

# EE3025 Assignment-2

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Download all python codes and latex-tikz codes from  
<https://github.com/saimehar31/EE3025/tree/master/Assignment2>

## 1 Introduction

We are supposed to design the equivalent FIR and IIR filter realizations for filter number 114. This is a bandpass filter whose specifications are available below.

## 2 Filter Specifications

The sampling rate for the filter has been specified as  $F_s = 48$  kHz. If the un-normalized discrete-time (natural) frequency is  $F$ , the corresponding normalized digital filter (angular) frequency is given by  $\omega = 2\pi \left( \frac{F}{F_s} \right)$ .

### 2.1 The Digital Filter

1. *Tolerances:* The passband ( $\delta_1$ ) and stopband ( $\delta_2$ ) tolerances are given to be equal, so we let  $\delta_1 = \delta_2 = \delta = 0.15$ .
2. *Passband:* The passband of filter number  $j$ ,  $j$  going from 109 to 135 is from  $\{3 + 0.6(j-109)\}$  kHz to  $\{3 + 0.6(j-107)\}$  kHz. Since our filter number is 114, substituting  $j = 114$  gives the passband range for our bandpass filter as 6 kHz - 7.2 kHz. Hence, the un-normalized discrete time filter passband frequencies are  $F_{p1} = 7.2$  kHz and  $F_{p2} = 6.0$  kHz. The corresponding normalized digital filter passband frequencies are  $\omega_{p1} = 2\pi \frac{F_{p1}}{F_s} = 0.3\pi$  and  $\omega_{p2} = 2\pi \frac{F_{p2}}{F_s} = 0.25\pi$  kHz. The centre frequency is then given by  $\omega_c = \frac{\omega_{p1} + \omega_{p2}}{2} = 0.275\pi$ .
3. *Stopband:* The *transition band* for bandpass filters is  $\Delta F = 0.3$  kHz on either side of the passband. Hence, the un-normalized *stopband* frequencies are  $F_{s1} = 7.2 + 0.3 = 7.5$  kHz and  $F_{s2} = 6.0 - 0.3 = 5.7$  kHz. The corresponding normalized frequencies are  $\omega_{s1} = 0.3125\pi$  and  $\omega_{s2} = 0.2375\pi$ .

## 2.2 The Analog filter

In the bilinear transform, the analog filter frequency ( $\Omega$ ) is related to the corresponding digital filter frequency ( $\omega$ ) as  $\Omega = \tan \frac{\omega}{2}$ . Using this relation, we obtain the analog passband and stopband frequencies as  $\Omega_{p1} = 0.5095$ ,  $\Omega_{p2} = 0.4142$  and  $\Omega_{s1} = 0.5345$ ,  $\Omega_{s2} = 0.3914$  respectively.

## 3 The IIR Filter Design

*Filter Type:* We are supposed to design filters whose stopband is monotonic and passband equiripple. Hence, we use the *Chebyshev approximation* to design our bandpass IIR filter.

### 3.1 The Analog Filter

1. *Low Pass Filter Specifications:* If  $H_{a,BP}(j\Omega)$  be the desired analog band pass filter, with the specifications provided in Section 2.2, and  $H_{a,LP}(j\Omega_L)$  be the equivalent low pass filter, then

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega} \quad (1)$$

where  $\Omega_0 = \sqrt{\Omega_{p1}\Omega_{p2}} = 0.4594$  and  $B = \Omega_{p1} - \Omega_{p2} = 0.0953$ . The low pass filter has the passband edge at  $\Omega_{Lp} = 1$  and stopband edges at  $\Omega_{Ls1} = 1.4653$  and  $\Omega_{Ls2} = -1.5511$ . We choose the stopband edge of the analog low pass filter as  $\Omega_{Ls} = \min(|\Omega_{Ls1}|, |\Omega_{Ls2}|) = 1.4653$ .

