

Abstract

This report is the discuss the procedure of designing a FIR filter to satisfy the given properties. Kaiser window is used for the implementation. All the operations were done in MATLAB 2016a (By mathworks).

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2.Filter specification

Property	Symbol	Value	Units
Maximum passband ripple,	\tilde{A}_p	0.06	dB
Minimum stopband attenuation,	\tilde{A}_a	49	dB
Lower passband edge,	Ω_{p1}	1200	rad/s
Upper passband edge,	Ω_{p2}	1500	rad/s
Lower stopband edge,	Ω_{a1}	1050	rad/s
Upper stop band edge,	Ω_{a2}	1600	rad/s
Sampling Frequency,	Ω_s	4000	rad/s

Table 1;Properties derived from index number

Parameter	Symbol	Method	Value	Unit
Lower transition width	B_{t1}	$\Omega_{p1} - \Omega_{a1}$	150	rad/s
Upper transition width	B_{t2}	$\Omega_{a2} - \Omega_{p2}$	100	rad/s
Critical transition width	B_t	$\text{Min}(B_{t1}, B_{t2})$	100	rad/s
Lower cutoff frequency	ω_{c1}	$\Omega_{p1} - B_t/2$	1150	rad/s
Upper Cutoff Frequency	ω_{c2}	$\Omega_{p1} + B_t/2$	1550	rad/s
Sampling Period	T	$2\pi/ \Omega_s$	1.570×10^{-3}	s

Table 2; derived parameters from table 1

3.Method

Step 1

Construct the impulse response of the bandpass filter $h(nT)$ using ideal conditions.

$$H(e^{j\omega T}) \begin{cases} 1 & ; \quad \omega_{c1} < |\omega| < \omega_{c2} \\ 0 & ; \quad \text{Otherwise} \end{cases}$$

Step 2

Choose δ such that actual pass band attenuation is less than or equal maximum pass band attenuation (\tilde{A}_p) and stopband attenuation is higher or equal than minimum stop band attenuation (\tilde{A}_a).

$$\delta = \min(\delta_p, \delta_a) \quad \delta_p = (10^{0.05 \tilde{A}_p} - 1) / (10^{0.05 \tilde{A}_p} + 1) \quad , \delta_a = 10^{-0.05 \tilde{A}_a}$$

Step 3

Actual stopband attenuation A is calculated as

$$A_a = -20 \log_{10}(\delta)$$

Step 4

Construct alpha such that

$$\alpha = \begin{cases} 0 & ; \text{for } A_a < 21 \\ 0.5842(A_a - 21)^{0.4} + 0.07886(A_a - 21) & ; \text{for } 21 < A_a < 50 \\ 0.1102(A_a - 8.7) & ; \text{for } A_a > 50 \end{cases}$$

Step 5

Choose D and N as following

$$D = \begin{cases} 0.9222 & ; \text{for } A_a \leq 21 \\ \frac{A_a - 7.95}{14.36} & ; \text{for } A_a > 21 \end{cases}$$

$$N \geq \frac{\omega_s D}{B_t} + 1$$

Step 6

Form kaiser window function as follows

$$w_k(nT) \begin{cases} \frac{I_0(\beta)}{I_0(\alpha)} & \text{for } |n| \leq N \\ 0 & \text{otherwise} \end{cases}$$

$$\beta = \sqrt{1 - \left(\frac{n}{M}\right)^2} \quad M = (N-1)/2$$

$$I_0(X) = 1 + \sum_{k=1}^{\infty} \left(\frac{1}{k!} \left(\frac{X}{2} \right)^k \right)^2$$

Step 7

Form the impulse response of the band pass filter

$$h_w(nT) = w_k(nT) \times h(nT)$$

Parameter	Value
δ_p	3.45×10^{-3}
δ_a	3.54×10^{-3}
Aa	49.236dB
α	4.45
D	2.87
N	117
M	58

