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Spatial AI & Robotics Lab

CSE 473/573-A

L4: WARPING

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Content

- Image Warping
  - Mirror, rotation, translation, scaling, skewing
  - Euclidean, similarity, affine, projective (homograph)
- Optics
  - Lens, aperture, focal length.
  - Field of view, depth of view.
  - Vignetting, chromatic/spherical aberration
  - Pincushion, barrel distortion

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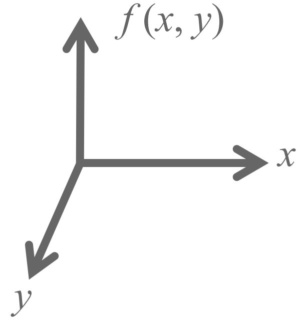
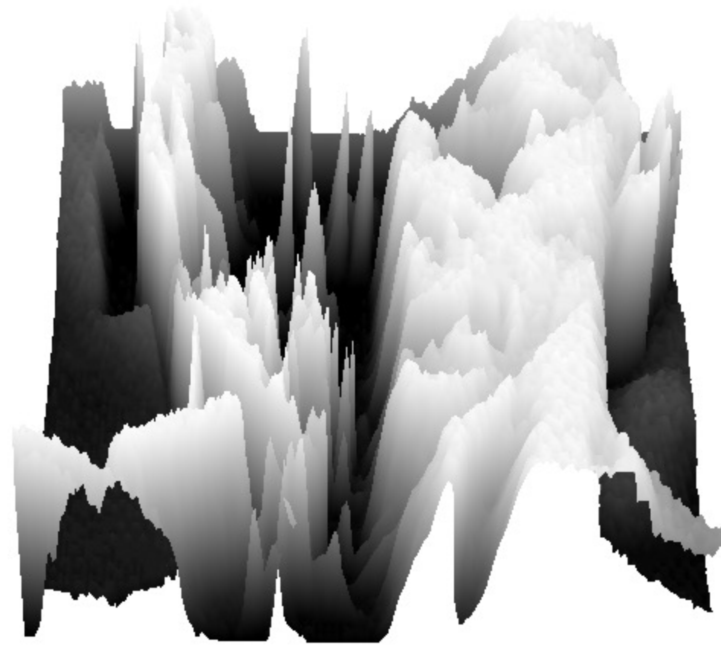
IMAGE FORMATION

Image Warping

Recap: Image representation

- A (grayscale) image as a **function**, *f*, from R^2 to R:
  - $f(x, y)$ gives the **intensity** at position (x, y).
  - A **digital image** is a discrete (**sampled**, **quantized**) version of this function.





Warping (Transformation)

image warping: change ***domain*** of image

$$g(x) = f(T(x))$$

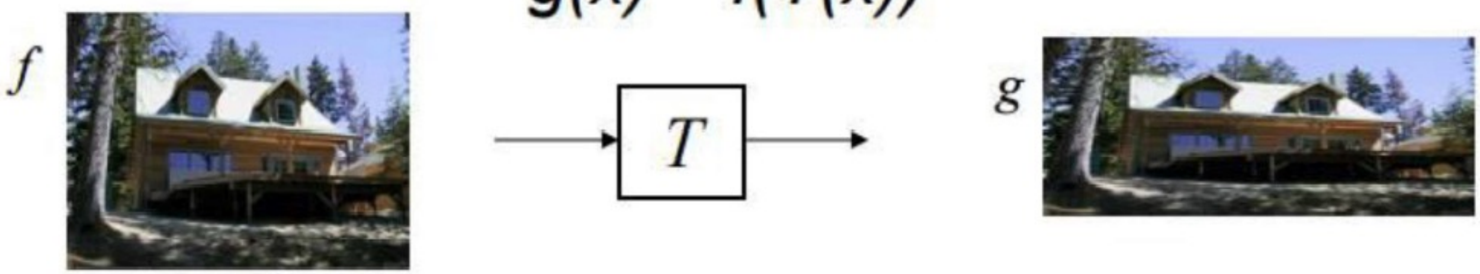
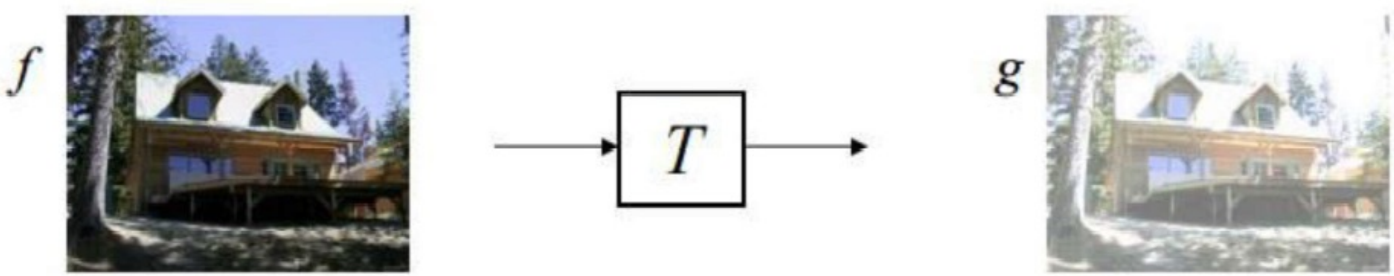


image filtering: change ***range*** of image (Next Week)

$$g(x) = T(f(x))$$



2D Planer Transformation

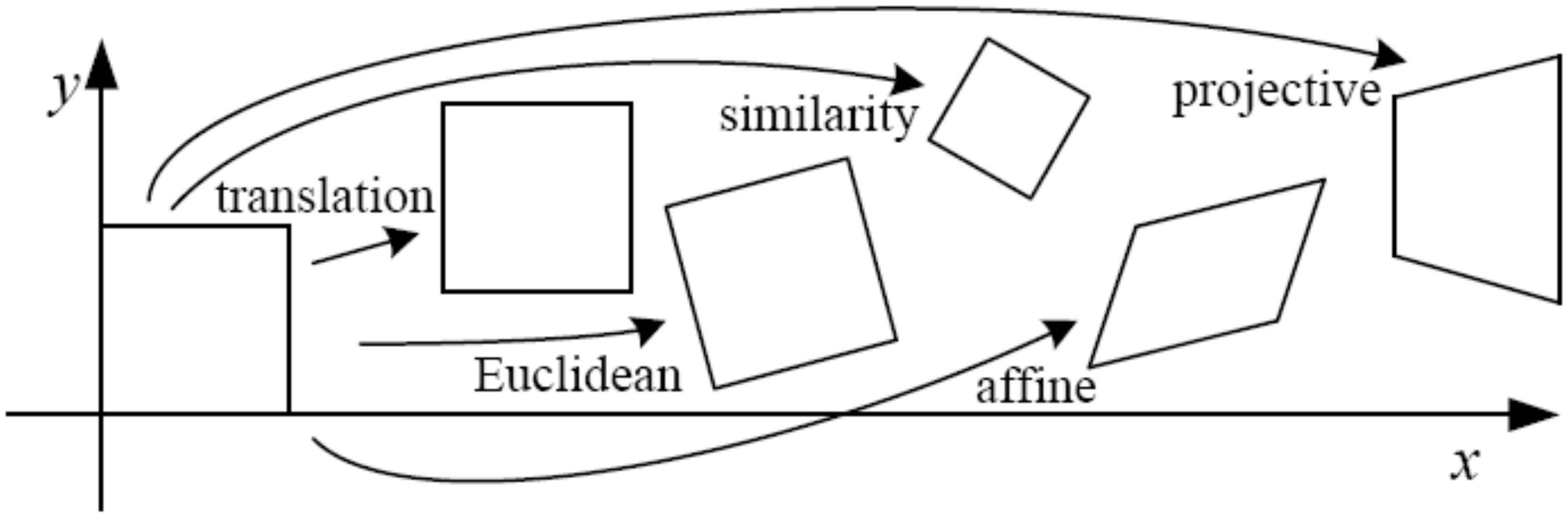
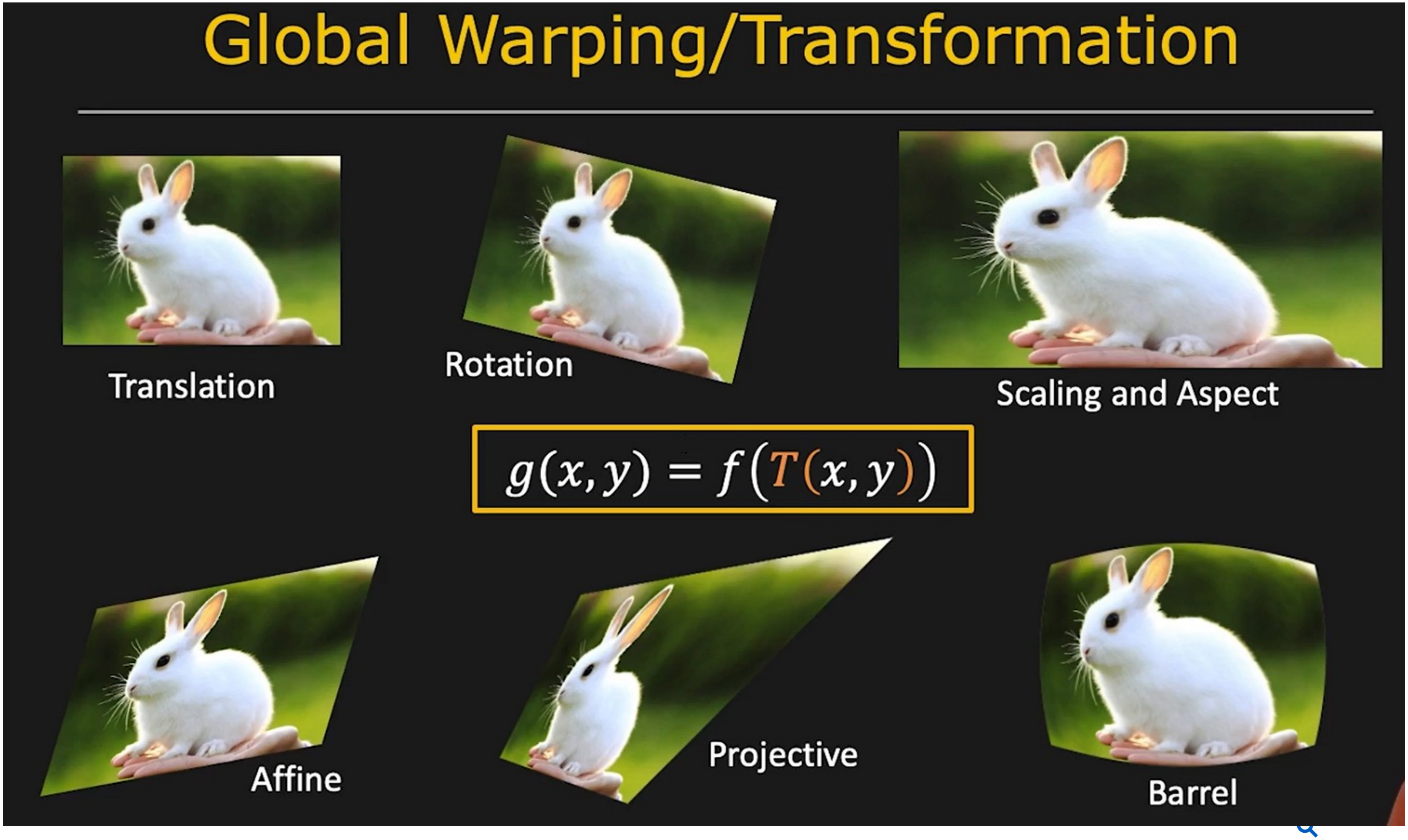
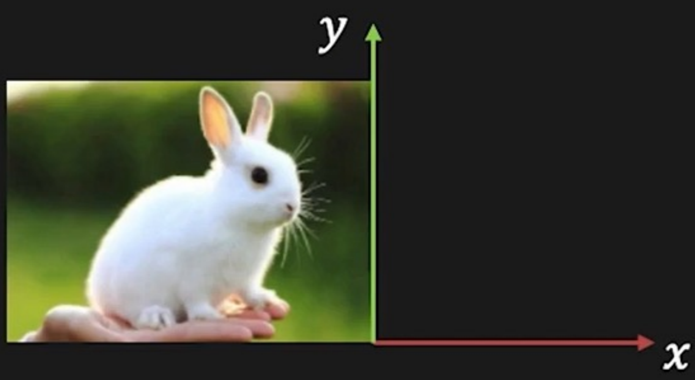


