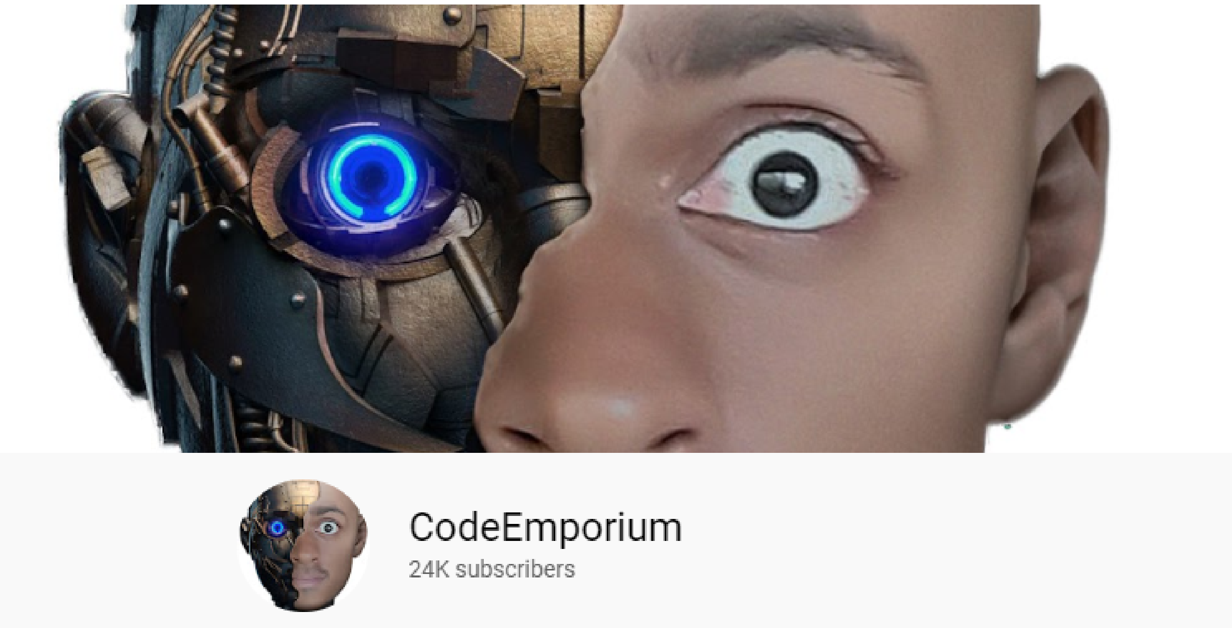
-->

thanks to code emporium for providing wonderfull tutorial and formulas and material in this video some materials have been borrowed from code emporium so courtesy code emporium youtube channel

Figure 1:C:\Users\saktheeswaran\Desktop\jil.png



--> y=f(x)+ε;

(%o19) $y=\epsilon+\mathrm{f}(x)$

-->

--> f(x)=β[0]+β[1]·x;

(%o20) $\mathrm{f}(x)=\beta_1 x+\beta_0$

-->

Figure 2:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\3est.png

$$\hat{y} = \widehat{f(X)}$$

-->

Figure 3:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\4est.png

$$\hat{y} = \widehat{\beta_0} + \widehat{\beta_1} X$$

Figure 4:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\7.png

$$\{\, (X_1, y_1), (X_2, y_2), \dots, (X_n, y_n) \,\}$$

Resudual Error of sample i :

$$e_i = y_i - \widehat{y_i}$$

Sum of Squared Residuals (RSS):

$$RSS = \sum_{i=1}^{n} e_i^2$$

Figure 5:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\8.png

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \widehat{y_i})^2$$

Figure 6:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\9.png

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \widehat{y_i})^2$$

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \left(y_i - (\widehat{\beta_0} + \widehat{\beta_1} X_i)\right)^2$$

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \left(y_i - \widehat{\beta_0} - \widehat{\beta_1} X_i\right)^2$$

Figure 7: C:\Users\saktheeswaran\Desktop\bcap.png

$$\widehat{\beta_1}$$

--> ;

<u>instead of beta cap using just beta α</u>

hence the residual error from β cap changed to α which canm be written as

Figure 8:C:\Users\saktheeswaran\Desktop\linear regression in deteil for deep learning\8.png

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

--> argmin[β[0],β[1]]·sum((y[i]−α[i])^2,i,1,n);

$$(\%o21) \quad argmin_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \alpha_i)^2$$

now substitute α with α[0]and α[1]

--> argmin[β[0],β[1]]·sum((y[i]−(α[0]+α[1]·x[i]))^2,i,1,n);

$$(\%o26) \quad argmin_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \alpha_1\, x_i - \alpha_0)^2$$

--> diff(sum((y[i]−α[1]·x[i]−α[0])^2,i,1,n) ,α[0],1);

$$(\%o31) \quad -2 \sum_{i=1}^{n} y_i - \alpha_1\, x_i - \alpha_0$$

--> −2·sum(y[i]−α[1]·x[i]−α[0],i,1,n)=0;

$$(\%o32) \quad -2 \sum_{i=1}^{n} y_i - \alpha_1\, x_i - \alpha_0 = 0$$

-->

--> sum(y[i]−α[1]·x[i]−α[0],i,1,n);

(%o33) $\sum_{i=1}^{n} y_i - \alpha_1\, x_i - \alpha_0$

--> sum(y[i],i,1,n)−sum(α[1]·x[i],i,1,n)−sum(α[0],i,1,n);

(%o36) $-\alpha_0 n + \left(\sum_{i=1}^{n} y_i\right) - \alpha_1 \sum_{i=1}^{n} x_i$

