SAIR

Spatial AI & Robotics Lab

CSE 473/573-A

L10: HOUGH TRANSFORM

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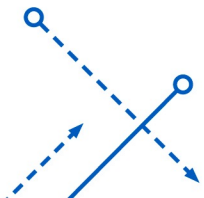
Content

- Hough Transform
  - Line Parameterization
    - Slope Intercept Form
    - Double Intercept Form
    - Normal Form
  - Line Detection
    - Image Space
    - Parameter Space
    - Hough Voting
  - Circles and Others



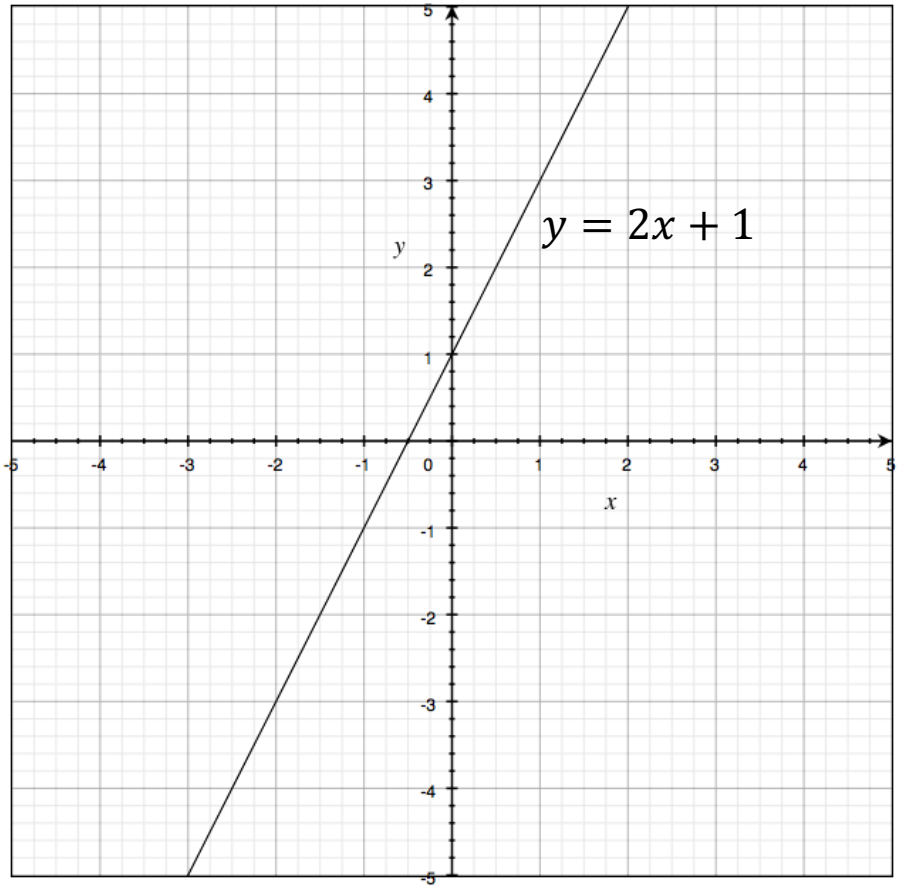
Hough Transform

- Hough Transform can detect basic shapes
  - Detect points/edges → Find shapes.
  - Lines, Circles, etc.
- Line parameterizations
  - Slope intercept form
  - Double intercept form
  - Normal Form

Slope intercept form

$$y = mx + b$$

slope y-intercept

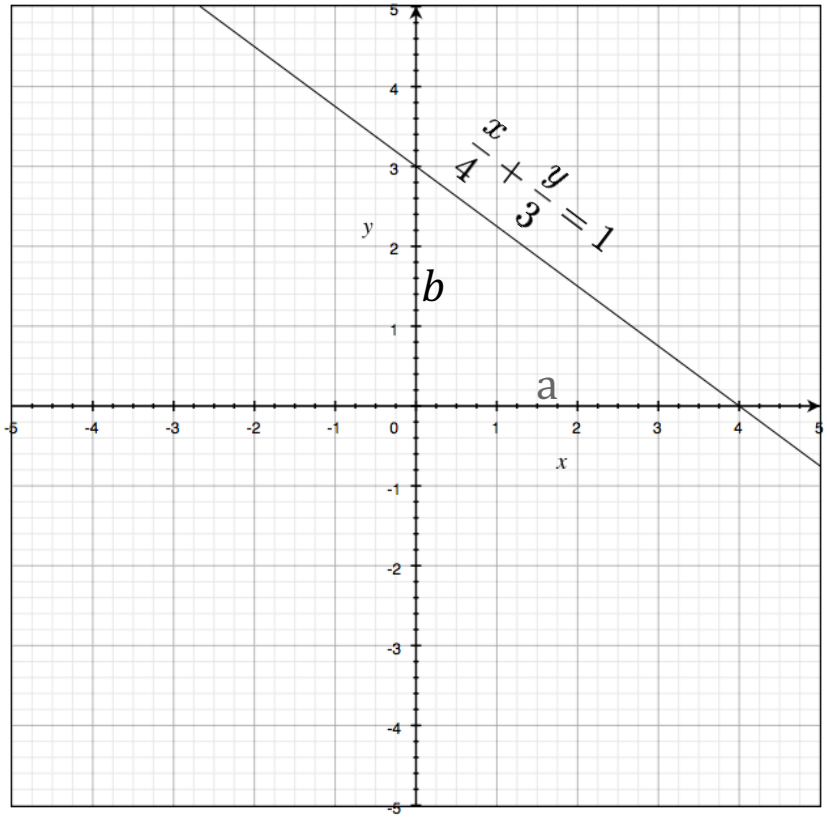


Double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

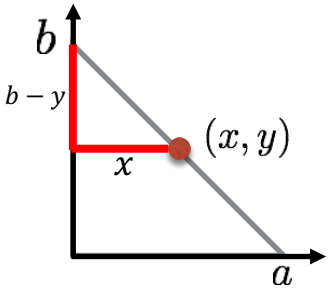
x-intercept

y-intercept



Derivation:

(Similar Triangles)



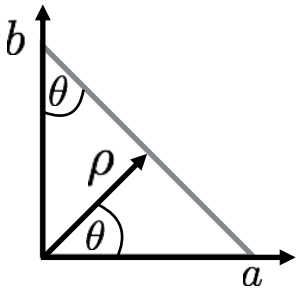
$$\frac{x}{a} = \frac{b-y}{b}$$

What are a and b?

Normal Form

$$x\cos\theta + y\sin\theta = \rho$$

Derivation:

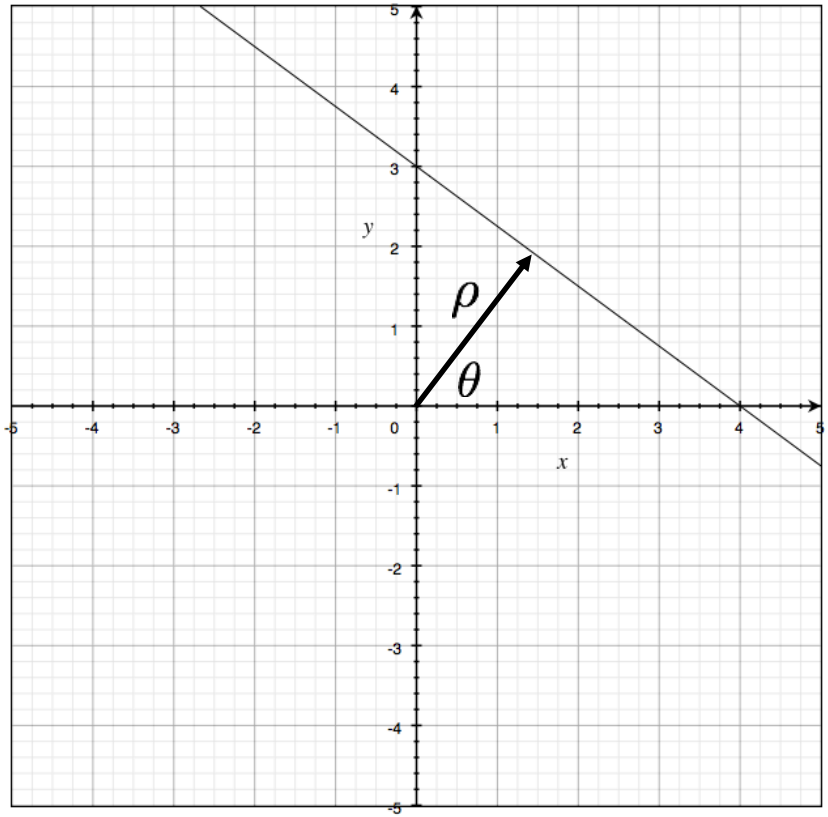


$$a = \frac{\rho}{\cos\theta}$$

$$b = \frac{\rho}{\sin\theta}$$

plug into:

$$\frac{x}{a} + \frac{y}{b} = 1$$



What are ρ and θ ?

Hough Transform

- Slope intercept form
- Normal Form

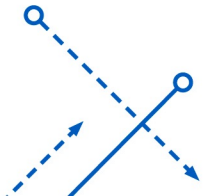


Image and parameter space

variables

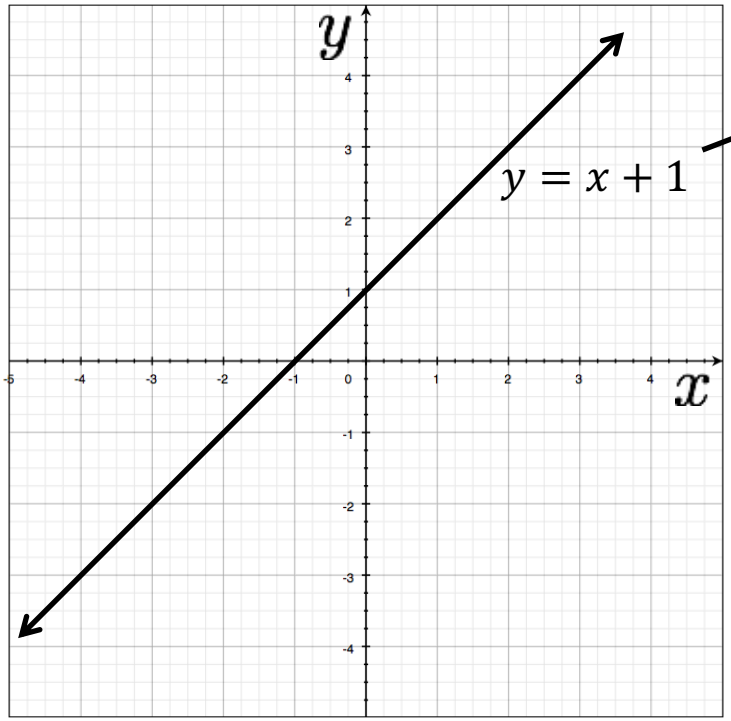
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



$m = 1$
$b = 1$

a line becomes a point

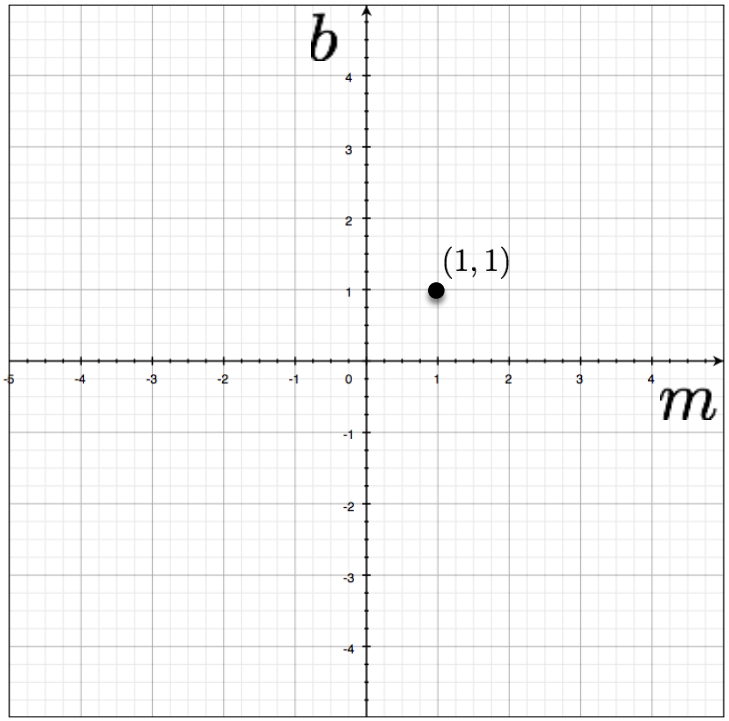


Image space

Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

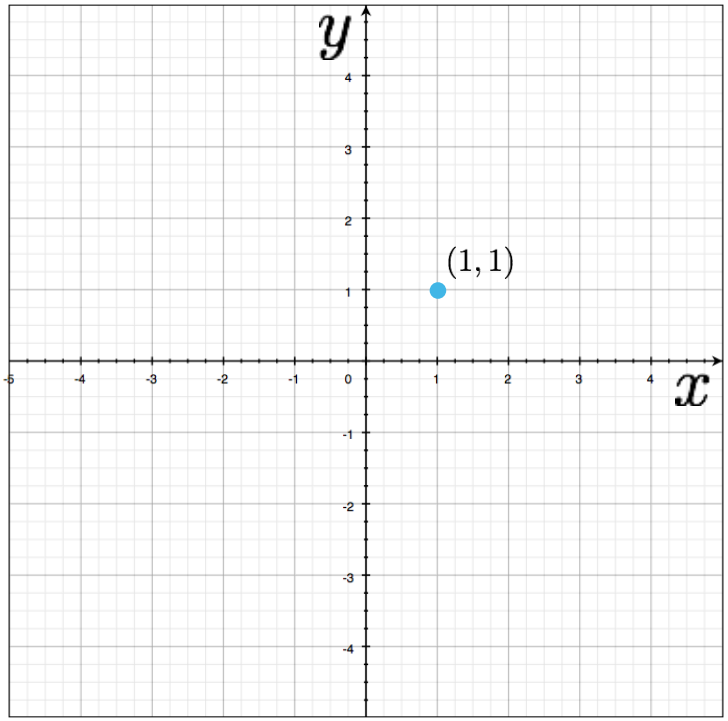


Image space

What would a point in image space become in parameter space?

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

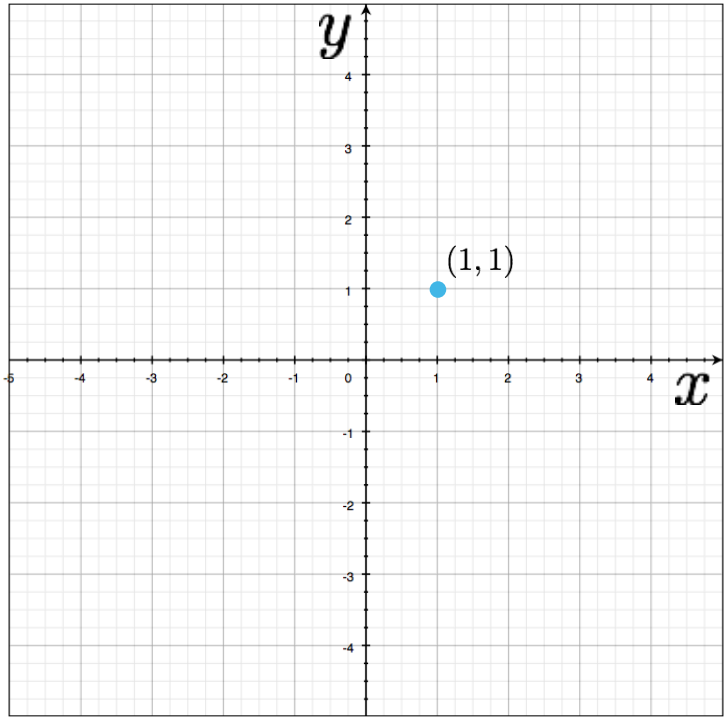
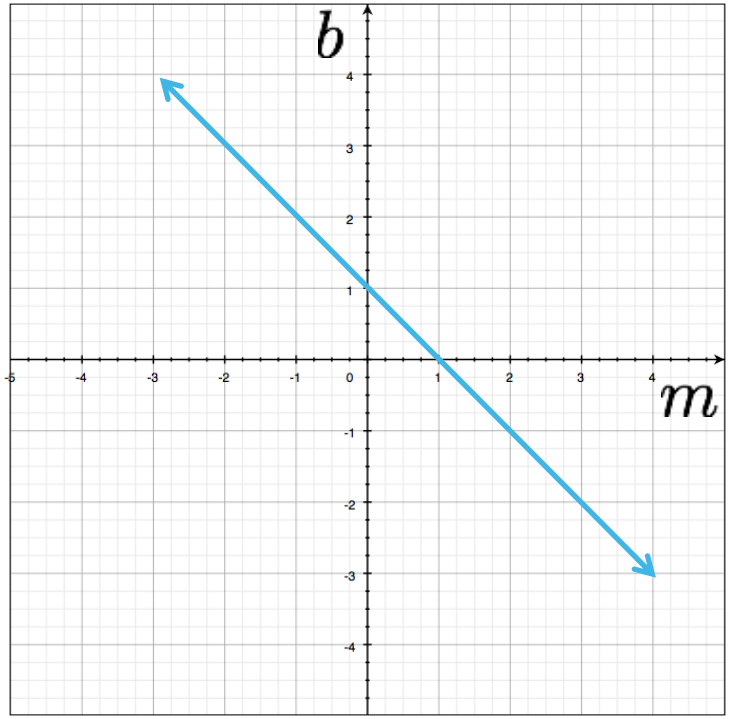


Image space

a point becomes a line



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

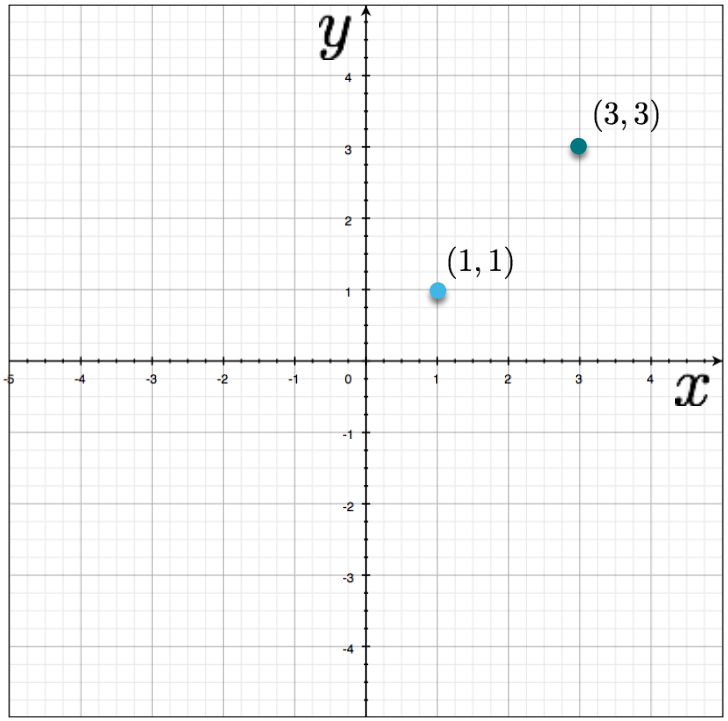
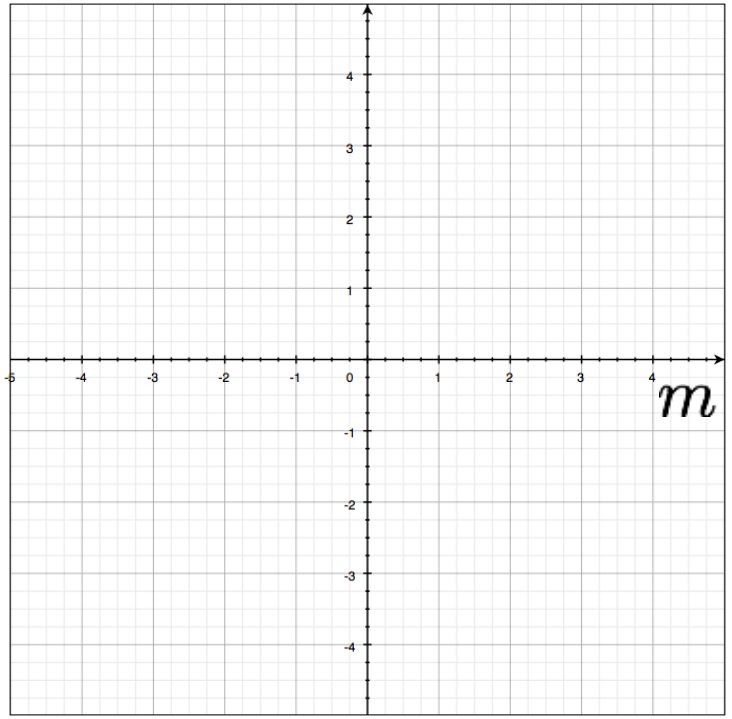


