Optimization

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1 Introduction

This report introduced several algorithm for optimization, including genetic algorithm and differential evolution algorithm. 18 functions are used as bench mark to test the performance of each algorithm.
The experiment is written in C++14, compiled with Clang 11.0.0. Random numbers are generated using Mersenne Twister(MT) pseudo-random number generator by Makoto Matsumoto.

System	macOS Catalina 10.15.3
Processor	2.5 GHz Dual-Core Intel Core i7
Memory	16GB 2133 MHz LPDDR3

Table 1: Hardware information

2 Algorithms

2.1 Genetic Algorithm

Genetic algorithm(GA) are the heuristic search and optimization techniques that mimic the process of natural evolution.
Basic steps of genetic algorithm consists of selection, crossover and mutation for each generation. Size of the population is kept constant by replacing the bad solution with a better solution. In selection step, parent with high fitness value are selected for crossover. In crossover step, both parents will be cut-off at a same randomly chosen point, and combining the inherited first half with the second half from the other parent to form two new individuals. Then the two individuals are mutated based on criteria.
For this experiment, genetic algorithm uses roulette wheel method during selection step, and differential evolution algorithm uses 10 different strategies. Each strategy shown below follows the convention of DE/x/y/z. x represents a string denoting the vector to be perturbed, y is the number of difference vectors considered for perturbation of x, and z is the type of crossover being used.

2.2 Differential Evolution algorithm

Differential evolution consist of three steps for each generation, including mutation, crossover and selection. During each generation, each vector generates a potential solution. During mutation step, a new parameter vector is generated by adding the weighted difference vector between two randomly selected population member to a third member. In exponential crossover, the crossover iterate through the loop of the vector within the CR bound. In binomial crossover, the crossover is performed on each variable with a probability determined by a random number compared against CR value. The potential solution is selected based on the cost compare to the current solution. Different strategies can be adopted depending on the type of problem applied.

Strategy	Formulation
Strategy 1: DE/best/1/exp	$v = x_{best}^G + F \cdot \left(x_{r2}^G - x_{r3}^G\right)$
Strategy 2: DE/rand/1/exp	$v = x_{r1}^G + F \cdot \left(x_{r2}^G - x_{r3}^G\right)$
Strategy 3: DE/rand-to-best/1/exp	$v = x_i^G + \lambda \cdot \left(x_{best}^G - x_i^G\right) + F \cdot \left(x_{r1}^G - x_{r2}^G\right)$
Strategy 4: DE/best/2/exp	$v = x_{best}^G + \lambda \cdot \left(x_{r1}^G + x_{r2}^G - x_{r3}^G - x_{r4}^G\right)$
Strategy 5: DE/rand/2/exp	$v = x_{r5}^G + \lambda \cdot \left(x_{r1}^G + x_{r2}^G - x_{r3}^G - x_{r4}^G\right)$
Strategy 6: DE/best/1/bin	$v = x_{best}^G + F \cdot \left(x_{r2}^G - x_{r3}^G\right)$
Strategy 7: DE/rand/1/bin	$v = x_{r1}^G + F \cdot \left(x_{r2}^G - x_{r3}^G\right)$
Strategy 8: DE/rand-to-best/1/bin	$v = x_i^G + \lambda \cdot \left(x_{best}^G - x_i^G\right) + F \cdot \left(x_{r1}^G - x_{r2}^G\right)$
Strategy 9: DE/best/2/bin	$v = x_{best}^G + \lambda \cdot \left(x_{r1}^G + x_{r2}^G - x_{r3}^G - x_{r4}^G\right)$
Strategy 10: DE/rand/2/bin	$v = x_{r5}^G + \lambda \cdot \left(x_{r1}^G + x_{r2}^G - x_{r3}^G - x_{r4}^G\right)$

Table 2: Differential Evolution strategies

2.3 Particle Swarm Optimization

Particle Swarm Optimization(PSO) is inspired by flocking and schooling patterns of bird and fish. Over a number of iterations, a group of variables have their value adjusted closer to the member who's value is closest to the target at any given moment. The algorithm keeps track of each particles best solution in its history, a global best solution found by particles in the population. For each iteration, a new velocity is calculated for each particle influenced by local optimal solution and global optimal solution so far. The current particle will be updated with the new velocity, local optimal and global optimal will also be updated if needed.

2.4 Moth-flame Optimization

Moth-flame Optimization(MFO) is inspired by navigation method of moth called traverse orientation, that it flies by maintaining a fixed angle with respect to the moon. A logarithmic spiral is the main update mechanism for this algorithm. Initial point of the spiral start from the moth, the final point ends at the position of the target, which is the flame. The moth can converge to any point in the neighbourhood of the flame, and the frequency of position update on the flame is increased as the moth get closer to the flame. For each iteration, position of the moth, position and number of flames are updated. Number of flame decreases linearly, removing the worst flame in each generation.

2.5 Cuckoo Search Optimization

Cuckoo Search Optimization(CS) is inspired by Cuckoo birds' reproductive strategy. For each iteration, one egg is altered based on Levy Flight random walk. In Cuckoo Search based on Gauss Distribution(CSG), gaussian distribution is used for random walk. Fitness of this egg will be compared with another egg randomly chosen from the nests, if the fitness of the new egg is higher, it will replace the other egg. Then a fraction p_a of worse nests will be abandoned and replaced with new eggs. Five algorithms used in this experiment are cuckoo search with levy flight random walk, cuckoo search with gaussian random walk with the basic p_a and three variants of p_a for CSG.p_{aCi} is the p_a for current iteration, p_{aMax} is the maximum value of p_a, C_i is current iteration and T_i is total number of iteration. Another one tested but results not included is the Enhanced Cuckoo Search(ECS).

Strategy	Formulation	Equation
Strategy CSG: CSCo	Constant	0.25
Strategy CSG1: CSCLI	Linear increasing	$p_{ac_i} = (p_{aMax}) * (C_i/T_i)$
Strategy CSG2: CSEI	Constant	$p_{ac_i} = (p_{aMax}) * Exp(C_i/T_i)$
Strategy CSG3: CSPI	Constant	$p_{ac_i} = (p_{aMax}) * (C_i/T_i)^3$

Table 3: Cuckoo Search switching paramter p_a

2.6 Whale Optimization Algorithm

Whale Optimization Algorithm(WOA) consists of two parts, bubble-net attacking method(exploitation phase) and search for prey(exploration phase). Every generation for every whale(search agent), a random p value will decide whether the whale goes for a encircling or spiral motion.For encircling motion, the whale assumes current best candidate solution is the prey and moves close to it. Another random number A will be generated for using either best or random solution to update the whale's position, and the step size is controlled by a decreasing number a. For spiral motion, the whale uses a spiral equation to mimic the behavior. In real world, a whale swims around the prey within a shrinking circle and along a spiral-shaped path simultaneously. A random value p_a is generated to simulate this movement.

3 Benchmarks

$f_i(x^*)$ represents the optimal value for each function.

3.1 Schwefel's function

$$\begin{aligned} f_1(x) = (418.9829 \cdot n) - \sum_{i=1}^{n} -x_i \cdot sin(\sqrt{|x_i|}), -512 \leq x_i \leq 512 \\ f_1(x^*) = 0 \end{aligned} \tag{1}$$

3.2 1st De Jong's function

$$\begin{aligned} f_2(x) = \sum_{i=1}^{n} x_i^2, -100 \leq x_i \leq 100 \\ f_2(x^*) = 0 \end{aligned} \tag{2}$$

3.3 Rosenbrock

$$\begin{aligned} f_3(x) = \sum_{i=1}^{n-1} 100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2, -100 \leq x_i \leq 100 \\ f_3(x^*) = 0 \end{aligned} \tag{3}$$

3.4 Rastrigin

$$\begin{aligned} f_4(x) = 10 \cdot n + \sum_{i=1}^{n} (x_i^2 - 10 \cdot cos(2\pi \cdot x_i)), -30 \leq x_i \leq 30 \\ f_4(x^*) = 0 \end{aligned} \tag{4}$$

3.5 Griewangk

$$\begin{aligned} f_5(x) = 1 + \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right), -500 \leq x_i \leq -500 \\ f_5(x^*) = 0 \end{aligned} \tag{5}$$

3.6 Sine Envelope Sine Wave

$$\begin{aligned} f_6(x) = -\sum_{i=1}^{n-1} 0.5 + \frac{sin(x_i^2 + x_{i+1}^2 - 0.5)^2}{(1 + 0.001(x_i^2 + x_{i+1}^2))^2}, -30 \leq x_i \leq 30 \\ f_6(x^*) = -1.495(n-1) \end{aligned} \tag{6}$$

3.7 Stretched V Sine Wave

$$\begin{aligned} f_7(x) = \sum_{i=1}^{n-1} \left(\sqrt[4]{x_i^2 + x_{i+1}^2} \cdot \left(sin50\left(\sqrt[10]{x_i^2 + x_{i+1}^2} \right) \right)^2 + 1 \right), -30 \leq x_i \leq 30 \\ f_7(x^*) = 0 \end{aligned} \tag{7}$$

3.8 Ackley's One

$$f_8(x) = \sum_{i=1}^{n-1} \frac{1}{e^{0.2}} \sqrt{x_i^2 + x_{i+1}^2} + 3(cos(2x_i) + sin(2x_{i+1})), -32 \leq x_i \leq 32$$
$$f_8(x^*) = -7.54276 - 2.91867(n-3) \tag{8}$$

3.9 Ackley's Two

$$f_9(x) = \sum_{i=1}^{n-1} \frac{1}{e^{0.2}} \sqrt{x_i^2 + x_{i+1}^2} + 3(cos(2x_i) + sin(2x_{i+1})), -32 \leq x_i \leq 32$$
$$f_9(x^*) = 0 \tag{9}$$

3.10 Egg Holder

$$f_10(x) = \sum_{i=1}^{n-1} -x_i \cdot sin\left(\sqrt{|x_i - x_{i+1} - 47|}\right) - (x_{i+1} + 47) \cdot$$
$$sin\left(\sqrt{|x_{i+1} + 47 + \frac{x_i}{2}|}\right), -500 \leq x_i \leq 500 \tag{10}$$

3.11 Rana

$$f_11(x) = \sum_{i=1}^{n-1} x_i \cdot sin\left(\sqrt{|x_{i+1} - x_i + 1|}\right) \cdot cos\left(\sqrt{|x_{i+1} + x_i + 1|}\right)$$
$$+(x_{i+1} + 1) \cdot cos\left(\sqrt{|x_{i+1} - x_i + 1|}\right) \cdot sin\left(\sqrt{|x_{i+1} + x_i + 1|}\right)$$
$$, -500 \leq x_i \leq 500 \tag{11}$$

3.12 Pathological

$$f_12(x) = \sum_{i=1}^{n-1} 0.5 + \frac{sin(\sqrt{100x_i^2 + x_{i+1}^2})^2 - 0.5}{1 + 0.001(x_i^2 - 2x_i \cdot x_{i+1} + x_{i+1}^2)^2}, -100 \leq x_i \leq 100 \tag{12}$$

3.13 Michalewicz

$$f_13(x) = \sum_{i=1}^{n} sin(x_i) \cdot \left(sin\left(\frac{i \cdot x_i^2}{\pi}\right)\right)^{20}, 0 \leq x_i \leq \pi$$
$$f_13(x^*) = 0.966n \tag{13}$$

3.14 Master's Cosine Wave

$$f_14(x) = -\sum_{i=1}^{n-1} e^{-\frac{1}{8}(x_i^2 + x_{i+1}^2 + 0.5x_{i+1} \cdot x_i)}$$
$$cos\left(4\sqrt{x_i^2 + x_{i+1}^2 + 0.5x_{i+1} \cdot x_i}\right), -30 \leq x_i \leq 30$$
$$f_14(x^*) = 1 - n \tag{14}$$

3.15 Quartic

$$f_15(x) = \sum_{i=1}^{n} (i \cdot x_i^4), -100 \leq x_i \leq 100$$
$$f_15(x^*) = 0 \tag{15}$$

3.16 Levy

$$f_16(x) = sin^2(\pi w_i) + \sum_{i=1}^{n-1}(w_i - 1)^2[1 + 10 \cdot sin^2(\pi w_i + 1)]$$
$$+(w_i - 1)^2[1 + sin^2(2\pi w_n)] \text{ , where: } w_i = 1 + \frac{x_i - 1}{4}, -10 \leq x_i \leq 10 \tag{16}$$
$$f_16(x^*) = 0$$

3.17 Step

$$f_17(x) = \sum_{i=0}^{n-1}(|x_i| + 0.5)^2, -100 \leq x_i \leq 100 \tag{17}$$
$$f_17(x^*) = 0$$

3.18 Alpine

$$f_18(x) = \sum_{i=0}^{n-1}|x_i \cdot sin(x_i) + 0.1 \cdot x_i|, -100 \leq x_i \leq 100 \tag{18}$$
$$f_18(x^*) = 0$$

4 Experimentation

Below are the parameters for experimentation. Random numbers are generated using Mersenne Twister. Results includes average, standard deviation, range, and median of the function value, and the execution time for each experiment in microseconds.

Parameters	GA and DE	PSO and MFO	CS	CSG	WOA
Experimentation	50	50	50	50	50
Population size	200	200	200	200	200
Generation	200	2000	8000	2000	2000
Dimension	30	30	30	30	30

Table 4: Parameters

4.1 PSO and MFO

Parameters used for PSO are $c1 = 0.5$, $c2 = 2.0$.

4.2 CS and CSG

Parameters used for CS are $alpha0 = 0.05$, $pa_{max} = 0.25$.
Parameters used for CSG are $alpha0 = 0.05$, $miu = 0.0001$, $sigma0 = 8$.

4.3 WOA

Parameters used for CS are $b = 0.8$.

