code

docs

ARCHModels.jl

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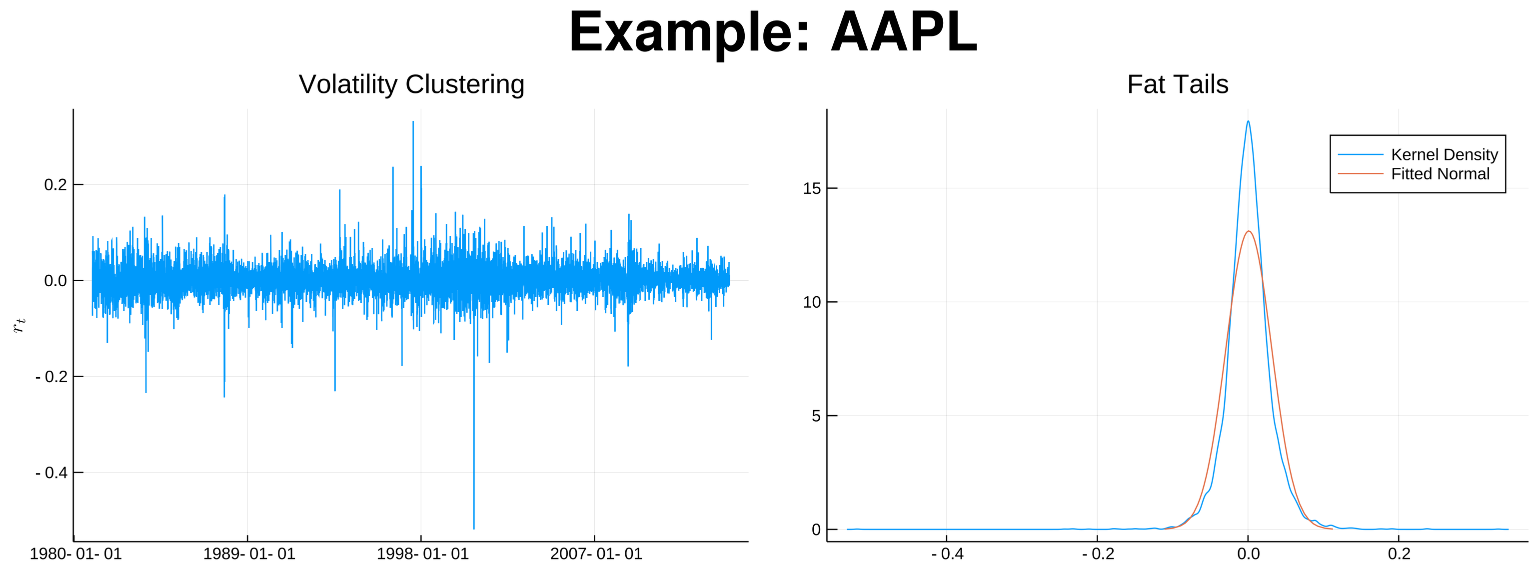
Introduction

- Daily financial returns data exhibit a number of stylized facts:
  - Volatility clustering
  - Non-Gaussianity, fat tails
  - Leverage effects: negative returns have larger effect on future volatility
- Similar for other data (e.g., changes in interest rates).
- Important throughout finance (risk management, derivative pricing, portfolio management, …).
- [G]ARCH ([**G**eneralized] **A**utoregressive **C**onditional **V**olatility) models are the most popular for modelling them.