2. *The Low Pass Chebyshev Filter Paramters:* The magnitude squared of the Chebyshev low pass filter is given by

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\Omega_L/\Omega_{Lp})} \quad (2)$$

where  $c_N(x) = \cosh(N \cosh^{-1} x)$  and the integer  $N$ , which is the order of the filter, and  $\epsilon$  are design paramters. Since  $\Omega_{Lp} = 1$ , (2) may be rewritten as

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + \epsilon^2 c_N^2(\Omega_L)} \quad (3)$$

Also, the design paramters have the following constraints

$$\frac{\sqrt{D_2}}{c_N(\Omega_{Ls})} \leq \epsilon \leq \sqrt{D_1},$$

$$N \geq \left\lceil \frac{\cosh^{-1} \sqrt{D_2/D_1}}{\cosh^{-1} \Omega_{Ls}} \right\rceil, \quad (4)$$

where  $D_1 = \frac{1}{(1-\delta)^2} - 1$  and  $D_2 = \frac{1}{\delta^2} - 1$ . After appropriate substitutions, we obtain  $N \geq 4$  and  $0.3184 \leq \epsilon \leq 0.6197$ . In Figure 1, we plot  $|H(j\Omega)|$

for a range of values of  $\epsilon$ , for  $N = 4$ . We find that for larger values of  $\epsilon$ ,  $|H(j\Omega)|$  decreases in the transition band. We choose  $\epsilon = 0.4$  for our IIR filter design.

3. *The Low Pass Chebyshev Filter:* Thus, we obtain

$$|H_{a,LP}(j\Omega_L)|^2 = \frac{1}{1 + 0.16c_4^2(\Omega_L)} \quad (5)$$

where

$$c_4(x) = 8x^4 + 8x^2 + 1. \quad (6)$$

The poles of the frequency response in (2) lying in the left half plane are in general obtained as  $r_1 \cos \phi_k + jr_2 \sin \phi_k$ , where

$$\begin{aligned} \phi_k &= \frac{\pi}{2} + \frac{(2k+1)\pi}{2N}, k = 0, 1, \dots, N-1 \\ r_1 &= \frac{\beta^2 - 1}{2\beta}, r_2 = \frac{\beta^2 + 1}{2\beta}, \beta = \left[ \frac{\sqrt{1 + \epsilon^2} + 1}{\epsilon} \right]^{\frac{1}{N}} \end{aligned} \quad (7)$$

Thus, for  $N$  even, the low-pass stable Chebyshev filter, with a gain  $G$  has the form

$$H_{a,LP}(s_L) = \frac{G_{LP}}{\prod_{k=1}^{\frac{N}{2}-1} (s_L^2 - 2r_1 \cos \phi_k s_L + r_1^2 \cos^2 \phi_k + r_2^2 \sin^2 \phi_k)} \quad (8)$$

Substituting  $N = 4$ ,  $\epsilon = 0.5$  and  $H_{a,LP}(j) = \frac{1}{\sqrt{1+\epsilon^2}}$ , from (7) and (8), we obtain

$$H_{a,LP}(s_L) = \frac{0.3125}{s_L^4 + 1.1068s_L^3 + 1.6125s_L^2 + 0.9140s_L + 0.3366} \quad (9)$$

In Figure 2 we plot  $|H(j\Omega)|$  using (5) and (9), thereby verifying that our low-pass Chebyshev filter design meets the specifications.

4. *The Band Pass Chebyshev Filter:* The analog bandpass filter is obtained from (9) by substituting  $s_L = \frac{s^2 + \Omega_0^2}{Bs}$ . Hence

$$H_{a,BP}(s) = G_{BP} H_{a,LP}(s_L) \Big|_{s_L = \frac{s^2 + \Omega_0^2}{Bs}}, \quad (10)$$

where  $G_{BP}$  is the gain of the bandpass filter. After appropriate substitutions, and evaluating the gain such that  $H_{a,BP}(j\Omega_{p1}) = 1$ , we obtain

$$H_{a,BP}(s) = \frac{2.7776 \times 10^{-5} s^4}{s^8 + 0.1055s^7 + 0.8589s^6 + 0.0676s^5 + 0.2735s^4 + 0.0143s^3 + 0.0383s^2 + 0.001s + 0.002} \quad (11)$$

In Figure 3, we plot  $|H_{a,BP}(j\Omega)|$  as a function of  $\Omega$  for both positive as well as negative frequencies. We find that the passband and stopband frequencies in the figure match well with those obtained analytically through the bilinear transformation.