5 Result

Below are the results for genetic algorithm and differential evolution algorithm for each benchmark. Result of the strategy with lowest mean value is selected as the optimal result for each function in differential algorithm.

f	Mean	Median	Std	Range(low)	Range(high)	Time(mus)
1	1296.11	1271.96	314.163	739.845	2070.52	294.02
2	1894.73	1690.94	1250.06	321.035	7274.19	257.76
3	2.52905e+08	1.67502e+08	2.0353e+08	4.17359e+07	7.68276e+08	264.98
4	278.735	263.301	101.944	107.095	493.46	376.76
5	11.1036	10.0315	5.32465	3.07357	28.045	316.46
6	-38.8408	-38.8981	1.05394	-40.5856	-36.1993	310.32
7	30.1287	30.042	0.520667	29.2761	31.5993	460.04
8	-40.6842	-44.1224	14.8503	-69.9276	-4.3997	398.74
9	133.586	132.211	21.5206	93.4007	181.566	422.92
10	-17272.7	-17380.3	1091.95	-19267.8	-14366.6	344.58
11	-10806.9	-10842.6	466.06	-11803.6	-9455.31	419.22
12	-13.7428	-13.7161	2.02614	-18.1283	-9.97208	344.34
13	-27.3242	-27.3095	0.518281	-28.2436	-26.3545	434.2
14	-12.8914	-13.0983	2.11936	-16.9444	-8.19672	391
15	4.861e+07	3.24196e+07	5.59477e+07	4.05516e+06	3.08714e+08	305.38
16	5.5246	5.39293	2.22272	1.3127	11.4731	513.62
17	2055.2	2005.97	1014.85	202.112	6041.29	247.7
18	10.9389	10.8302	3.19496	4.91395	21.5856	274.84

Table 5: Result for genetic algorithm

f	Mean	Median	Std	Range(low)	Range(high)	Time(mius)
de1	1361.04	1299	357.356	810	2191	756.28
de2	0	0	0	0	0	677.1
de3	1900.34	104.5	7110.6	26	42807	681.76
de4	76.76	70.5	31.4443	36	167	1028.86
de5	1.02	1	0.14	1	2	700.16
de6	-30.3	-30	0.806226	-32	-29	735.24
de7	39.58	40	1.29754	37	43	804.36
de8	-75.76	-76.5	3.41503	-80	-63	1119.14
de9	34.66	30	12.743	29	88	755.38
de10	-16139.9	-15987	1218.48	-19188	-14117	1094.56
de11	-6921.74	-6882.5	289.615	-7740	-6402	860.92
de12	0.44	0	0.80399	-1	3	756.58
de13	-16.18	-16	0.653911	-18	-15	790.66
de14	-6.3	-6	0.608276	-8	-5	778.9
de15	0	0	0	0	0	698.84
de16	0	0	0	0	0	790.64
de17	8.1	8	0.3	8	9	674.02
de18	10.42	6.5	11.7611	0	49	1066.92

Table 6: Result for differential evolution algorithm

Function	Mean	Median	Std	Range(low)	Range(high)	Time(mus)
1	3545.68	3327.85	733.627	2513.31	6582.94	1269.5
2	2202.88	0.035084	4140.95	1.29566e-08	10000	1057.26
3	2.1131e+09	2.14748e+09	1.28e+08	1.40105e+09	2.14748e+09	1210.34
4	424.131	244.757	371.633	98.5007	1333.78	1203.2
5	11.4042	0.0695535	24.0638	6.03239e-08	63.3302	1224.6
6	-30.8243	-30.8322	1.5458	-33.7274	-27.3299	1419.5
7	30.9457	30.376	1.71878	29.0223	36.1815	1980.68
8	68.0722	72.9753	44.2199	-2.88217	160.005	1711.16
9	271.719	272.37	48.8113	171.601	404.636	1814.88
10	-20236.8	-20414.2	1764.62	-23531.1	-16987.6	1406.46
11	-13578.9	-13562.8	81.7008	-13850.3	-13463.9	1639.3
12	1.7744	1.61673	2.1524	-2.45413	6.99135	1466.88
13	-13.986	-13.8096	1.83518	-18.9814	-11.2246	1583.32
14	-1.63977	-1.58492	0.330773	-2.5299	-0.97323	1647.34
15	2.35387e+08	2.00422e+08	2.62477e+08	1435.67	1.2e+09	1298.08
16	26.3169	26.7163	8.71392	8.88689	58.7391	1915.98
17	3855.12	13.6614	6960.21	7.59859	30307.5	1045.82
18	68.5264	46.0625	64.9844	0.00378576	284.456	1238.84

Table 7: Result for particle swarm optimization algorithm

Function	Mean	Median	Std	Range(low)	Range(high)	Time(mus)
1	3370.73	3327.87	62.8939	3213.1	3442.61	1462.38
2	115.696	98.5253	54.2801	25.164	243.056	1293.98
3	1.53612e+06	1.09448e+06	1.4897e+06	193907	8.20714e+06	1418.3
4	194.931	187.514	37.4124	115.32	277.372	1570.38
5	1.699	1.61184	0.353409	1.17113	2.58277	1623.7
6	-22.6323	-22.5794	0.711159	-25.3752	-21.4409	1603.68
7	36.004	35.2345	3.5191	30.5548	43.7506	2280.84
8	-23.1218	-24.8292	14.3918	-63.2611	9.20829	1947.34
9	188.088	184.429	44.0895	103.116	346.892	2194.94
10	-17439.4	-17603.8	1066.36	-19643	-14720.1	1901.4
11	-13542.3	-13557.6	172.801	-13741.2	-12483.9	2022.7
12	-9.6468	-9.05438	1.8695	-16.4302	-6.65341	1716.24
13	-15.0251	-14.7642	1.6782	-18.5771	-11.9855	2155.16
14	-4.97637	-4.6903	1.00159	-7	-3.47618	2055.22
15	2.24617e+06	165755	1.39795e+07	26718.6	1.0009e+08	1639.16
16	9.25173	8.23516	5.49194	1.84777	31.4718	2331.18
17	156.395	148.736	45.6156	66.9605	262.177	1340.64
18	195.437	192.788	50.277	88.7991	315.518	1500.8

Table 8: Result for moth-flame optimization algorithm

Function	Mean	Median	Std	Range(low)	Range(high)	Time(mus)
1	1087.93	1264.89	981.329	-1801.95	2565.92	1823.84
2	734.868	703.207	239.534	402.68	1819.89	352.26
3	3.92783e+06	3.53619e+06	2.49069e+06	663903	1.45042e+07	358.36
4	275.008	266.363	26.3797	233.086	347.909	386.2
5	5.44217	5.14344	1.40782	2.81493	9.57782	386.64
6	-27.6244	-27.4838	1.00252	-31.4717	-26.1886	380.58
7	39.8372	39.9585	0.961172	37.0386	42.199	486.08
8	9.51631	11.133	11.0459	-15.8874	30.291	299.48
9	173.938	174.665	15.6316	143.755	208.23	418.6
10	-26892.7	-26693	3812.24	-36039.2	-18723.3	1980.34
11	-12794.2	-12547.5	988.082	-15280.5	-11389.8	2146.02
12	-5.59986	-5.28983	1.88067	-10.9379	-2.30982	1827.78
13	-14.1686	-13.9198	0.894458	-16.4578	-12.8318	2206.62
14	-4.20339	-4.08156	0.543363	-5.9579	-3.37735	413.64
15	584042	535834	329795	96821.2	2.06917e+06	364.76
16	4.61967	4.64413	1.39778	2.50004	8.86857	553.58
17	796.993	816.324	219.934	325.522	1330.07	333.58
18	52.8558	52.9469	6.37478	41.6003	66.5181	357.28

Table 9: Result for cuckoo search optimization algorithm

Function	Mean	Median	Std	Range(low)	Range(high)	Time(mus)
1	1950	1952.68	615.736	118.798	3098.35	1020.14
2	0	0	0	0	0	974.54
3	23.9174	24.0866	1.10022	22.2508	26.9657	826.4
4	69.9268	70.9846	39.6363	0	176.704	998.5
5	7.54952e-17	0	1.05184e-16	0	2.22045e-16	1060.24
6	-29.2017	-29.1782	1.99596	-33.2183	-25.2228	1134.42
7	33.962	33.5439	2.46315	29.9639	42.0306	1844.02
8	-63.7284	-66.6805	12.1979	-81.2184	-31.8776	1459.04
9	29.7443	29.7343	0.0621078	29.6881	29.8793	1615.46
10	-24242.5	-24409.6	1149.65	-25866.4	-20937.9	1435.28
11	-13971.1	-13991.2	155.957	-14230.5	-13637.6	1641.28
12	-16.4097	-16.043	2.40194	-23.3872	-11.7032	1258.2
13	-17.6906	-17.6462	1.58967	-20.9505	-14.4481	1718.08
14	-26.3033	-29	4.44371	-29	-11.3634	1552.3
15	0	0	0	0	0	1072.48
16	0.687824	0.72435	0.183576	0.270877	1.00439	1784.36
17	7.5	7.5	1.65181e-07	7.5	7.5	801.9
18	55.1107	52.2699	23.0505	9.23946	160.26	1005.52

Table 10: Result for whale optimization algorithm

5.1 Schwefel's

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	1296.11	1271.96	314.163	739.845	2070.52	294.02
DE1	1361.04	1299	357.356	810	2191	756.28
DE2	5810.08	5786	291.114	4779	6374	1146.72
DE3	6011.68	6089	373.894	5073	6851	807.62
DE4	5110.1	5199	464.51	3801	6226	1322.56
DE5	6076.24	6114.5	299.313	5126	6626	1814.16
DE6	2273.2	2187	506.5	1340	3665	1100.16
DE7	7602.32	7624.5	323.222	6505	8197	1702.28
DE8	7060.9	7262.5	844.801	4002	8505	1190.26
DE9	6128.28	6317	931.86	3631	8170	1866.18
DE10	7803.58	7874	277.954	6760	8190	2492.78
PSO	3545.68	3327.85	733.627	2513.31	6582.94	1269.5
MFO	3370.73	3327.87	62.8939	3213.1	3442.61	1462.38
CS	1087.93	1264.89	981.329	-1801.95	2565.92	1823.84
CSG	7671.02	7758.93	425.429	6167.59	8470.32	428.82
CSG1	9074.62	9132.11	323.767	8349.26	9642.19	388
CGS2	9123.18	9125.95	234.588	8562.35	9581.41	371.6
CSG3	9549.23	9621.94	426.599	8026.82	10292.5	303.5
WOA	1950	1952.68	615.736	118.798	3098.35	1020.14

Table 11: Results for Schwefel

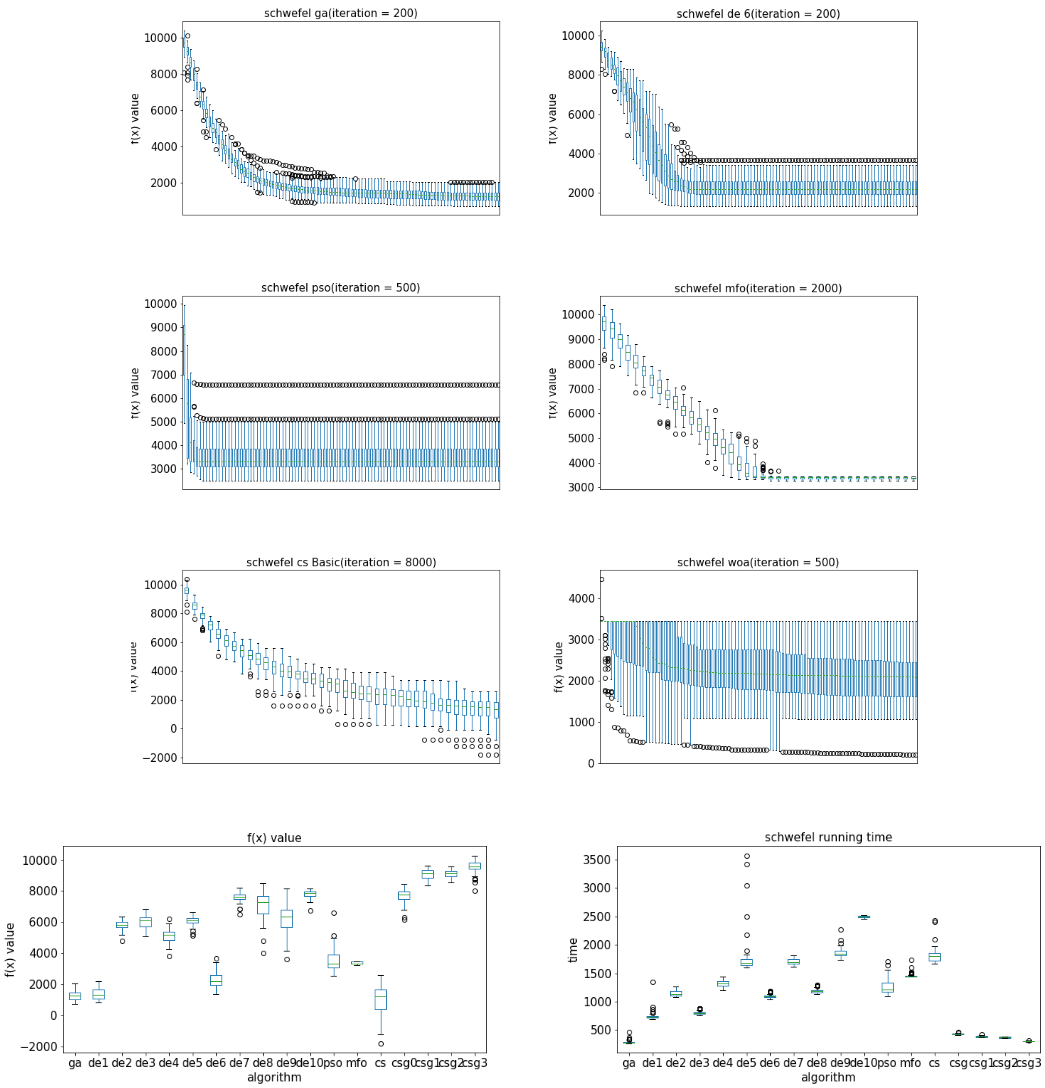

Figure 1: Results for Schwefel's function

5.2 1st De Jong's

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	1894.73	1690.94	1250.06	321.035	7274.19	257.76
DE1	0	0	0	0	0	677.1
DE2	70.62	72	10.2662	45	91	908.68
DE3	0	0	0	0	0	709.08
DE4	26.42	24.5	9.02683	16	56	993.34
DE5	2078.28	2093.5	219.837	1522	2649	1279.12
DE6	0.8	1	0.6	0	2	1009.7
DE7	190.68	182	34.7888	115	275	1393.92
DE8	0.02	0	0.14	0	1	1057.42
DE9	27.04	23.5	19.6662	6	111	1520.9
DE10	8135.12	8207.5	963.839	5380	10278	2016.84
PSO	2202.88	0.035084	4140.95	1.29566e-08	10000	1057.26
MFO	115.696	98.5253	54.2801	25.164	243.056	1293.98
CS	1898.17	1738.44	506.225	1061.32	3198.81	1514.38
CSG	772.023	744.105	191.823	409.438	1211.73	367.2
CSG1	747.323	672.191	235.953	309.81	1369.83	350.32
CSG2	734.868	703.207	239.534	402.68	1819.89	352.26
CSG3	845.67	799.32	269.993	352.16	1450.1	307.86
WOA	0	0	0	0	0	974.54

