

1.4 – Utility Maximization

ECON 306 • Microeconomic Analysis • Spring 2023

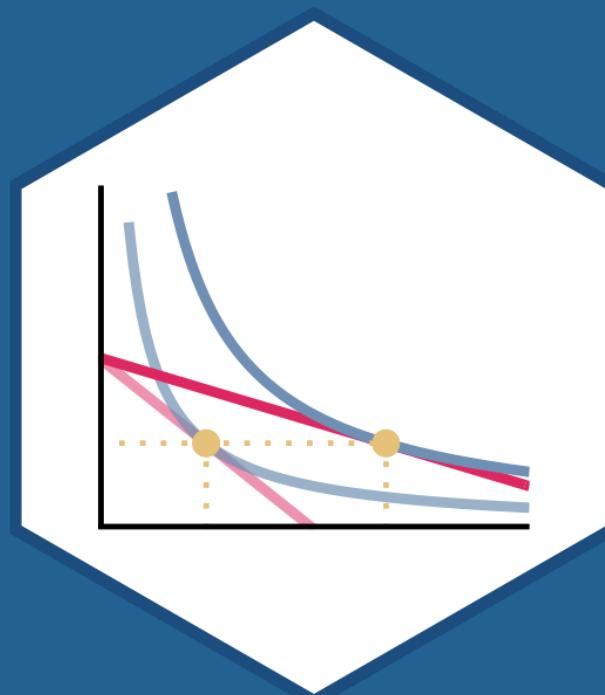
Ryan Safner

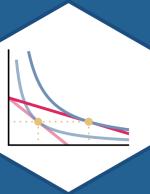
Associate Professor of Economics

 safner@hood.edu

 [ryansafner/microS23](https://github.com/ryansafner/microS23)

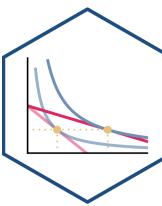
 microS23.classes.ryansafner.com





Constrained Optimization

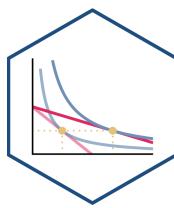
Constrained Optimization I



- We model most situations as a **constrained optimization problem**:
- People **optimize**: make tradeoffs to achieve their **objective** *as best as they can*
- Subject to **constraints**: limited resources (income, time, attention, etc)



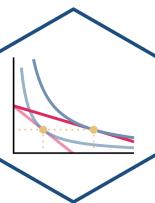
Constrained Optimization II



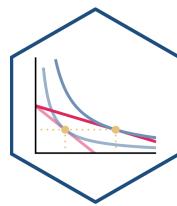
- One of the most generally useful mathematical models
- *Endless applications:* how we model nearly every decision-maker
 - consumer, business firm, politician, judge, bureaucrat, voter, dictator, pirate, drug cartel, drug addict, parent, child, etc
- **Key economic skill: recognizing how to apply the model to a situation**



Remember!



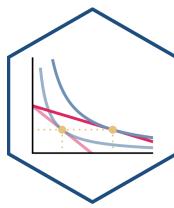
Constrained Optimization III



- All constrained optimization models have three moving parts:



Constrained Optimization III

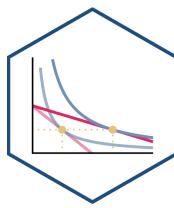


- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >



Constrained Optimization III

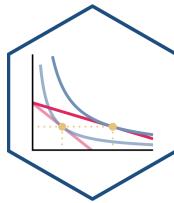


- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >
2. **In order to maximize:** < some objective >



Constrained Optimization III

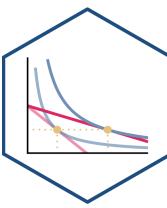


- All constrained optimization models have three moving parts:

1. **Choose:** < some alternative >
2. **In order to maximize:** < some objective >
3. **Subject to:** < some constraints >



Constrained Optimization: Example I

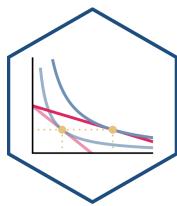


Example: A Hood student picking courses hoping to achieve the highest GPA while getting an Econ major.

- 1. Choose:**
- 2. In order to maximize:**
- 3. Subject to:**



Constrained Optimization: Example II

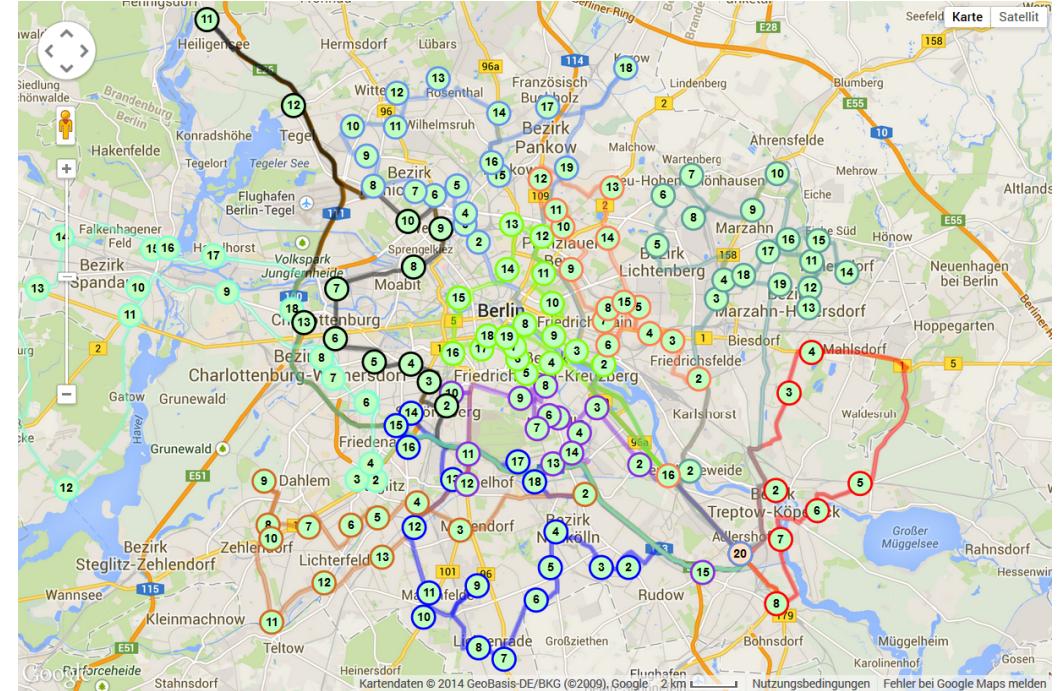


Example: How should FedEx plan its delivery route?