Example: AAPL



[G]ARCH Models

- Basic setup: given a sample of financial returns $\{r_t\}_{t\in\{1,\ldots,T\}}$, decompose r_t as
$$r_t = \mu_t + \sigma_t z_t, \quad z_t \overset{i.i.d.}{\sim} (0,1),$$
where $\mu_t \equiv \mathbb{E}[r_t \mid \mathcal{F}_{t-1}]$ and $\sigma_t^2 \equiv \mathbb{E}[(r_t-\mu_t)^2 \mid \mathcal{F}_{t-1}]$.
- Assume $\mu_t = 0$ for simplicity. Focus is on the volatility σ_t.
- [G]ARCH models make σ_t a function of *past* returns and variances. Examples:
  - ARCH(q) (Engle, 1982):
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i r_{t-i}^2$$
  - GARCH(p, q) (Bollerslev, 1986):
$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2$$
  - TGARCH (Glosten et al., 1993):
$$\sigma_t^2 = \omega + \sum_{i=1}^{o} \gamma_i a_{t-i}^2 1_{a_{t-i}<0} + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2.$$
  - EGARCH(o, p, q) (Nelson, 1991):
$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{o} \gamma_i z_{t-i} + \sum_{i=1}^{p} \beta_i \log(\sigma_{t-i}^2) + \sum_{i=1}^{q} \alpha_i(|z_t| - \mathbb{E}|z_t|)$$

Estimation

- [G]ARCH models are usually estimated by maximum likelihood: with f_z denoting the density of z_t,
$$\max \prod_t f(r_t \mid \mathcal{F}_{t-1}) = \max \prod_t \frac{1}{\sigma_t} f_z(r_t/\sigma_t).$$
- σ_t is recursive $\Rightarrow$ not “vectorizable” $\Rightarrow$ loops.
- Matlab, Python's `rugarch` have to implement the likelihood in C.

`ARCHModels.jl` Highlights

- `ARCHModels.jl` is registered and supports Julia 1.0 and later.
- Extensive documentation available at `https://s-broda.github.io/ARCHModels.jl/stable/`.
- Implements estimation and inference for ARCH, GARCH, TGARCH, and EGARCH models of arbitrary orders.
- Supports Gaussian, GED, and Student's t errors natively, plus any continuous distribution from `Distributions.jl`.
- Mean equations can be specified as ARMA(p, q) models, or a regression model from `GLM.jl`.
- Also: automatic model selection, risk measure calculation, forecasting, simulation, model diagnostics, specification tests.
- Most importantly, it's

FAAAAAAAAAAAAST!

Implementation

- Designed to be easily extensible with new models, distributions.
- Volatility specifications subtype `VolatilitySpec`. Parametrized on (o, p, q) to facilitate loop unrolling.
- Simulation and estimation return instances of `UnivariateARCHModel`, which implements `StatisticalModel` from `StatsBase`.
- ML estimation via `Optim.jl`, standard errors obtained by AD via `ForwardDiff.jl`.

Benchmarks

```
julia> using ARCHModels, MATLAB, BenchmarkTools

julia> mat"version"
"9.4.0.813654 (R2018a)"

julia> @btime fit(GARCH{1, 1}, $BG96, meanspec=NoIntercept)
  3.865 ms (1778 allocations: 354.42 KiB)

ARCHModels.TGARCH{0,1,1} model with Gaussian errors, T=1974.


Volatility parameters:
──────────────────────────────────────────────────────
     Estimate    Std.Error   z value  Pr(>|z|)
──────────────────────────────────────────────────────
ω    0.0108661   0.00657449   1.65277   0.0984
β₁   0.804431    0.0730395   11.0136   <1e-27
α₁   0.154597    0.0539319    2.86651   0.0042
──────────────────────────────────────────────────────


julia> mat"tic; estimate(garch(1, 1), $BG96); toc; 0";

    GARCH(1,1) Conditional Variance Model (Gaussian Distribution):

                 Value      StandardError    TStatistic      PValue
               _________    _____________    __________    __________

    Constant    0.010868      0.0012972        8.3779      5.3896e-17
    GARCH{1}     0.80452       0.016038        50.162               0
    ARCH{1}      0.15433       0.013852        11.141      7.9448e-29

Elapsed time is 0.067641 seconds.
```

Results

- `BG96` are daily DM/GBP exchange rate data from Bollerslev & Ghysels (1996), the de-facto standard for testing implementations of [G]ARCH models.
- `ARCHModels.jl` beats Matlab by a factor of about 10, despite Matlab implementing the likelihood in C.
- Estimates are similar, but standard errors and t-stats are not.
- `ARCHModels.jl` matches the benchmark results given in Brooks et al. (2001) to the published precision.

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