Table 12: Results for 1st De Jong

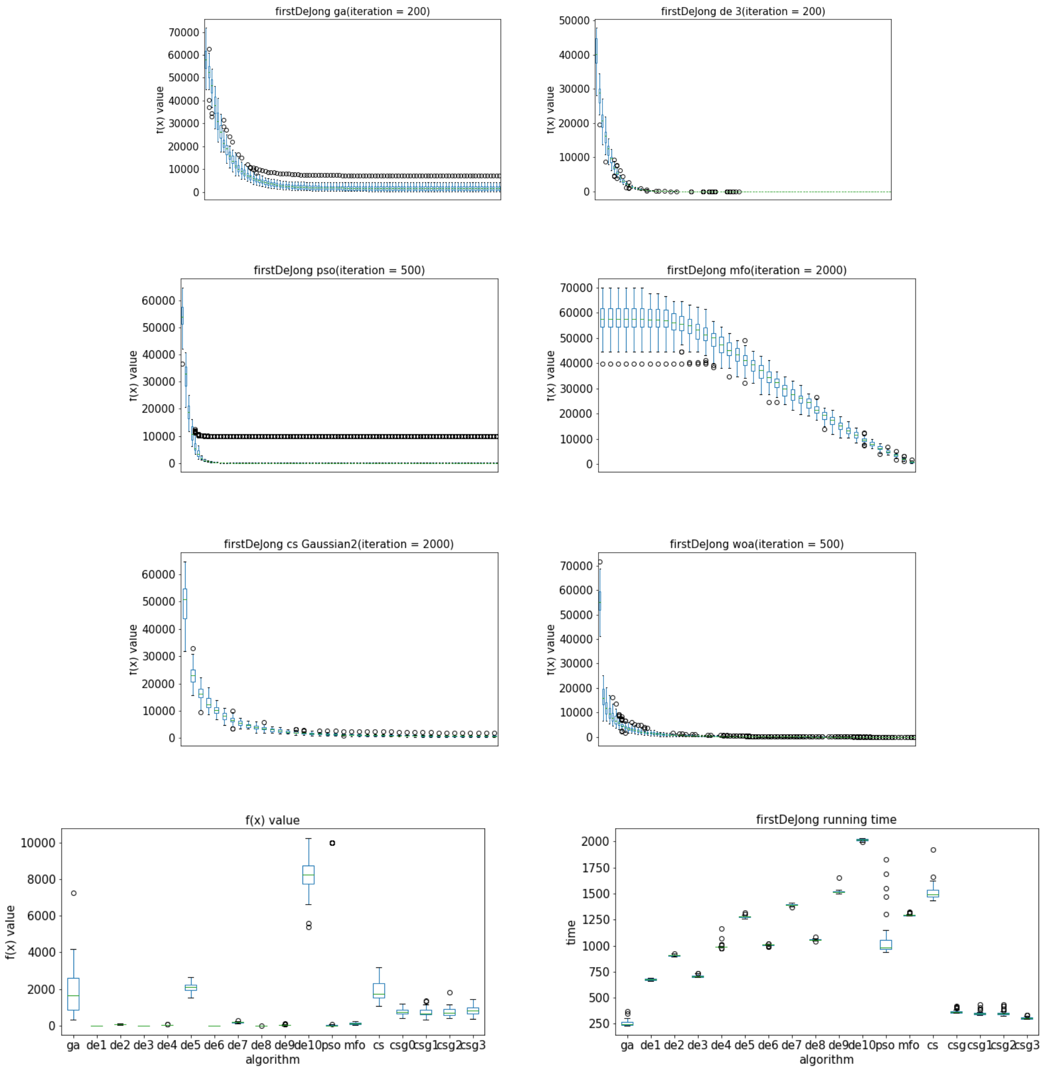

Figure 2: Results for 1st De Jong's function

5.3 Rosenbrock

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	2.52905e+08	1.67502e+08	2.0353e+08	4.17359e+07	7.68276e+08	264.98
DE1	1900.34	104.5	7110.6	26	42807	681.76
DE2	782047	790196	221985	350889	1.54090e+06	912.86
DE3	2.51927e+07	1728	1.11715e+08	25	7.58991e+08	712.58
DE4	2.08876e+06	858254	3.73367e+06	101758	2.18171e+07	1008.84
DE5	9.58863e+07	9.47195e+07	2.43273e+07	4.20972e+07	1.47739e+08	1292
DE6	2.14748e+09	2.14748e+09	0	2.14748e+09	2.14748e+09	1009.82
DE7	8.41088e+06	8.2488e+06	2.43292e+06	3.29429e+06	1.57714e+07	1399.94
DE8	1.89475e+09	2.14748e+09	6.47555e+08	223044	2.14748e+09	1093.4
DE9	6.87684e+08	4.69213e+07	9.61227e+08	483719	2.14748e+09	1565.5
DE10	1.00718e+09	9.56769e+08	2.33404e+08	5.17987e+08	1.50138e+09	2050.76
PSO	2.1131e+09	2.14748e+09	1.28e+08	1.40105e+09	2.14748e+09	1210.34
MFO	1.53612e+06	1.09448e+06	1.4897e+06	193907	8.20714e+06	1418.3
CS	2.31456e+07	1.56375e+07	2.3556e+07	3.01797e+06	1.34864e+08	1591.8
CSG	4.64314e+06	3.97286e+06	2.68892e+06	1.3043e+06	1.5343e+07	397.88
CSG1	3.92783e+06	3.53619e+06	2.49069e+06	663903	1.45042e+07	358.36
CSG2	4.29288e+06	3.22959e+06	2.81307e+06	595050	1.30176e+07	368.74
CSG3	5.04272e+06	4.57379e+06	2.60745e+06	1.11104e+06	1.21742e+07	319.44
WOA	23.9174	24.0866	1.10022	22.2508	26.9657	826.4

Table 13: Results for Rosenbrock

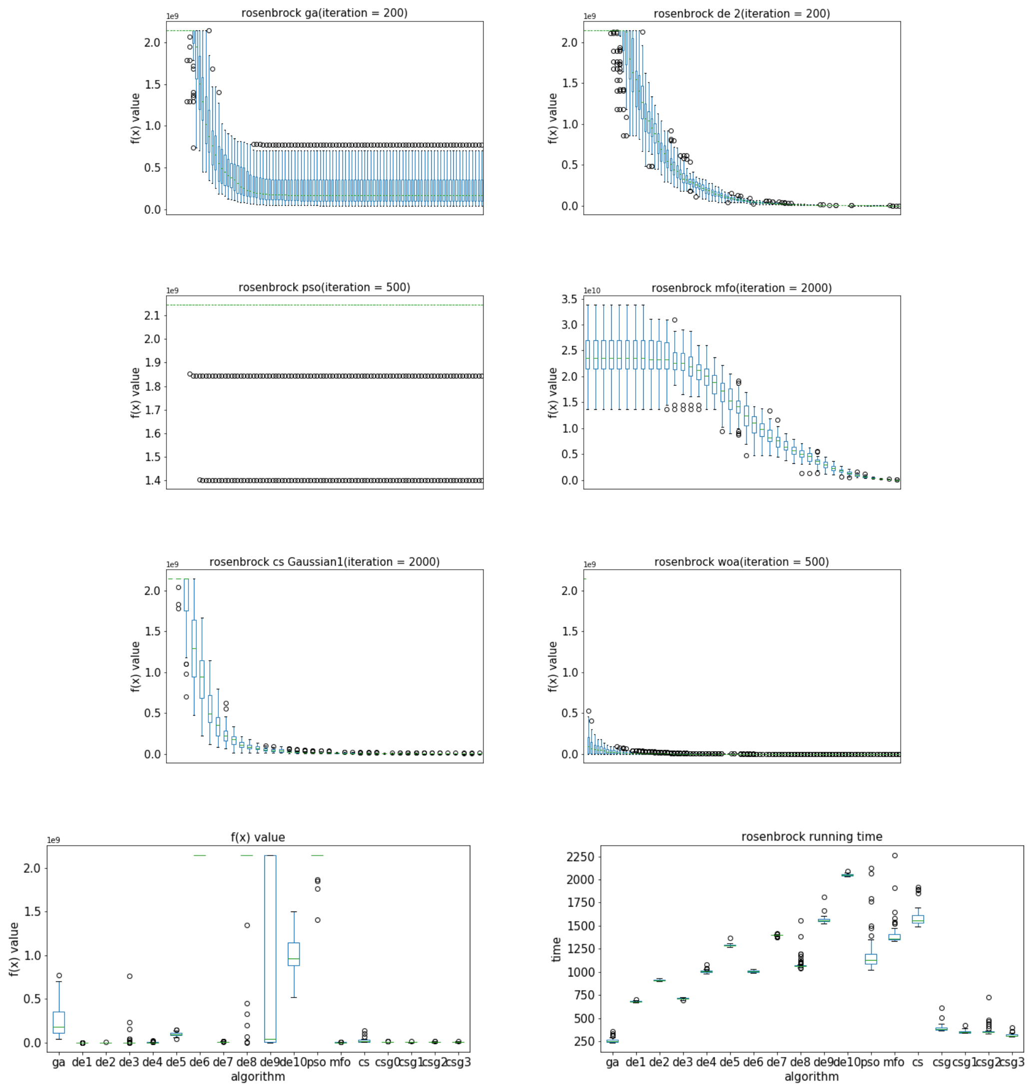

Figure 3: Results for Rosenbrock function

5.4 Rastrigin

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	278.735	263.301	101.944	107.095	493.46	376.76
DE1	128.76	130	16.2241	81	158	700.98
DE2	210.38	211.5	11.1156	177	227	933.8
DE3	126.18	128	10.0253	101	158	731.72
DE4	219.82	222.5	12.978	174	238	1009.68
DE5	459.64	457	26.529	405	523	1303.96
DE6	76.76	70.5	31.4443	36	167	1028.86
DE7	275.94	277	15.0777	223	303	1418.12
DE8	179.9	180	11.3494	163	212	1078.7
DE9	281.26	282	21.163	215	332	1549.6
DE10	1035.5	1055.5	95.1767	772	1189	2053.34
PSO	424.131	244.757	371.633	98.5007	1333.78	1203.2
MFO	194.931	187.514	37.4124	115.32	277.372	1570.38
CS	331.79	312.915	64.037	230.384	544.31	1946.22
CSG	280.995	281.118	21.5007	225.532	329.612	428.78
CSG1	275.008	266.363	26.3797	233.086	347.909	386.2
CSG2	276.155	274.297	24.1272	225.02	349.621	376.96
CSG3	277.916	274.202	23.1309	229.448	334.21	317.54
WOA	69.9268	70.9846	39.6363	0	176.704	998.5

Table 14: Results for Rastrigin

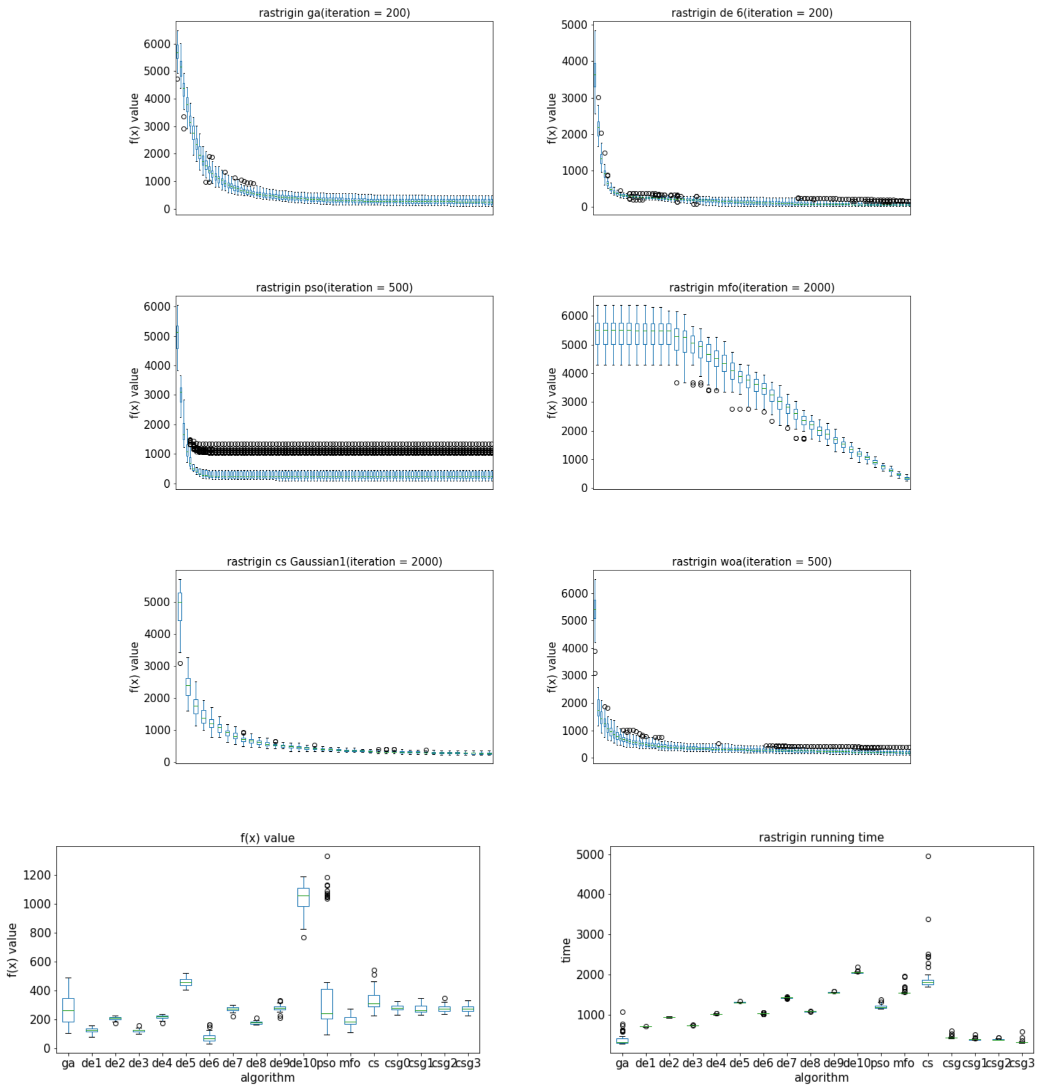

Figure 4: Results for Rastrigin function

5.5 Griewangk

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	11.1036	10.0315	5.32465	3.07357	28.045	316.46
DE1	1.02	1	0.14	1	2	700.16
DE2	1	1	0	1	1	942.04
DE3	1	1	0	1	1	732.2
DE4	1	1	0	1	1	1019.82
DE5	13.06	13	1.52853	9	16	1310.66
DE6	1.68	2	0.507543	1	3	1027.62
DE7	1.84	2	0.366606	1	2	1415.14
DE8	1	1	0	1	1	1064.96
DE9	1	1	0	1	1	1535.2
DE10	50.74	49.5	5.98936	36	62	2041.66
PSO	11.4042	0.0695535	24.0638	6.03239e-08	63.3302	1224.6
MFO	1.699	1.61184	0.353409	1.17113	2.58277	1623.7
CS	11.6185	11.356	3.30956	4.06017	19.7802	1765.98
CSG	6.09	6.04532	1.52392	2.92509	10.2576	432.98
CSG1	5.44217	5.14344	1.40782	2.81493	9.57782	386.64
CSG2	5.52049	5.50506	1.33302	3.2123	8.68589	371.24
CSG3	6.16549	5.72344	1.67286	3.86187	11.7753	301.06
WOA	7.54952e-17	0	1.05184e-16	0	2.22045e-16	1060.24