--> solve(−α[0]·n+(sum(y[i],i,1,n))−α[1]·sum(x[i],i,1,n), [α[0]]);

(%o38) $[\alpha_0 = \frac{\left(\sum_{i=1}^{n} y_i\right) - \alpha_1 \sum_{i=1}^{n} x_i}{n}]$

-->

--> α[0]=sum(y[i]−α[1]·x[i],i,1,n)/n;

(%o39) $\alpha_0 = \frac{\sum_{i=1}^{n} y_i - \alpha_1\, x_i}{n}$

-->

Figure 9:

$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

--> argmin[β[0],β[1]]·sum((y[i]−α[i])^2,i,1,n);

(%o40) $argmin_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \alpha_i)^2$

-->

--> argmin[β[0],β[1]]·sum((y[i]−α[1]·x[i]−α[0])^2,i,1,n);

(%o41) $argmin_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \alpha_1\, x_i - \alpha_0)^2$

-->

--> sum((y[i]−α[1]·x[i]−α[0])^2,i,1,n);

(%o42) $$\sum_{i=1}^{n} (y_i - \alpha_1\, x_i - \alpha_0)^2$$

--> diff(sum((y[i]−α[1]·x[i]−α[0])^2,i,1,n) ,α[1],1);

(%o44) $$-2\sum_{i=1}^{n} x_i\,(y_i - \alpha_1\, x_i - \alpha_0)$$

--> −2·sum(x[i]·(y[i]−α[1]·x[i]−α[0]),i,1,n)=0;

(%o45) $$-2\sum_{i=1}^{n} x_i\,(y_i - \alpha_1\, x_i - \alpha_0) = 0$$

-->

--> sum(x[i]·(y[i]−α[1]·x[i]−α[0]),i,1,n);

(%o46) $$\sum_{i=1}^{n} x_i\,(y_i - \alpha_1\, x_i - \alpha_0)$$

--> sum(x[i]·y[i],i,1,n)−α[1]·sum(x[i]^2,i,1,n)−α[0]·sum(x[i],i,1,n);

(%o47) $$\left(\sum_{i=1}^{n} x_i\, y_i\right) - \alpha_1 \left(\sum_{i=1}^{n} x_i^2\right) - \alpha_0 \sum_{i=1}^{n} x_i$$

--> α[0]=sum(y[i]−α[1]·x[i],i,1,n)/n;

(%o52) $$\alpha_0 = \frac{\sum_{i=1}^{n} y_i - \alpha_1\, x_i}{n}$$

--> α[0]=sum(y[i],i,1,n)/(n)−α[1]·sum(x[i],i,1,n)/n;

(%o53) $$\alpha_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\alpha_1 \sum_{i=1}^{n} x_i}{n}$$

--> α[0]=sum(y[i]−α[1]·x[i],i,1,n)/n;

(%o70) $$\alpha_0 = \frac{\sum_{i=1}^{n} y_i - \alpha_1\, x_i}{n}$$

--> α[0]=sum(y[i]/(n),i,1,n)−α[1]·sum(x[i]/(n),i,1,n);

(%o74) $$\alpha_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\alpha_1 \sum_{i=1}^{n} x_i}{n}$$

--> (sum(x[i]·y[i],i,1,n))−α[1]·(sum(x[i]^2,i,1,n))−α[0]·sum(x[i],i,1,n);

(%o55) $$\left(\sum_{i=1}^{n} x_i\, y_i\right) - \alpha_1 \left(\sum_{i=1}^{n} x_i^2\right) - \alpha_0 \sum_{i=1}^{n} x_i$$

\- (sum(x[i]·y[i],i,1,n))−α[1]·(sum(x[i]^2,i,1,n))−(sum(y[i],i,1,n)/n−(α[1]·sum(x[i],i,1,n))/n)·sum(x[i],i,1,n);
\-
\>

$$(\%o75)\quad \left(\sum_{i=1}^{n} x_i\right)\left(\frac{\alpha_1 \sum_{i=1}^{n} x_i}{n} - \frac{\sum_{i=1}^{n} y_i}{n}\right) + \left(\sum_{i=1}^{n} x_i\, y_i\right) - \alpha_1 \sum_{i=1}^{n} x_i^2$$

--> (sum(x[i],i,1,n))·(((1/n)·α[1]·sum(x[i],i,1,n))−(1/n)·sum(y[i],i,1,n))+(sum(x[i]·y[i],i,1,n))−α[1]·sum(x[i]^2,i,1,n);

$$(\%o7)\quad \left(\sum_{i=1}^{n} x_i\right)\left(\frac{\alpha_1 \sum_{i=1}^{n} x_i}{n} - \frac{\sum_{i=1}^{n} y_i}{n}\right) + \left(\sum_{i=1}^{n} x_i\, y_i\right) - \alpha_1 \sum_{i=1}^{n} x_i^2$$