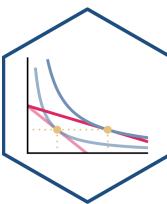
- 1. Choose:**

- 2. In order to maximize:**

- 3. Subject to:**



Constrained Optimization: Example III

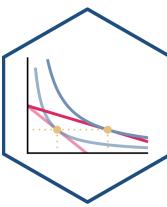


Example: The U.S. government wants to remain economically competitive but reduce emissions by 25%.

- 1. Choose:**
- 2. In order to maximize:**
- 3. Subject to:**



Constrained Optimization: Example IV

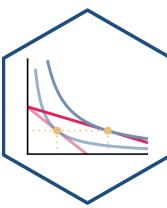


Example: How do elected officials make decisions in politics?

- 1. Choose:**
- 2. In order to maximize:**
- 3. Subject to:**



The Utility Maximization Problem

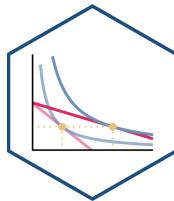


- The individual's **utility maximization problem** we've been modeling, finally, is:

1. **Choose:** < a consumption bundle >
2. **In order to maximize:** < utility >
3. **Subject to:** < income and market prices >



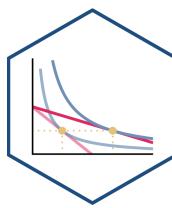
The Utility Maximization Problem: Tools



- We now have the tools to understand individual choices:
- **Budget constraint**: individual's **constraints** of income and market prices
 - How **market** trades off between goods
 - **Marginal cost** (of good x , in terms of y)
- **Utility function**: individual's **objective** to maximize, based on their preferences
 - How **individual** trades off between goods
 - **Marginal benefit** (of good x , in terms of y)



The Utility Maximization Problem: Verbally

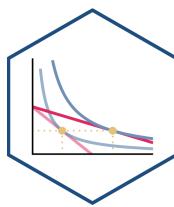


- The **individual's constrained optimization problem**:

choose a bundle of goods to maximize utility, subject to income and market prices



The Utility Maximization Problem: Mathematically



$$\max_{x,y \geq 0} u(x, y)$$

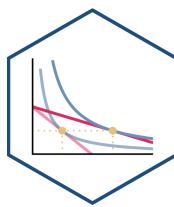
$$s.t. p_x x + p_y y = m$$

- This requires calculus to solve.[†] We will look at **graphs** instead!

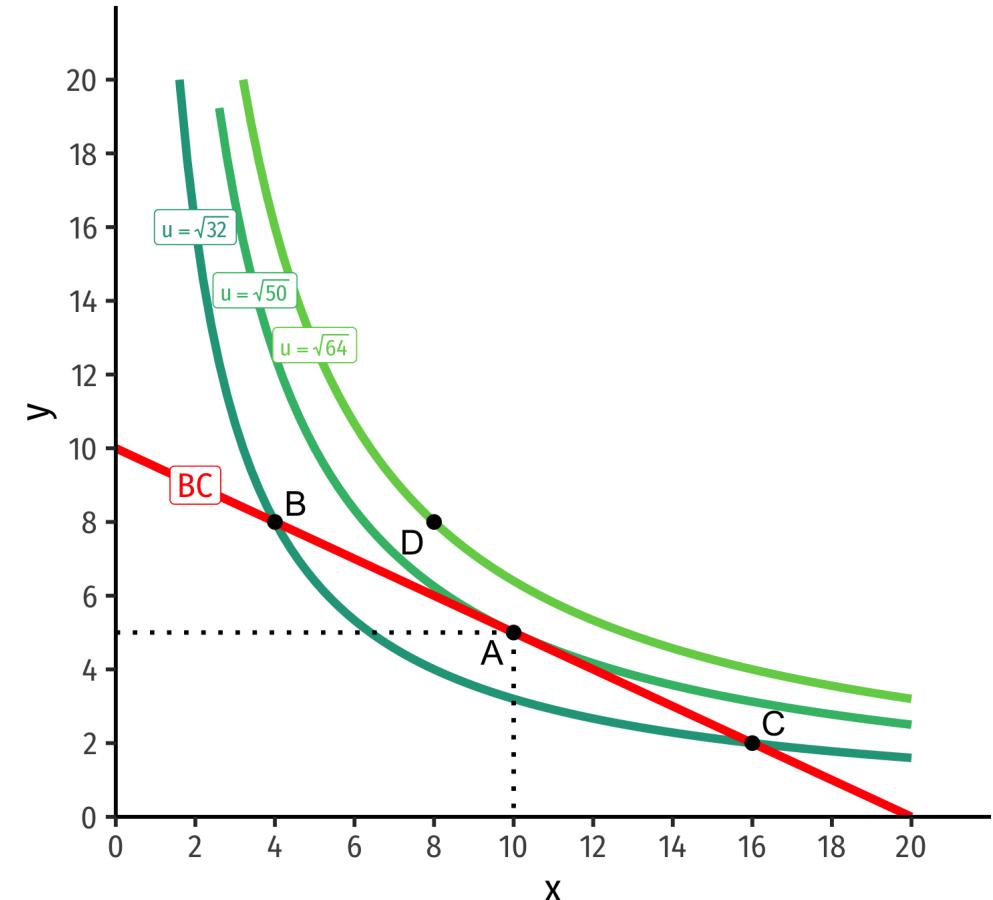


[†] See the [mathematical appendix](#) in today's class notes on how to solve it with calculus, and an example.

The Individual's Optimum: Graphically

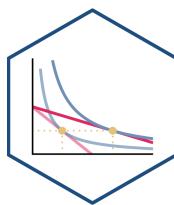


- **Graphical solution: Highest indifference curve tangent to budget constraint**
 - Bundle A!

