ISE5113
Friday, March 3, 2017
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Homework #2 – Network Flow

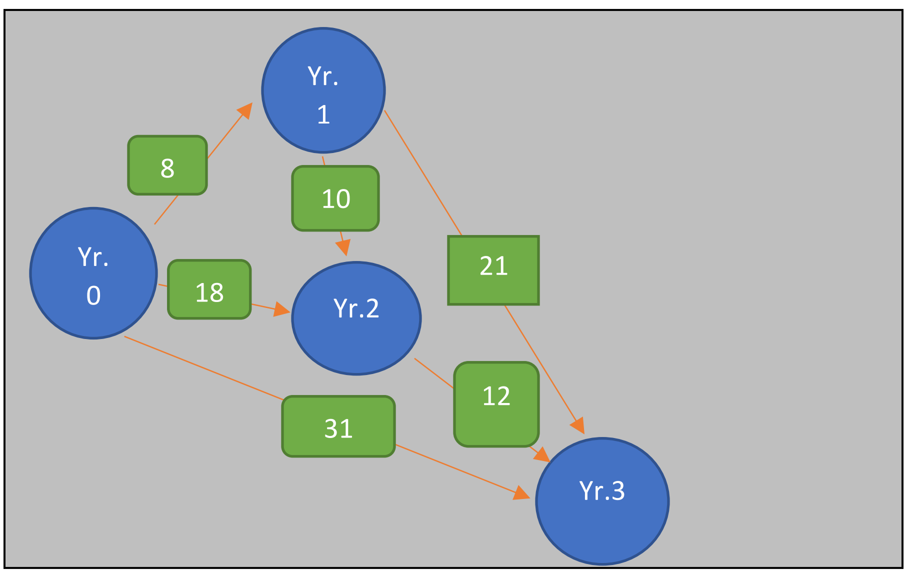
Question 1: Totally Unimodular Matrices (5 pts).

A1 – For this matrix we have 2 columns (3rd and 4th) where numbers of the same sign within each column cannot be divided into different sets to uphold the Total Unimodularity theorem. Therefore, the matrix A1 is not TU.

A2 – Column 2 in this matrix is the only one with two adjacent numbers of the same sign within the entire matrix. You can separate that column to evaluate all submatrices for unimodularity. All submatrices have a determinant that meets all unimodular properties. Thus, this table is TU.

Question 2: Airport Tractor (8 pts).

The graph below shows the network path for the tractor maintenance cost.



Some of the iterations starting with the optimal solutions are:

Yr. 0 --- Yr. 1, Yr. 1 --- Yr. 3; **$8k + $21k = $29K**

Yr. 0 --- Yr. 2, Yr. 2 --- Yr. 3; $18k + $12k = $30K

Yr. 0 --- Yr. 1, Yr. 1 --- Yr. 2, Yr. 2 --- Yr. 3; $8k + $10k + $12k = $30K

Results by modelling in ampl

```
CPLEX 12.6.3.0: optimal solution; objective 29
0 dual simplex iterations (0 in phase I)
x :=
```

```
0 1   1
0 2   0
0 3   0
1 2   0
1 3   1
2 3   0
;

sum{(i,j) in ARCS} c[i,j]*x[i,j] = 29
```

Question 3: Christmas party and summer camp (14 pts).

A- Christmas party

You have two decision variables for this problem: the number of people and rooms. Each decision variable is binary with the objective function being to maximize the likelihood of coworkers who like each other sharing the same room.

Objective F(x):

$$maximize\ z = \sum_{i=1}^{m} \sum\nolimits_{i=1}^{n} x_{ij}$$

St:

$$\sum_{i=1}^{15} x_{ij} = 1\ and\ x_i = \begin{cases} 1\ if\ two\ people\ are\ paired\ to\ a\ room \\ 0\ if\ otherwise \end{cases}$$

$$\sum_{j=1}^{30} x_{ij} = 1\ and\ x_i = \begin{cases} 1\ if\ people\ like\ each\ other \\ 0\ if\ otherwise \end{cases}$$

$$x_{ij} \geq 0$$

As an example, the table below shows that you cannot create a scenario where every coworker will only like no more than two of their cohorts per column and in a pattern needed to uphold the third property of unimodularity.

