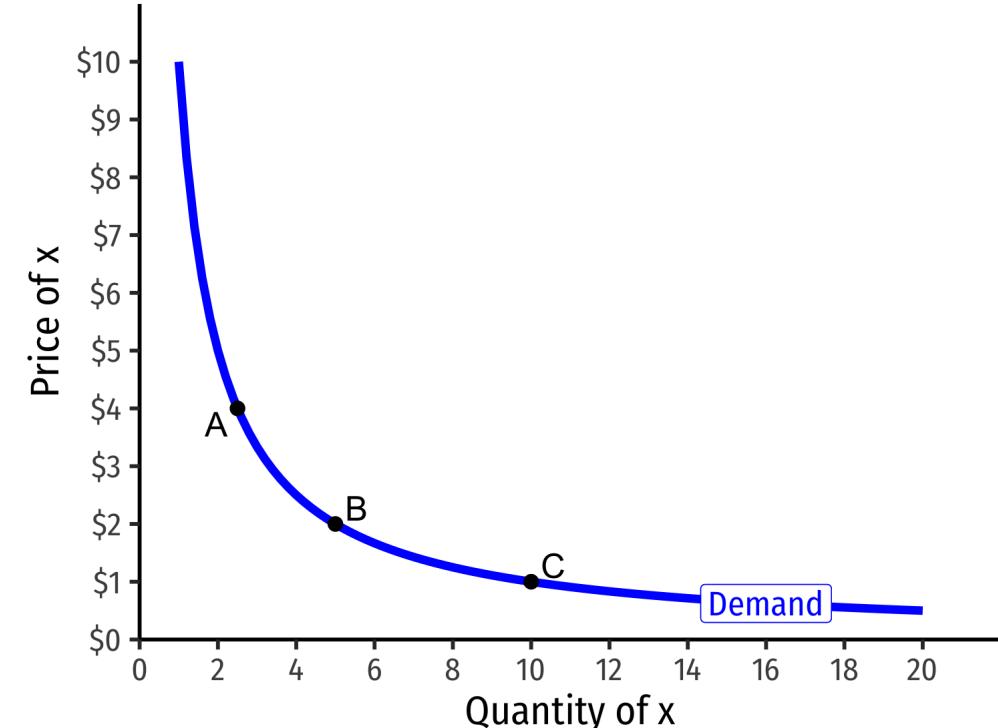
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- Demand curve for x relates optimal consumption of x ("quantity") as price of x changes
- At $p_x = 4$, consumer buys 2 x ; at $p_x = 2$, consumer buys 5 x ; at $p_x = 1$, consumer buys 10 x