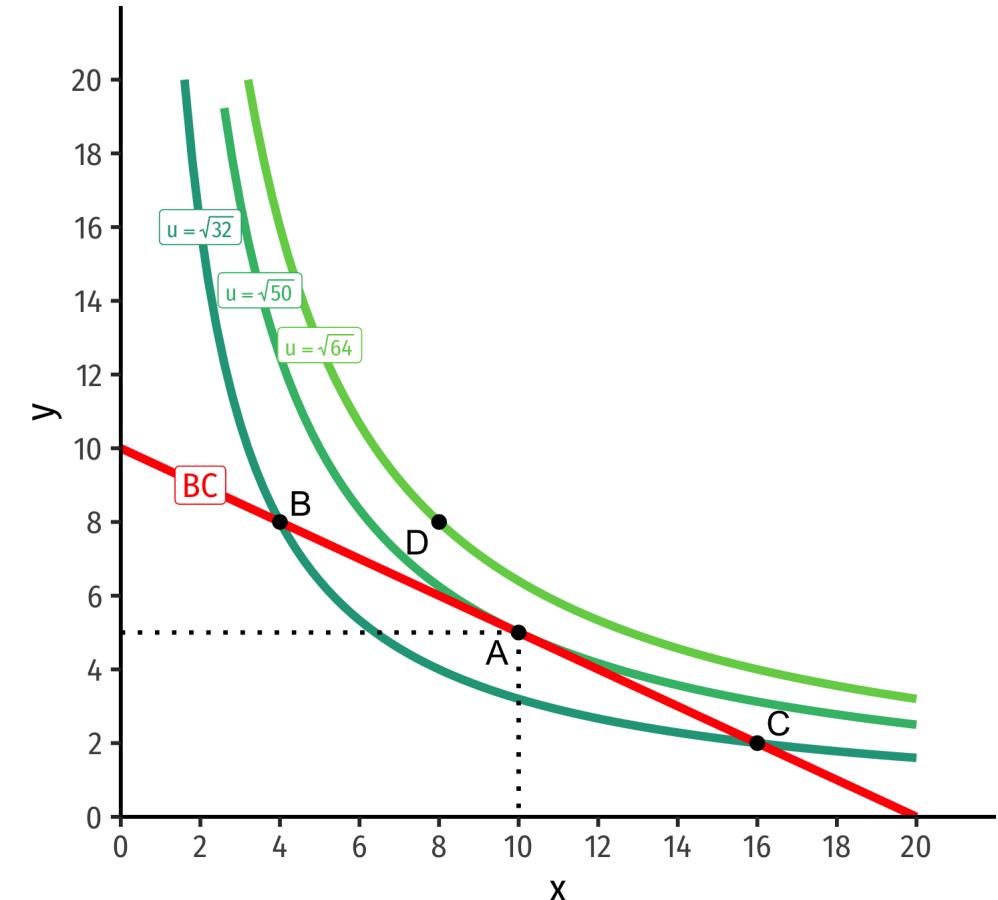


$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Graphically

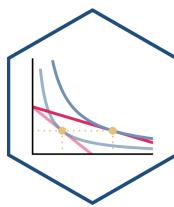


- **Graphical solution: Highest indifference curve tangent to budget constraint**
 - Bundle A!
- B or C spend all income, but a better combination exists

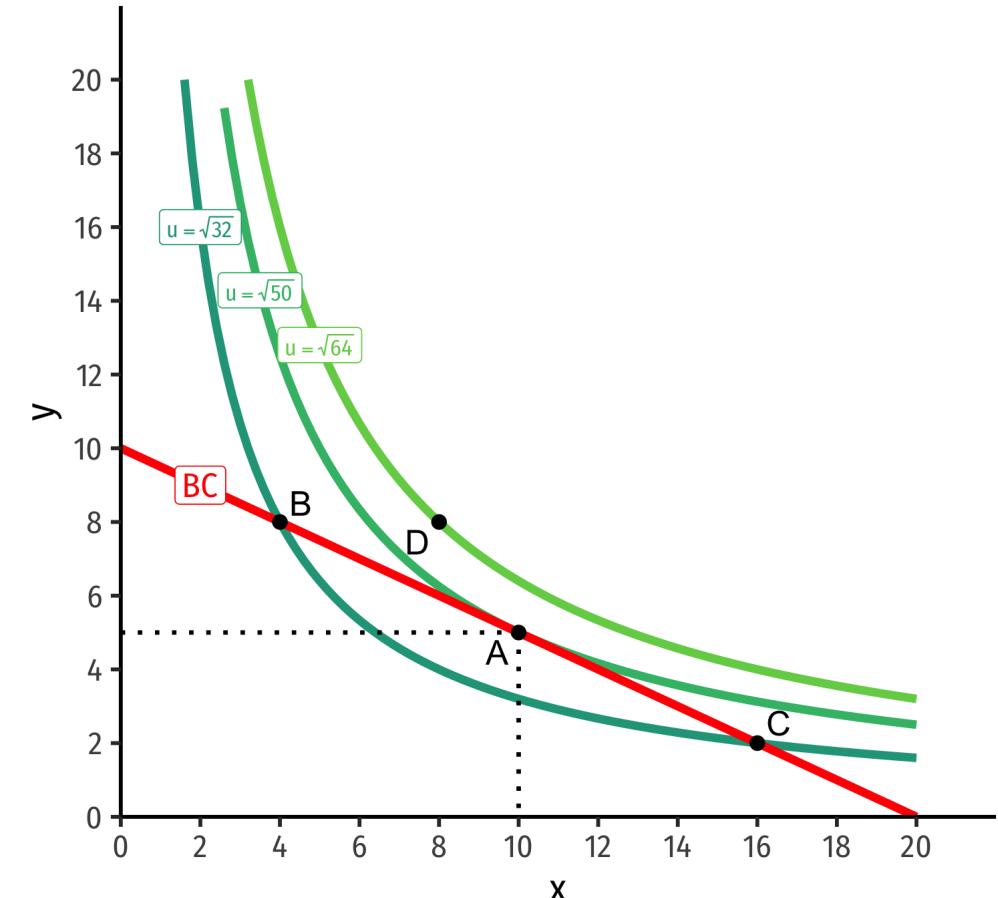


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The Individual's Optimum: Graphically

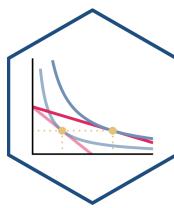


- **Graphical solution: Highest indifference curve tangent to budget constraint**
 - Bundle A!
- B or C spend all income, but a better combination exists
- D is higher utility, but *not affordable* at current income & prices

