2017

FINAL EXAM

ADVANCE ANALYTICS AND METAHEURISTICS

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Question 1: Breakfast of Champions (15 points)

Formulate this problem (generally) as a mathematical programming problem. Make sure that you identify and label all decision variables, parameters, and sets; write the objective function; write and label all constraints.

Objective is to minimize cost while satisfying demand and supply constraints.

Let

S_i *be a set where* $i = [1,2,3 \ldots .n]$ *be set up cost for each supplier* i.

VC_i *be a set where* $i = [1,2,3 \ldots .n]$ *be variable cost per* 100 *boxes for supplier* i

Mq_i *be a set where* $i = [1,2,3 \ldots .n]$ *be maximum quantity available from each supplier* i.

Decision Variable:

X_i *be a set where* $i = [1,2,3 \ldots ..n]$ *be no of boxes selected from each supplier* i.

Binary Variable:

Let y_{ji} *be the binary variable for set* $j = [0,1]$ *and* $i = [1,2,3 \ldots \ldots .n]$ *here*

$$j = o \; if \; x_i = 0$$

$$j = 1 \; if \; x_i \geq 1$$

and i *is the set for suppliers* 1 *through* n

Objective function:

$$\boldsymbol{Min}\ (\sum_{i=1}^{n} \boldsymbol{S_i * y_{ij}} + \sum_{i=1}^{n} \boldsymbol{X_i * \frac{VC_i}{100}})$$

Constraints:

$\boldsymbol{X_i \leq Mq_i * y_{ij}\ for\ all\ i = 1\ to\ n}$ **Supply constraint.**

$\boldsymbol{\sum_{i=1}^{n} X_i \geq 25000}$ **Demand constraint.**

$\boldsymbol{X_i \geq 0\ non\ negative\ constraint}$.

Question 2: Portfolio Optimization (20 points)

Aim is to minimize risk and variance of the portfolio.

Let's take a set i = [1,2,3] where the set I represents i = [L,D,W]

Parameters:

Here I am considering everything in percentage of stock

Co_{ij} *is the covariance of the two Stocks i and j*

V_i *is the variance of stock i,* M_i *is the expected mean return of investment*

Decision variables:

$$X_i\ is\ the\ percentage\ of\ stock\ invested\ in\ the\ portfolio$$

Objective Function:

$$minimize\ risk\ \sigma^2(portfolio) = \sum_{i=1}^{n} X_i^2 V_i + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} X_i X_j Co_{ij}$$

Constraints:

$$\sum_{i=1}^{3} X_i = 100$$

$$\sum_{i=1}^{3} X_i M_i \geq 12.5$$

$X_i \geq 0 \; and \; X_i \leq 100$

(b)

i)

We can consider **Swap neighborhood** for this problem. i.e., if we generate an initial solution we will use this neighborhood to swap the positions of variables in our solution. This can be done in many ways for example in this 3-dimensional problem we can swap positions for 1 and 2 elements or 2 and 3 or 3 and 1 to form a new solution and then we can evaluate that solution.

ii)

given solution (30%,30%,40%) so, using the above neighborhood definition we shall swap 1 and 4 elements so our new solution is (40%,30%,30%)

when we evaluate this solution:

$$\begin{aligned} var^2 = 0.16 * 0.04 + 0.09 * 0.0225 + 0.09 * 0.0064 + 2 * 0.4 * 0.3 \\ * 0.018 + 2 * 0.4 * 0.3 * 0.0064 + 2 * 0.3 * 0.3 * 0.0084 \\ = 0.016369 \end{aligned}$$

$$std = \sqrt{0.016369} = 0.127$$

That is expected return of 12.7%.

