# EE3025 Assignment-1

M SAI MEHAR - EE18BTECH11029

Download all python codes from

```
https://github.com/saimehar31/EE3025/tree/main/
    Assignment1/codes
```

and latex-tikz codes from

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https://github.com/saimehar31/EE3025/blob/main/
    Assignment1/ee18btech11029.tex
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## 1 Problem

1.1. Let

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{1.1.1}$$

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{1.1.2}$$

1.2. Compute $X(k)$, $H(k)$ and $y(n)$ using FFT and IFFT methods

## 2 Solution

2.1. Input signal x(n) is given as

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \right\} \tag{2.1.1}$$

2.2. Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{2.2.1}$$

2.3. FFT of a Input Signal $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1 \ldots N-1 \tag{2.3.1}$$

2.4. FFT of a Impulse Response $h(n)$ is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.4.1}$$

2.5. FFT of output Signal $y(n)$ can be calculated by

$$Y(k) = X(k)H(k) \tag{2.5.1}$$

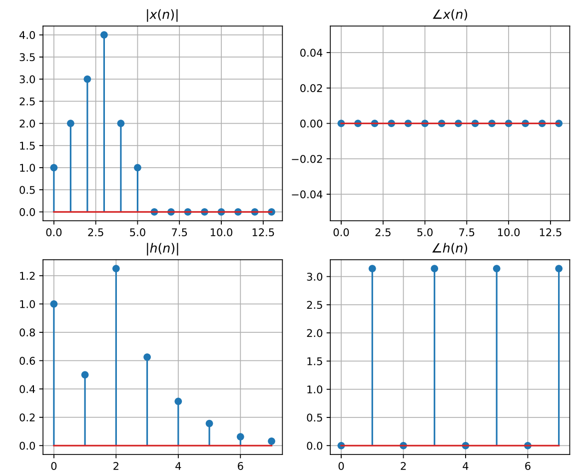


Fig. 2.1: Input signal x(n) and Impulse response h(n)

2.6. $y(n)$ can be calculated by doing IFFT for $Y(k)$

$$y(n) = \frac{1}{N} \sum_{n=0}^{N-1} Y(k) e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.6.1}$$

2.7. Plotting FFT of output signal

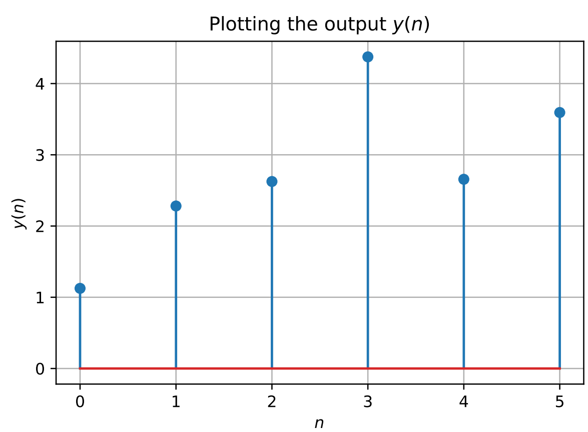


Fig. 2.7: Output signal $y(n)$

## 3 Problem

3.1. Wherever possible, express all the above equations as matrix equations.

## 4 Solution

4.1. FFT of signal X(n)

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{4.1.1}$$

4.2. Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be ex− pressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} \tag{4.2.1}$$

4.3. Given that $x(n) = \{1, 2, 3, 4, 2, 1\}$ and As, N = 6 then above equation on multiplying matrices becomes

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3} + \ldots + (1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3} + \ldots + (1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3} + \ldots + (1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3} + \ldots + (1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3} + \ldots + (1)e^{-5j5\pi/3} \end{bmatrix} \tag{4.3.1}$$

