SAIR
Spatial AI & Robotics Lab

CSE 473/573-A

L6: MORPHOLOGY

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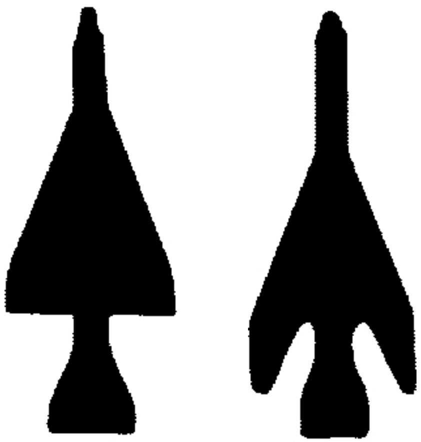
Morphology – Introduction

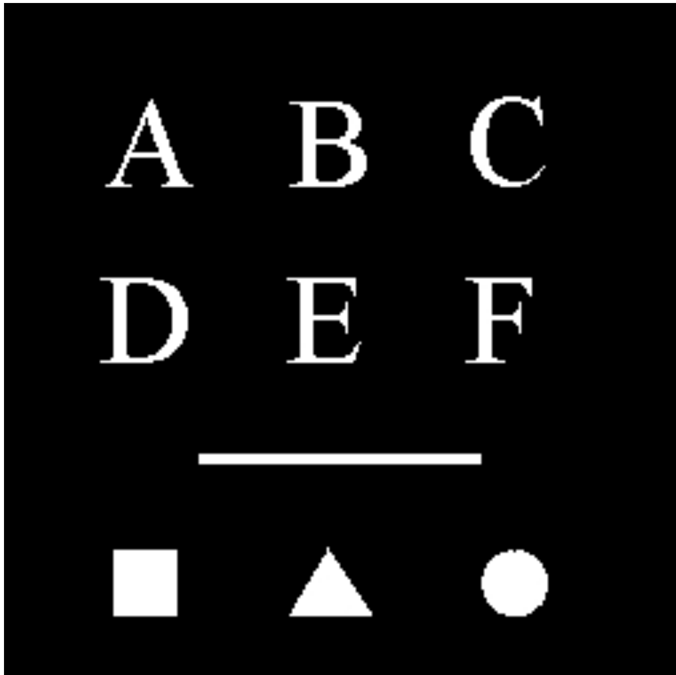
Looking at these images……

What is interesting, important or useful information we care about?

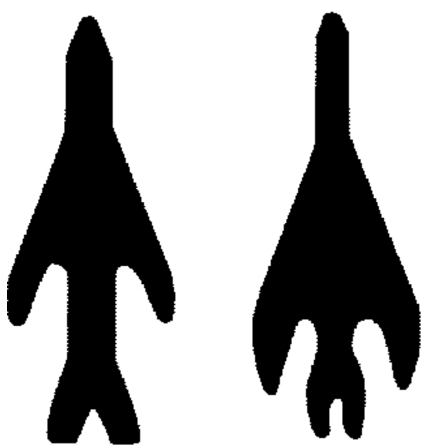


The pixel value of the image is not important as there are only two different values.

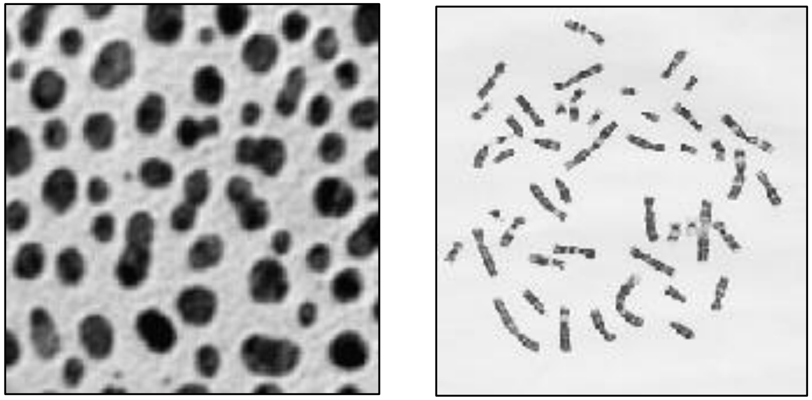
- Region shape and boundaries of object are important.



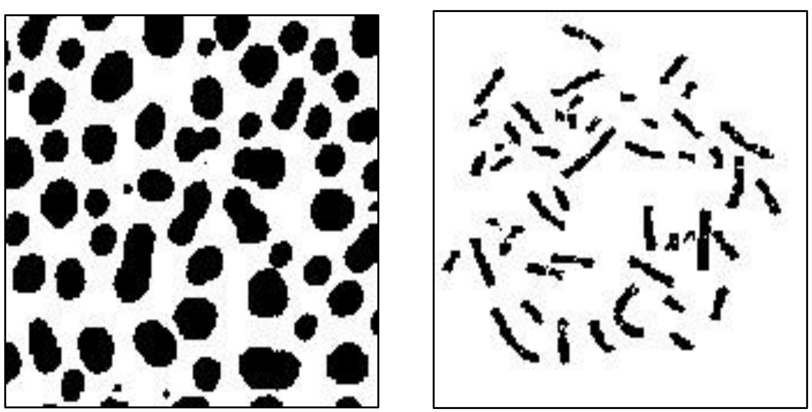
- Form and structure can be represented by object pixel set.



Morphology – Introduction



Grayscale Images



Binary Images

Image analysis needs to measure the characteristics of objects in the images.

Geometric measurements are important objects characteristics

- location, orientation, area, length of perimeter

These geometric characteristics are often easier to be measured from binary images.

Morphology – Introduction

- Visual perception requires image processing to extract **shape information**.

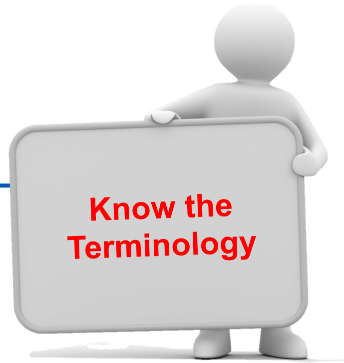
- **Goal:**
  - Distinguish meaningful shape information from irrelevant one.

- The vast majority of shape processing and analysis techniques are **based on designing a shape operator** which satisfies desirable properties.

Morphology –Introduction

- Morphology deals with form and structure
- Mathematical morphology is a tool for extracting image components useful in:
  - representation and description of region shape (e.g. boundaries)
  - pre- or post-processing (filtering, thinning, etc.)
- Morphological operations usually follow a segmentation task or an edge detection task.
  - Thus, often operate on binary images.
- Based on set theory and logic operations

Morphology –Set Theory



- A two dimensional integer space is denoted by $\mathbf{Z}^2$.
- An element in this space has two components $a=(a_1, a_2)$.
- For image representation, $a=(a_1, a_2)$ are the x- and y-coordinates of a pixel.
- Let A be a set in $\mathbf{Z}^2$. If $a=(a_1, a_2)$ is an element of A, we denote

$$a \in A$$

- If not, then

$$a \notin A$$

Morphology –Set Theory

- $\varnothing$ denotes null (empty) set

- An example that specifies a set C:

$$C = \{ \, w \mid w = a + d, \, a \in A \, \}, \, d = (8, 5).$$

- If a set A is a subset of B, we denote:

$$A \subseteq B$$

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Morphology –Set Theory

- Union of A and B: $C = A \cup B$
- Intersection of A and B: $D = A \cap B$
- Disjoint sets: $A \cap B = \varnothing$
- Complement of A: $A^c = \{w \mid w \notin A\}$
- Difference of A and B: $A - B = \{w \mid w \in A,\ w \notin B\} = A \cap B^c$

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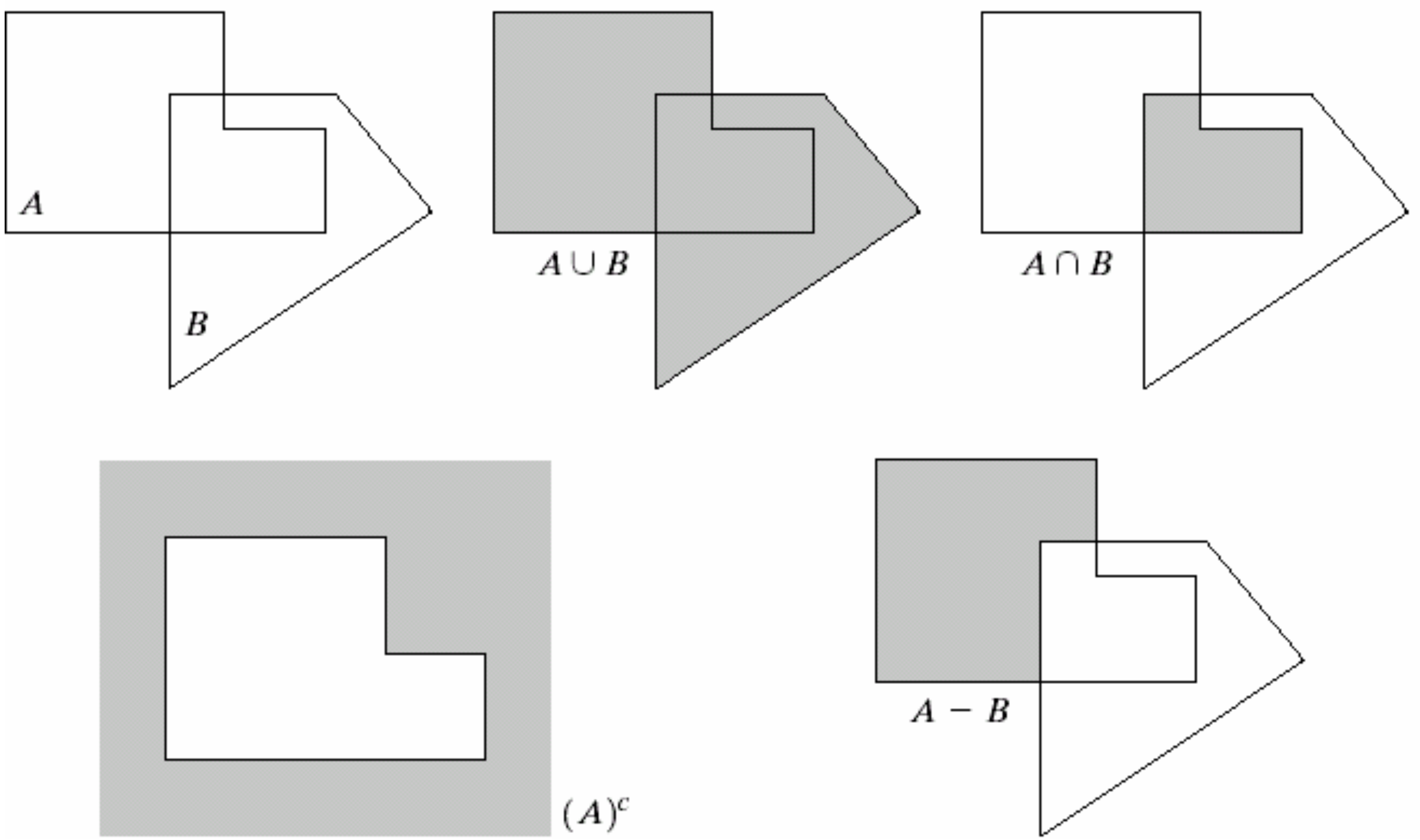
Morphology –Set Theory