Question 3: Power Generation (30 points)

A)

This is formulated as minimum cost network flow problem (MCNF)

Decision variables:

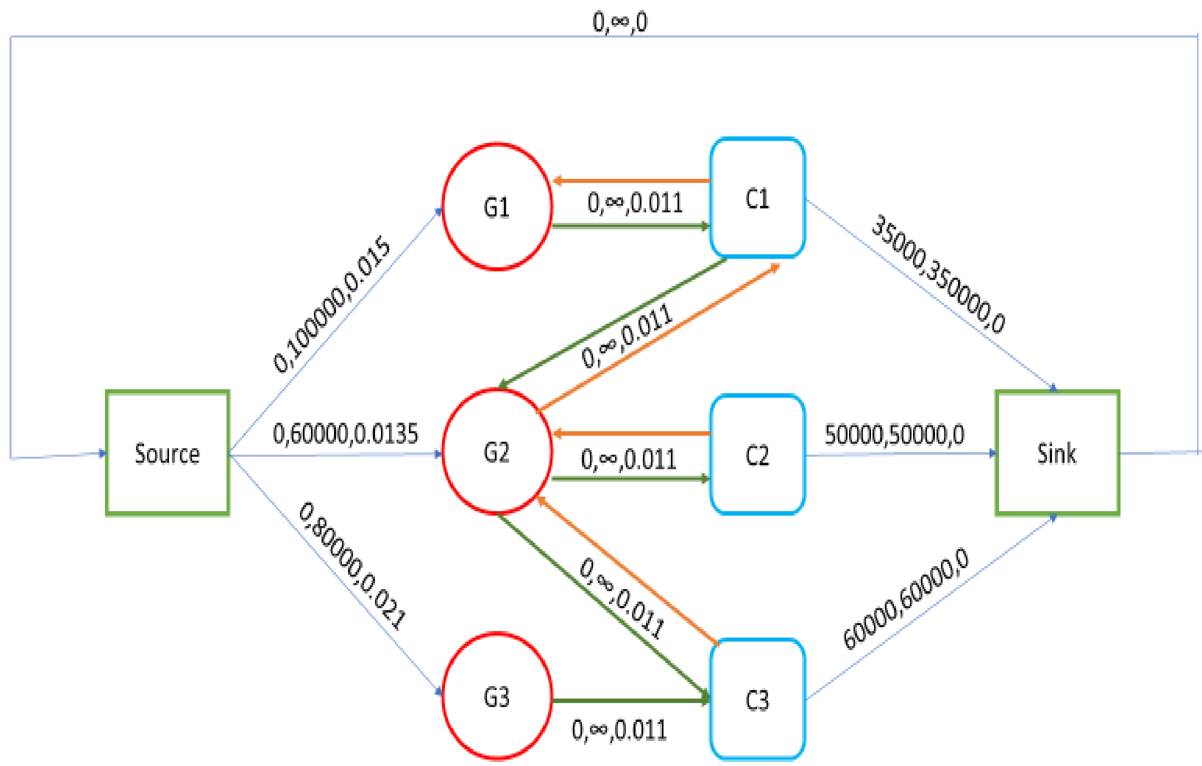
Amount of current flow through Arcs.

Objective:

Minimize the cost of flow through network while satisfying demand.

$$\sum_{i=1}^{m} c_{ij}x_{ij}$$

Network Flow diagram:



ARC Values in order (Lower Bound, Upper Bound, cost)

In the above diagram the green arc represents flow from source node to destination node.

The red arc represents flow from Destination node to source node.

Constraints:

- Maximum supply of current at generator G1 = 100000, G2 = 60000 and G3 = 80000.
- Demand of current at Consumption point C1 = 35000, C2 = 50000 and C3 = 60000.
- Non-negative constraints.

(b).

Optimal Solution:

Objective: 3980$

Generator	Amount Generated
G1	35000
G2	60000
G3	50000

Flow From	Flow To	Amount
G1	C1	35000
G2	C1	0
G2	C2	50000
G2	C3	10000
G3	C3	50000

Code: model

```
# AMPL model for the Minimum Cost Network Flow Problem
#
# By default, this model assumes that b[i] = 0, c[i,j] = 0,
# l[i,j] = 0 and u[i,j] = Infinity.
#
# Parameters not specified in the data file will get their default values.

# model file for problem 3
reset;
options solver cplex;

set NODES;                          # nodes in the network
set ARCS within {NODES, NODES};     # arcs in the network

param b {NODES} default 0;          # supply/demand for node i
param c {ARCS}  default 0;          # cost of one of flow on arc(i,j)
param l {ARCS}  default 0;          # lower bound on flow on arc(i,j)
```

```
param u {ARCS}  default Infinity; # upper bound on flow on arc(i,j)

var x {ARCS};                       # flow on arc (i,j)
# importing data
data Problem_3.dat;

# objective function
minimize cost: sum{(i,j) in ARCS} c[i,j] * x[i,j];  #objective: minimize arc flow cost


# Flow Out(i) - Flow In(i) = b(i)
# flow constraints

subject to flow_balance {i in NODES}:
sum{j in NODES: (i,j) in ARCS} x[i,j] - sum{j in NODES: (j,i) in ARCS} x[j,i] = b[i];

subject to capacity {(i,j) in ARCS}: l[i,j] <= x[i,j] <= u[i,j];

solve;

# to Display the results
display x;
display sum{(i,j) in ARCS} c[i,j] * x[i,j];
```

Data file:

```
#NSC MCNFP Problem Formulation - data file for problem instance
#Charles Nicholson, ISE 5113, 2015

#use with MCNFP.txt model
#note: default arc costs and lower bounds are 0
#      default arc upper bounds are infinity
#      default node requirements are 0


# data file for problem 3


set NODES :=
       source #source node
       g1 g2 g3 # generator nodes
       c1 c2 c3 # consuming nodes
       sink; # sink node


set ARCS :=  (source, *) g1 g2 g3        #Arcs from source to generators
```

```
            (g1, c1)                          #Arcs connecting generators and consumption
units
            (g2, *) c1 c2 c3
            (g3, c3)
            (c3, *)g3 g2
            (c2, g2)
            (c1, *)g1 g2
            (c1, sink)                    #Arcs from consumption node to sink
            (c2, sink)
            (c3, sink)
            (sink, source);


param c:=
      [source, *] g1 0.015 g2 0.0135 g3 0.021      #Generating costs
      [g1, c1] 0.011                            #Flow through arcs cost
      [g2, *] c1 0.011 c2 0.011 c3 0.011
      [g3, c3] 0.011
      [c3, g2] 0.011
      [c2, g2] 0.011
      [c1, g1] 0.011;

param l:=
      [c1, sink] 35000           #lower bounds consumption points to sink
      [c2, sink] 50000
      [c3, sink] 60000
      [sink, source] 145000;           #lower bound sink to source

param u:=
      [c1, sink] 35000           #upper bounds consumption points to sink
      [c2, sink] 50000
      [c3, sink] 60000
      [sink, source] 145000      #Upper bound sink to source
      [source, g1] 100000        #Upper bounds source to generator
      [source, g2] 60000
      [source, g3] 80000;
```

Question 4: Flipping the bits (20 points)

(a)

i.

n = 20 dimensions and binary problem {0,1}

The size of the solution space is 2^{20} because here we have 20 dimensions and for each dimension we have a possibility of 0 or 1 and solution space is unrestricted.

So, size of the solution space is **1048578.**

ii.

1 - flip neighborhood.

We can choose 1 bit out of 20 and flip it. Now the size of 1 – flip neighborhood can be given as

$$20c_1 = \frac{20!}{1!\,(20-1)!} = \frac{20!}{19!} = 20$$

2 – flip neighborhood

Here we can choose 2 bit out of 20 and flip them. Now the size of 2 – flip neighborhood can be given as

$$20c_2 = \frac{20!}{2!\,(20-2)!} = \frac{(20*19)}{2} = 190$$

3 – flip neighborhood

Here we can choose 3 bits out of 20 and flip them. Now the size of 3 – flip neighborhood can be given as

$$20c_3 = \frac{20!}{3!\,(20-3)!} = \frac{20*19*18}{3*2} = 1140$$

(b)

Given n = 20 dimensions, for a knapsack problem we will have 2 possibilities 0,1 for each element and neighborhood is 2- flip.

i.

for the 2 – flip neighborhood we must select 2 bits out of 20 bits and those 2 will be appended to tabu list.