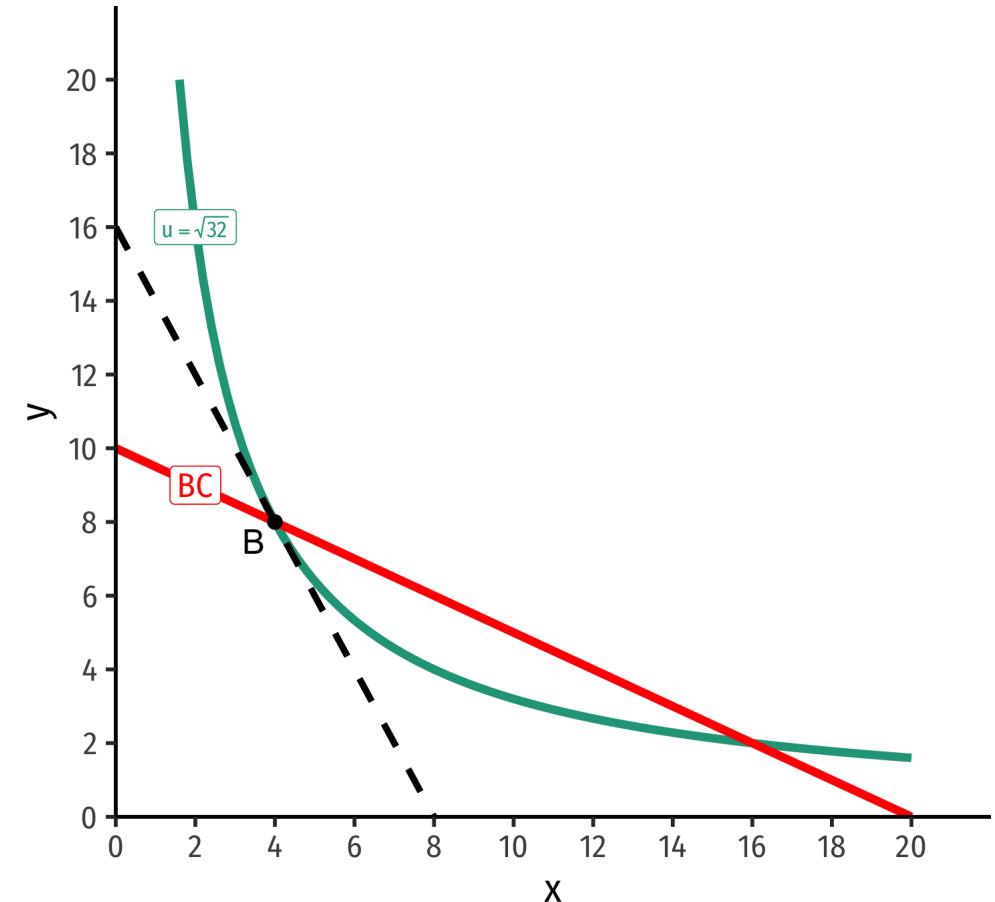


$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not B?

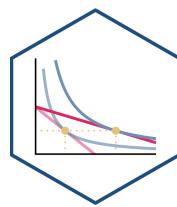


indiff. curve slope > budget constr. slope



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not B?

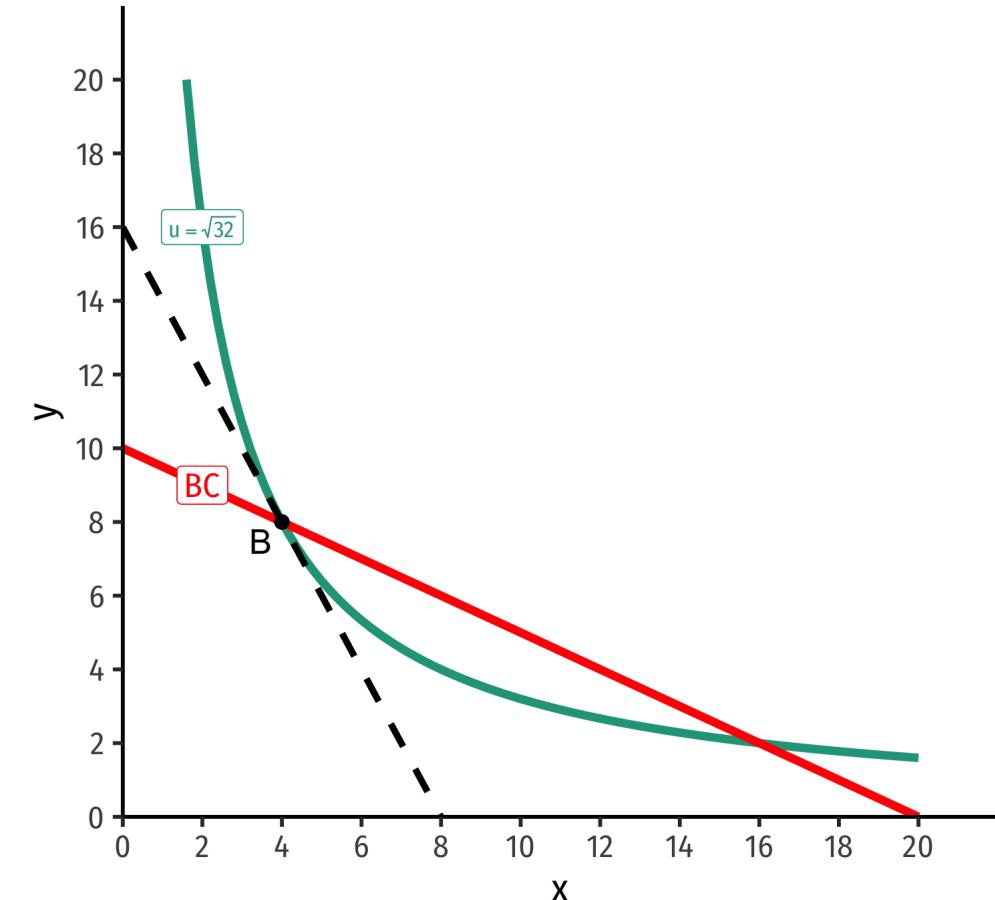


indiff. curve slope > budget constr. slope

$$\frac{MU_x}{MU_y} > \frac{p_x}{p_y}$$

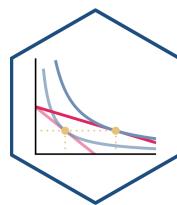
$$2 > 0.5$$

- **Consumer** views MB of x is 2 units of y
 - Consumer's "exchange rate:" **2Y:1X**
- **Market**-determined MC of x is 0.5 units of y
 - Market exchange rate is **0.5Y:1X**



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not B?

