Discrete Maths :

I) Mathematical Logic

Proposition : stmt that can be true or false

- **Simple**: which can't be broken down
- **Compound**: which can be broken down to multiple simple stmts. → joined by
  - Modifier: Not
  - Connectives: AND ($\wedge$) But, OR ($\vee$), Conditional ($\rightarrow$), biconditional ($\leftrightarrow$)

*) n→proposition variable.

$\boxed{\text{Num of proposition fns} = 2^{2^n}}$

Implication : $p \rightarrow q$

Premise	Conclusion
antecedent	Consequent

*) If p then q	*) q if p
*) p Implies q	*) q when p
*) p only if q	*) q whenever p
*) if p, q	*) q unless $\neg p$

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

*) Implication : $p \rightarrow q$
Converse : $q \rightarrow p$
Inverse : $\neg p \rightarrow \neg q$
Contrapositive : $\neg q \rightarrow \neg p$.

Converse & Inverse: logically Equivalent
Implication & Contrapositive: logically Equivalent

def: Two Compound stmts if they have same behavior then they are logically equivalent

Satisfiable Expression : Atleast one true in the result
Tautology / valid Expr : Always true in the result — Every tautology is satisfiable.
Contradiction / absurdity : Always False in the result.
Contigency : If a Expr is neither tautology nor Contradiction.

Biconditional : $\boxed{p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)}$
*) iff
*) if and only if

Rules of Mathematical logic :

① $p \wedge T \equiv p$
$p \vee F \equiv p$

② $p \vee T \equiv T$
$p \wedge F \equiv F$

③ $p \vee p \equiv p$
$p \wedge p \equiv p$

④ $\neg(\neg p) \equiv p$

⑤ $p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$

⑥ $p \vee (q \vee r) \equiv (p \vee q) \vee r$
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

⑦ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

⑧ $\neg(p \vee q) \equiv \neg p \wedge \neg q$
$\neg(p \wedge q) \equiv \neg p \vee \neg q$

⑨ $p \rightarrow q \equiv \neg p \vee q$
Imp $p \rightarrow q \equiv \neg q \rightarrow \neg p$
$\neg p \rightarrow q \equiv p \vee q$

⑩ $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow q) \wedge (r \rightarrow q) \equiv (p \vee r) \rightarrow q$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow q) \vee (r \rightarrow q) \equiv (p \wedge r) \rightarrow q$

⑪ $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$\boxed{p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)}$

⑫ $p \wedge \neg p \equiv F$
$p \vee \neg p \equiv T$

Rules of Inference : Templates for constructing valid arguments.

→ Considering all the premises are true then the conclusion must be true.

$$\left.\begin{array}{l} P_1 \\ P_2 \\ \vdots \\ P_n \end{array}\right\} \rightarrow \text{premises} \quad \Rightarrow \quad (P_1 \wedge P_2 \wedge \cdots \wedge P_n) \longrightarrow Q$$

$\underline{\qquad}$

$Q \longrightarrow$ Conclusion

Arguments.

Types of Inference rules :

① Modus ponens :

$$\boxed{\begin{array}{c} P \rightarrow q \\ P \\ \hline q \end{array}}$$

$((P \rightarrow q) \wedge P) \rightarrow q$

② Modus Tollens :

$$\boxed{\begin{array}{c} P \rightarrow q \\ \sim q \\ \hline \sim p \end{array}}$$

$((P \rightarrow q) \wedge \neg q) \rightarrow \neg p$

③ Hypothetical syllogism :

$$\boxed{\begin{array}{c} P \rightarrow q \\ q \rightarrow r \\ \hline P \rightarrow r \end{array}}$$

$((P \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (P \rightarrow r)$

④ Disjunctive Syllogism :

$$\boxed{\begin{array}{c} P \vee q \\ \sim p \\ \hline q \end{array}}$$

$((P \vee q) \wedge \sim p) \rightarrow q$

⑤ Addition :

$$\boxed{\begin{array}{c} P \\ \hline P \vee q \end{array}}$$

$P \rightarrow (P \vee q)$

⑥ Simplification :

$$\boxed{\begin{array}{c} P \wedge q \\ \hline P \end{array} \quad (\text{or}) \quad \begin{array}{c} P \wedge q \\ \hline q \end{array}}$$

$P \wedge q \rightarrow P$ (or) $P \wedge q \rightarrow q$

⑦ Conjunction :

$$\boxed{\begin{array}{c} P \\ q \\ \hline P \wedge q \end{array}}$$

$(P) \wedge (q) \rightarrow P \wedge q$

⑧ Resolution :

$$\boxed{\begin{array}{c} P \vee q \\ \sim p \vee r \\ \hline q \vee r \end{array}}$$

$(P \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

*) If A then B else C ≡ if A then B and If ¬A then C

Imp:

$$(A \rightarrow B) \wedge (\neg A \rightarrow C)$$

Inconsistency :

A set of premises is said to be inconsistent, if all the premises cannot be simultaneously true ie

The conjunction of all the premises is a contradiction.

$$(P_1 \wedge P_2 \wedge \cdots P_n) \Leftrightarrow F$$

Imp

*) In any argument if the premises are inconsistent then the argument is valid.

