İNTEGRAL

$\forall x \in [a,b]$ için $f(x) \leq g(x)$ ise

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

$\forall x \in [a,b]$ için $m \leq f(x) \leq L$ ise

$$m.(b-a) \leq \int_a^b f(x)\,dx \leq L.(b-a)$$

$\pi \leq \int_0^{\pi} \sqrt{1+3\cos^2 x}\,dx \leq 2\pi$ ifadesinin ispatı, $\forall x \in [0,\pi]$ için

$-1 \leq \cos x \leq 1$ ise, $\forall x \in [0,\pi]$ için $1 \leq \sqrt{1+3\cos^2 x} \leq 2$

$$\pi = \int_0^{\pi} dx \leq \int_0^{\pi} \sqrt{1+3\cos^2 x} \leq \int_0^{\pi} 2\,dx = 2\pi$$

Ortalama Değer

Eğer $f: [a,b] \longmapsto \mathbb{R}$ sürekli ise

$\int_a^b f(x)\,dx = f(c).(b-a)$ | olacak şekilde (a,b) aralığında en az bir $c \in \mathbb{R}$ sayısı vardır.

$\forall x \in [a,b]$ için $g(x) = \int_a^x f(t)\,dt$ ve $\forall x \in [a,b]$ için $g'(x) = f(x)$ olur.

$\int_a^b f(x)\,dx = F(a) - F(b)$ | Calculus'un Temel Teoremi |

$$\frac{d}{dt}\left(\int_a^x f(t)\,dt\right) = f(x)$$

$f:[a,b] \longmapsto \mathbb{R}$ fonksiyonunun sürekli ve $h:[c,d] \longrightarrow [a,b]$ fonksiyonun türevlenebilir olduğunu kabul edelim. Eğer $x \in [c,d]$ olmak üzere

$$F(x)=\int_a^{h(x)} f(t)\,dt$$

ise $F(x)$ türevlenebilir ve $\forall x \in [c,d]$ için $F'(x)=(f \circ h)(x) \cdot h'(x)$

yani
$$\frac{d}{dt}\left(\int_{g(x)}^{h(x)} f(t)\,dt\right)=f(h(x))\cdot\frac{dh(x)}{dx}-f(g(x))\cdot\frac{dg(x)}{dx}$$

$\int \sec^2 x\,dx = \tan x + c$ $\qquad$ $\int \csc^2 x\,dx = -\cot x + C$

$\int \sec x \tan x\,dx = \sec x + c$ $\qquad$ $\int \csc x \cot x\,dx = -\csc x + C$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right)+c$ $\qquad$ $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\cdot \arctan\left(\frac{x}{a}\right)+c$

$$\int_0^{\pi/2} \cos^3 x \, dx = ?$$

$\cos^2 x = 1 - \sin^2 x$

$u = \sin x$

$du = \cos x \, dx$

$$= \int_0^{\pi/2} \cos^2 x \cdot \cos x \, dx = \int_0^{\pi/2} (1-\sin^2 x) \cos x \, dx = \int_0^1 (1-u^2) \cdot du = u - \frac{u^3}{3} \Big|_0^1 = \frac{2}{3}$$

$$\int \frac{dx}{\cos x - \sin x}$$

$$\int \cos\sqrt{x} \, dx$$

$a = \cos$

$a = \sqrt{x}$

$da = \frac{dx}{2\sqrt{x}}$

$2a \, da = dx$

$$\int 2a \cos a \, da$$

$\cos a = v$

$2a \, da = du$

$-\sin a \, da = du$

$a^2 = u$

LAPTÜ

u	dv
	$\cos a \, da$
$+ 2a$	$\sin a$
$- 2$	$-\cos a$
$+ 0$	

$2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

$$\cos a \cdot a^2 - \int a^2 \cdot -\sin a \, da$$

$-\sin a = v$

$a^2 da = du$

$dv = -\cos a \, da$

$\frac{a^3}{3} = u$

$$-\sin a \cdot \frac{a^3}{3} - \int \frac{a^3}{3}$$

$$\int_0^{\pi/2} f(\sin x)\,dx = \int_0^{\pi/2} f(\cos x)\,dx$$ old. göster.