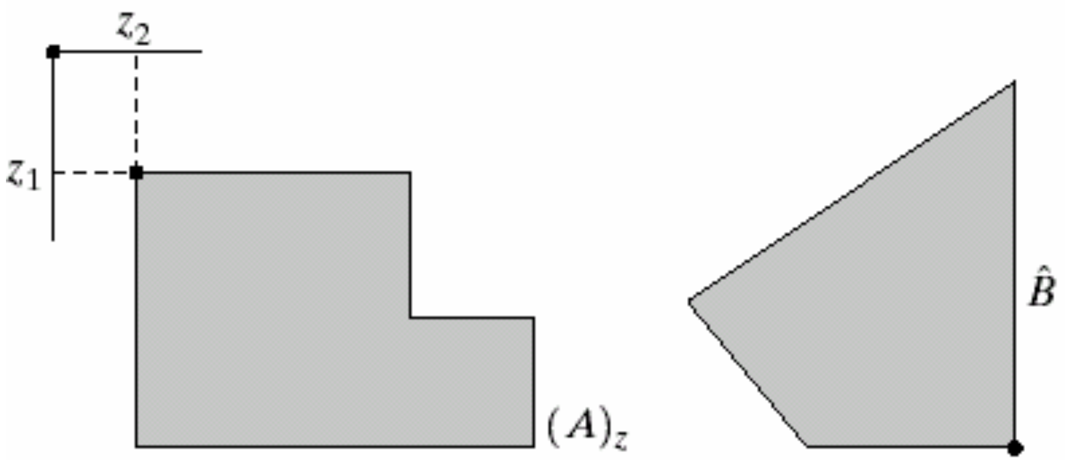


a b c
d e

(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Morphology –Set Theory

- Translation of A by $z=(z_1, z_2)$: $(A)_z = \{c \mid c = a+z, a \in A\}$



a b

(a) Translation of A by z.
(b) Reflection of B.

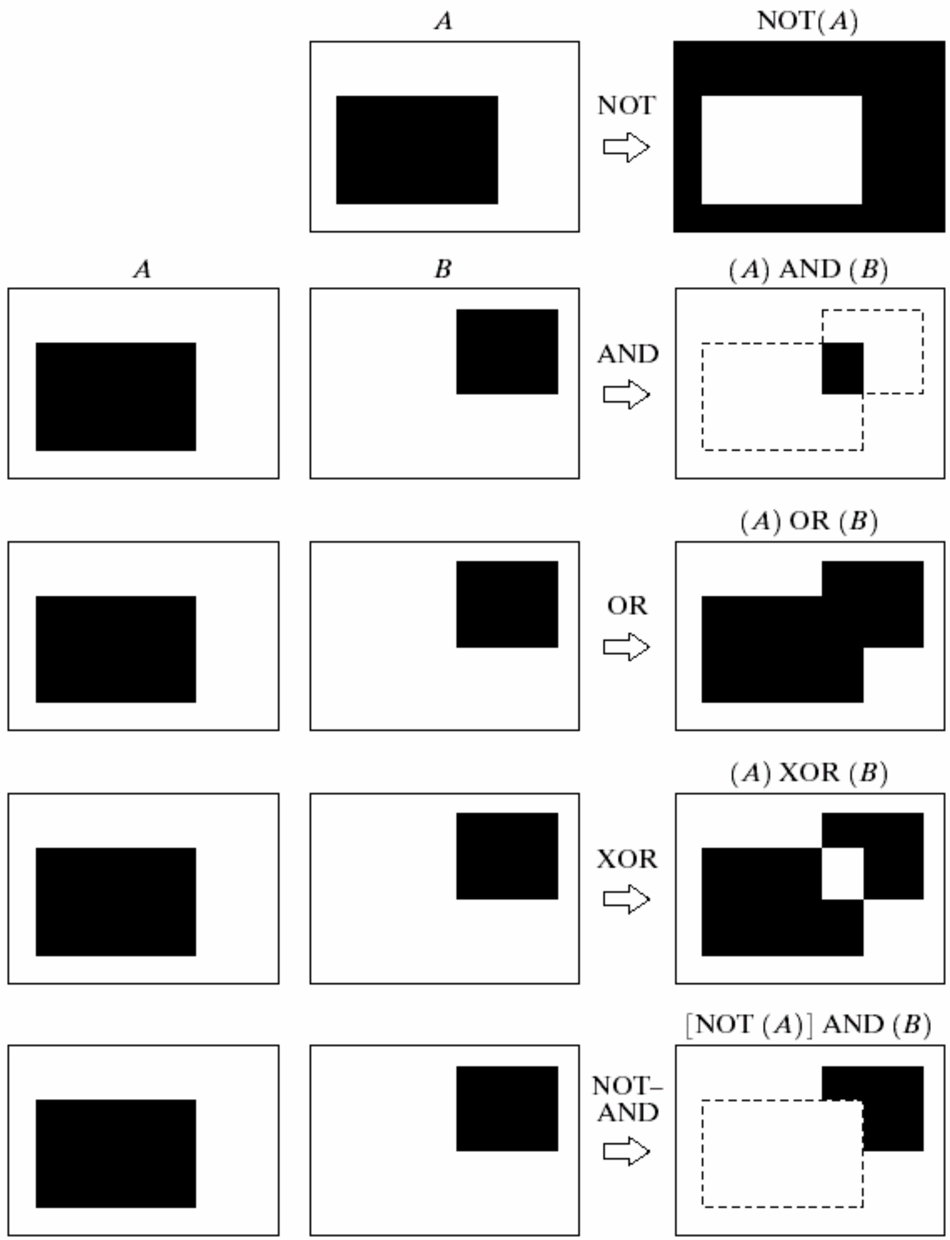
- Reflection of B: $\hat{B} = \{w \mid w = -b, \ b \in B\}$

Morphology –Set Theory

- Three basic logical operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

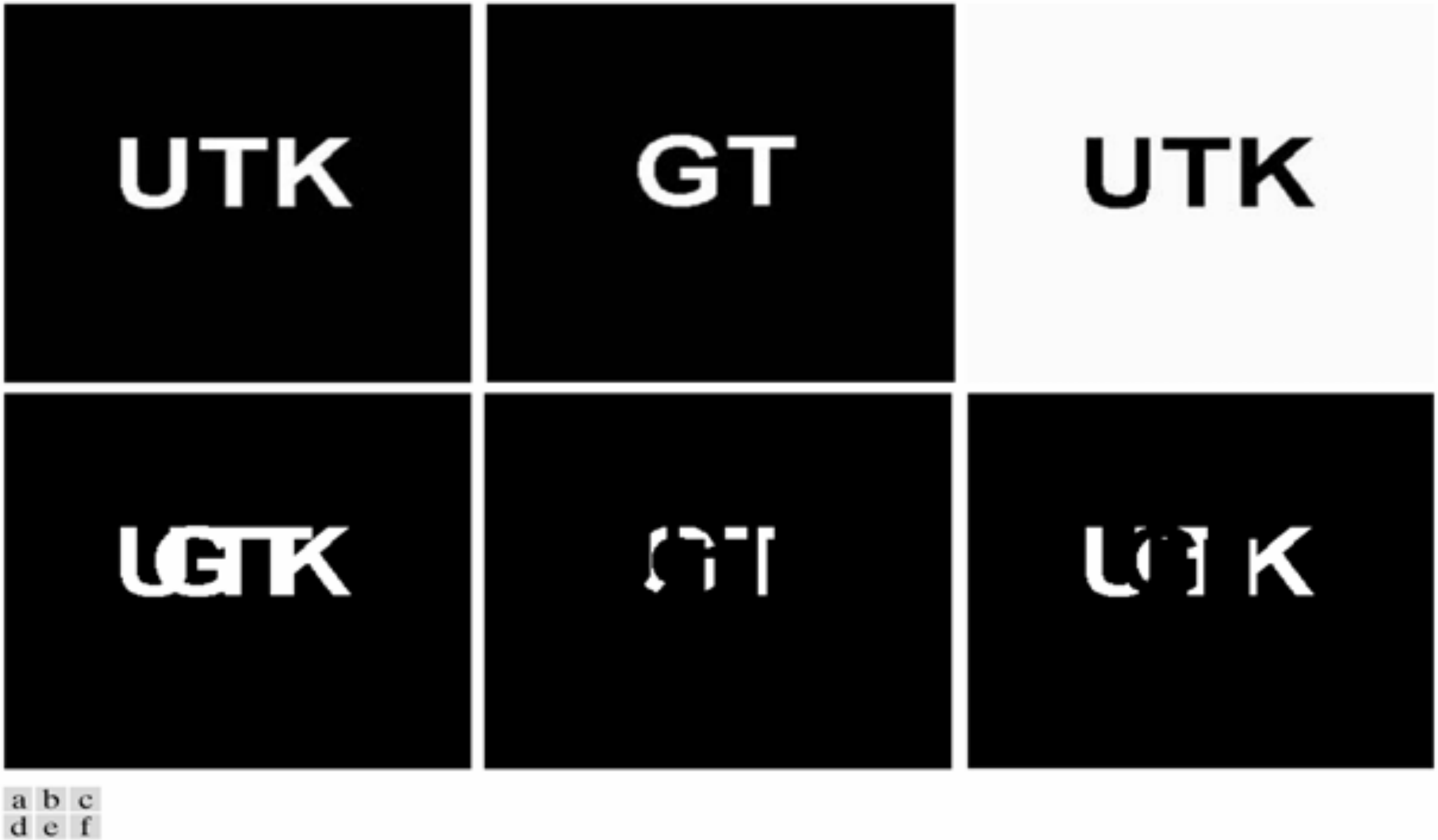
Morphology



Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

inputs are differ

Morphology



(a) Binary image A. (b) Binary image B. (c) Complement ~A. (d) Union A | B. (e) Intersection A & B. (f) Set difference $A - B$

Morphology – Operators

- Primary morphological operations are Dilation and Erosion
- More complicated morphological operators such as Opening and Closing can be designed by combining erosions and dilations
- Opening generally **smoothes the contour** of an image and **eliminates protrusions**
- Closing **smoothes sections of contours**, but it generally **fuses** breaks, holes and gaps

Morphology – Dilation

- **Dilation** of A by B, denoted by $A \oplus B$, is defined as:

$$A \oplus B = \left\{ z \,\middle|\, \left[\left(\hat{B} \right)_z \cap A \right] \neq \varnothing \right\}$$

- **Interpretation:**

  Obtaining the reflection of B about its origin and then shifting by z.