Table 15: Results for Griewangk

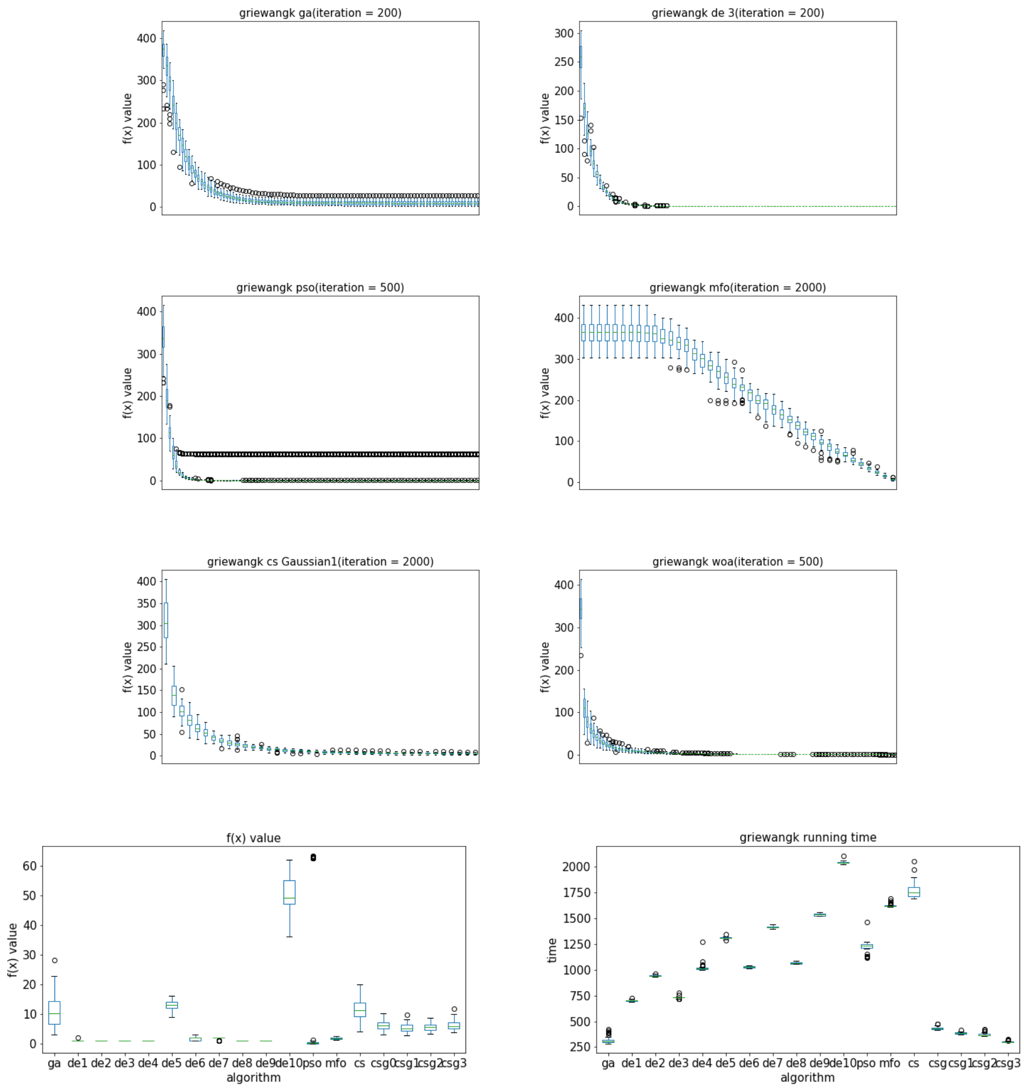

Figure 5: Results for Greiwangk function

5.6 Sine Envelope Sine Wave

Below are results for Sine Envelope Sine Wave.

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-38.8408	-38.8981	1.05394	-40.5856	-36.1993	310.32
DE1	-28.9	-29	0.67082	-30	-28	732.16
DE2	-28.76	-29	0.736478	-31	-27	990.82
DE3	-30.3	-30	0.806226	-32	-29	735.24
DE4	-28.38	-28	0.718053	-31	-27	1119.8
DE5	-28.14	-28	0.774855	-30	-27	1462.58
DE6	-24.08	-24	0.820731	-26	-22	1081.84
DE7	-24.16	-24	0.731027	-26	-23	1533.38
DE8	-25.36	-25	0.866256	-27	-24	1092.38
DE9	-23.68	-24	0.705408	-26	-23	1751.9
DE10	-23.72	-24	0.567098	-25	-23	2350.52
PSO	-30.8243	-30.8322	1.5458	-33.7274	-27.3299	1419.5
MFO	-22.6323	-22.5794	0.711159	-25.3752	-21.4409	1603.68
CS	-23.8208	-23.719	0.609572	-26.2173	-22.7224	1976.34
CSG	-25.7576	-25.7567	0.831377	-28.0545	-24.4725	456.64
CSG1	-27.2466	-27.06	0.970649	-30.6462	-25.7363	385.42
CSG2	-27.6244	-27.4838	1.00252	-31.4717	-26.1886	380.58
CSG3	-25.4542	-25.4056	1.04125	-28.3041	-23.6854	294.84
WOA	-29.2017	-29.1782	1.99596	-33.2183	-25.2228	1134.42

Table 16: Results for Sine Envelope Sine Wave

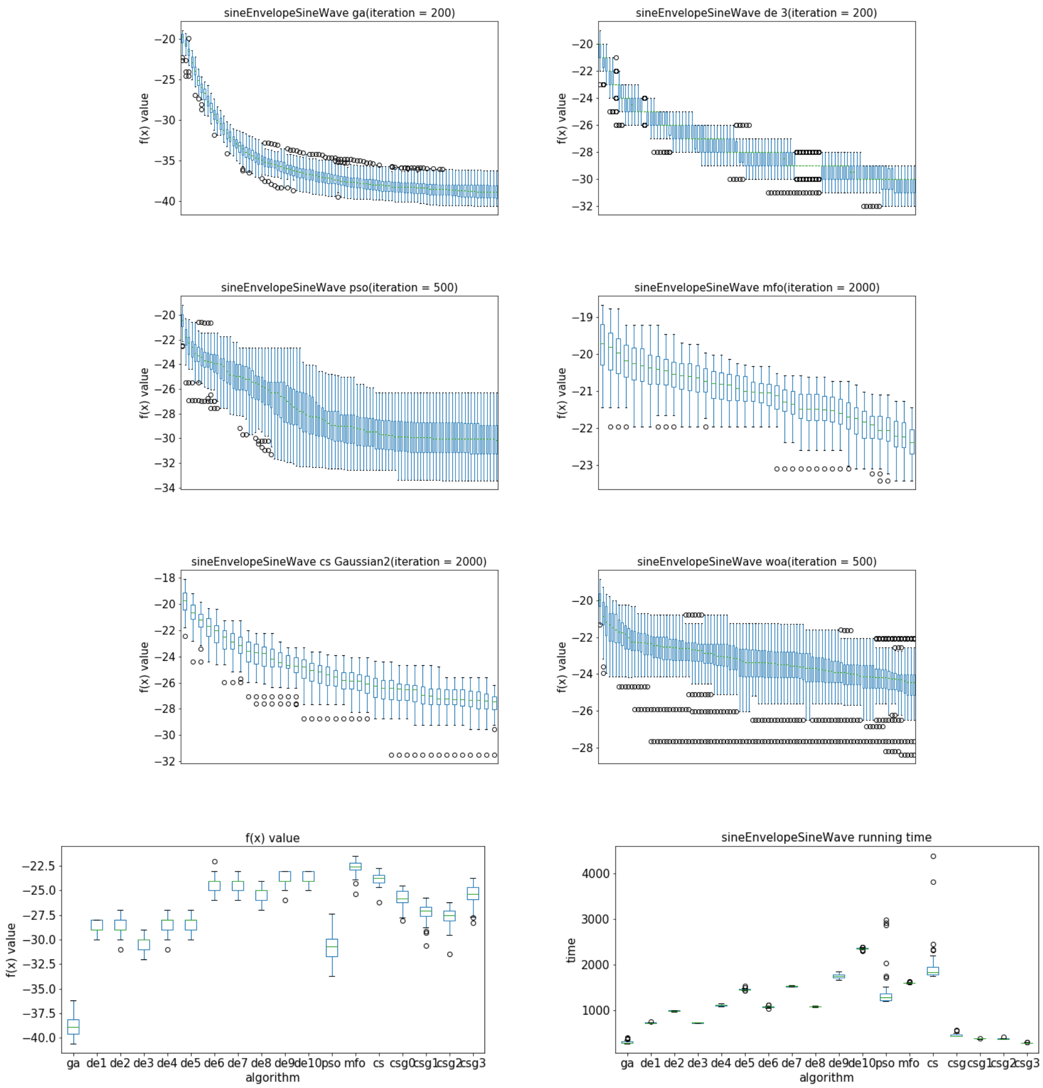

Figure 6: Results for Sine Envelope Sine Wave function

5.7 Stretched V Sine Wave

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	30.1287	30.042	0.520667	29.2761	31.5993	460.04
DE1	41.72	42	1.96	37	46	807.8
DE2	41.76	42	1.50413	35	45	1044.6
DE3	39.58	40	1.29754	37	43	804.36
DE4	42.42	43	1.48445	39	45	1192.46
DE5	42.34	42	1.25873	40	45	1516.12
DE6	52.08	52	3.49193	44	59	1137.86
DE7	52.5	53	1.73494	46	55	1564.52
DE8	48.56	48	2.40133	44	54	1162.62
DE9	54.32	54	2.29295	50	60	1791.46
DE10	54.08	54.5	2.18028	48	57	2347.08
PSO	30.9457	30.376	1.71878	29.0223	36.1815	1980.68
MFO	36.004	35.2345	3.5191	30.5548	43.7506	2280.84
CS	43.0424	43.0109	2.1146	38.3917	46.8022	2280
CSG	42.2551	42.5181	1.17864	38.5682	44.9525	568.52
CSG1	40.2375	40.2617	1.00822	38.0322	42.2373	455.7
CSG2	39.8372	39.9585	0.961172	37.0386	42.199	486.08
CSG3	41.8961	41.832	1.22126	39.3105	44.7782	311.8
WOA	33.962	33.5439	2.46315	29.9639	42.0306	1844.02

Table 17: Results for Stretched V Sine Wave

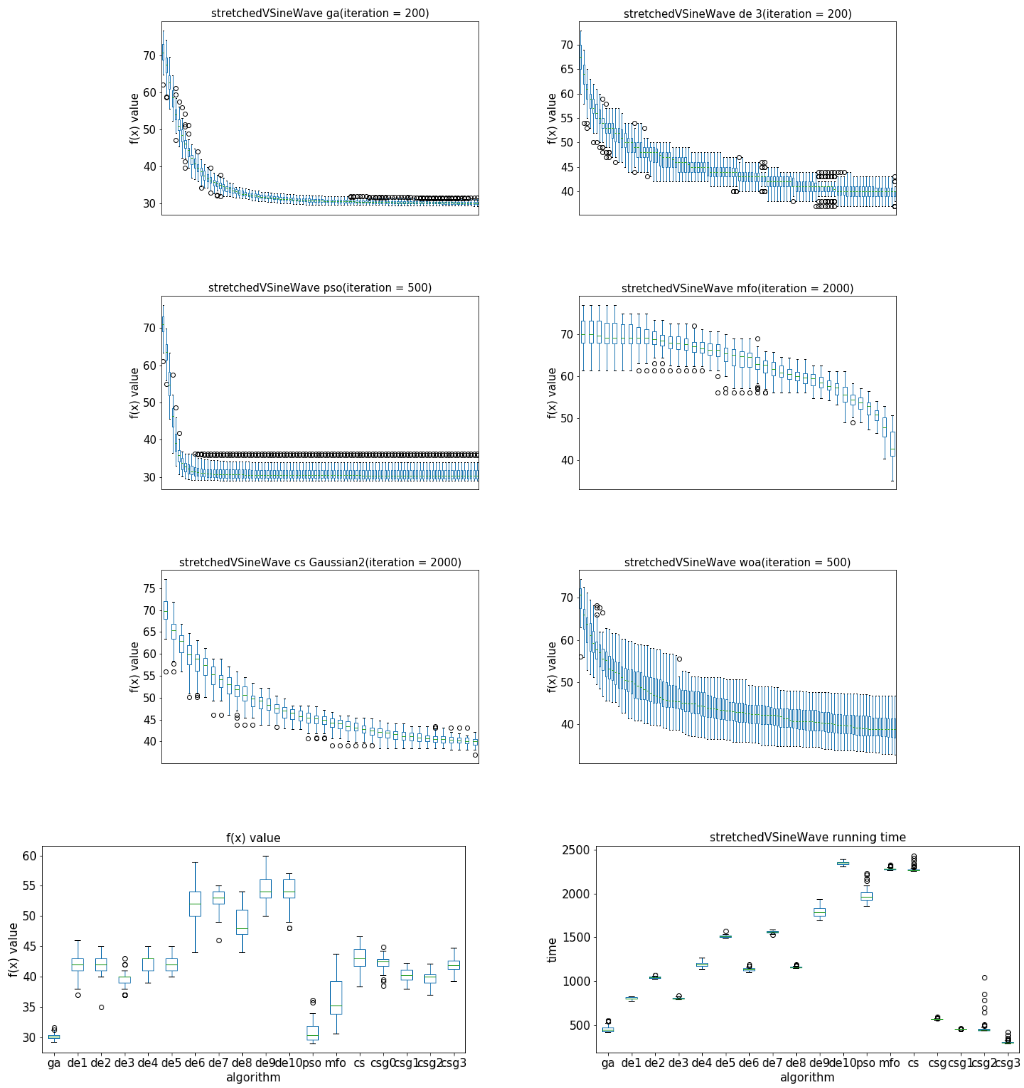

Figure 7: Results for Stretched V Sine Wave function

5.8 Ackley's One

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-40.6842	-44.1224	14.8503	-69.9276	-4.3997	398.74
DE1	-73.84	-75	5.97113	-83	-54	740.8
DE2	-2.46	-1	5.61502	-25	6	966.08
DE3	-43.64	-44	4.83636	-52	-28	769.4
DE4	5.46	4.5	7.16438	-17	20	1046.38
DE5	59.2	60.5	6.4436	44	70	1344.5
DE6	-49.86	-50.5	15.1947	-75	-11	1068.96
DE7	33.22	34	6.78613	10	47	1450.64
DE8	-75.76	-76.5	3.41503	-80	-63	1119.14
DE9	38.76	39	10.5633	15	61	1580.22
DE10	145.04	148.5	12.0399	112	164	2086.44
PS0	68.0722	72.9753	44.2199	-2.88217	160.005	1711.16
MFO	-23.1218	-24.8292	14.3918	-63.2611	9.20829	1947.34
CS	17.5691	17.463	10.5636	-7.19003	41.0382	2119.8
CSG	17.4481	17.8522	8.64625	-4.57381	35.3841	579.2
CSG1	12.7897	11.9038	8.04251	-0.790027	30.6617	457.76
CSG2	9.60244	10.2867	8.42987	-18.7279	26.184	410.7
CSG3	9.51631	11.133	11.0459	-15.8874	30.291	299.48
WOA	-63.7284	-66.6805	12.1979	-81.2184	-31.8776	1459.04

