Advance Analytics

Homework _ 3

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Question 1: Mr. X (12 points)

Assume a continuous decision variable $0 \leq x \leq 1000$. Write constraint(s) to meet the following requirements. You can add any auxiliary variables as you like, but you must define them. Make sure your constraints are as "tight" as possible.

a) x=0 or x≥10

let y be a binary variable than adding this inequality makes sure that

$$x \geq 10y$$

This will make sure that for all values of y, x is 0 or 10.

b) 0 ≤ x ≤ 15 or 30 ≤ x ≤ 100

Let y1 and y2 be 2 binary variables such that y1 + y2 = 1 than adding this inequality makes sure that

$$30y_2 \leq x \leq 15y_1 + 100y_2$$

Value of x is between 0 ≤ x ≤ 15 or 30 ≤ x ≤ 100 for all values of y

Y1	Y2	X
1	0	$0 \leq x \leq 15$
0	1	$30 \leq x \leq 100$

c) x ∈ {12, 12.3, 87, 99.1}

Here the x takes only certain values for this to work we will add a variable zi which will take the values from set of x i.e., $z_1 = 12, z_2 = 12.3, z_3 = 87, z_4 = 99.1$ and let yi be a binary variable and n = 4, i = {1,2,3,4}

Now

$$\mathrm{x} = \sum_{i=1}^{n} y_i z_i$$

$$y_1 + y_2 + y_3 + y_4 \geq 4$$

This will make sure that value of x takes only the values of the set.

Question 2: Valid inequalities (12 points)

$$\begin{aligned} \min\ & 14x_1 + 2x_2 + 11x_3 + 9x_4 + x_5 \\ \text{s.t. } & 3x_1 - 4x_2 + 2x_3 - 3x_4 + x_5 \leq -2 \\ & x_i \in \{0,1\} \text{ for } i = 1, \ldots, 5 \end{aligned}$$

$$Valid\ inequalities$$

$$x_1 + x_3 + x_5 \leq 1$$

Here x is a binary variable this inequality will make at most 1 variables to assume value of 1

$$x_2 + x_4 \geq 2$$

This inequality will make sure that value of x2 and x4 is always 1.

So, now if we go through all the possible cases and constraint will always be satisfied <= -2.

X1	X2	X3	X4	X5	Constraint
1	1	0	1	0	**-4**
0	1	0	1	1	**-6**
0	1	1	1	0	**-5**

Question 3: Gizmos and Gadgets (30 points)

Suppliers:

1. Widgets International Incorporated (W2).
2. Widgets ‘R Us (WRS).

3. Widgets Unlimited (WU).
4. World of Widgets (WOW).

Decision variables: w2, Wrs, Wu, yWu, Wow1, Wow2, Wow3.

Objective: minimize cost = 4.25*w2 + 3.15*wrs + 1.90*wu + yWu*15000 + 5.50*wow1 + 3.50*wow2 + 2*wow3;

Result:

Demand	W2	WRS	WU	WOW	TOTAL COST
17000	2000	15000	0	0	**55750**
18000	3000	15000	0	0	**60000**
19000	0	10000	9000	0	**63600**
28000	0	7500	0	20500	**93375**
32000	0	7500	0	24500	**101375**

This Indicates that selection of supplier entirely depends on the demand. So, it changes accordingly.

Question 4: Facility Location (46 points)

(a)

Model File

```
# Sets I: Districts and J: FireHouse
set I;
set J;

# Parameters for Distance, Population, Budget, Fixed cost, Variable cost
param D{I, J} >=0;  # Distance b/w district <- i,site <- j
param P{I} >=0;     # Population of district i
param B >= 0;             # Allocated budget
param F{J} >=0;     # Fixed cost for building/maintaining site J
param V{J} >=0;     # Variable cost for building site J


# Binary Variables
var y{J}    binary; #its value is 1 if site j is selected or else 0
var x{I, J} binary; #its value is 1 if district i is assigned to site j or else 0
var z       binary;

# Variables for Population, Maximum distance and Total cost
var s{J} integer;   # Population assigned to site j
var MaxD >= 0;      # Maximum distance among all the districts and the sites
```

```
var TC >= 0;        # Total cost

# Objective Function
minimize Distance_Between_I_J: MaxD;

# Constraints

# Assigning district to 1 firehouse
subject to only_one_site {i in I}: sum{j in J} x[i,j] = 1;
# making sure that a district is not assigned to unused site
subject to unused_site {j in J}: (sum{i in I} x[i,j]) <= y[j]*45;
# Either sites 1 and 2 / sites 3 and 4 are selected
subject to either_Site12 : y[1]+y[2] >= 2*z;
subject to either_Site34 : y[3]+y[4] >= 2*(1-z);
# Population at a particular district
subject to population {j in J}: s[j] = sum{i in I} x[i,j]*P[i];
# cost associated with building a fire house at a site
subject to TotalCost : sum{j in J} (F[j]*y[j]+V[j]*s[j]) = TC;
# making sure that we are within budget
subject to WithinBudget : sum{j in J} (F[j]*y[j]+V[j]*s[j]) <= B;
# Maximum distance
subject to Distance {i in I}: MaxD >= sum{j in J} D[i,j]*x[i,j];
```

(b)

i. (8 points) Solve the provided instance of the facility location

a) Determine optimal solution and objective value

Optimal Solution:

```
  y [*] :=
 1 1   4 1   7 0   10 0   13 0   16 0   19 1   22 1   25 0
 2 0   5 1   8 0   11 0   14 0   17 1   20 0   23 0
 3 1   6 0   9 0   12 0   15 0   18 0   21 0   24 0
;
```

Total sites used = 7

This means site 1, 3, 4, 17, 19, 22 are used to set up Firehouse.