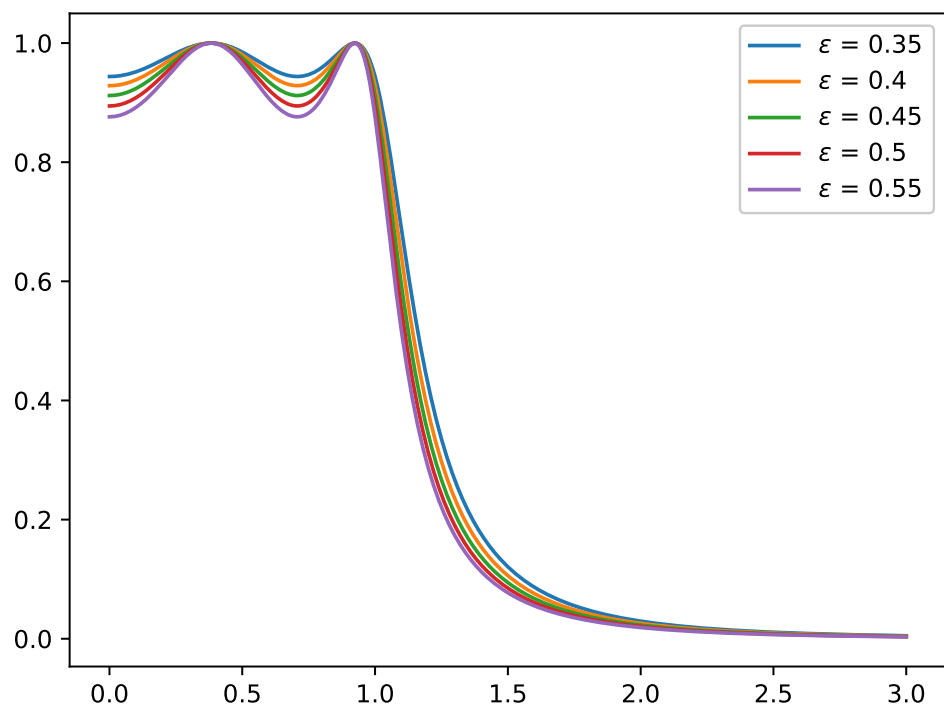


Figure 1: The Analog Low-Pass Frequency Response for  $0.35 \leq \epsilon \leq 0.6$

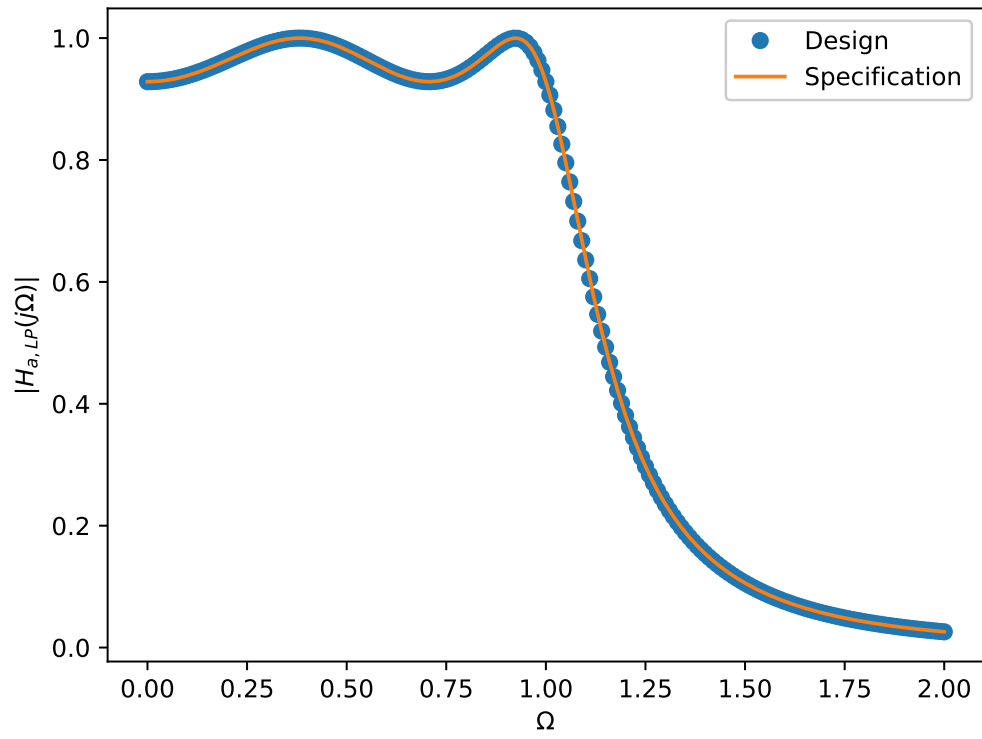


Figure 2: The magnitude response plots from the specifications in Equation 5 and the design in Equation 9