4.4. On solving we get,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 13 \\ -4-\sqrt{3}j \\ 1 \\ -1 \\ 1 \\ -4+\sqrt{3}j \end{bmatrix} \tag{4.4.1}$$

$$\implies X(0) = 13 + 0j, \tag{4.4.2}$$
$$X(1) = -4 - 1.732j, \tag{4.4.3}$$
$$X(2) = 1 + 0j, \tag{4.4.4}$$
$$X(3) = -1 + 0j, \tag{4.4.5}$$
$$X(4) = 1 + 0j, \tag{4.4.6}$$
$$X(5) = -4 + 1.732j \tag{4.4.7}$$

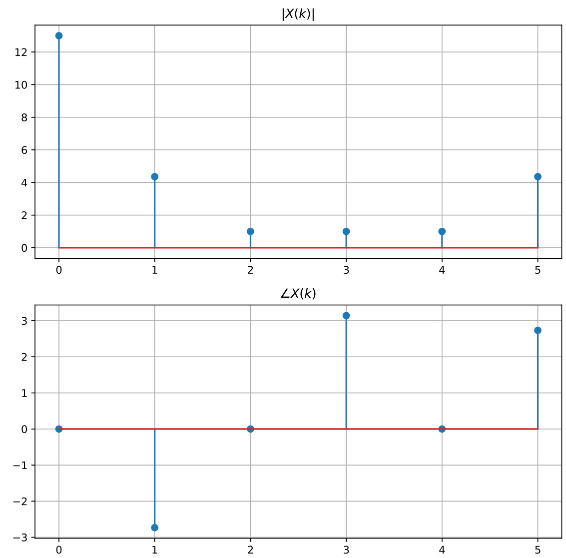


Fig. 4.4: FFT of $x(n)$

4.5. Now to find $H(k)$ we need to know the h(n).For that we need to first find the Y(z) by applying Z-transform on equation i.e.,

$$Y(z) + \frac{1}{2} z^{-1} Y(z) = X(z) + z^{-2} X(z) \tag{4.5.1}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)} X(z) \tag{4.5.2}$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.5.3}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{4.5.4}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{4.5.5}$$

By applying inverse Z - transform we can find the value of $h(n)$

$$h(n) = Z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{4.5.6}$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \quad (4.5.7)$$

Assuming that length of h(n) is same as length of x(n) i.e., N = 6. Now finding the value of $h(n)$ using matrix method we get

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \end{bmatrix} \quad (4.5.8)$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0) + h(1) + h(2) + h(3) + h(4) + h(5) \\ h(0) + h(1)e^{-j\pi/3} + ... + h(5)e^{-j5\pi/3} \\ h(0) + h(1)e^{-2j\pi/3} + ... + h(5)e^{-2j5\pi/3} \\ h(0) + h(1)e^{-3j\pi/3} + ... + h(5)e^{-3j5\pi/3} \\ h(0) + h(1)e^{-4j\pi/3} + ... + h(5)e^{-4j5\pi/3} \\ h(0) + h(1)e^{-5j\pi/3} + ... + h(5)e^{-5j5\pi/3} \end{bmatrix}$$

4.6. On solving we get

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125 \\ 0.51625 - 0.5142j \\ -0.07813 + 1.1096j \\ 3.84375 \\ -0.07183 - 1.1096j \\ 0.51625 + 0.5142j \end{bmatrix} \quad (4.6.1)$$

$$\implies H(0) = 1.28125 + 0j, \quad (4.6.2)$$
$$H(1) = 0.51625 - 0.5141875j, \quad (4.6.3)$$
$$H(2) = -0.078125 + 1.1095625j, \quad (4.6.4)$$
$$H(3) = 3.84375 + 0j, \quad (4.6.5)$$
$$H(4) = -0.071825 - 1.1095625j. \quad (4.6.6)$$
$$H(5) = 0.515625 + 0.5141875j \quad (4.6.7)$$

So,These values which we got are same as that of from the plots.

