**Input:**  $x \in V^*$ , a sequence of token IDs.

**Output:**  $P \in [0, 1]^{N_V \times \text{length}(x)}$ , where  $P_t$  represents the conditional distribution  $p(x_{t+1} | x_{1:t})$ .

**Hyperparameters:**  $N_V$ ,  $\theta_{\text{max}}$ , L, H,  $d_e$ ,  $d_{\text{mlp}}$ , MLP hidden dimension.

**Parameters:**  $\theta$  includes all the following parameters:

```
\begin{aligned} & \boldsymbol{W}_{e} \in \mathbb{R}^{d_{e} \times N_{V}} & \text{token embeddings matrix.} \\ & \boldsymbol{W}_{p} \in \mathbb{R}^{d_{e} \times \theta_{\text{max}}} & \text{positional embeddings matrix.} \\ & \boldsymbol{W}_{l}, \boldsymbol{W}_{e2d_{l}} & \text{attention and MLP weights.} \\ & \text{For } l \in [L] & \text{See paper for details.} \\ & \boldsymbol{\gamma}, \boldsymbol{\beta} \in \mathbb{R}^{d_{e}} & \text{output normalization parameters.} \\ & \boldsymbol{W}_{u} \in \mathbb{R}^{N_{V} \times d_{e}} & \text{unembedding matrix.} \end{aligned}
```

```
\theta \leftarrow \text{length}(x)
       for t \in \theta : \boldsymbol{e}_t \leftarrow \boldsymbol{W}_e[:,x[t]] + \boldsymbol{W}_p[:,t]
3
       X \leftarrow [\boldsymbol{e}_1, \boldsymbol{e}_2, \dots \boldsymbol{e}_{\theta}]
4
       for l = 1, 2, ..., L do
5
                  X \leftarrow \text{layer norm}(X[:,t] \mid \gamma_l^1, \beta_l^1)
                  X \leftarrow X + \text{MHAttention}(\tilde{X} \mid W_I, \text{Mask}[t, t'] = [[t \le t']]
6
7
                   X \leftarrow \text{layer norm}(X \mid \gamma, \beta)
8
                  X \leftarrow X + \text{MLP}(X \mid W_I)
9
       X \leftarrow \text{layer norm}(X \mid \gamma, \beta)
10 end
11 for t \in [\theta]: P_t \leftarrow \text{softmax}(W_uX[:,t])
      return P
```

This implements an autoregressive decoder-only transformer similar to GPT with causal self-attention masking. It processes the input sequence, applies multiple transformer layers with causal self-attention and MLP blocks, and produces a conditional distribution over the next token at each position. The specifics like layer normalization and multi-head attention are as defined in the "Formal Algorithms for Transformers" article.