Table 18: Results for Ackley's One

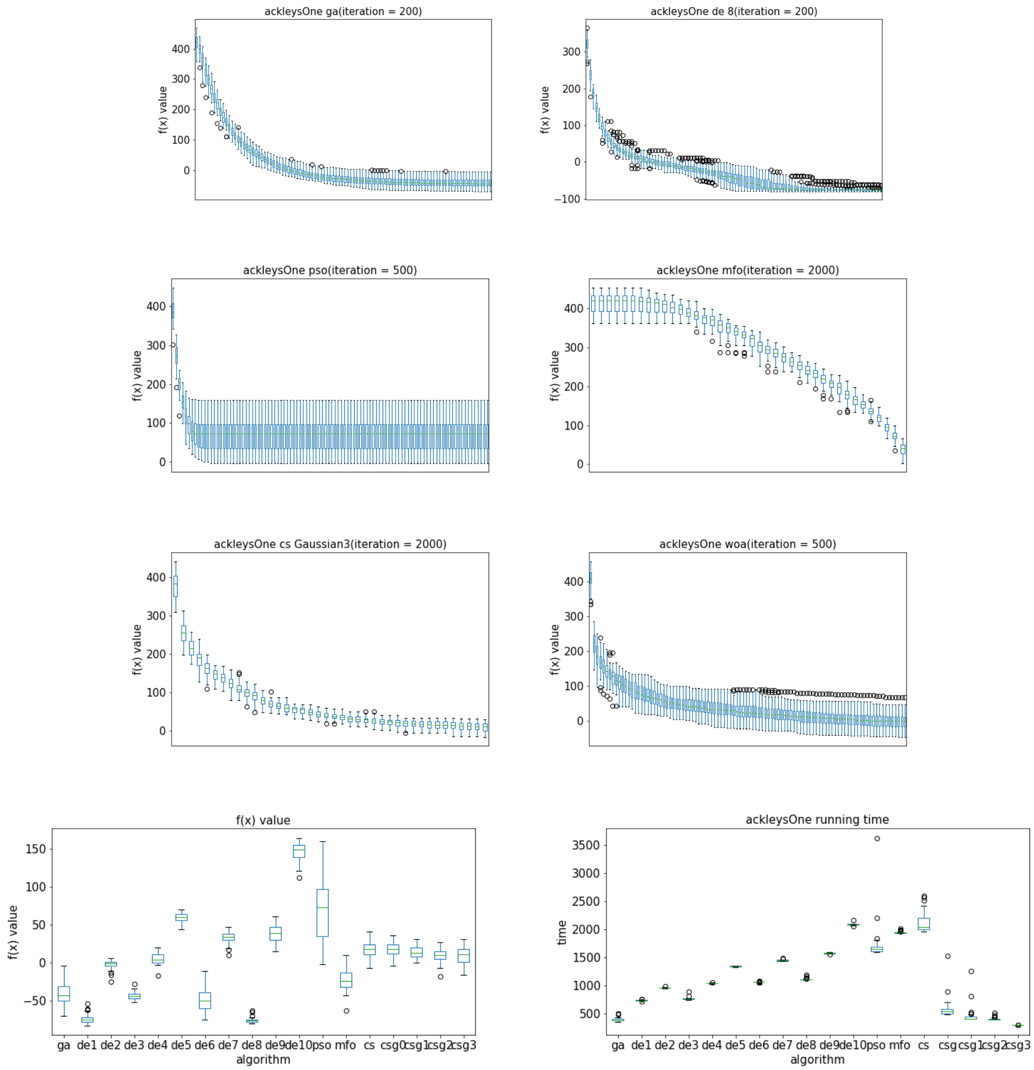

Figure 8: Results for Ackley's One function

5.9 Ackley's Two

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	133.586	132.211	21.5206	93.4007	181.566	422.92
DE1	34.66	30	12.743	29	88	755.38
DE2	80.94	81	3.8388	75	91	1004.78
DE3	29.74	30	0.82	29	33	804.26
DE4	80.9	81	6.56125	66	97	1081.68
DE5	235.3	234.5	9.41966	214	258	1380.78
DE6	67.22	69.5	26.7517	30	147	1078.72
DE7	126.58	126.5	6.40341	112	144	1488.8
DE8	32.98	30	7.08376	29	68	1131.58
DE9	95.64	95.5	15.7147	70	140	1616.18
DE10	376.3	380	13.2759	336	397	2155.32
PS0	271.719	272.37	48.8113	171.601	404.636	1814.88
MFO	188.088	184.429	44.0895	103.116	346.892	2194.94
CS	221.701	221.072	19.8724	180	264.631	2207.88
CSG	182.134	179.942	13.8731	156.513	217.592	540.32
CSG1	176.732	178.409	17.9353	136.804	213.831	438.68
CSG2	173.938	174.665	15.6316	143.755	208.23	418.6
CSG3	176.382	176.353	18.0855	136.578	220.766	300.96
WOA	29.7443	29.7343	0.0621078	29.6881	29.8793	1615.46

Table 19: Results for Ackley's Two

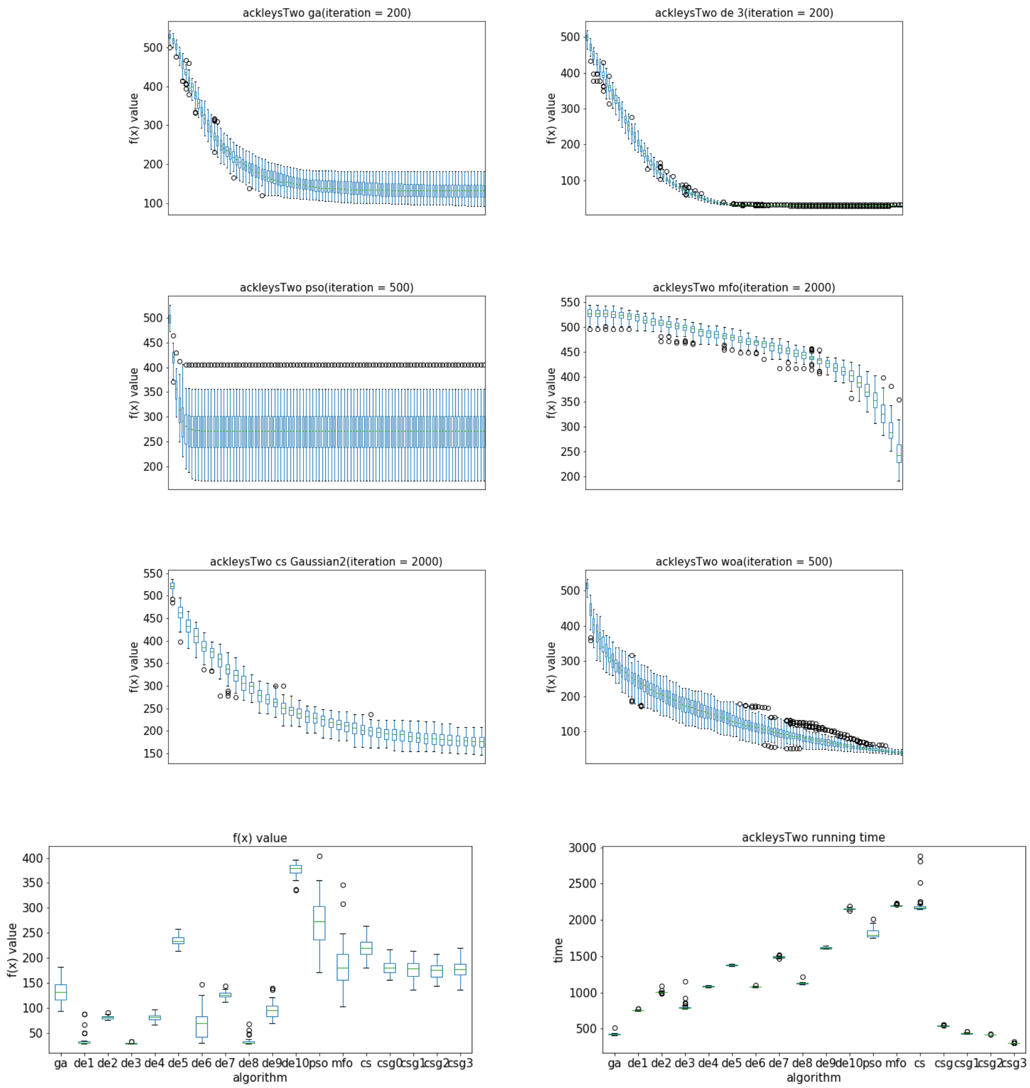

Figure 9: Results for Ackley's Two function

5.10 Egg Holder

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-17272.7	-17380.3	1091.95	-19267.8	-14366.6	344.58
DE1	-12622.6	-12472.5	1840.68	-19529	-9843	835.08
DE2	-9527.9	-9492	522.409	-11337	-8381	1112.78
DE3	-9064	-9070.5	564.985	-10512	-8020	811.26
DE4	-10238.1	-10181	535.434	-12391	-9402	1334.88
DE5	-9560.76	-9493	407.05	-10693	-8941	1648.38
DE6	-16139.9	-15987	1218.48	-19188	-14117	1094.56
DE7	-6319.88	-6246	448.459	-7504	-5612	1646.8
DE8	-6451.02	-6351	658.159	-8498	-5191	1174.82
DE9	-7304.56	-7200.5	614.045	-9054	-6243	1931.28
DE10	-6482.92	-6387	485.883	-8260	-5679	2532.08
PSO	-20236.8	-20414.2	1764.62	-23531.1	-16987.6	1406.46
MFO	-17439.4	-17603.8	1066.36	-19643	-14720.1	1901.4
CS	-26892.7	-26693	3812.24	-36039.2	-18723.3	1980.34
CSG	-7435.89	-7316.65	609.048	-8729.11	-6054.02	491.58
CSG1	-5437.88	-5382.01	497.192	-7216.03	-4467.64	414.2
CSG2	-5447.39	-5472.6	496.971	-6793.54	-4418.45	405.38
CSG3	-4526.74	-4487.76	561.248	-5813.92	-3245.88	292.4
WOA	-24242.5	-24409.6	1149.65	-25866.4	-20937.9	1435.28

Table 20: Results for Egg Holder

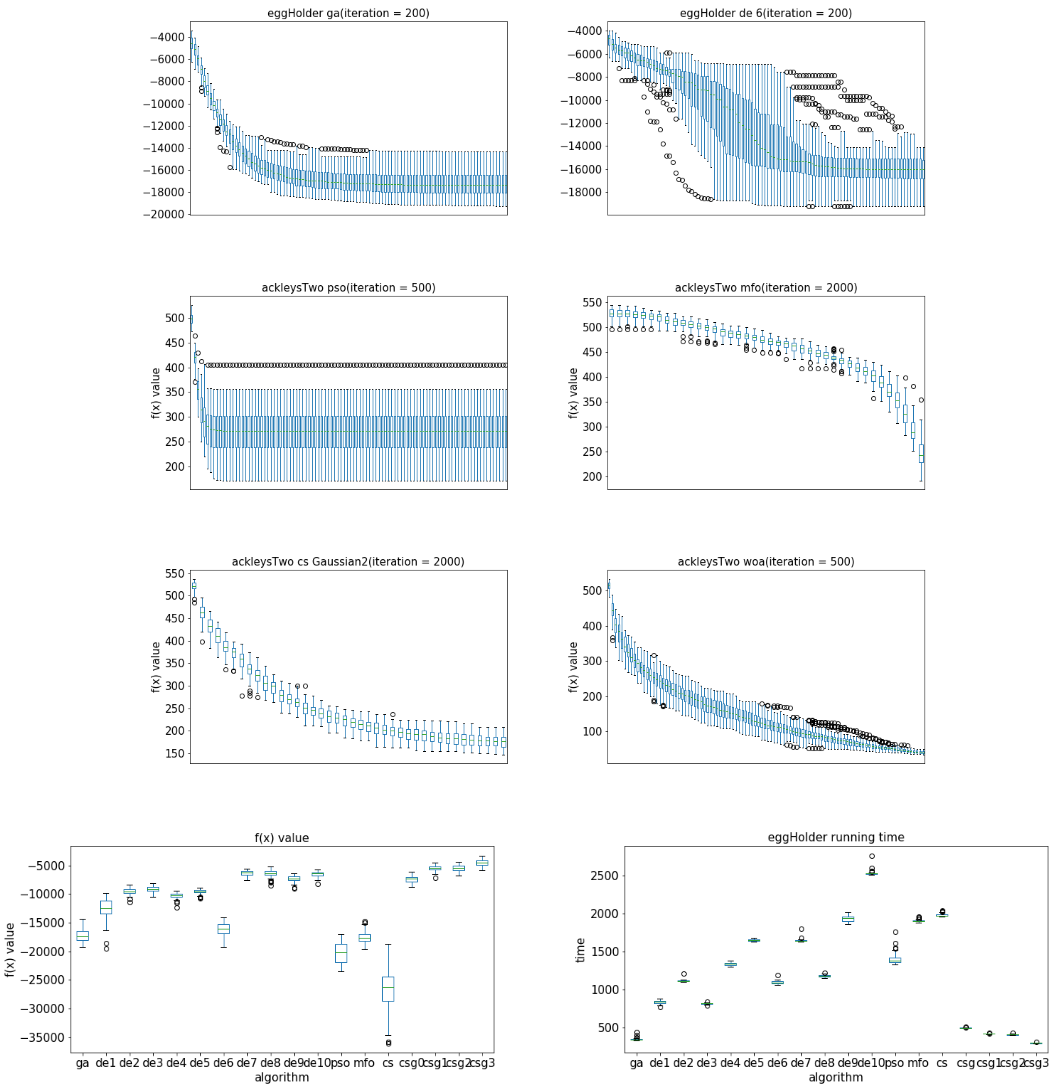

Figure 10: Results for Egg Holder function

5.11 Rana

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-10806.9	-10842.6	466.06	-11803.6	-9455.31	419.22
DE1	-6921.74	-6882.5	289.615	-7740	-6402	860.92
DE2	-6229.62	-6159.5	246.317	-6825	-5707	1121.9
DE3	-5980.32	-5996.5	274.006	-6705	-5259	830.28
DE4	-6561.34	-6528	290.008	-7371	-6002	1342.46
DE5	-6227.1	-6220	226.735	-6795	-5706	1655
DE6	-5514.84	-5471	705.905	-8132	-4595	1184.66
DE7	-4309.44	-4311	293.502	-5109	-3658	1655.42
DE8	-4286.62	-4258.5	363.627	-5446	-3722	1184.28
DE9	-4780.58	-4736	253.022	-5415	-4285	1953.88
DE10	-4385.7	-4318	300.093	-5454	-3891	2519.96
PSO	-13578.9	-13562.8	81.7008	-13850.3	-13463.9	1639.3
MFO	-13542.3	-13557.6	172.801	-13741.2	-12483.9	2022.7
CS	-12794.2	-12547.5	988.082	-15280.5	-11389.8	2146.02
CSG	-5645.13	-5640.4	474.101	-7111.98	-4619.97	533.86
CSG1	-3882.88	-3855.65	336.747	-4850.16	-3340.82	437.88
CSG2	-3797.86	-3710.36	419.526	-5386.72	-3111.4	421.92
CSG3	-3002.44	-2944.97	432.193	-4015.1	-2313.69	294.16
WOA	-13971.1	-13991.2	155.957	-14230.5	-13637.6	1641.28