4.7. Now to find $H(k)$ we need to know the h(n).For that we need to first find the Y(z) by applying Z-transform on equation

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} \times \begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} \quad (4.7.1)$$

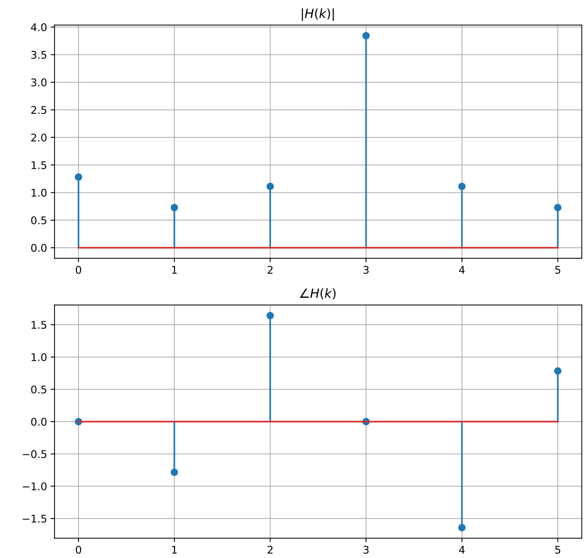


Fig. 4.6: FFT of $h(n)$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.65625 \\ -2.95312 + 1.16372j \\ -0.07813 + 1.1096j \\ -3.84375 \\ -0.07813 - 1.1096j \\ -2.95312 - 1.16372j \end{bmatrix} \quad (4.7.2)$$

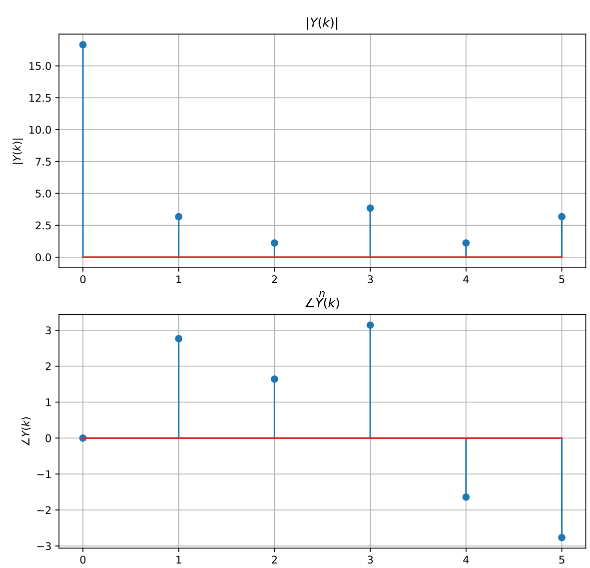


Fig. 4.7: FFT of $y(n)$

4.8. $y(n)$ can be calculated by applying IFFT for the above Y matrix and is calculated by python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 \\ 2.28125071 \\ 2.6250019 \\ 4.37499667 \\ 2.6562481 \\ 3.59375262 \end{bmatrix}$$

These are the same values that we have plotted in the above plot (2.7)

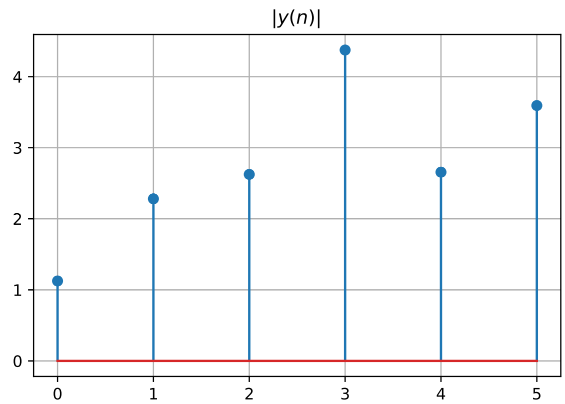


Fig. 4.8: IFFT of $Y(k)$

4.9. Properties :

a) Symmetry property :

$$W_N^{k+N/2} = -W_N^k$$

b) Periodicity property :

$$W_N^{k+N} = W_N^k$$

c)

$$W_N^2 = W_{N/2}$$

4.10. Using properties to derive FFT from DFT :

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn}, \quad k = 0, 1, \ldots, N-1 \tag{4.10.1}$$