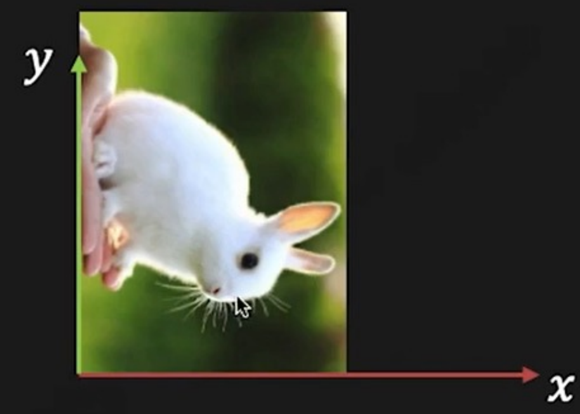
Figure 2.4: *Basic set of 2D planar transformations*

Image Warping



Mirror





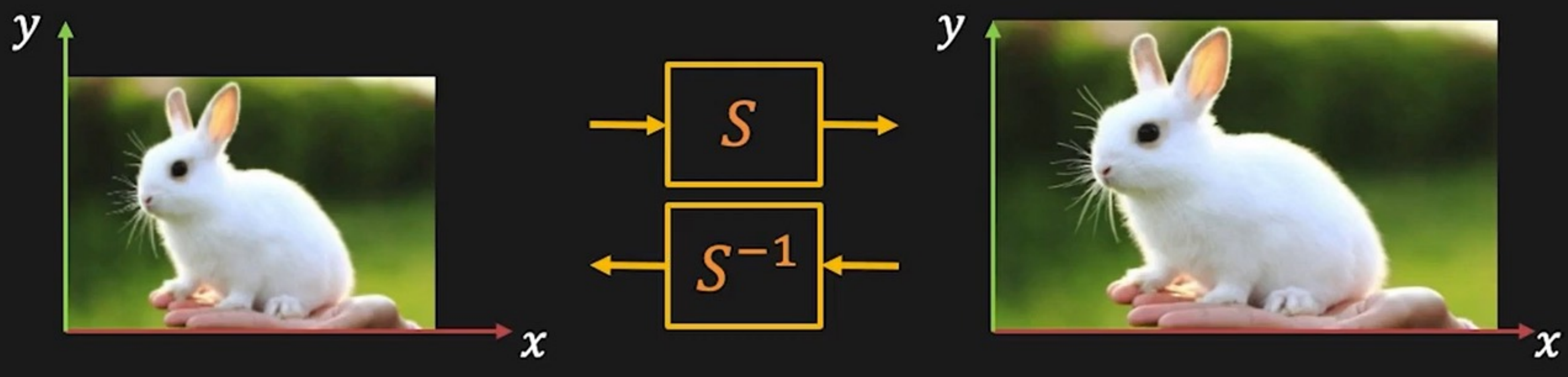
Mirror about Y-axis:

$$x_2 = -x_1$$
$$y_2 = y_1$$

Mirror about line $y = x$:

$$x_2 = y_1$$
$$y_2 = x_1$$

Scaling (Stretching or Squashing)



Forward:

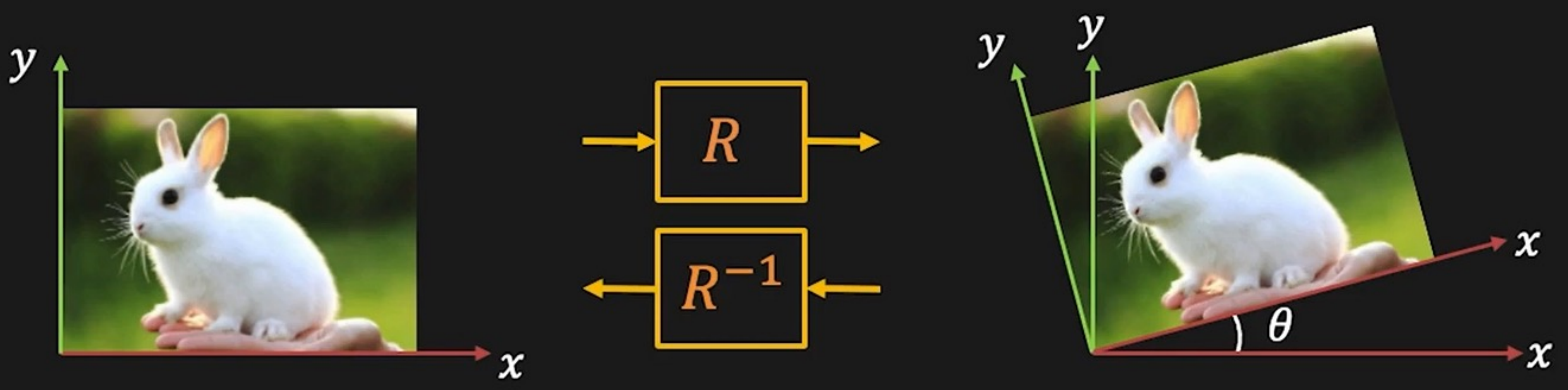
$$x_2 = a x_1 \qquad y_2 = b y_1$$

Inverse:

$$x_1 = \frac{1}{a} x_2 \qquad y_1 = \frac{1}{b} y_2$$

- Scaling a coordinate means multiplying each component by a scalar
- Uniform scaling means this scalar is the same for all component.

Rotation



Forward:

$$x_2 = x_1 cos\theta - y_1 sin\theta$$

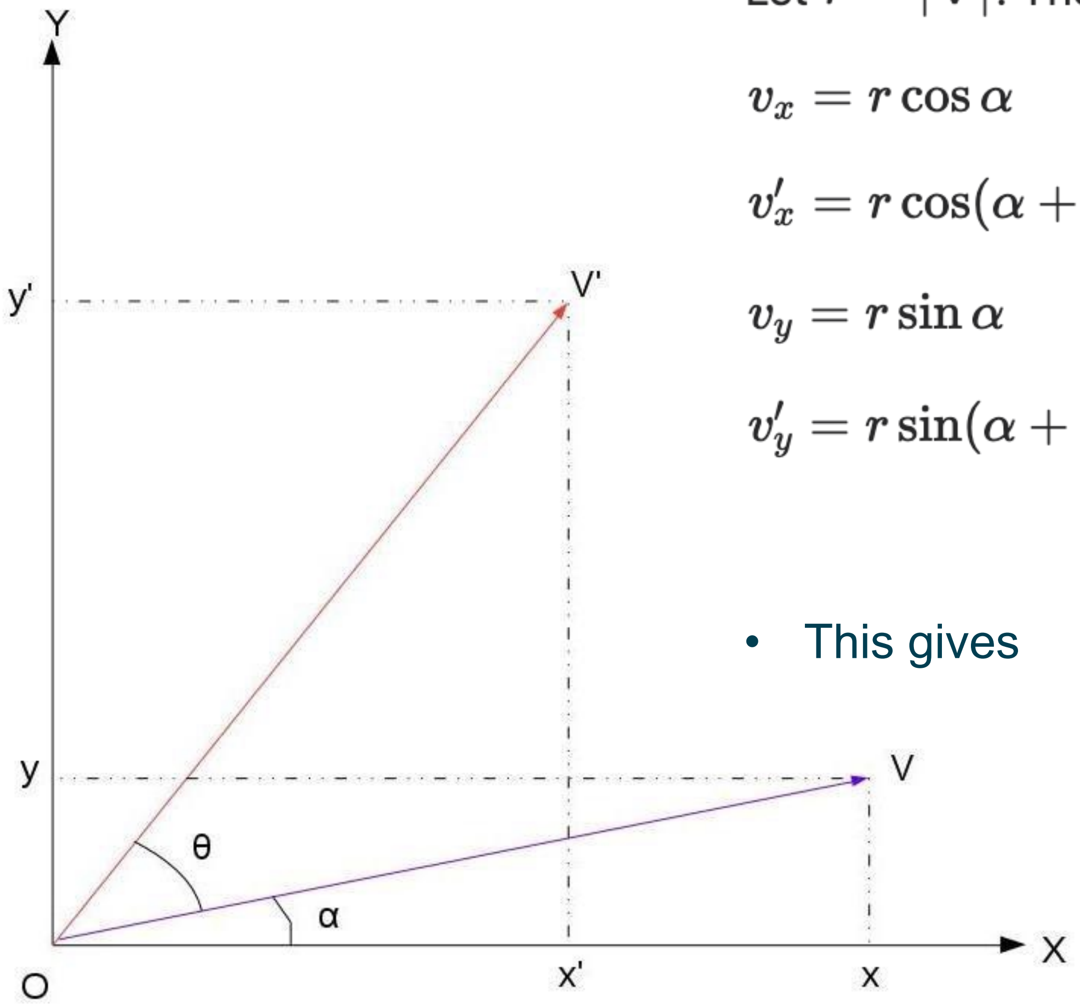
$$y_2 = x_1 sin\theta + y_1 cos\theta$$

Inverse:

$$x_1 = x_2 cos\theta + y_2 sin\theta$$

$$y_1 = -x_2 sin\theta + y_2 cos\theta$$

How to derive rotation matrix?



Let $r = |\mathbf{V}|$. Then, we have the relations:

$$v_x = r\cos\alpha$$

$$v'_x = r\cos(\alpha+\theta)$$

$$v_y = r\sin\alpha$$

$$v'_y = r\sin(\alpha+\theta)$$

$$v'_x = v_x\cos\theta - v_y\sin\theta$$

$$v'_y = v_x\cos\theta + v_y\sin\theta$$

- This gives $\begin{pmatrix} v'_x \\ v'_y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix}$

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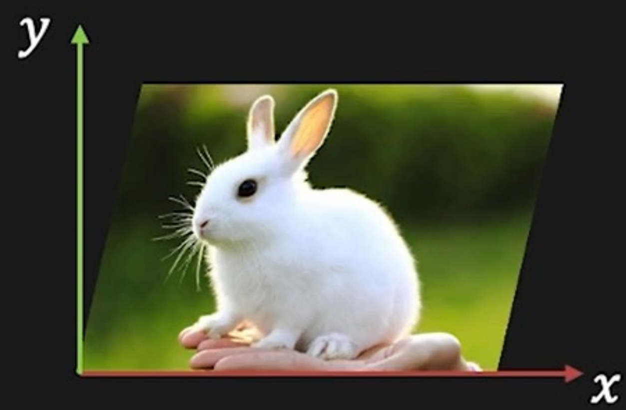
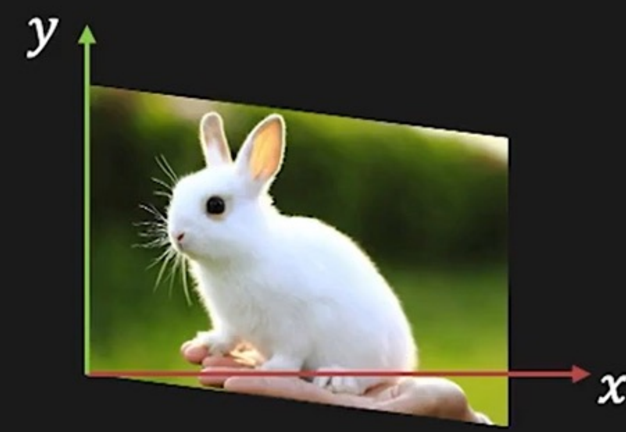
Skew