Image space

two points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

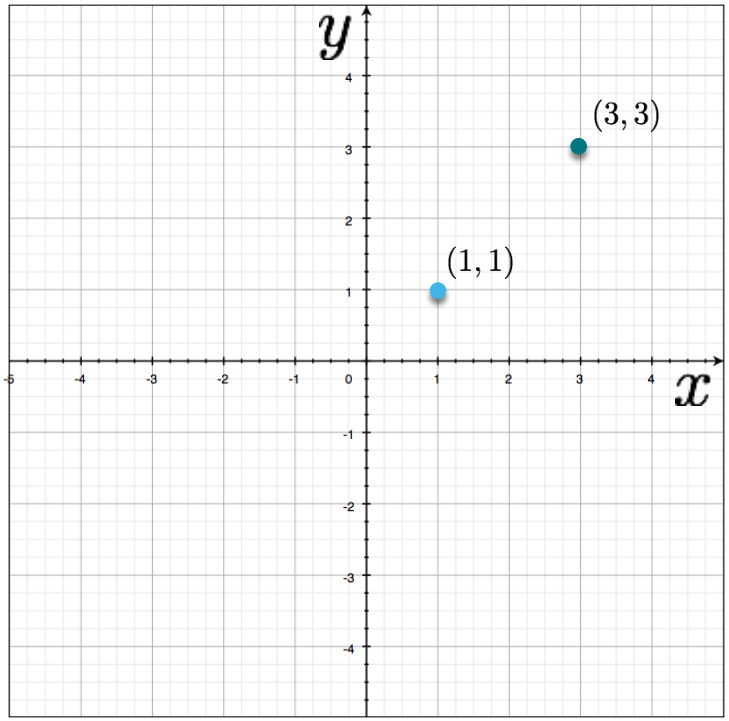
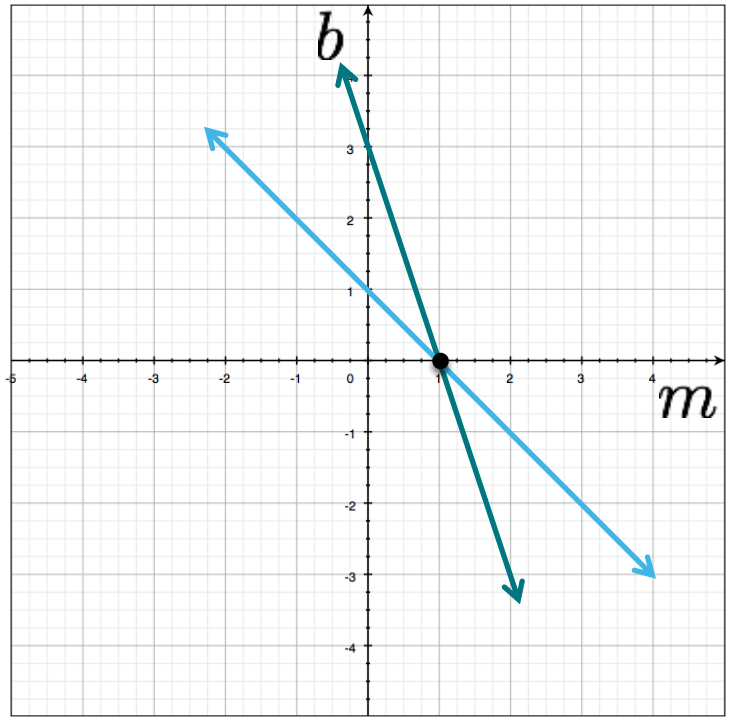


Image space

two points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

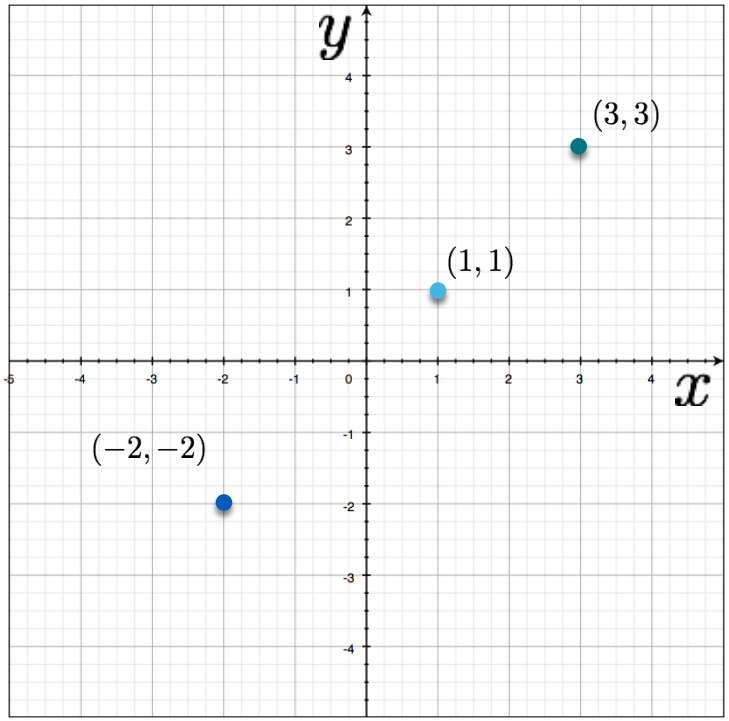
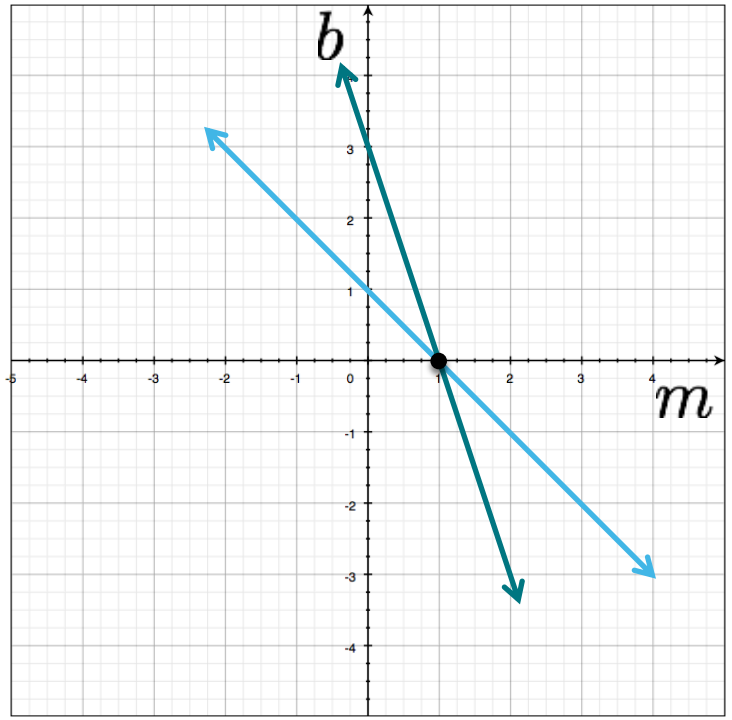


Image space

three points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

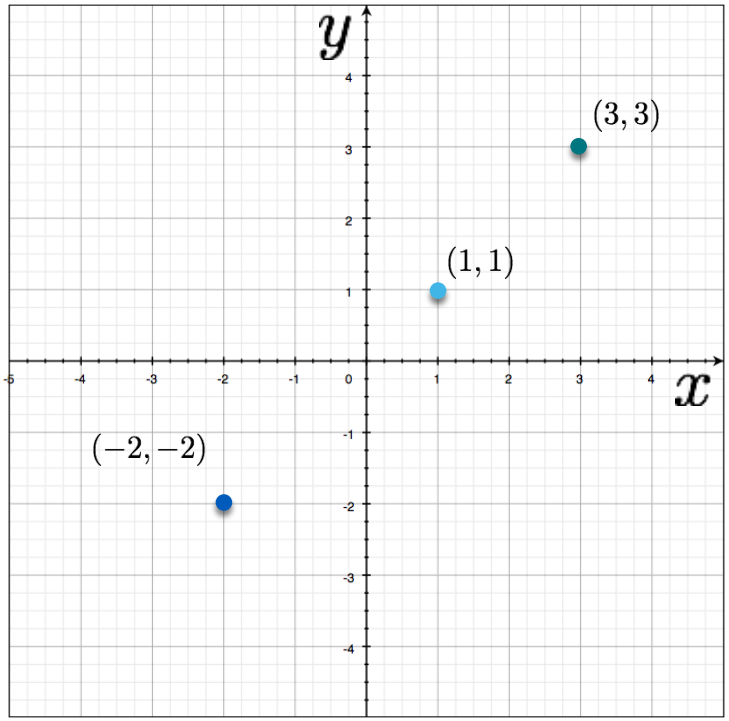
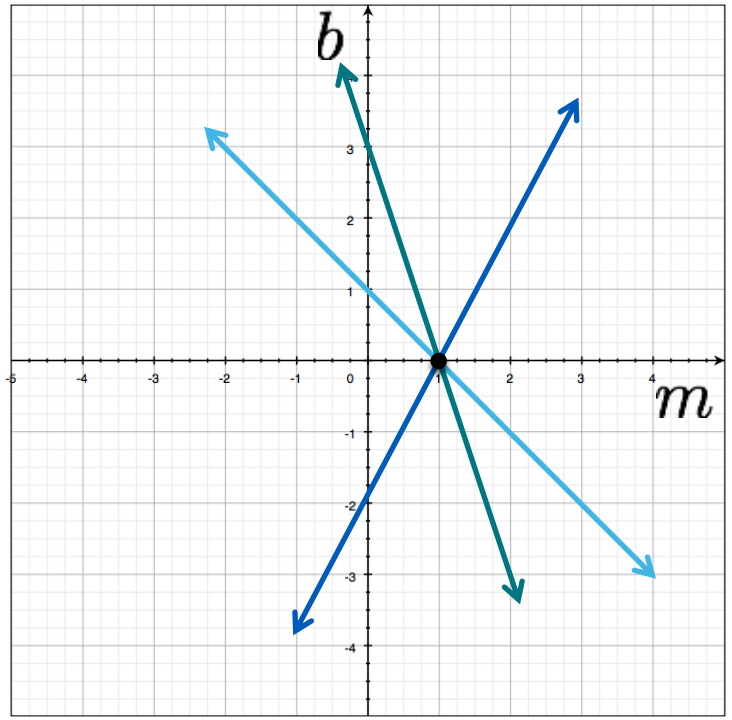


Image space

three points become ?



Parameter space

Image and parameter space

variables

$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters

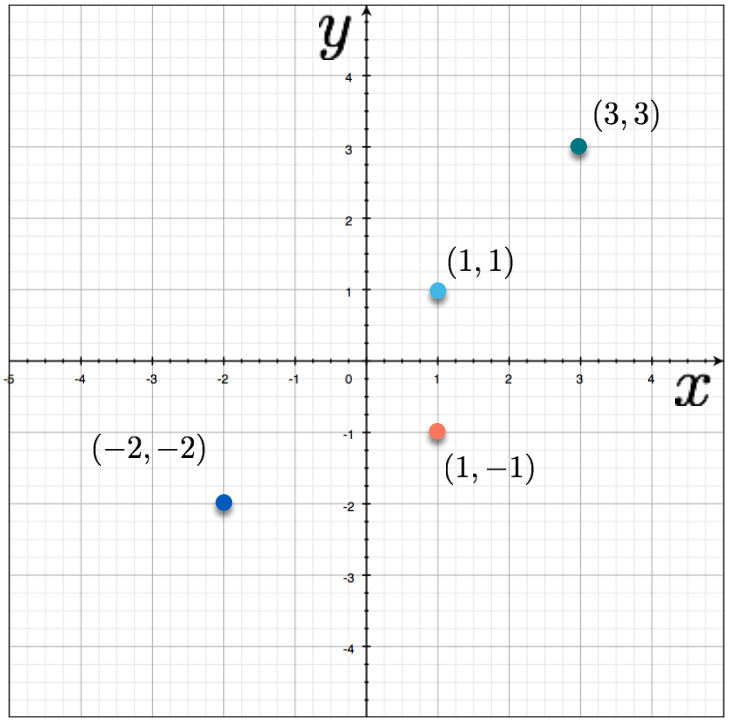
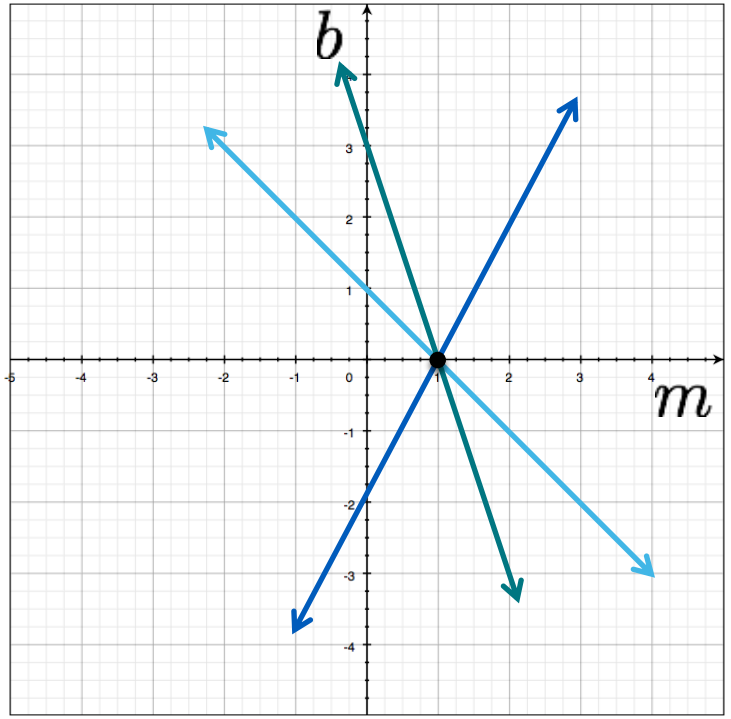


Image space

four points become ?



Parameter space

Image and parameter space

variables

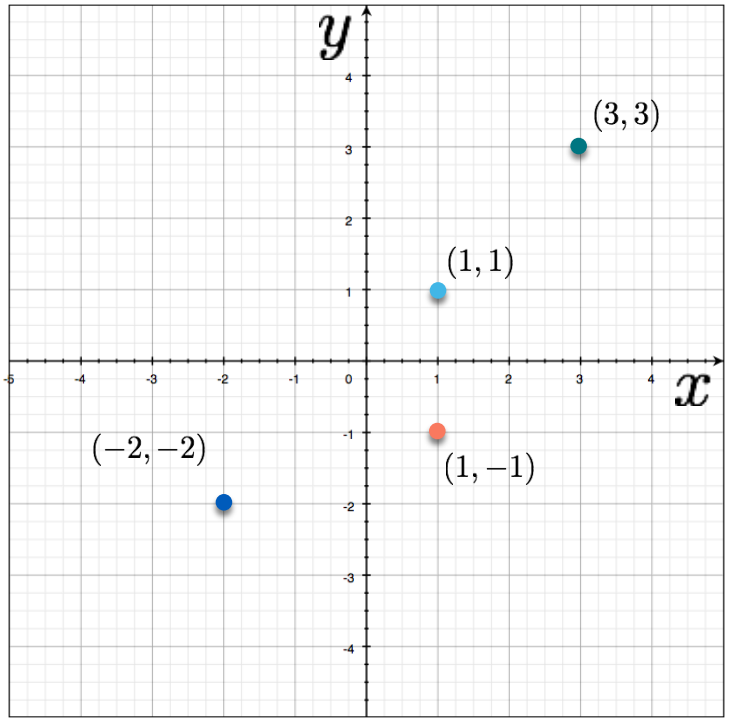
$$y = mx + b$$

parameters

variables

$$b = -xm + y$$

parameters



four points become ?

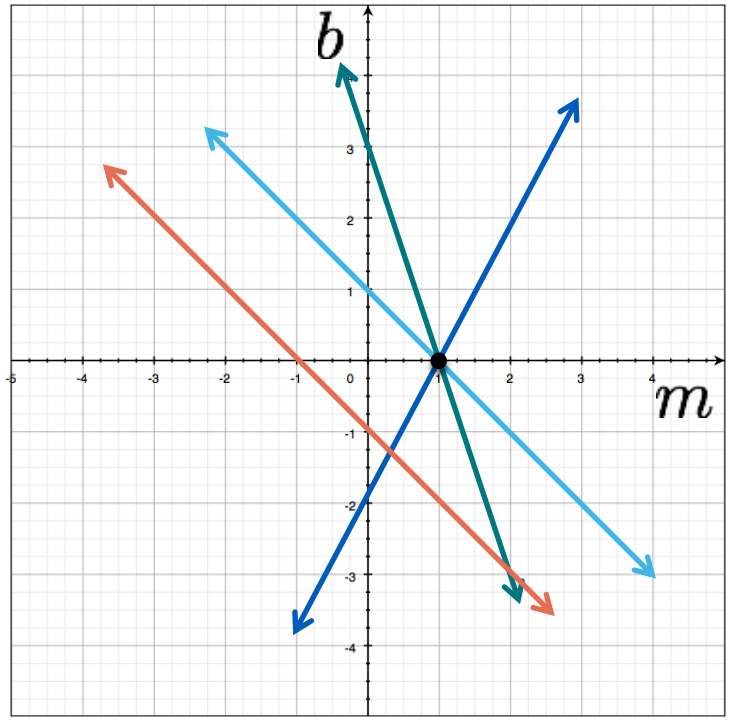
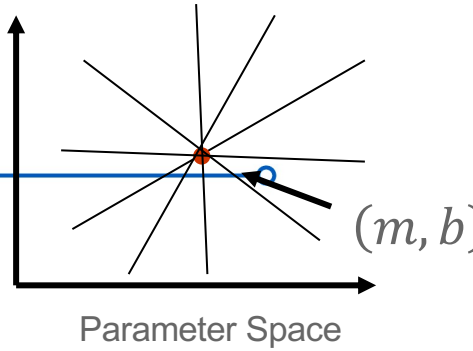


Image space

Parameter space

Is this method robust to noise?

Hough **Voting**



Line Detection Algorithm:

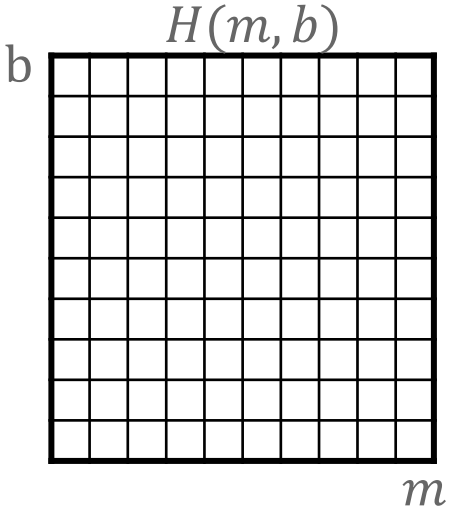
1. Quantize Parameter Space (m, b).

2. Create Hough Space Array $H(m, b) = 0$.

3. For each image point (x_i, y_i):
   For all points (m, b) on $b = -x_i m + y_i$:
   $H(m, b) = H(m, b) + 1$

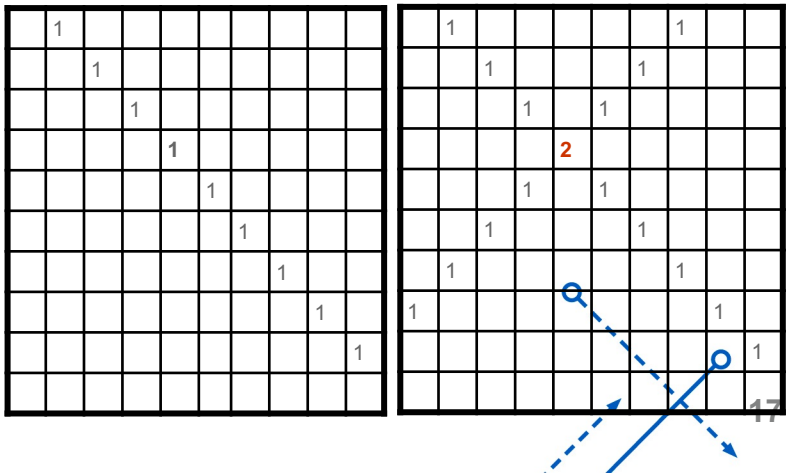
4. Find local maxima in $H(m_m, b_m)$.

5. The detected line: $y = m_m x + b_m$.



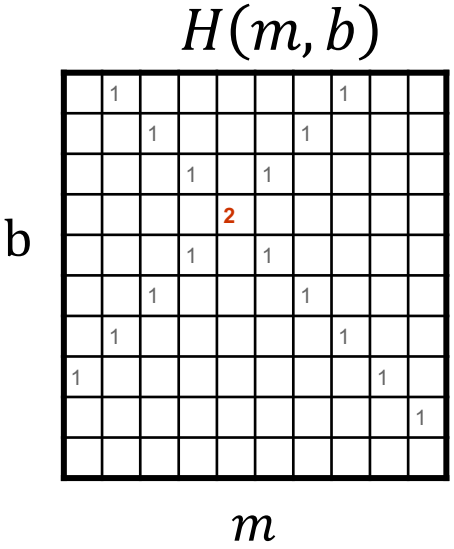
Is it able to detect multiple lines?

Is this solution good enough?



Problems with slope intercept form

How big does the Hough array have to be?



The space of m is huge!

$$-\infty \leq m \leq \infty$$

The space of b is huge!