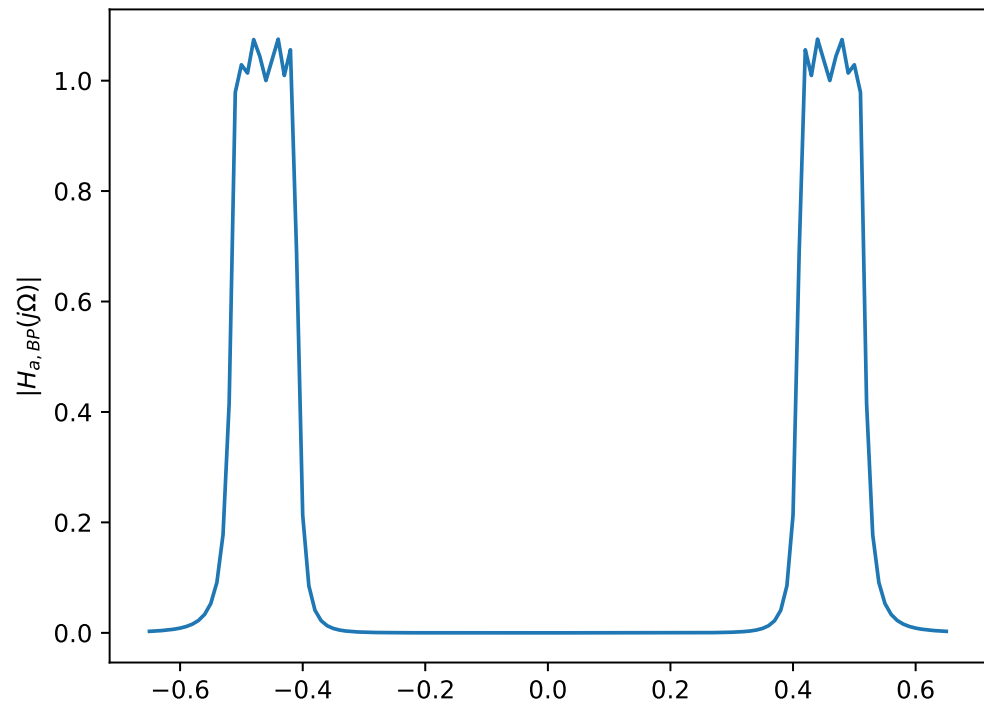


Figure 3: The analog bandpass magnitude response plot from Equation 11

### 3.2 The Digital Filter

From the bilinear transformation, we obtain the digital bandpass filter from the corresponding analog filter as

$$H_{d,BP}(z) = GH_{a,BP}(s) \Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} \quad (12)$$

where  $G$  is the gain of the digital filter. From (11) and (12), we obtain

$$H_{d,BP}(z) = G \frac{N(z)}{D(z)} \quad (13)$$

where  $G = 2.7776 \times 10^{-5}$ ,

$$N(z) = 1 - 4z^{-2} + 6z^{-4} - 4z^{-6} + z^{-8} \quad (14)$$

and

$$\begin{aligned} D(z) = & 2.3609 - 12.0002z^{-1} + 31.8772z^{-2} - 53.7495z^{-3} + 62.8086z^{-4} \\ & - 51.4634z^{-5} + 29.2231z^{-6} - 10.5329z^{-7} + 1.9842z^{-8} \end{aligned} \quad (15)$$

The plot of  $|H_{d,BP}(z)|$  with respect to the normalized angular frequency (normalizing factor  $\pi$ ) is available in Figure 4. Again we find that the passband and stopband frequencies meet the specifications well enough.

## 4 The FIR Filter

We design the FIR filter by first obtaining the (non-causal) lowpass equivalent using the Kaiser window and then converting it to a causal bandpass filter.