Horizontal Skew:

$$x_2 = x_1 + m_x y_1$$

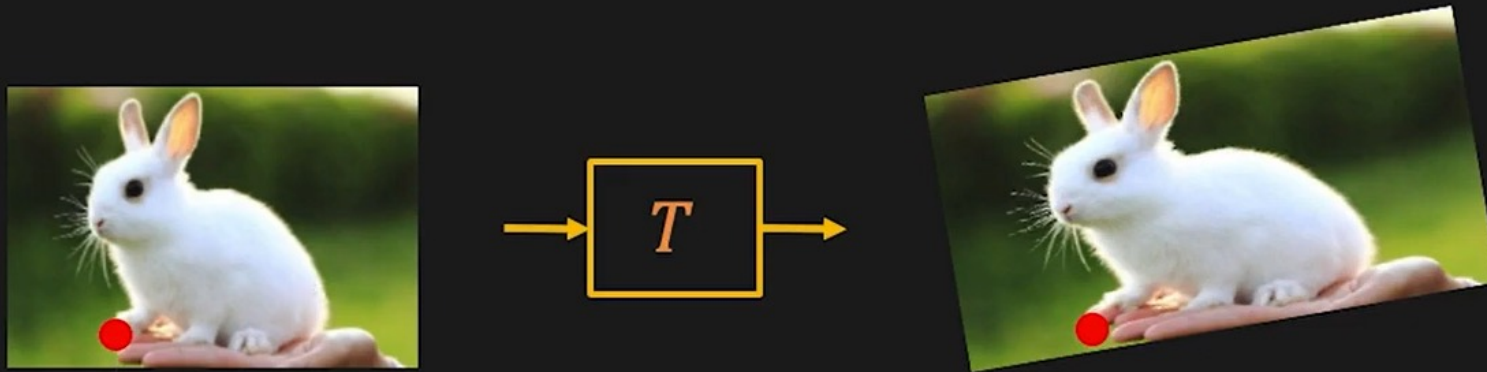
$$y_2 = y_1$$

Vertical Skew:

$$x_2 = x_1$$

$$y_2 = m_y x_1 + y_1$$

Linear Transformations



$\mathbf{p}_1 = (x_1, y_1)$ $\mathbf{p}_2 = (x_2, y_2)$

T can be represented by a matrix.

$$\mathbf{p}_2 = T\mathbf{p}_1 \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

2 x 2 Matrix Transformations

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

- Origin maps to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

Scaling, Rotation, Skew, Translation

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}s_x & 0 & 0\\ 0 & s_y & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Scaling

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & m_x & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Skew

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}1 & 0 & t_x\\ 0 & 1 & t_y\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Translation

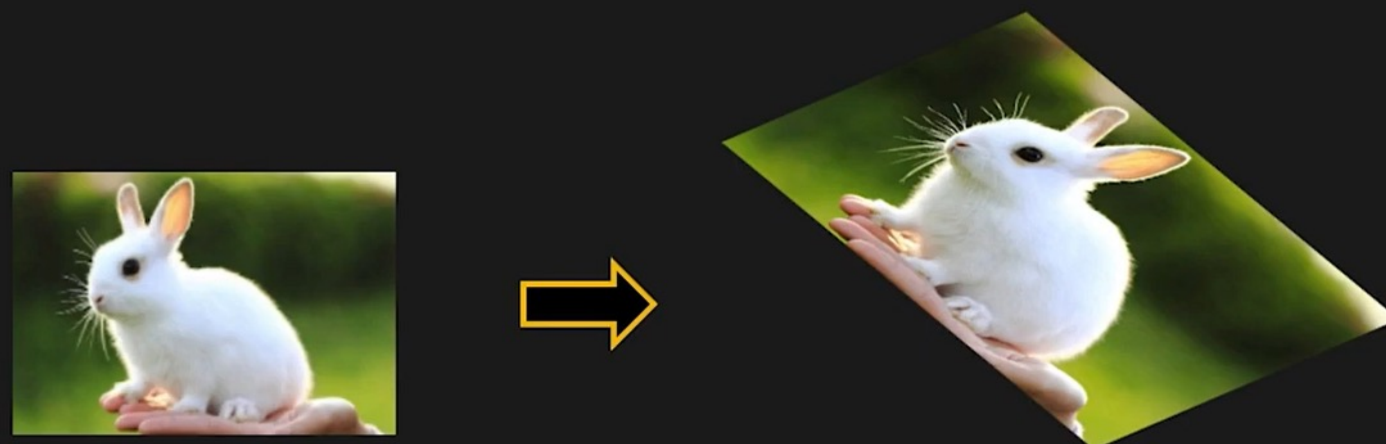
$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}cos\theta & -sin\theta & 0\\ sin\theta & cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ y_1\\ 1\end{bmatrix}$$

Rotation

Affine Transformation

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \boxed{0 \quad 0 \quad 1} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$



parallelogram

Affine Transformation

Any transformation of the form:

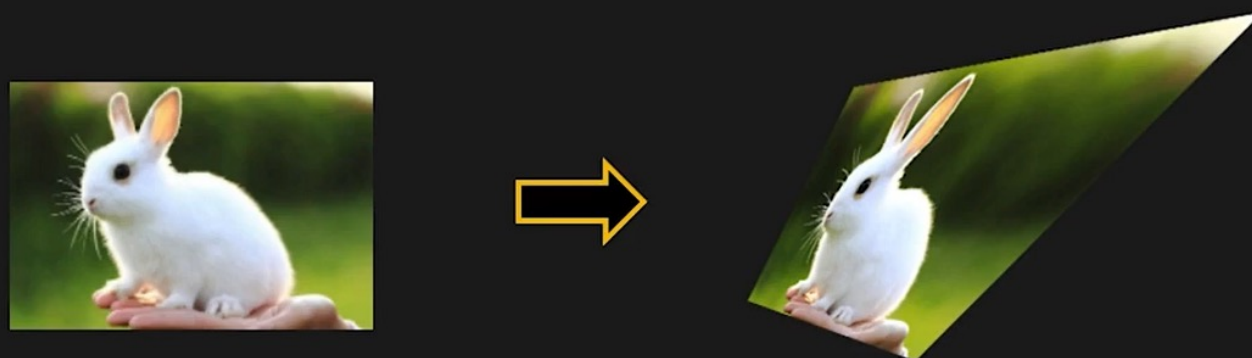
$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

Projective Transformation

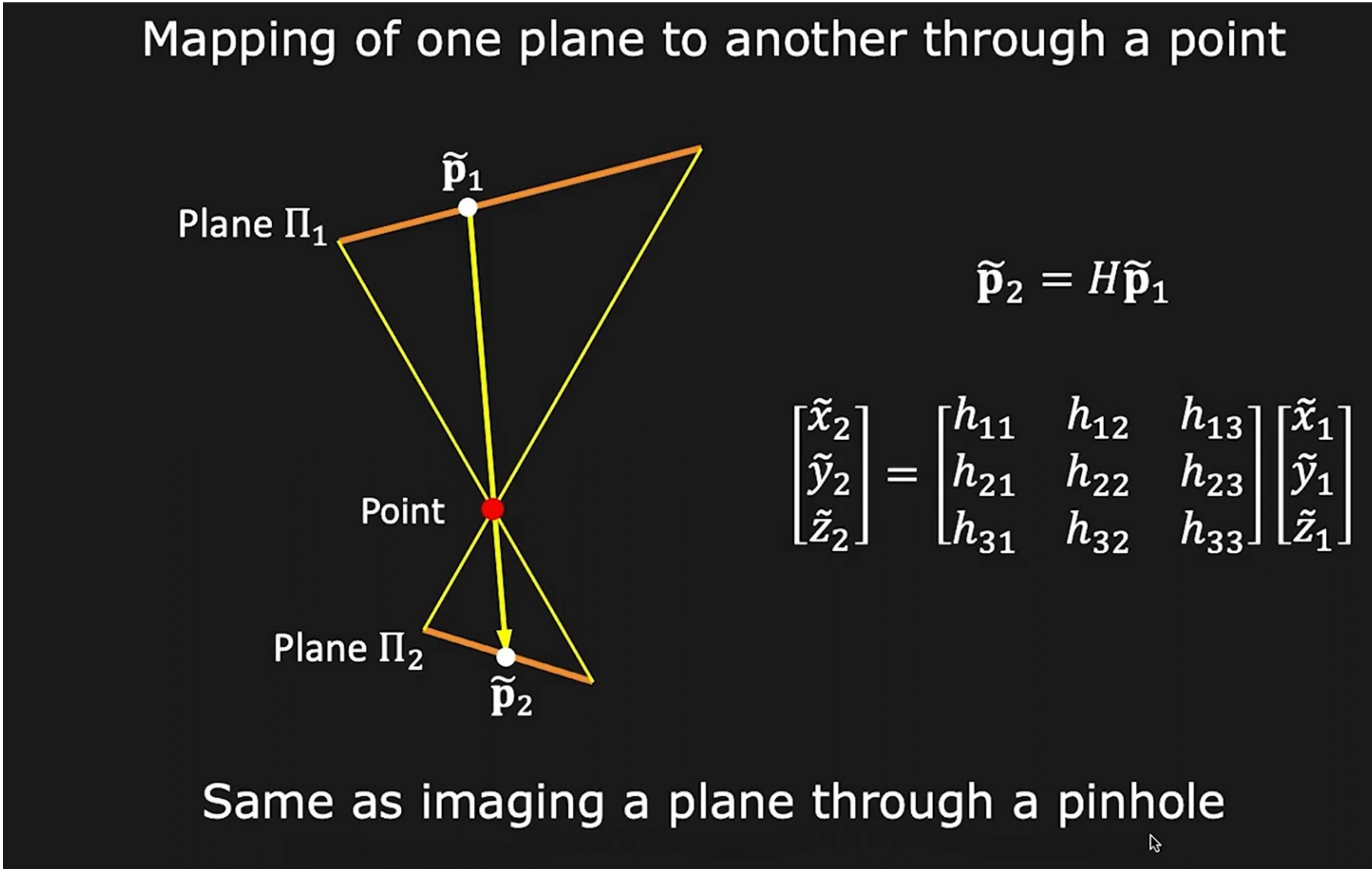
Any transformation of the form:

$$\begin{bmatrix}\tilde{x}_2\\ \tilde{y}_2\\ \tilde{z}_2\end{bmatrix} = \begin{bmatrix}h_{11} & h_{12} & h_{13}\\ h_{21} & h_{22} & h_{23}\\ h_{31} & h_{32} & h_{33}\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{y}_1\\ \tilde{z}_1\end{bmatrix} \qquad \widetilde{\mathbf{p}}_2 = H\widetilde{\mathbf{p}}_1$$



Also called Homography, or perspective transform

Projective Transformation



Projective Transformation

Homography can only be defined up to a scale.

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

If we fix scale such that $\sqrt{\Sigma(h_{ij})^2} = 1$ then 8 free parameters

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Closed under composition

2D vs 3D transform

2D

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

3D

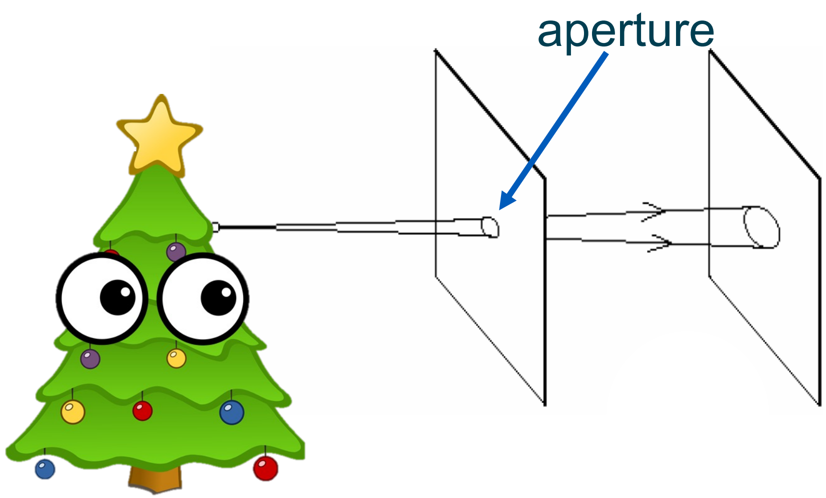
Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{3\times 4}$	3	orientation	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{3\times 4}$	6	lengths	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{3\times 4}$	7	angles	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{3\times 4}$	12	parallelism	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{4\times 4}$	15	straight lines	

IMAGE FORMATION

Optics

Previous Model - Pinhole

How does the size of the “Pinhole” affect the image we’d get?



