p_1: $\forall x \in \mathbb{R}$ $\exists y \in \mathbb{R}$ tale che $x^2 = y^2 - 1$ vera

$y^2 = x^2 + 1$ $\qquad y = \pm\sqrt{x^2+1}$ ok.

$x^2 + 1 > 0$

p_2: $\exists x \in \mathbb{R}$ tale che $\forall y \in \mathbb{R}$ $x^2 = y^2 - 1$ falsa

$y_1 = 2 \qquad y_2 = 4$

se esiste un tale x $\qquad x = 4 - 1 \qquad x = 16 - 1$ contraddizione

$(\exists x \in \mathbb{R})(\forall y \in \mathbb{R} \quad x^2 = y^2 - 1)$ $\qquad 0$

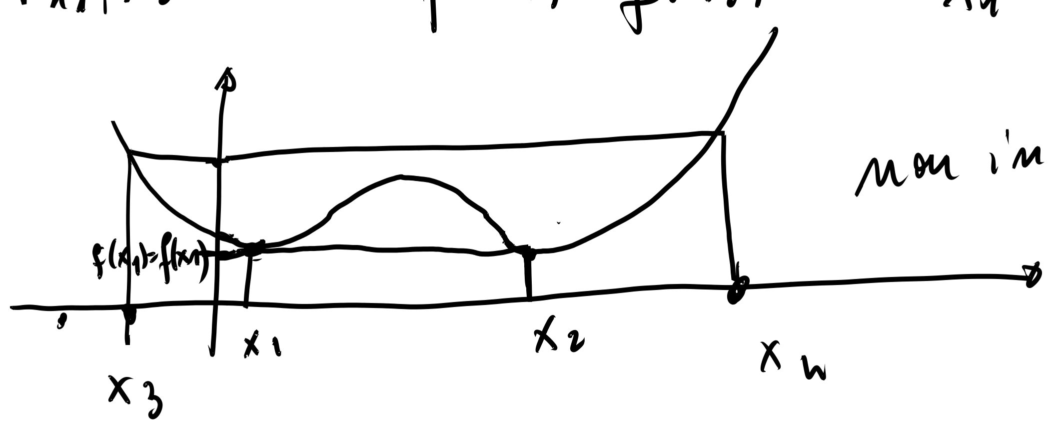
$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R} \quad x^2 = y^2 - 1)$ $\qquad 1$

$\neg p_1$: $\forall x \in \mathbb{R}$ $\exists y \in \mathbb{R}$ tale che $x^2 \neq y^2 - 1$ $\qquad 1$

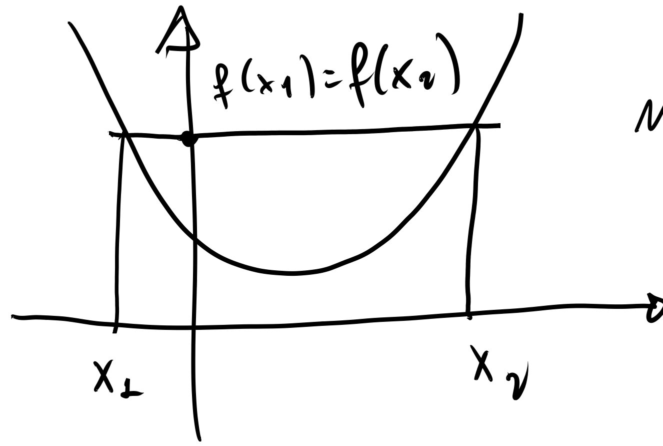
$\neg p_2$: $\exists x \in \mathbb{R}$ tale che $\forall y \in \mathbb{R}$ $x^2 \neq y^2 - 1$ $\qquad 0$

$f : A \to B$ $\qquad \forall x_1, x_2 \in A \quad x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

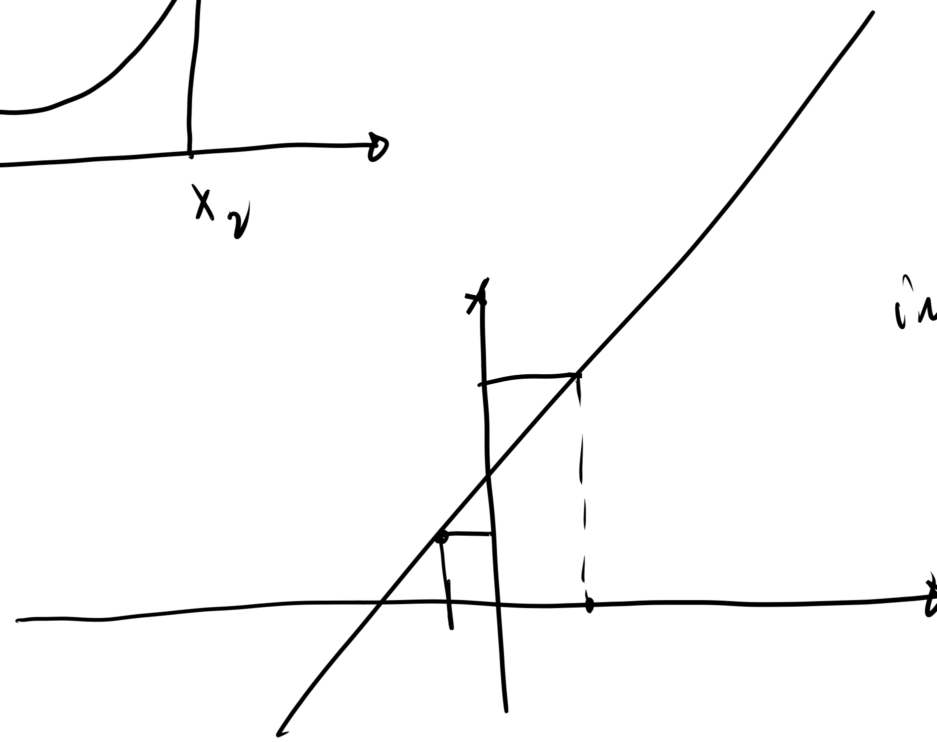
$\forall x_1, x_2 \in A \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$



