EE3025 Assignment-1

M SAI MEHAR - EE18BTECH11029

Download all python codes from

https://github.com/saimehar31/EE3025/tree/main/Assignment1/codes

and latex-tikz codes from

https://github.com/saimehar31/EE3025/blob/main/Assignment1/ee18btech11029.tex

1 Problem

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \quad (1.1.1)$$

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \quad (1.1.2)$$

1.2. Compute $X(k)$, $H(k)$ and $y(n)$ using FFT and IFFT methods

2 Solution

2.1. Input signal x(n) is given as

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \quad (2.1.1)$$

2.2. Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \quad (2.2.1)$$

2.3. FFT of a Input Signal $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1 \ldots N-1 \quad (2.3.1)$$

2.4. FFT of a Impulse Response $h(n)$ is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \quad (2.4.1)$$

2.5. FFT of output Signal $y(n)$ can be calculated by

$$Y(k) = X(k)H(k) \quad (2.5.1)$$

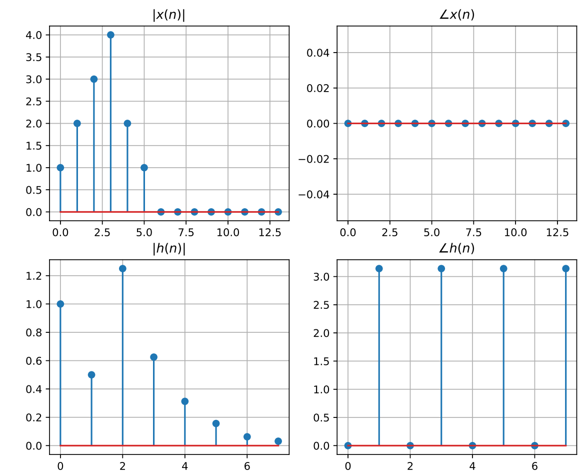

Fig. 2.1: Input signal x(n) and Impulse response h(n)

2.6. $y(n)$ can be calculated by doing IFFT for $Y(k)$

$$y(n) = \frac{1}{N} \sum_{n=0}^{N-1} Y(k) e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1 \quad (2.6.1)$$

2.7. Plotting FFT of output signal

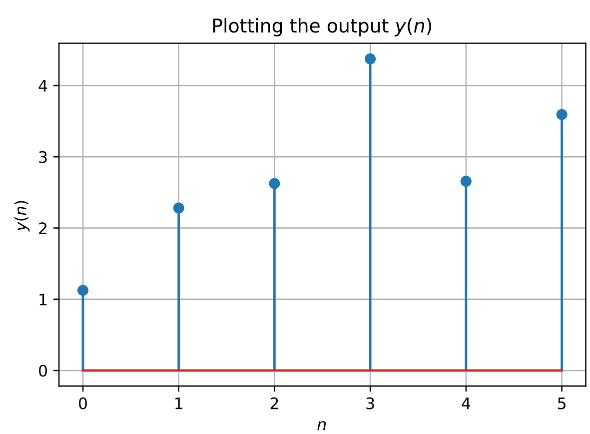

Fig. 2.7: Output signal $y(n)$

3 Problem

3.1. Wherever possible, express all the above equations as matrix equations.

4 Solution

4.1. FFT of signal X(n)

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{4.1.1}$$

4.2. Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be ex− pressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} \tag{4.2.1}$$

4.3. Given that $x(n) = \{1, 2, 3, 4, 2, 1\}$ and As, N = 6 then above equation on multiplying matrices becomes

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3} + \ldots + (1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3} + \ldots + (1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3} + \ldots + (1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3} + \ldots + (1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3} + \ldots + (1)e^{-5j5\pi/3} \end{bmatrix} \tag{4.3.1}$$

4.4. On solving we get,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 13 \\ -4-\sqrt{3}j \\ 1 \\ -1 \\ 1 \\ -4+\sqrt{3}j \end{bmatrix} \tag{4.4.1}$$

$$\implies X(0) = 13 + 0j, \tag{4.4.2}$$
$$X(1) = -4 - 1.732j, \tag{4.4.3}$$
$$X(2) = 1 + 0j, \tag{4.4.4}$$
$$X(3) = -1 + 0j, \tag{4.4.5}$$
$$X(4) = 1 + 0j, \tag{4.4.6}$$
$$X(5) = -4 + 1.732j \tag{4.4.7}$$