Deriving a Demand Curve Graphically



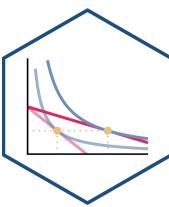
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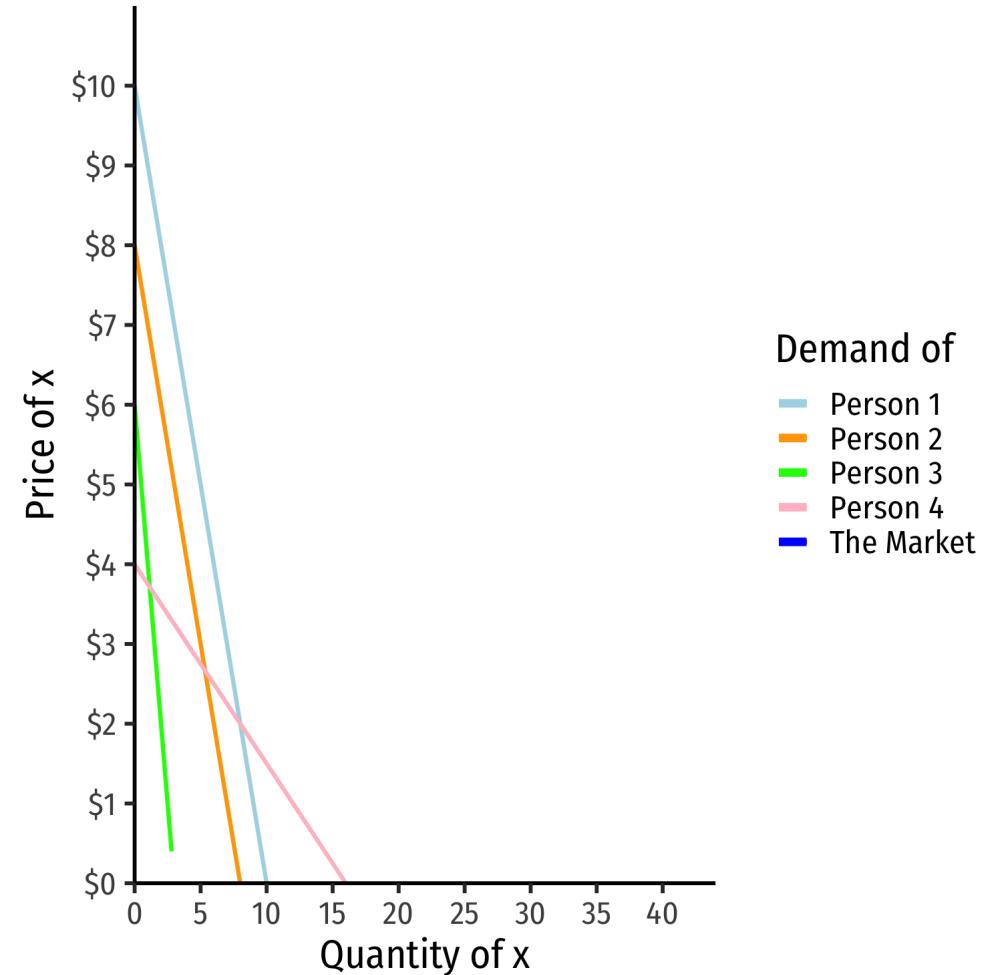
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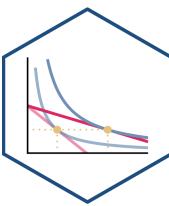
From Individual Demand to Market Demand



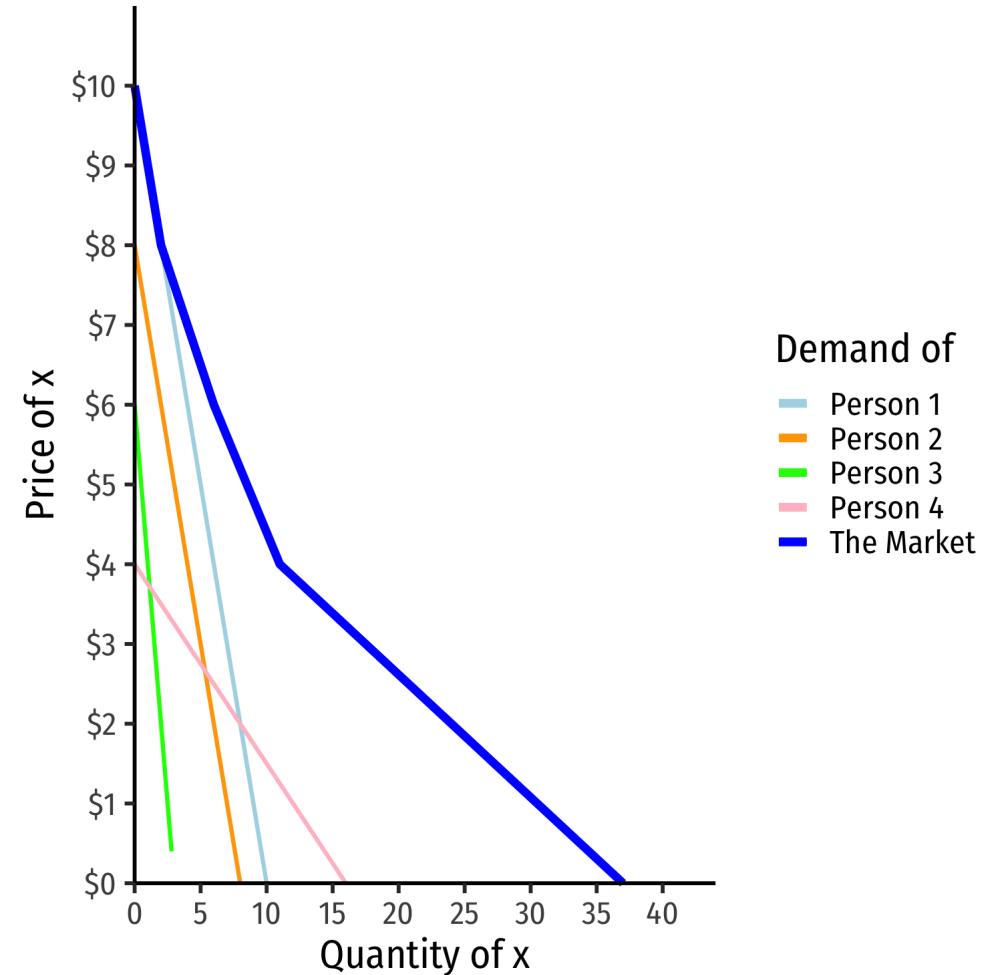
- Note so far we have been talking about *an individual person's demand*
- In principles, you learned about the entire **market demand**