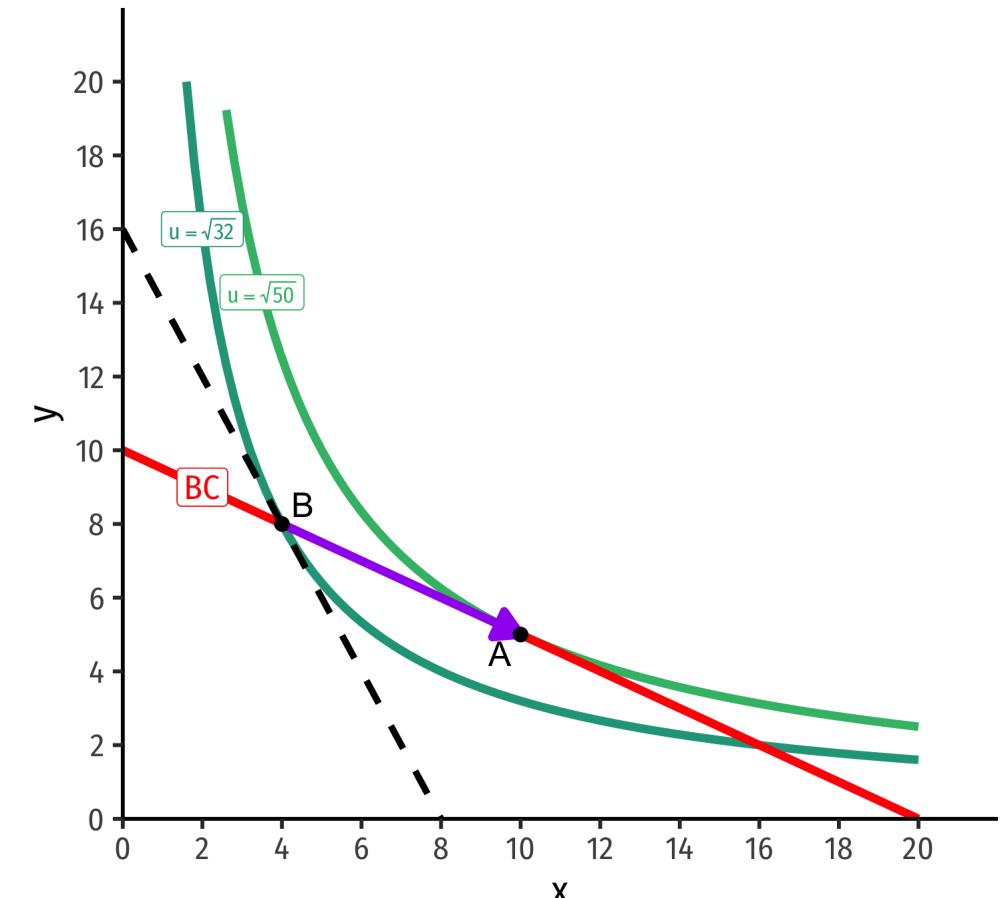


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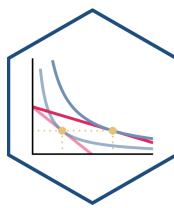
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 - Market exchange rate is **0.5Y:1X**
- Can **spend less on y, more on x for more utility!**

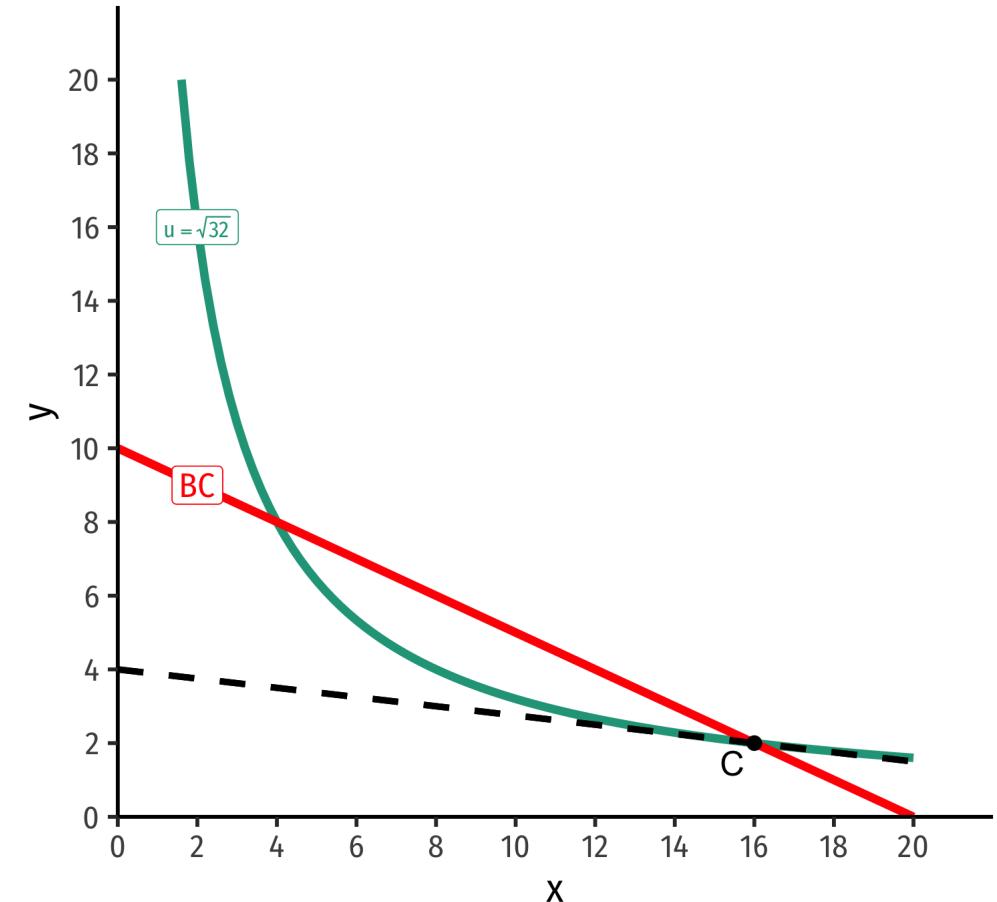


$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not C?

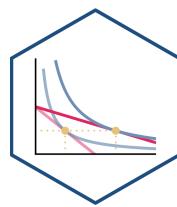


indiff. curve slope < budget constr. slope



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not C?