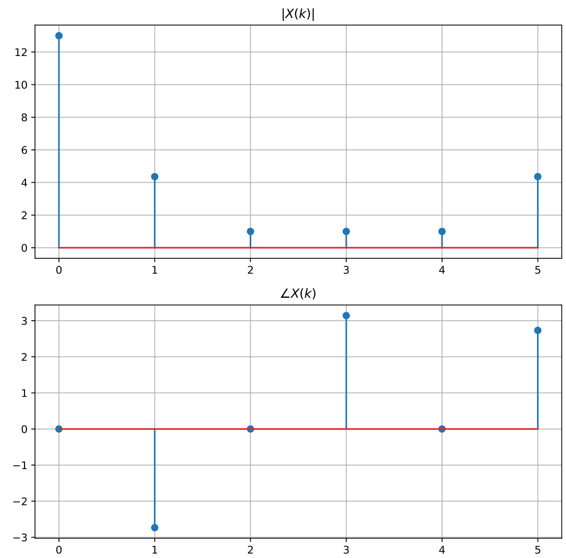

Fig. 4.4: FFT of $x(n)$

4.5. Now to find $H(k)$ we need to know the h(n).For that we need to first find the Y(z) by applying Z-transform on equation i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.5.1}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{4.5.2}$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.5.3}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{4.5.4}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{4.5.5}$$

By applying inverse Z - transform we can find the value of $h(n)$

$$h(n) = Z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{4.5.6}$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \quad (4.5.7)$$

Assuming that length of h(n) is same as length of x(n) i.e., N = 6. Now finding the value of $h(n)$ using matrix method we get

$$\begin{bmatrix} H(0)\\ H(1)\\ H(2)\\ H(3)\\ H(4)\\ H(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1\\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5\\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10}\\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15}\\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20}\\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} h(0)\\ h(1)\\ h(2)\\ h(3)\\ h(4)\\ h(5) \end{bmatrix} \quad (4.5.8)$$

$$\begin{bmatrix} H(0)\\ H(1)\\ H(2)\\ H(3)\\ H(4)\\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5)\\ h(0)+h(1)e^{-j\pi/3}+...+h(5)e^{-j5\pi/3}\\ h(0)+h(1)e^{-2j\pi/3}+...+h(5)e^{-2j5\pi/3}\\ h(0)+h(1)e^{-3j\pi/3}+...+h(5)e^{-3j5\pi/3}\\ h(0)+h(1)e^{-4j\pi/3}+...+h(5)e^{-4j5\pi/3}\\ h(0)+h(1)e^{-5j\pi/3}+...+h(5)e^{-5j5\pi/3} \end{bmatrix}$$

4.6. On solving we get

$$\begin{bmatrix} H(0)\\ H(1)\\ H(2)\\ H(3)\\ H(4)\\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125\\ 0.51625-0.5142j\\ -0.07813+1.1096j\\ 3.84375\\ -0.07183-1.1096j\\ 0.51625+0.5142j \end{bmatrix} \quad (4.6.1)$$

$$\implies H(0) = 1.28125 + 0j, \quad (4.6.2)$$
$$H(1) = 0.51625 - 0.5141875j, \quad (4.6.3)$$
$$H(2) = -0.078125 + 1.1095625j, \quad (4.6.4)$$
$$H(3) = 3.84375 + 0j, \quad (4.6.5)$$
$$H(4) = -0.071825 - 1.1095625j. \quad (4.6.6)$$
$$H(5) = 0.515625 + 0.5141875j \quad (4.6.7)$$

So,These values which we got are same as that of from the plots (4.6)

4.7. Now to find $H(k)$ we need to know the h(n).For that we need to first find the Y(z) by applying Z-transform on equation

$$\begin{bmatrix} Y(0)\\ Y(1)\\ Y(2)\\ Y(3)\\ Y(4)\\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0)\\ X(1)\\ X(2)\\ X(3)\\ X(4)\\ X(5) \end{bmatrix} \times \begin{bmatrix} H(0)\\ H(1)\\ H(2)\\ H(3)\\ H(4)\\ H(5) \end{bmatrix} \quad (4.7.1)$$