$x = \pi/2 - v$

$dx = -dv$

$$\int_{\pi/2}^{0} f\left(\sin(\pi/2 - v)\right) \cdot -dv = \int_0^{\pi/2} f(\cos v)\,dv = \int_0^{\pi/2} f(\cos x)\,dx$$

$0 \leq x \leq 1$ için

$\ln(x+1) \leq \arctan x$ old göster

$x^2 \leq x$

$x^2+1 \leq x+1$

$\frac{1}{x+1} \leq \frac{1}{x^2+1}$

$$\int \frac{dx}{x+1} \leq \int \frac{dx}{x^2+1} = \arctan x \geq \ln(x+1)$$

$\forall x \geq 1$ için $\ln x \leq 2(\sqrt{x} - 1)$ old göster

$x \geq \sqrt{x}$

$\frac{1}{\sqrt{x}} \geq \frac{1}{x}$

$$\int_1^a \frac{dx}{\sqrt{x}} \geq \int_1^a \frac{dx}{x}$$

$x^{-\frac{1}{2}}$

$2 . x^{1/2} \geq \ln x$

$\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$

$2\sqrt{x} - 2 \geq \ln x$

$$\int \frac{x^2 . e^{-x}\,dx}{(2-x)^2}$$

$u = x^2 . e^{-x}$ $\quad du = -e^{x}(2x - x^2)\,dx$

$dv = \frac{dx}{(2-x)^2}$ $\quad \int dv = \int \frac{dx}{(2-x)^2} = v = \frac{1}{(2-x)}$

$v . du$

$= u . v - \int v . du$

$e^{-x}dx = da \quad a = -e^{-x}$

$x = b$

$$\frac{x^2 . e^{-x}}{(2-x)} - \int x . e^{-x} dx$$

e^{x}

$$-e^{-x} . x - \int -e^{-x} . e^{-x} dx$$

$$\left[\frac{x^2 . e^{-x}}{(2-x)} + e^{-x} . x + \int . e^{2x} dx = e^{-x} . x - \frac{e^{2x}}{2} \right]$$

$$\int \frac{2\cos^2 x}{\sqrt{2x+\sin 2x}}\,dx \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \int \frac{1+\cos 2x}{\sqrt{2x+\sin 2x}}\,dx$$

$$u^2 = 2x+\sin 2x$$

$$2u\,du = 2 + 2.\sin 2x.\cos 2x$$

$$u.du = (1+\cos 2x)\,dx$$

$$\int \frac{u\,du}{u} = \int du = u = \sqrt{2x+\sin 2x} + C$$

$$\int_0^1 \frac{2x\,dx}{\sqrt{x^2+1}-x} \qquad x^2+1 = u^2 \qquad u^2-1=x^2$$

$$2x\,dx = 2u\,du$$

$$\int_1^{\sqrt{2}} \frac{2u\,du}{u-\sqrt{u^2-1}} = \int_1^{\sqrt{2}} \frac{2u.(u+\sqrt{u^2-1})}{1}\,du = \int_1^{\sqrt{2}} 2u^2 + 2u\sqrt{u^2-1}\,du$$

$$= \int_1^{\sqrt{2}} 2u\,du + \int_1^{\sqrt{2}} 2u\sqrt{u^2-1}\,du$$

$$\int_0^1 2a^2\,da = \frac{2}{3}a^3\Big|_0^1 = \boxed{\frac{2}{3}}$$

$$u^2-1 = a^2$$

$$2u\,du = 2a\,da$$

$$\int_1^{\sqrt{2}} 2u\,du = u^2\Big|_1^{\sqrt{2}} = 1 \qquad \frac{2}{3}+1 = \frac{5}{3}$$

$$\int \frac{dx}{\cos x - \sin x}$$

$$\int \frac{\cos x + \sin x}{\cos 2x} \, dx$$

$U = \tan x/2$

$$\cos(x + \pi/4) = \cos x . \frac{\sqrt{2}}{2} - \sin x . \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} (\cos x - \sin x)$$

$$du = \frac{1}{2} . \sec^2 \frac{x}{2} \, dx$$

$$= \frac{1}{2} . \frac{1}{\cos^2 x/2}$$

$$\int \frac{\sqrt{2}}{\cos(x + \pi/4)} = \sqrt{2} \int \sec(x + \pi/4) \, dx$$

$$\frac{\sin x + \pi/4}{\cos x + \pi/4} . \frac{1}{\cos x + \pi/4}$$

$$\int \frac{[\sec(x + \pi/4) + \tan(x + \pi/4)] . \sec(x + \pi/4)}{\sec(x + \pi/4) + \tan(x + \pi/4)} \, dx$$