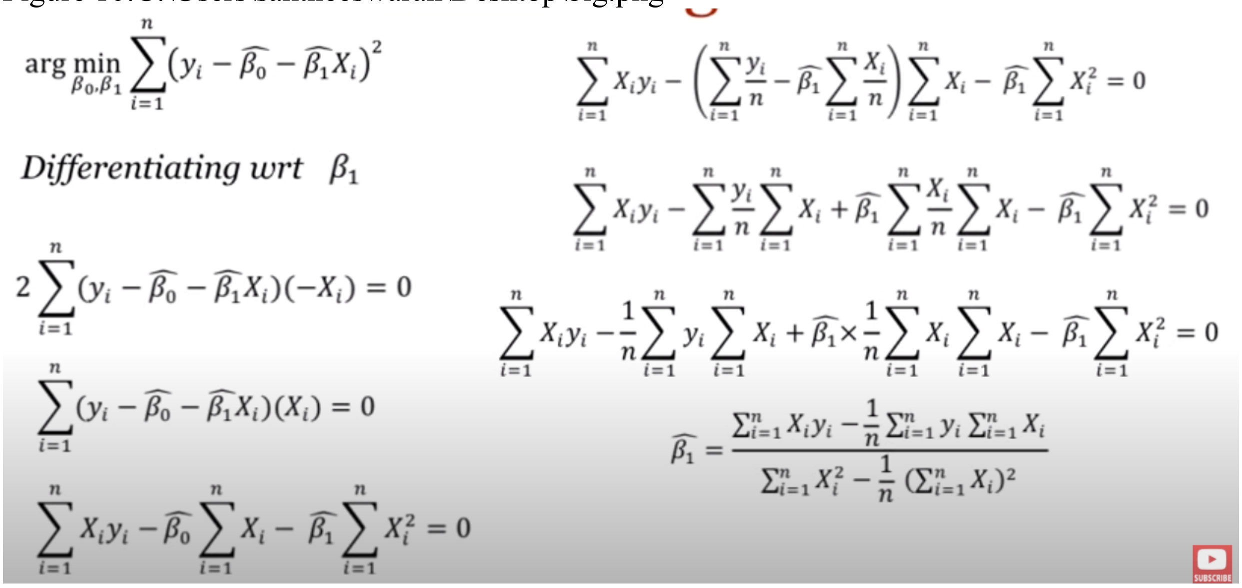
--> solve([(%o7)], [α[1]]);

$$(\%o8)\quad [\alpha_1 = \frac{\left(\sum_{i=1}^{n} x_i\, y_i\right) n - \left(\sum_{i=1}^{n} x_i\right) \sum_{i=1}^{n} y_i}{\left(\sum_{i=1}^{n} x_i^2\right) n - \left(\sum_{i=1}^{n} x_i\right)^2}]$$

-->

but wait a minute this was not the result in the code emporium channel ok let me prove it to you here is the numerator

Figure 10:C:\Users\saktheeswaran\Desktop\big.png



$$\arg\min_{\beta_0,\beta_1} \sum_{i=1}^{n} (y_i - \widehat{\beta_0} - \widehat{\beta_1} X_i)^2$$

Differentiating wrt β_1

$$2\sum_{i=1}^{n} (y_i - \widehat{\beta_0} - \widehat{\beta_1} X_i)(-X_i) = 0$$

$$\sum_{i=1}^{n} (y_i - \widehat{\beta_0} - \widehat{\beta_1} X_i)(X_i) = 0$$

$$\sum_{i=1}^{n} X_i y_i - \widehat{\beta_0}\sum_{i=1}^{n} X_i - \widehat{\beta_1}\sum_{i=1}^{n} X_i^2 = 0$$

$$\sum_{i=1}^{n} X_i y_i - \left(\sum_{i=1}^{n}\frac{y_i}{n} - \widehat{\beta_1}\sum_{i=1}^{n}\frac{X_i}{n}\right)\sum_{i=1}^{n} X_i - \widehat{\beta_1}\sum_{i=1}^{n} X_i^2 = 0$$

$$\sum_{i=1}^{n} X_i y_i - \sum_{i=1}^{n}\frac{y_i}{n}\sum_{i=1}^{n} X_i + \widehat{\beta_1}\sum_{i=1}^{n}\frac{X_i}{n}\sum_{i=1}^{n} X_i - \widehat{\beta_1}\sum_{i=1}^{n} X_i^2 = 0$$

$$\sum_{i=1}^{n} X_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} X_i + \widehat{\beta_1}\times\frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i - \widehat{\beta_1}\sum_{i=1}^{n} X_i^2 = 0$$

$$\widehat{\beta_1} = \frac{\sum_{i=1}^{n} X_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}(\sum_{i=1}^{n} X_i)^2}$$

Figure 11:C:\Users\saktheeswaran\Desktop\biger.png

$$\sum_{i=1}^{n} X_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} X_i + \widehat{\beta_1} \times \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} X_i - \widehat{\beta_1}\sum_{i=1}^{n} X_i^2 = 0$$

$$\widehat{\beta_1} = \frac{\sum_{i=1}^{n} X_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2}$$

here is the numerator from the code emporium channel

```
--> sum(x[i]·y[i],i,1,n)−(1/n)·(sum(y[i],i,1,n)·sum(x[i],i,1,n));
```

(%o16) $$\left(\sum_{i=1}^{n} x_i\, y_i\right) - \frac{\left(\sum_{i=1}^{n} x_i\right)\sum_{i=1}^{n} y_i}{n}$$

```
-->
```

here is the denominator now i will prove to you the results we got are the same as code emporium channel has derived

```
--> sum(x[i]^2,i,1,n)−(1/n)·(sum(x[i],i,1,n)^2);
```

(%o17) $$\left(\sum_{i=1}^{n} x_i^2\right) - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$$

```
-->
```

divide numerator by denominator you get

```
--> ((sum(x[i]·y[i],i,1,n))\−((sum(x[i],i,1,n))·sum(y[i],i,1,n))/n)/((sum(x[i]^2,i,1,n))−(sum(x[i],i,1,n))^2/n)
;
```

$$(\%o19)\quad \frac{\left(\sum_{i=1}^{n} x_i\, y_i\right) - \frac{\left(\sum_{i=1}^{n} x_i\right) \sum_{i=1}^{n} y_i}{n}}{\left(\sum_{i=1}^{n} x_i^2\right) - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}$$

now expand it it gets big

--> expand((%o19));

$$(\%o20)\quad \frac{\sum_{i=1}^{n} x_i\, y_i}{\left(\sum_{i=1}^{n} x_i^2\right) - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}} - \frac{\left(\sum_{i=1}^{n} x_i\right) \sum_{i=1}^{n} y_i}{\left(\sum_{i=1}^{n} x_i^2\right) n - \left(\sum_{i=1}^{n} x_i\right)^2}$$