Imp

Functional completeness :

A set of connectives is said to be functionally complete, if every propositional fn is equivalent to a propositional fn using connectives from the set

Ex :- $\{\vee, \sim\}$, $\{\wedge, \sim\}$, $\{\rightarrow \sim\}$, $\{\wedge, \vee, \sim\}$

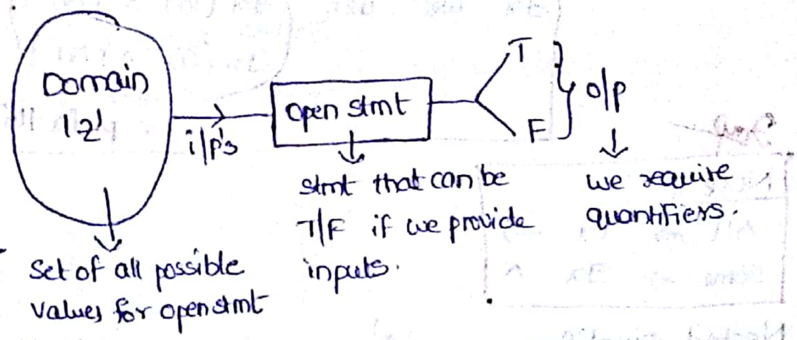
(*) A set is said to be functionally complete if we can derive a set which is already functionally complete.

Predicate logic

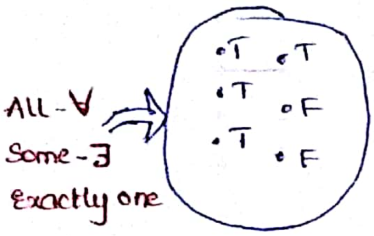
:- x $\boxed{\text{is a Even number}}$: openstmt
↳ property ⇒ predicate

P(x)
↓ Predicate
↳ variable.

Notation ←



Quantifiers : Based on Quantity

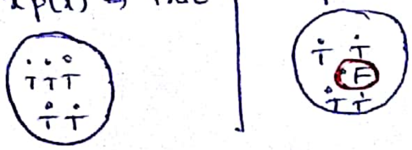


Quantifiers
- Universal Quantifier (∀)
  - for all
  - for every
- Existential Quantifier (∃)
  - for some
  - there exist

① Universal Quantifier :

- Universal Quantifier will be True if predicate is true for each and every element.
- Universal Quantifier will be False if atleast one stmt is false.

$\forall x\, p(x) \Rightarrow$ True | $\forall x\, p(x) \rightarrow$ false

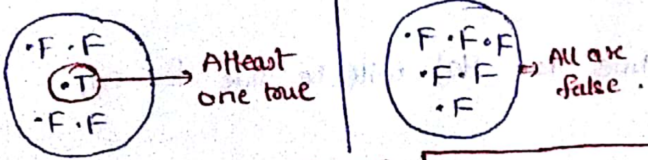


UQ

$$\boxed{P(x_1) \wedge p(x_2) \wedge p(x_3) \wedge \cdots p(x_n) \quad x_1, x_2 \cdots x_n \in \text{Domain}(z)}$$

② Existential Quantifier :

- $\exists x\, p(x) \Rightarrow$ True. | $\exists x\, p(x) \Rightarrow$ false.



EQ

$$\boxed{p(x_1) \vee p(x_2) \vee \cdots \vee p(x_n) \quad x_1, x_2 \cdots x_n \in \text{Domain}(z)}$$

Note :

$\neg(\forall x\, p(x)) \equiv \exists x\, \neg p(x)$

$\neg(\exists x\, p(x)) \equiv \forall x\, \neg p(x)$

Imp

Note :

- $\exists x(P(x) \wedge Q(x)) \longrightarrow \exists x\, p(x) \wedge \exists x\, q(x)$
- $\exists x(P(x) \vee Q(x)) \equiv \exists x\, p(x) \vee \exists x\, q(x)$
- $\forall x(P(x) \wedge Q(x)) \equiv \forall x\, p(x) \wedge \forall x\, q(x)$
- $\forall x(P(x) \vee Q(x)) \longleftarrow \forall x\, p(x) \vee \forall x\, q(x)$
- $\forall x(P(x) \rightarrow Q(x)) \longrightarrow \forall x\, p(x) \longrightarrow \forall x\, q(x)$

F T   T F : F   F   F : F

- All Comedians are funny
- Every Comedians are funny $\rightarrow$ Every x, x is a comedian, x is funny

- C-T
- C-F
- X-Some

- Whenever we have the stmt like this we left with two choices.