  Dilation of A by B is the set of all z displacements such that $\hat{B}$ and A overlap by at least one nonzero element.

- B is called the **structuring element** in Dilation.

Morphology – Dilation

- Dilation of A by B can also be expressed as:

$$A \oplus B = \{ z \mid \left[\left(\hat{B} \right)_z \cap A \right] \subseteq A \}$$

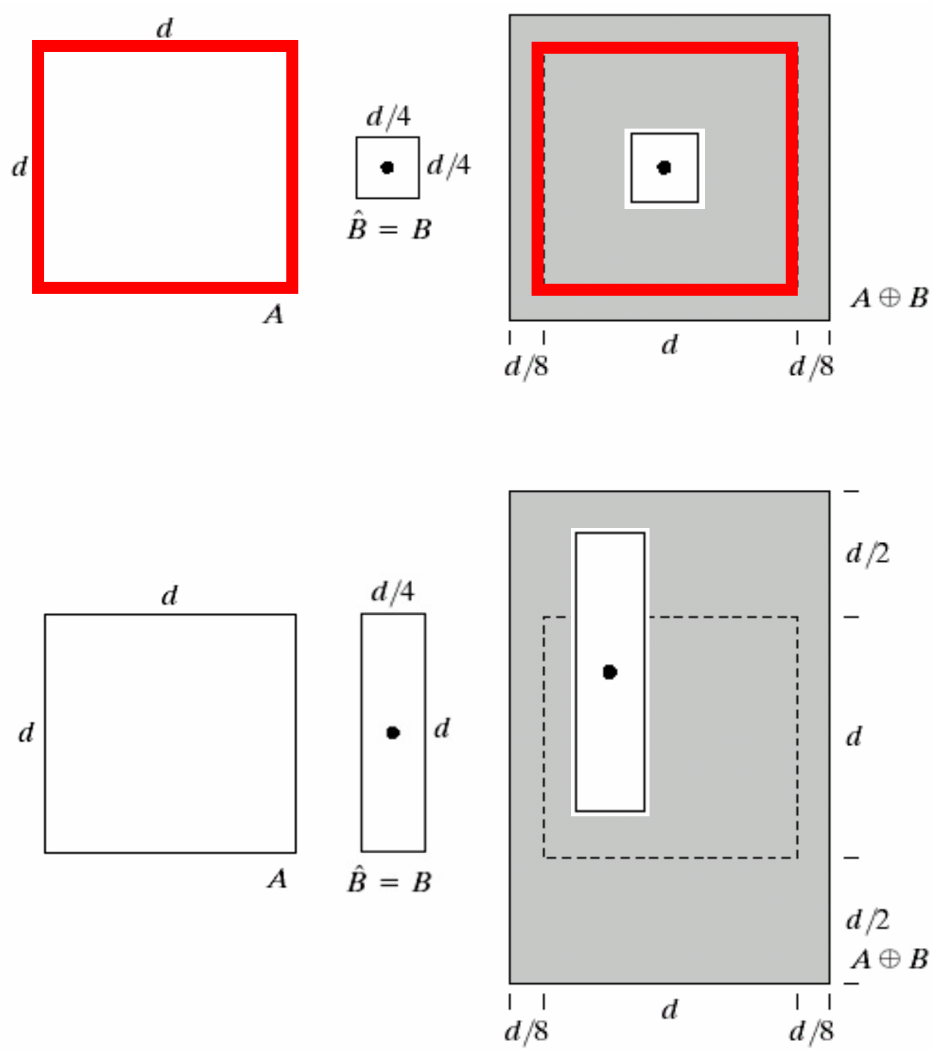
- Further Interpretation:
  Set B can be viewed as a convolution mask.

  The process of "flipping" B and then successively displace it so that it slides over set (image) A is analogous to the convolution.

Morphology – Dilation

a b c
d e

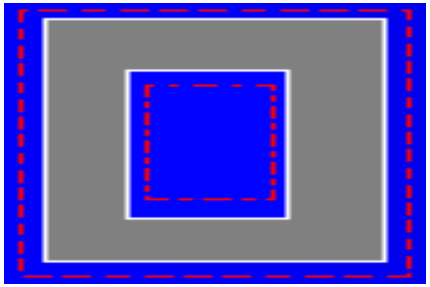
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.



Morphology – Dilation

- The dilation morphological operation generates an output image g from an input image f using a structuring element h:

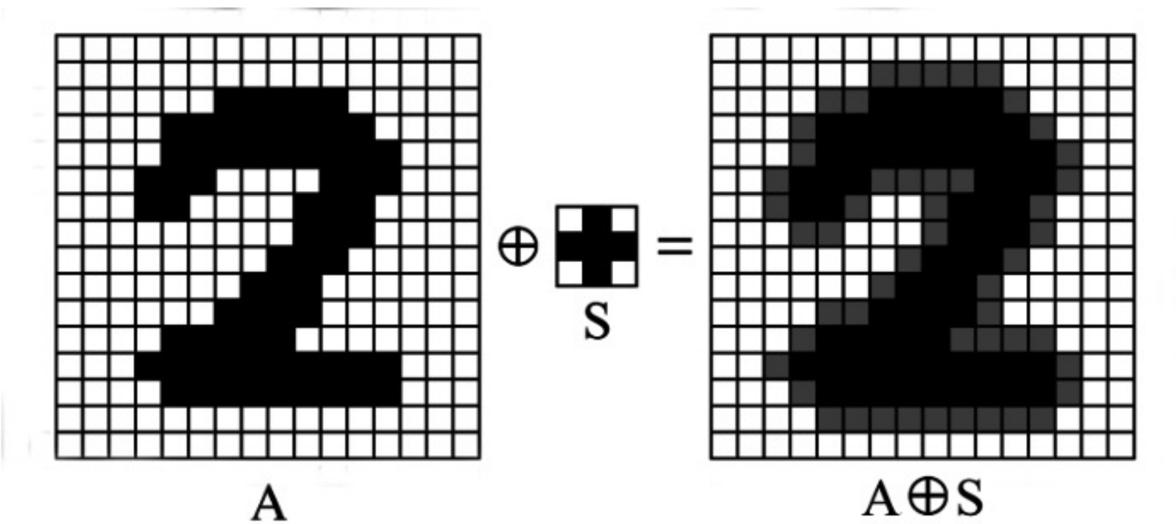
$$g(x,y) = \begin{cases} 1, & \text{if } h \text{ hints } f \\ 0, & \text{else} \end{cases}$$

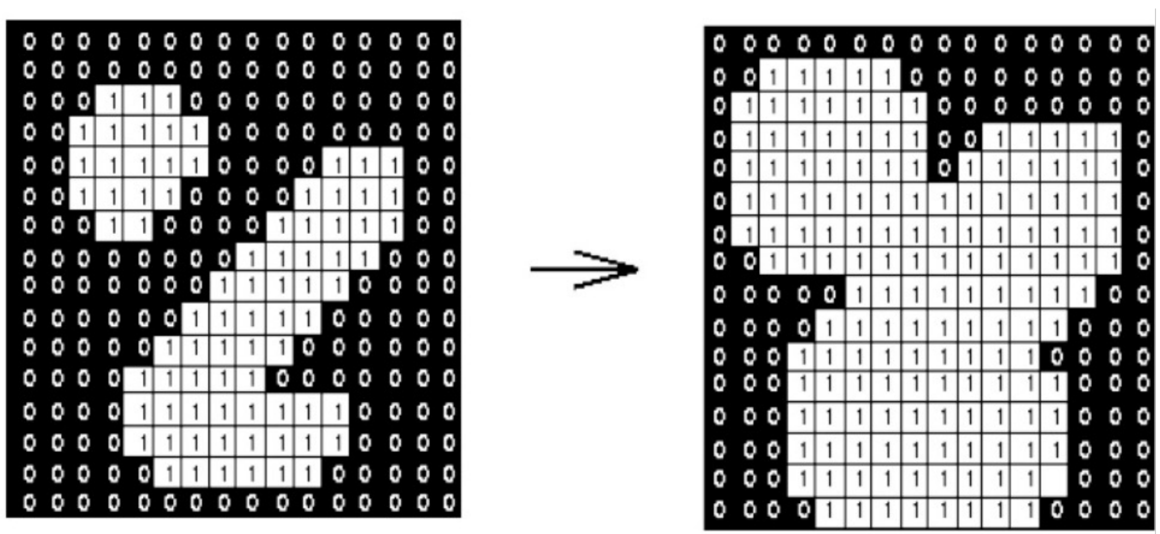


- The effect of dilation with 3 x 3 mask is to add a single layer of pixels to the outer edge of an object and to decrease by a single layer of pixels to the holes in the object.
- A 5 x 5 mask will add two layers of pixels which is equivalent to applying a 3 x 3 mask twice.

- The main application of dilation is to remove small holes from the interior of an object.