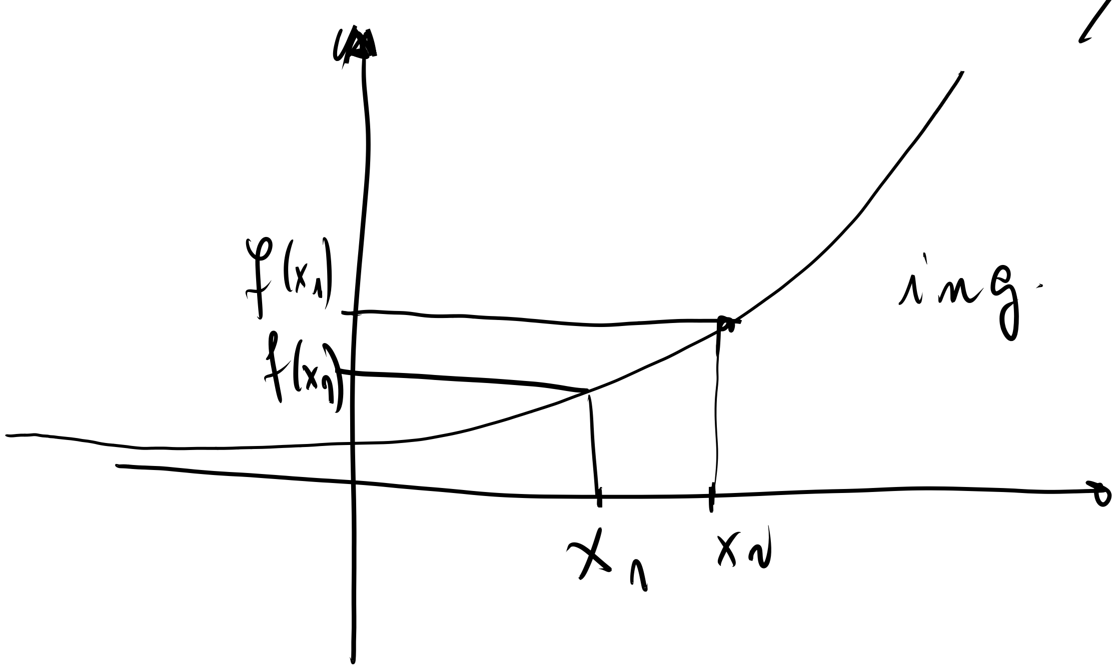
non iniett.



non ing.



ing.



ing.

$A \qquad \mathcal{P}(A) \qquad \Pi \subset \mathcal{P}(A)$

$\Pi = \{A_i \ : i \in I\}$

X_1, X_2 $\quad X_1 \cup X_2 = \{x \in A : x \in X_1 \vee x \in X_2\}$

$= \{x \in A : \exists\, i = 1, 2 \text{ tale che } x \in X_i\}$

X_1, X_2, X_3 $\quad X_1 \cup X_2 \cup X_3 = \{x \in A : x \in X_1 \vee x \in X_2 \vee x \in X_3\}$

$= \{x \in A : \exists\, i = 1, 2, 3 \text{ tale che } x \in X_i\}$

$X_1, X_2, \dots, X_n$ $\quad X_1 \cup X_2 \cup \dots \cup X_n = \{x \in A : x \in X_1 \vee \dots \vee x \in X_n\}$

$= \{x \in A : \exists\, i = 1, \dots, n \text{ tale che } x \in X_i\}$

$X_i,\ i \in I$ $\quad \bigcup_{i \in I} X_i = \{x \in A : \exists\, i \in I \text{ tale che } x \in X_i\}.$

$\mathcal{R} = \{(a, b) \in \mathbb{Q}^* \times \mathbb{Q}^* : \exists\, h \in \mathbb{Z} \text{ tale che } a = 5^h b\}$

$\mathcal{R}$ è riflessiva $\quad a \in \mathbb{Q}^*$ $\quad \exists\, h = 0 \in \mathbb{Z}$ tale che $a = 5^0 \cdot a \Rightarrow$ $(a, a) \in \mathcal{R}$.

$\mathcal{R}$ è simmetrica $\quad a, b \in \mathbb{Q}^*$ tali che $(a, b) \in \mathcal{R}$

allora $\exists\, h \in \mathbb{Z}$ tale che $a = 5^h b$ e quindi

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$\mathbb{Q}$ = insieme dei numeri razionali $\quad \mathbb{Q}^* = \mathbb{Q} - \{0\}$

$$5^{-h} a = \underbrace{5^{-h} \cdot 5^{h}}_{=1} b \Rightarrow b = 5^{-h} \cdot a$$

quindi esiste $-h \in \mathbb{Z}$ tale che $b = 5^{-h} a$

$(b, a) \in R$.

R è transitiva siano $a, b, c \in \mathbb{Q}^*$ tali che
$\underline{(a, b) \in R} \wedge \underline{(b, c) \in R}$. Allora $(\exists h \in \mathbb{Z}$ tale che $a = 5^h b) \wedge$
$(\exists k \in \mathbb{Z}$ tale che $b = 5^k c)$

$$a = 5^h \cdot b = 5^h (5^k c) = (5^h \cdot 5^k) \cdot c = 5^{h+k} c$$

quindi $\exists t = h+k \in \mathbb{Z}$ tale che $a = 5^t c$ e quindi

$\underline{(a, c) \in R}$.