$$\int \frac{\sec^2(x + \pi/4) + \sec(x + \pi/4) . \tan(x + \pi/4)}{\sec(x + \pi/4) + \tan(x + \pi/4)} \, dx$$

$$U = \sec(x + \pi/4) + \tan(x + \pi/4)$$

$$du = \sec(x + \pi/4) . \tan(x + \pi/4) + \sec^2(x + \pi/4) \, dx$$

$$\sqrt{2} \int \frac{dx}{U} = \ln|U| + c$$

$$= \ln\left|\sec(x + \pi/4) + \tan(x + \pi/4)\right|^{\sqrt{2}}$$

Riemann Toplamı

$$\int_a^b f(x)dx = \lim_{n\to\infty} \left\{ \frac{b-a}{n} \cdot \sum_{k=1}^{n} f\left(a + \frac{k-1}{n} \cdot (b-a)\right) \right\}$$

$\int_0^2 x^2 = \frac{8}{3}$ old göster

$$\lim_{n\to\infty} \left\{ \frac{2}{n} \cdot \sum_{k=1}^{n} f\left(0 + \frac{2}{n} \cdot (k-1)\right) \right\} = \lim_{n\to\infty} \frac{2}{n} \cdot \sum_{k=1}^{n} \frac{4 \cdot (k-1)^2}{n^2}$$

$$4 \cdot \sum_{k=1}^{n} (k^2 - 2k + 1) = \frac{2 \cdot n \cdot (n+1) \cdot (2n+1)}{6} - \frac{2 \cdot n(n+1)}{2} + n = \frac{n \cdot (2n^2 - 3n + 1)}{6}$$

$$\int_0^2 x^2 dx = \lim_{n\to\infty} \left(\frac{8}{n^3} \cdot \frac{n \cdot (2n^2 - 3n + 1)}{6} \right) = \frac{16}{6} = \frac{8}{3}$$

Ortalama Değer

$f(x) = 4 - x^2$ f'nin $[-1, 2]$ arası ort değerini ve c sayısını bul.

$$\int_{-1}^{2} 4 - x^2 \, dx = 9$$

$$\frac{1}{b-a} \int_{-1}^{2} 4 - x^2 dx = 3 = f_{avg}$$

$c = \pm 1$

Alt Üst Toplam Örneği

$\int_0^3 2x+1 = 12$ olduğunu göster.

$[x_{k-1}, x_k]$ aralığında $M_k = 2x_k+1$ maximumu ve $m_k = 2x_{k-1}+1$ minimumu vardır.

O(nlogn)

$$A = \sum_{k=1}^{n} (2x_{k-1}+1)\Delta x_k \qquad B = \sum_{k=1}^{n} (2x_k+1)\Delta x_k$$

$$2x_{k-1}+1 \le x_k + x_{k-1}+1 \le 2x_k+1$$

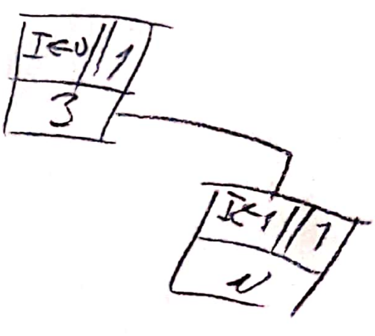
$$(2x_{k-1}+1)\Delta x \le (x_k+x_{k-1}+1)\Delta x \le (2x_k+1)\Delta x$$

$$A \le \sum_{k=1}^{n} (x_k+x_{k-1}+1)(x_k-x_{k-1}) \le B$$

$$\downarrow$$

$$\sum_{k=1}^{n} (x_k^2 - x_{k-1}^2 + x_k - x_{k-1}) = 3^2 - 0^2 - 3 - 0 = 12$$

$$\sum \frac{1}{n^p} = 0$$



$x^n + y^n = z^n$

$n > 2$

improrer integral

Sınırsız aralık

- $\int_0^\infty \frac{dx}{e^{-x}+e^x} = ?$

$$\int \frac{dx}{e^{-x}+e^x} = \int \frac{e^x dx}{1+e^{2x}} = \arctan e^x + c$$

$$\lim_{b\to\infty} \int_0^b \frac{dx}{e^{-x}+e^x} = \lim_{b\to\infty} \arctan e^x \Big|_0^b = \lim_{b\to\infty} \arctan e^b - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