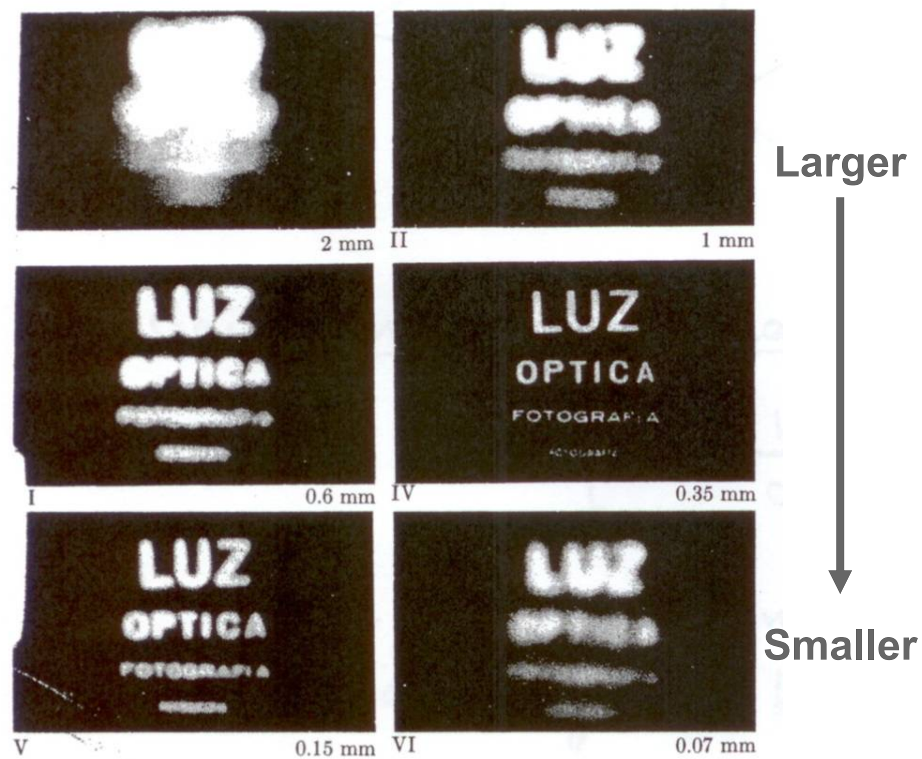
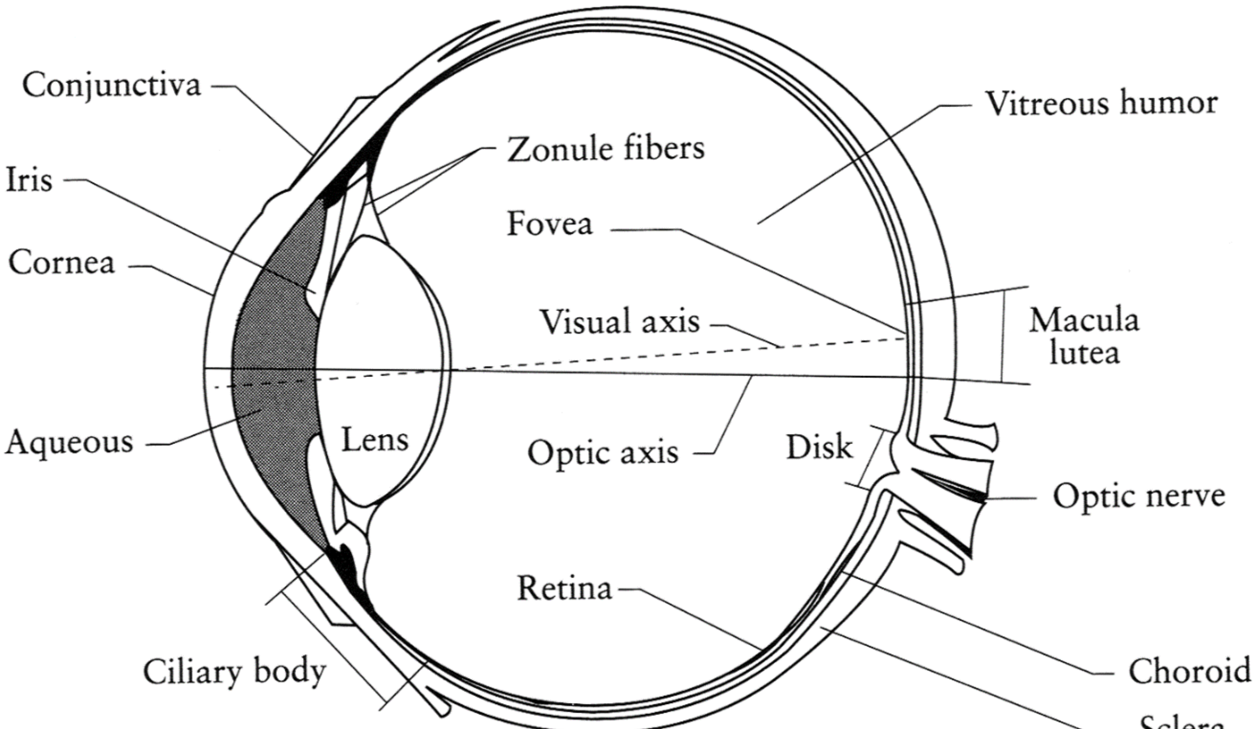


Fig. 5.96 The pinhole camera. Note the variation in image clarity as the hole diameter decreases. [Photos courtesy Dr. N. Joel, UNESCO.]

- Make the aperture smaller?
  - How small?
  - Like a pinhole? Infinitely Small?
- What is the problem then?
  - How much light do we need?
- Solution?
  - Use a Lens

Human Eye

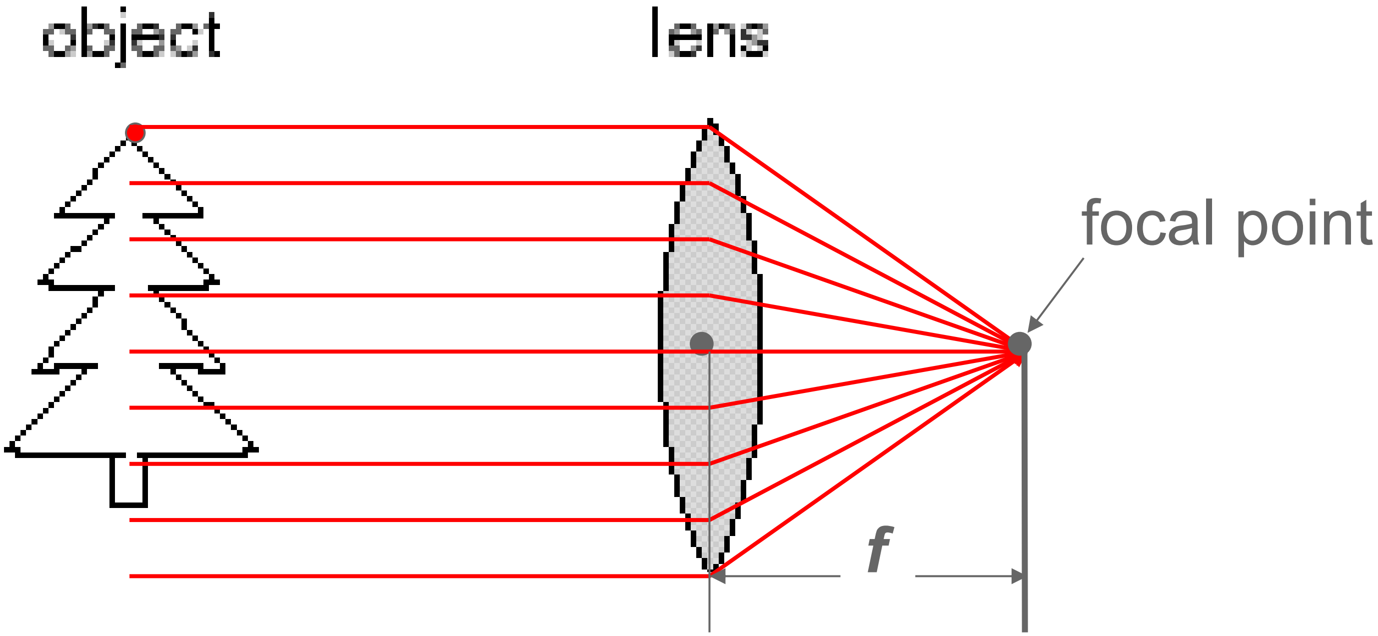
- **Iris:** Colored annulus with radial muscles
- **Pupil:** the hole (aperture) whose size is controlled by the iris.
- What's the "film"?



– photoreceptor cells (rods and cones) in the **retina**

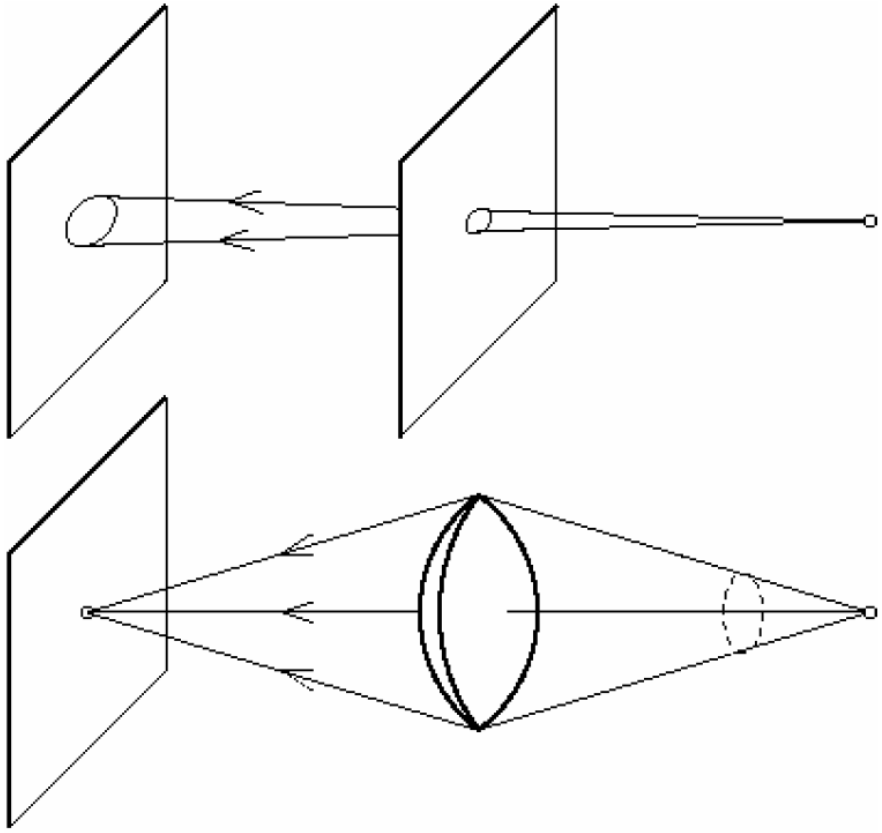
Lens

- A lens focuses parallel rays onto a single focal point
  - Gathers more lights; while keeping focus;
  - Make pinhole perspective projection practical