-->

here is the beautiful part now it gets to the same equation as before

--> factor((%o20));

$$(\%o21)\quad \frac{\left(\sum_{i=1}^{n} x_i\, y_i\right) n - \left(\sum_{i=1}^{n} x_i\right) \sum_{i=1}^{n} y_i}{\left(\sum_{i=1}^{n} x_i^2\right) n - \left(\sum_{i=1}^{n} x_i\right)^2}$$

-->

ok did we see this before yes which we got by nor mal substitution hence are results are correct but we just verfied it now go and refer α[1] it will be the same

Figure 12:C:\Users\saktheeswaran\Desktop\fdefd.png

still not finished in this manner the multiple regression can also be derived and lets now go to matrix calculus .org where no computer algebra is capable of dealing with

tensor and matrix calculus now lets see how it can be done
you can also use sympy but for me iam not good at it but i will figure it soon so lets do our matrix calculus there but first lets finish this multiple regression here

Figure 13:C:\Users\saktheeswaran\Desktop\linearregression\11.png

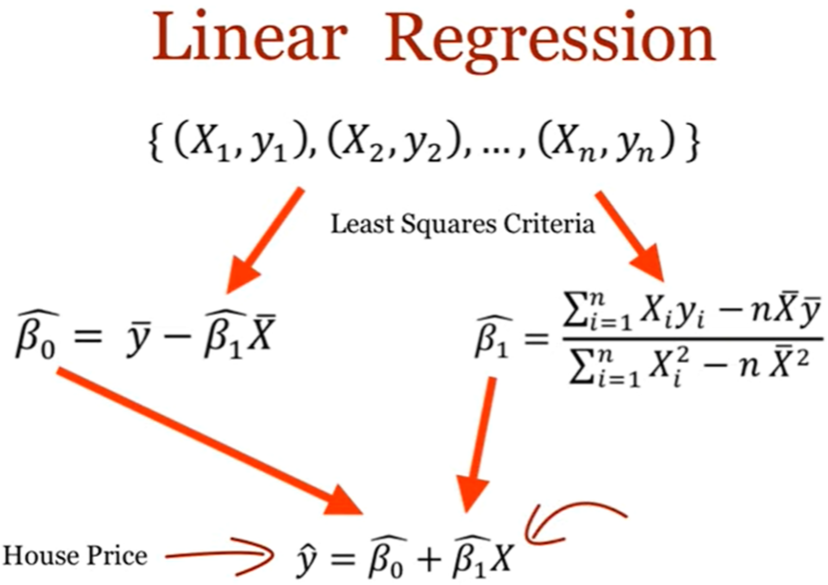


Figure 14:C:\Users\saktheeswaran\Desktop\linearregression\22.png

Multiple Regression

General Parametric Equation:

$$y = \underbrace{f(\boldsymbol{X})}_{\text{Depends on Statistical Method}} + \epsilon$$

$$f(\boldsymbol{X}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p$$

$$\hat{y} = \widehat{\beta_0} + \widehat{\beta_1} X_1 + \cdots + \widehat{\beta_p} X_p$$

Figure 15:C:\Users\saktheeswaran\Desktop\linearregression\33.png

Multiple Regression

nx1 column matrix

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix}, \boldsymbol{X} = \begin{bmatrix} 1 & X_{1,1} & X_{1,2} & . & . & X_{1,p} \\ 1 & X_{2,1} & X_{2,2} & . & . & . \\ 1 & X_{3,1} & X_{3,2} & . & . & . \\ 1 & . & . & . & . & . \\ 1 & . & . & . & . & . \\ 1 & X_{n,1} & X_{n,2} & . & . & X_{n,p} \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_p \end{bmatrix}, \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ . \\ . \\ . \\ \epsilon_n \end{bmatrix}$$

n dimensional column vector or an matrix nx1

matrix

n dimensional column vector or a matrix nx1

Figure 16:C:\Users\saktheeswaran\Desktop\linearregression\44.png

Multiple Regression

column vector or nx1 matrix

eppisilon is an nx1 column vector or nx 1 matrix

$$
\begin{array}{cccc}
n\,x\,1 & n\,x\,(p{+}1) & (p{+}1)\,x\,1 & n\,x\,1 \\
\boldsymbol{y}=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix},
& \boldsymbol{X}=\begin{bmatrix} 1 & X_{1,1} & X_{1,2} & . & . & X_{1,p} \\ 1 & X_{2,1} & X_{2,2} & . & . & . \\ 1 & X_{3,1} & X_{3,2} & . & . & . \\ 1 & . & . & . & . & . \\ 1 & . & . & . & . & . \\ 1 & X_{n,1} & X_{n,2} & . & . & X_{n,p} \end{bmatrix}
& ,\boldsymbol{\beta}=\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ . \\ . \\ . \\ \beta_p \end{bmatrix}
& ,\boldsymbol{\epsilon}=\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ . \\ . \\ . \\ \epsilon_n \end{bmatrix}
\end{array}
$$

this is an matrix with nx (p+1) p+1 because multiplicant thats aded

column vector or (p+1)x1 or n dimensional coulmn vector

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon} \qquad \hat{\boldsymbol{y}} = \boldsymbol{X}\hat{\boldsymbol{\beta}}$$