Table 21: Results for Rana

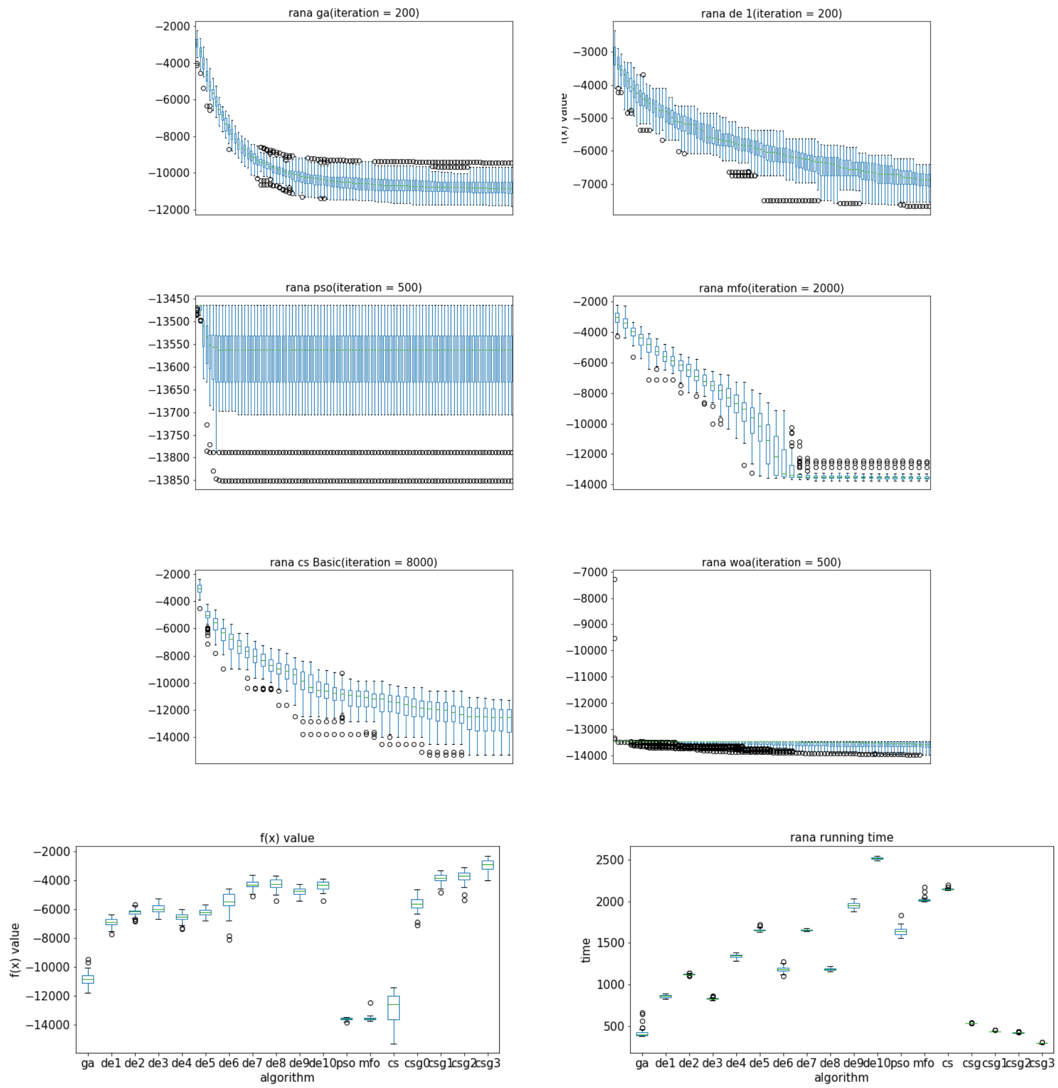

Figure 11: Results for Rana function

5.12 Pathological

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-13.7428	-13.7161	2.02614	-18.1283	-9.97208	344.34
DE1	0.44	0	0.80399	-1	3	756.58
DE2	1.56	2	0.92	-1	3	1014.5
DE3	0.52	0	0.727736	0	2	751.3
DE4	1.34	1	0.815107	-1	3	1169.34
DE5	1.66	2	0.764461	0	3	1511.32
DE6	3.92	4	1.56	0	7	1082.36
DE7	6.06	6	0.675574	5	7	1563.02
DE8	5	5	0.774597	3	6	1118.88
DE9	5.82	6	0.817068	4	8	1770.8
DE10	6.16	6	0.856971	4	7	2379.84
PSO	1.7744	1.61673	2.1524	-2.45413	6.99135	1466.88
MFO	-9.6468	-9.05438	1.8695	-16.4302	-6.65341	1716.24
CS	-5.59986	-5.28983	1.88067	-10.9379	-2.30982	1827.78
CSG	3.80814	4.06331	0.943962	1.43118	5.41819	458.68
CSG1	0.735516	0.929011	1.24477	-2.08742	3.29143	395.98
CSG2	-0.0368201	0.0044997	1.12373	-3.73864	2.57455	386.96
CSG3	0.924233	1.07809	1.52058	-2.54995	3.88231	294.4
WOA	-16.4097	-16.043	2.40194	-23.3872	-11.7032	1258.2

Table 22: Results for Pathological

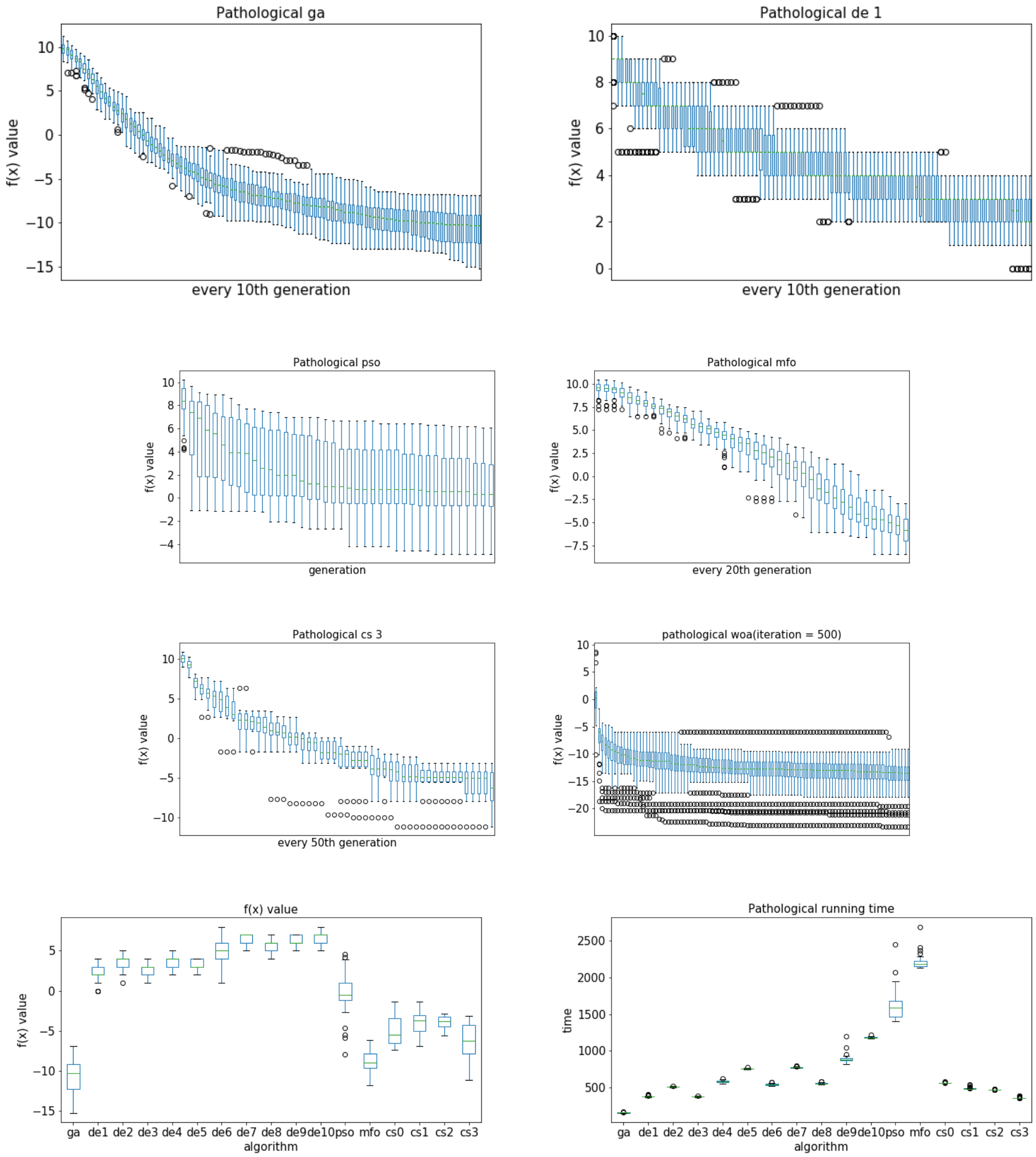

Figure 12: Results for Pathological function

5.13 Michalewicz

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-27.3242	-27.3095	0.518281	-28.2436	-26.3545	434.2
DE1	-16.18	-16	0.653911	-18	-15	790.66
DE2	-15.22	-15	0.60959	-17	-14	1043.66
DE3	-16.02	-16	0.616117	-18	-15	797.04
DE4	-14.68	-15	0.64622	-16	-14	1175.06
DE5	-14.54	-14	0.639062	-16	-14	1508.08
DE6	-13	-13	0.916515	-15	-11	1130.76
DE7	-11.86	-12	0.447661	-13	-11	1577.92
DE8	-13.08	-13	0.688186	-15	-12	1151.6
DE9	-11.32	-11	0.733212	-13	-10	1793.62
DE10	-11.14	-11	0.663626	-13	-10	2356.26
PSO	-13.986	-13.8096	1.83518	-18.9814	-11.2246	1583.32
MFO	-15.0251	-14.7642	1.6782	-18.5771	-11.9855	2155.16
CS	-14.1686	-13.9198	0.894458	-16.4578	-12.8318	2206.62
CSG	-12.5577	-12.4782	0.683028	-14.6661	-11.2692	552.5
CSG1	-12.7946	-12.6722	0.5618	-14.1556	-11.9103	449.36
CSG2	-12.7797	-12.7552	0.605906	-14.2362	-11.6675	429.26
CSG3	-11.5173	-11.4541	0.708627	-13.7144	-9.93078	294.78
WOA	-17.6906	-17.6462	1.58967	-20.9505	-14.4481	1718.08