$$= \sum_{n=even} x(n)W_N^{kn} + \sum_{n=odd} x(n)W_N^{kn} \tag{4.10.2}$$

$$= \sum_{m=0}^{2} x(2m)W_N^{2mk} + \sum_{m=0}^{2} x(2m+1)W_N^{(2m+1)k} \tag{4.10.3}$$

using property c, we get,

$$X(k) = \sum_{m=0}^{2} x(2m)W_{N/2}^{mk} + W_N^k \sum_{m=0}^{2} x(2m+1)W_{N/2}^{mk} \tag{4.10.4}$$

$$= X_1(k) + W_N^k X_2(k) \tag{4.10.5}$$

4.11.
- $X_1$(k) and $X_2$(k) are 3 point DFTs of x(2m) and x(2m+1) , m=0,1,2.
- $X_1$(k) and $X_2$(k) are periodic, Hence $X_1$(k+3) = $X_1$(k) and $X_2$(k+3) = $X_1$(k).
- By performing this step once we can see that number of operations have been reduced from $N^2$ to $\frac{N^2}{2}$.

4.12. Let us take $F_N$ as the N-point DFT Matrix. By the property of Complex Exponentials we can write $F_N$ in terms of $F_{N/2}$

$$F_N = \begin{bmatrix} I_{N/2} & D_{N/2} \\ I_{N/2} & -D_{N/2} \end{bmatrix} \begin{bmatrix} F_{N/2} & 0 \\ 0 & F_{N/2} \end{bmatrix} P_N \tag{4.12.1}$$

For N = 6

$$\implies F_6 = \begin{bmatrix} I_3 & D_3 \\ I_3 & -D_3 \end{bmatrix} \begin{bmatrix} F_3 & 0 \\ 0 & F_3 \end{bmatrix} P_6 \tag{4.12.2}$$

Here $I_3$ is the 3x3 identity matrix. Writing matrices in block form :

$$D_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^1 & 0 \\ 0 & 0 & W_3^2 \end{bmatrix} \tag{4.12.3}$$

$$P_6 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.12.4}$$

$$\implies P_6 \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} = \begin{bmatrix} x(0) \\ x(2) \\ x(4) \\ x(1) \\ x(3) \\ x(5) \end{bmatrix} \tag{4.12.5}$$

Let

$$\begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{bmatrix} = F_{N/2} \begin{bmatrix} x(0) \\ x(2) \\ x(4) \end{bmatrix} \tag{4.12.6}$$

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{bmatrix} = F_{N/2} \begin{bmatrix} x(1) \\ x(3) \\ x(5) \end{bmatrix} \tag{4.12.7}$$

be the N/2 point DFTs.

4.13. By replacing the above results in the equation $X = F_N x$, we get

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & W_6^0 & 0 & 0 \\ 0 & 1 & 0 & 0 & W_6^1 & 0 \\ 0 & 0 & 1 & 0 & 0 & W_6^2 \\ 1 & 0 & 0 & -W_6^0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -W_6^1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -W_6^2 \end{bmatrix} \begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \\ X_2(0) \\ X_2(1) \\ X_2(2) \end{bmatrix} \tag{4.13.1}$$

Using the above method breaking the N-point DFT into two N/2-point DFT's we get

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \end{bmatrix} = \begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{bmatrix} + \begin{bmatrix} W_6^0 & 0 & 0 \\ 0 & W_6^1 & 0 \\ 0 & 0 & W_6^2 \end{bmatrix} \begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{bmatrix} \tag{4.13.2}$$

$$\begin{bmatrix} X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} X_1(0) \\ X_1(1) \\ X_1(2) \end{bmatrix} - \begin{bmatrix} W_6^0 & 0 & 0 \\ 0 & W_6^1 & 0 \\ 0 & 0 & W_6^2 \end{bmatrix} \begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \end{bmatrix} \tag{4.13.3}$$

Hence the time complexity is reduced from O($N^2$) to O(NlogN).