- $\int_0^\infty \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx \longrightarrow \int \frac{e^{-x} dx}{\sqrt{1-e^{-2x}}}$ $\quad u = e^{-x} \quad du = -e^{-x} dx$ $\quad u^2 = e^{-2x}$

$$= -\int \frac{du}{\sqrt{1-u^2}} = -\arcsin u + c$$

$$= \lim_{b\to\infty} \int_0^b \frac{e^{-x} dx}{\sqrt{1-e^{-2x}}} = \lim_{b\to\infty} -\arcsin e^{-x} \Big|_0^b$$

$$= -\left(\underbrace{\lim_{b\to\infty} \arcsin e^{-b}}_{\to 0} - \frac{\pi}{2}\right) = \frac{\pi}{2}$$

- $\int_{-\infty}^\infty \frac{2x dx}{(x^2+1)^2} = \lim_{a\to-\infty} \int_a^0 \frac{2x dx}{(x^2+1)^2} + \lim_{b\to\infty} \int_0^b \frac{2x dx}{(x^2+1)^2}$

$$\int \frac{2x dx}{(x^2+1)^2} \quad x^2+1 = u \quad 2x dx = du \longrightarrow \int \frac{du}{u^2} = -\frac{1}{u}$$

$$\lim_{a\to-\infty} \int_a^0 \frac{2x dx}{(x^2+1)^2} + \lim_{b\to\infty} \int_0^b \frac{2x dx}{(x^2+1)^2} = \lim_{a\to-\infty} \frac{-1}{x^2+1}\Big|_a^0 + \lim_{b\to\infty} \frac{-1}{x^2+1}\Big|_0^b$$

$$-1+1=0$$

$$\int_0^{\infty} \frac{3dx}{x^3+1} \to \int \frac{3dx}{x^3+1} = \int \frac{1}{x+1} - \frac{x-2}{x^2-x+1}\,dx$$

$$= \int \frac{dx}{x+1} - \int \frac{x-2}{x^2-x+1}\,dx$$

$\frac{1}{t}$

$$\to \int \frac{\frac{1}{2}(2x-1)-\frac{3}{2}}{x^2-x+1} = \frac{1}{2}\int \frac{2x-1}{x^2-x+1}\,dx - \frac{3}{2}\int \frac{dx}{x^2-x+1}$$

$$\int \frac{dx}{x^2-x+1} = \int \frac{dx}{\frac{3}{4}+\left(x-\frac{1}{2}\right)^2}$$

$$= \int \frac{dx}{x+1} - \frac{1}{2}\int \frac{2x-1\,dx}{x^2-x+1} + \frac{3}{2}\int \frac{dx}{\frac{3}{4}+\left(x-\frac{1}{2}\right)^2}$$

$$= \ln|x+1| - \ln\sqrt{x^2-x+1} + \sqrt{3}\cdot \arctan \frac{2x-1}{\sqrt{3}} + C$$

$$\lim_{b\to\infty} \ln|x+1| - \ln\sqrt{x^2-x+1} + \sqrt{3}\cdot \arctan \frac{2x-1}{\sqrt{3}} \Bigg|_0^b$$

$$\lim_{b\to\infty} \ln \frac{x+1}{\sqrt{x^2-x+1}} + \sqrt{3}\arctan \frac{2x-1}{\sqrt{3}} \Bigg|_0^b = \frac{2\pi}{\sqrt{3}}$$

Tanımsız Integraldlar I

- $\displaystyle\int_0^1 \frac{dx}{x^{2/3}} = ?$

$$\int_0^1 \frac{dx}{x^{2/3}} = \lim_{a\to 0^+} \int_a^1 \frac{dx}{x^{2/3}} = 3.\lim_{a\to 0^+} x^{1/3}\Big|_a^1 = 3-0=3$$

- $\displaystyle\int_0^{\pi/2} \frac{\sin x\, dx}{\sqrt{e^{\cos x}-1}}$

$\cos x = U$

$-\sin x\, dx = du$

$$\int \frac{\sin x\, dx}{\sqrt{e^{\cos x}-1}} = -\int \frac{du}{\sqrt{e^{U}-1}}$$

$e^U - 1 = \theta^2$

$e^U du = 2\theta d\theta$

$e^U = \theta^2+1$

$\theta^2+1 \cdot du = 2\theta d\theta$

$du = \dfrac{2\theta d\theta}{\theta^2+1}$

$$= -\int \frac{2\cancel{\theta} d\theta}{\cancel{\theta}\cdot \theta^2+1} = -\int \frac{2\,d\theta}{\theta^2+1}$$