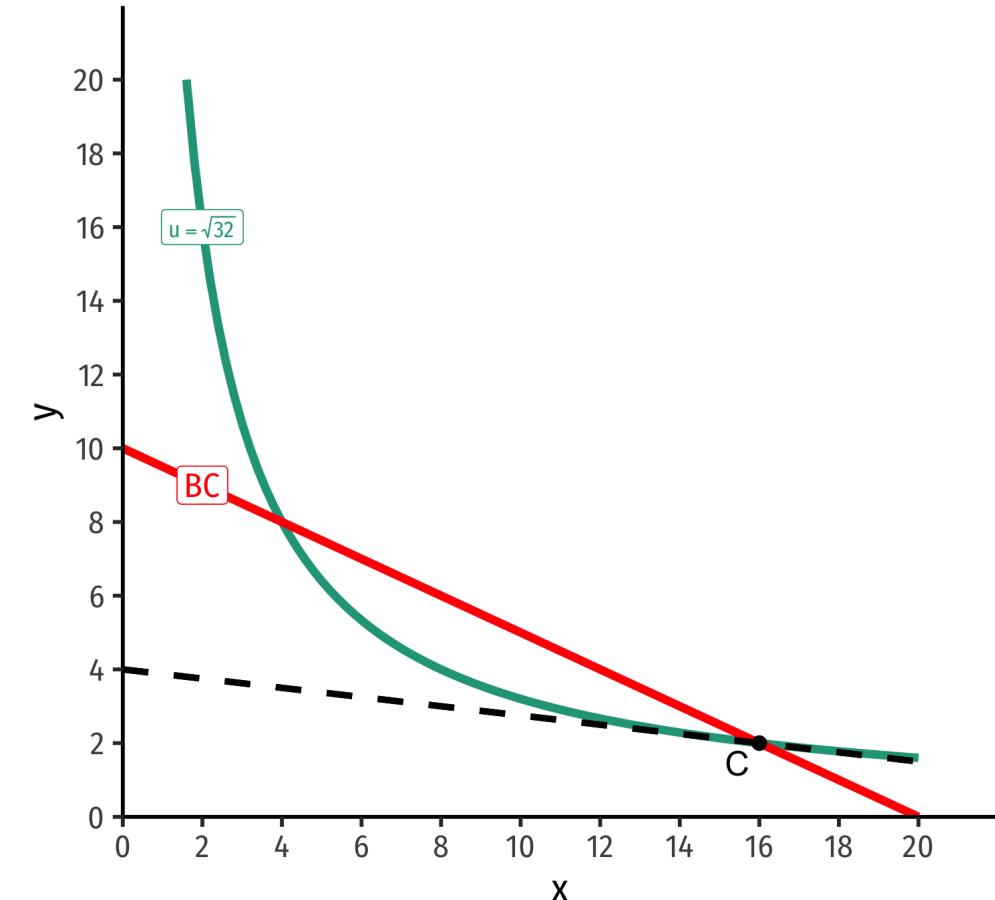


indiff. curve slope < budget constr. slope

$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$

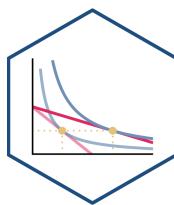
$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y
 - Consumer's "exchange rate:" **0.125Y:1X**
- **Market**-determined MC of x is 0.5 units of y
 - Market exchange rate is **0.5Y:1X**



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why Not C?

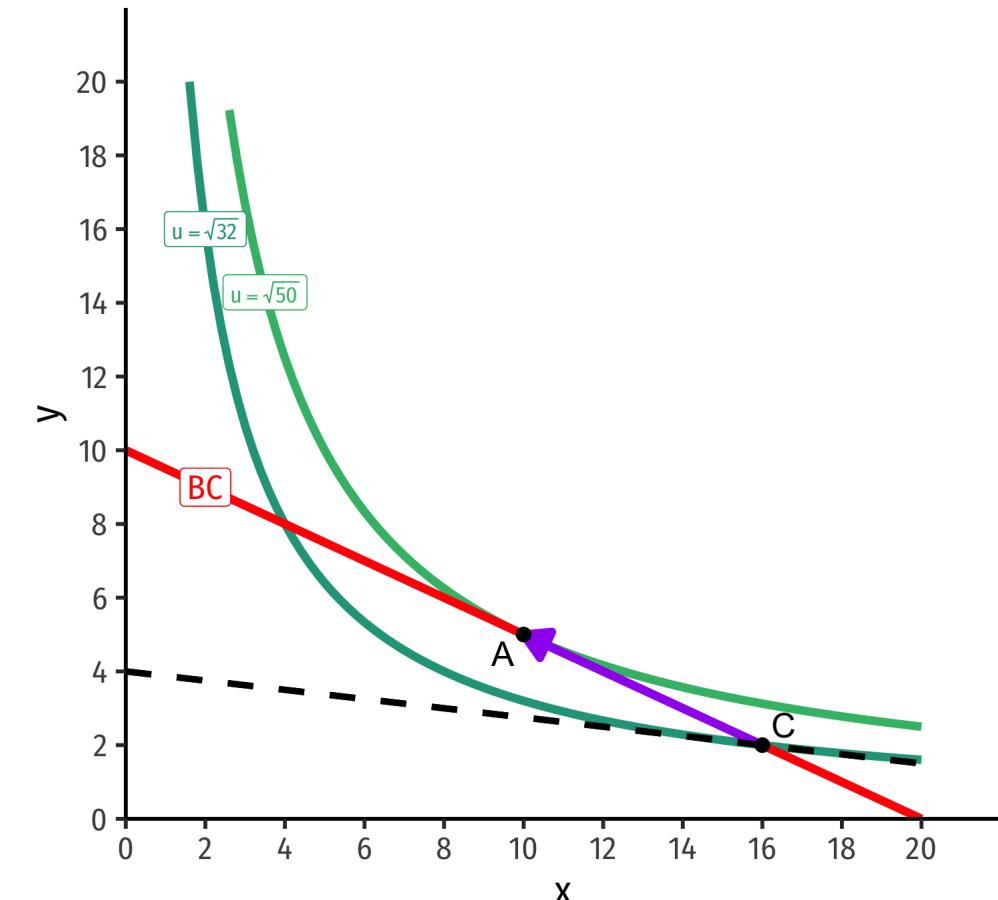


indiff. curve slope < budget constr. slope

$$\frac{MU_x}{MU_y} < \frac{p_x}{p_y}$$

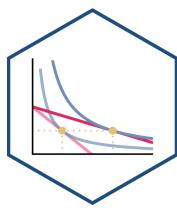
$$0.125 < 0.5$$

- **Consumer** views MB of x is 0.125 units of y
 - Consumer's "exchange rate:" **0.125Y:1X**
- **Market**-determined MC of x is 0.5 units of y
 - Market exchange rate is **0.5Y:1X**
- Can **spend less on y, more on x for more utility!**