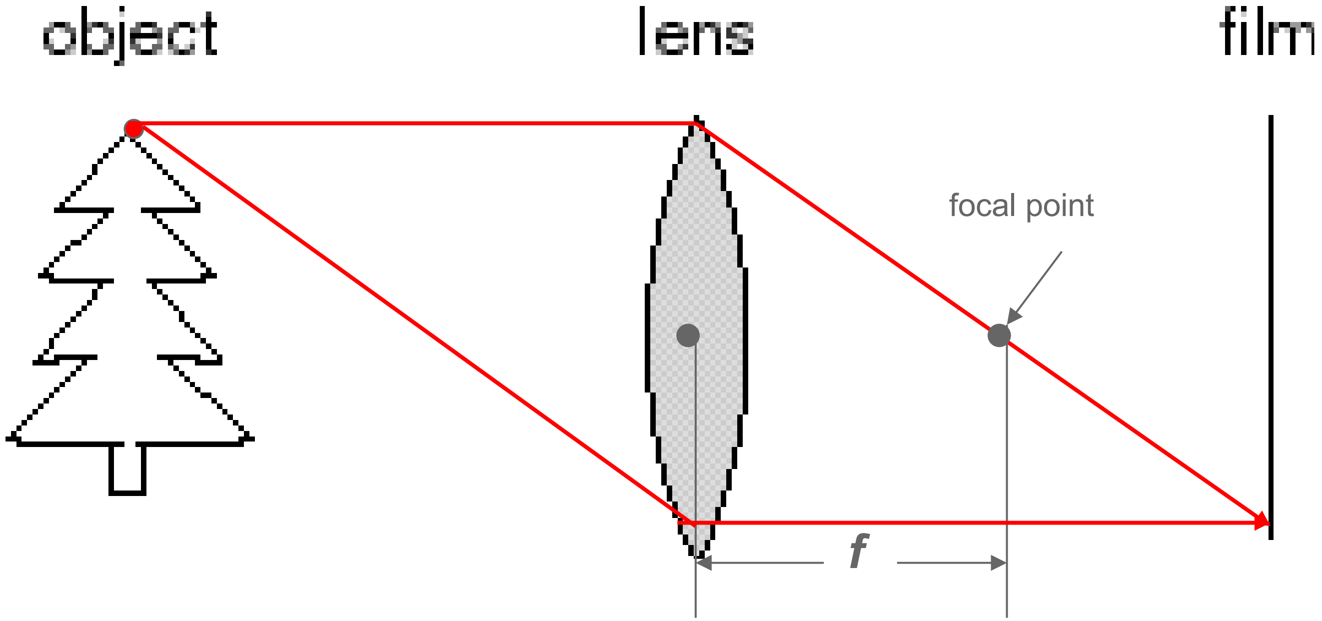
Benefits and challenges of adding lens

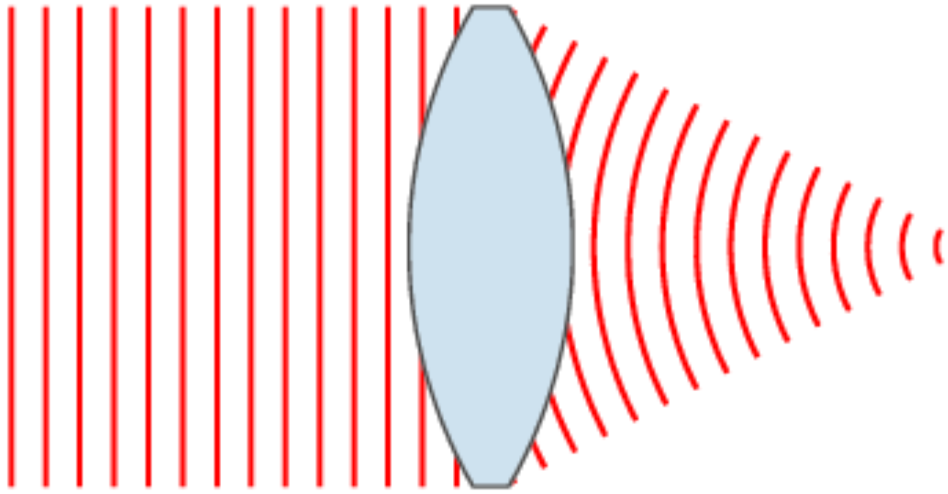
- Benefits
  - Light Concentration
  - Change the Focus
  - Depth of Field
  - Field of View



- Problems
  - Vignetting
  - Aberration

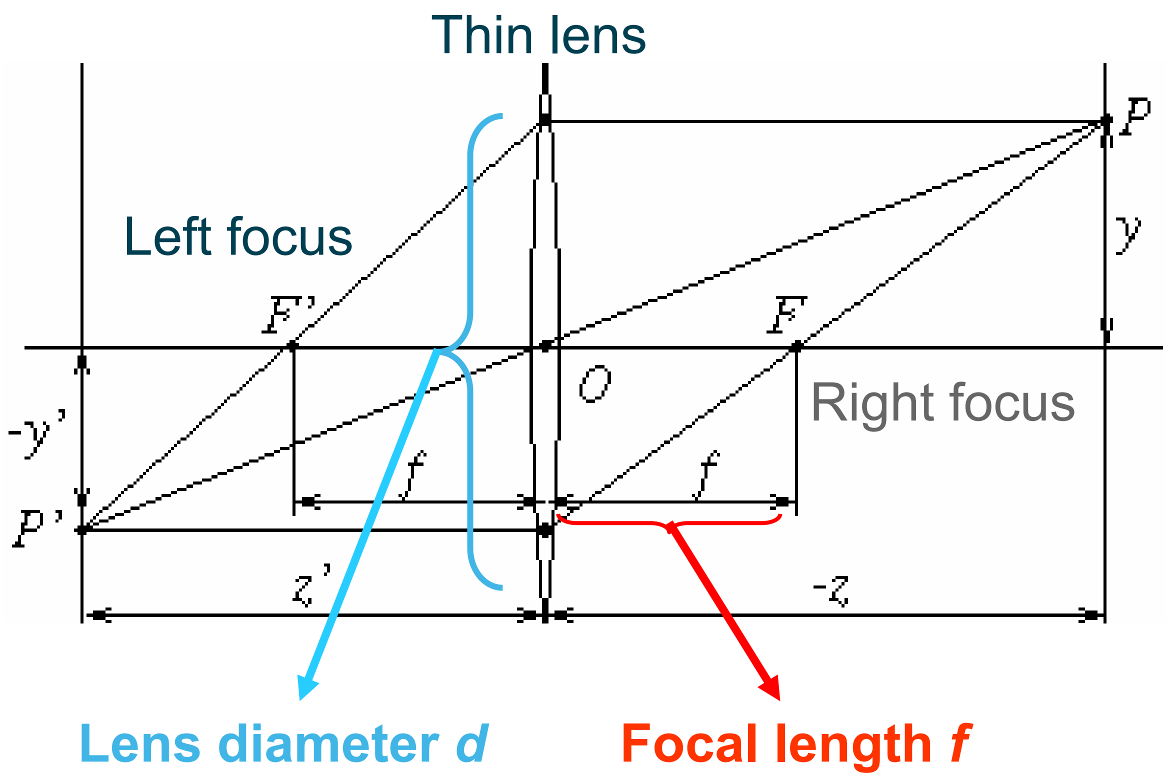
Light Concentration





Thin lens

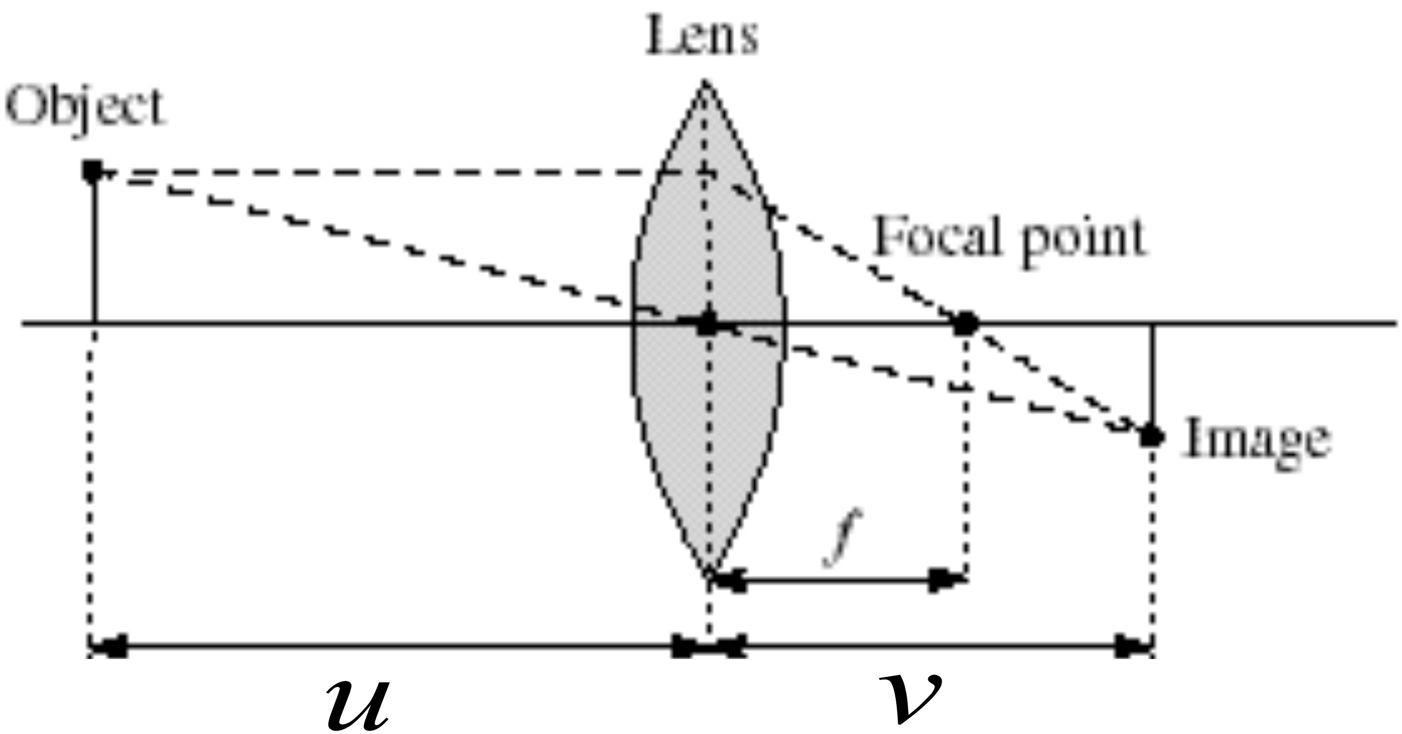
- Rays entering parallel on one side go through focus on other, and vice versa.
- In ideal case, all rays from P imaged at P'.