(i) $\forall x (C(x) \wedge F(x))$

(ii) $\forall x (C(x) \rightarrow F(x))$

In order to use which expression

If we have

$\forall x$ we use $\forall x (C(x) \rightarrow F(x))$

$\exists x$ we use $\exists x (C(x) \wedge F(x))$; $\exists x (C(x) \rightarrow F(x))$: True

$\exists x (C(x) \wedge F(x))$: False.

we prefer this.

Imp

Note:

All $\Rightarrow$ $\forall x$ $\rightarrow$

Some $\Rightarrow$ $\exists x$ $\wedge$

Nested Quantifiers

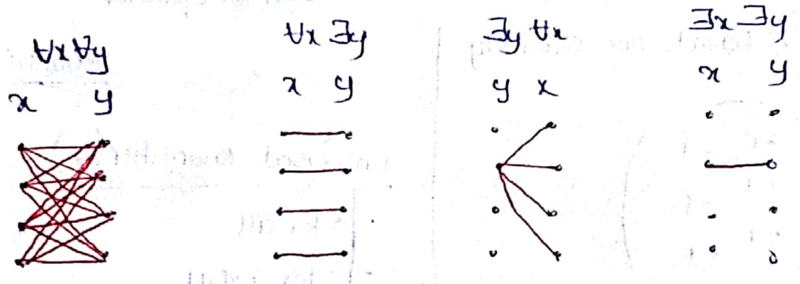
*) $\forall x \forall y \equiv \forall y \forall x$

$\forall x \exists y \leftarrow \forall y \exists x$

$\exists x \forall y$ $\quad$ $\exists y \forall x$

$\exists x \exists y \equiv \exists y \exists x$

*) $\forall x \forall y$ $\quad$ $\forall x \exists y$ $\quad$ $\exists y \forall x$ $\quad$ $\exists x \exists y$



Note:

$\forall x \forall y \Leftrightarrow \forall y \forall x$

$\downarrow$ $\quad$ $\downarrow$

$\exists y \forall x$ $\quad$ $\exists x \forall y$

$\downarrow$ $\quad$ $\downarrow$

$\forall x \exists y$ $\quad$ $\forall y \exists x$

$\downarrow$ $\quad$ $\downarrow$

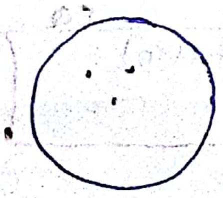
$\exists x \exists y \Leftrightarrow \exists y \exists x$

Negation of Nested Quantifiers :

- $\neg \forall x \forall y\, p(x,y) \equiv \exists x \exists y\, \neg p(x,y)$
- $\neg \exists x \exists y\, p(x,y) \equiv \forall x \forall y\, \neg p(x,y)$
- $\neg \forall x \exists y\, p(x,y) \equiv \exists x \forall y\, \neg p(x,y)$

Inference rule with Quantifier :

① Universal specification :



$\forall x p(x) \longrightarrow p(a)$, a is random

Considering $\forall x p(x)$ is true then $p(a)$ will be true for any value of a.

ie $\dfrac{\forall x p(x)}{p(a)}$ $\rightarrow$ a is random.

② Universal generalization :

$p(a) \longrightarrow \forall x p(x)$, a is random

Considering Every Element is true in the domain then we can say $\forall x p(x)$ is true.

$\dfrac{p(a)}{\forall x p(x)}$ a is random

3) Existential specification :

$\exists x\, p(x) \longrightarrow p(a)$, a is fixed and single element

Considering $\exists x\, p(x)$ is true then $p(a)$ will be true for a single element

$\boxed{\dfrac{\exists x\, p(x)}{p(a)}}$ a is fixed

4) Existential generalization

$p(a) \longrightarrow \exists x\, p(x)$, a is fixed & single element

If $p(a)$ is true for single element then $\exists x\, p(x)$ will be true.

$\boxed{\dfrac{p(a)}{\exists x\, p(x)}}$ a is fixed

x—— The End ——x