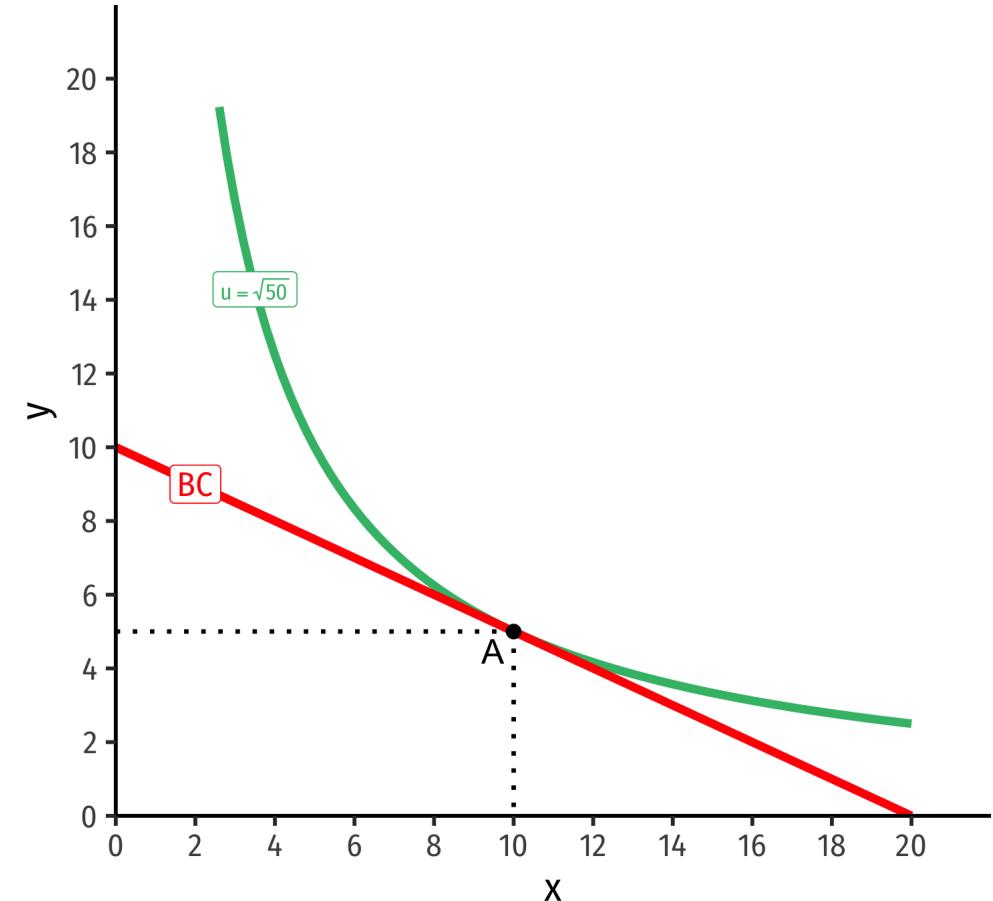


$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Why A?

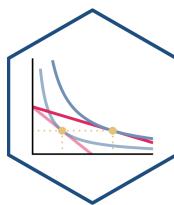


indiff. curve slope = budget constr. slope



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The Individual's Optimum: Why A?

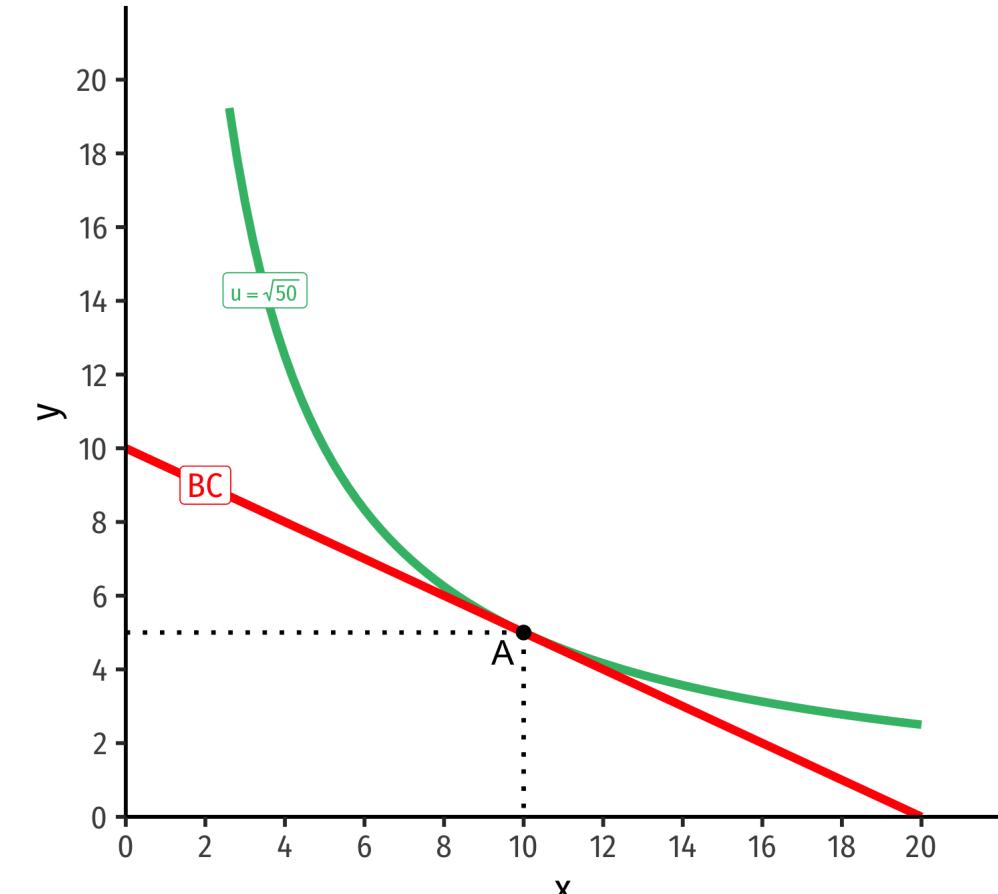


indiff. curve slope = budget constr. slope

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$0.5 = 0.5$$

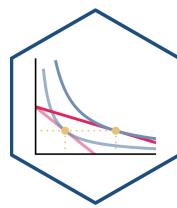
- Marginal benefit = Marginal cost
 - Consumer exchanges at same rate as market
- No other combination of (x,y) exists that could increase utility![†]



[†] At current income and market prices!

$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

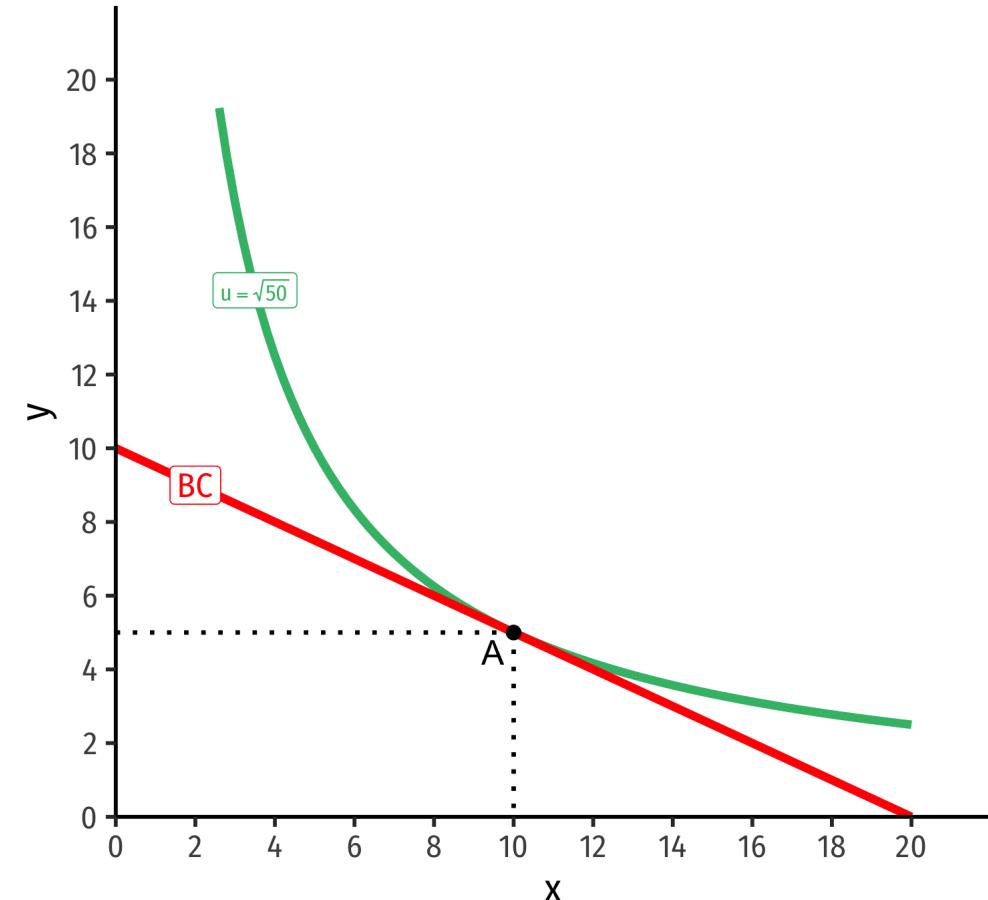
The Individual's Optimum: Two Equivalent Rules