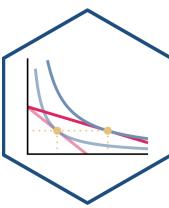
From Individual Demand to Market Demand



- Note so far we have been talking about *an individual person's demand*
- In principles, you learned about the entire **market demand**
- This is simply the sum of all individuals' demands



Demand Schedule (For Individual Or Market)



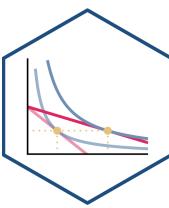
- **Demand schedule** expresses the quantity of good a person(s) would be willing to buy (q_D) at any given price (p_x)

- Holding constant all other prices (p_y) and income (m)! (“**ceterus paribus**”)

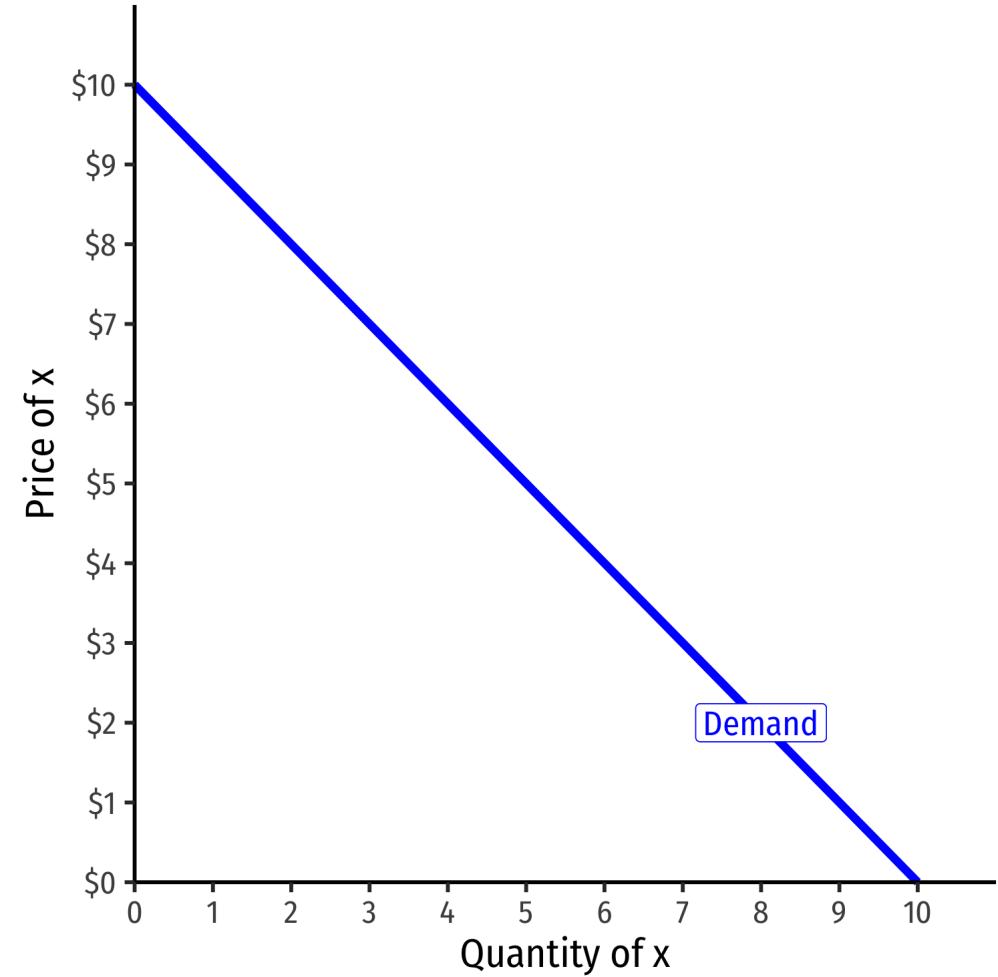
- Note: each of these is a consumer's optimum at a given price!