Employee	1	2	3	4
1	0	1	1	1
2	1	0	1	0
3	1	1	0	0
4	1	0	0	1

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B- Summer camp

The only decision variable to account for when formulating this problem is the willingness of each child. The model is as below:

Objective Function F(x):

$$maximize\ z = \sum_{i=1}^{m} x_{ij}$$

St:

$$\sum_{i=1}^{10} x_{ij} = 1 \ and\ x_i = \begin{cases} 1\ for\ willingness\ to\ participate \\ 0\ if\ otherwise \end{cases}$$

$$x_{ij} \geq 0$$

In the case of the participation model, the student is either willing or unwilling to participate. The degree of willingness to participate is irrelevant to the creation of the square matrix. Each student is an independent variable without relation to the other. Given this information the matrix should be TU.

Student	1	2
1	1	0
2	0	1

C- The constraint matrix for part A and B

As explained in section A and B we do not think that you can come up with a Totally Unimodular scenario for the Christmas party given the relationship of the variables but you can have a Totally Unimodular matrix for the French and English speaking student model.

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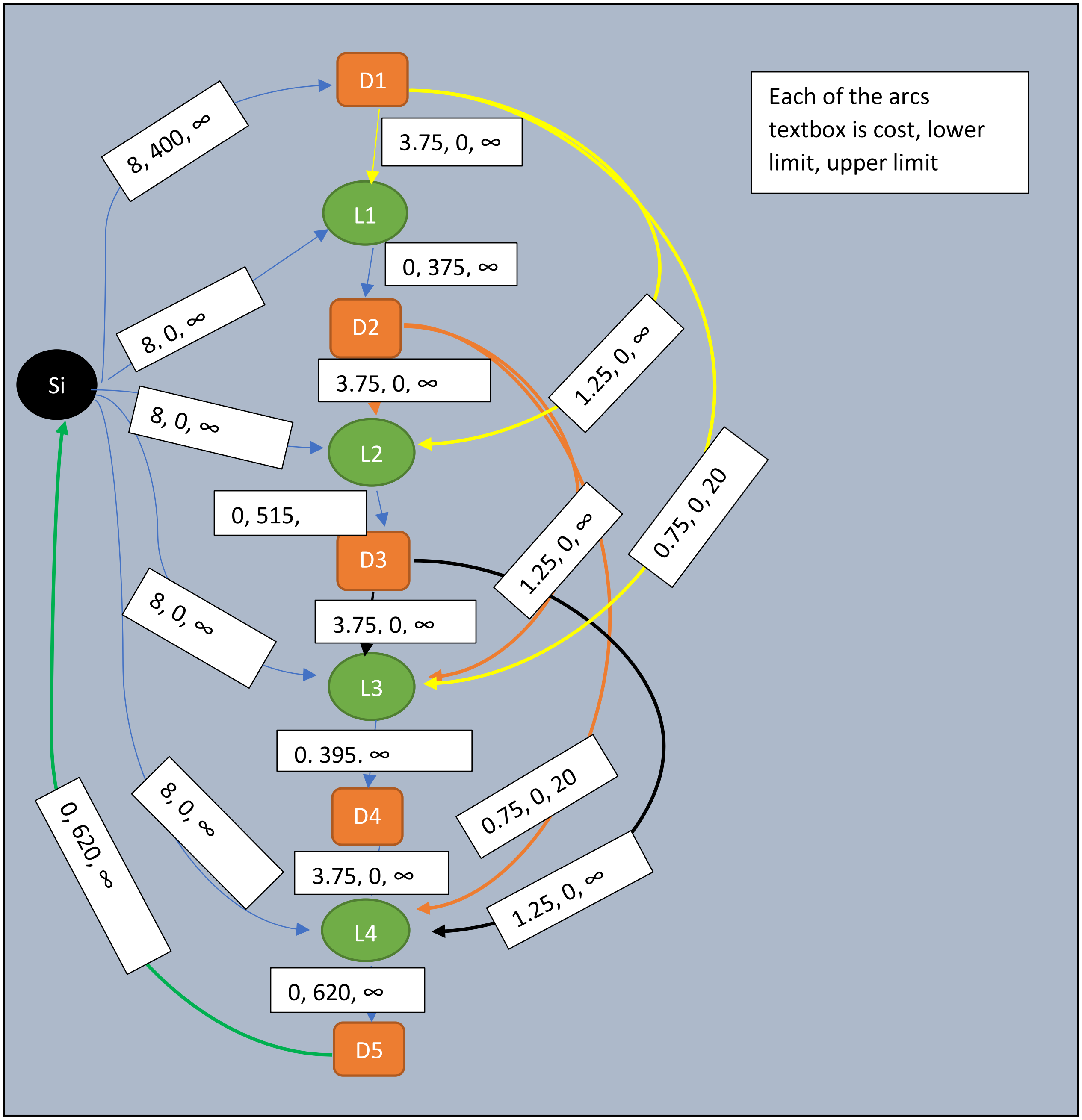
Question 4: Clean table cloth problem (25 pts.)

