3. Sequence modeling

1) three states (R, A, B)

→ learn transition matrix using MLE

R-A-A-B-A-B-B-A-A-A-R

	A	B	R	
A	3	2	1	6
B	2	1	0	3
R	1	0	0	1

⇒

	A	B	R
A	3/6	2/6	1/6
B	2/3	1/3	0
R	1	0	0

MLE=

2) Likelihood of sequence

R-B-B-B-A-A-B-A-B-R

* ADD one pseudo count to each transition

	A	B	R
A	4/9	3/9	2/9
B	3/6	2/6	1/6
R	2/3	1/3	1/3

$$\text{Likelihood} = \left(\frac{1}{3}\right)^1 \cdot \left(\frac{2}{6}\right)^2 \cdot \left(\frac{3}{6}\right)^2 \cdot \left(\frac{4}{9}\right)^1 \cdot \left(\frac{3}{9}\right)^2 \cdot \left(\frac{1}{6}\right)^1$$

$$= 0{,}00000762$$

$$\text{MLE} = 1 \cdot \ln\left(\frac{1}{3}\right) + 2\ln\left(\frac{2}{6}\right) + 2\ln\left(\frac{3}{6}\right) + \ln\frac{4}{9} + 2\ln\frac{3}{9} + \ln\frac{1}{6}$$

$$= -9{,}482$$

3) Given the transition params, predict next state

PREDICT R-B-B-? ⇒ A

$P(A|B) = 2/3$ → MORE LIKELY

$P(B|B) = 1/3$

PREDICT R-A-B-A-B-A-? ⇒ A

$P(A|A) = 3/6$ → MORE LIKELY

$P(B|A) = 2/6$

4) FIRST ORDER ASSUMPTION.

FIRST ORDER MARKOV CHAIN ASSUMES THAT THE FUTURE IS INDEPENDENT FROM THE PAST (OR DISTANT PAST) IF PRESENT IS GIVEN (OR RECENT PAST).

THE PRESENT HAS A BIGGER INFLUENCE ON THE FUTURE THAN THE PAST.

IF WE LESSEN THIS ASSUMPTION, WE CAN CREATE A 2ND ORDER MARKOV CHAIN, WHICH DEPENDS ON THE NEAR PAST, AND THE CURRENT STATE.

WE CAN COMPARE 1ST AND 2ND ORDER MODELS STATISTICALLY TO DETERMINE WHICH IS BETTER FOR A CERTAIN APPLICATION.

5) ZERO ORDER MARKOV CHAIN. IF WE DO NOT RELY ON THE PAST AT ALL, THE OUTCOME WILL BE RANDOM.