price	quantity
10	0
9	1
8	2
7	3
6	4
5	5
4	6
3	7
2	8
1	9
0	10

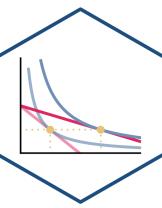
Demand Curve



- **Demand curve** graphically represents the demand schedule
- Also measures a person's **maximum willingness to pay (WTP)** for a given quantity
- **Law of Demand (price effect)** \implies demand curves always slope downwards



Demand Function



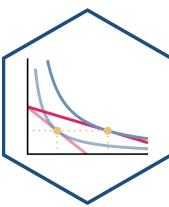
- **Demand function** relates quantity to price

Example:

$$q = 10 - p$$

- Not graphable (wrong axes)!

Inverse Demand Function

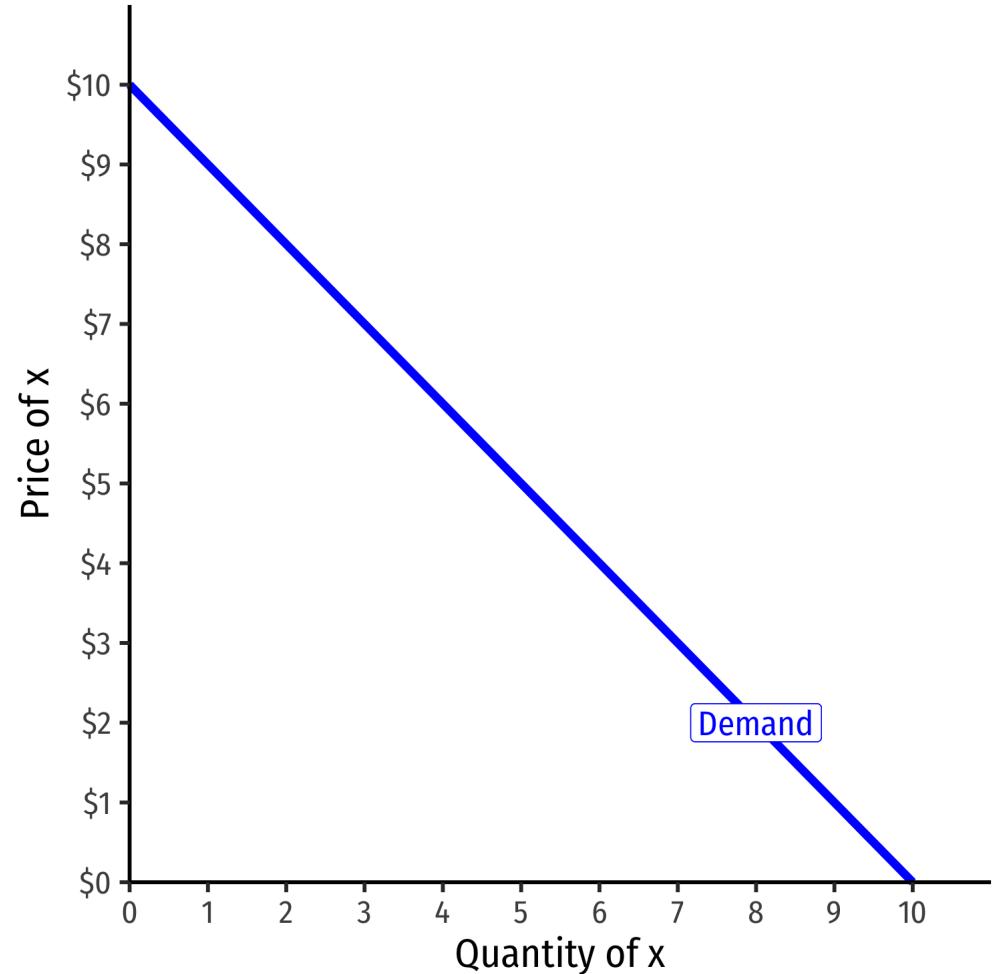


- ***Inverse demand function*** relates price to quantity
 - Take demand function and solve for p

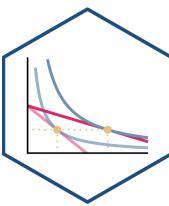
Example:

$$p = 10 - q$$

- Graphable (price on vertical axis)!



Inverse Demand Function

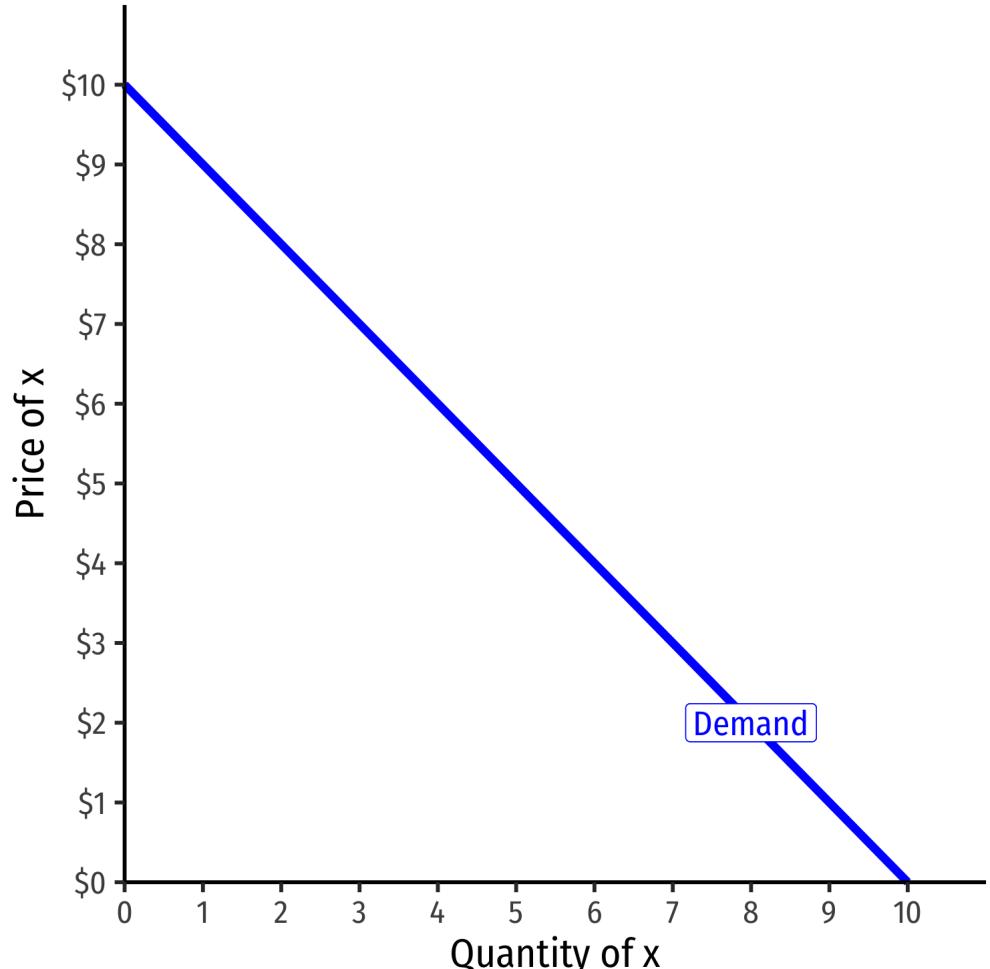


- ***Inverse demand function*** relates price to quantity
 - Take demand function and solve for p