Table 23: Results for Michalewicz

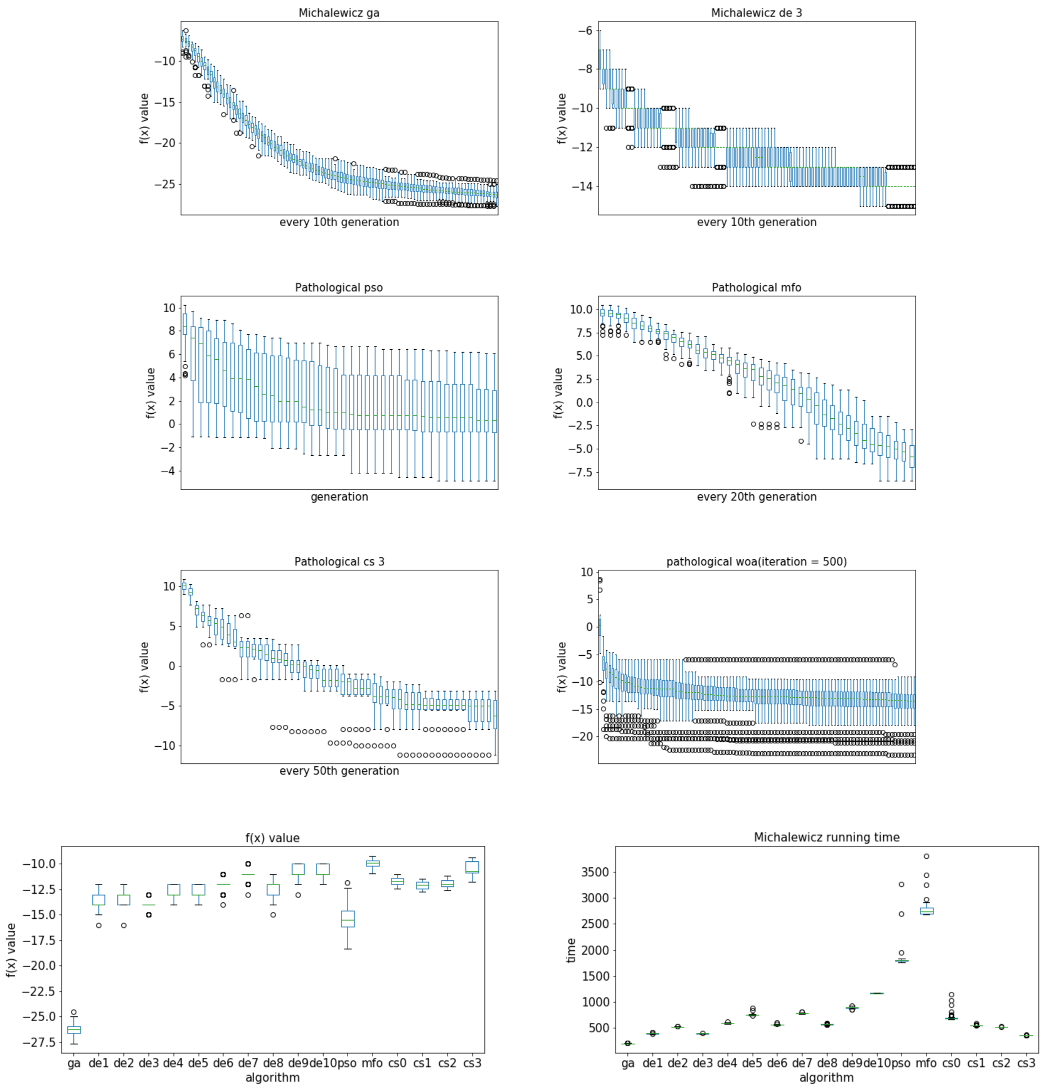

Figure 13: Results for Michalewicz function

5.14 Master's Consine Wave

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	-12.8914	-13.0983	2.11936	-16.9444	-8.19672	391
DE1	-6.3	-6	0.608276	-8	-5	778.9
DE2	-4.64	-5	0.557136	-6	-4	1020.84
DE3	-5.64	-6	0.52	-7	-5	778.5
DE4	-4.2	-4	0.447214	-6	-4	1165.64
DE5	-4.02	-4	0.423792	-5	-3	1487.84
DE6	-3.18	-3	0.712461	-5	-2	1158.74
DE7	-2.68	-3	0.507543	-4	-2	1564.28
DE8	-3.32	-3	0.507543	-5	-3	1137
DE9	-2.38	-2	0.485386	-3	-2	1830.1
DE10	-2.26	-2	0.438634	-3	-2	2377.38
PSO	-1.63977	-1.58492	0.330773	-2.5299	-0.97323	1647.34
MFO	-4.97637	-4.6903	1.00159	-7	-3.47618	2055.22
CS	-3.24952	-3.07935	0.471737	-4.84261	-2.61368	2075
CSG	-3.09135	-3.04168	0.399467	-4.15449	-2.21824	516.92
CSG1	-3.98917	-3.94674	0.473495	-5.03915	-3.16634	429.7
CSG2	-4.20339	-4.08156	0.543363	-5.9579	-3.37735	413.64
CSG3	-3.66577	-3.56542	0.693042	-7.14807	-2.89985	293.74
WOA	-26.3033	-29	4.44371	-29	-11.3634	1552.3

Table 24: Results for Master's Consine Wave

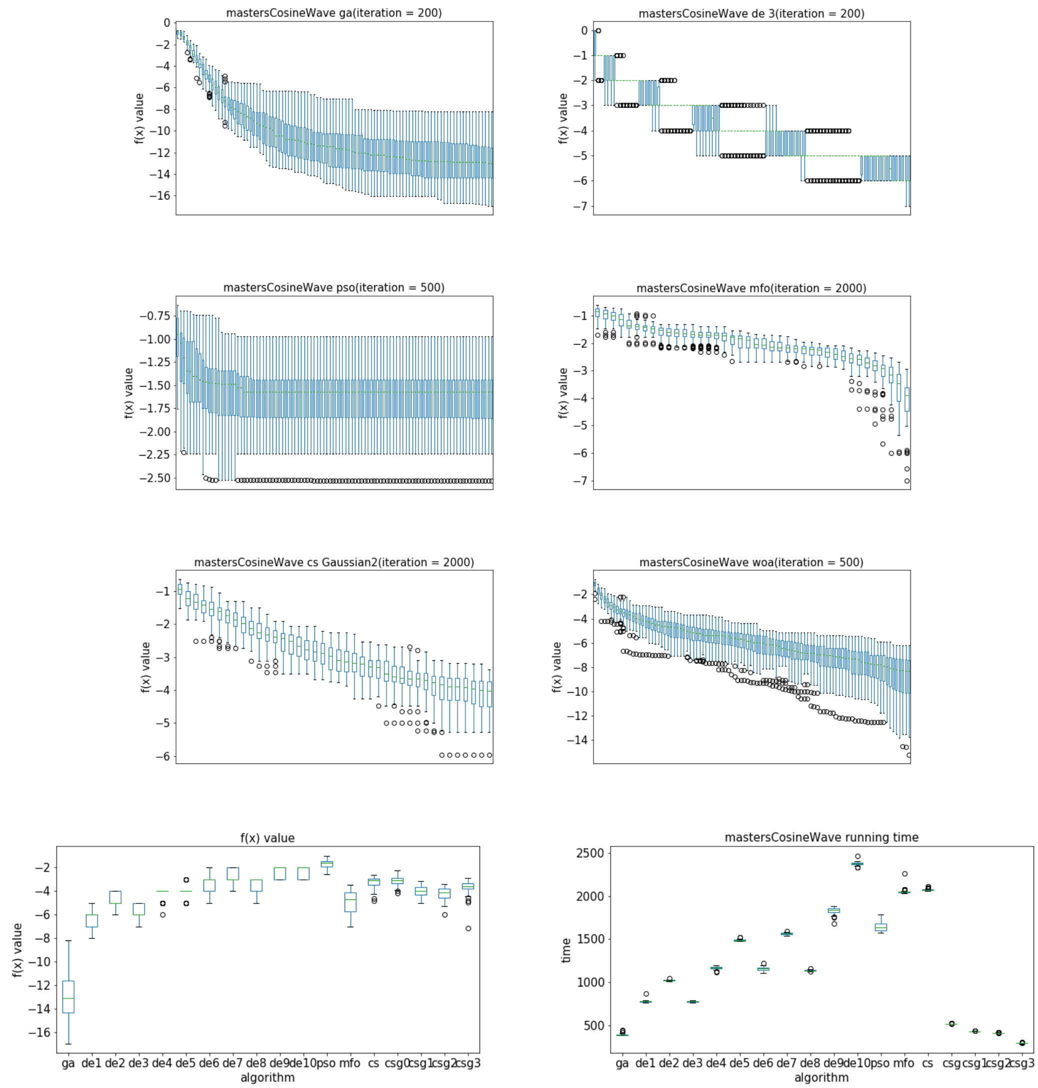

Figure 14: Results for Master's Cosine Wave function

5.15 Quartic

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	4.861e+07	3.24196e+07	5.59477e+07	4.05516e+06	3.08714e+08	305.38
DE1	0	0	0	0	0	698.84
DE2	127476	127692	37980.7	46664	215190	933.18
DE3	0	0	0	0	0	724.16
DE4	22178.6	17784	14381.7	2832	73806	1009.38
DE5	1.45695e+07	1.48634e+07	3.19486e+06	8.01007e+06	2.21765e+07	1313.34
DE6	0.14	0	0.346987	0	1	1037.82
DE7	1.21549e+06	1.18309e+06	321852	511087	2.62293e+06	1429.88
DE8	0.08	0	0.337046	0	2	1083.32
DE9	54875	33093	74223.7	4877	447640	1550.56
DE10	1.2305e+08	1.27472e+08	2.58943e+07	5.2815e+07	1.74217e+08	2092.84
PSO	2.35387e+08	2.00422e+08	2.62477e+08	1435.67	1.2e+09	1298.08
MFO	2.24617e+06	165755	1.39795e+07	26718.6	1.0009e+08	1639.16
CS	3.42597e+06	2.76366e+06	2.21697e+06	760168	8.52673e+06	1668.72
CSG	775830	606891	481542	146346	2.08824e+06	418.72
CSG1	584042	535834	329795	96821.2	2.06917e+06	364.76
CSG2	691353	523965	420505	85927.6	1.85951e+06	356.5
CSG3	762281	677146	466795	72008.3	2.49487e+06	294.92
WOA	0	0	0	0	0	1072.48

Table 25: Results for Quartic

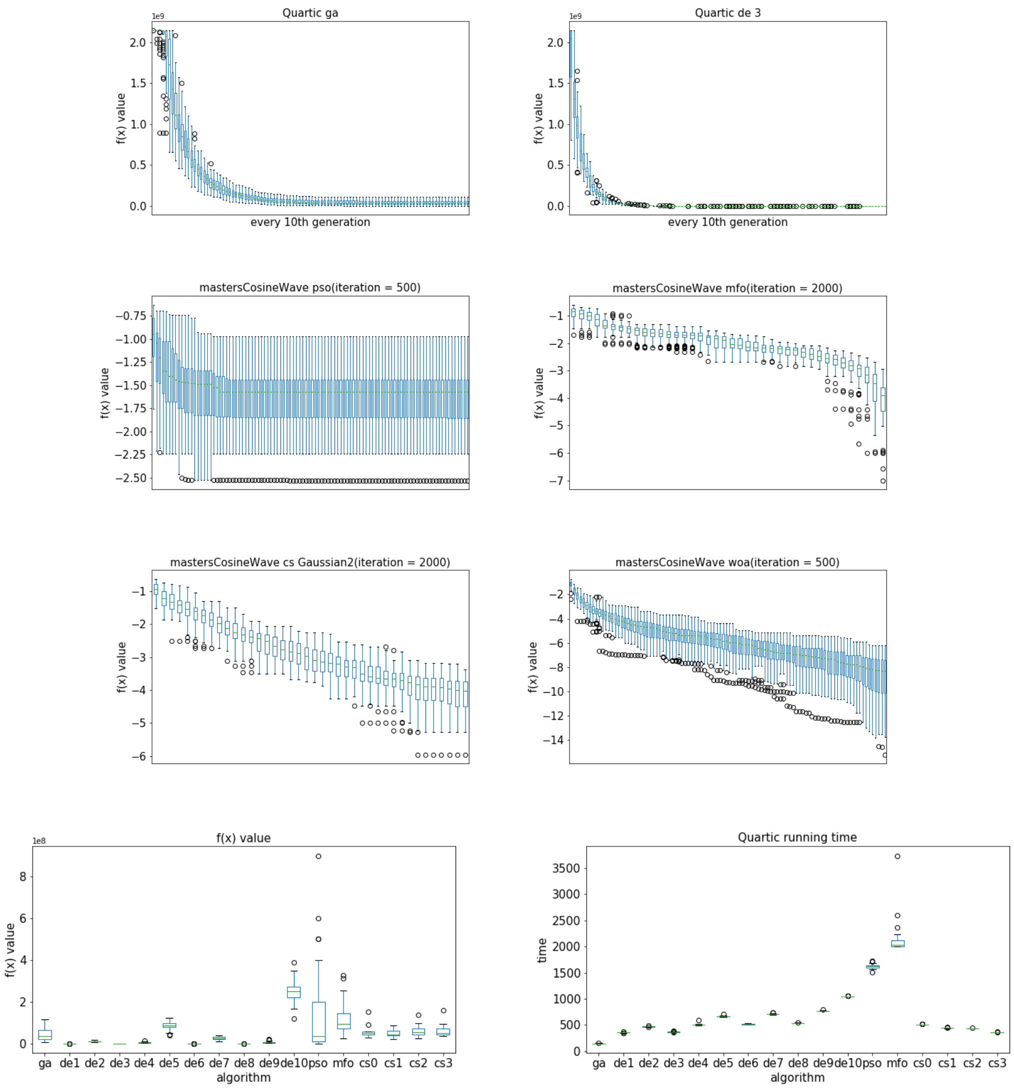

Figure 15: Results for Quartic function

5.16 Levy

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	5.5246	5.39293	2.22272	1.3127	11.4731	513.62
DE1	3.08	3	2.31378	0	11	767.54
DE2	1.46	1	0.498397	1	2	998.42
DE3	0	0	0	0	0	790.64
DE4	1.82	2	0.993781	0	5	1079.22
DE5	13.84	14	1.84781	10	18	1407.38
DE6	13.24	10.5	11.4901	1	55	1105.92
DE7	6.2	6	0.87178	4	8	1507.62
DE8	0.74	0	1.18	0	7	1154.58
DE9	9.54	8	7.87962	2	49	1638.6
DE10	41.06	41	4.8761	28	52	2191.82
PSO	26.3169	26.7163	8.71392	8.88689	58.7391	1915.98
MFO	9.25173	8.23516	5.49194	1.84777	31.4718	2331.18
CS	8.54	9.04002	3.04567	2.51758	14.8605	2207.1
CSG	4.61967	4.64413	1.39778	2.50004	8.86857	553.58
CSG1	5.21365	4.92608	2.14307	1.44442	13.2173	442.22
CSG2	5.71375	5.15724	2.4024	2.11104	14.1057	423.46
CSG3	8.4039	6.9788	4.37741	2.7157	21.7341	297.78
WOA	0.687824	0.72435	0.183576	0.270877	1.00439	1784.36

