Multiple Qubits Representation

Two-qubit states are represented by taking the tensor product of single-qubit state vectors. For example,

$$\begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}.$$

In general, tensor multiplication works as follows:

$$\begin{bmatrix}a\\b\end{bmatrix} \otimes \begin{bmatrix}c\\d\end{bmatrix} = \begin{bmatrix}ac\\ad\\bc\\bd\end{bmatrix}.$$

For conciseness, we will express multi-qubit systems as strings. So, the first tensor product mentioned is equivalent to 00.

The sum of the squares of the probability amplitudes in a multi-qubit system must equal 1. Referring to the first tensor product,

$$1^2 + 0^2 + 0^2 + 0^2 = 1.$$

Entangled Qubits Revisited

In the 01 notes, we learned the concept of two qubits being entangled. That is, regardless of the distance between the qubits, they share state information. As it turns out, we can express this scientific idea mathematically. If the state of a two-qubit system cannot be expressed as the tensor product of two single-qubit state vectors, then they exist in entanglement. There is some information stored outside of the individual qubits.

If this seems too abstract, consider the vector

$$\begin{bmatrix}1/\sqrt{2}\\0\\0\\1/\sqrt{2}\end{bmatrix}.$$

The tensor product of two single-qubit state vectors cannot represent this.

When a multi-qubit system is measured, its state as a WHOLE will collapse, just like our single qubit scenario. However, we can also measure the state of a single qubit in a multi-qubit system without collapsing the entire system.

Quantum Computation vs. Classical Computation

This is how we will approach problems on a quantum machine:

1. Represent the qubits in a known state, like $|000\cdots\rangle$.
2. Then, apply a unitary* transformation matrix, which is formed from the product of multiple quantum gates (each of which acts on a small number of qubits).

3. Measure and record the states of the qubits following the matrix multiplication.

Because measuring discrete outputs of a quantum system is probabilistic (different outputs from different trials), quantum computers return a distribution of outputs.

There is a common misconception that quantum computers can solve problems that classical computers cannot solve, irrespective of resources. This is FALSE. Classical machines can theoretically model quantum systems, but this becomes far more complex as the number of qubits modeled grows. A classical computer representing N bits can model 2^N individual bit strings. However, each bit string is trivial to model: just a string of N classical bits! On the other hand, a SINGLE quantum bit sequence of length N encodes a combination of 2^N possible states. Furthermore, any model of a quantum system must consider the correlations between qubits. We call these correlations entanglement.

We will soon introduce the idea of quantum parallelism, which gives quantum computers their exponential speedup.

Unitary Matrices

A matrix is unitary if the product between it and the transpose of its complex conjugate is the identity matrix (a matrix where 1s run down the diagonal from the top left to the bottom right). I put an example of a unitary matrix in the notes section.

Any operation represented by a unitary matrix can be implemented on a quantum computer. This is because they are reversible. This means that we can pass f a qubit string represented by x and derive an output y. Then, we can derive x by computing $x = f^{-1}(y)$. Engineers build gates that don't lose information.

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