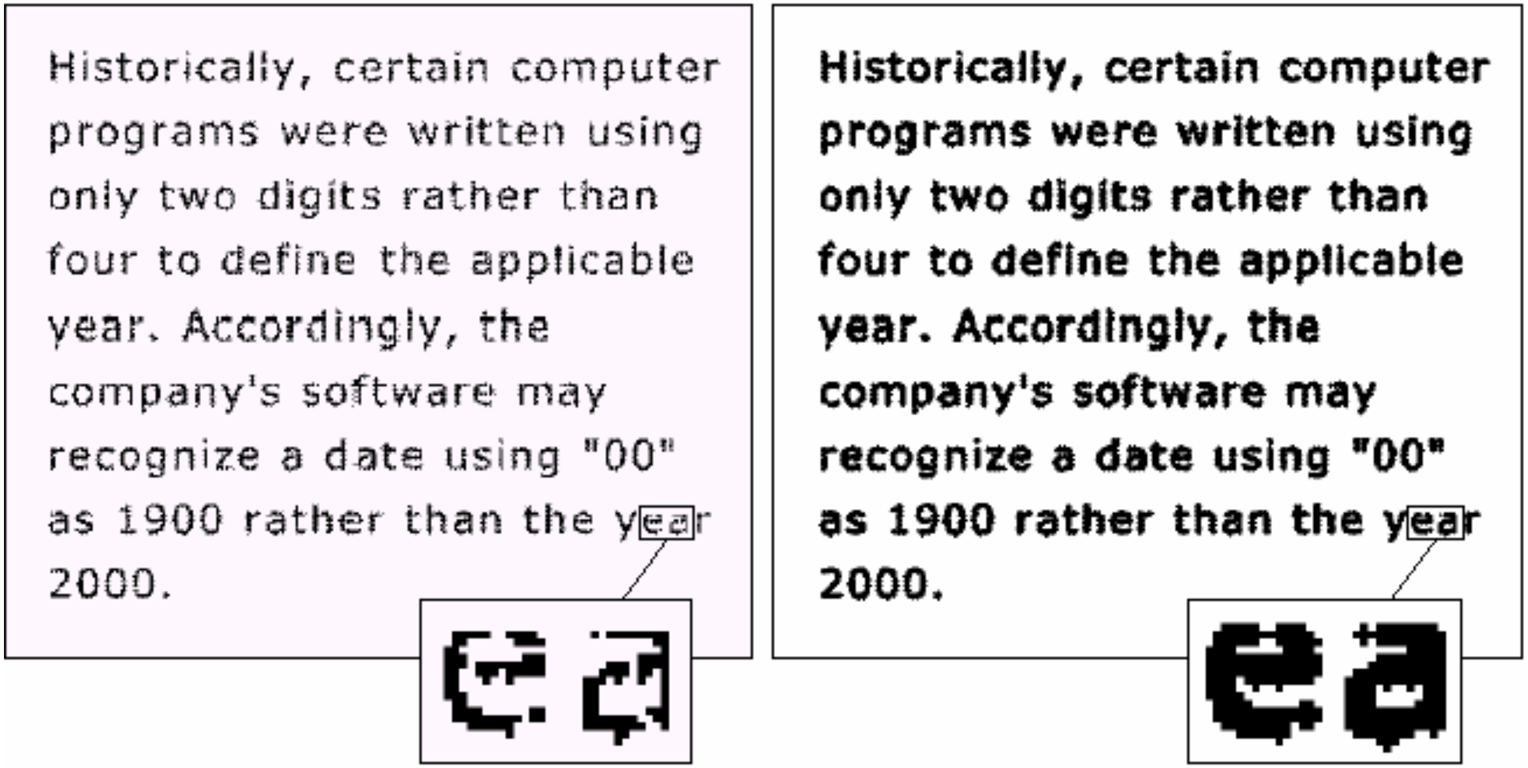
Dilation - Example





Effect of dilation using a 3×3 square structuring element

Dilation - Application



0	1	0
1	1	1
0	1	0

a c
b

(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Morphology - Erosion

- Erosion of A by B, denoted $A \ominus B$, is defined as:

$$A \ominus B = \left\{ z \middle| (B)_z \subseteq A \right\}$$

```
[[0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 1, 1, 1, 1, 0, 0],
 [0, 0, 1, 1, 1, 1, 0, 0],
 [0, 0, 1, 1, 1, 1, 0, 0],
 [0, 0, 1, 1, 1, 1, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0]]
```

Original

$\ominus$

```
[[1, 1, 1],
 [1, 1, 1],
 [1, 1, 1]]
```

Structuring Element

$\rightarrow$

```
[[0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 1, 1, 0, 0, 0],
 [0, 0, 0, 1, 1, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0],
 [0, 0, 0, 0, 0, 0, 0, 0]]
```

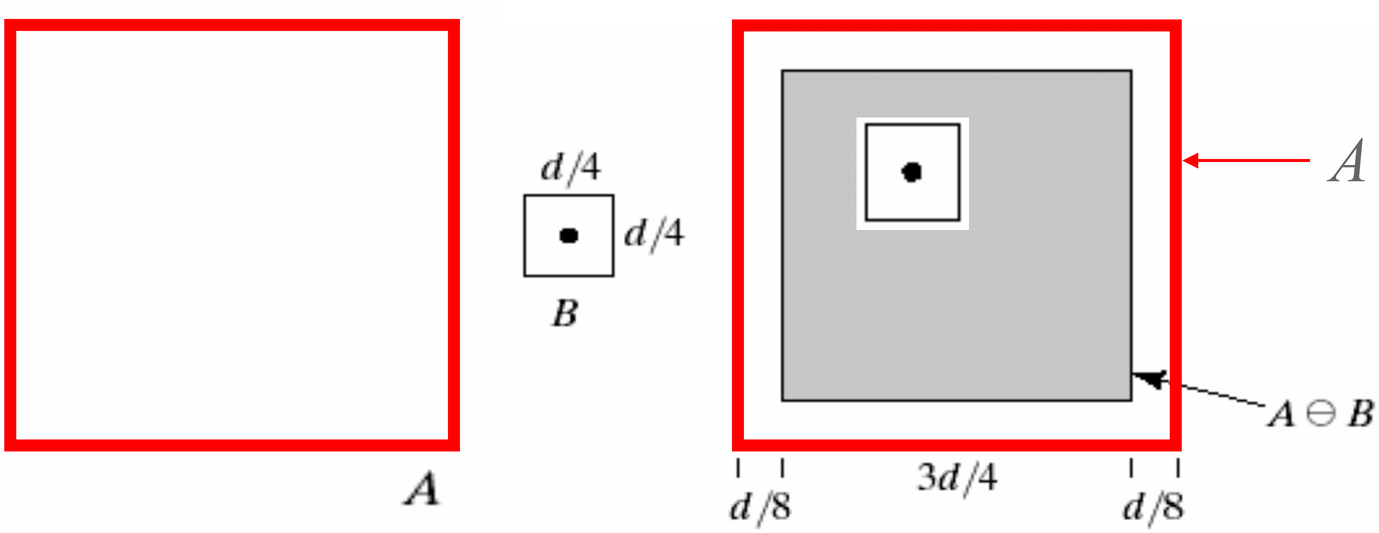
Eroded

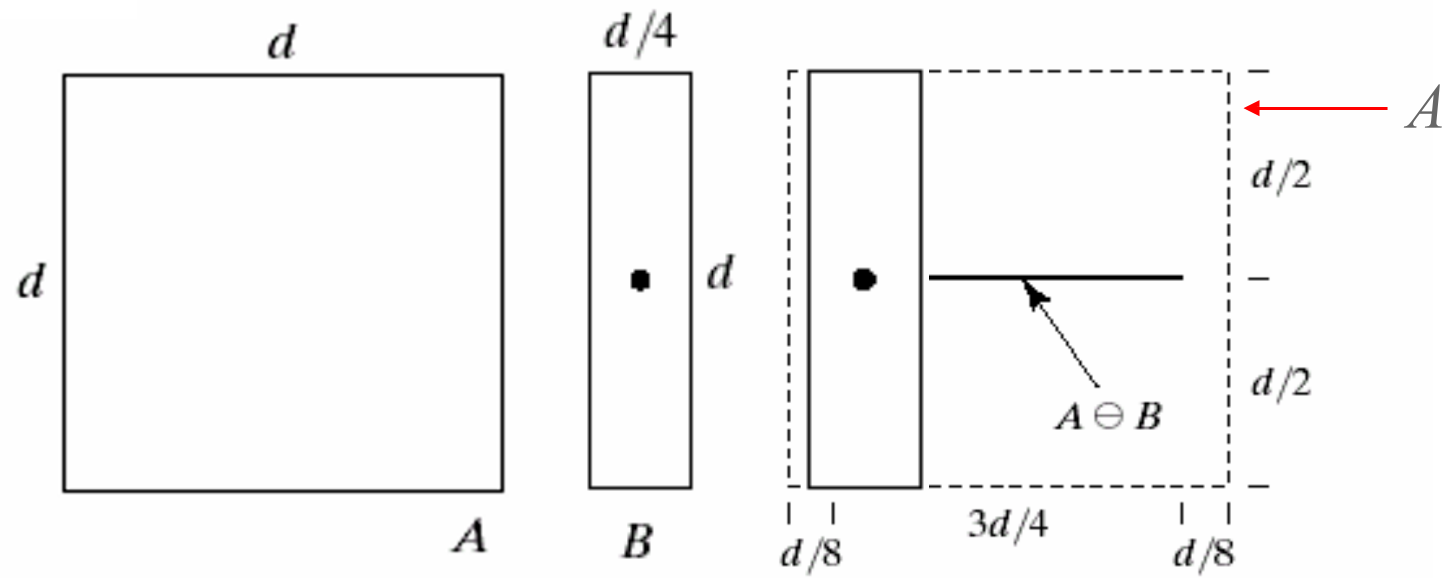
- Erosion of A by B is the set of all points z such that B, translated by z, is contained in A.

- Dilation and erosion are duals of each other with respect to set complementation and reflection. That is,

$$\left(A \ominus B \right)^c = A^c \oplus \hat{B}$$

Erosion - Example

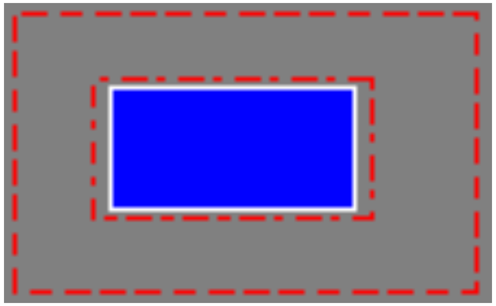




Erosion

- The erosion operation generates an output g from an input f using a structuring element h where :

$$g(x, y) = \begin{cases} 1, & \text{if } h \text{ completly falls in } f \\ 0, & \text{else} \end{cases}$$

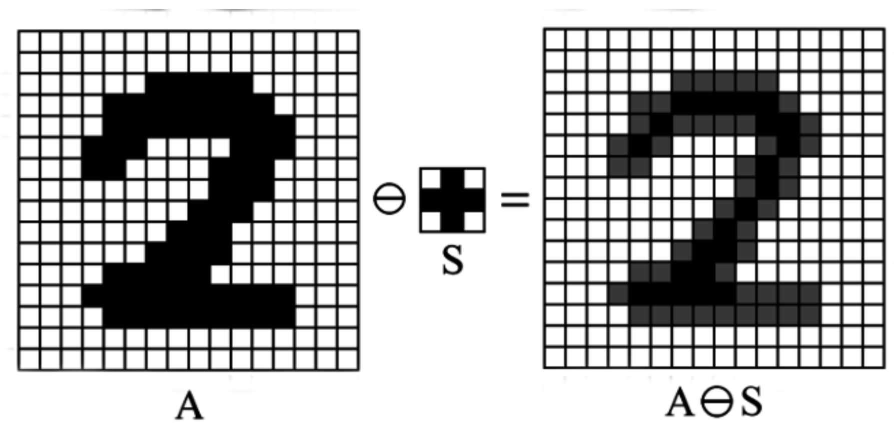


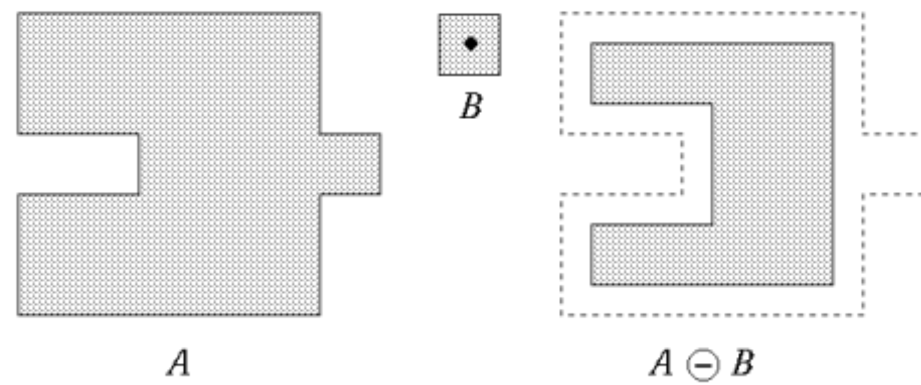
- The effect of an erosion with 3 x 3 mask is to strip a single layer of pixels from the outer edge of an object and to increase by a single layer of pixels to holes in the object.