$\tan x = 0$

$\dfrac{\sin x}{\cos x} = 0$

$x = 0$

$$= -2\arctan\theta = -2\arctan\sqrt{e^{\cos x}-1}$$

$$\lim_{a\to \pi/2^-} \int_0^a \frac{\sin x\, dx}{\sqrt{e^{\cos x}-1}} = \lim_{a\to \pi/2^-} -2\arctan\sqrt{e^{\cos x}-1}\Big|_0^a = 0-(-2\arctan\sqrt{e-1})$$

$$= 2.\arctan\sqrt{e-1}$$

- $$\int_0^2 \frac{dx}{\sqrt{2x-x^2}}$$ iki uç noktada da süreksiz. integral improper

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(1-x)^2}} = \arcsin (x-1)^2 + C$$

$$\lim_{a\to 0^+} \int_a^1 \frac{dx}{\sqrt{2x-x^2}} + \lim_{b\to 2^-} \int_1^b \frac{dx}{\sqrt{2x-x^2}}$$

$$= \lim_{a\to 0^+} \arcsin \Big|_a^1 + \lim_{b\to 2^-} \arcsin (x-1)^2 \Big|_1^b = \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Tanımsız İntegral II

- $$\int_{-1}^{1} \frac{dx}{\sqrt[3]{x}}$$

$$= \int_{-1}^{0} \frac{dx}{x^{1/3}} + \int_{0}^{1} \frac{dx}{x^{1/3}}$$

$$\lim_{a \to 0^-} \int_{-1}^{a} \frac{dx}{x^{1/3}} + \lim_{b \to 0^+} \int_{b}^{1} \frac{dx}{x^{1/3}} = -\frac{3}{2} + \frac{3}{2} = 0$$

Yakınsaklık Araştırmak

$\int_{0}^{1} \frac{dx}{x^2+\sqrt{x}}$ integralinin yakınsaklığını araştırınız

$\forall x \in [0,1] \quad x^2+\sqrt{x} \geq 0$

$$\frac{1}{x^2+\sqrt{x}} \leq \frac{1}{\sqrt{x}}$$

$$\int_{0}^{1} \frac{dx}{x^2+\sqrt{x}} \leq \int_{0}^{1} \frac{dx}{\sqrt{x}} \longrightarrow \lim_{a \to 0^+} \int_{a}^{1} \frac{dx}{\sqrt{x}} = \lim_{a \to 0^+} 2\sqrt{x} \Big|_{a}^{1} = 2$$

$$0 \leq \int_{0}^{1} \frac{dx}{x^2+\sqrt{x}} \leq 2 \longrightarrow$$ yakınsaktır.

- $\displaystyle\int_0^1 \frac{dx}{e^{\sqrt{x}}-1}$ yakınsaklığı ?

$\forall x \in [0,1]$

$e^{\sqrt{x}}-1 \leq e-1$

$e^{\sqrt{x}}-1 \geq \sqrt{x}$

$$\frac{1}{e-1} \leq \int_0^1 \frac{dx}{e^{\sqrt{x}}-1} \leq \int_0^1 \frac{dx}{\sqrt{x}} = 2$$

- $\displaystyle\int_0^\infty \frac{\sin x \, dx}{x^{3/2}}$ yakınsaklığı ?

$x^{-\frac{3}{2}+1}$ $\quad x^{-\frac{1}{3}}$

$\forall x \in [0,\infty)$ $\dfrac{\sin x}{x^{3/2}} > 0$

$-1 < \sin x < 1$

$\sin x < 1 \rightarrow \dfrac{\sin x}{x^{3/2}} < \dfrac{1}{x^{\frac{1}{2}}}$

$$\int_0^\infty \frac{\sin x \, dx}{x^{3/2}} = \int_0^\infty \frac{dx}{x^{3/2}}$$

$$\lim_{b\to\infty} \int_0^b \frac{dx}{x^{3/2}} = \infty$$

$$0 < \int_0^\infty \frac{\sin x}{x^{3/2}} dx < 2$$

- $\displaystyle\int_1^\infty \frac{\sqrt{t}}{\ln t}\,dt$ ıraksaklığını göster

$$\frac{1}{t} < \frac{1}{\ln t} < \frac{\sqrt{t}}{\ln t}$$

$$\lim_{a\to\infty}\int_1^a \frac{1}{t} = \infty \qquad \text{o zaman ıraksak}$$

- $\displaystyle\int_0^\infty \frac{x.\ln x}{(1+x^2)^2}\,dx = 0$ old göster.