Figure 17:C:\Users\saktheeswaran\Desktop\linearregression\55.png

Multiple Regression

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$$

$$
\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} =
\begin{bmatrix}
\beta_0 + \beta_1 X_{1,1} + \beta_2 X_{1,2} + \cdots + \beta_p X_{1,p} + \epsilon_1 \\
\beta_0 + \beta_1 X_{2,1} + \beta_2 X_{2,2} + \cdots + \beta_p X_{2,p} + \epsilon_2 \\
\beta_0 + \beta_1 X_{3,1} + \beta_2 X_{3,2} + \cdots + \beta_p X_{3,p} + \epsilon_3 \\
. \\
. \\
. \\
\beta_0 + \beta_1 X_{n,1} + \beta_2 X_{n,2} + \cdots + \beta_p X_{n,p} + \epsilon_n
\end{bmatrix}
$$

Figure 18:C:\Users\saktheeswaran\Desktop\linearregression\66.png

Multiple Regression

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$$

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ . \\ e_n \end{bmatrix} = \begin{bmatrix} y_1 - \widehat{y_1} \\ y_2 - \widehat{y_2} \\ y_3 - \widehat{y_3} \\ . \\ . \\ . \\ y_n - \widehat{y_n} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} - \begin{bmatrix} \widehat{y_1} \\ \widehat{y_2} \\ \widehat{y_3} \\ . \\ . \\ . \\ \widehat{y_n} \end{bmatrix} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$$

Figure 19:C:\Users\saktheeswaran\Desktop\linearregression\77.png

Multiple Regression

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\epsilon}$$

$$\boldsymbol{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ . \\ . \\ . \\ e_n \end{bmatrix} = \begin{bmatrix} y_1 - \widehat{y_1} \\ y_2 - \widehat{y_2} \\ y_3 - \widehat{y_3} \\ . \\ . \\ . \\ y_n - \widehat{y_n} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} - \begin{bmatrix} \widehat{y_1} \\ \widehat{y_2} \\ \widehat{y_3} \\ . \\ . \\ . \\ \widehat{y_n} \end{bmatrix} = \boldsymbol{y} - \widehat{\boldsymbol{y}}$$