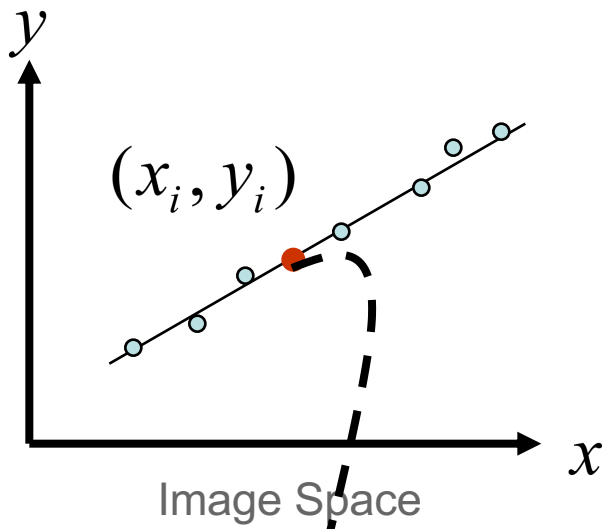
$$-\infty \leq b \leq \infty$$

Hough Transform with Normal Form

Use normal form:

$$x\cos\theta + y\sin\theta = \rho$$

The Hough space become $H(\rho, \theta)$



Hough Space

$$0 \le \theta \le \pi$$
$$0 \le \rho \le \rho_{max}$$

(Finite Hough Array Size)

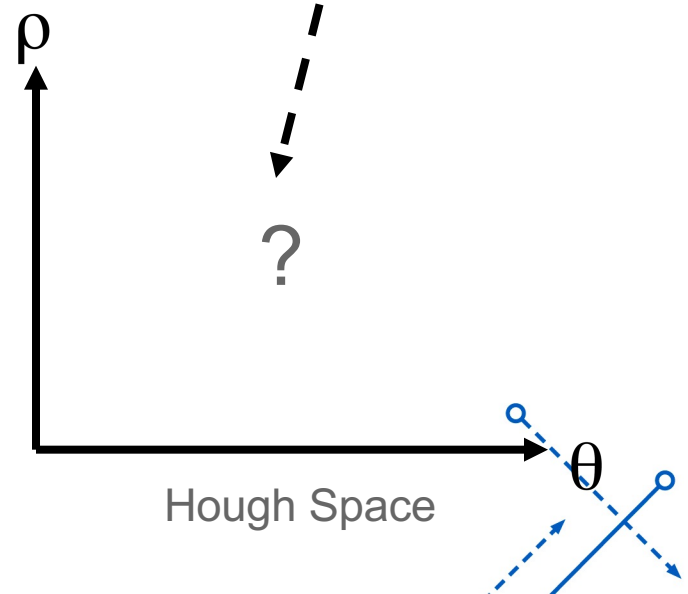


Image and parameter space

variables

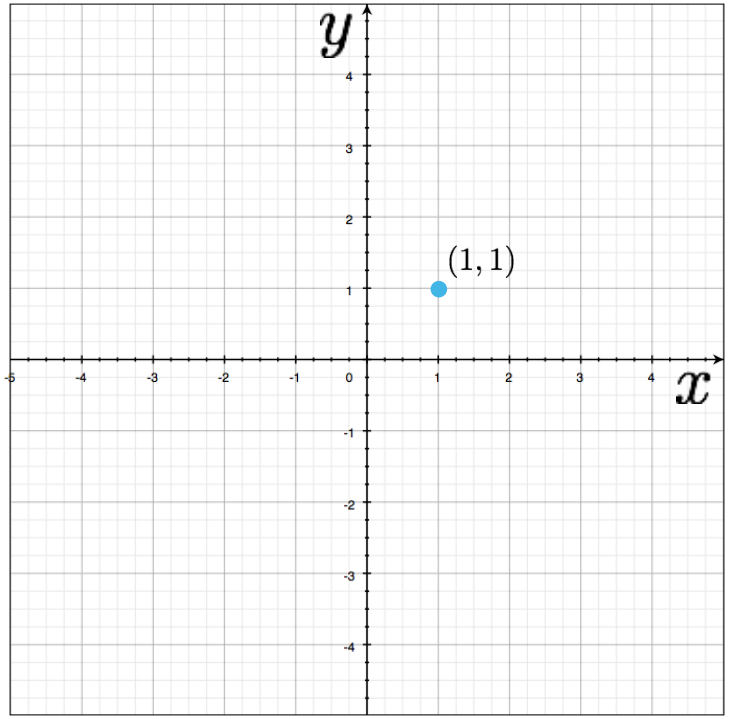
$$x\cos\theta + y\sin\theta = \rho$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



a point becomes?

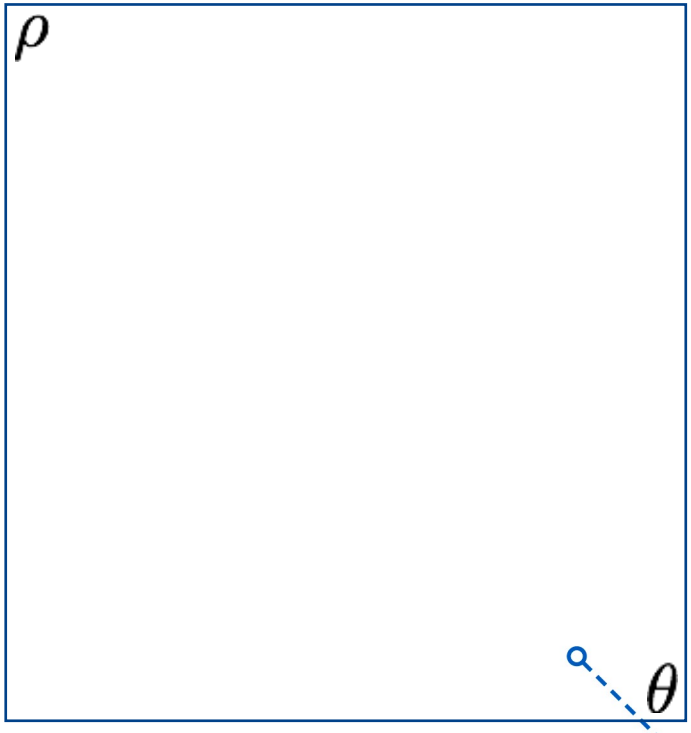


Image space

Parameter space

Image and parameter space

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$$x\cos\theta + y\sin\theta = \rho$$

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$$x\cos\theta + y\sin\theta = \rho$$

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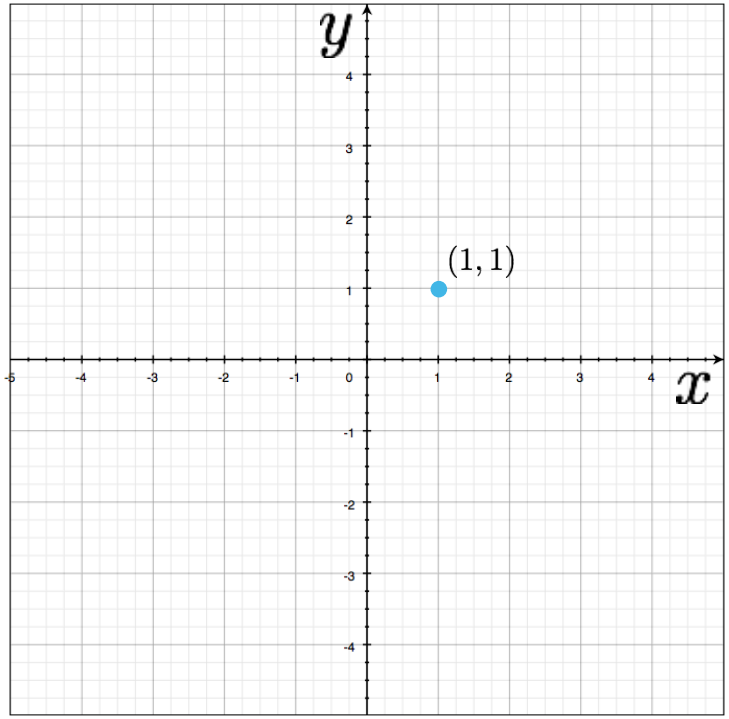

a point becomes a wave

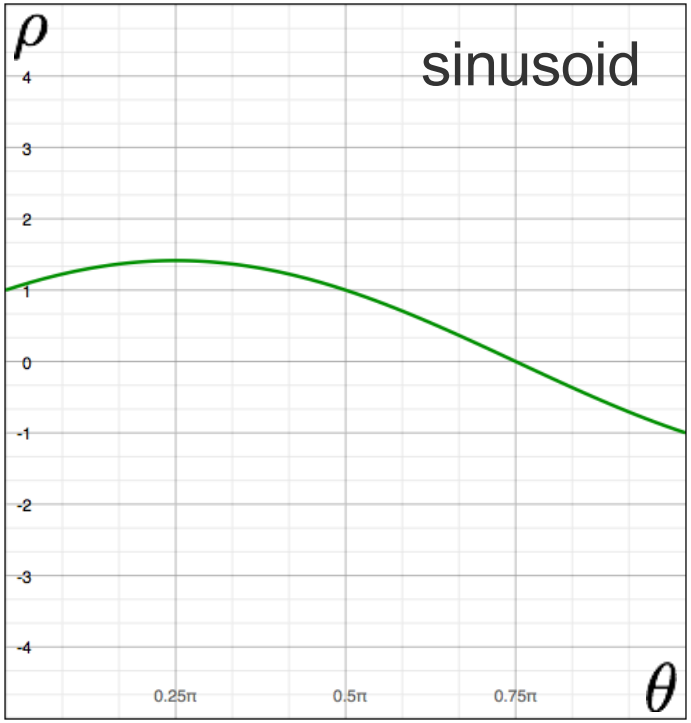

Image space

Parameter space

Image and parameter space

variables

$$x\cos\theta + y\sin\theta = \rho$$

parameters

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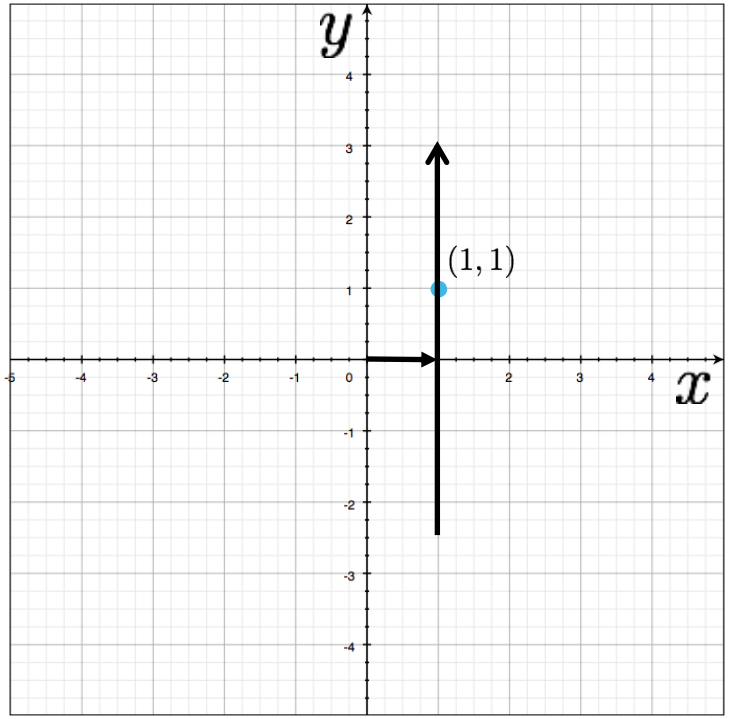
$$x\cos\theta + y\sin\theta = \rho$$

variables



a line becomes?

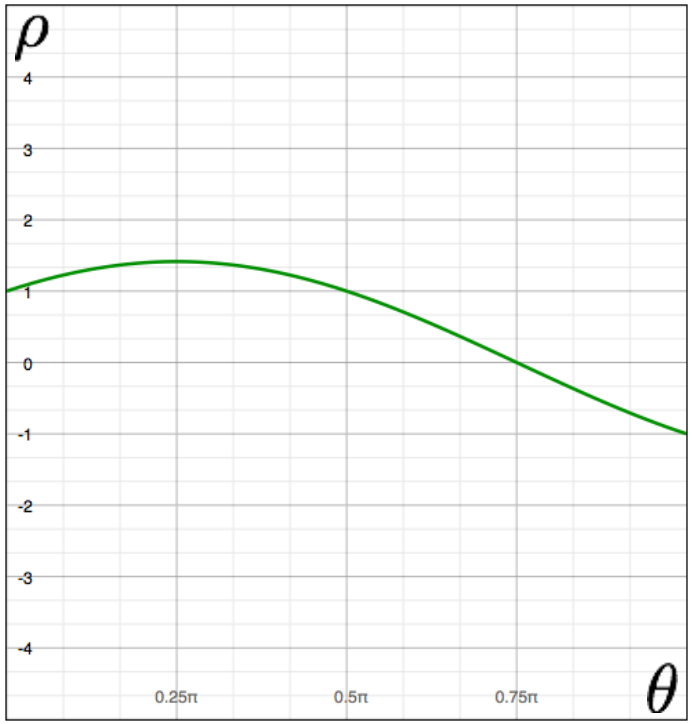


Image space

Parameter space

Image and parameter space

variables

$$x\cos\theta + y\sin\theta = \rho$$

parameters

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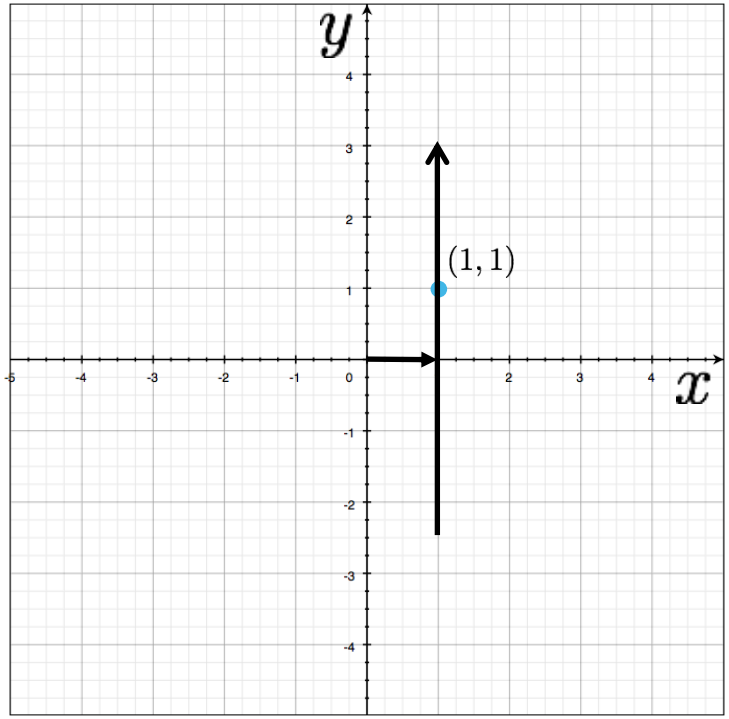
$$x\cos\theta + y\sin\theta = \rho$$

variables



a line becomes a point

Image space

Parameter space

Image and parameter space

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$$x\cos\theta + y\sin\theta = \rho$$

parameters

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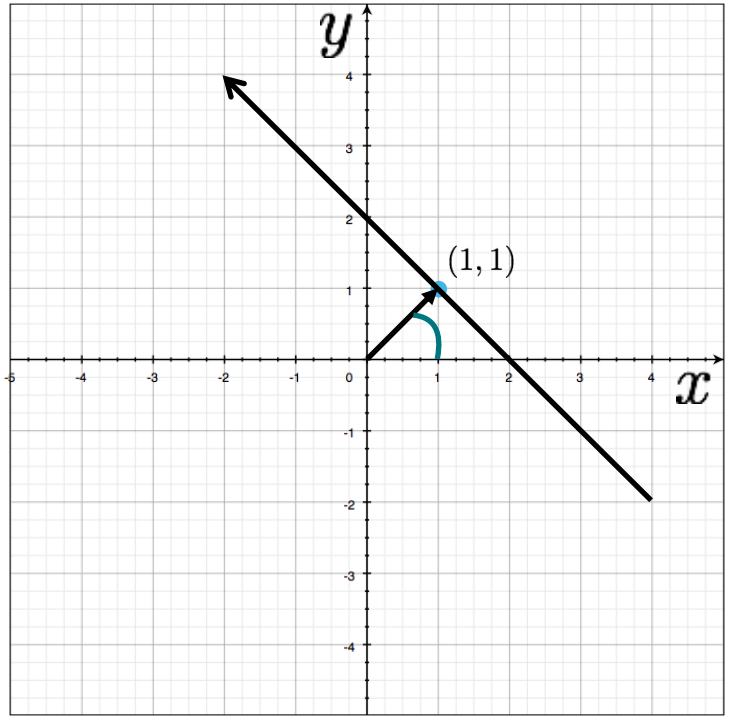
$$x\cos\theta + y\sin\theta = \rho$$

variables



a line becomes?

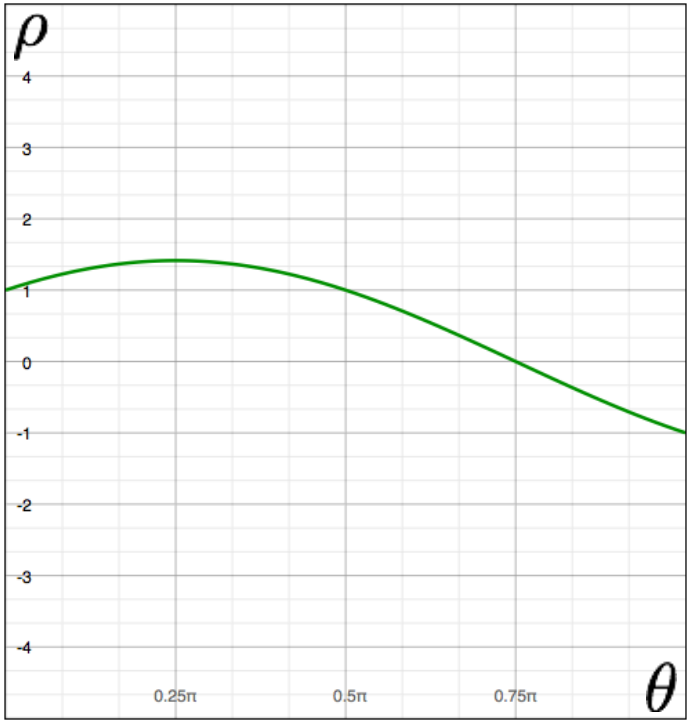


Image space

Parameter space

Image and parameter space

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$$x\cos\theta + y\sin\theta = \rho$$

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$$x\cos\theta + y\sin\theta = \rho$$

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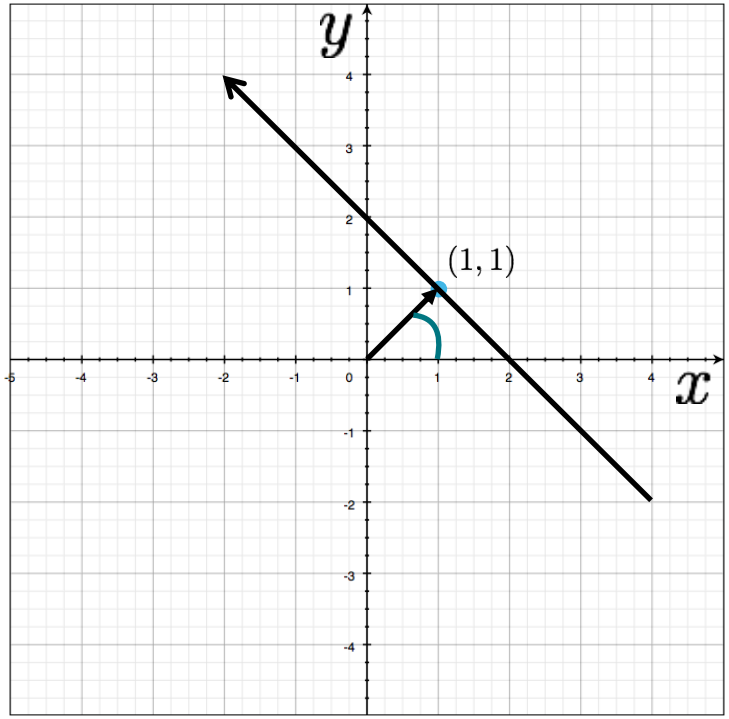

a line becomes a point

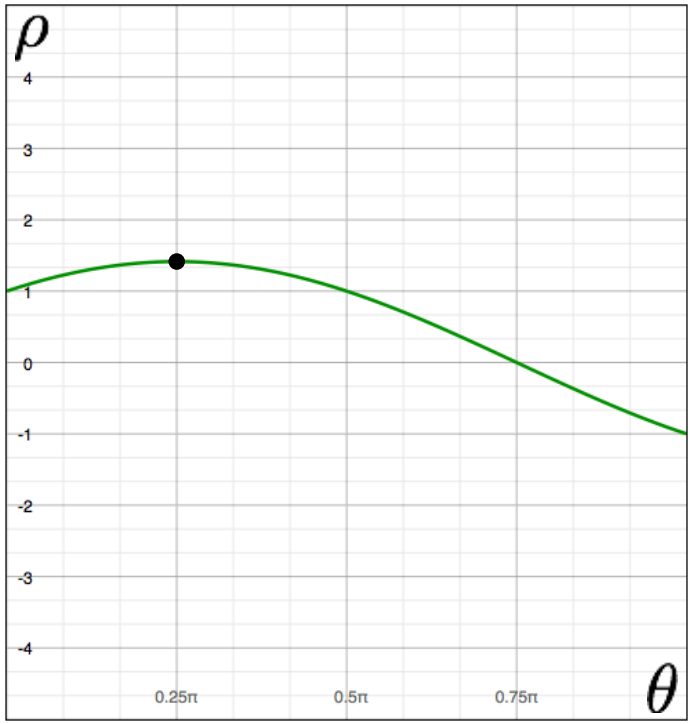

Image space

Parameter space

Image and parameter space

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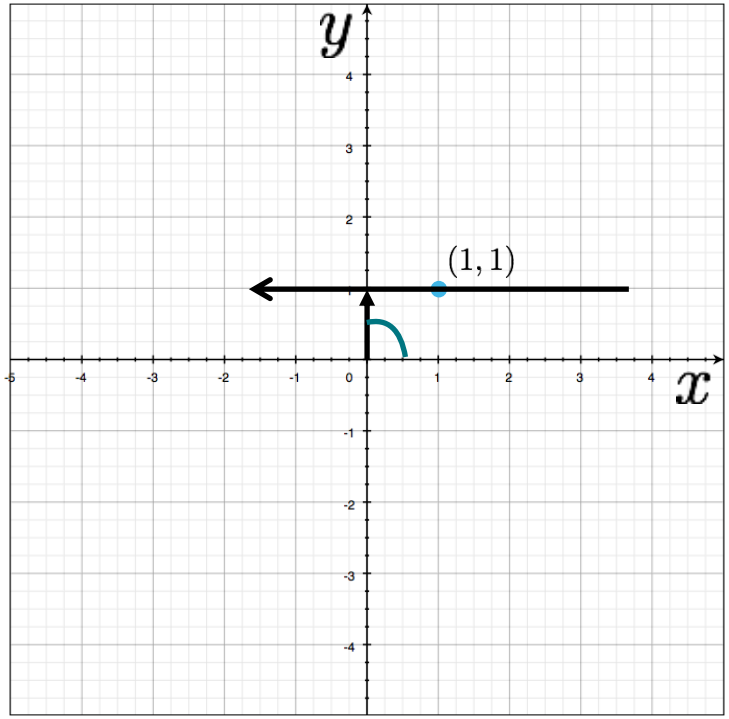
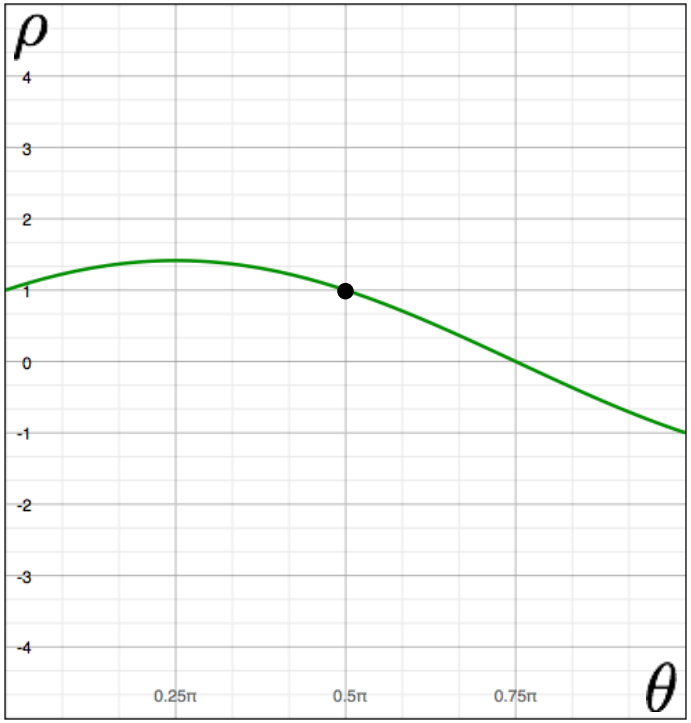