For this selection, initially we have
$20c_2$ *posiibilities for the solution space and than after*

$$1st\ tabu\ iteration\ we\ have\ 18c\_2\ as\ solution\ space$$

$$no\ of\ neighbors\ of\ x_1 are\ tabu\ is\ 20c_2 - 18c_2 = 190 - 153 = 37$$

ii.

immediately after first move appending 2 more elements to tabu list we have 16 elements so number of neighbors x2 in tabu is $20c_2 - 16c_2 = 190 - 120 = 70$

(c)

Given evaluation function $\boldsymbol{f(y) = y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5}$

i.

Probability $= \left(\frac{f_i}{\sum_{j=1}^{n} f_i}\right)$

Roulette wheel probabilities

10001

$$f(y) = 1 + 2 * 0 + 3 * 0 + 4 * 0 + 5 * 1 = 6$$

$$Probablity = \frac{6}{28} = 0.214$$

00101

$$f(y) = 0 + 2 * 0 + 3 * 1 + 4 * 0 + 5 * 1 = 8$$

$$Probablity = \frac{8}{28} = 0.285$$

01011

$$f(y) = 0 + 2 * 1 + 3 * 0 + 4 * 1 + 5 * 1 = 11$$

$$Probablity = \frac{11}{28} = 0.392$$

11000

$$f(y) = 1 + 2 * 1 + 3 * 0 + 4 * 0 + 5 * 0 = 3$$

$$Probablity = \frac{3}{28} = 0.1071$$

ii.

So here the highest probability is for 01011

$$f(y) = 0 + 2 * 1 + 3 * 0 + 4 * 1 + 5 * 1 = 11$$

$$Probablity = \frac{11}{28} = 0.392$$

So here the lowest probability is for 11000

$$f(y) = 1 + 2 * 1 + 3 * 0 + 4 * 0 + 5 * 0 = 3$$

$$Probablity = \frac{3}{28} = 0.1071$$

When we crossover 2 parents 01011 and 11000 at 2 and 3 bit than we get 2 children

01000

$$f(y) = 0 + 2 * 1 + 3 * 0 + 4 * 0 + 5 * 0 = 2$$

11011

$$f(y) = 1 + 2 * 1 + 3 * 0 + 4 * 1 + 5 * 1 = 12$$

Question 5: Swarming (15 points)

(a)

The formulae for the velocity and the position are given:

$$V_i^{t+1} = V_i^t + \phi_1 r_1 (P_i - X_i^t) + \phi_2 r_2 (P_g - X_i^t)$$
$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

Given $r_1 = 0.25, r_2 = 0.5, \emptyset_1 = \emptyset_2 = 1$

$$P_i = (10,13,8), P_g = (8,0,2) \; and \; X_i^t = (14,5,2)$$

$$V_1^{t+1} = (1,0,1) + 0.25 * [(10,13,8) - (14,5,2)] + 0.5 * [(8,0,2) - (14,5,2)]$$

$$= (0,0,1) + (0.25) * (-4,8,6) + 0.5 * (-6,-3,-2)$$

$$= (1,0,1) + (-1,2,1.5) + (-3,-1.5,-1) = (\mathbf{-3}, \mathbf{0.5}, \mathbf{1.5})$$

So, now the position is

$$X_1^t = X_1^t + V_1^{t+1} = (14,5,2) + (-3,0.5,1.5) = (\mathbf{11}, \mathbf{5.5}, \mathbf{3.5})$$

Final **velocity** and **position** for particle 1 are **(-3,0.5,1.5)** and **(11,5.5,3.5).**

(b)

$$P_i = (10,13,8), P_g = (18,7,5) \; and \; X_i^t = (14,5,2)$$

Position and velocity for particle 1 using Ring Topology.

$$V_1^{t+1} = (1,0,1) + 0.25 * [(10,13,8) - (14,5,2)] + 0.5 * [(18,7,5) - (14,5,2)]$$

$$= (1,0,1) + 0.25 * (-4,8,6) + 0.5 * (4,2,3)$$

$$= (1,0,1) + (-1,2,1.5) + (2,1,1.5)$$

$$= (2,3,4)$$

$$X_1^t = X_1^t + V_1^{t+1} = (14,5,2) + (-1.5,3.5,4) = (16,8,6)$$

Final **Velocity** and **position** for particle 1 are **(2,3,4)** and **(16,8,6).**

---End---