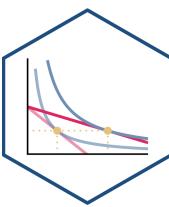
Example:

$$p = 10 - q$$

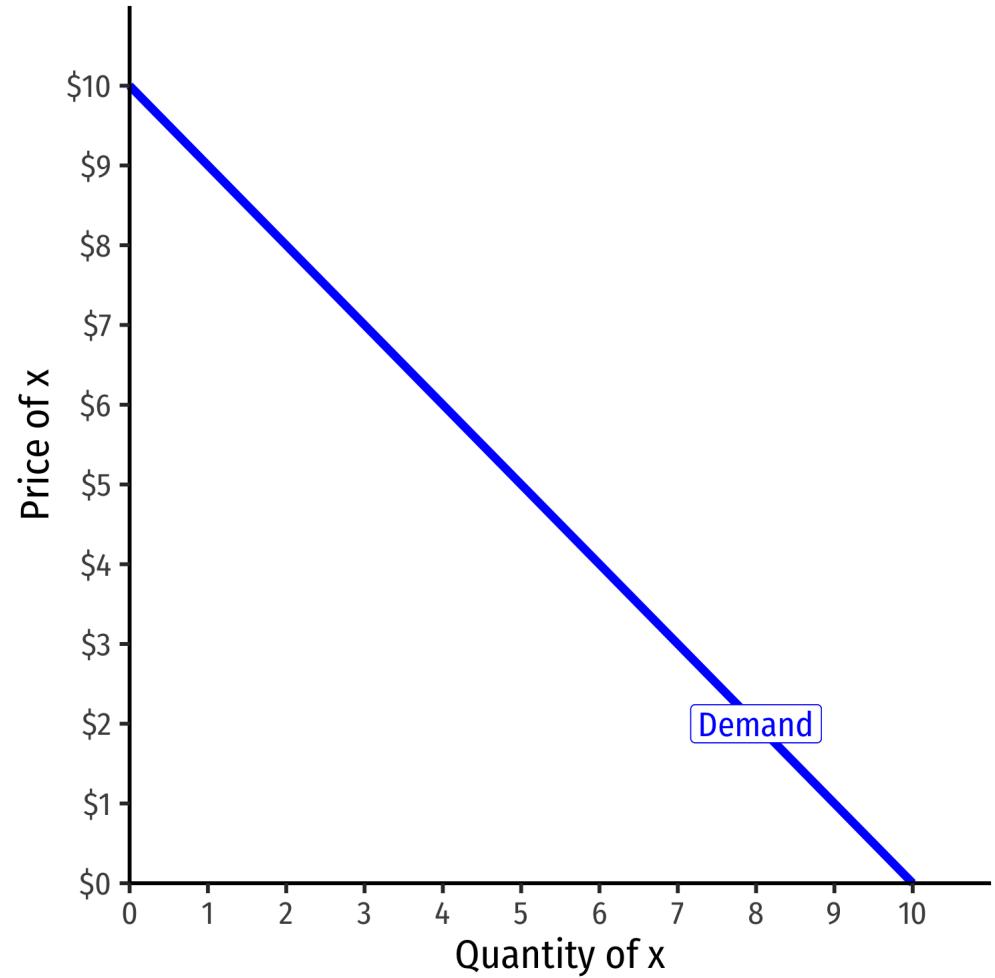
- Vertical intercept ("**Choke price**"): price where $q_D = 0$ (\$10), just high enough to discourage *any* purchases



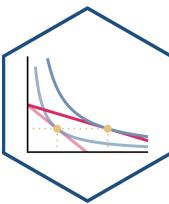
Inverse Demand Function



- Read two ways:
- Horizontally: at any given price, how many units person wants to buy
- Vertically: at any given quantity, the **maximum willingness to pay (WTP)** for that quantity
 - This way will be very useful later