$$RSS = \sum_{i=1}^{n} e_i^2$$

Figure 20:C:\Users\saktheeswaran\Desktop\linearregression\88.png

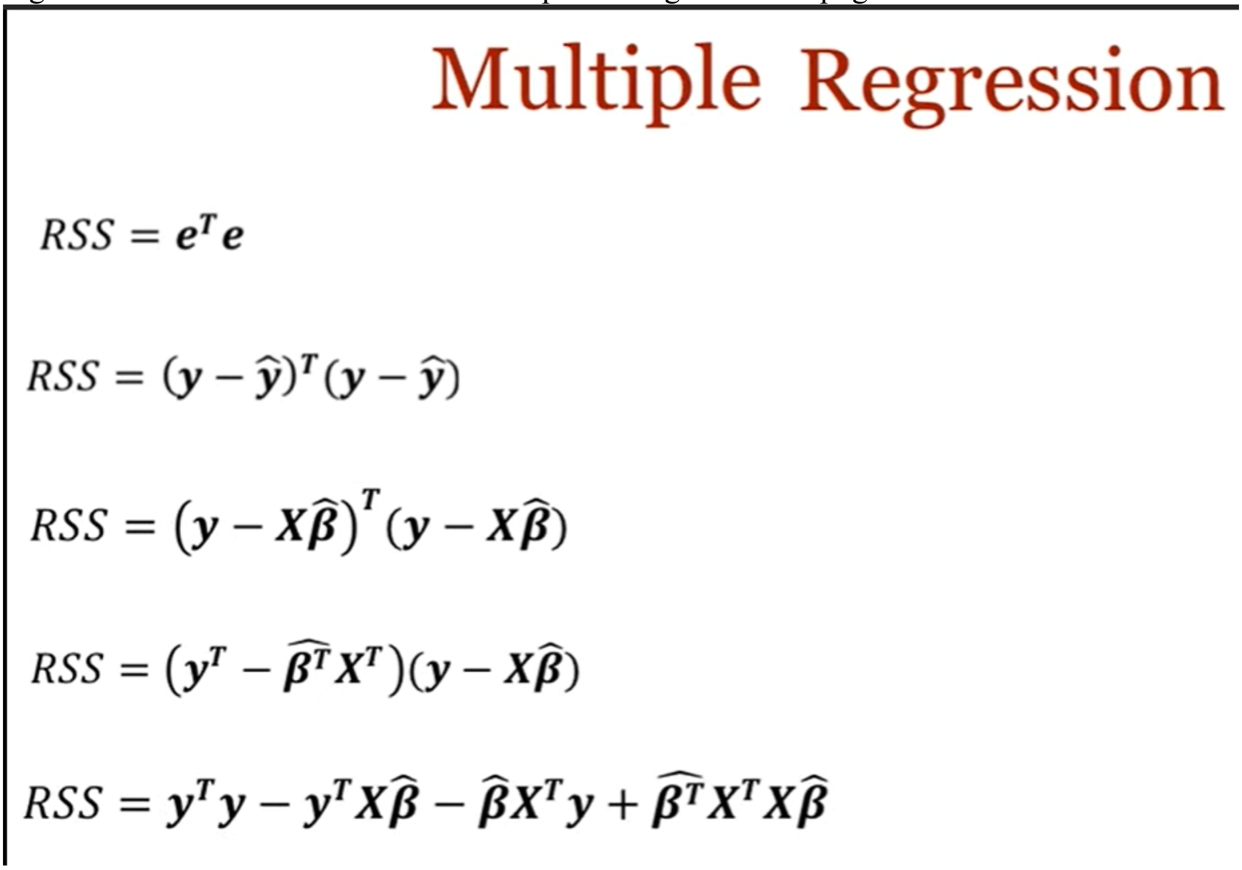


Figure 21:C:\Users\saktheeswaran\Desktop\linearregression\99.png

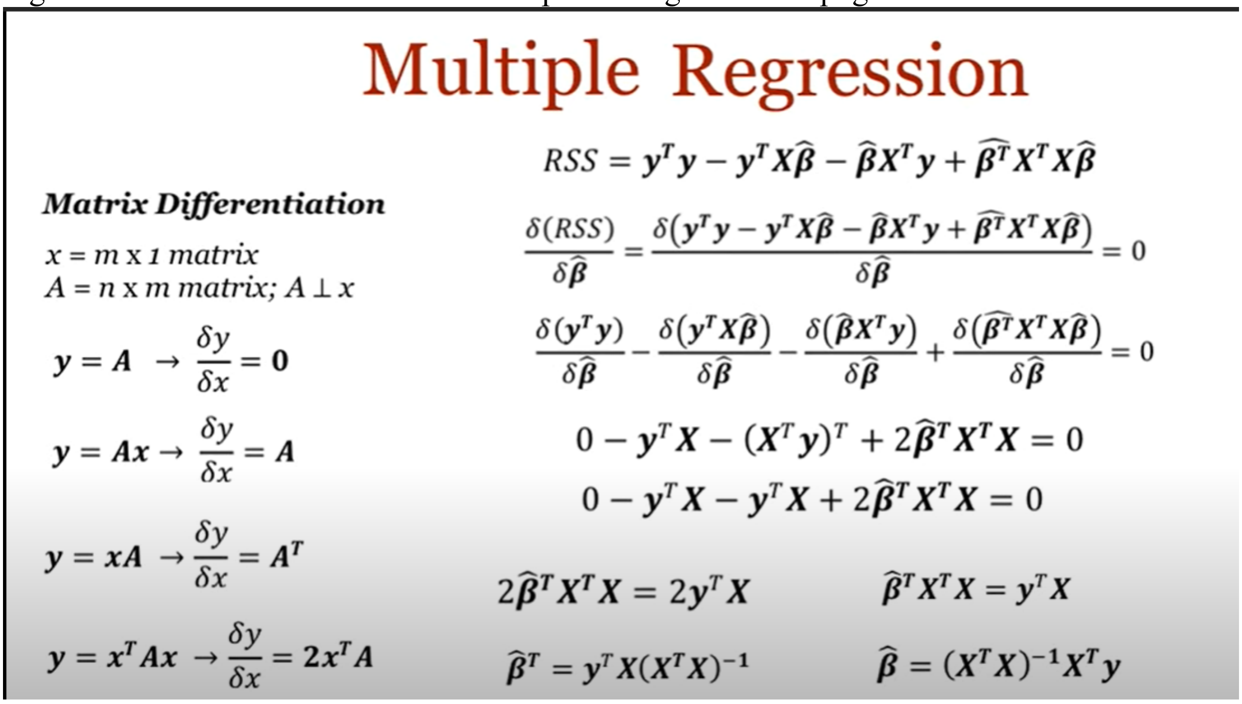


Figure 22:C:\Users\saktheeswaran\Desktop\linearregression\100.png