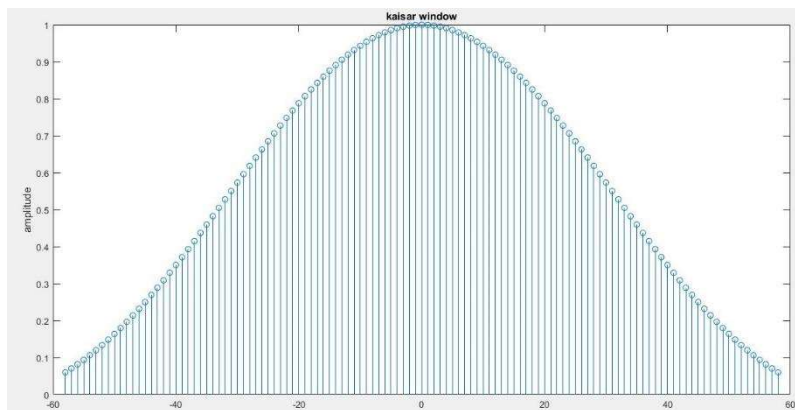


Figure 1.Kaiser window for N=117

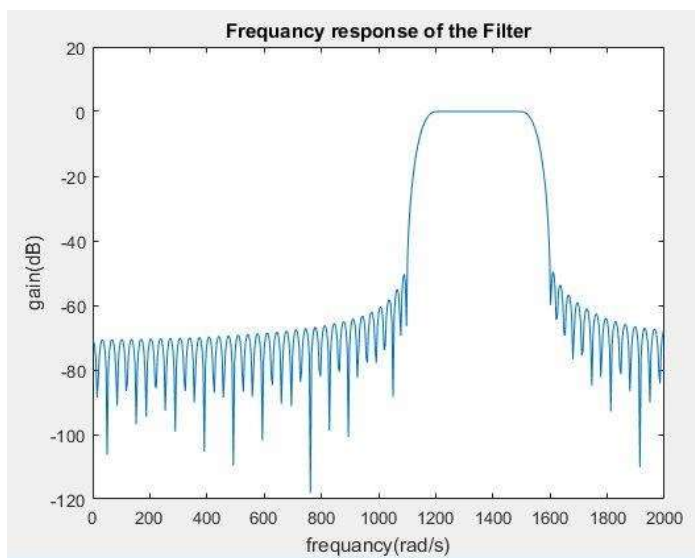


Figure 2. Frequency responseof the FIR filter

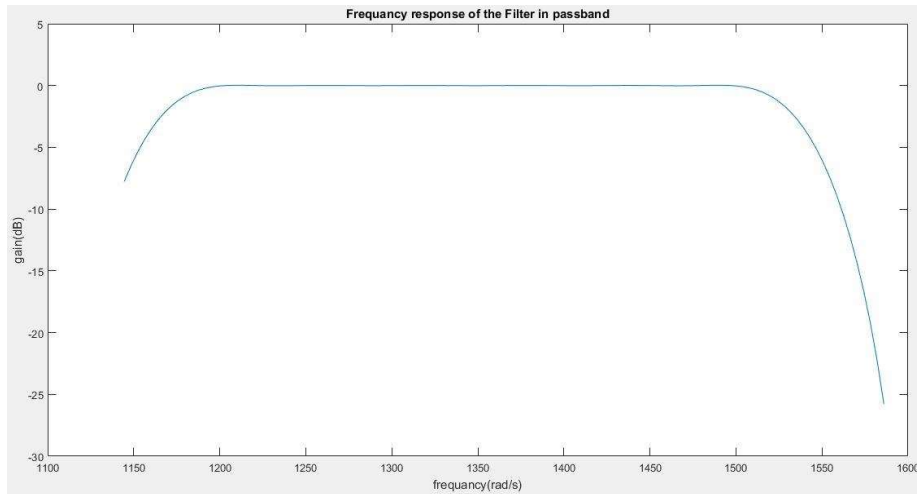


Figure 3 passband of the FIR filter

4. Testing the filter

Testing of the filter was done by giving a sum of three sinusoids as input.

$$\sum_{i=1}^3 \sin(\Omega_i nT)$$

Where Ω_1 is the mid frequency of the lower stop band, Ω_3 is the mid frequency of upper stopband and Ω_2 is the mid frequency of the passband.