$$U = \frac{1}{x}$$

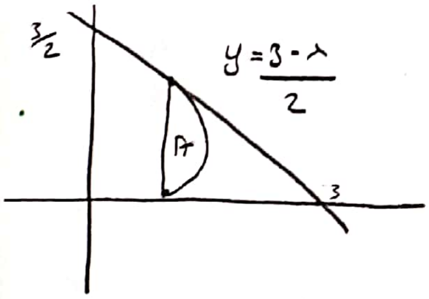
$$\int_0^\infty \frac{x\ln x}{(1+x^2)^2} = \int_\infty^0 \frac{\frac{1}{u}\cdot\ln\frac{1}{u}}{\left(1+\frac{1}{u^2}\right)^2}\cdot\left(-\frac{du}{u^2}\right) = \int_\infty^0 \frac{u.\ln u}{(1+u^2)^2}\,du$$

$$\int_0^\infty \frac{x\ln x}{(1+x^2)^2} = -\int_0^\infty \frac{x.\ln x}{(1+x^2)^2} = 0$$

İntegralin Uygulamaları

$V=\int_a^b A(x)\,dx$	$V=\int_a^b \pi\cdot[f(x)]^2$

- Bir cismin tabanı $x+2y=3$ ve koordinat ekseni ile oluşturulmuştur. X eksenine dik alınmış dik kesitler yarım daireler ise $V=?$

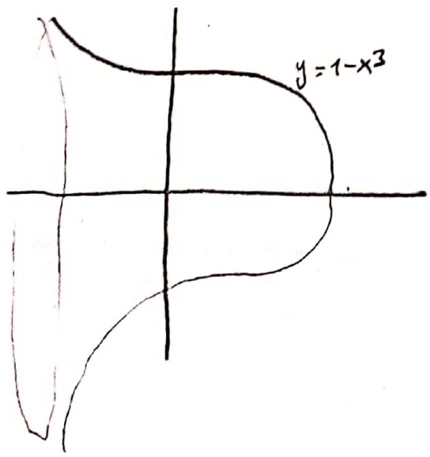


$A=\frac{\pi r^2}{2}$ $\quad r=\frac{3-x}{4}$

$A(x)=\frac{\pi}{2}\cdot\left(\frac{3-x}{4}\right)^2$

$$V=\int_0^3 \frac{\pi}{2}\cdot\left(\frac{3-x}{4}\right)^2=\frac{9\pi}{32}\text{ br}^3$$

$y=1-x^3$ eğrisinin x ekseni ve $x=\pm 1$ ile oluşturduğu bölgenin X ekseni etrafında dönmesi ile



$A(x)=\pi\cdot(1-x^3)^2$

$$V=\int_{-1}^{1} A(x)=\frac{9\pi}{14}$$

Disk Tekniği

(x-ekseni etrafında)

$$V=\int_a^b A(x)=\int_a^b \pi.[f(x)]^2 dx$$

(y-ekseni etrafında)

$$V=\int_c^d A(y)dy=\int_c^d \pi.[g(y)]^2 dy$$

(y = y₀ doğrusu etrafında)

$$V=\int_a^b \pi.|y_0-f(x)|^2 dx$$

(x = x₀ doğrusu etrafında)

$$V=\int_c^d \pi.|x_0-g(y)|^2 dy$$

Pul Tekniği

(x-ekseni etrafında)

$f_1(x) \geq f_2(x)$

$$V=\int_a^b \pi.\{[f_1(x)]^2-[f_2(x)]^2\}dx$$

(y-ekseni etrafında)

$$V=\int_c^d \pi.\{[g_1(y)]^2-[g_2(y)]^2\}.dy$$

Silindir Kabuğu Tekniği

(y-ekseni)

$x=a$, $x=b$ ile oluşan bölgeyi y ekseni etrafında döndürmek

$$V=\int_a^b 2\pi.x.f(x).dx$$

(x-ekseni)

$y=c$, $y=d$ ile oluşan bölgeyi x ekseni etrafında döndürmek

$$V=\int_c^d 2\pi.y.g(y).dy$$

Yay Uzunluğu ve Fonksiyonu

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$$

$$S(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$$

Dönel yüzeyin alanı

(x - ekseni)

$$S_A = \int_a^b 2\pi f(x) . \sqrt{1 + [f'(x)]^2}\, dx$$

(y - ekseni)

$$S_A = \int_c^d 2\pi . g(y) . \sqrt{1 + [g'(y)]^2}\, dy$$

Limit NOTLARI

Ortalama Değişim Oranı

$y=f(x)$ fonksiyonu, $[x_1, x_2]$ aralığındaki ortalama değişim oranı.