Objective Value = 31.3

b) Determine how much of the budget was used

Budget used = 14968700

c) Determine total solution time; determine the total number of branch-and-bound nodes used in the algorithm

Total solution time = 0.69sec.

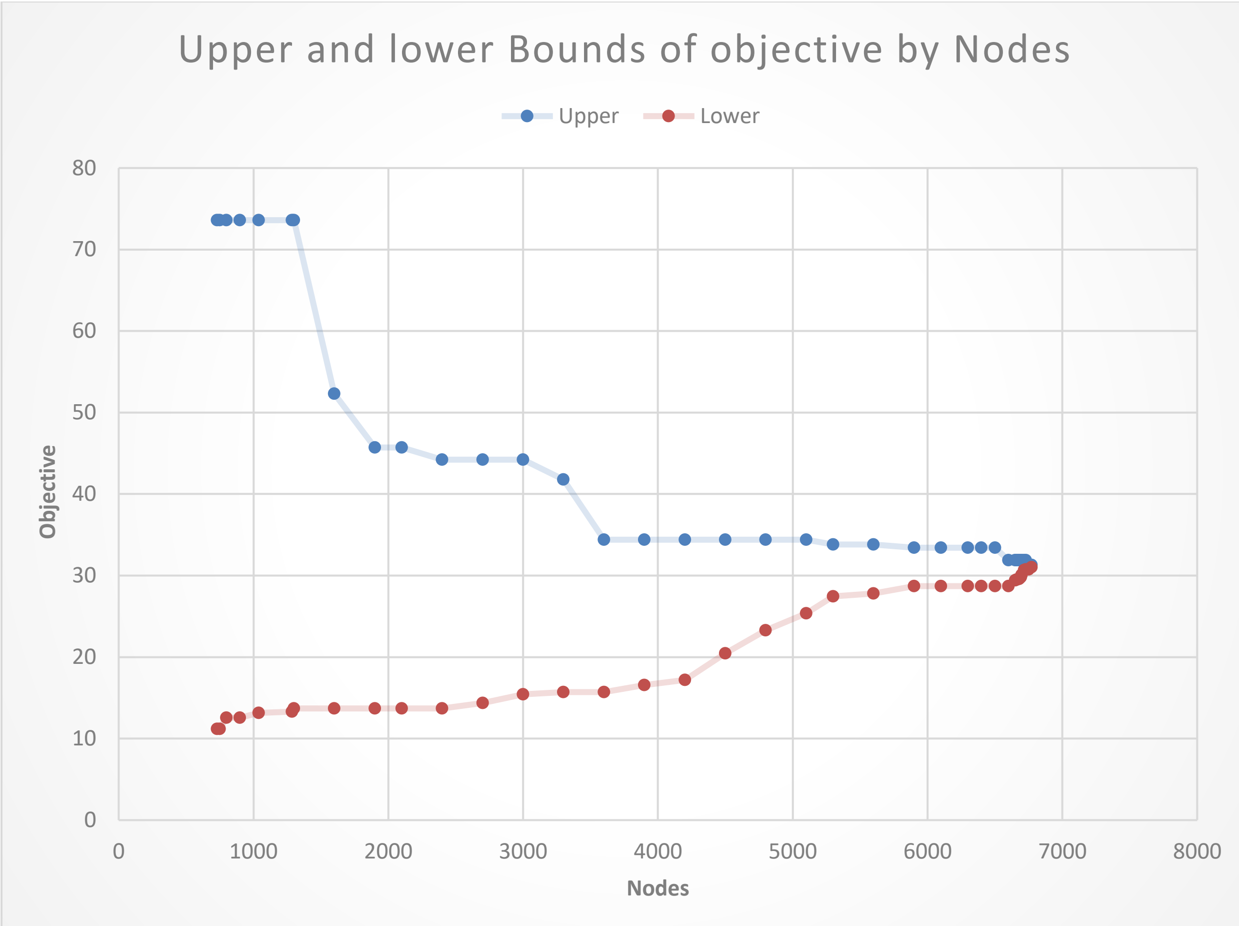
Total no of Branch and Bound nodes = 6772.

d) What was the root relaxation value? What and when (node number) was the first incumbent value found?

Root relaxation value = 6.1359

First incumbent value = 73.6 at node 730

ii. (12 points) Create a graph of upper bound and lower bound objective values by node number (if there are too many nodes, you collect data and plot values for every k nodes, where k > 1 according to the size of the node tree.).



iii. (6 points) Turn the cutting plane options on (mipcuts and splitcuts) and re-solve. Describe how this affects the enumeration tree and solution time.

The solution time increases, it is now 1.63sec and the no of branch & bound nodes also increased to 8632. So, we can say that turning this option will increase the enumerations.

(c) (12 points) : Reformulate the problem to minimize the average distance (instead of minimizing the worst-case distance) and resolve. Note: the original problem is called the p-center problem, the new variation is called the p-median problem. How does your solution change?

When we reformulate the problem the solution time changes to 0.23sec so the time is decreasing for the average case.

And the objective value is 10.9km which decreased by 20 units. But in this case, we are taking the average of distances between the district and the site so, we are taking the value closer to median

No of MIP simplex iterations also decreased greatly to 5344, no of branch and bound nodes are 1578 which also decreased significantly.

Root relaxation value is 2.61.

First incumbent value was found at node 242 and the value is 14.2067.

The no of sites increased to 9 from 7.

Optimal sites selection 1,3,4,5,12,17,21,22,24 this time.

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