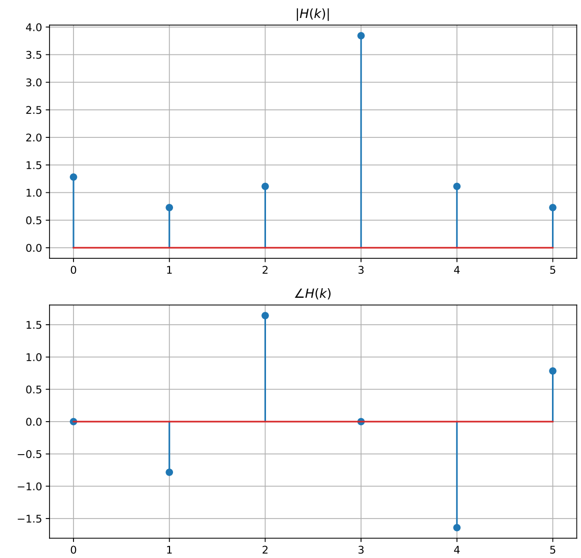

Fig. 4.6: FFT of $h(n)$

$$\begin{bmatrix} Y(0)\\ Y(1)\\ Y(2)\\ Y(3)\\ Y(4)\\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.65625\\ -2.95312+1.16372j\\ -0.07813+1.1096j\\ -3.84375\\ -0.07813-1.1096j\\ -2.95312-1.16372j \end{bmatrix} \quad (4.7.2)$$

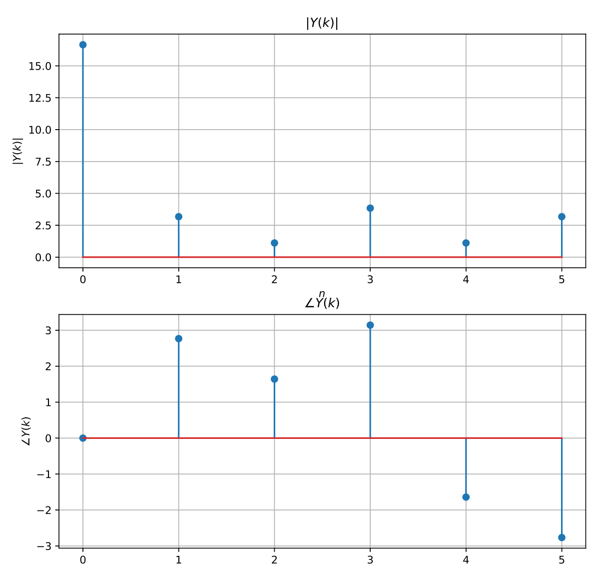

Fig. 4.7: FFT of $y(n)$

4.8. $y(n)$ can be calculated by applying IFFT for the above Y matrix and is calculated by python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 \\ 2.28125071 \\ 2.6250019 \\ 4.37499667 \\ 2.6562481 \\ 3.59375262 \end{bmatrix}$$

These are the same values that we have plotted in the above plot (2.7)

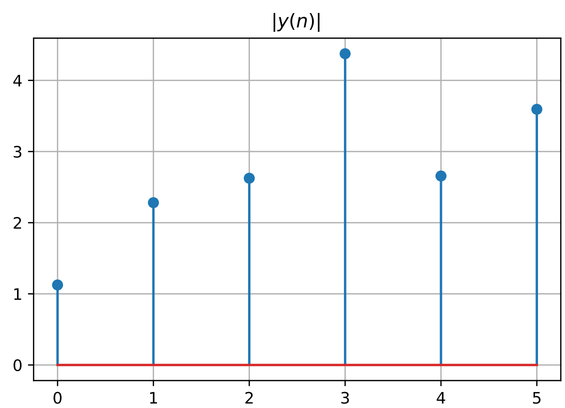

Fig. 4.8: IFFT of $Y(k)$