a line becomes a point



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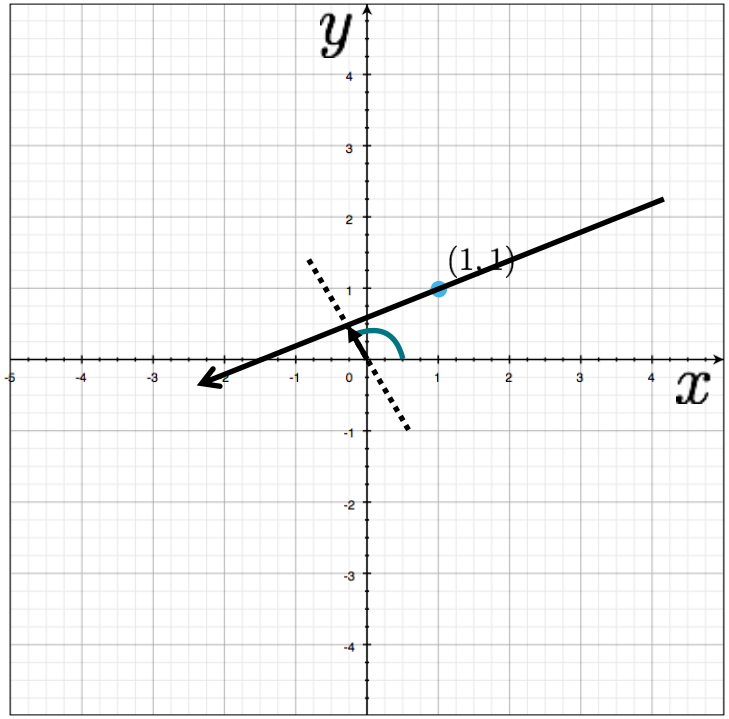

a line becomes a point

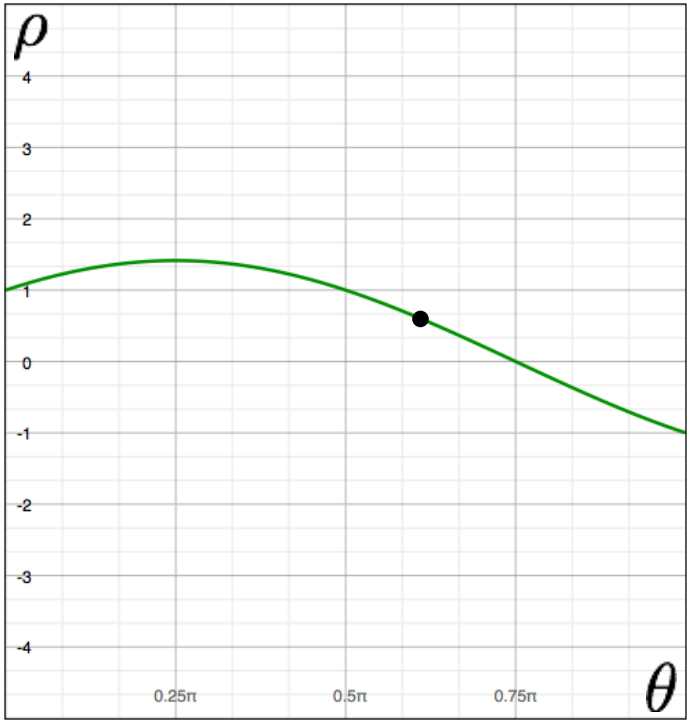

Image space

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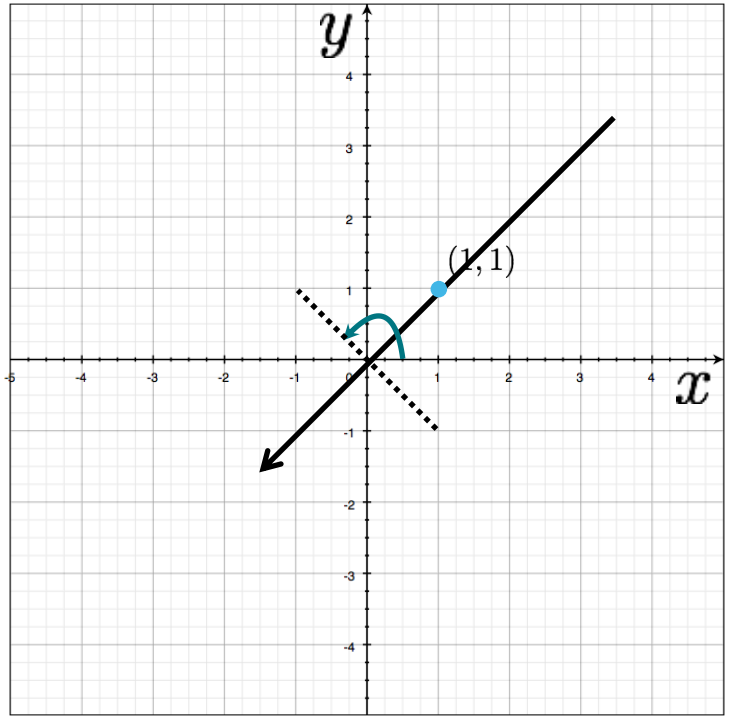
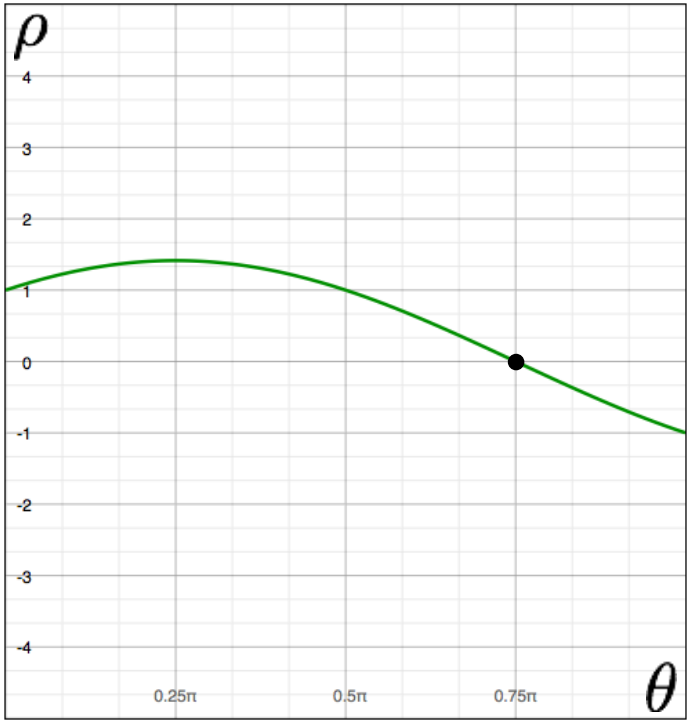


a line becomes a point



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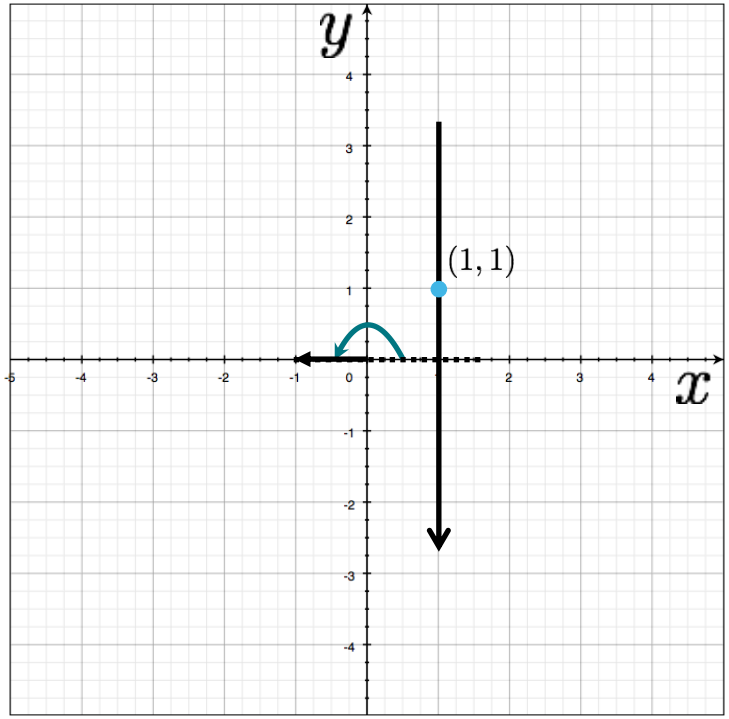


a line becomes a point

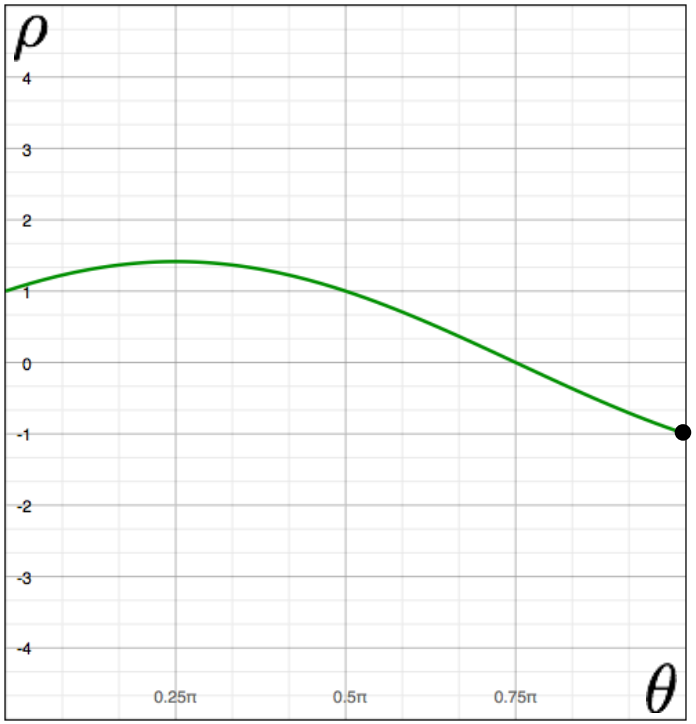


Image space

Parameter space

Image and parameter space

variables

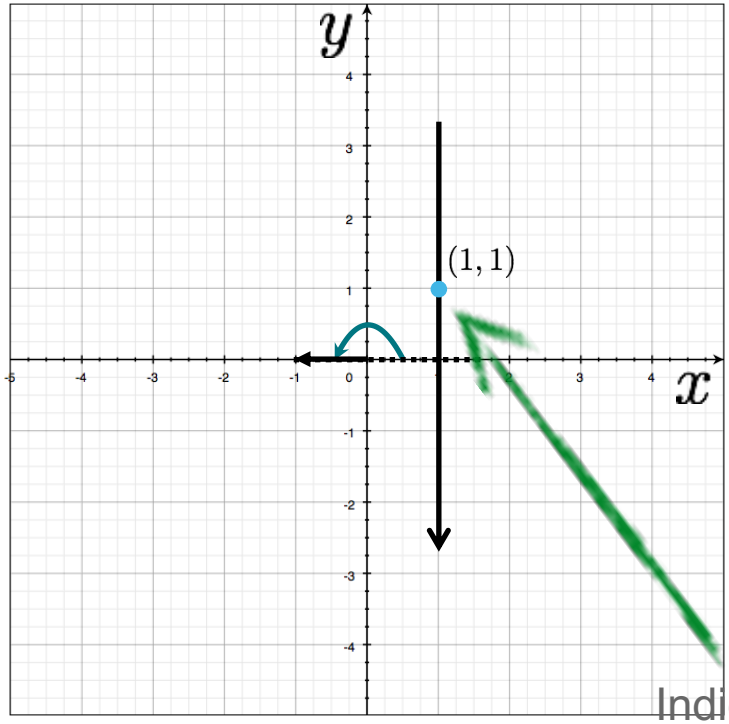
$$x\cos\theta + y\sin\theta = \rho$$

parameters

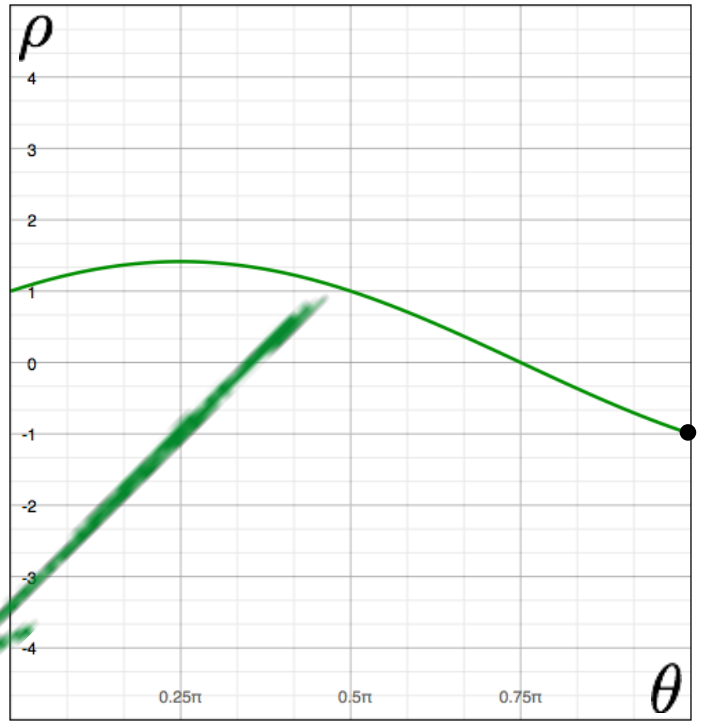
parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



a line becomes a point



Indicate all lines pass through (1,1)

Image space

Parameter space

Image and parameter space

variables

$$x\cos\theta + y\sin\theta = \rho$$

parameters

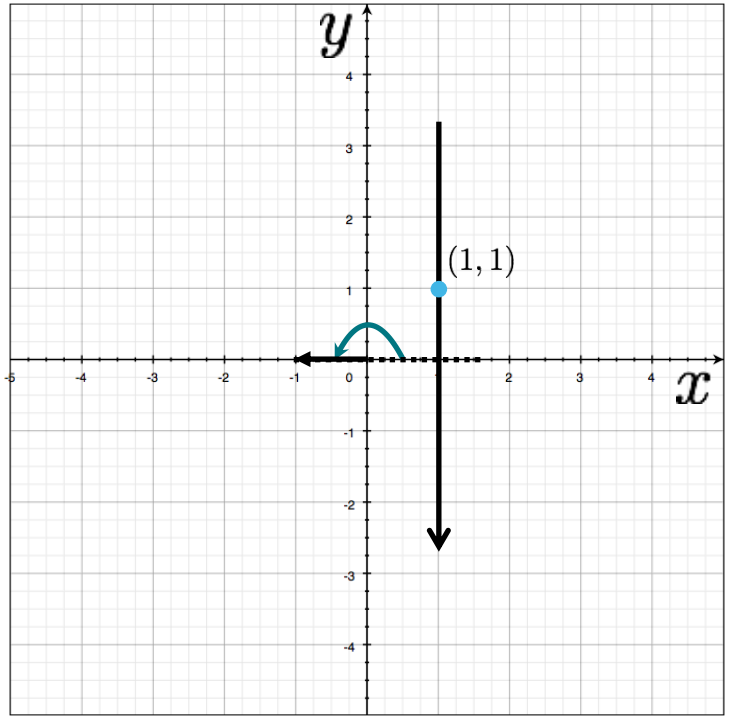


Image space

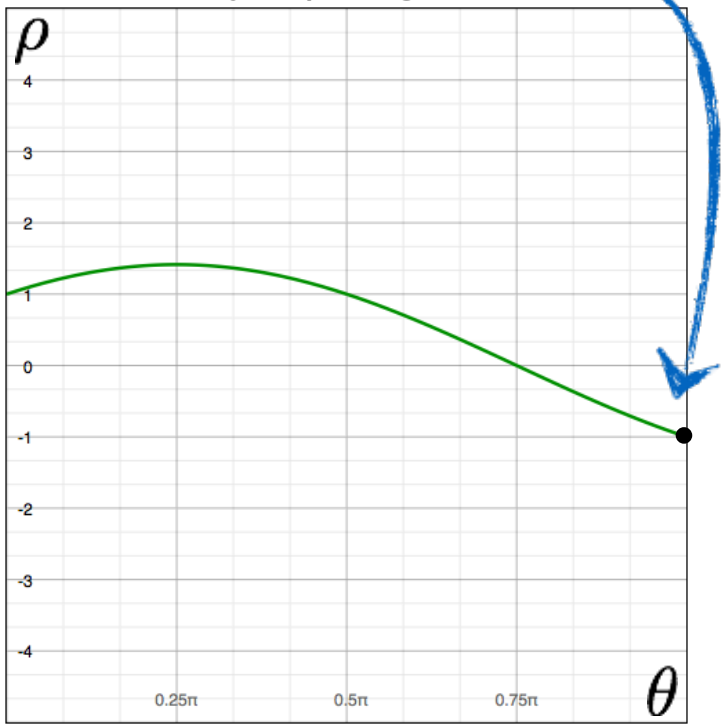
a line becomes a point

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables

Wait …why is ρ negative?



Parameter space

Image and parameter space

variables

$$x\cos\theta + y\sin\theta = \rho$$

parameters

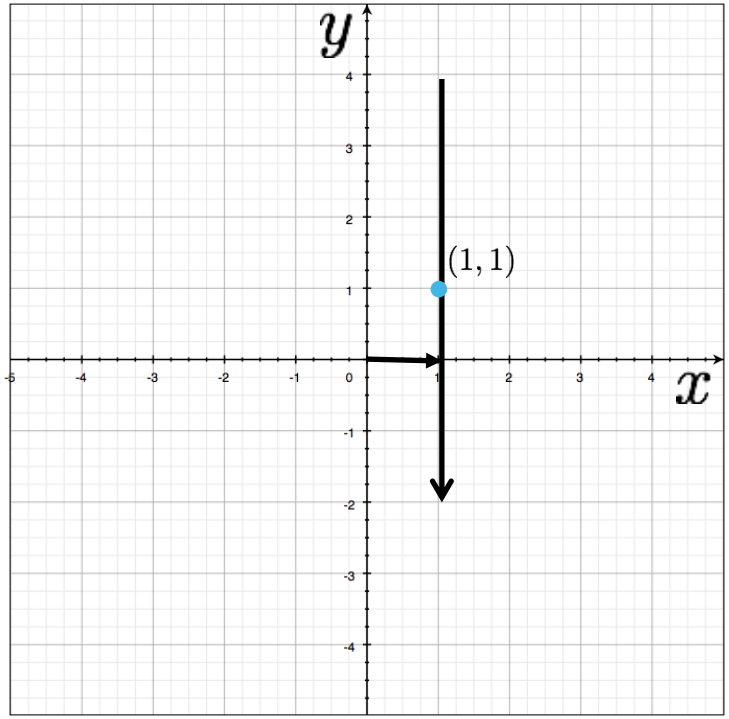


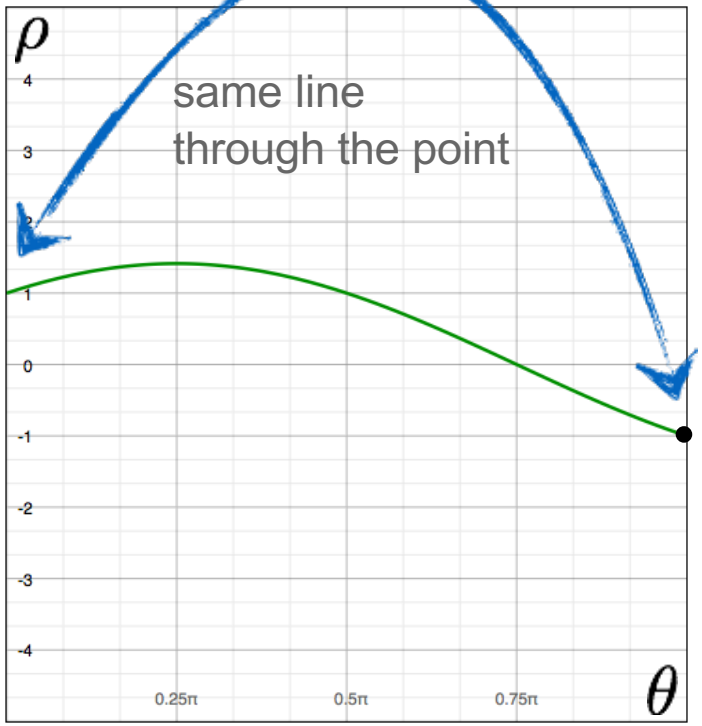
Image space

a line becomes a point

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



Parameter space

There are two ways to write the same line

Positive ρ version:

