Permittivity and Source Charge Estimation using Force Measurements using Neural Networks

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***Question*— In Coulomb's law, force exerted on test charge is dependent on the field at the location of test charge, and medium permittivity. Source of the field (i.e. first charge) and permittivity are unknown. Only measurable parameter is force on the test charge. You have no control over the medium and first charge, however you can play with test charge. Measured force have contribution from both (i.e. first charge and permittivity). You have to propose an algorithm to segregate the contribution of permittivity and first charge on the force. This is also equivalent to mapping force data to corresponding medium permittivity and charge. Additionally, this proposed algorithm must be implemented using any machine learning tool of your choice.**

Keywords*— *Coulomb's Law, Electrostatics, Permittivity, Python Implementation

I. Coulomb's Law

Coulomb's law, or *Coulomb's inverse-square law*, is an experimental law of physics that quantifies the amount of force between two stationary, electrically charged particles. The electric force between charged bodies at rest is conventionally called electrostatic force or Coulomb force. The quantity of electrostatic force between stationary charges is always described by Coulomb's law. The law was was essential to the development of the theory of electromagnetism, maybe even its starting point, because it was now possible to discuss quantity of electric charge in a meaningful way.[1]

$$F = K\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1q_2}{r^2}$$

$$\varepsilon_0 = 8.8541878128(13)\times 10^{-12}\,F\cdot m^{-1}$$

where q_1 and q_2 are the signed magnitudes of the charges, and the scalar r is the distance between the charges. The force of the interaction between the charges is attractive if the charges have opposite signs (i.e., F is negative) and repulsive if like-signed (i.e., F is positive).

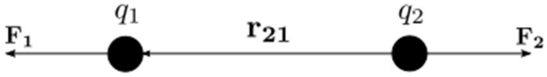


Figure 1: Direction of Coulomb Force

The constant ε_0 is the vacuum electric permittivity (also known as "electric constant") in $C^2\cdot m^{-2}\cdot N^{-1}$. It should not be confused with ε_r, which is the dimensionless relative permittivity of the material in which the charges are immersed, or with their product $\varepsilon_a = \varepsilon_0\varepsilon_r$, which is called "absolute permittivity of the material".

II. Force due to Charge inside Spherical Dielectric

In a dielectric, these charges cannot move separately from each other through any macroscopic distance, so when an electric field is applied there is no net electric current. However, it does push the positive charges a bit in its direction while the negative charges are pushed in the opposite directions. Thus, the molecules comprising the dielectric acquire dipole moments in the direction of the field. As the result of this, the bulk of the dielectric remains electrically neutral. But in a surface layer on the side of the dielectric there are only negative charges while in a similar layer on the other side there are only positive charge

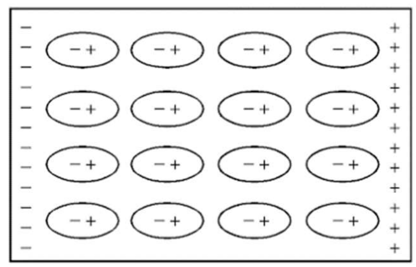


Figure 2:Dielectric Polarization due to Induced Electric Field

In a dielectric sphere with a positive charge at the centre, a negative surface charge is introduced on the inner surface and an equal positive charge is induced on the outer surface. The net effect of these induced charges cancels out in the region outside the sphere due to a corollary of the shell theorem. [2]

$$F = K\frac{1}{4\pi\epsilon_0}\cdot\frac{q_1q_2}{r^2} \quad \text{(Inside Sphere)}$$

$$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1q_2}{r^2} \quad \text{(Outside Sphere)}$$

So, effectively speaking a force measurement outside the sphere is the as if there was no dielectric sphere and ε_r=1. Inside the sphere, $\varepsilon_r = \varepsilon_{dielectric}$ as is expected inside any given dielectric.