The network below represents our model



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<u>The Assumptions</u> are:

- You can reuse all the tablecloths from the previous day.
- No tablecloths in inventory and any leftover tablecloths has any value.
- There is no reason to carry inventory because the cost of new tablecloth remains the same every day.
- There is no limit to how many new tablecloths you can purchase.

<u>The Decision Variables</u> are:

- The number of tablecloth to purchase each day.
- The daily cost of sending tablecloths to each laundromat.

<u>Objective function</u>: Minimize the total cost of purchasing and cleaning the tablecloths for the five days' event.

$$minimize\ z = \sum_{i=1}^{m} c_{ij}\, x_{ij}$$

<u>Constraints</u>:

- The daily tablecloth demand must be met
- The capacity for Jimmy R US laundromat is 20 cloths a day
- Carl's Casino laundromat takes 2 days to clean up the tablecloths
- All decision variables are non-negatives

The result leading to an Objective value of $9,841.25 is below.

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```
CPLEX 12.6.3.0: optimal solution; objective 9841.25
7 dual simplex iterations (0 in phase I)
x :=

d1 l1   155
d1 l2   245
d1 l3     0
d2 l2   270
d2 l3   105
d2 l4     0
d3 l3   290
d3 l4   225
d4 l4   395
d5 s    620
l1 d2   375
l2 d3   515
l3 d4   395
l4 d5   620
s  d1   400
s  l1   220
s  l2     0
s  l3     0
s  l4     0
;