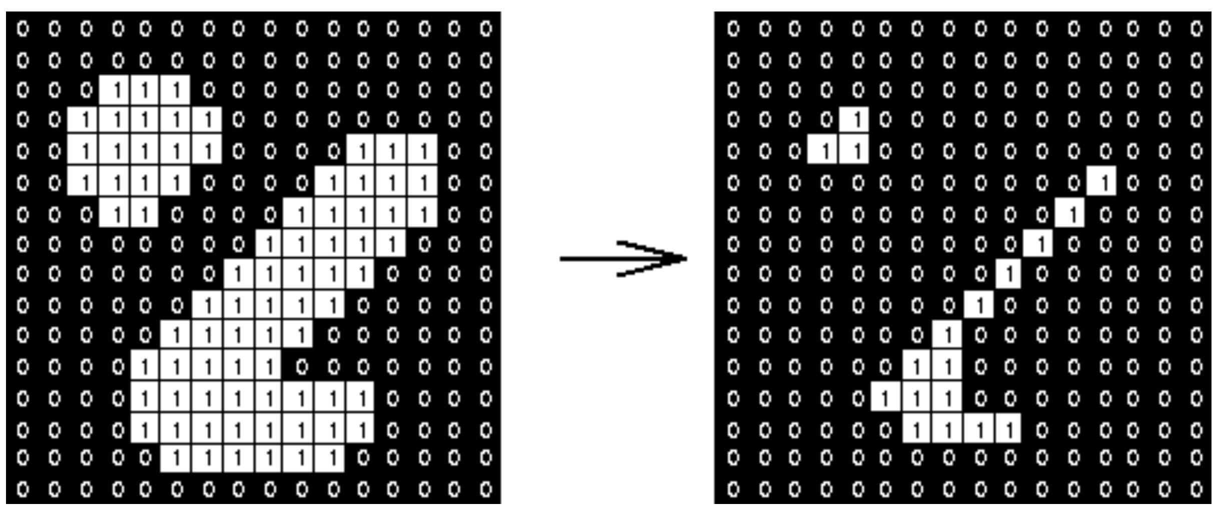
- A 5 x 5 mask will strip off two layers of pixels which is equivalent to applying a 3 x 3 mask twice.

- The main application of erosion is to remove small noise artifacts from an image.

Erosion - Example

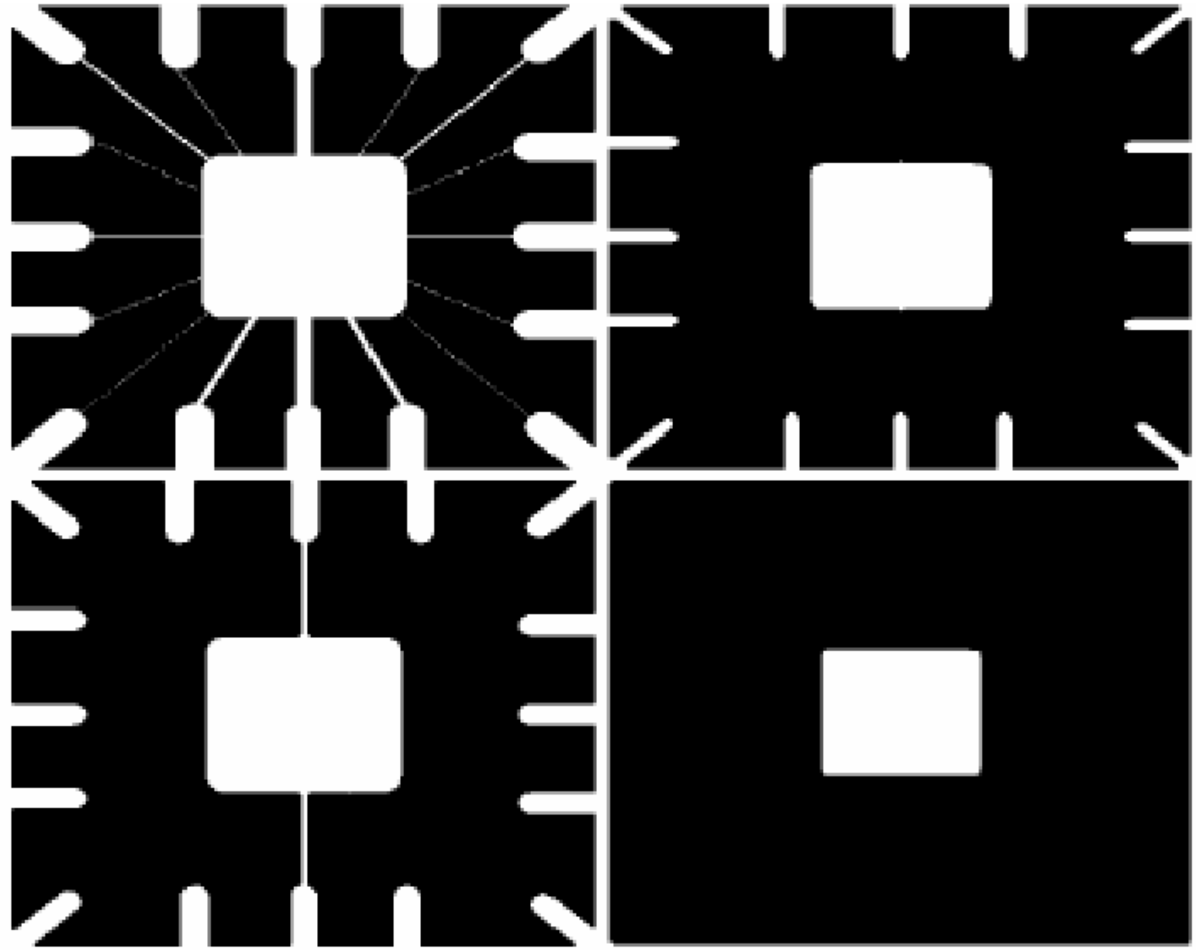






Effect of erosion using a 3×3 square structuring element

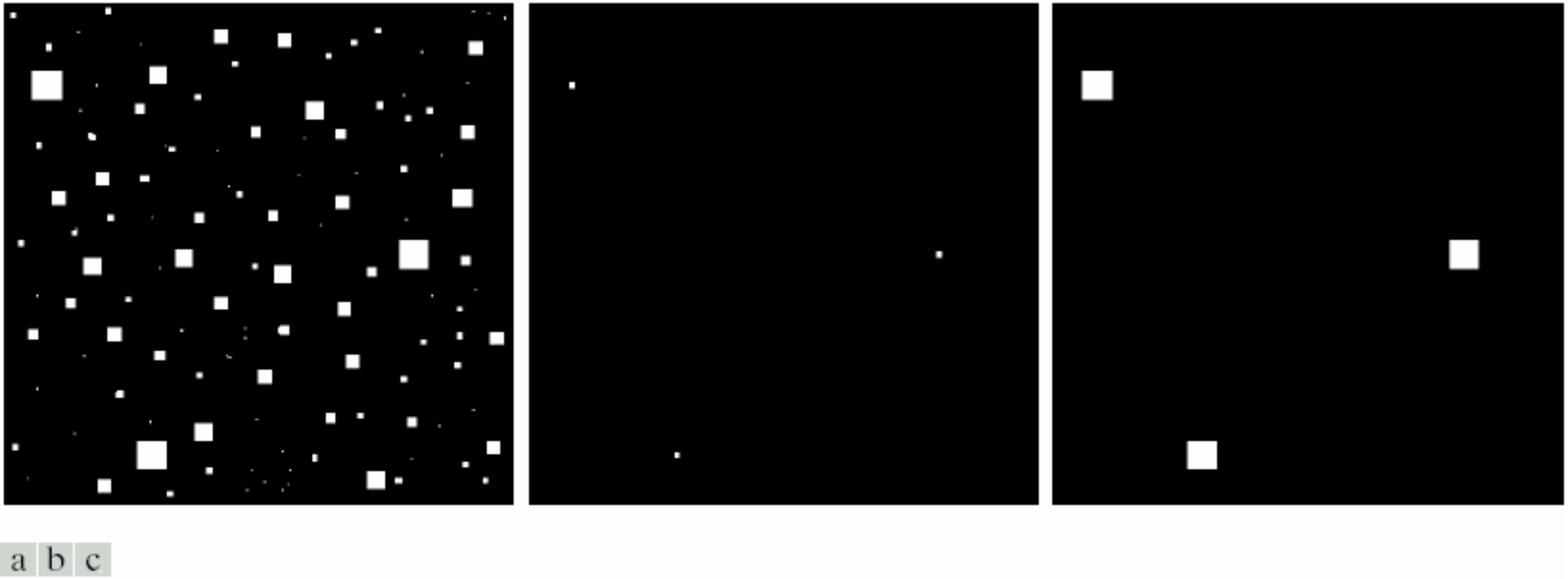
Morphology – Erosion



a b
c d

An illustration of erosion.
(a) Original image.
(b) Erosion with a disk of radius 10.
(c) Erosion with a disk of radius 5.
(d) Erosion with a disk of radius 20.

