Combinatorics :

Product rule :

A Task T can be divided into T_1 and T_2 and there are n_1 ways to do task T_1 and n_2 ways to do task T_2 and for each n_1 ways we can perform n_2 operations and both the task can happen simultaneously

$\boxed{\text{Hence total ways } = n_1 * n_2}$

Sum rule :

A Task T can be divided into two tasks T_1 and T_2 and there are n_1 ways to do task T_1 and there are n_2 ways to do task T_2 and both tasks cannot be performed simultaneously

$\boxed{\text{Hence total ways } = n_1 + n_2}$

Euler ϕ-function :

$\phi(n)$ = num of elements which is less than or equal to n having GCD with 'n' will be '1'.

Ex:- $\phi(8) = 4$

1	2	3	4	5	6	7	8
✓	×	✓	×	✓	×	✓	×

Note :

Relative prime } → GCD with n is 1
Co-prime }

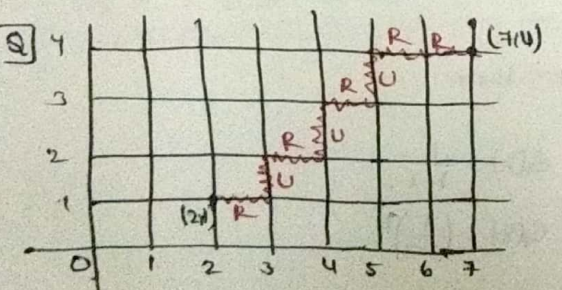
Properties of ϕ function :

- $\phi(P) = p-1$; p is a prime num
- $\phi(p^a) = p^{a-1}(p-1)$
- $\phi(A * B) = \phi(A) * \phi(B)$
- If $n = p_1^a p_2^b p_3^c \ldots$ then $\phi(n) = \dfrac{n * (P_1-1)(P_2-1)(P_3-1)\ldots}{P_1 P_2 P_3 \ldots}$

Permutation with Identical objects :

- n Elements Contain k same type of elements then the total ways we can arrange the num of elements $\dfrac{n!}{k!}$

Q



Num of possible ways

RURURURR → Arrange this letters

$\dfrac{8!}{5!\,3!} \to 56$ ways

Combinations with repetitions :

Q If $x_1+x_2+x_3+x_4 = 10$ How many sols are possible $x_i \geq 0$

x_1	x_2	x_3	x_4
11	111	111	11

$^{13}C_3$ ways.

Q) How many ways we can distribute 10 similar coins among 4 children such that each child has atleast one coin.

c_1	c_2	c_3	c_4
×	×	×	×
××	×	×	××

$^9C_3$ ways

Binomial Coefficients:

$$\boxed{(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \cdots + {}^nC_n b^n}$$

Binomial Expansion

Binomial Coefficients | ∴ It will work only for two items.

Extended binomial Coefficient:

$$\boxed{{}^{-n}C_k = \frac{(-n)(-n-1)(-n-2)\cdots(-n-k+1)}{k!} = (-1)^k \, {}^{n+k-1}C_k}$$

$$\boxed{{}^nC_k = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}}$$

$$\boxed{T_r = {}^nC_r x^{n-r} y^r}$$ rth term.

If $x<0$

① $\frac{1}{1-x} = (1-x)^{-1} = 1+x+x^2+x^3+\cdots$

② $\frac{1}{1+x} = (1+x)^{-1} = 1-x+x^2-x^3+\cdots$

③ $\frac{1}{(1-x)^2} = \frac{d}{dx}\left[\frac{1}{1-x}\right] = 1+2x+3x^2+4x^3+\cdots$

④ $\frac{1}{1-ax} = 1+ax+(ax)^2+(ax)^3+\cdots$

⑤ $\frac{1}{1+ax} = 1-ax+(ax)^2-(ax)^3+\cdots$

Generating functions:

Consider we have a sequence $a_0\ a_1\ a_2\ a_3 \ldots$ then

$G(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$$\boxed{G(x) = \sum_{i=0}^{\infty} a_i x^i}$$

$a_i = 1$ $\quad G(x) = \frac{1}{1-x}$.

$a_i = i+1$ $\quad G(x) = \left(\frac{1}{1-x}\right)^2$

Q) $x_1+x_2+x_3+x_4 = 16 \quad 1 \le x_i \le 5$

$(x+x^2+x^3+x^4+x^5)^4 = 16$

$x^4\left[1+x+x^2+x^3+x^4\right]^4 = 16$

$x^4\left[\frac{1-x^5}{1-x}\right]^4 = 16.$

$x^4\left[1-x^5\right]^4 (1-x)^{-4} = 16.$

Coeff of x^{16} $\Rightarrow x^4\left[{}^4C_0 1 - {}^4C_1 x^5 + {}^4C_2 x^{10} - {}^4C_3 x^{15} + {}^4C_4 x^{20}\right](1-x)^{-4}$

$\Rightarrow \left[x^4 - 4x^9 + 6x^{14} - 4x^{19} + x^{24}\right]\left[1-x\right]^{-4}$

1. ${}^{-4}C_{12} - 4\,{}^{-4}C_7 + 6\,{}^{-4}C_2$

$\Rightarrow {}^{15}C_{12} - 4 \cdot {}^{10}C_7 + 6\,{}^5C_2.$

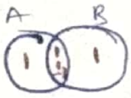
${}^{15}C_3 - 4\,{}^{10}C_3 + 6\,{}^5C_2$

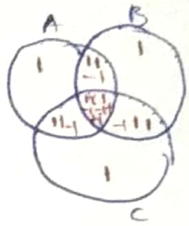
Pigeon hole principle :

Theorem1 : If there are n+1 pigeons and only n holes then one pigeon hole contains 2 pigeons.

Theorem2 : N = total num of pigeons
K = total num of holes
then one hole contains atleast $\lceil \frac{N}{K} \rceil$ pigeons.

Inclusion and Exclusion :





- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Derangements :

Num of arrangements such that no Element is at right position

$$\boxed{D_n = n! \left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \dots + (-1)^n \frac{1}{n!} \right]}$$

Note :

$D_1 = 0$	$D_4 = 9$
$D_2 = 1$	$D_5 = 44$
$D_3 = 2$	$D_6 = 265$

Recurence relation :

It is an Expr that is happening repetively and the next term is based on prev term

Note :

		Sol (Char eqn)
Type 1	$a_n = 2a_{n-1}$	$a_n = 2^n a_0$
Type 2	a, b are roots	$a_n = c_1 a^n + c_2 b^n$
Type 3	a, a are roots	$a_n = c_1 a^n + c_2 n a^n$
Type 4	a, b, c are roots	$a_n = c_1 a^n + c_2 b^n + c_3 c^n$
Type 5	a, a, b are roots	$a_n = c_1 a^n + c_2 n a^n + c_3 b^n$
Type 6	a, a, a are roots	$a_n = c_1 a^n + c_2 n a^n + c_3 n^2 a^n$

x — The End — x