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(x_1+h)-f(x_1)}{h} \quad , h \neq 0$$

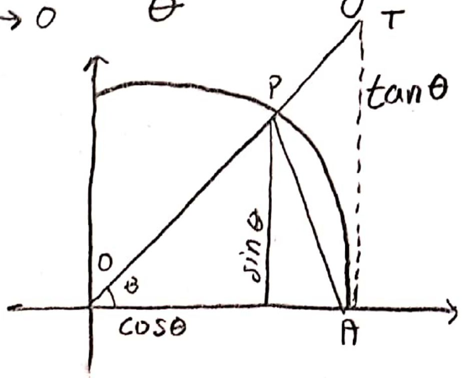
Teğet Doğrusu Ve Eğimi

$y=x^2$ parabolünün $P(2,4)$ noktasındaki eğimi ve teğet doğrusu?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(2+h)^2-4}{h} = 4+h = 4$$

$$4 = \frac{y-4}{x-2} \longrightarrow y = 4x-4$$

$\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ göster.



$$A(\overset{\triangle}{OPA}) \leq A(\overset{\frown}{OPA}) \leq A(\overset{\triangle}{OTA})$$

$$\frac{1}{2}.\sin\theta \leq \frac{1}{2}.\theta \leq \frac{1}{2}.\tan\theta$$

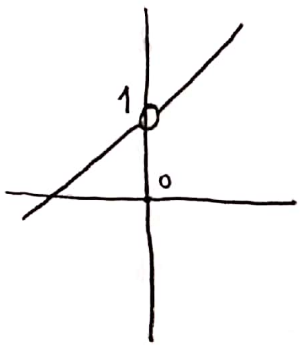
$$1 \leq \frac{\theta}{\sin\theta} \leq \frac{1}{\cos\theta}$$

$$\lim_{\theta\to 0} \cos\theta \leq \frac{\sin\theta}{\theta} \leq 1 = 1 \leq a \leq 1$$

$$a=1$$

Süreksizlik Çeşitleri

Kaldırılabilir Süreksizlik



$f(x),\ x \neq 0$

$f(0)$, 0'da sürekli değil

$$f(x) = \begin{cases} f(x), & x \neq 0 \\ 1, & x = 0 \end{cases} \longrightarrow f(x),\ x=0\text{'da süreklidir.}$$

Esas Süreksizlik

$$\lim_{n \to a^+} f(n) = +\infty \qquad \lim_{n \to a^-} f(n) = -\infty$$

Sıçramalı Süreksizlik

$$\lim_{n \to a^+} f(n) = b \qquad \lim_{n \to a^-} f(n) = c$$

$b \neq c$

Lineerleştirme

$f(x) \approx L(x)$

$L(x) = f(x_0) + f'(x_0) \cdot (x - x_0)$

$\sqrt{24}$ 'ün yaklaşık değeri = ?

$f(x) = \sqrt{x}$ $\qquad x = 25$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad L(x) = 5 + \frac{1}{10} \cdot (x - 25)$

$f(24) \approx L(24) = 5 - \frac{1}{10} \cdot 1 = \frac{49}{10} = 4{,}9$

Diferansiyel

$dy = f'(x)\,dx$

$\Delta y = f(x_0 + \Delta x) - f(x_0)$

$f(x_0 + \Delta x) = \Delta y + f(x_0)$

$\Delta y \approx dy$

$\Delta x \approx dx$

$f(x) = \sqrt{x}$ $\qquad \sqrt{26} = ?$

$f'(x) = \frac{1}{2\sqrt{x}}$ $\qquad f'(25) = \frac{1}{10}$ $\qquad \Delta x = 1$

$dy \approx \Delta x = f'(x) \cdot dx \approx f'(x)\Delta x$

$= \frac{1}{10} \cdot 1$

$f(25+1) = f(26) = \sqrt{26} = \frac{1}{10} + 5 = 5{,}1$

TRIGONOMETRIK ŞEYLER 1

$1+\tan^2\theta = \sec^2\theta$

$1+\cot^2\theta = \csc^2\theta$

$\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A+(-B)) = \cos A\cos B + \sin A\sin B$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A+(-B)) = \sin A\cos B - \cos A\sin B$

$\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$

$\cos A+\cos B = 2.\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$

$\cos A-\cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\sin A+\sin B = 2.\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)$