Erosion then Dilation



(a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Morphology - Opening

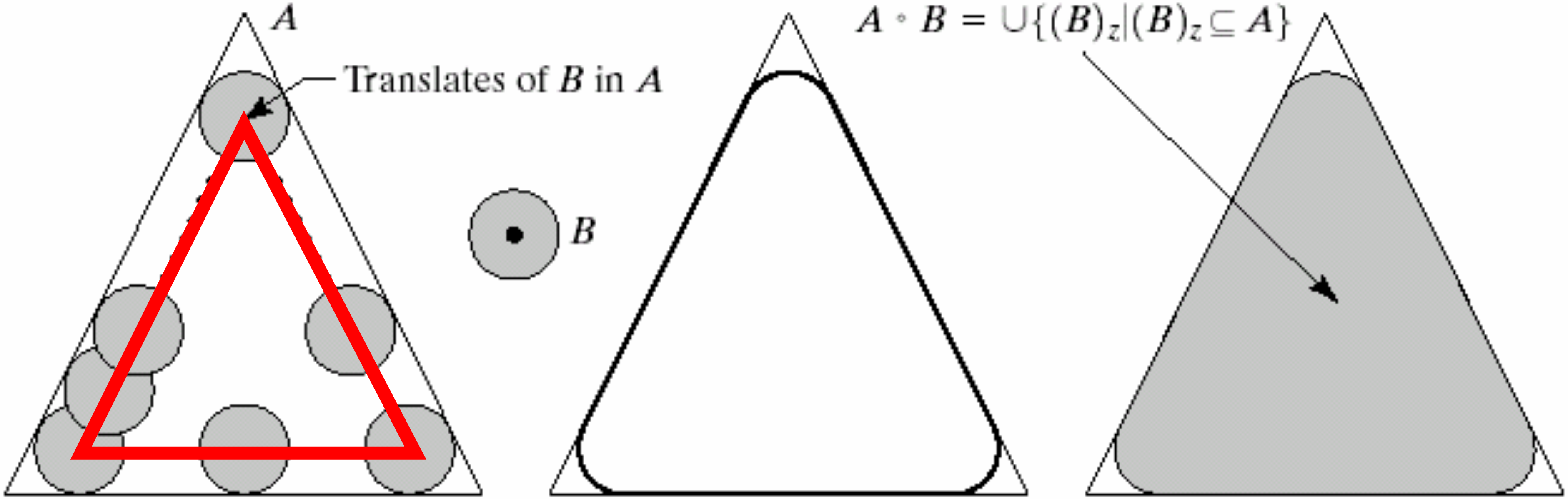
- Compound operations – Opening
- A **compound** operation is when **two or more** morphological operations are performed in **succession**.
- A common example is opening which is an erosion followed by a dilation:

$$A \circ B = (A \ominus B) \oplus B$$

> The opening A by B is obtained by taking the union of all translates of B that fit into A. This can be expressed as a fitting processing such that:

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

Morphology – Opening

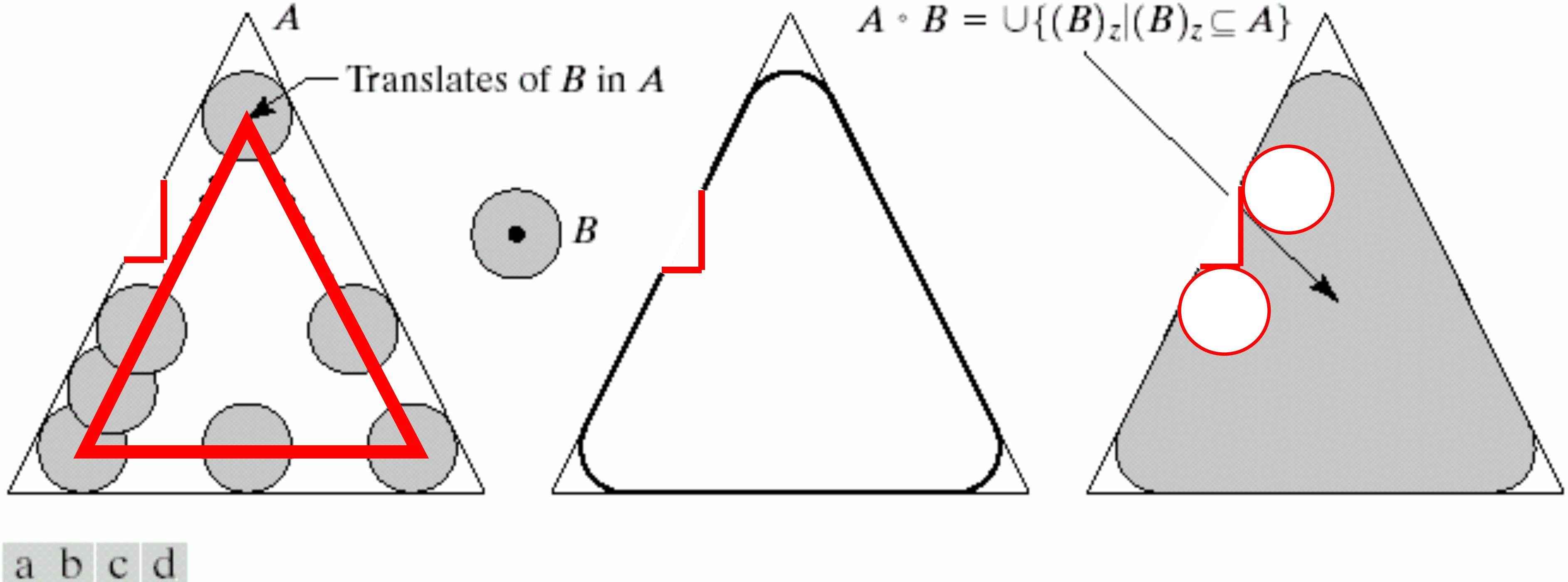


a b c d

(a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

- Note that the outward pointing corners are rounded, where the inward pointing corners remain unchanged.

Morphology – Opening



(a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

- Note that the outward pointing corners are rounded, where the inward pointing corners remain unchanged.

Morphology – Opening

$$A \circ B = (A \ominus B) \oplus B \qquad A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

- Opening is often performed to clear an image of noise whilst retaining the original object size.
- The opening operation tends to flatten the sharp peninsular projections on the object.
- Care must be taken that the operation does not distort the shape size of the object if this is significant.
- A useful way to see the effects is to look for differences between before and after opening by projecting these differences onto the original image.

Morphology – Closing

- Compound operations – Closing

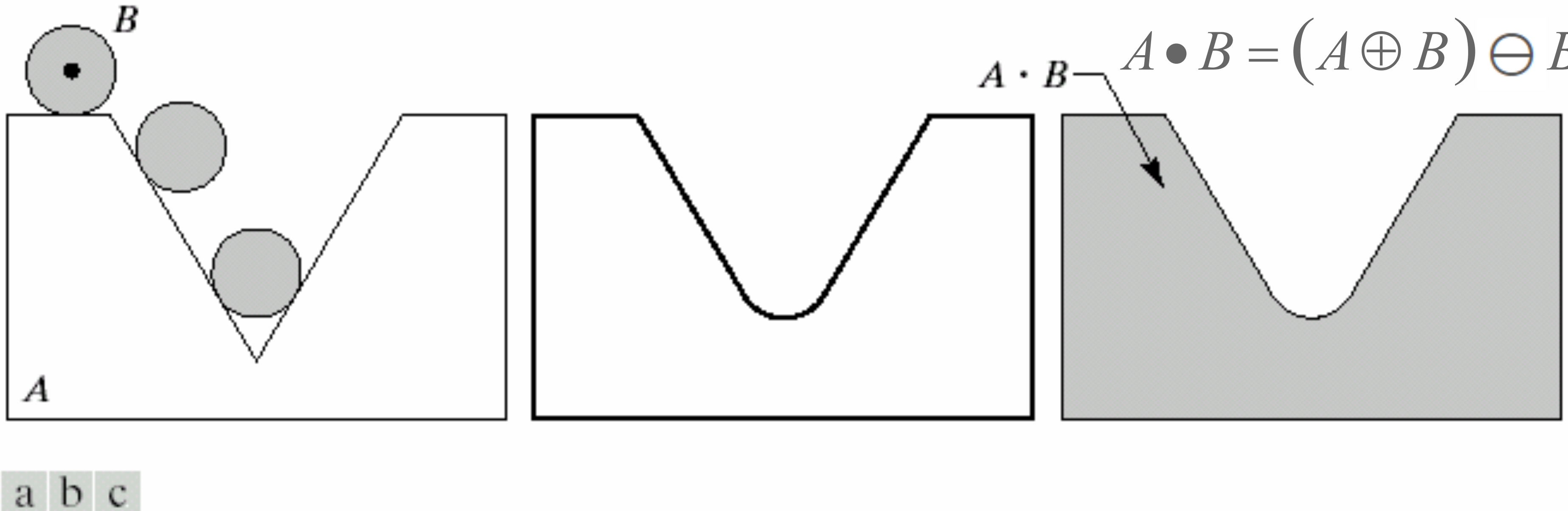
- Closing is the complementary operation of opening, defined as dilation followed by erosion.

$$A \bullet B = \left(A \oplus B \right) \ominus B$$

➢ Opening and closing are duals of each other as:

$$\left(A \bullet B \right)^c = A^c \circ \hat{B}$$

Morphology –Closing



$$A \bullet B = (A \oplus B) \ominus B$$

(a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

- Note that the inward pointing corners are rounded, where the outward pointing corners remain unchanged.