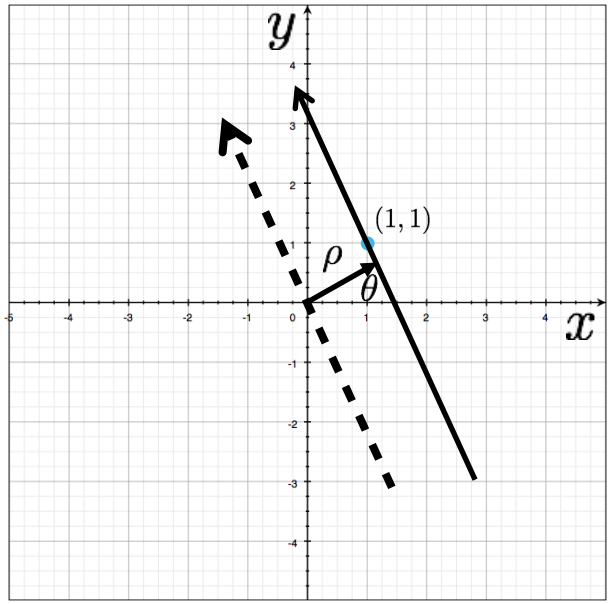
$$x\cos\theta + y\sin\theta = \rho$$

Negative ρ version:

$$x\cos(\theta+\pi) + y\sin(\theta+\pi) = -\rho$$

Recall:

$$\sin(\theta) = -\sin(\theta+\pi)$$
$$\cos(\theta) = -\cos(\theta+\pi)$$



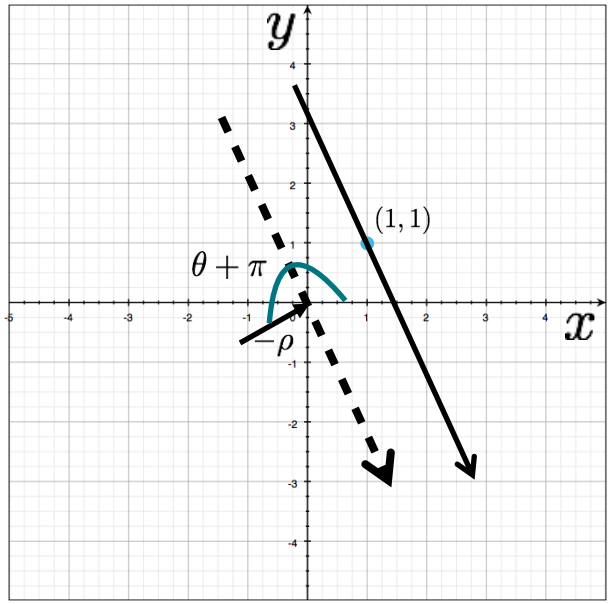


Image and parameter space

variables

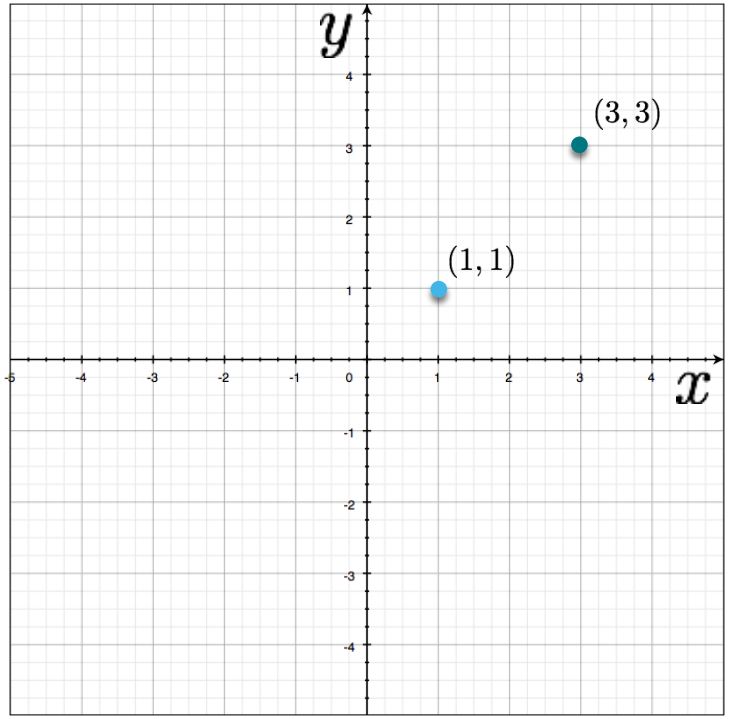
$$x\cos\theta + y\sin\theta = \rho$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



two points become ?

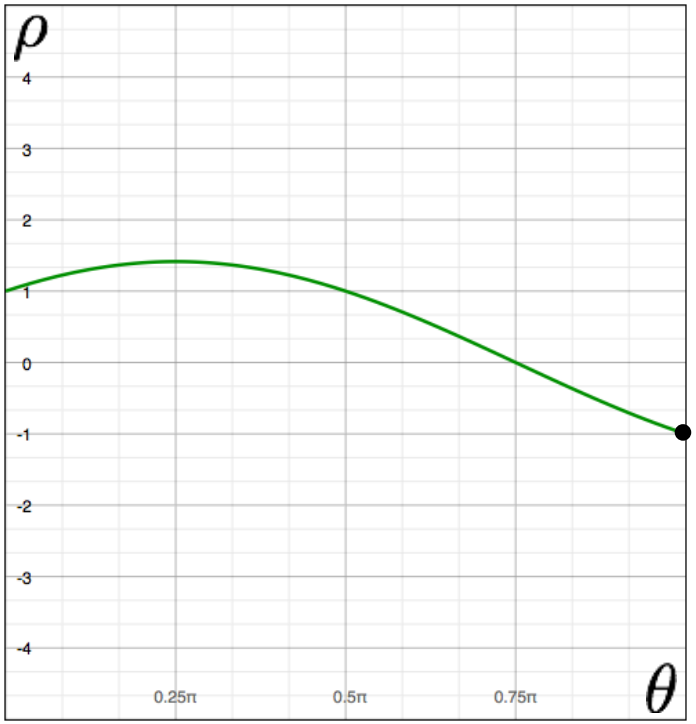


Image space

Parameter space

Image and parameter space

variables

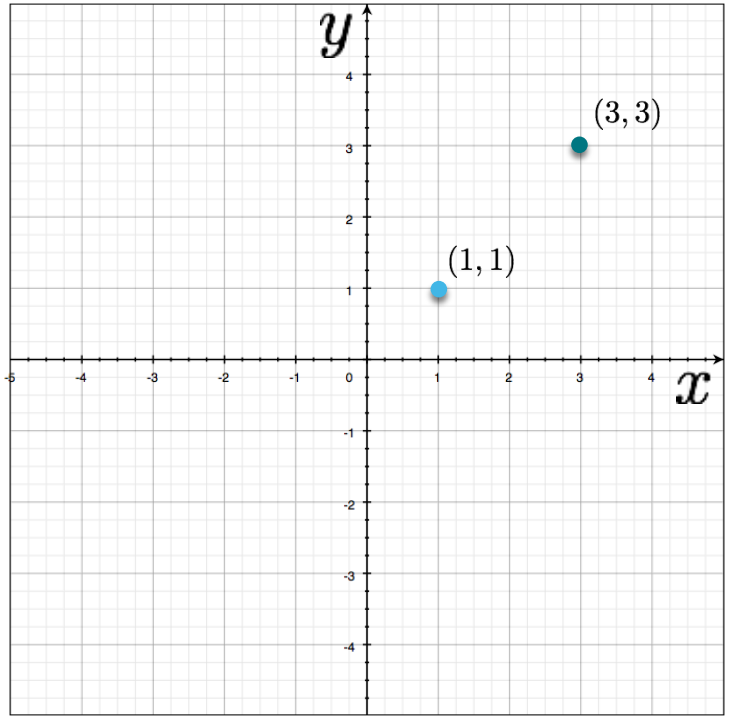
$$x\cos\theta + y\sin\theta = \rho$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



two points become ?

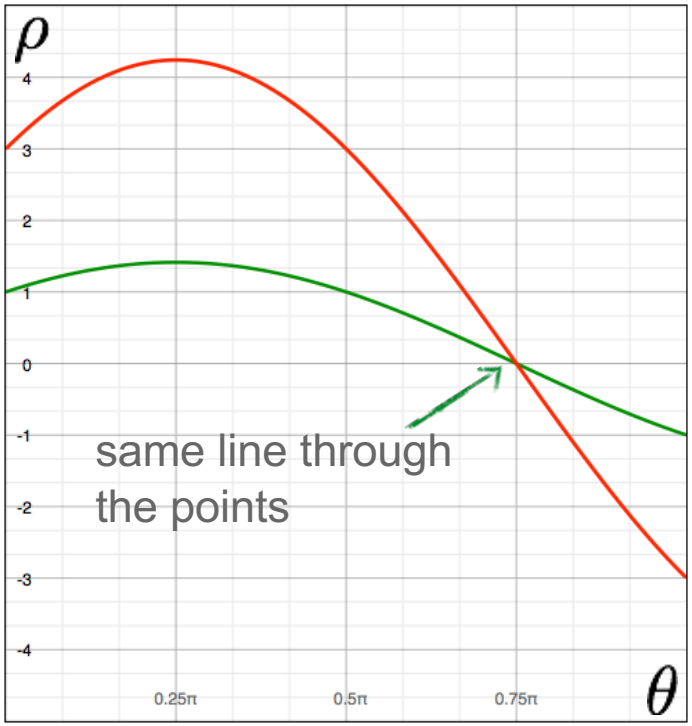


Image space

Parameter space

Image and parameter space

variables

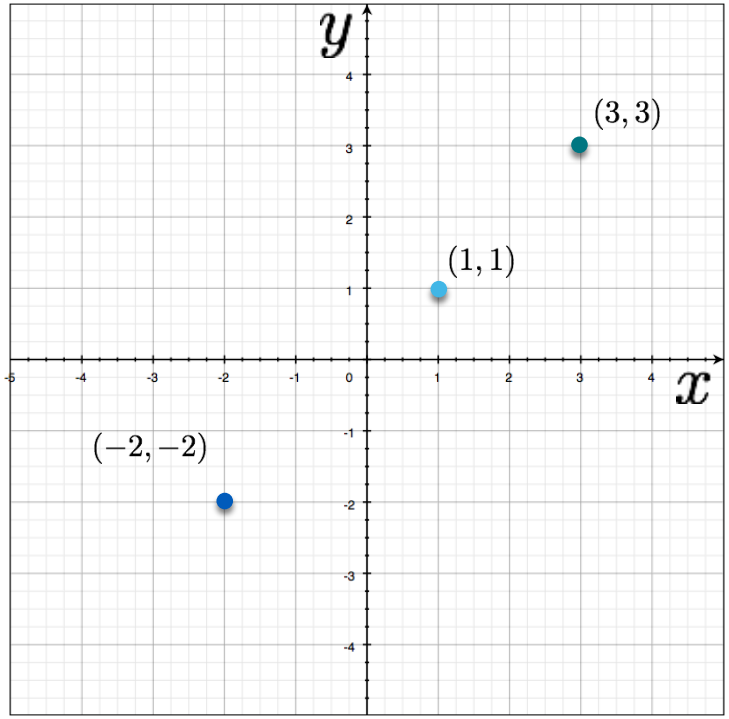
$$x\cos\theta + y\sin\theta = \rho$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



three points become ?

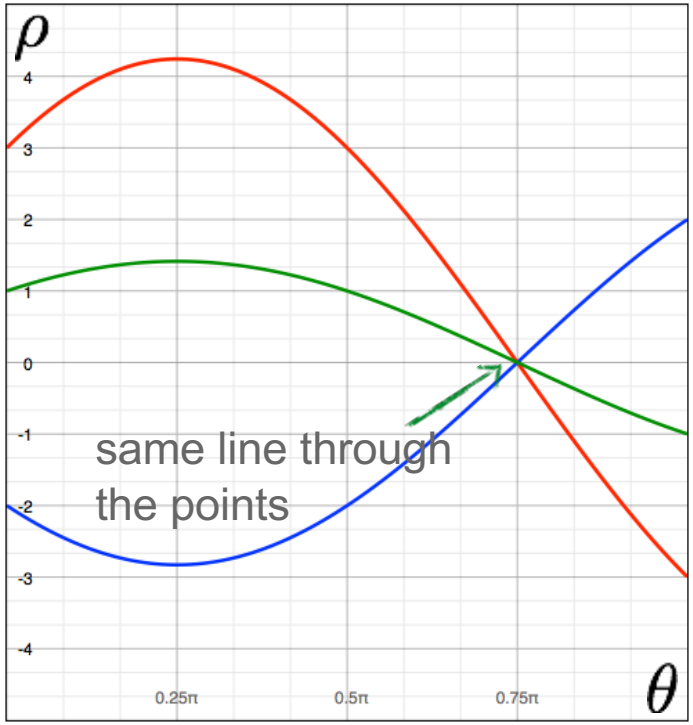


Image space

Parameter space

Image and parameter space

variables

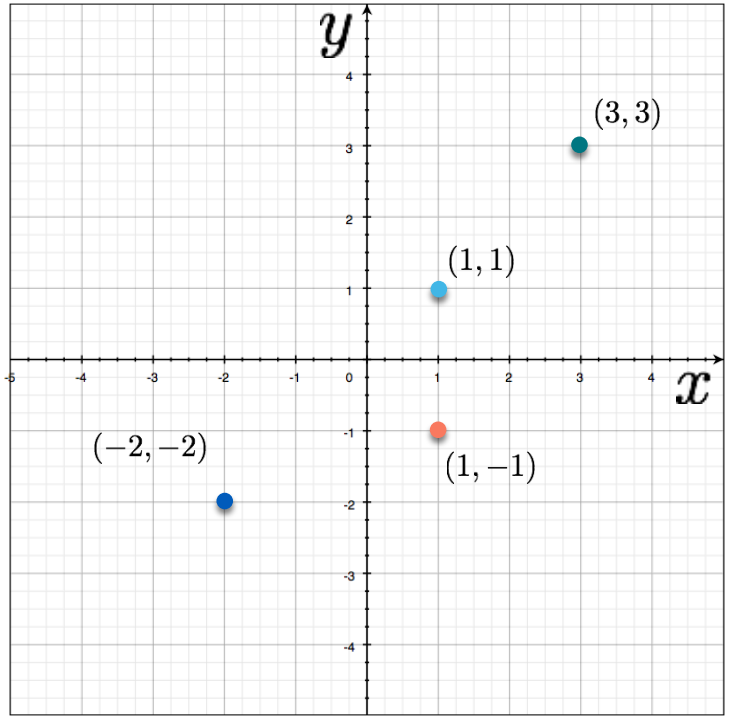
$$x\cos\theta + y\sin\theta = \rho$$

parameters

parameters

$$x\cos\theta + y\sin\theta = \rho$$

variables



four points become ?

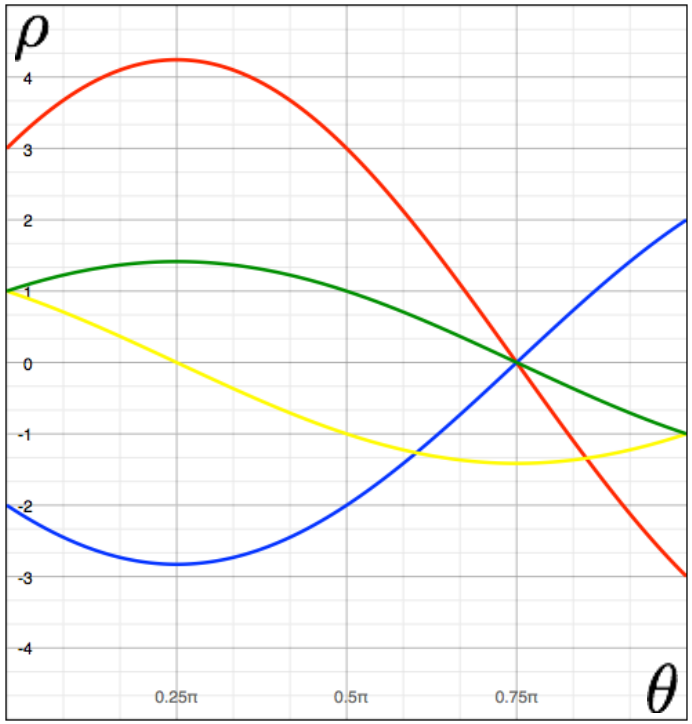
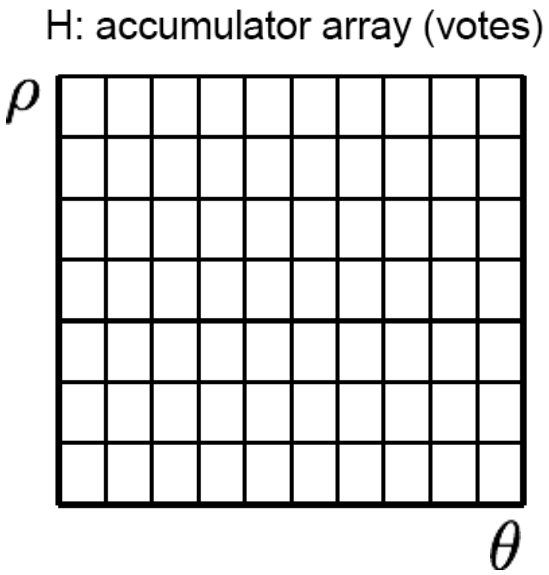


Image space

Parameter space

Line Detection by Hough Voting



Algorithm:

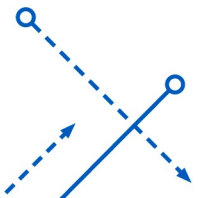
1. Quantize Parameter Space (θ, ρ).

2. Create Hough Space Array $H(\theta, \rho) = 0$.

3. For each image point (x_i, y_i):
   For all points (θ, ρ) on $\rho = x_i \cos\theta + y_i \sin\theta$:
   $H(\theta, \rho) = H(\theta, \rho) + 1$

4. Find local maxima $H(\theta_m, \rho_m)$.

5. The detected line: $x \cos\theta_m + y \sin\theta_m = \rho_m$



Line Detection by Hough Voting

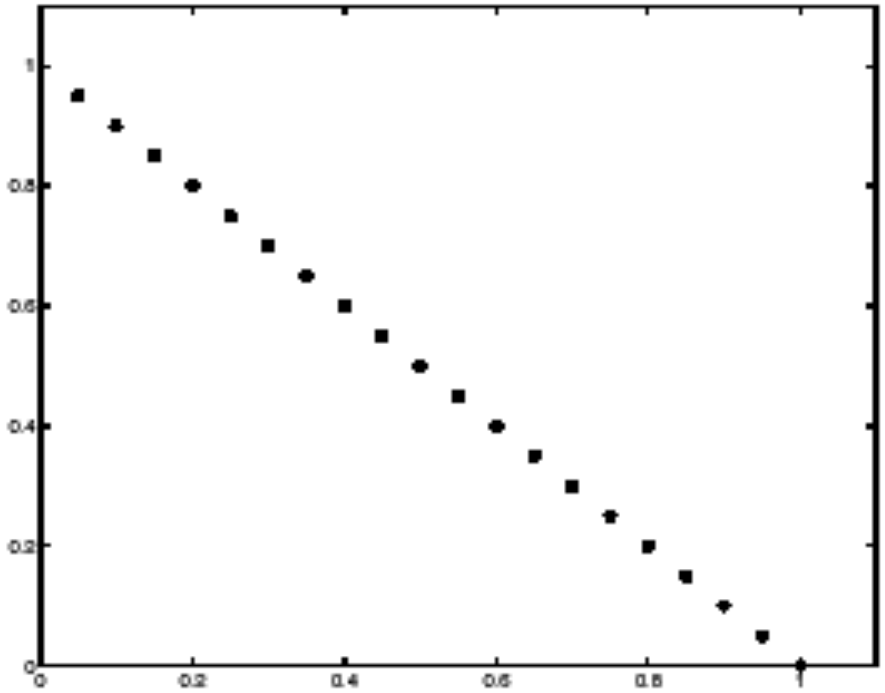
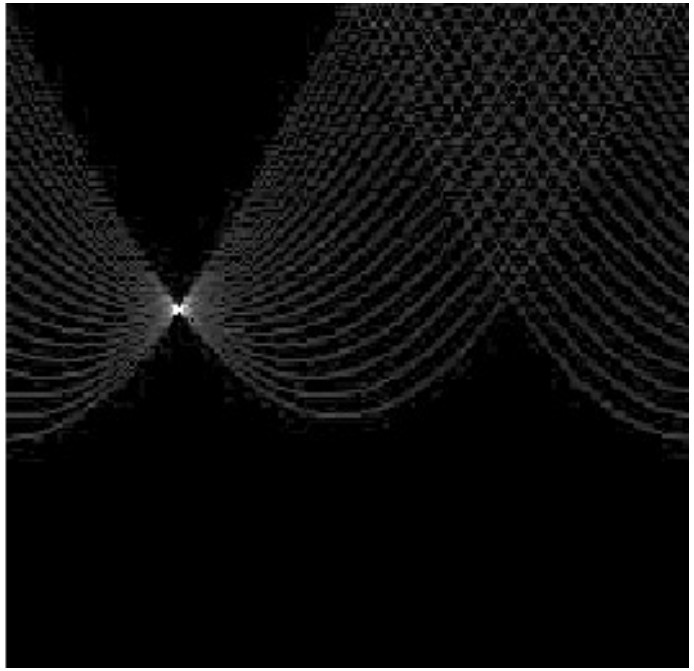