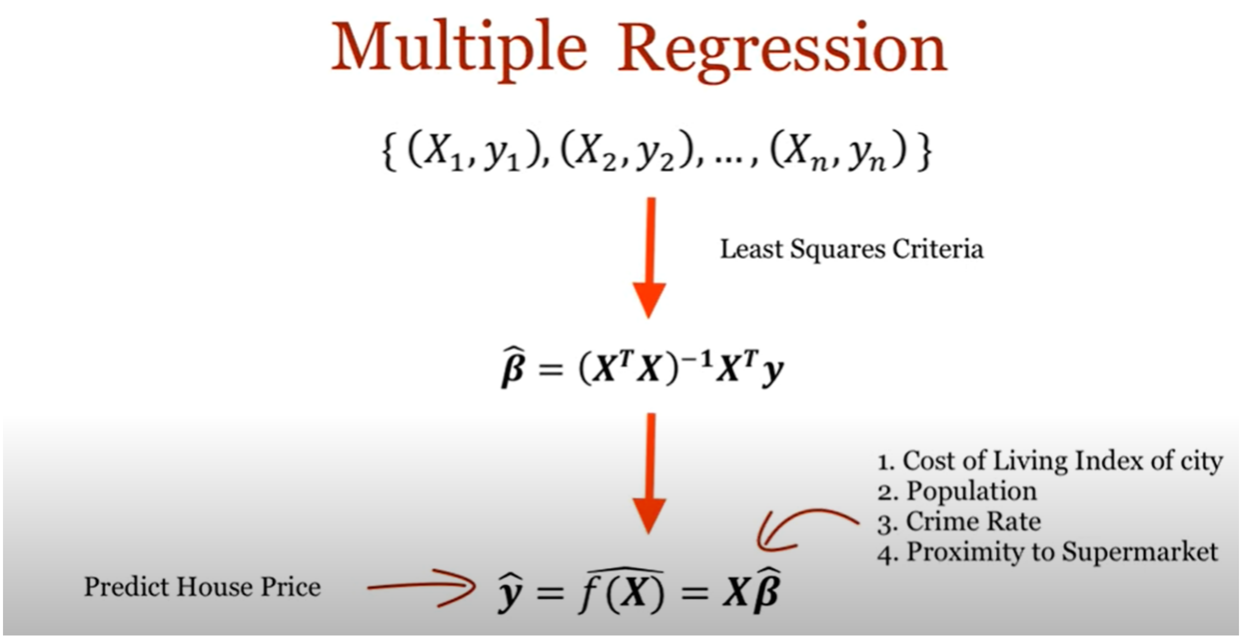


Figure 23:C:\Users\saktheeswaran\Desktop\linearregression\100.png

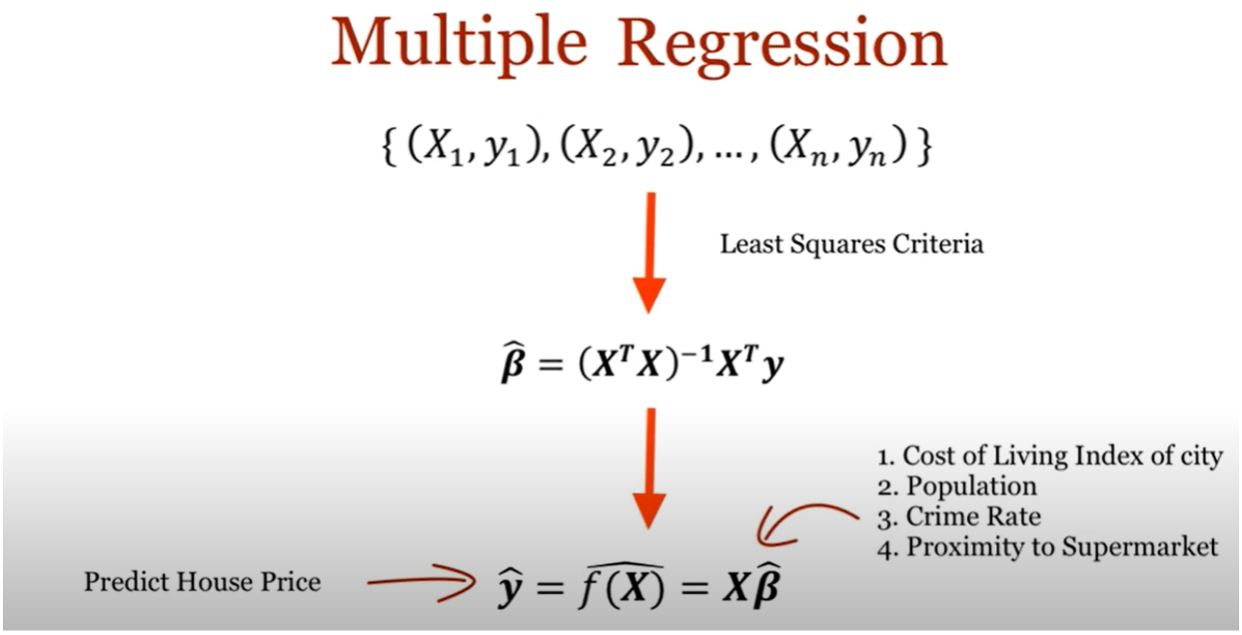


Figure 24:C:\Users\saktheeswaran\Desktop\linearregression\0100u.png

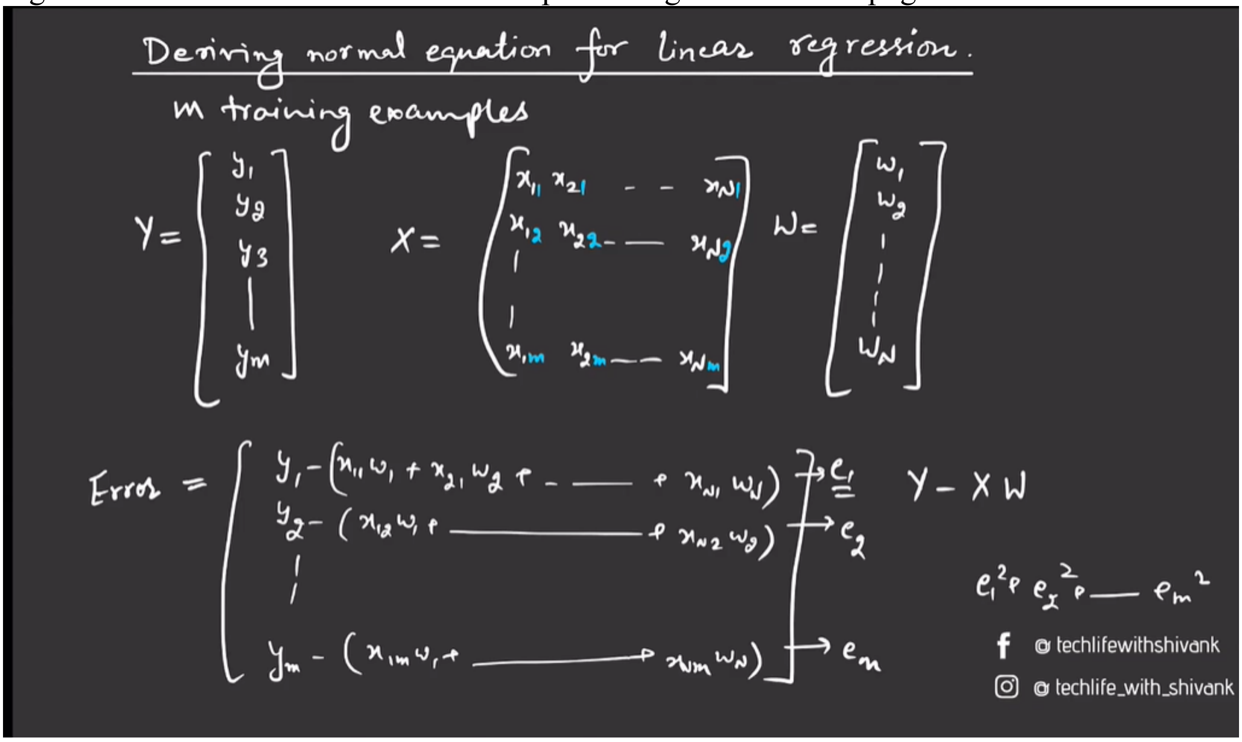