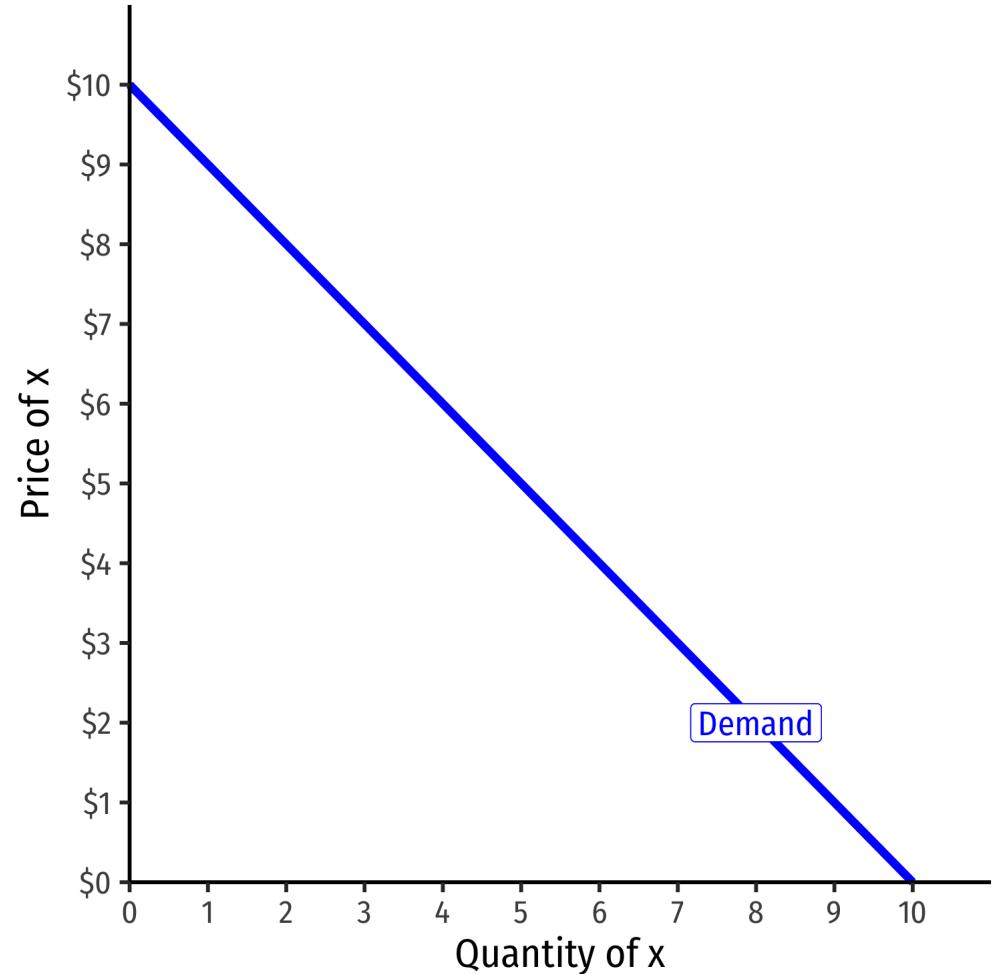
Shifts in Demand I



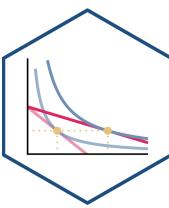
- Note a simple (inverse) demand function only relates (own) **price** and **quantity**

Example: $q = 10 - p$ or
 $p = 10 - q$

- What about all the other "**determinants of demand**" like income and other prices?
- They are captured in the vertical intercept (choke price)!



Shifts in Demand II



- A change in one of the "**determinants of demand**" will **shift** demand curve!
 1. Change in **income** m
 2. Change in **price of other goods** p_y
 3. Change in **preferences** or **expectations** about good x
- Shows up in (inverse) demand function by a **change in intercept (choke price)**!
- See my [Visualizing Demand Shifters](#)