K. Grauman

Thin Lens

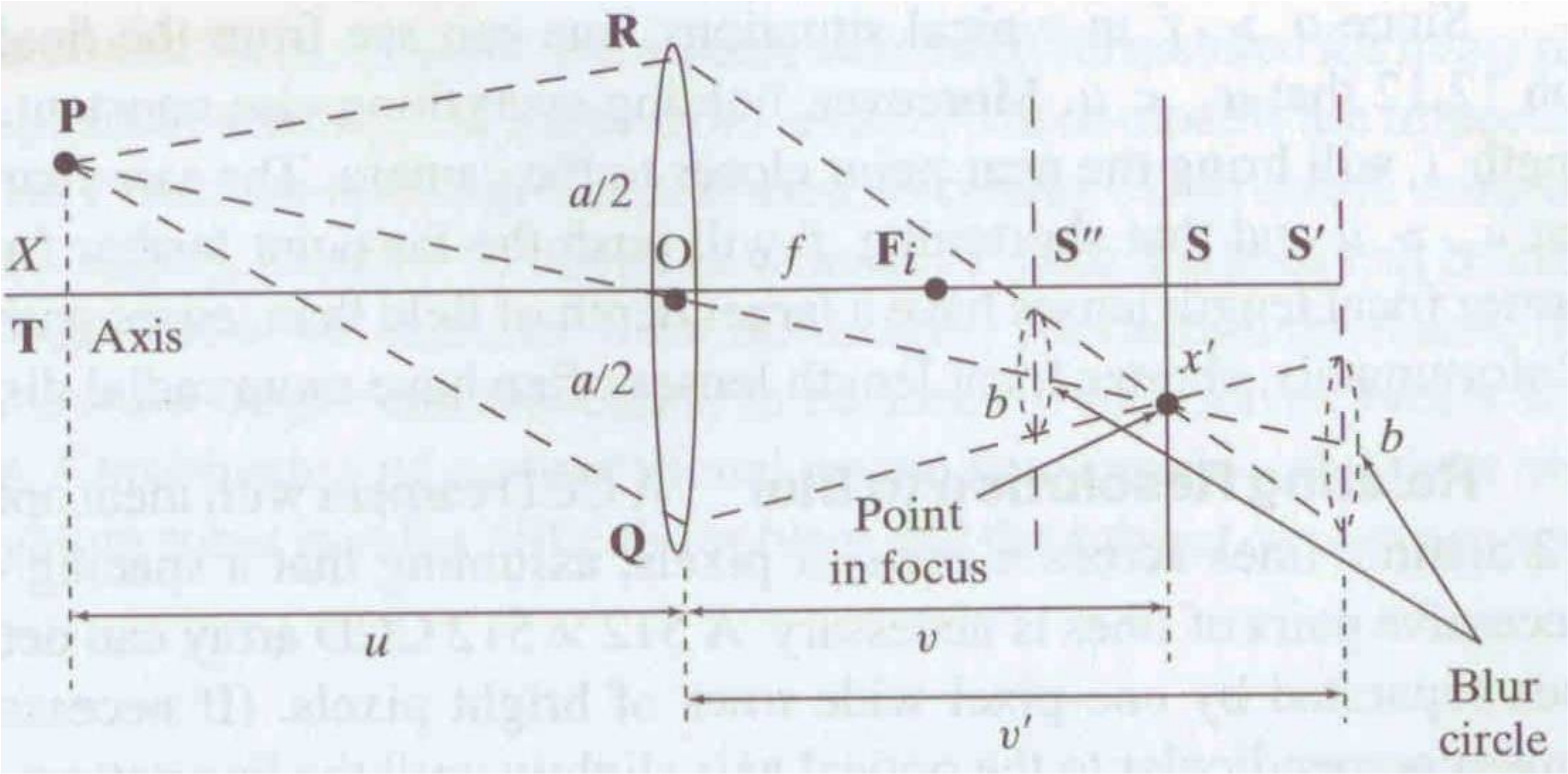
- Scene points at distinct depths come in focus at different image planes.



$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

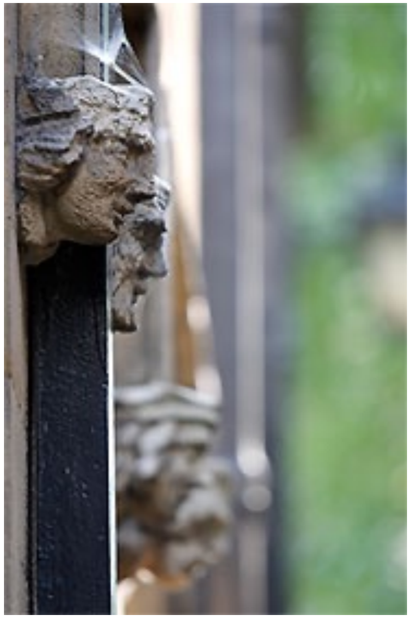
Depth of field

- Distance between image planes where blur is tolerable
- (Real camera lens systems have greater depth of field)



K. Grauman

Focus and depth of field



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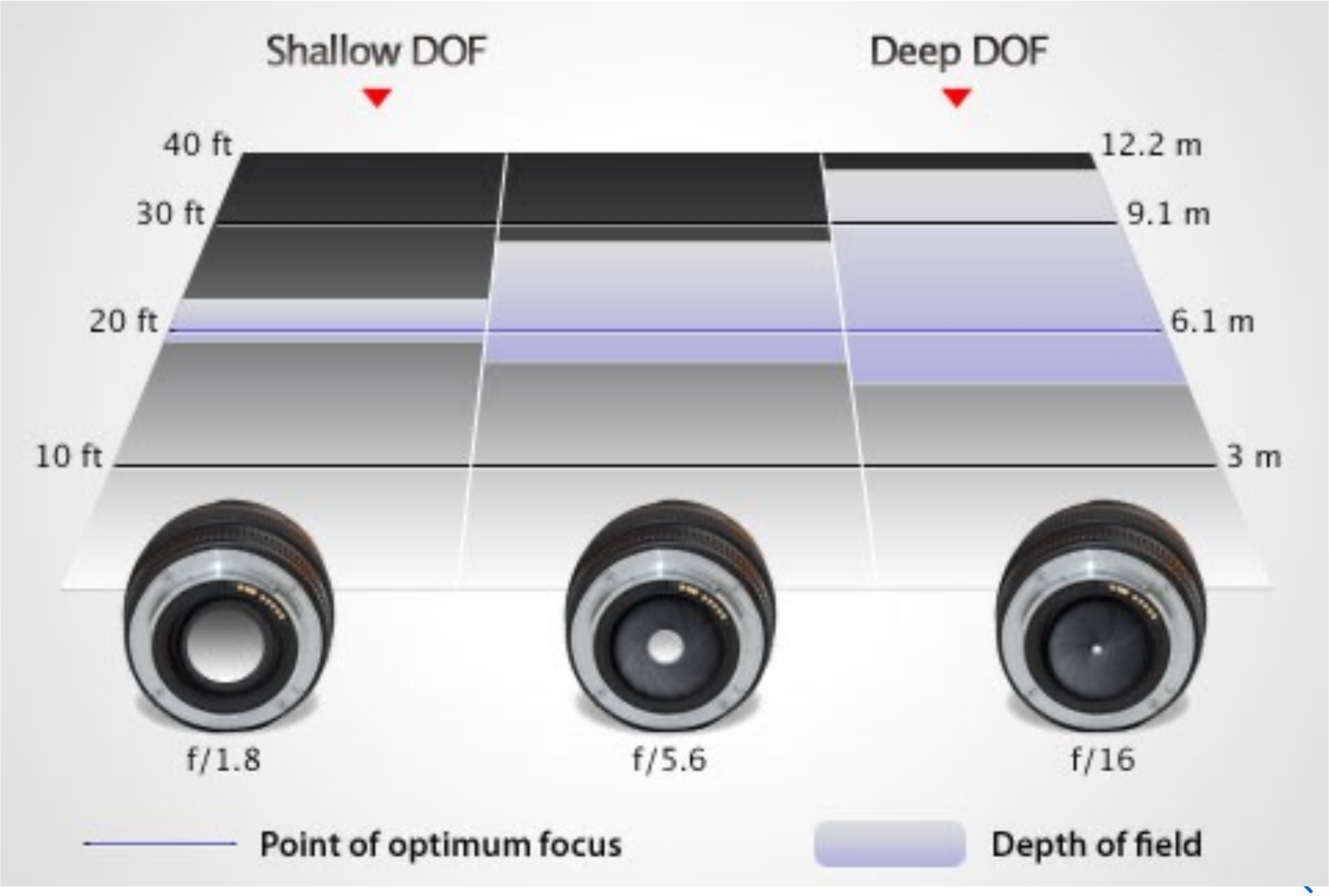
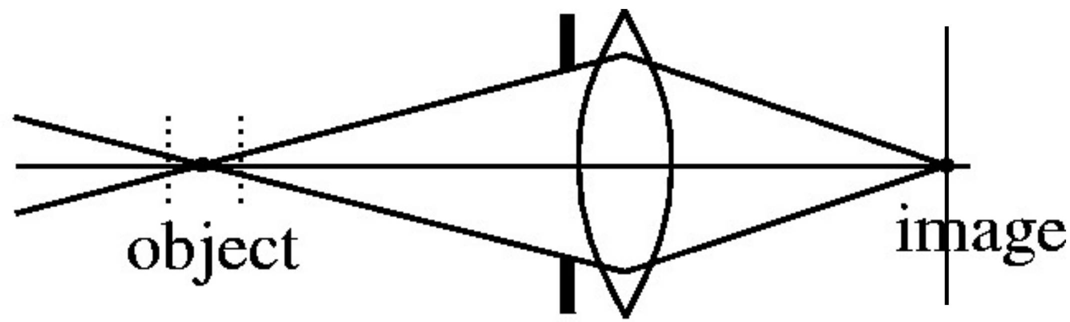
Depth of field

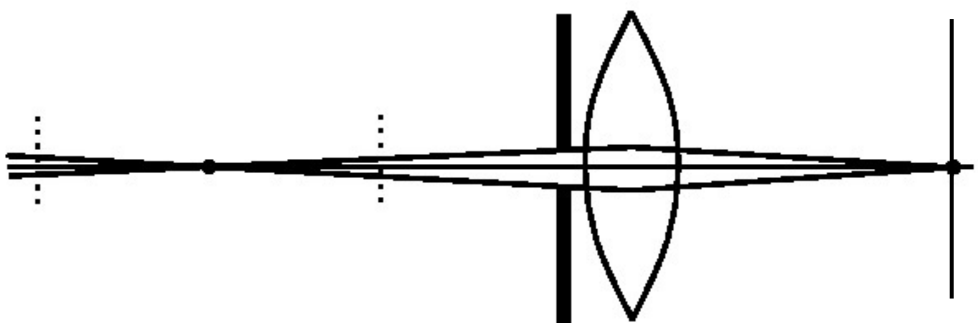


http://www.exposureguide.com/focusing-basics.htm

Aperture affects the depth of field

- A smaller aperture increases the range in which the object is approximately in focus