4.14. If we have $N = 2^M$ where $M \in \mathbb{Z}^+$ then we can recursively breakdown N/2 point DFT Matrix into N/4 point DFT Matrix and so on till we reach 2-point DFT Matrix.
for N = 8, we can write,

$$F_8 = \begin{bmatrix} I_4 & D_4 \\ I_4 & -D_4 \end{bmatrix} \begin{bmatrix} F_4 & 0 \\ 0 & F_4 \end{bmatrix} P_8 \tag{4.14.1}$$

$$F_4 = \begin{bmatrix} I_2 & D_2 \\ I_2 & -D_2 \end{bmatrix} \begin{bmatrix} F_2 & 0 \\ 0 & F_2 \end{bmatrix} P_4 \tag{4.14.2}$$

Finally, the 2-point DFT Matrix is the base case

$$F_2 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_1 - x_2 \end{bmatrix} \tag{4.14.3}$$

4.15. Step by Step visualization of computing 8-Point DFT recursively using 4-point DFT's and 2-point DFT's.Expressing 8-point DFT's in terms of 4-point DFT's.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} X_e(0) \\ X_e(1) \\ X_e(2) \\ X_e(3) \end{bmatrix} + \begin{bmatrix} W_8^0 & 0 & 0 & 0 \\ 0 & W_8^1 & 0 & 0 \\ 0 & 0 & W_8^2 & 0 \\ 0 & 0 & 0 & W_8^3 \end{bmatrix} \begin{bmatrix} X_o(0) \\ X_o(1) \\ X_o(2) \\ X_o(3) \end{bmatrix} \tag{4.15.1}$$

$$\begin{bmatrix} X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} X_e(0) \\ X_e(1) \\ X_e(2) \\ X_e(3) \end{bmatrix} - \begin{bmatrix} W_8^0 & 0 & 0 & 0 \\ 0 & W_8^1 & 0 & 0 \\ 0 & 0 & W_8^2 & 0 \\ 0 & 0 & 0 & W_8^3 \end{bmatrix} \begin{bmatrix} X_o(0) \\ X_o(1) \\ X_o(2) \\ X_o(3) \end{bmatrix} \tag{4.15.2}$$

4-point FFTs into 2-point FFTs

$$\begin{bmatrix} X_1(0) \\ X_1(1) \end{bmatrix} = \begin{bmatrix} X_3(0) \\ X_3(1) \end{bmatrix} + \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_4(0) \\ X_4(1) \end{bmatrix} \tag{4.15.3}$$

$$\begin{bmatrix} X_1(2) \\ X_1(3) \end{bmatrix} = \begin{bmatrix} X_3(0) \\ X_3(1) \end{bmatrix} - \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_4(0) \\ X_4(1) \end{bmatrix} \tag{4.15.4}$$

$$\begin{bmatrix} X_2(0) \\ X_2(1) \end{bmatrix} = \begin{bmatrix} X_5(0) \\ X_5(1) \end{bmatrix} + \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_6(0) \\ X_6(1) \end{bmatrix} \tag{4.15.5}$$

$$\begin{bmatrix} X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} X_5(0) \\ X_5(1) \end{bmatrix} - \begin{bmatrix} W_4^0 & 0 \\ 0 & W_4^1 \end{bmatrix} \begin{bmatrix} X_6(0) \\ X_6(1) \end{bmatrix} \tag{4.15.6}$$

$$P_8 \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix} = \begin{bmatrix} x(0) \\ x(2) \\ x(4) \\ x(6) \\ x(1) \\ x(3) \\ x(5) \\ x(7) \end{bmatrix} \tag{4.15.7}$$

$$P_4 \begin{bmatrix} x(0) \\ x(2) \\ x(4) \\ x(6) \end{bmatrix} = \begin{bmatrix} x(0) \\ x(4) \\ x(2) \\ x(6) \end{bmatrix} \tag{4.15.8}$$

$$P_4 \begin{bmatrix} x(1) \\ x(3) \\ x(5) \\ x(7) \end{bmatrix} = \begin{bmatrix} x(1) \\ x(5) \\ x(3) \\ x(7) \end{bmatrix} \tag{4.15.9}$$