sum{(i,j) in ARCS} c[i,j]*x[i,j] = 9841.25 dollars
```

The rows for each day show the quantity of tablecloth to each node. For example, on day 1 you are sending 155 tablecloths to Willie's laundromat at the end of day 1.

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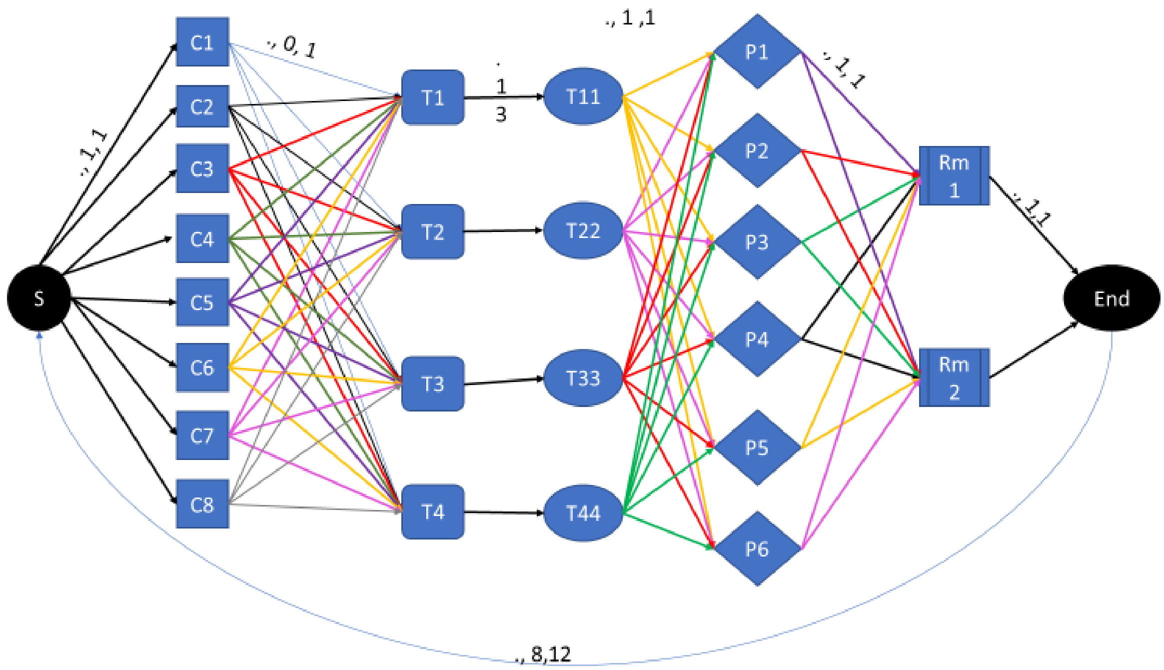
Question 5: Teachers and Classes Assignment (18 pts.)

Our model for this problem is below.



The Assumptions are:

- All classes must be taught every day.
- Every teacher is qualified to teach every class.
- Teachers will never take a sick day and will always be available to teach a class
- The facilities will always be functional.

The Decision Variables are:

- They are no decision variables

Objective function: Maximize the network flow to teach all eight classes in two rooms across six periods.

$$Maximize\ v$$

Constraints:

- All classes must be taught
- Each teacher cannot teach more than 3 classes a day
- There are only two rooms that are available for each period.

- All decision variables are non-negatives

The results maximized flow is expressed in the table below.

Teachers	Classes	Periods/Room
Teacher1	C1, C2, C3	P2,rm1; P3,rm3; P6,rm1
Teacher2	C4,C5,C6	P2,rm2; P3,rm2; P6,rm6
Teacher3	C7	P1,rm1
Teacher4	C8	P1,rm2

As you can see we do not need every period to meet our objective of teaching all 8 classes.

Question 6: Boomer Global Air Service (30 pts.)

(A) Formulate the problem and solve for an optimum fuel plan for Chase for the upcoming trip.

The Assumptions are:

- We are using only 1 aircraft for the entire trip.
- Crew is not changing.
- No passenger will weigh more than the given average of 200 pounds
- Carrying extra passengers does not influence the fuel burn rate
- There are no weather effects

The Decision Variables are:

- How much fuel to purchase at each airport