### 4.1 The Equivalent Lowpass Filter

The lowpass filter has a passband frequency  $\omega_l$  and transition band  $\Delta\omega = 2\pi \frac{\Delta F}{F_s} = 0.0125\pi$ . The stopband tolerance is  $\delta$ .

1. The *passband frequency*  $\omega_l$  is defined as  $\omega_l = \frac{\omega_{p1} - \omega_{p2}}{2}$ . Substituting the values of  $\omega_{p1}$  and  $\omega_{p2}$  from section 2.1, we obtain  $\omega_l = 0.025\pi$ .
2. The *impulse response*  $h_{lp}(n)$  of the desired lowpass filter with cutoff frequency  $\omega_l$  is given by

$$h_l(n) = \frac{\sin(n\omega_l)}{n\pi} w(n), \quad (16)$$

where  $w(n)$  is the Kaiser window obtained from the design specifications.

## 4.2 The Kaiser Window

The Kaiser window is defined as

$$w(n) = \frac{I_0 \left[ \beta N \sqrt{1 - \left( \frac{n}{N} \right)^2} \right]}{I_0(\beta N)}, \quad -N \leq n \leq N, \quad \beta > 0$$

$$= 0 \quad \text{otherwise,} \quad (17)$$

where  $I_0(x)$  is the modified Bessel function of the first kind of order zero in  $x$  and  $\beta$  and  $N$  are the window shaping factors. In the following, we find  $\beta$  and  $N$  using the design parameters in section 2.1.

1.  $N$  is chosen according to

$$N \geq \frac{A - 8}{4.57\Delta\omega}, \quad (18)$$

where  $A = -20 \log_{10} \delta$ . Substituting the appropriate values from the design specifications, we obtain  $A = 16.4782$  and  $N \geq 48$ .

2.  $\beta$  is chosen according to

$$\beta N = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ 0.5849(A - 21)^{0.4} + 0.07886(A - 21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases} \quad (19)$$

In our design, we have  $A = 16.4782 < 21$ . Hence, from (19) we obtain  $\beta = 0$ .

3. We choose  $N = 100$ , to ensure the desired low pass filter response. Substituting in (17) gives us the rectangular window

$$w(n) = \begin{cases} 1, & -100 \leq n \leq 100 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

From (16) and (20), we obtain the desired lowpass filter impulse response

$$h_{lp}(n) = \begin{cases} \frac{\sin(\frac{n\pi}{40})}{n\pi} & -100 \leq n \leq 100 \\ 0, & \text{otherwise} \end{cases} \quad (21)$$

The magnitude response of the filter in (21) is shown in Figure 5.

## 4.3 The FIR Bandpass Filter

The centre of the passband of the desired bandpass filter was found to be  $\omega_c = 0.275\pi$  in Section 2.1. The impulse response of the desired bandpass filter is obtained from the impulse response of the corresponding lowpass filter as