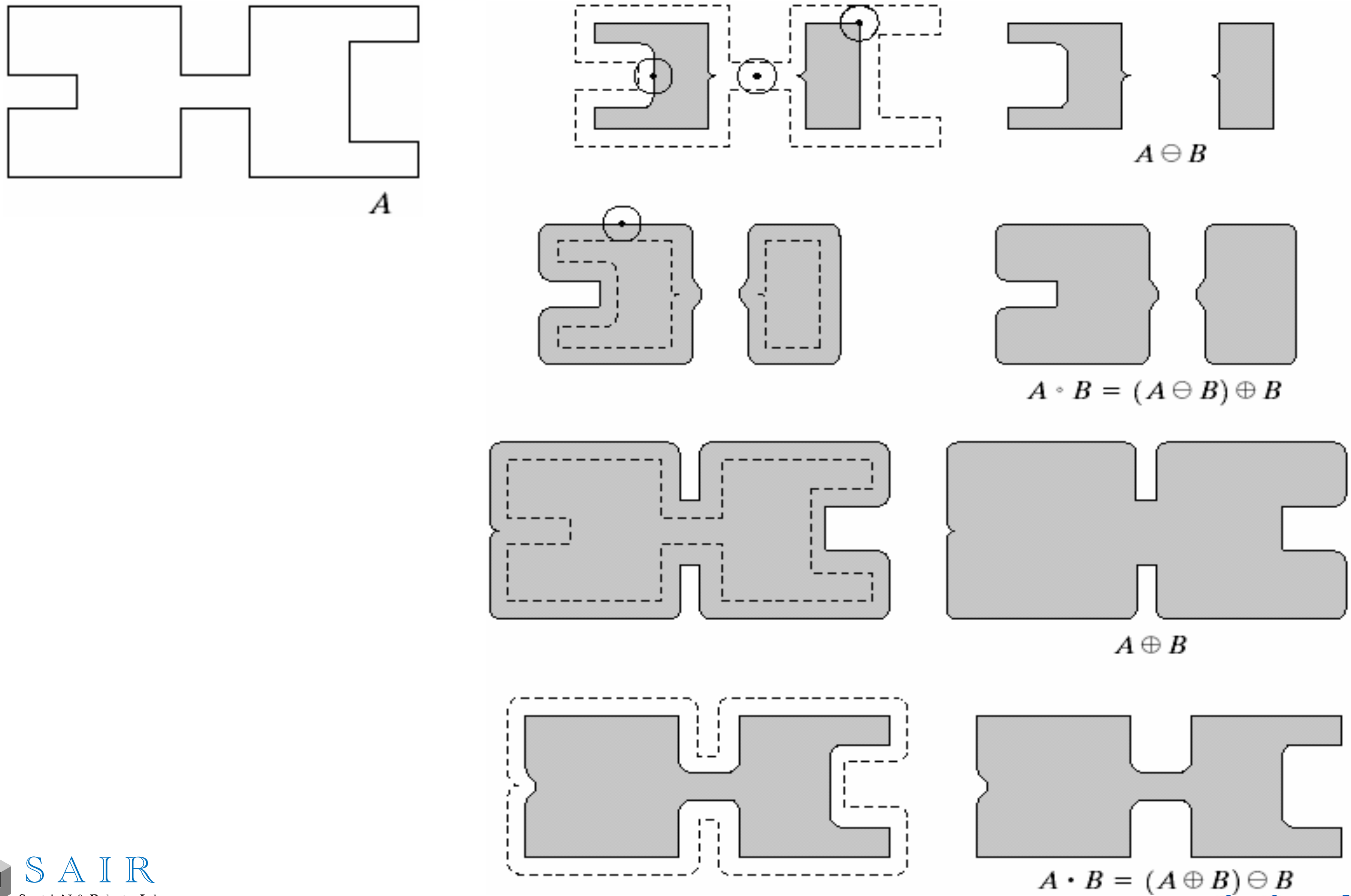
Morphology –Closing

- The classic application of closing is to fill holes in a region whilst retaining the original object size.

- Dilation fills the holes and erosion restores the original region size.

- In addition to filling holes the closing operation tends to fill the 'bays (凹)' on the edge of a region.

Erosion, Opening, Dilation, Closing



Morphology –Opening and Closing

- The opening operation satisfies the properties:
  - $A \circ B$ is a subset (subimage) of A
  - If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  - $(A \circ B) \circ B = A \circ B$
- The closing operation satisfies the properties:
  - A is a subset (sub image) of $A \bullet B$
  - If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
  - $(A \bullet B) \bullet B = A \bullet B$

Algorithms and Applications

- Morphology can be used for many applications in image processing, pattern recognition, computer vision.
    - Boundary Extraction
    - Region Filling
    - Connected Components Extraction
    - Denoising

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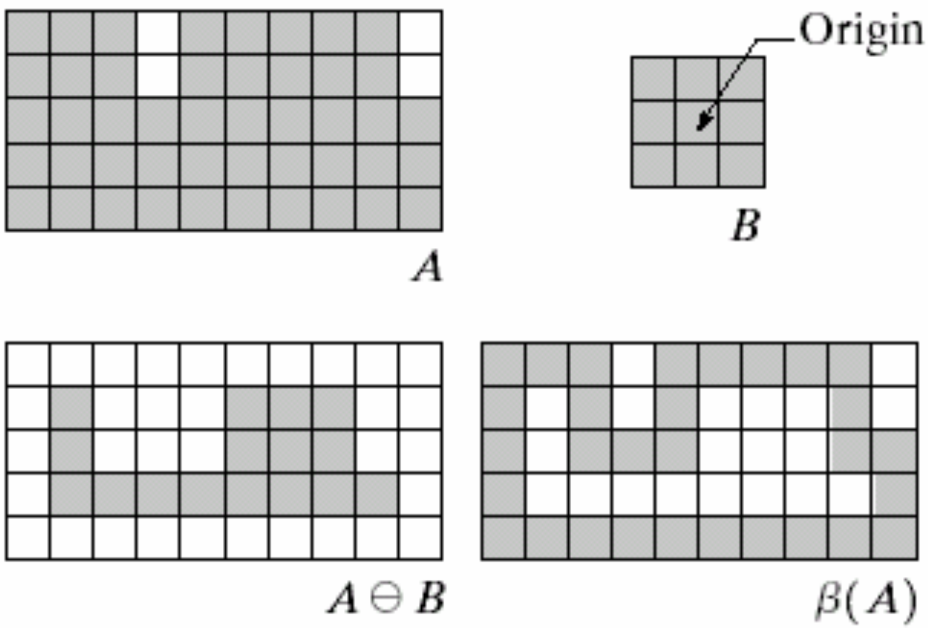
Boundary Extraction

- The boundary of a set A, denoted by $\beta(A)$:

$$\beta(A) = A - (A \ominus B)$$



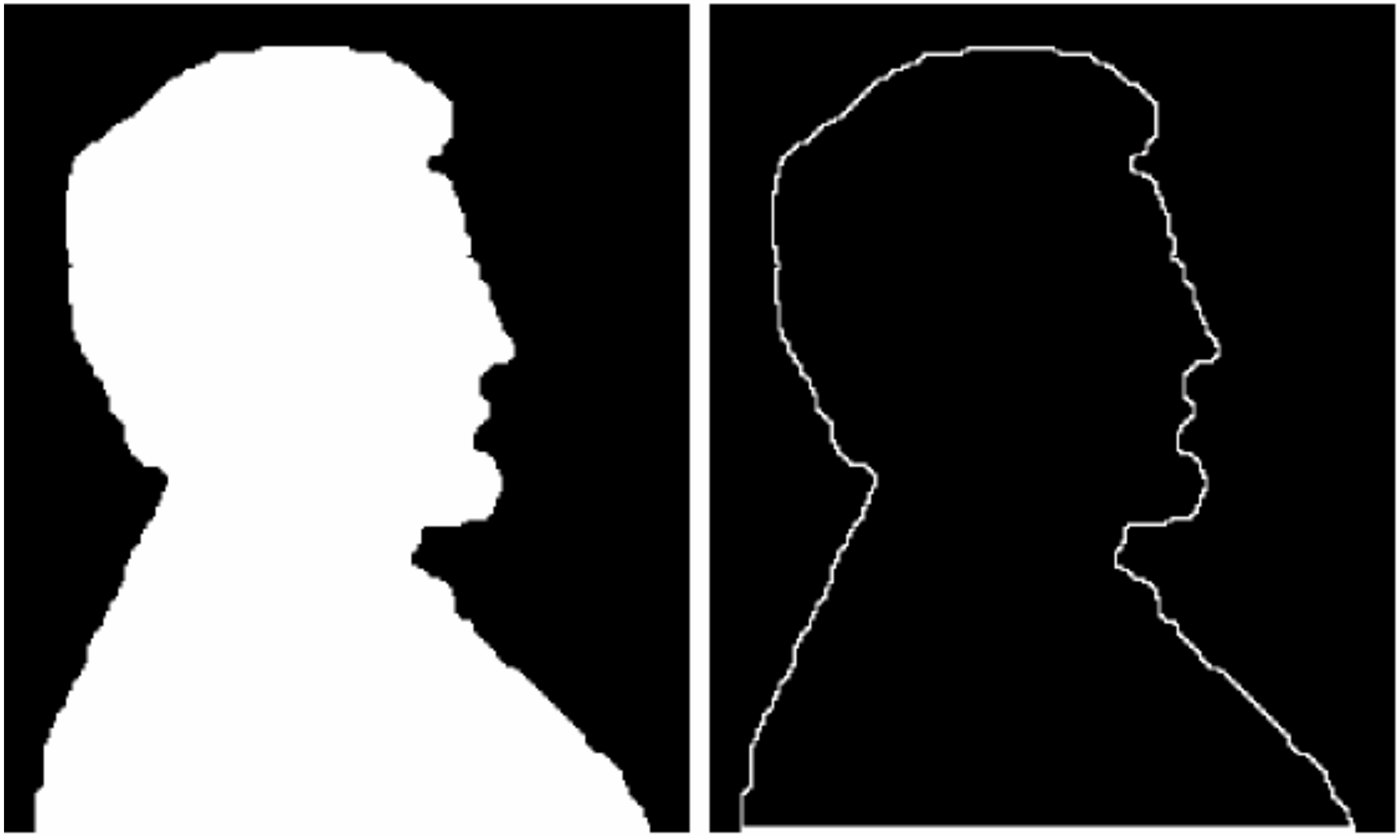
a	b
c	d

(a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

Boundary Extraction - Example

a b

(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

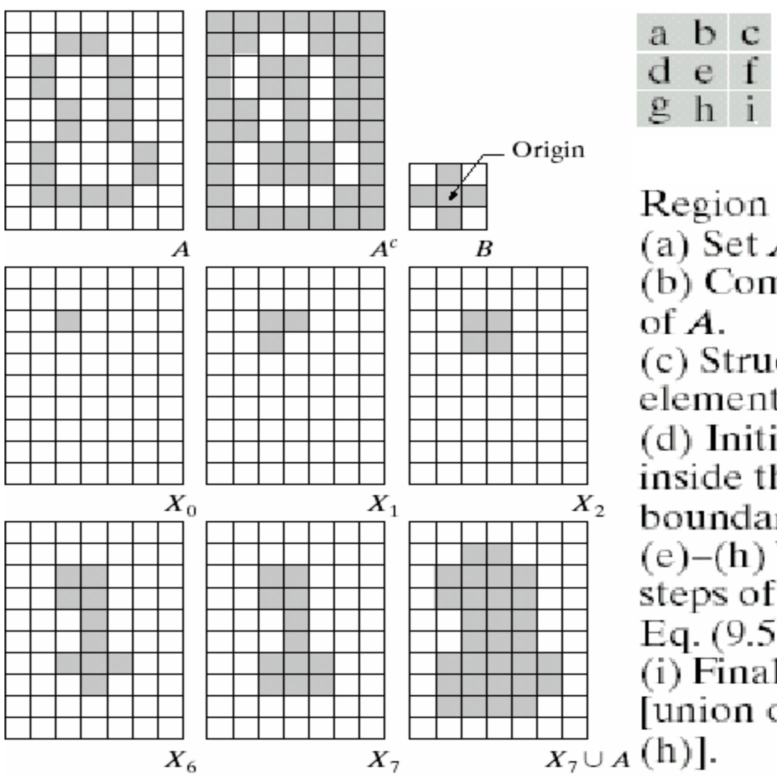


Region Filling

- Beginning with a point X_0 inside the boundary, the entire region inside the boundary is filled by the procedure:

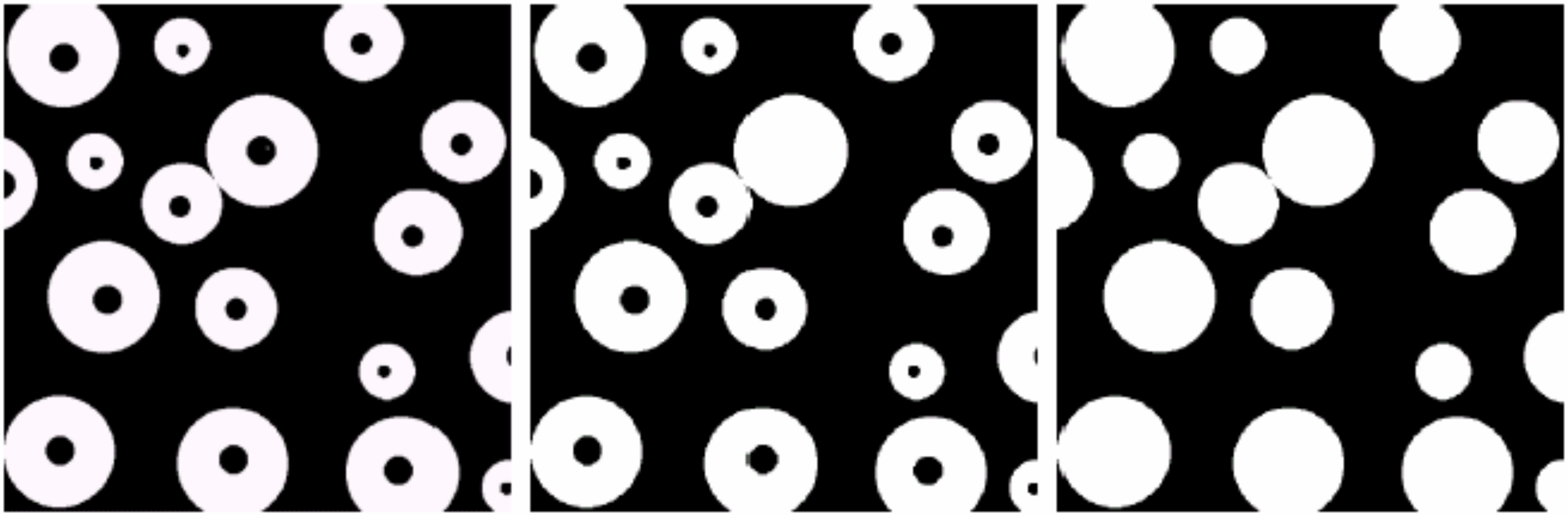
$$A^F = X_k \cup A,$$

$$X_k = (X_{k-1} \oplus B) \cap A^c, \qquad k = 1, 2, 3 \ldots$$



Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Region Filling - Example

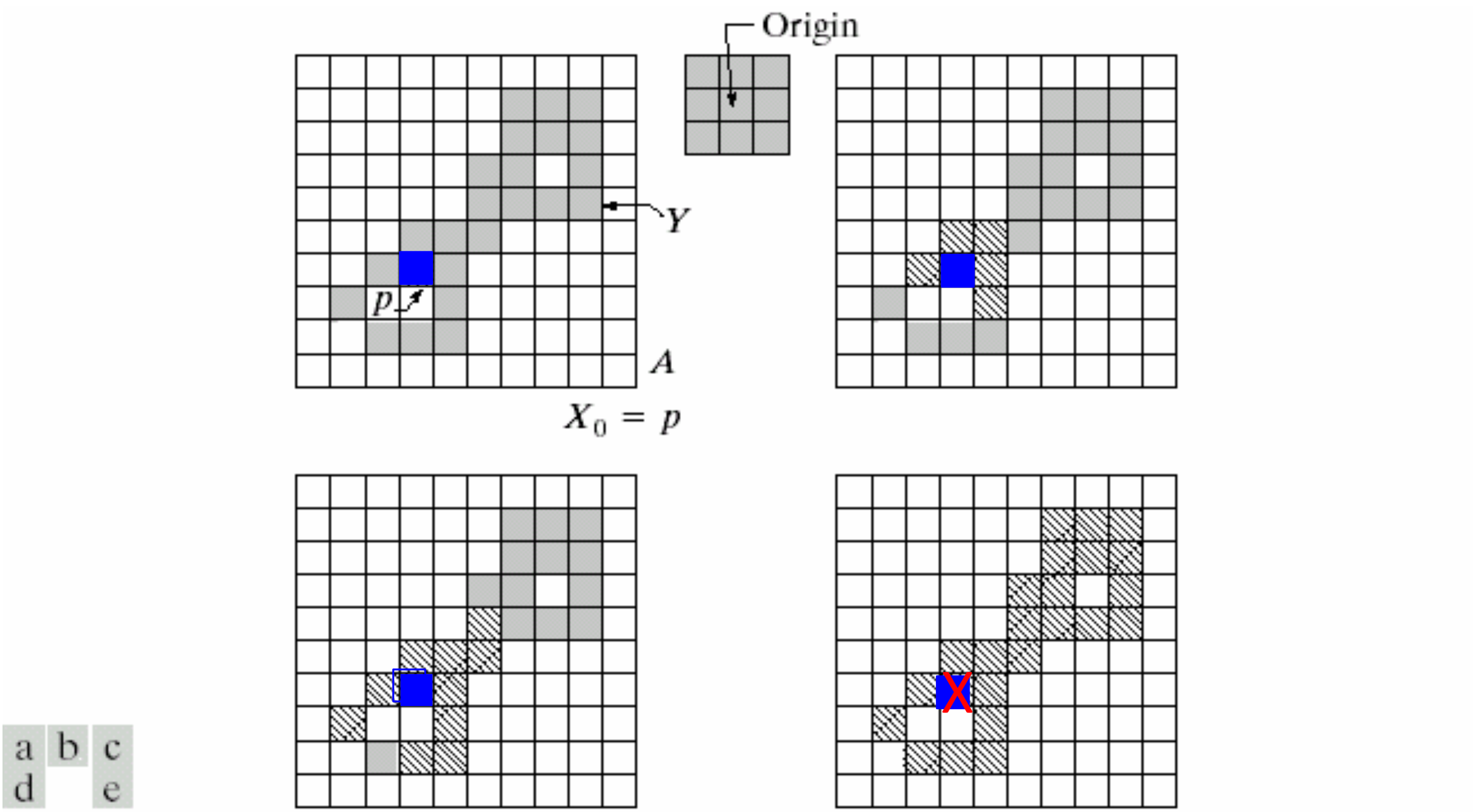


a b c

(a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extract connected components

- $X_k = (X_{k-1} \oplus B) \cap A, k = 1, 2, 3, \ldots$



(a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

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Denoising

- Closing and Opening can be used to **eliminate noise**.

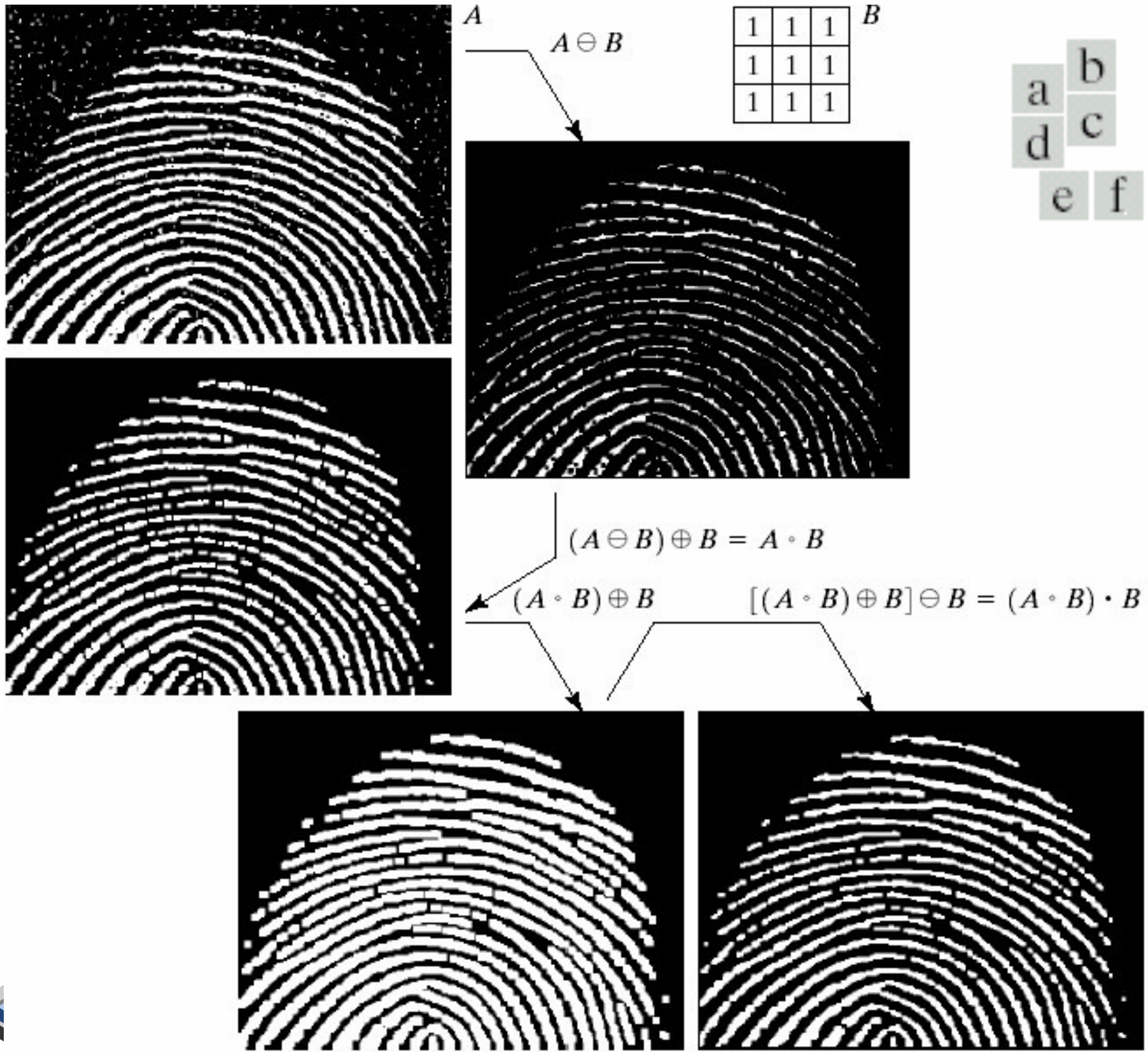
$$(A \circ B) \bullet B$$

or

$$(A \bullet B) \circ B$$

- Noise **outside** the object are removed by **opening** with B
- Noise **inside** the object are removed by **closing** with B.

Algorithms and Applications



(a) Noisy image.
(c) Eroded image.
(d) Opening of *A*.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Morphology –Summary

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)

Morphology –Summary

Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)