- Ramp fee which is a binary value.

Objective function: Minimize the cost of fuel for the entire roundtrip.

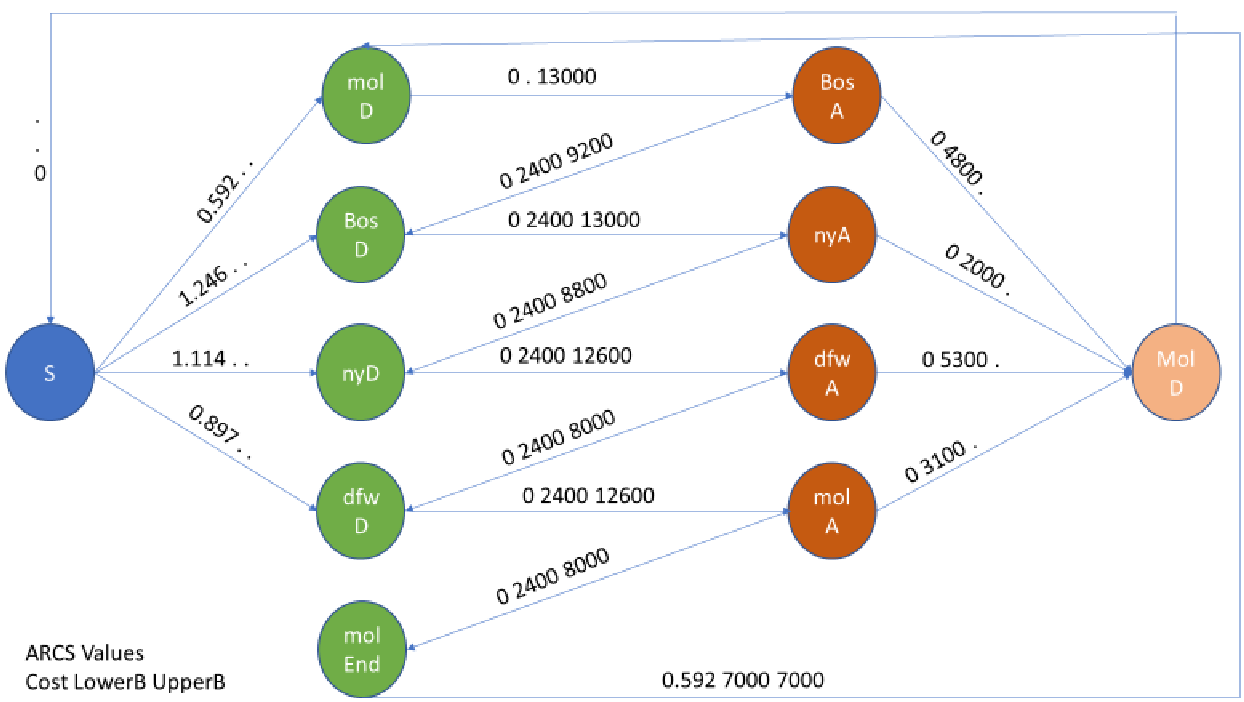
$$minimize\ z = \sum_{i=1}^{m} c_{ij}\, x_{ij} + 800 * B + 450 * N + 400 * D$$

Constraints:

- Maximum landing weight is 31,800 lbs.
- Maximum ramp weight is 36,400 lbs.
- Fuel tank capacity is 13,000 lbs.
- All decision variables are non-negatives.
- Boston minimum gallons to avoid ramp fees is 500 at $8.35/gal or $1.246/lbs.
- New York minimum gallons to avoid ramp fees is 450 at $7.47/gal or $1.114/lbs.
- Dallas minimum gallons to avoid ramp fees is 400 at $6.01/gal or $0.897/lbs.

- You must level back up to 7,000 lbs. after reaching Moline
- You must have 2,400 lbs. at each landing.
- You need 4,800 lbs. of fuel from Mol. – Bos.
- You need 2,000 lbs. of fuel from Bos. – NY.
- You need 5,300 lbs. of fuel from NY. – Dal.
- You need 3,100 lbs. of fuel from Dal. – Mol.

The network flow model with Tankering is:



The table below shows the upper and lower limit for tankering.

	With tankering	
	L	U
Mol- Bos	.	13000
Bos-NY	9200	13000
NY-DFW	8800	12600
DFW-Mol	8000	12600

The results are:

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```
ampl: model Prob6T.txt
CPLEX 12.7.0.0: optimal integer solution; objective 11989
2 MIP simplex iterations
0 branch-and-bound nodes
No basis.
B = 1
N = 1
D = 0

x :=
bosA   bosD      8200
bosA   e         4800
bosD   nyA       8200
dfwA   dfwD      2400
dfwA   e         5300
dfwD   molA      5500
e      s        15200
molA   e         3100
molA   molEnd    2400
molD   bosA     13000
molEnd molD      2400
nyA    e         2000
nyA    nyD       6200
nyD    dfwA      7700
s      bosD         0
s      dfwD      3100
s      molD     10600
s      nyD       1500
;

sum{(i,j) in ARCS} c[i,j]*x[i,j] = 10739
```