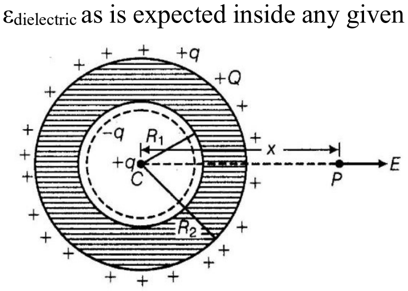


Figure 3: Dielectric Sphere with charge inside cavity

III. Problem Statement

A simpler statement of the problem is as follows: We are given the force measurements inside and outside of the dielectric sphere that is, the value of the force (F) and the respective radius(r) and test charge (q_2) values for that measurement. We have to calculate the permittivity (ε_r) and original charge(q_1) using only this force data.

Here this calculation is extremely easy. Simply calculate the charge from a single force measurement outside the sphere (where ε_r =1) and then knowing this charge, substitute it in a force measurement inside the sphere to get ε_r.

$$q_1 = \frac{F_{mesured,outside} * 4\pi\epsilon_0 * r^2}{q_2} \text{ (Known Quantities)}$$

$$\epsilon_r = \frac{F_{mesured,inside} * 4\pi\epsilon_0 * r^2}{q_2 q_1} \text{ (Known Quantities)}$$

However, instead of manually calculating these values and then manually substituting them and so on, we train an artificial neural network to do these calculations for us.

IV. Neural Network

Artificial neural networks (ANN) computing systems inspired by the biological neural networks that constitute animal brains.[3] Such systems "learn" to perform tasks by considering examples, generally without being programmed with task-specific rules. We use a simple feed forward neural network with the following architecture.

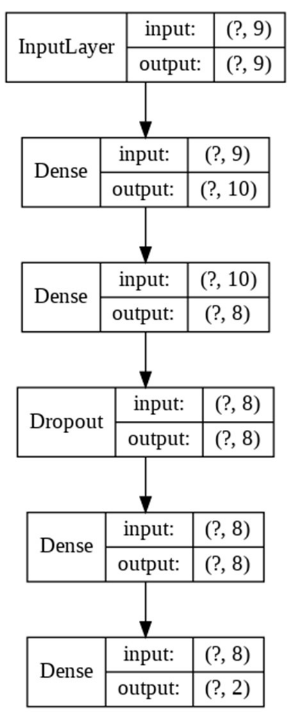
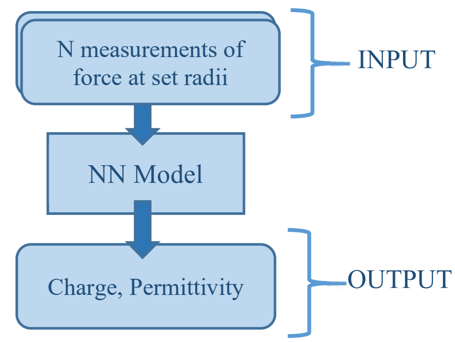


Figure 4: Description of Model Architecture (Here N =9)

V. Algorithm

For the inputs of the neural network, N measurements of the force at set distances were given for each expected output consisting of the permittivity and charge.



Implementation Details

The code [1] for the implementation was written in the Python3 version of the language and the neural network was implemented with the Keras Framework [4] using the TensorFlow Backend. It was tested on Google Colab server on a GPU.

VI. Conclusion and Result

We were able to predict the permittivity and source charge given only the force measurements by using a neural network. The predictions for q_1 taken as 50 mC and dielectric radius as 2m are as follows:

Quantity	Actual	Predicted
Source Charge	0.0677	0.0753
Permittivity	0.7800	0.7690

References

[1] Wikipedia, "Coulomb's law," *www*. [Online]. Available: https://en.wikipedia.org/wiki/Coulomb%27s_law.

[2] Wikipedia, "Shell Theorem," *www*. [Online]. Available: https://en.wikipedia.org/wiki/Shell_theorem.

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[1] https://github.com/s-ankur/permittivity-charge-estimation