Image space



Votes

If images are noisy…

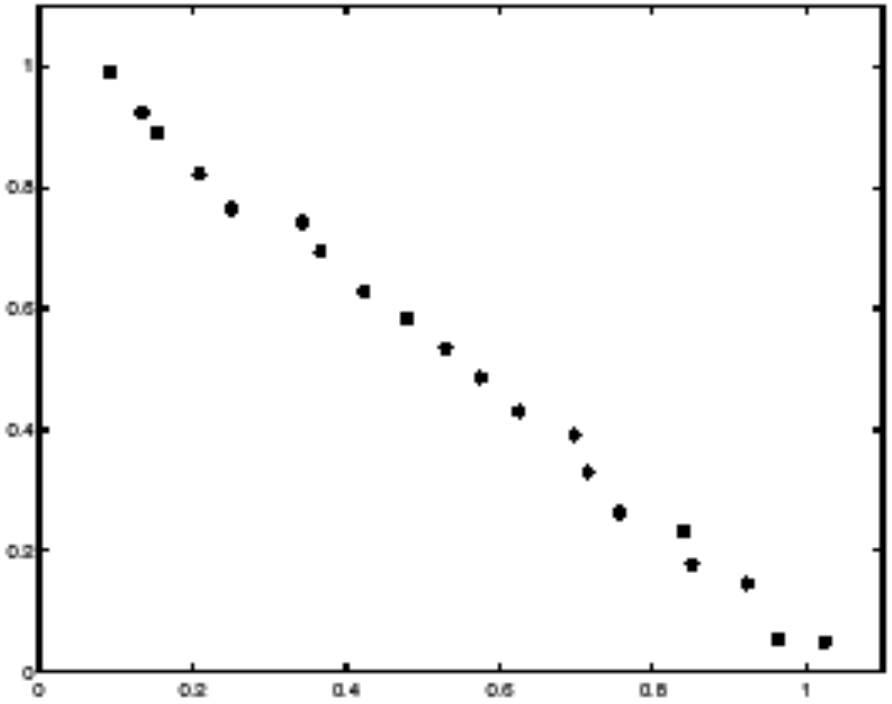
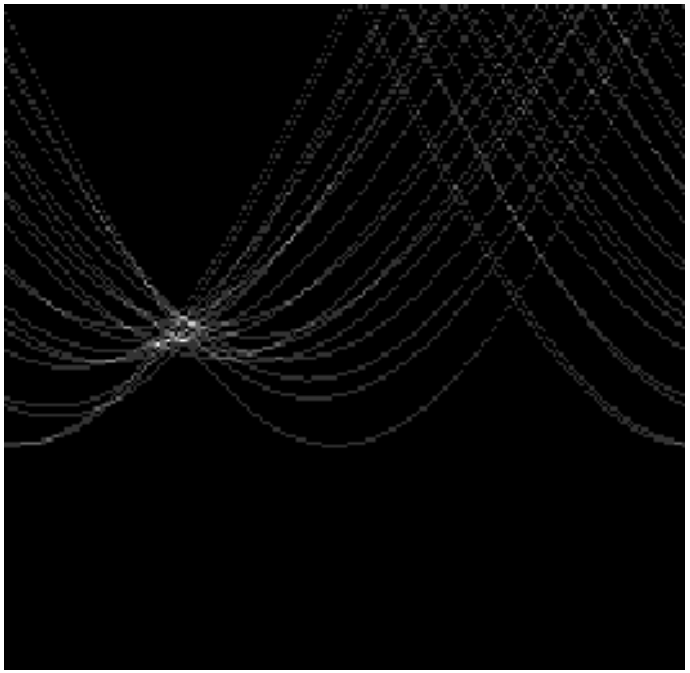


Image space



Votes

Too much noise

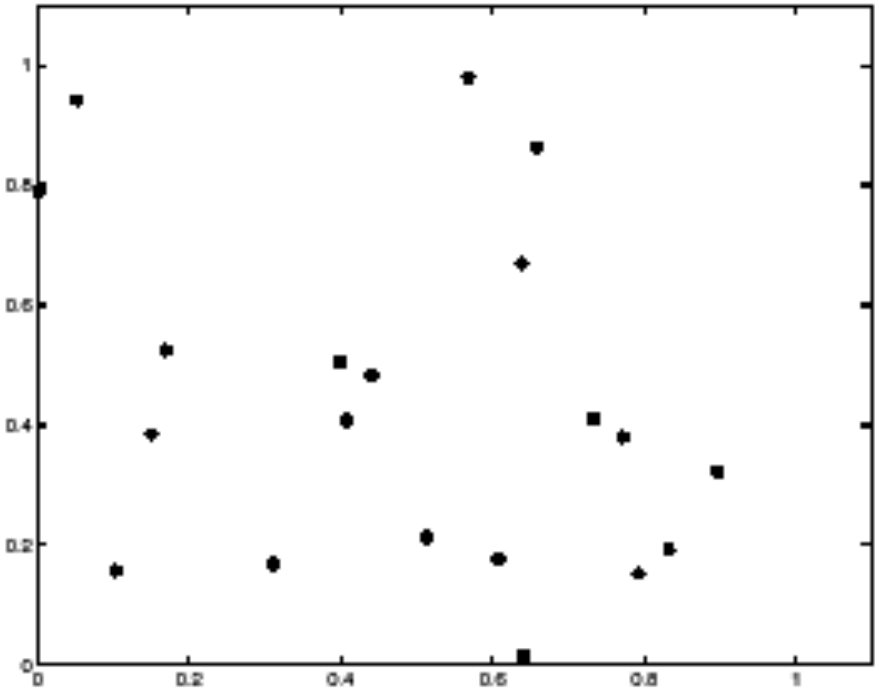
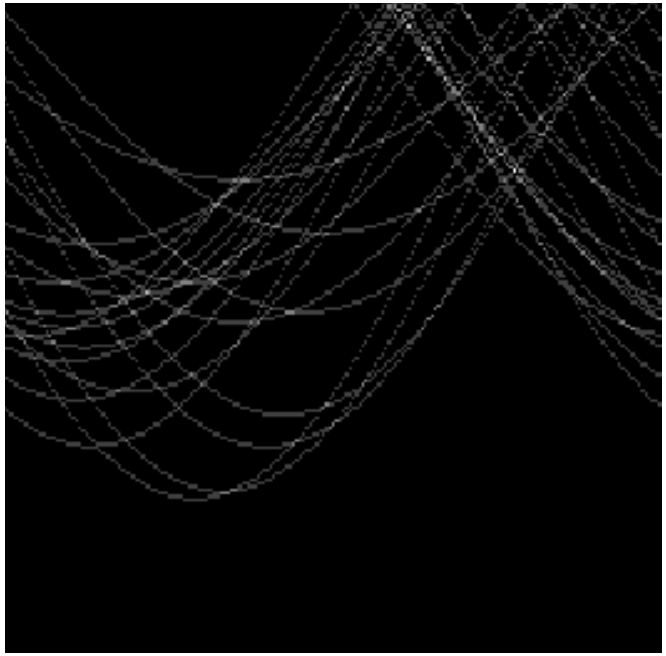


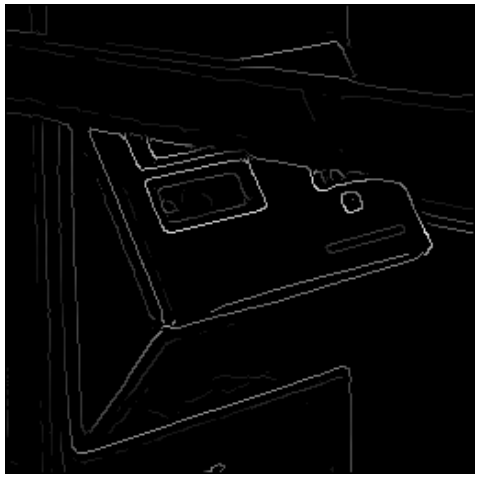
Image space



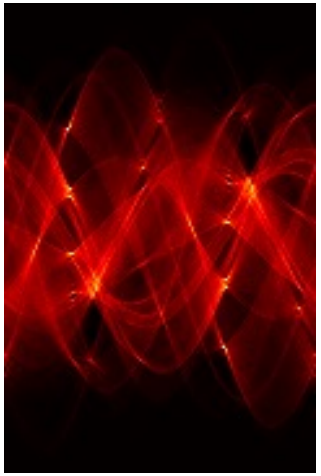
Votes

Real-world example

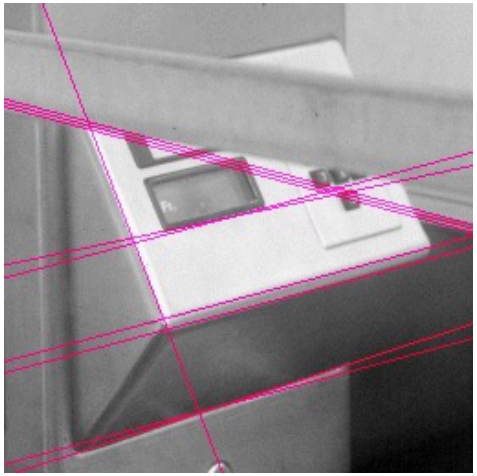
Original



Edges

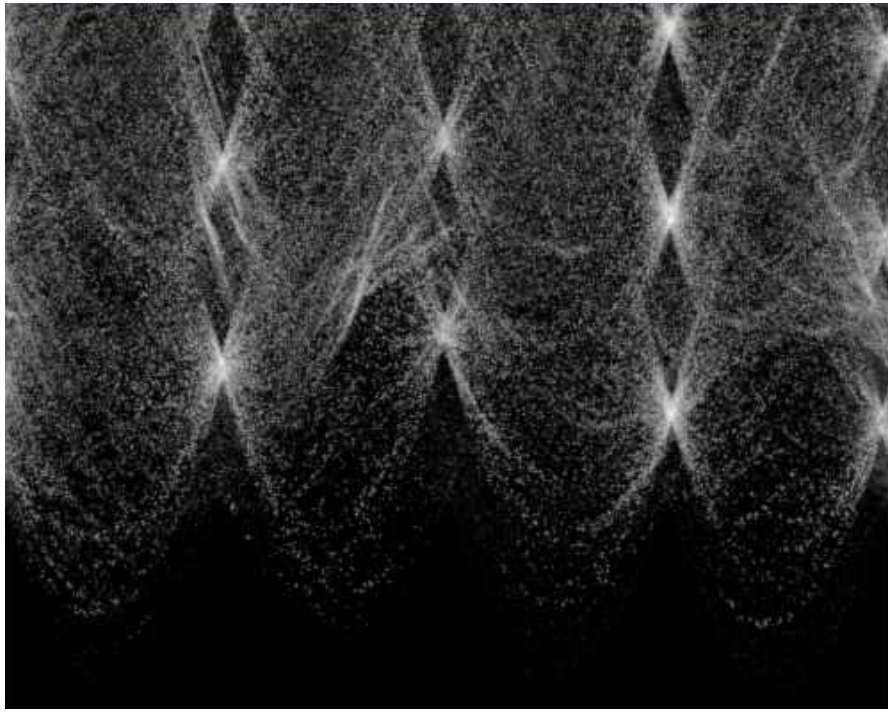


Parameter Space



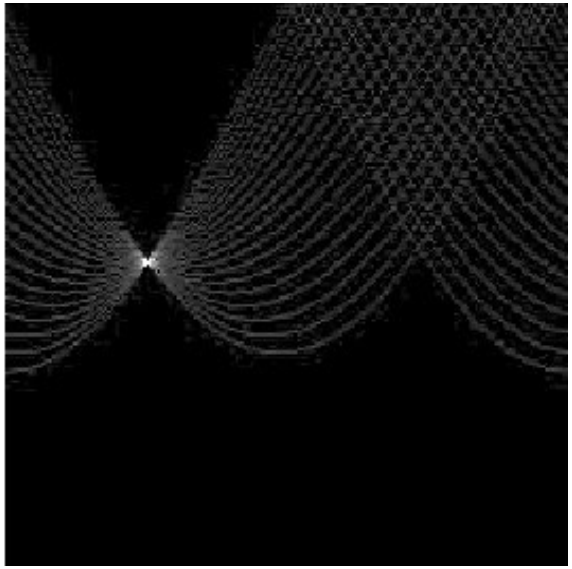
Hough Lines

More complex image

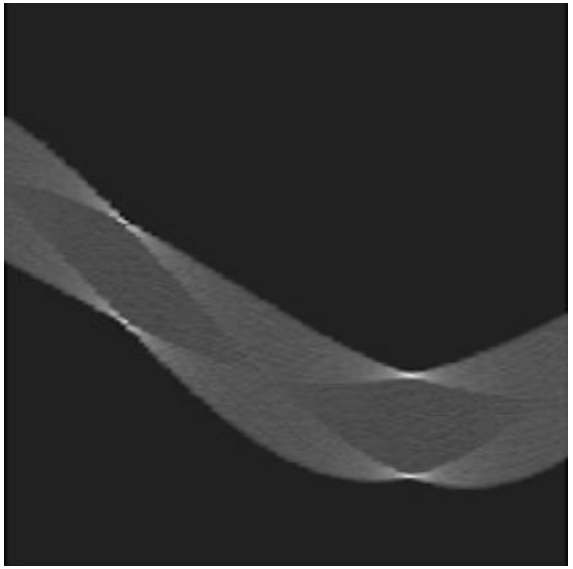


Basic Shapes

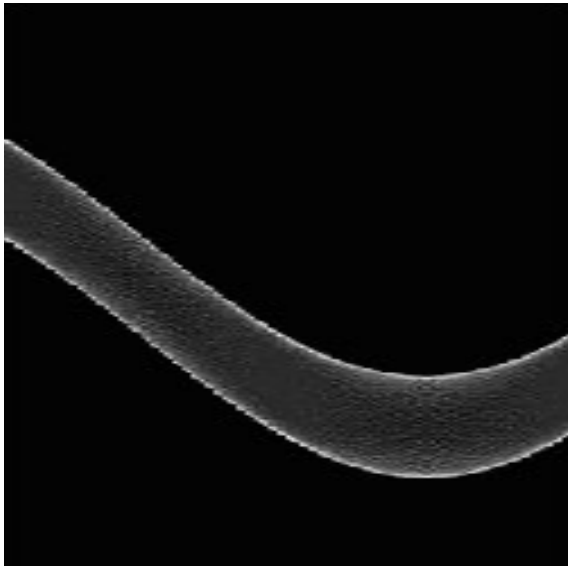
Parameter space



Line



rectangle
(parallelogram)



Circle

Can you guess the shape in image space?

Hough Circles

Let's assume **known** radius

parameters

$$(x-a)^2+(y-b)^2=r^2$$

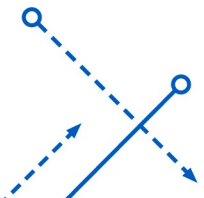
variables Fixed

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables Fixed

What is the dimension of the parameter space?



Hough Circles

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

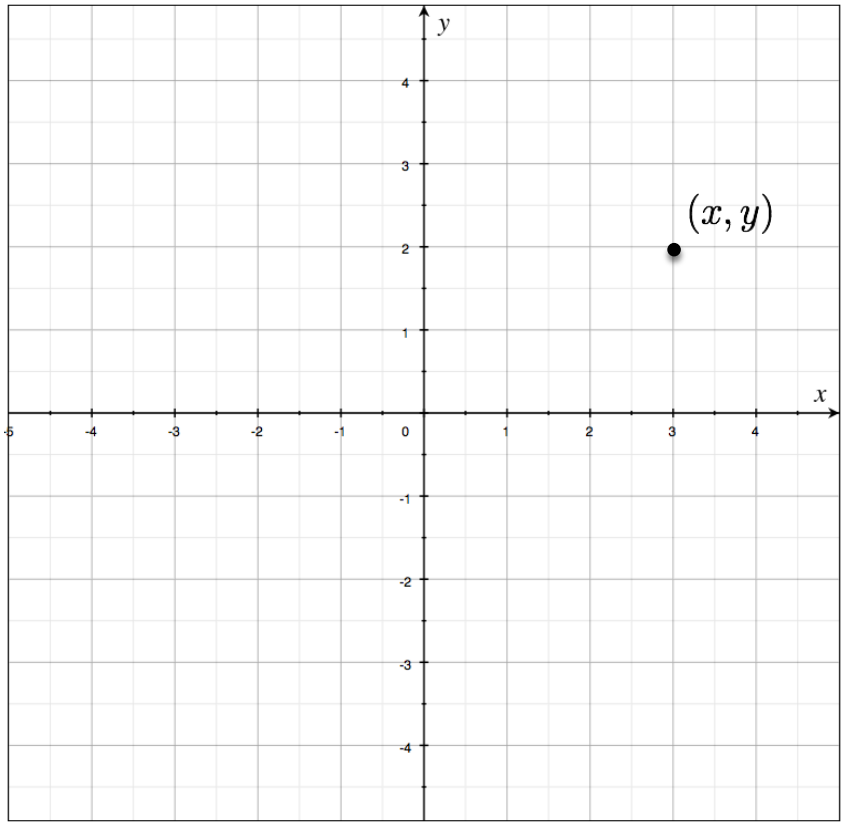
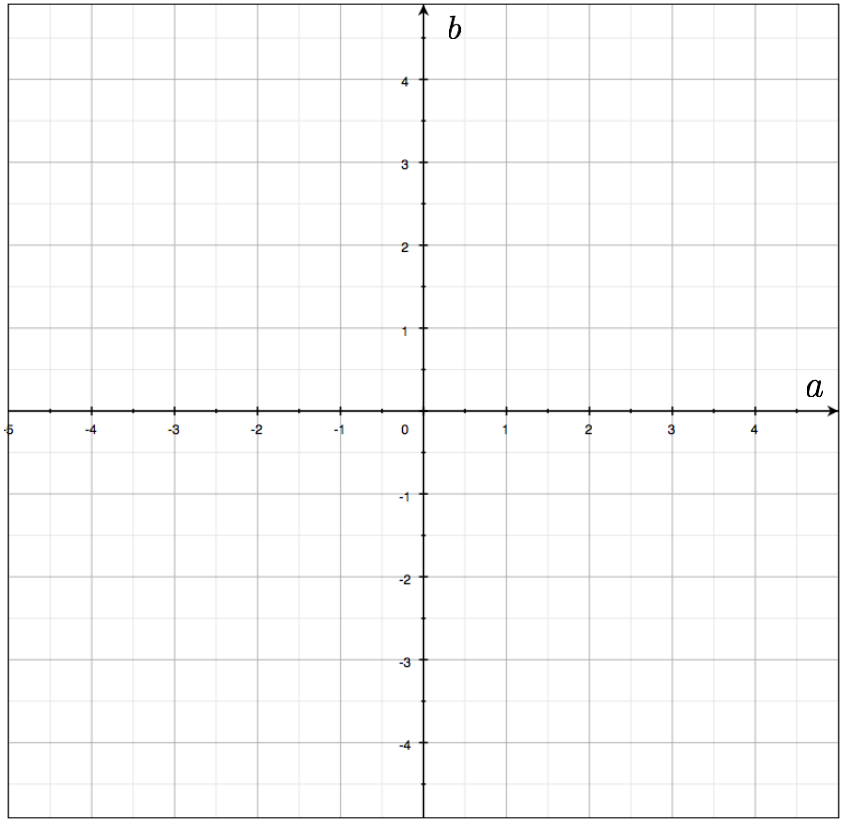


Image space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



Parameter space

What does a point in image space correspond to in parameter space?

Hough Circles

parameters

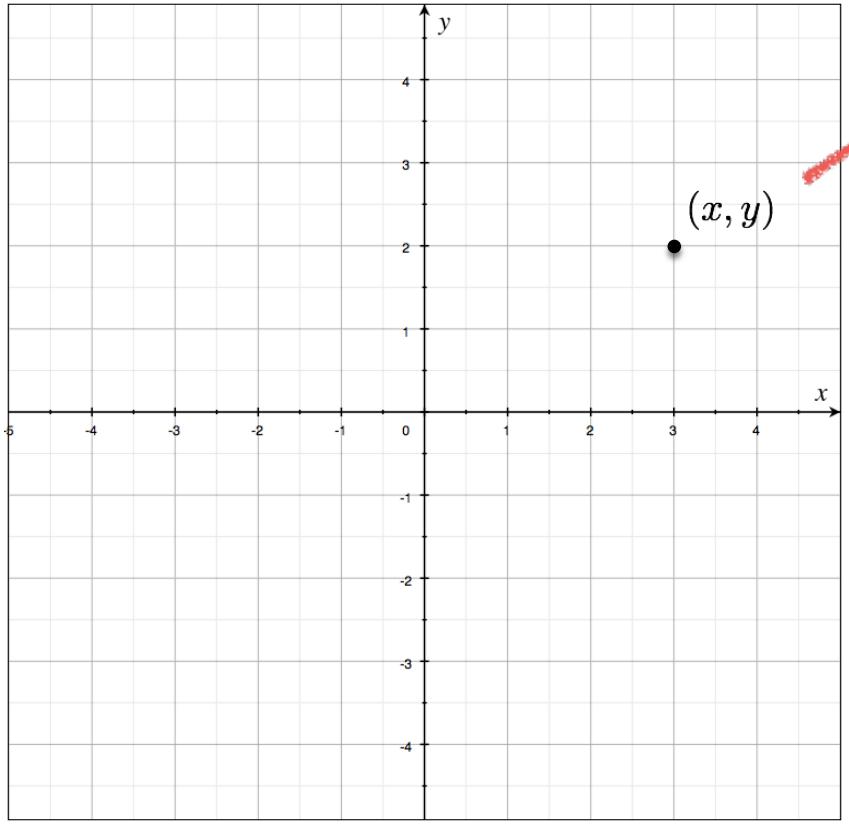
$$(x-a)^2+(y-b)^2=r^2$$

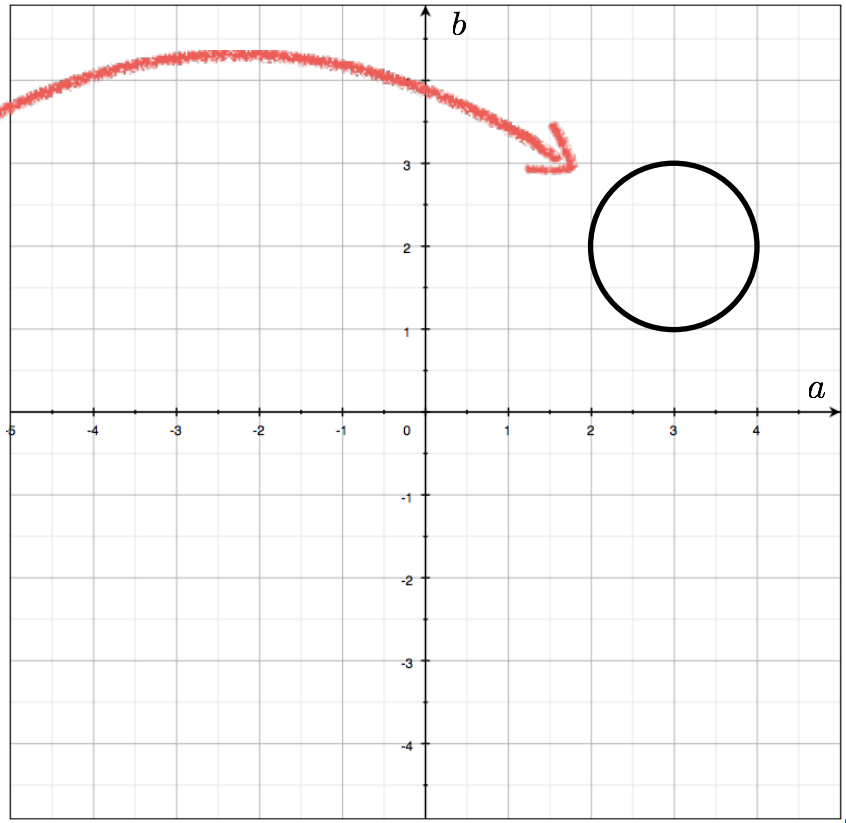
variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