$$h_{bp}(n) = 2h_{lp}(n)\cos(n\omega_c) \quad (22)$$



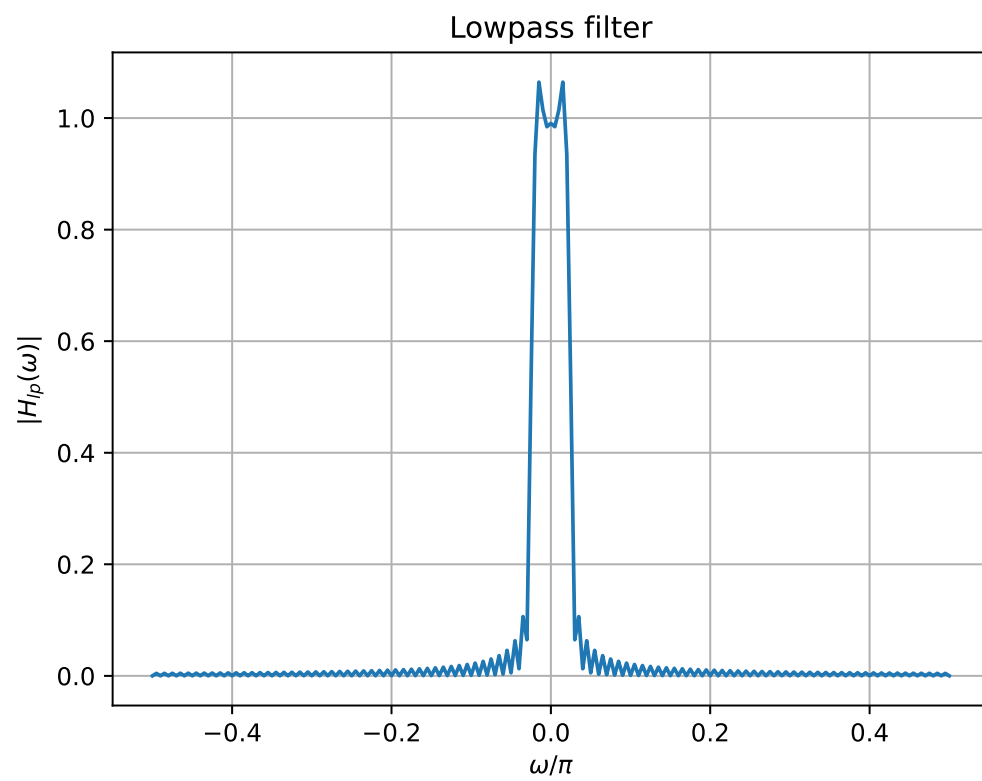


Figure 4: The magnitude response of the FIR lowpass digital filter designed to meet the given specifications

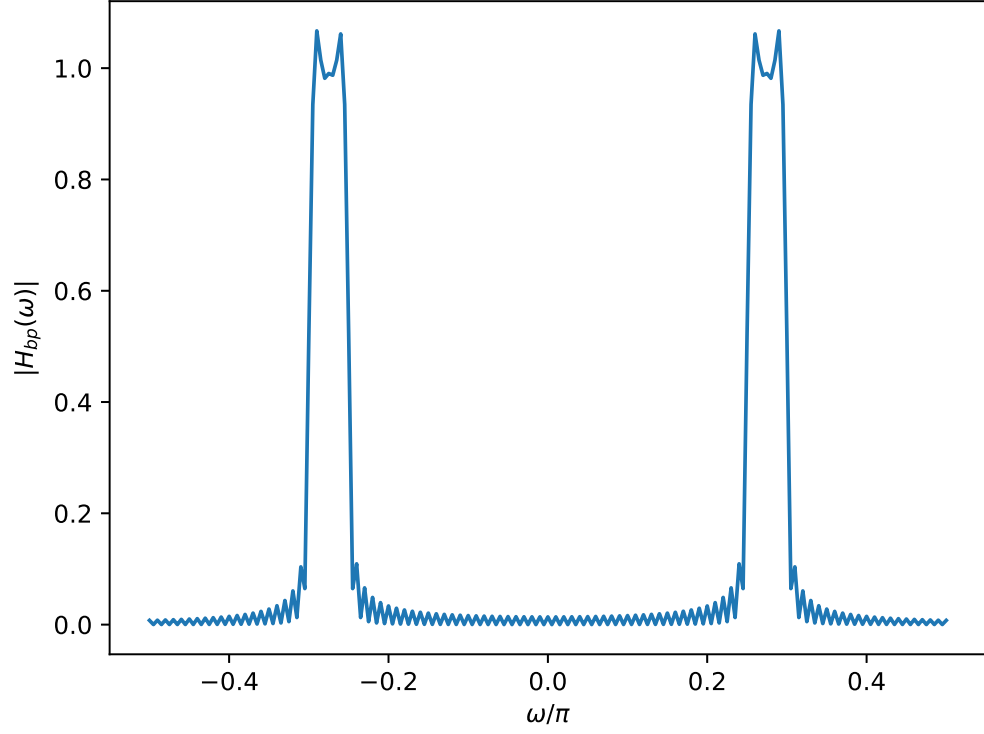


Figure 5: The magnitude response of the FIR bandpass digital filter designed to meet the given specifications

Thus, from (21), we obtain

$$\begin{aligned}
 h_{bp}(n) &= \frac{2 \sin(\frac{n\pi}{40}) \cos(\frac{11n\pi}{40})}{n\pi} & -100 \leq n \leq 100 \\
 &= 0, & \text{otherwise}
 \end{aligned} \tag{23}$$

The magnitude response of the FIR bandpass filter designed to meet the given specifications is plotted in Figure 6.