Rule 1

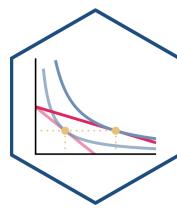
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

The Individual's Optimum: Two Equivalent Rules



Rule 1

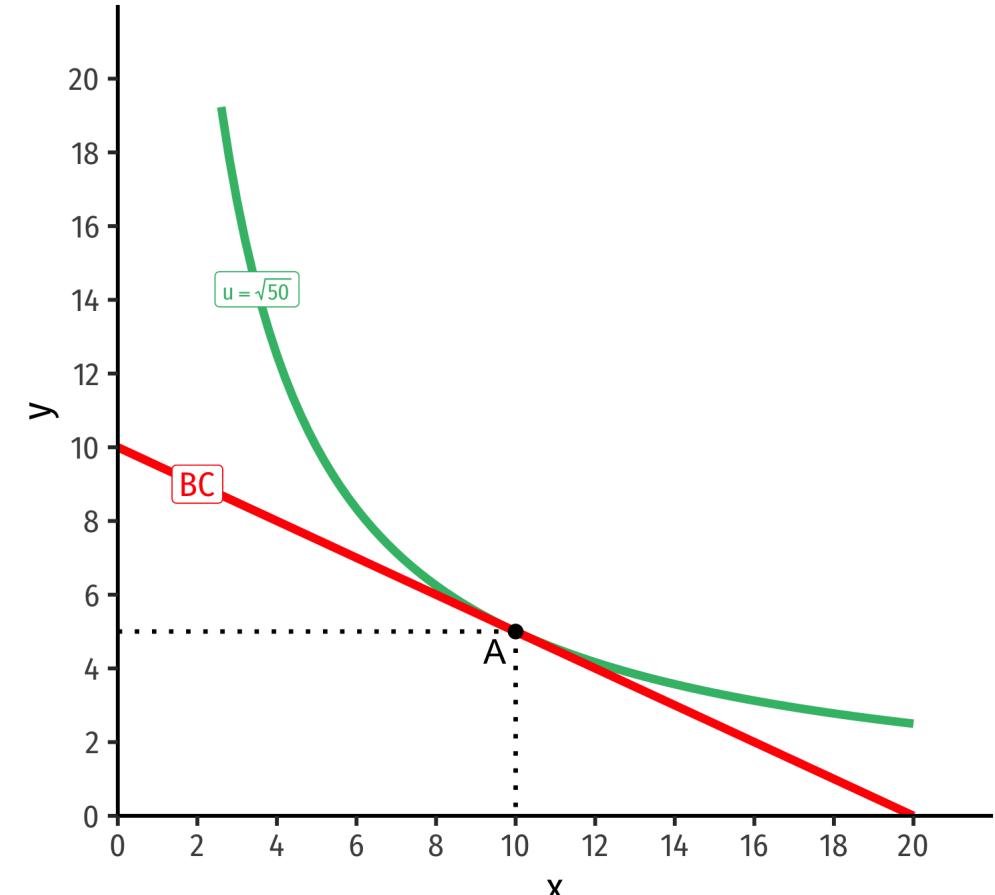
$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

- Easier for calculation (slopes)

Rule 2

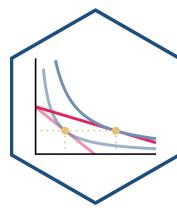
$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y}$$

- Easier for intuition (next slide)



$$u(x, y) = \sqrt{xy}, m = \$20, p_x = \$1, p_y = \$2$$

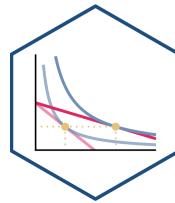
The Individual's Optimum: The Equimarginal Rule



$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} = \dots = \frac{MU_n}{p_n}$$

- **Equimarginal Rule:** consumption is optimized where the **marginal utility per dollar spent is equalized** across all n possible goods/decisions
- Always choose an option that gives higher marginal utility (e.g. if $MU_x < MU_y$), consume more y !
 - But each option has a different price, so weight each option by its price, hence $\frac{MU_x}{p_x}$

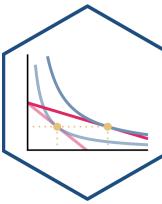
An Optimum, By Definition



- Any **optimum** in economics: no better alternatives exist under current constraints
- No possible change in your consumption that would increase your utility



Practice I



Example: You can get utility from consuming bags of Almonds (a) and bunches of Bananas (b), according to the utility function:

$$u(a, b) = ab$$

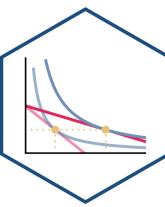
$$MU_a = b$$

$$MU_b = a$$

You have an income of \$50, the price of Almonds is \$10, and the price of Bananas is \$2. Put Almonds on the horizontal axis and Bananas on the vertical axis.

1. What is your utility-maximizing bundle of Almonds and Bananas?
2. How much utility does this provide? [Does the answer to this matter?]

Practice II, Cobb-Douglas!



Example: You can get utility from consuming Burgers (b) and Fries (f), according to the utility function:

$$u(b, f) = \sqrt{bf}$$

$$MU_b = 0.5b^{-0.5}f^{0.5}$$

$$MU_f = 0.5b^{0.5}f^{-0.5}$$

You have an income of \$20, the price of Burgers is \$5, and the price of Fries is \$2. Put Burgers on the horizontal axis and Fries on the vertical axis.

1. What is your utility-maximizing bundle of Burgers and Fries?
2. How much utility does this provide?