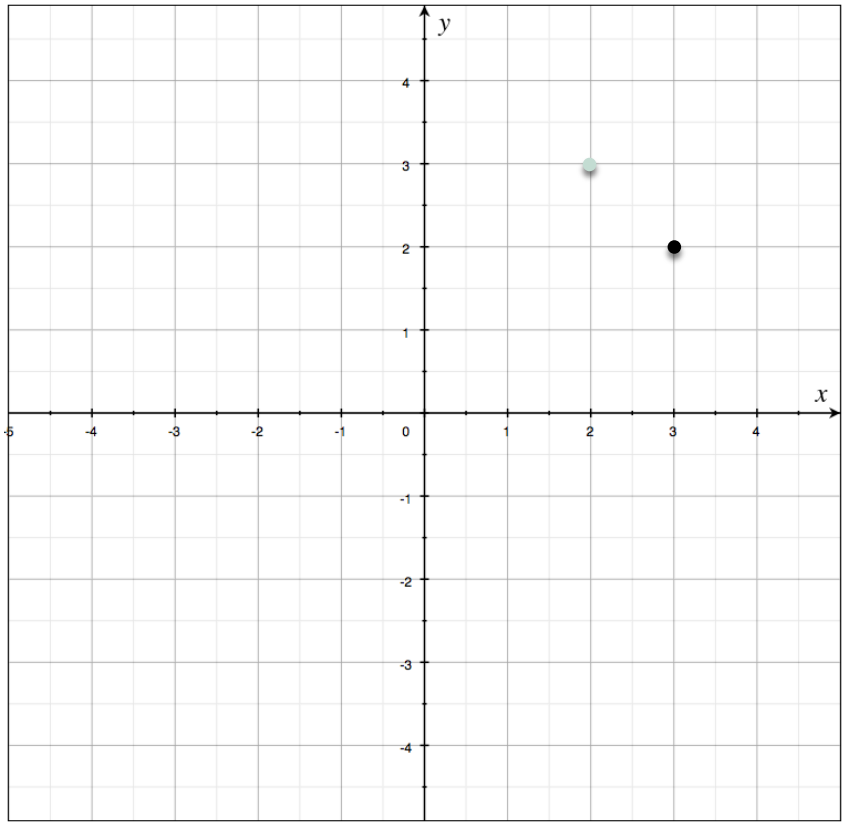


Hough Circles

parameters

$$(x-a)^2+(y-b)^2=r^2$$

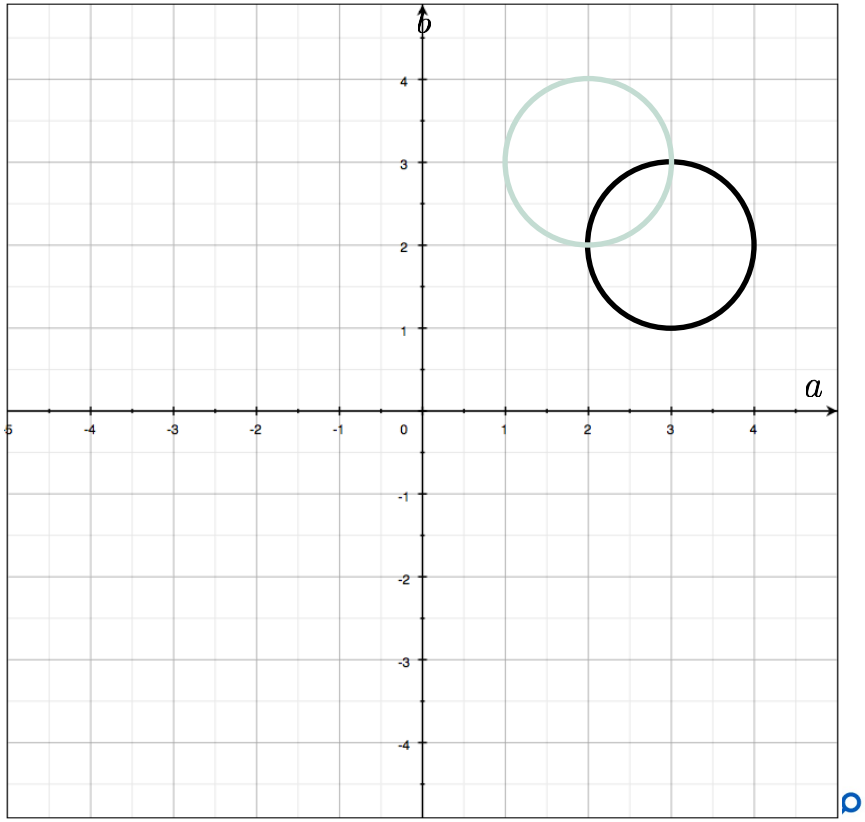
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



Hough Circles

parameters

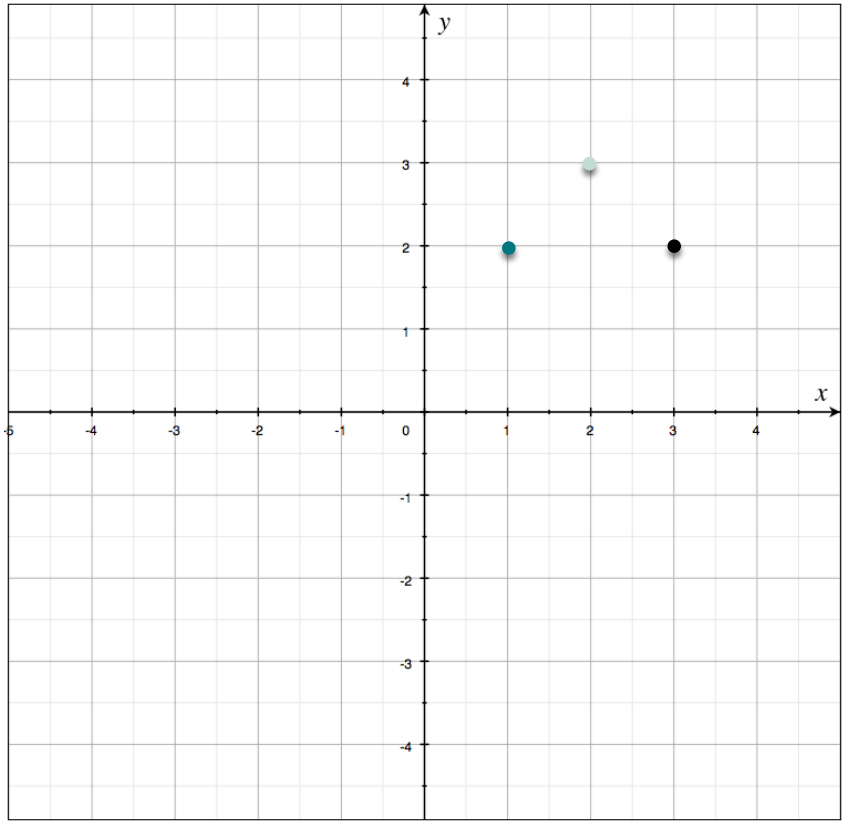
$$(x-a)^2+(y-b)^2=r^2$$

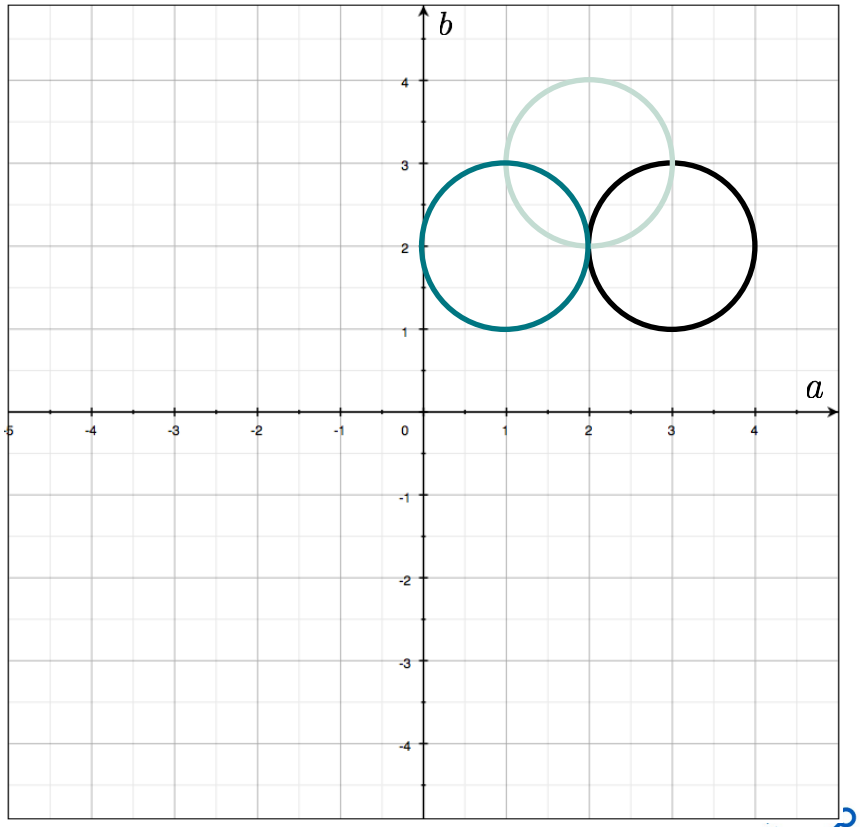
variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables





Hough Circles

parameters

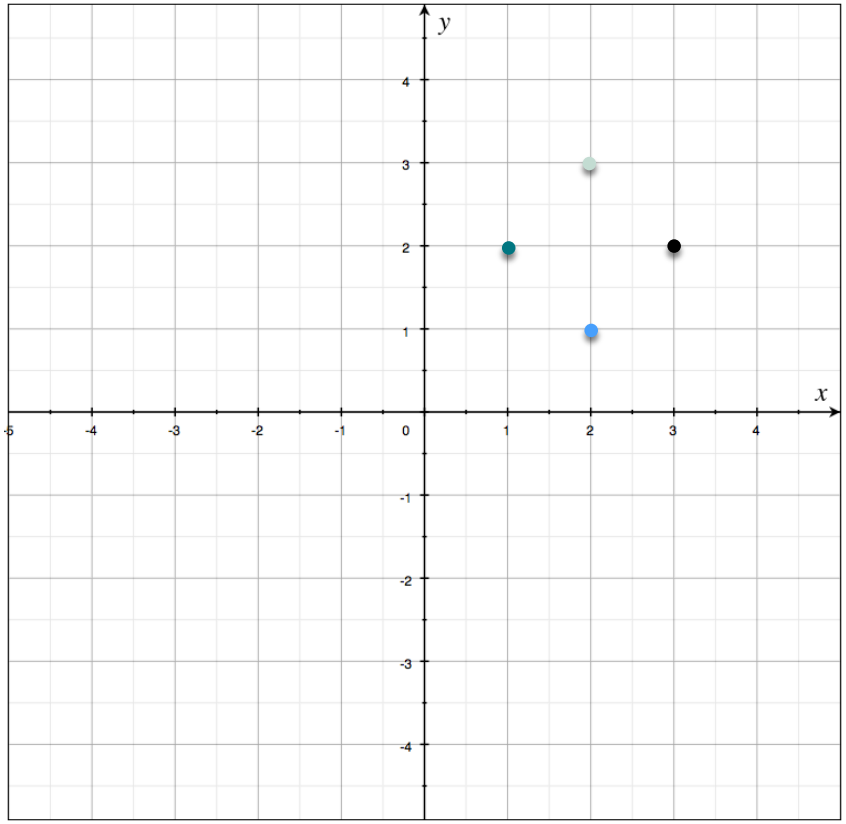
$$(x-a)^2+(y-b)^2=r^2$$

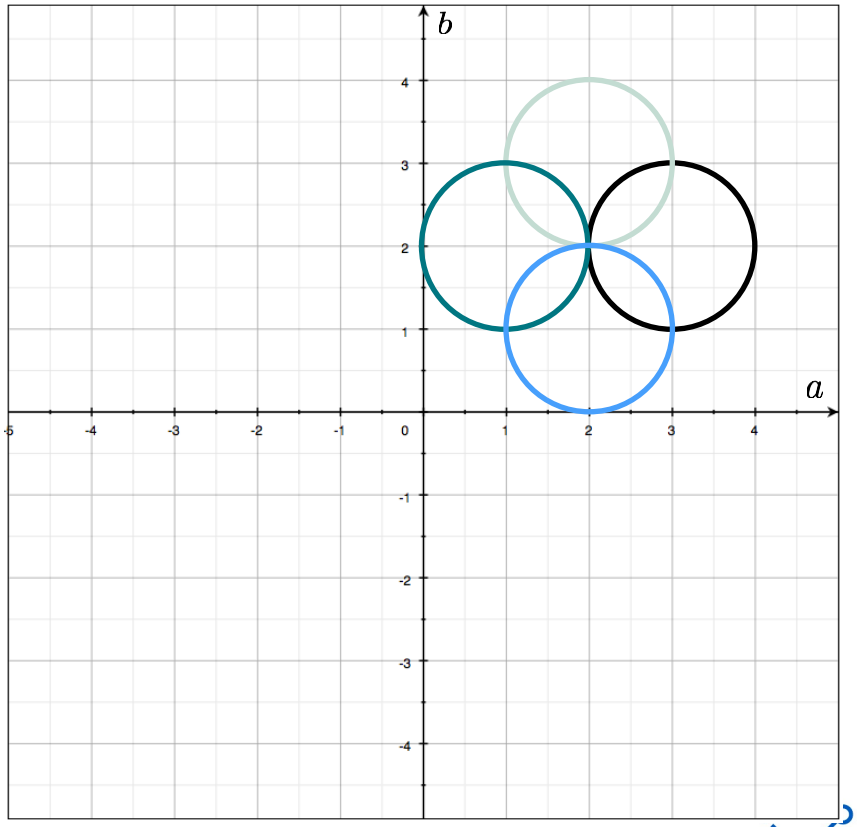
variables

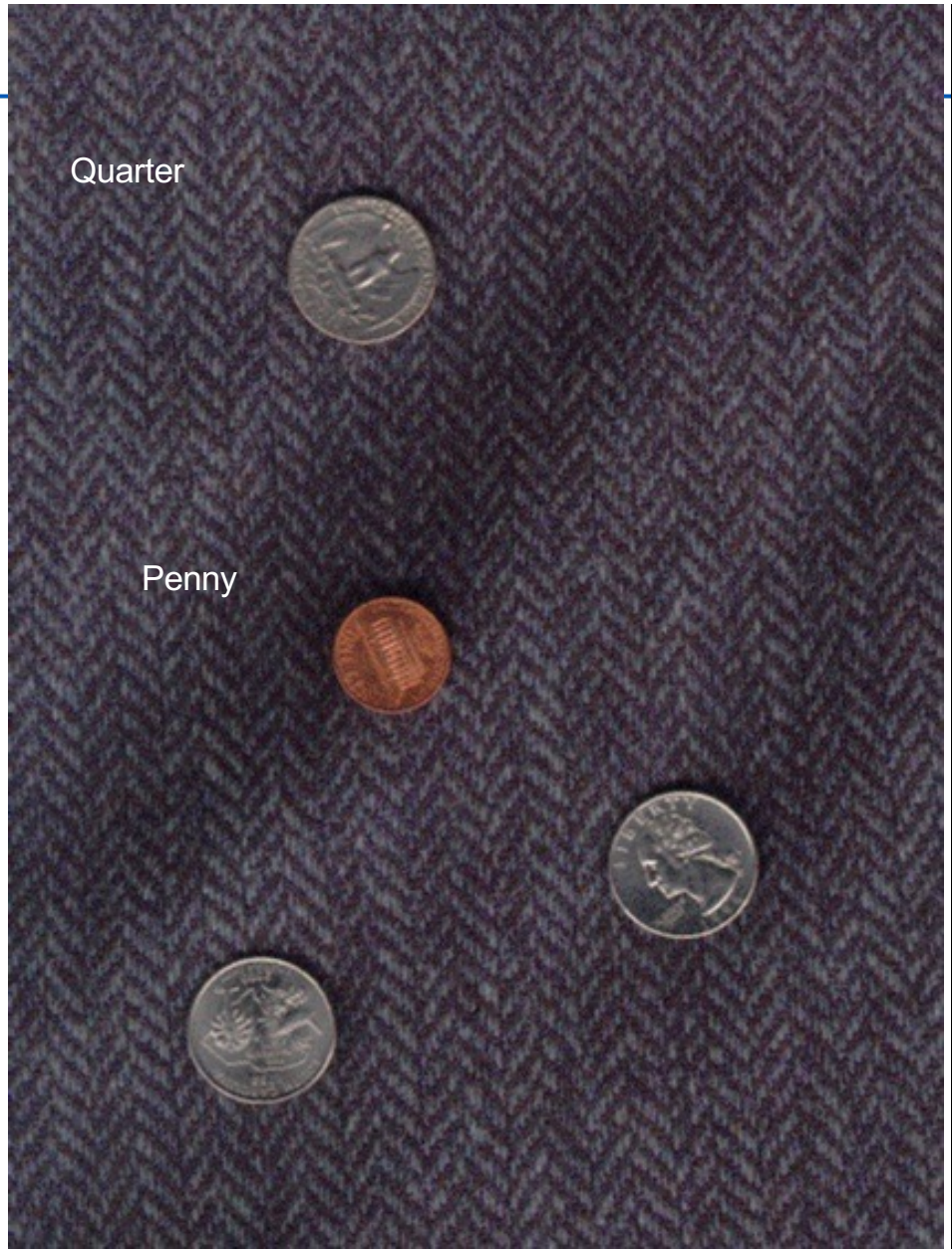
parameters

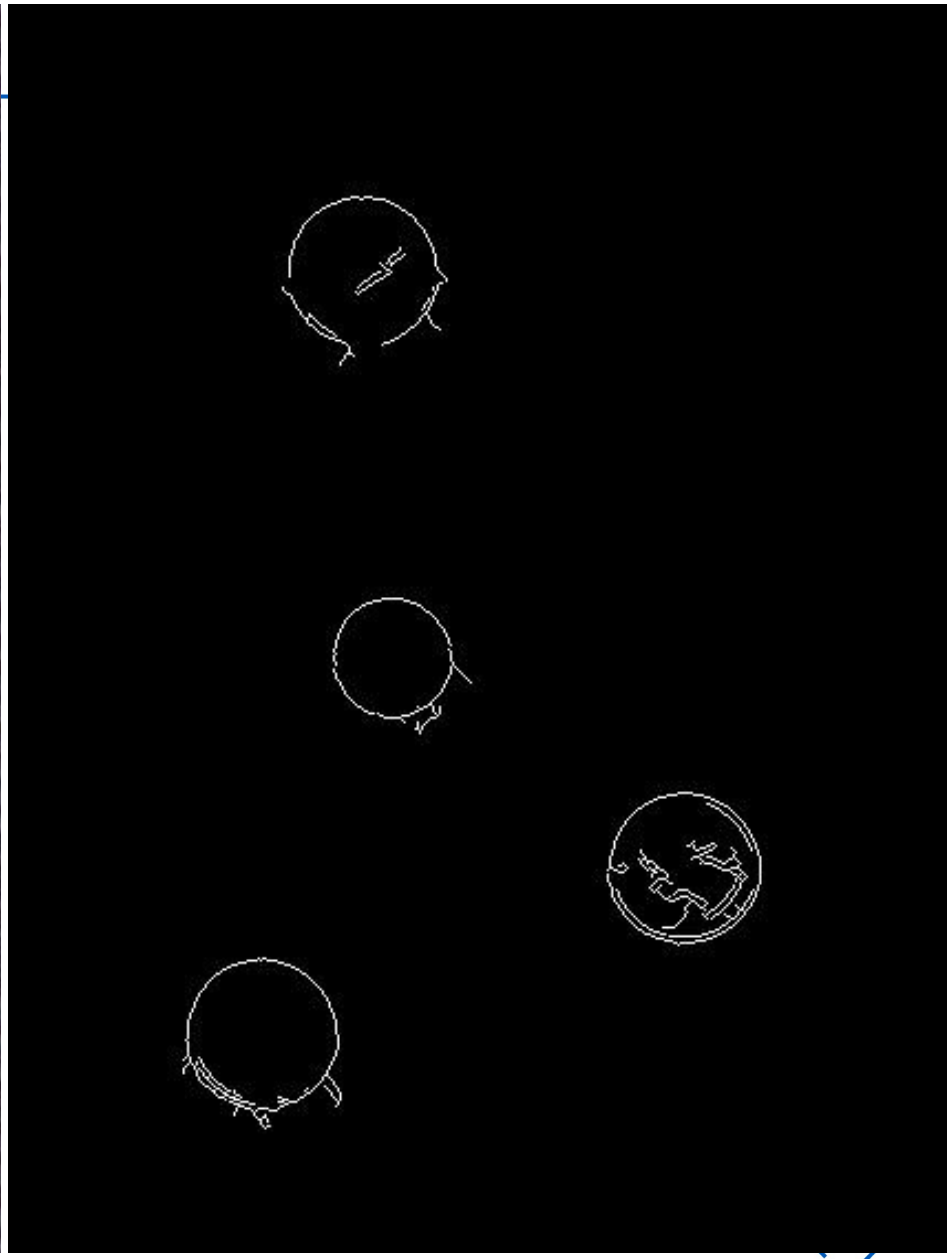
$$(x-a)^2+(y-b)^2=r^2$$

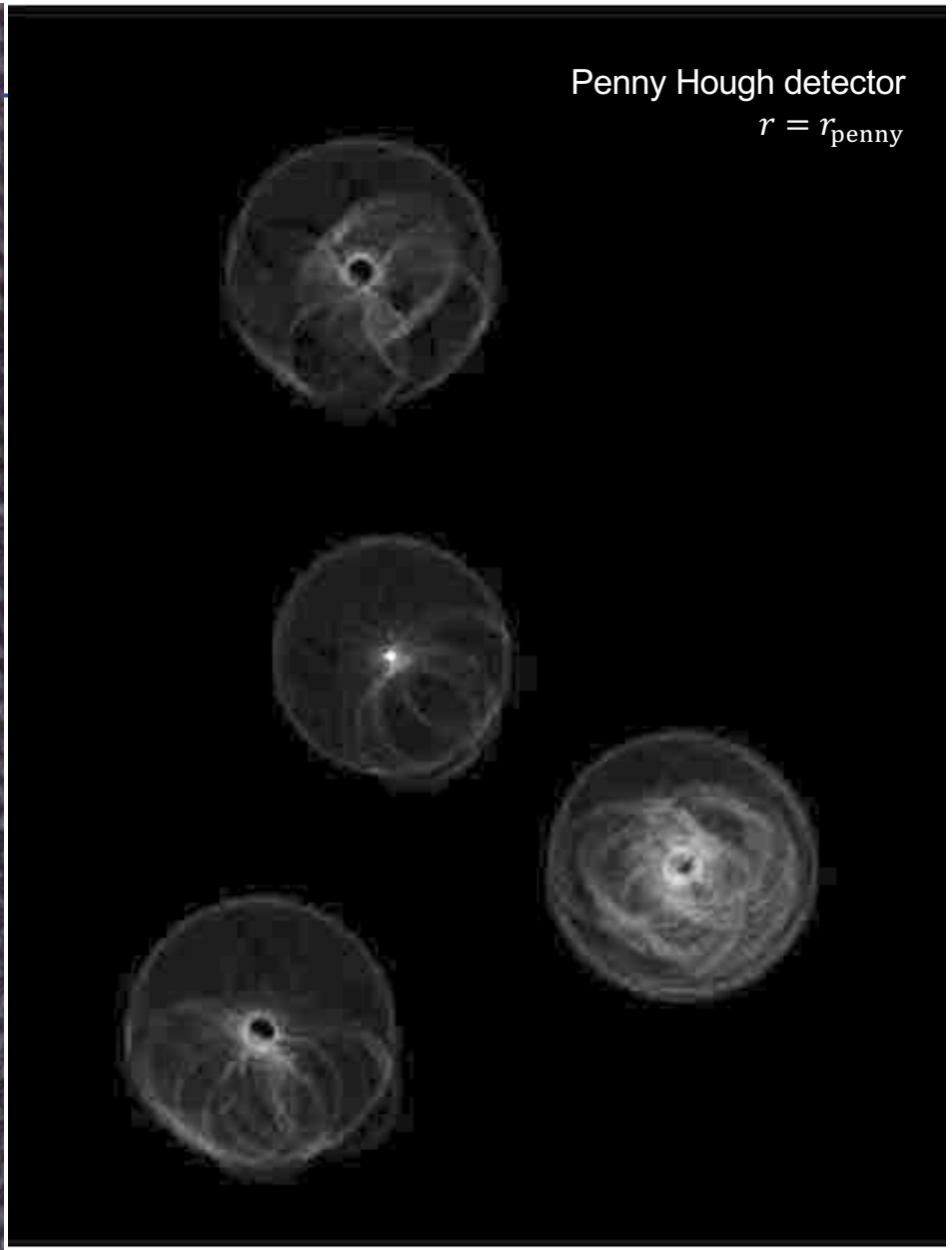
variables

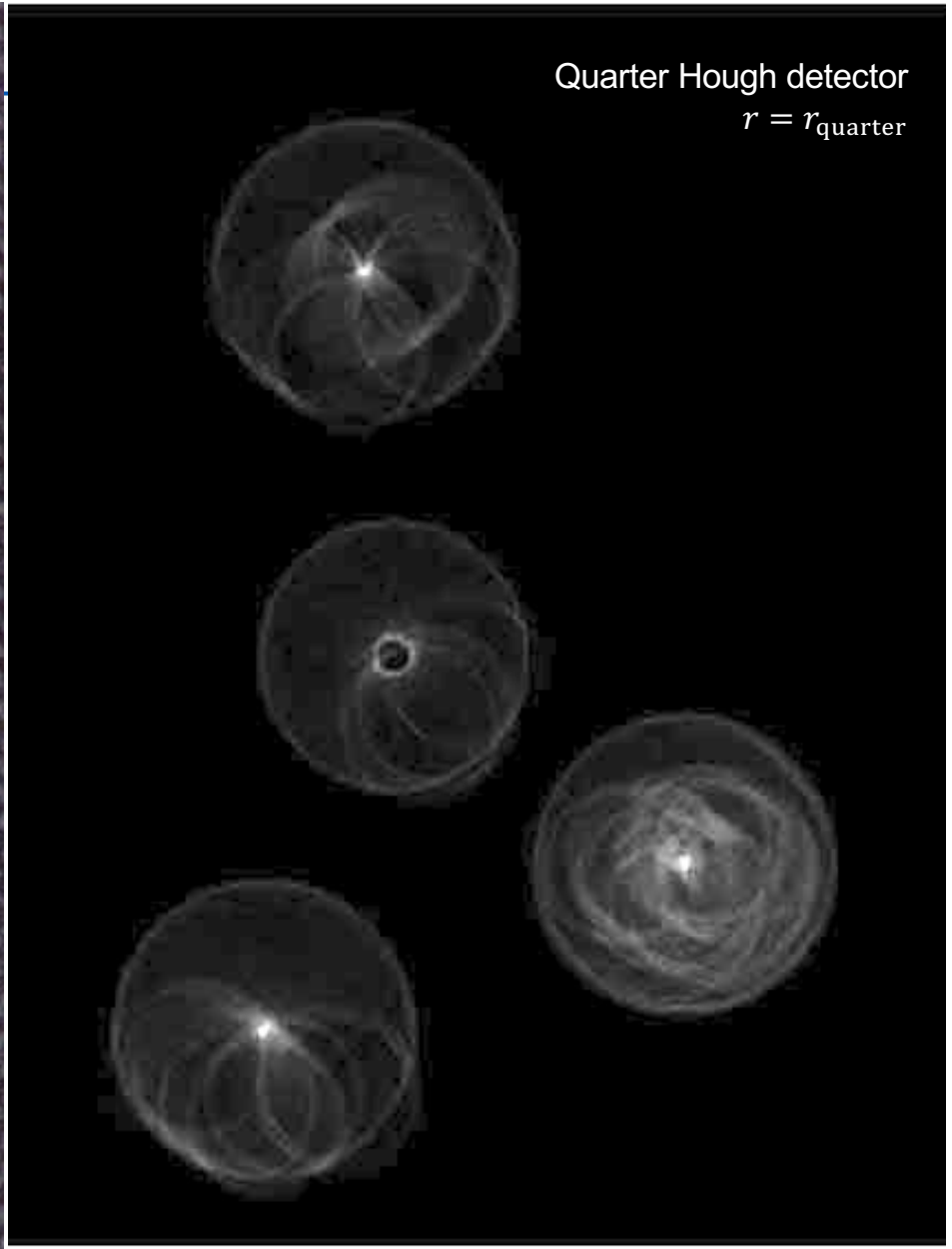


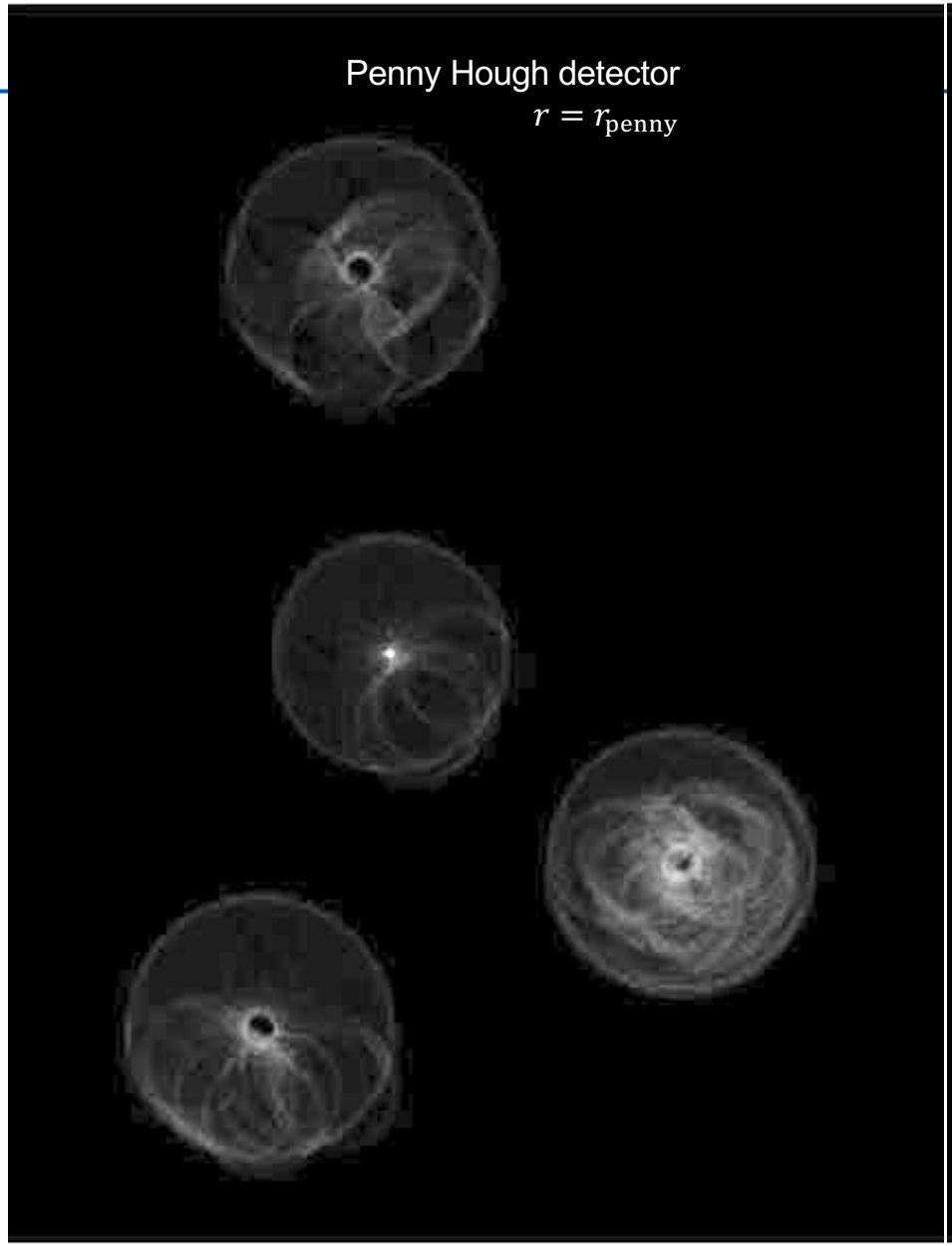


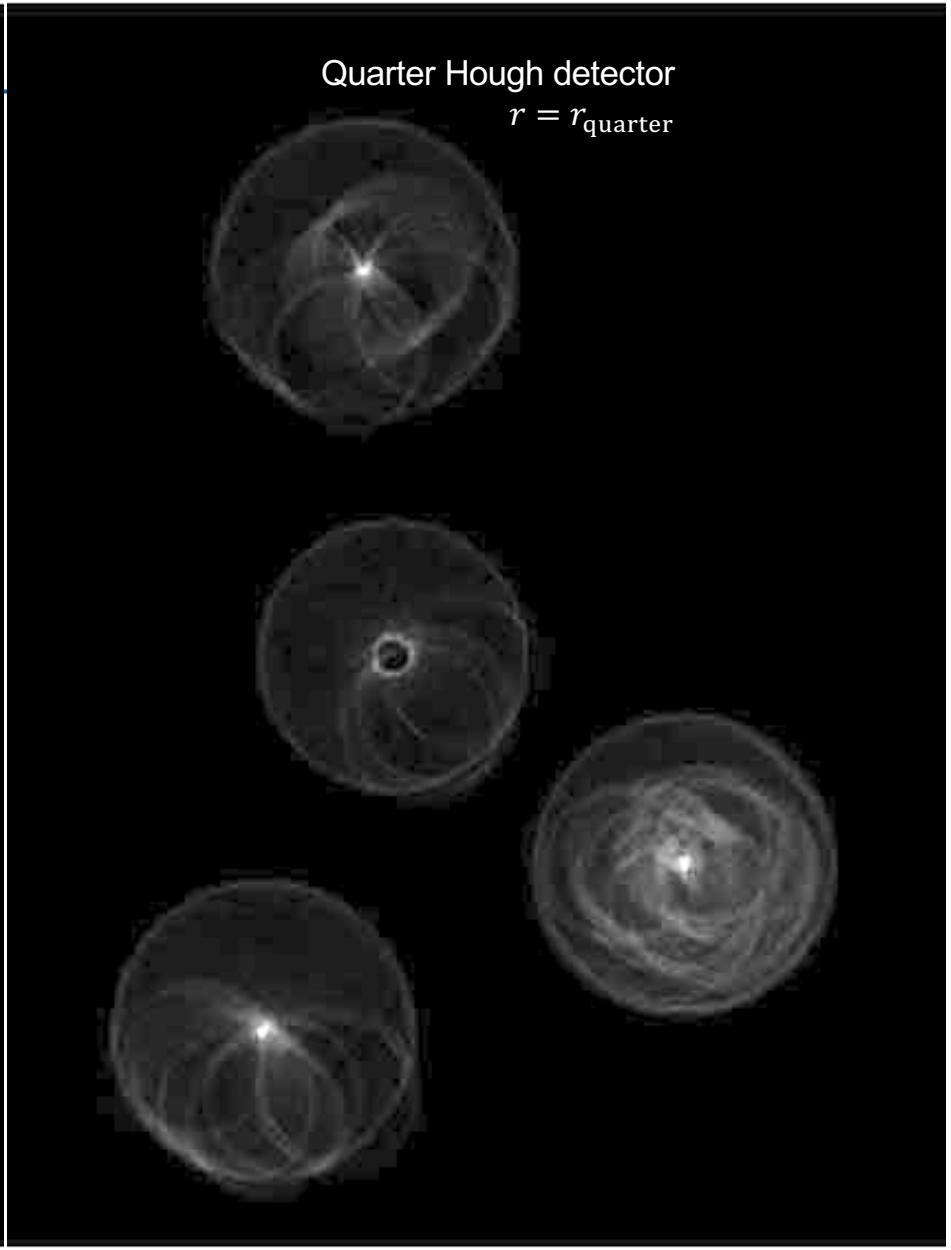

Quarter
Penny




Penny Hough detector
$r = r_{\text{penny}}$


Quarter Hough detector
$r = r_{\text{quarter}}$


Penny Hough detector
$r = r_{\text{penny}}$


Quarter Hough detector
$r = r_{\text{quarter}}$


Penny Hough detector
$r = r_{\text{penny}}$


Quarter Hough detector
$r = r_{\text{quarter}}$

What if radius is unknown?

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

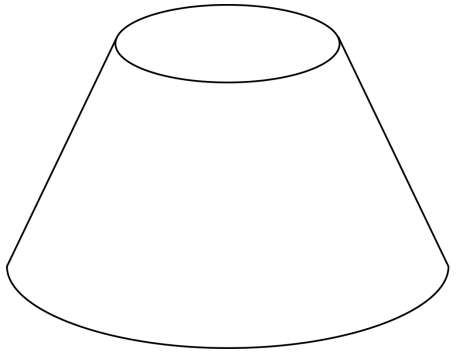
variables

If radius is unknown:

3D Hough Space!

Use Hough array $H(a,b,r)$.

Surface shape in Hough space is complicated.



Frustum of cone

Other Shapes?

Vertical Ellipse:

parameters

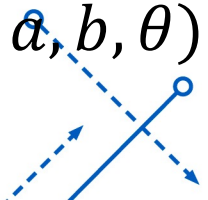
$$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$$

$$H(x_0, y_0, a, b)$$

Ellipse:

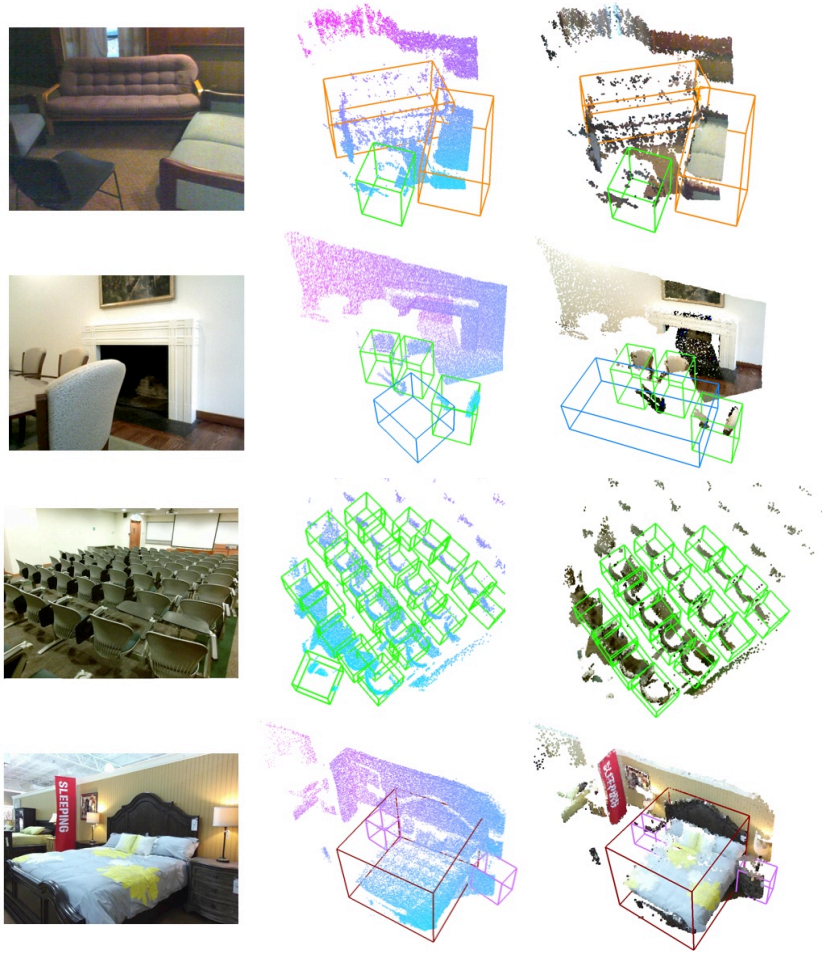
$$\frac{[(x-x_0)\cos\theta+(y-y_0)\sin\theta]^2}{a^2}+\frac{[-(x-x_0)\sin\theta+(y-y_0)\cos\theta]^2}{b^2}=1$$

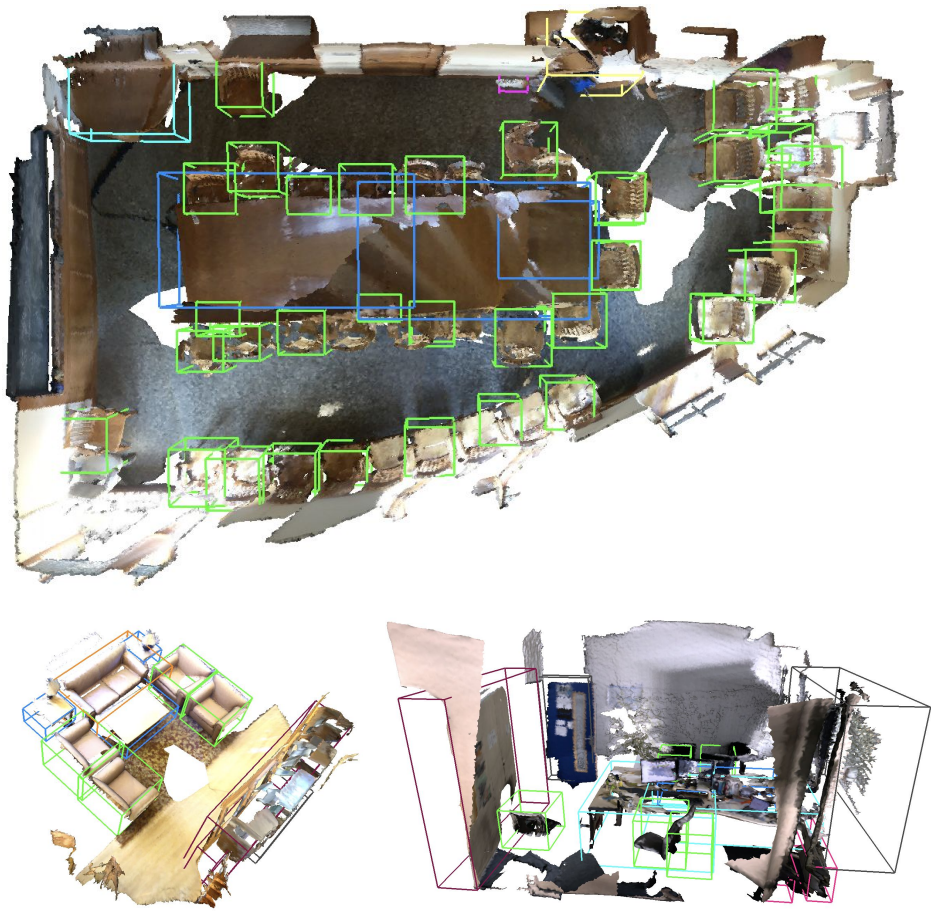
$$H(x_0, y_0, a, b, \theta)$$



Applications of Hough Voting

Scenes Prediction Ground Truth





Qi, Charles R., et al. "Deep **Hough Voting** for 3D Object Detection in Point Clouds." *IEEE/CVF International Conference on Computer Vision (CVPR)*, 2019

Conclusion

Is the following correct about Hough transform …

- Detects multiple instances (lines/circles)?

- Robust to noise?

- Can be used for other shapes beyond lines/circles?

- Good computational complexity?

- Deals with occlusion well?