Therefore,

$$\begin{bmatrix} X_3(0) \\ X_3(1) \end{bmatrix} = F_2 \begin{bmatrix} x(0) \\ x(4) \end{bmatrix} \tag{4.15.10}$$

$$\begin{bmatrix} X_4(0) \\ X_4(1) \end{bmatrix} = F_2 \begin{bmatrix} x(2) \\ x(6) \end{bmatrix} \tag{4.15.11}$$

$$\begin{bmatrix} X_5(0) \\ X_5(1) \end{bmatrix} = F_2 \begin{bmatrix} x(1) \\ x(5) \end{bmatrix} \tag{4.15.12}$$

$$\begin{bmatrix} X_6(0) \\ X_6(1) \end{bmatrix} = F_2 \begin{bmatrix} x(3) \\ x(7) \end{bmatrix} \tag{4.15.13}$$

$X_3$ and $X_4$ both combined would give $X_1$
$X_5$ and $X_6$ both combined would give $X_2$

4.16. The following C program will compute and print the FFT (N-point where N is of the form $2^n$)

https://github.com/saimehar31/EE3025/tree/master/Assignment1/codes/fft.c

4.17. Using the above properties recursively we have implemented radix-2 Fast-Fourier transform algorithm.

- In a N-point DFT matrix multiplication consists of $2N^2$ multiplication so the overall time complexity is in the order of O($N^2$)
- In FFT we recursively break down each stage into two N/2-point FFT's resulting in logN term and we do this for entire N so combining both we get O(NlogN)

4.18. Taking an example of 8-point function,

$$x(n) = \left\{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1, 0, 0 \right\} \tag{4.18.1}$$

We know that,

$$X(k) \triangleq W_N^{nk} x(n), \quad k = 0, 1, \ldots, N-1 \tag{4.18.2}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \\ x(7) \end{bmatrix} \tag{4.18.3}$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{4.18.4}$$

$$\implies \begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix} = \begin{bmatrix} 13 \\ -3.121 - 6.535j \\ 1j \\ 1.121 - 0.535j \\ -1 \\ 1.121 + 0.535j \\ -1j \\ -3.121 + 6.535j \end{bmatrix} \tag{4.18.5}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \\ H(6) \\ H(7) \end{bmatrix} = \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\ W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\ W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\ W_8^0 & W_8^3 & W_8^6 & W_8^9 & W_8^{12} & W_8^{15} & W_8^{18} & W_8^{21} \\ W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\ W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\ W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\ W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49} \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 1.25 \\ -0.65 \\ 0.3125 \\ -0.15625 \\ 0.078125 \\ -0.0390625 \end{bmatrix} \tag{4.18.6}$$

$$\implies \begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \\ H(6) \\ H(7) \end{bmatrix} = \begin{bmatrix} 1.32 \\ 0.858 - 0.514j \\ -0.015 - 0.007j \\ 0.516 + 1.829j \\ 3.96 \\ 0.516 - 1.829j \\ -0.015 + 0.007j \\ 0.858 + 0.514j \end{bmatrix} \tag{4.18.7}$$

So,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \\ Y(6) \\ Y(7) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \\ X(6) \cdot H(6) \\ X(7) \cdot H(7) \end{bmatrix} \tag{4.18.8}$$

Solving,

$$\implies \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \\ Y(6) \\ Y(7) \end{bmatrix} = \begin{bmatrix} 17.16 \\ -6.04 - 4j \\ -0.007 - 0.015j \\ 1.55 + 1.77j \\ -3.96 \\ 1.55 - 1.77j \\ 0.007 + 0.015j \\ -6.04 + 4j \end{bmatrix} \tag{4.18.9}$$

Similary we get

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} 0.53125 \\ 1.69 \\ 3.09 \\ 4.375 \\ 2.773 \\ 3.593 \\ 0.203 \\ 0.8984 \end{bmatrix} \tag{4.18.10}$$