Table 26: Results for Levy

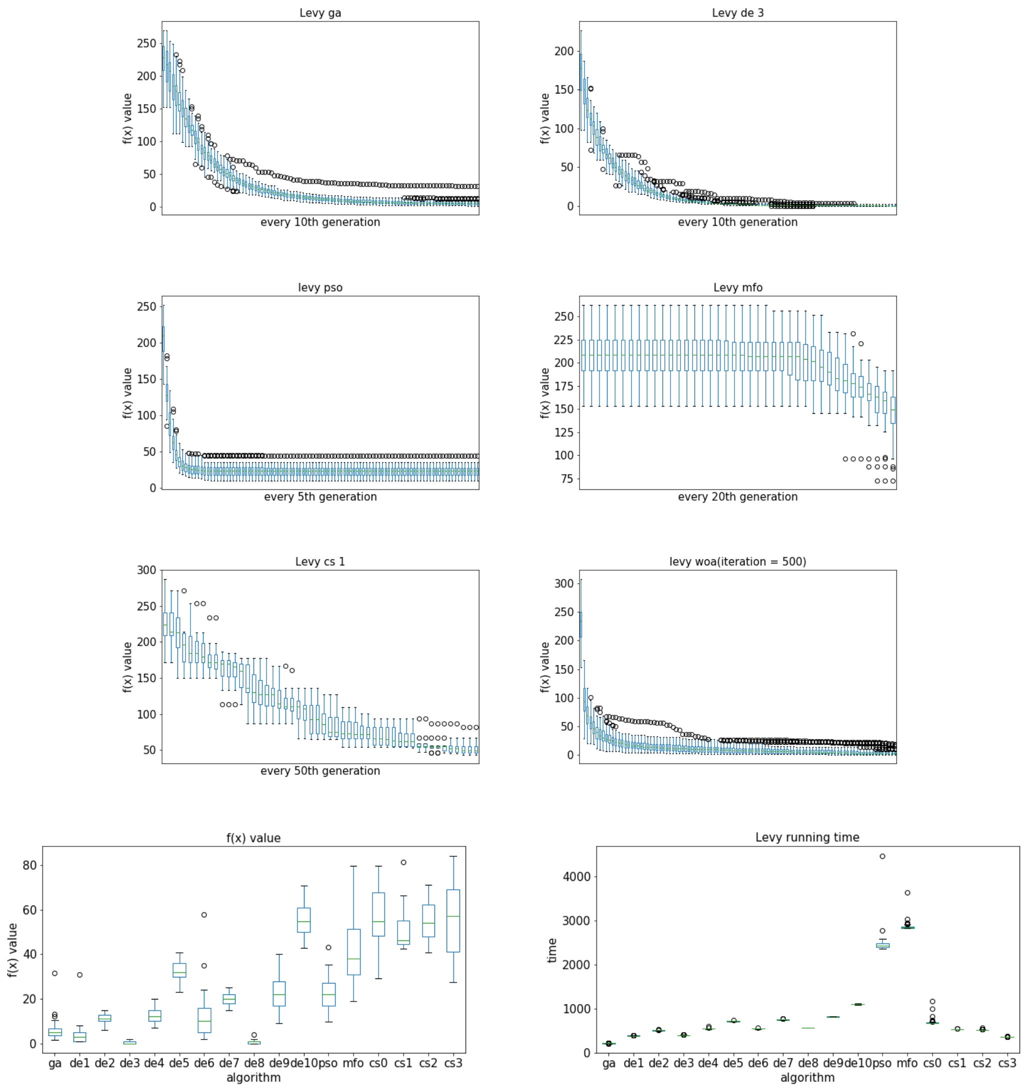

Figure 16: Results for Levy function

5.17 Step

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	2055.2	2005.97	1014.85	202.112	6041.29	247.7
DE1	8.1	8	0.3	8	9	674.02
DE2	112.9	114.5	13.0495	74	144	897.54
DE3	7.82	8	0.384187	7	8	700.38
DE4	53.68	52.5	13.0299	32	90	974.76
DE5	2253.54	2294	307.628	1202	2826	1274.76
DE6	10	10	0.916515	9	12	998.58
DE7	252.88	250.5	32.9464	186	344	1386.74
DE8	8.08	8	0.271293	8	9	1051.98
DE9	47.92	44.5	14.0653	27	88	1504.5
DE10	8701.14	8648	801.013	6415	10239	2001.98
PSO	3855.12	13.6614	6960.21	7.59859	30307.5	1045.82
MFO	2741.05	2774.96	880.051	834.494	4947.81	736.28
CS	1865	1963.65	602.451	768.055	3255.83	1460.78
CSG	978.475	956.223	232.831	558.891	1737.81	362.78
CSG1	796.993	816.324	219.934	325.522	1330.07	333.58
CSG2	841.136	818.119	247.938	396.18	1360.8	330
CSG3	959.082	936.342	264.999	534.529	1747.09	296.58
WOA	7.5	7.5	1.65181e-07	7.5	7.5	801.9

Table 27: Results for Step

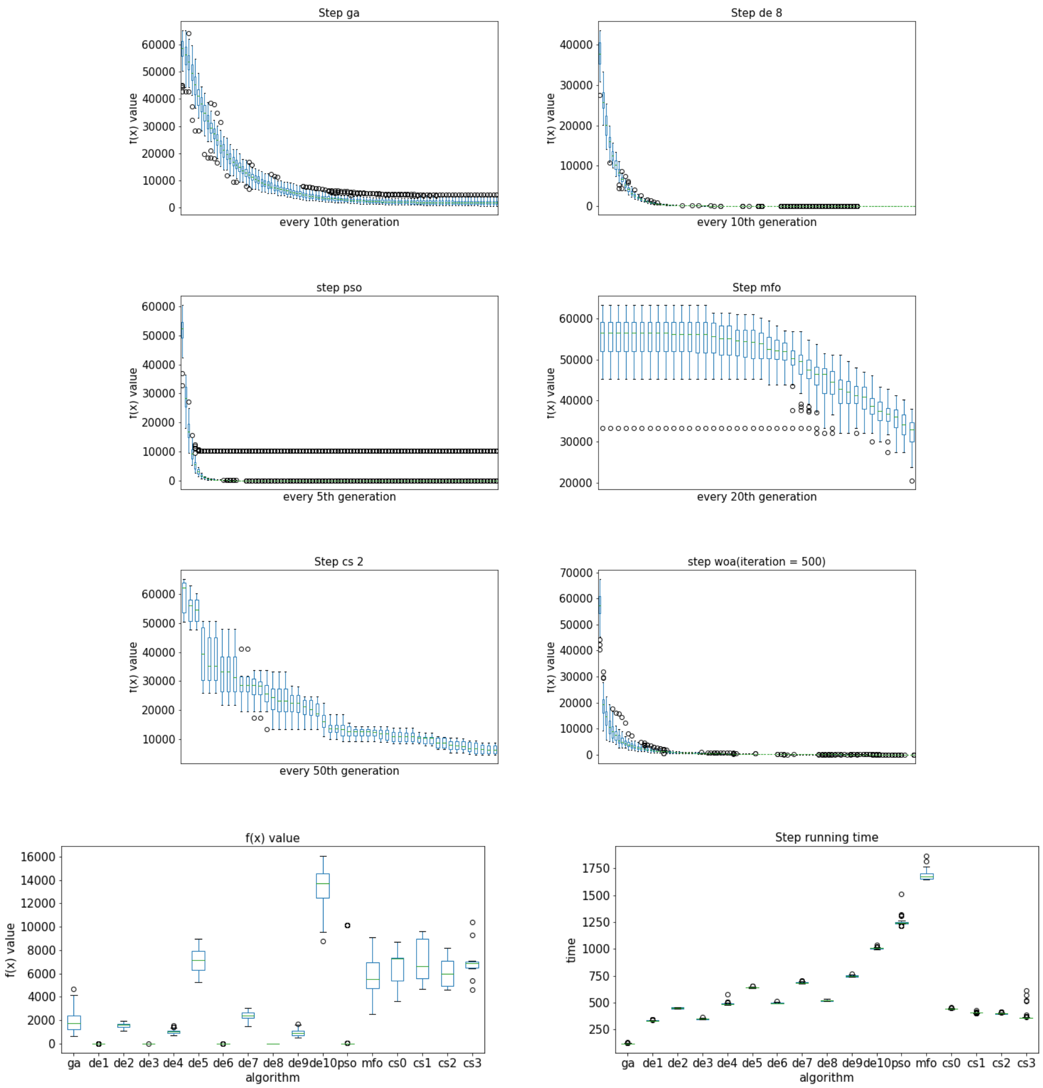

Figure 17: Results for Step function

5.18 Alpine

Strategy	Mean	Median	Std	low	high	Time(mus)
GA	10.9389	10.8302	3.19496	4.91395	21.5856	274.84
DE1	35.16	35	17.7937	1	78	688.18
DE2	88.18	89	7.1125	70	104	922.54
DE3	20.68	19	6.86277	9	40	712.94
DE4	123.66	122	15.6456	88	156	1023.98
DE5	154.44	155.5	11.5398	131	177	1332.32
DE6	80.58	75.5	57.9093	1	311	1027.06
DE7	148.1	150	10.4408	109	170	1416.7
DE8	10.42	6.5	11.7611	0	49	1066.92
DE9	201.58	200.5	33.0552	142	279	1581.62
DE10	264.18	265	18.8464	212	303	2115.18
PSO	68.5264	46.0625	64.9844	0.00378576	284.456	1238.84
MFO	195.437	192.788	50.277	88.7991	315.518	1500.8
CS	63.649	63.3767	9.99975	44.9947	82.1508	1654.48
CSG	60.9393	61.5391	7.35845	44.2559	78.3244	412.26
CSG1	54.665	55.2829	7.55401	41.4199	71.6279	367.48
CSG2	52.8558	52.9469	6.37478	41.6003	66.5181	357.28
CSG3	58.0249	58.0492	8.72982	36.7332	73.8833	294.38
WOA	55.1107	52.2699	23.0505	9.23946	160.26	1005.52

Table 28: Results for Alpine

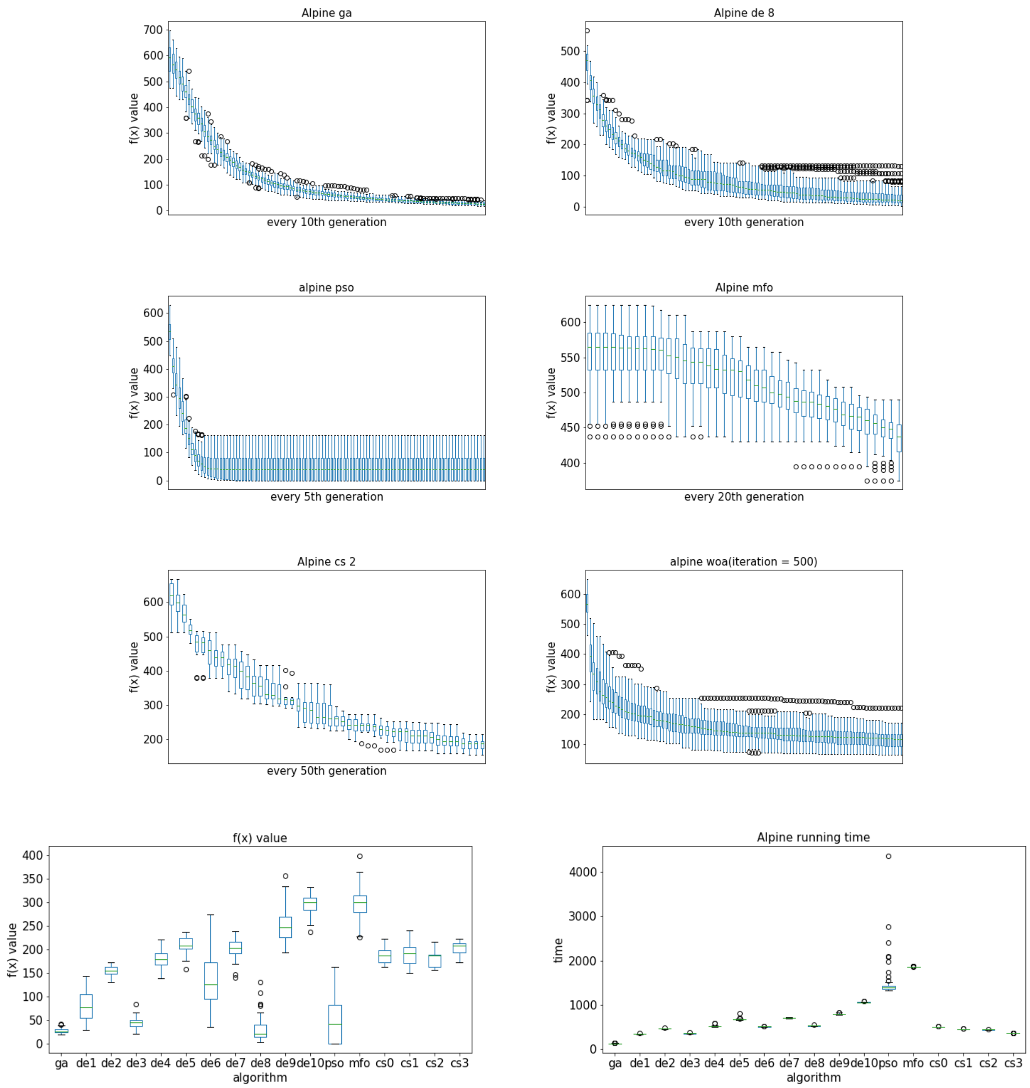

Figure 18: Results for Alpine function

6 Conclusion

Function	Best Mean	Best Low
Schwefel	CS	WOA
1st De Jong	DE/rand-to-best/1/., DE/best/1/., WOA	DE/rand-to-best/1/., DE/best/1/., WOA
Rosenbrock	WOA	WOA
Rastrigin	DE, WOA	DE, WOA
Griewangk	DE/best/./., DE/rand-to-best/./., MFO, WOA	PSO,WOA
Sine Envelope Sine Wave	GA	GA
Stretched V Sine Wave	GA, PSO	GA,PSO, MFO, WOA
Ackley's One	DE/best/1/exp, WOA	DE/best/1/exp, WOA
Ackley's Two	DE/rand-to-best/1/.,WOA	WOA
Egg Holder	WOA, CS	WOA
Rana	WOA	CS
Pathological hline Michalewicz	WOA GA	WOA GA
Master's Consine Wave	WOA	WOA
Quartic	DE/rand-to-best/1/., DE/best/1/., WOA	WOA
Levy	DE/rand-to-best/1/., WOA	DE/rand-to-best/1/., DE/best/./exp, WOA
Step	DE/rand-to-best/1/., DE/best/1/exp, WOA	DE/rand-to-best/1/., DE/best/1/., WOA
Alpine	DE/rand-to-best/1/bin	DE/best/1/.,DE/rand-to-best/1/bin, PSO

Table 29: Differential Evolution algorithm Strategies

6.1 GA and DE

Overall differential evolution algorithm performs better for half of the functions. Most commonly seen better strategies for differential algorithms uses the best or rand-to-best for vector to be perturbed which produced better fitness, and choosing 1 difference vector for perturbation which both produced better fitness and lower execution time. Binomial crossover has a comparatively lower execution time compare to exponential crossover due to the time cost for random number generation.Execution time of differential evolution algorithm can be improved by implementing parallelism.

6.2 PSO and MFO

Both algorithms has a low cost for several benchmark functions. They both have a comparatively good result for functions does not have known global optimal.

6.3 CS and CSG

Time for cuckoo search to reach a stable cost is longer compare to other algorithms done in this experiment. A possible reason is that for each generation, fitness improvement relies on one new solution accepted based on function cost, and the worst solution abandoned are substituted with global random solution. Trade-off between diversification and intensification can be improved for basic cuckoo search. Two possible improvement that can be tried are using Gaussian normal distribution and Gamma distribution instead of levy flight to determine the random walk size. In the basic cuckoo search, u and v chosen for step size calculation is drawn from $Norm(0, \sigma^2)$ and $Norm(0, 1)$ for Levy flight, in Gaussian distribution X_i is updated by drawing from $Gaussian(Best, \sigma)$, which could improve the intensification of the search. From the data we can see that their is a significant time improvement of using gaussian random walk. σ_0 that changes the step size contribute most to the improvement.

Three strategies that changes pa doesn't have significant difference in running time and cost of the function compare to the basic cuckoo search and each other. Thus changing switching parameter p_a might not have a significant improvement upon cuckoo search.

6.4 WOA

Whale optimization algorithm has achieved most of the best mean value and lowest value.