$\sin A-\sin B = 2.\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

TRIGONOMETRIK ŞEYLER 2

$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} = \sec^2 x = 1+\tan^2 x$$

$$\frac{d}{dx}\cot x = -\frac{1}{\sin^2 x} = -\csc^2 x = -(1+\cot^2 x)$$

$\arcsin u \rightarrow \dfrac{u'}{\sqrt{1-u^2}}$ $\qquad$ $\arccos u \longrightarrow \dfrac{-u'}{\sqrt{1-u^2}}$

$\arctan u \longrightarrow \dfrac{u'}{1+u'}$ $\qquad$ $\text{arccot}\, u \longrightarrow \dfrac{-u'}{1+u^2}$

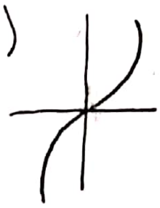
HIPERBOLIK ŞEYLER 1

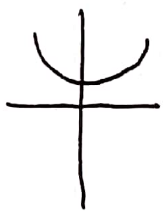
a) $\sinh x = \dfrac{e^x - e^{-x}}{2}$

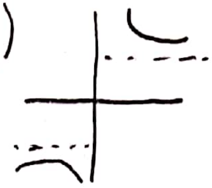
b) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

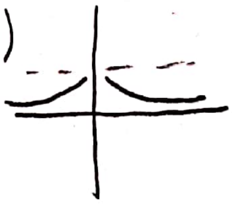
c) $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

d) $\coth x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

a)


b)


c)


d)


$\cos^2 hx - \sin^2 hx = 1$

$\sin h2x = 2\cdot \sinh x \cdot \cosh x$

$\cos h2x = \sin^2 hx + \cos^2 hx$

$\cos^2 hx = \dfrac{\cosh 2x + 1}{2}$

$\sin^2 hx = \dfrac{\cosh 2x - 1}{2}$

$\tan^2 hx = 1 - \text{sech}^2 x$

$\cot^2 hx = 1 + \csc^2 hx$

$\text{asech}\, x = \text{acosh}\, \dfrac{1}{x}$

$\text{acsch}\, x = \text{asinh}\, \dfrac{1}{x}$

$\text{acoth}\, x = \text{atanh}\, \dfrac{1}{x}$

Hiperbolik Şeyler 2

$\text{asinh}\,u \rightarrow \dfrac{u'}{\sqrt{1+u^2}}$, $u \in \mathbb{R}$ $\quad$ $\text{atanh}\,u \rightarrow \dfrac{u'}{1+u^2}$, $|u|<1$

$\text{acosh}\,u \rightarrow \dfrac{u'}{\sqrt{u^2-1}}$ $\quad u>1$ $\quad$ $\text{acoth}\,u \rightarrow \dfrac{u'}{1-u^2}$ $\quad |u|>1$

$\text{acsch}\,u \rightarrow \dfrac{-u'}{|u|\sqrt{1+u^2}}$, $u \in \mathbb{R}\setminus\{0\}$ $\quad$ $\text{asech}\,u \rightarrow \dfrac{u'}{u.\sqrt{1-u^2}}$ $\quad 0<u<1$

$\dfrac{d}{dx}\,\text{acosh}\,u = \dfrac{u'}{\sqrt{u^2-1}}$ old göster.

$f(x) = \cosh x \quad f'(x) = \sinh x$

$$(f^{-1})' = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sinh(\text{acosh}\,x)} \longrightarrow \begin{array}{l} \cosh^2 x - \sinh^2 x = 1 \\ \sinh x = \sqrt{\cosh^2 - 1} \end{array}$$

$$= \frac{1}{\sqrt{\cosh^2(\text{acosh}\,x)-1}} = \frac{1}{\sqrt{x^2-1}}$$

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asinh'in logaritmik ifadesi = ?

$y = \text{asinh} \quad x = \sinh y = \dfrac{e^y - e^{-y}}{2} = \dfrac{e^{2y}-1}{2.e^y}$

$e^y . x . 2 = e^{2y} - 1$

$e^{2y} - 2xe^y - 1 = 0 \qquad \ln e^y = \ln(x+\sqrt{x^2+1})$

$e^y = x + \sqrt{x^2+1} \qquad y = \ln(x+\sqrt{x^2+1})$

$y = \text{atanh}\,x$

$e^{2y} = \dfrac{1+x}{1-x}$

$y = \left[\ln\left(\dfrac{1+x}{1-x}\right)\right] \cdot \dfrac{1}{2}$