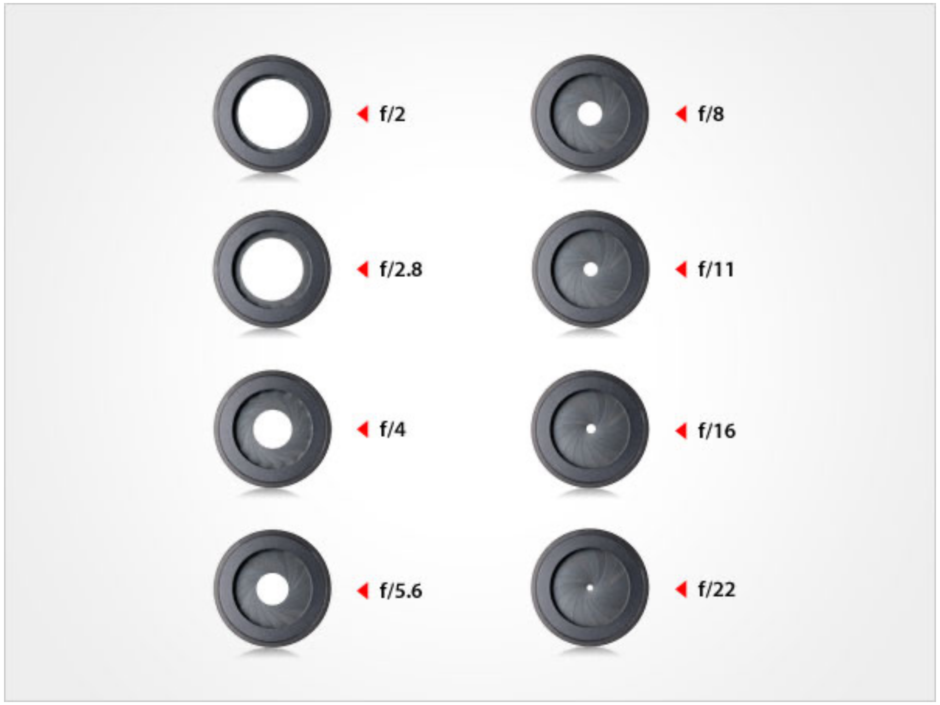




Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

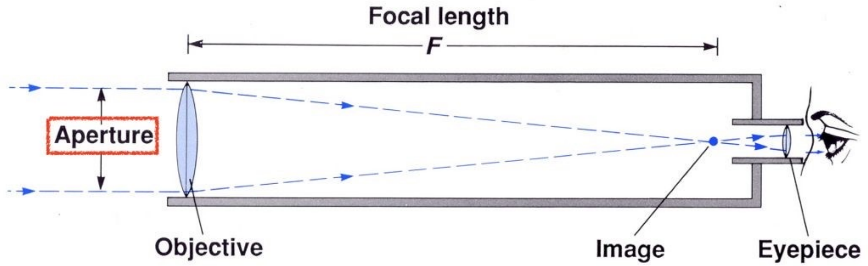
Slide from S. Seitz

Aperture settings



http://www.exposureguide.com/focusing-basics.htm

Brightness vs Aperture

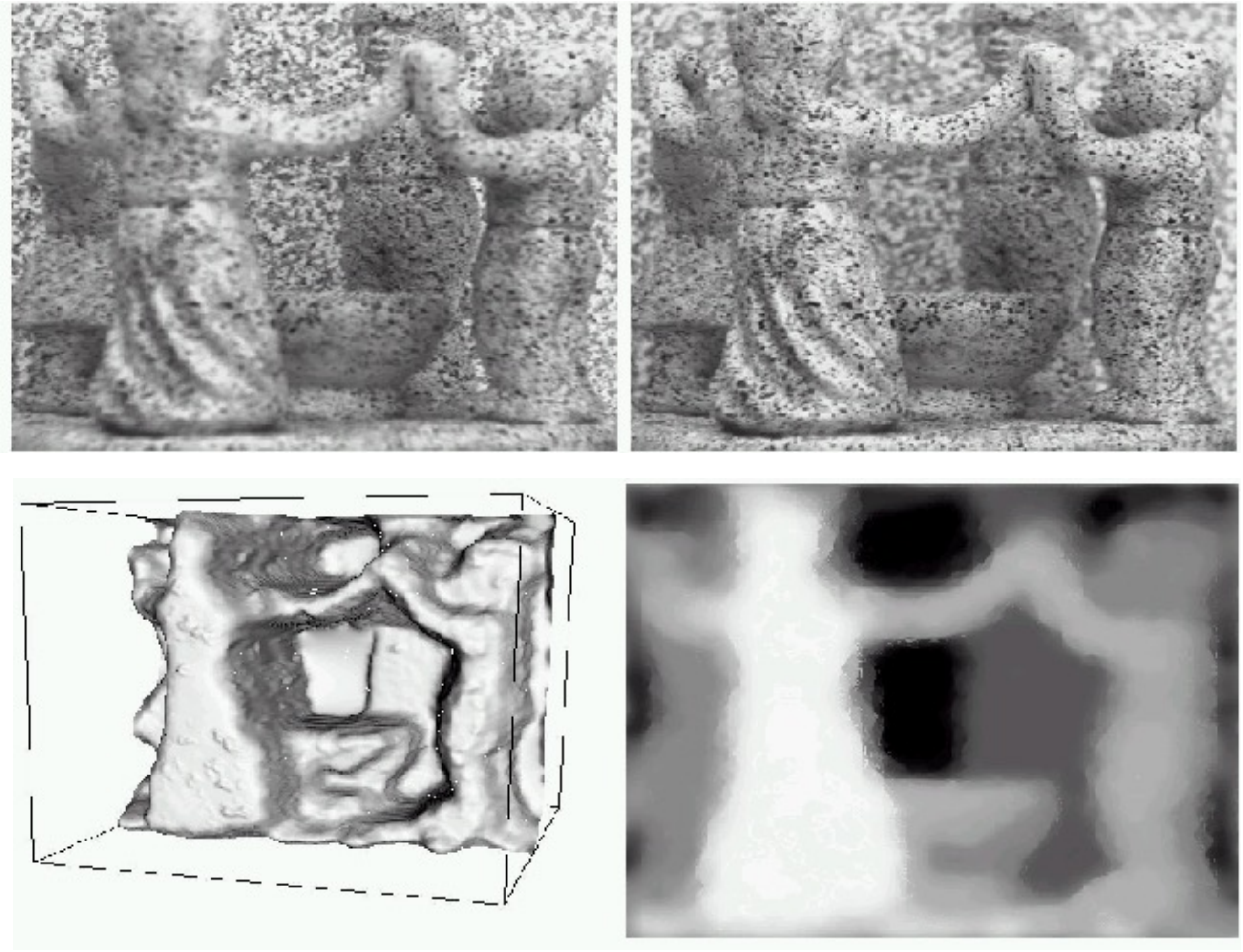


- **F-number**
  - Ratio of focal length to the diameter of the lens.
    - F-number = $\frac{f}{d}$
  - Greater f-number projects darker images.
- Brightness of the projected image (illuminance):
  - relative to the brightness of the scene (luminance)
  - decreases with the **square** of the f-number.
- Doubling the f-number decreases:
  - the relative brightness by a factor of four.
  - To maintain the same photographic exposure
  - The exposure time would need to be four times as long.

https://en.wikipedia.org/wiki/F-number

Depth from focus

- Same point of view, different camera parameters
- 3d shape / depth estimates



[figs from H. Jin and P. Favaro, 2002]

Field of view

- Angular measure of portion of 3D space seen by the camera

28 mm lens, 65.5° × 46.4°

50 mm lens, 39.6° × 27.0°

70 mm lens, 28.9° × 19.5°

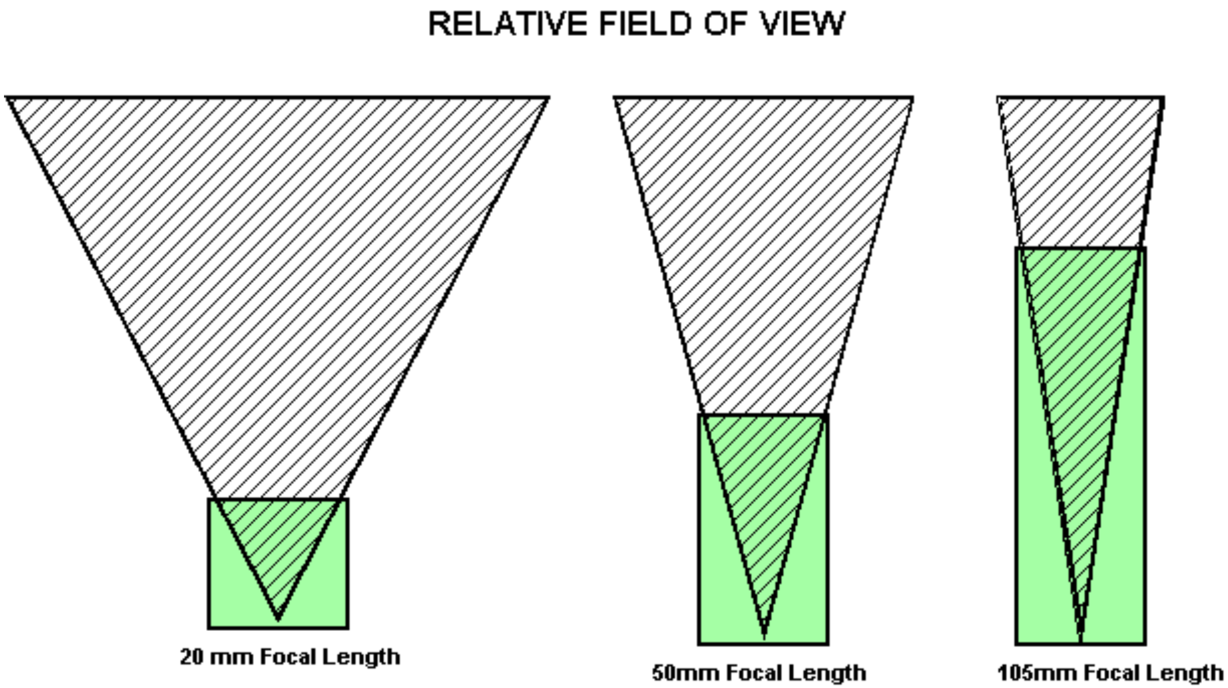
210 mm lens, 9.8° × 6.5°

http://en.wikipedia.org/wiki/Angle_of_view

K. Grauman

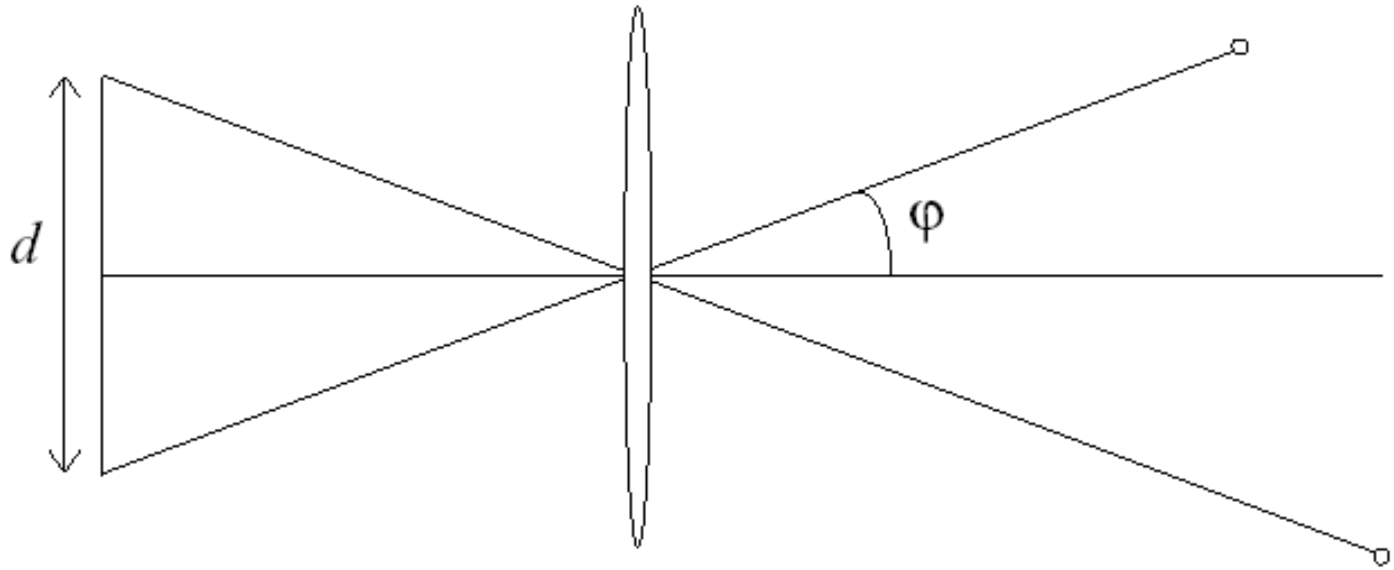
Field of view depends on focal length

- As ***F*** gets smaller, image becomes **wider angle**.
  - More world points project onto the image
- As ***F*** gets larger, image becomes **more *telescopic.***
  - Less world points project onto the image