Our Objective Value is $11,989.
The ramp fee has a binary value of 1 for Bos. – NY and NY – Dal. which is factored into obtaining our objective value.

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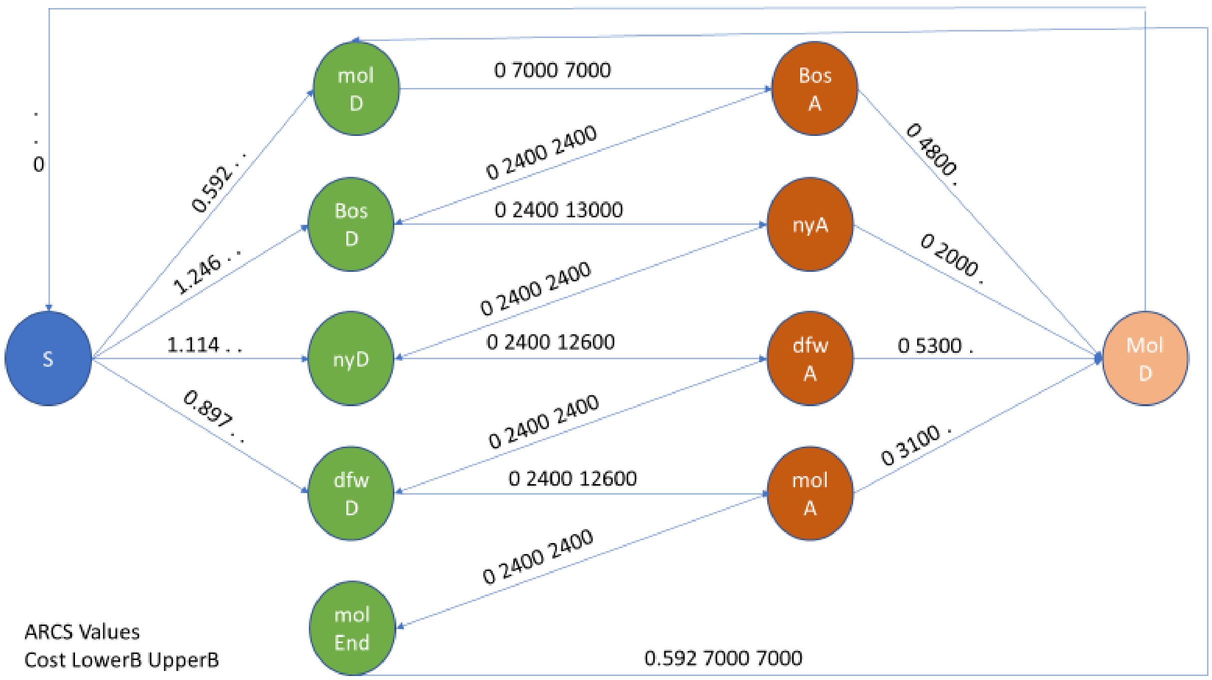
i. Compare your results with a no \tankering" solution.

Our network flow model for NO-Tankering is:



Our results are:
The objective value is $14,828.6 and includes ramp fees in Boston.

```
ampl: model Prob6NT.mod
CPLEX 12.7.0.0: optimal integer solution; objective 14828.6
0 MIP simplex iterations
0 branch-and-bound nodes
No basis.
x :=
bosA   bosD     2400
bosA   e        4800
bosD   nyA      4400
dfwA   dfwD     2400
dfwA   e        5300
dfwD   molA     5500
e      s       15200
molA   e        3100
molA   molEnd   2400
molD   bosA     7200
molEnd molD     2400
nyA    e        2000
nyA    nyD      2400
nyD    dfwA     7700
s      bosD     2000
s      dfwD     3100
s      molD     4800
s      nyD      5300
;

B = 1
N = 0
D = 0

sum{(i,j) in ARCS} c[i,j]*x[i,j] = 14028.6
```

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ii. What are the most important limitations of the model? How might these be addressed?

The most important limitation for this model is the maximum weight that you can have at each leg given the number of passengers that must be added throughout the trip and still meet maximum ramp weight and maximum landing weight limits.

The best way to address this limitation is take the aircraft with higher capacity, in this case the Gulfstream GV.

(B) Suppose the BGAS department manager wished to modify the model to require that \if you buy any gas, you must buy at least 100 gallons".

i. Formulate the problem with this modification.

Our network model remains the same as with Tankering. Without tankering we are buying more than 100 gallons of fuel or 670 lbs. at each leg and do not need to consider this for part B.

Our results are in the picture below.

```
ampl: model Prob6LMT.mod
CPLEX 12.7.0.0: optimal integer solution; objective 12076.77
2 MIP simplex iterations
0 branch-and-bound nodes
No basis.
x :=
bosA   bosD     8200
bosA   e        4800
bosD   nyA      8870
dfwA   dfwD     2400
dfwA   e        5300
dfwD   molA     5500
e      s       15200
molA   e        3100
molA   molEnd   2400
molD   bosA    13000
molEnd molD     2400
nyA    e        2000
nyA    nyD      6870
nyD    dfwA     7700
s      bosD      670
s      dfwD     3100
s      molD    10600
s      nyD       830
;

sum{(i,j) in ARCS} c[i,j]*x[i,j] = 10826.8

ampl:
```

<u>Our objective value total is $12,076.77</u>

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ii. How does this change the solution and cost?

The solution requires us to buy 100 gallons of fuel in Boston which adds cost to our objective value. The cost increase from $11,989 to $12,076.77 or a $87.77 difference.