Field of view depends on camera retina

- Smaller FOV = larger Focal Length

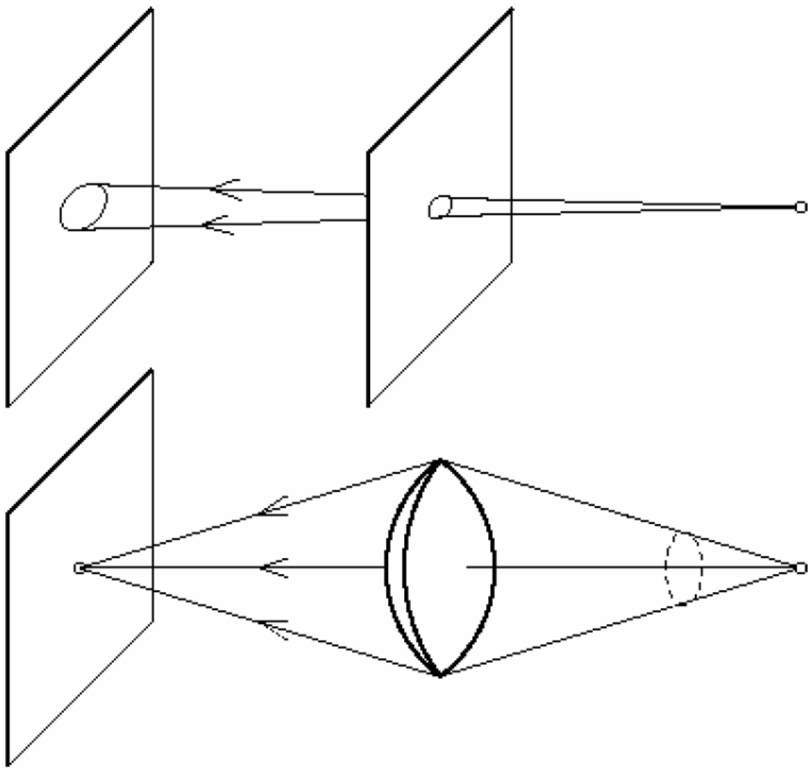


Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Benefits and challenges of adding lens

- Benefits
  - Light Concentration
  - Change the Focus
  - Depth of Field
  - Field of View



- Problems
  - Vignetting
  - Aberration

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Vignetting

- Lens vignetting is **a reduction in brightness or saturation on the periphery of an image**.

http://www.tlucretius.net/Photo/eHolga.html
http://www.ptgui.com/examples/vigntutorial.html

Vignetting

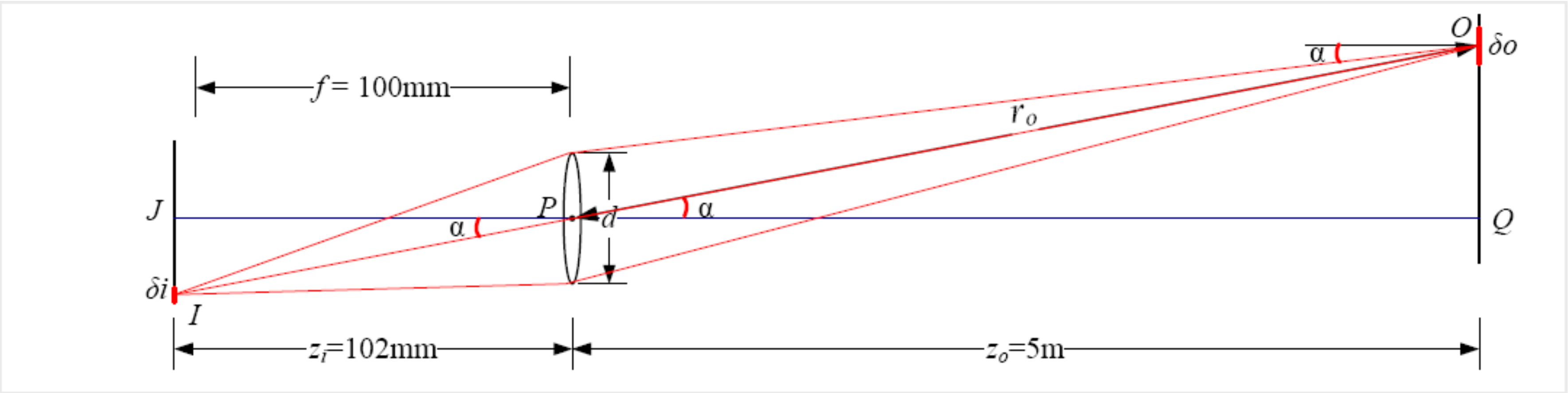


Figure 2.23: *The amount of light hitting a pixel of surface area* δi *depends on the square of the ratio of the aperture diameter* d *to the focal length* f*, as well as the fourth power of the off-axis angle* α *cosine,* $\cos^4 \alpha$.

$$l \propto \left(\frac{d}{f}\right)^2 \cos^4 \alpha$$

Chromatic aberration

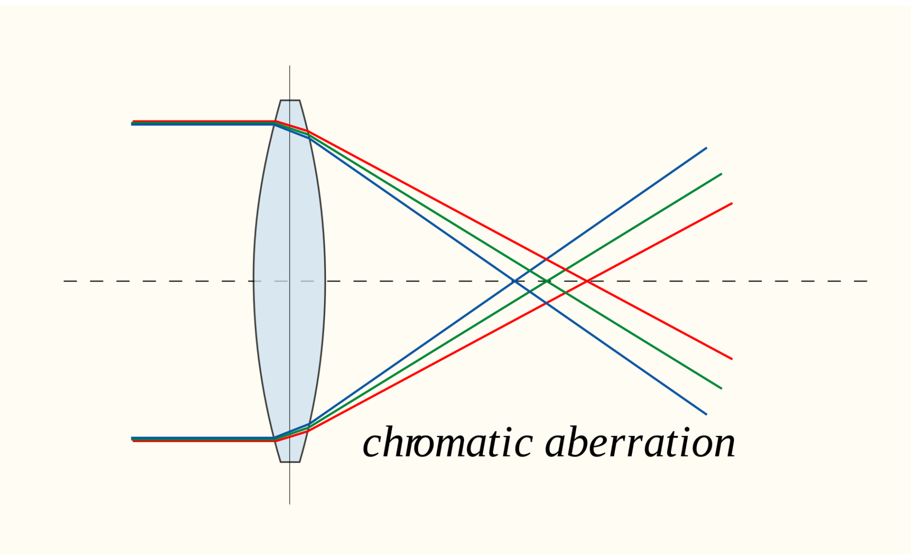
- Transverse chromatic aberration
  - a blur and a rainbow edge in areas of contrast.

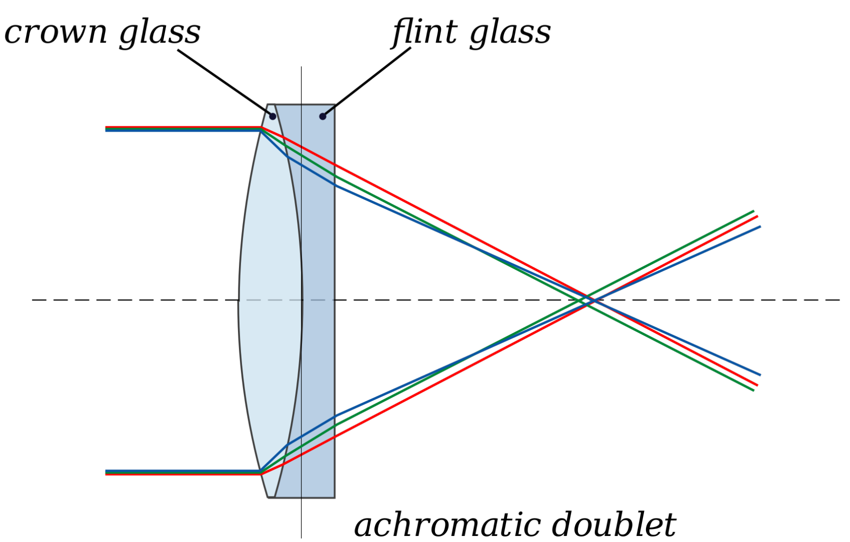
low quality lens

high quality lens

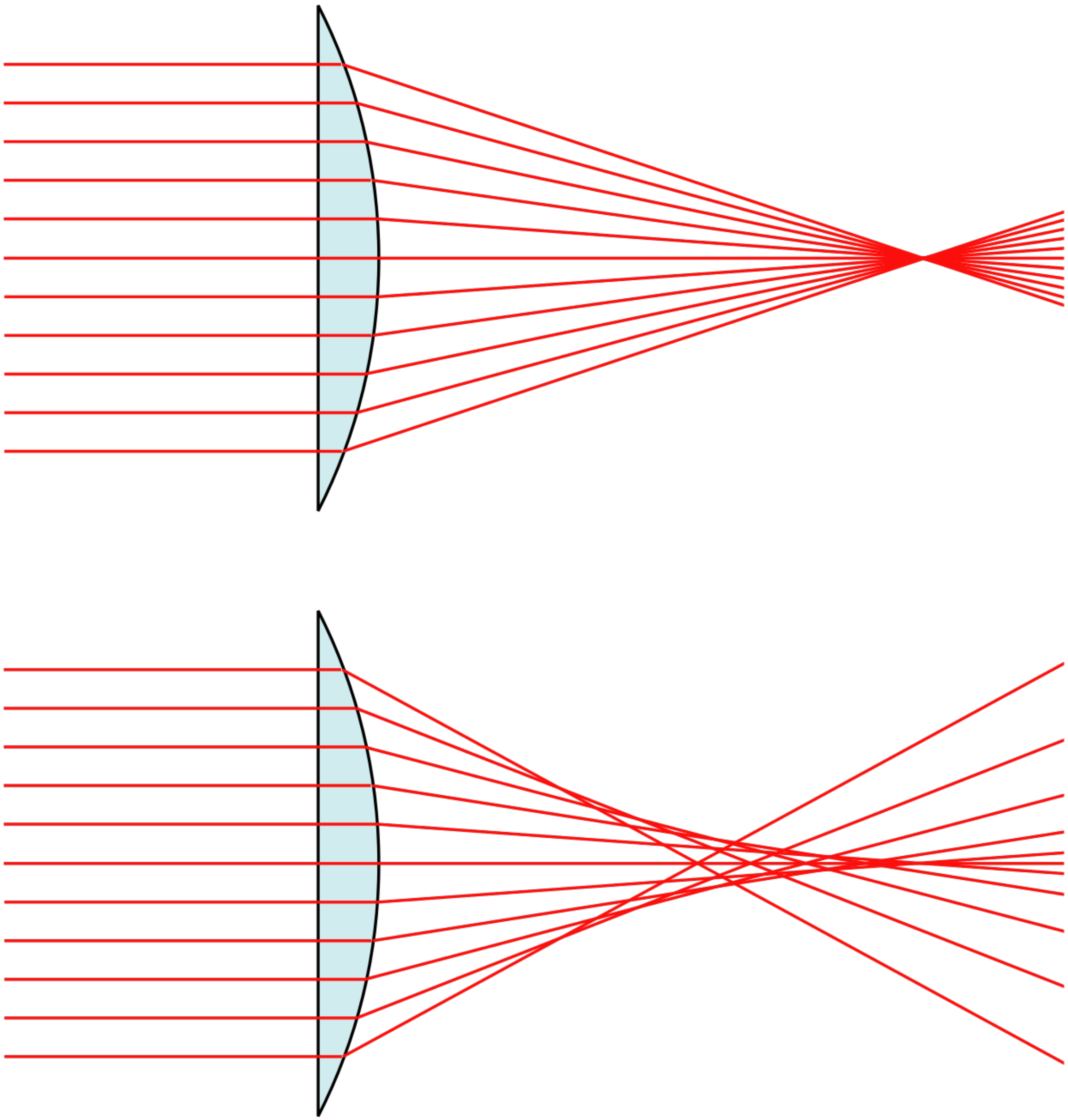
Chromatic aberration

- Dispersion of light





Spherical aberration



Other Distortions

- Imperfect Lenses
  - Deviations are most noticeable for rays passing through the edge of lens