Figure 25:C:\Users\saktheeswaran\Desktop\linearregression\01000v.png

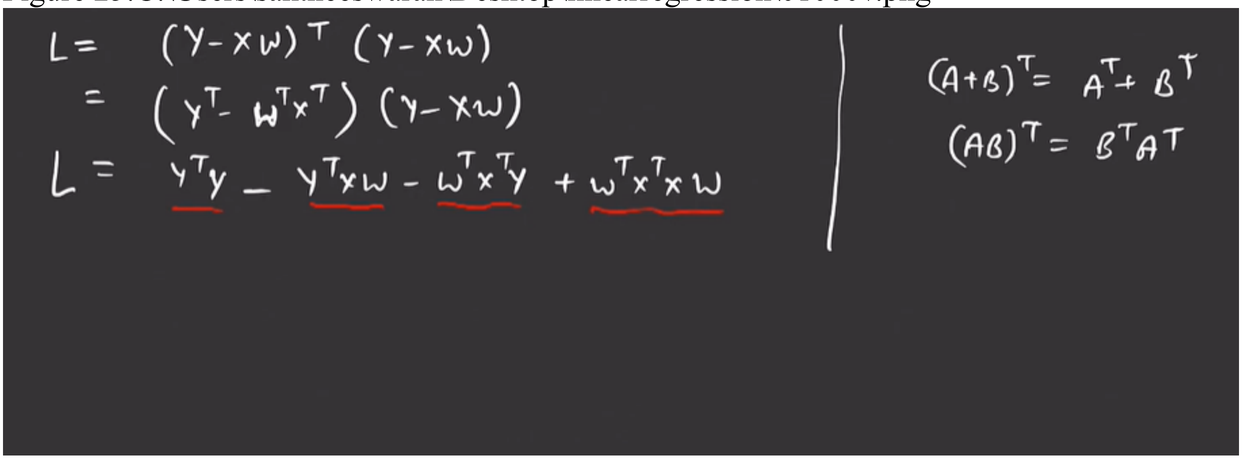


Figure 26:C:\Users\saktheeswaran\Desktop\linearregression\2u.png

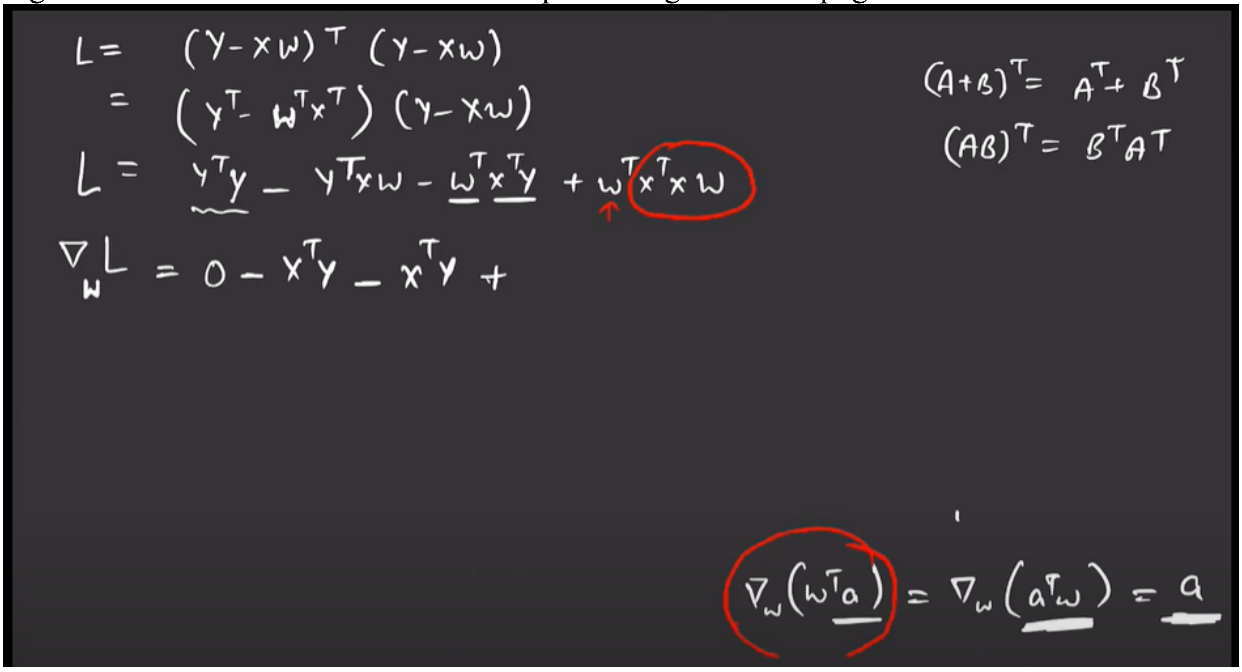


Figure 27:C:\Users\saktheeswaran\Desktop\linearregression\3v.png

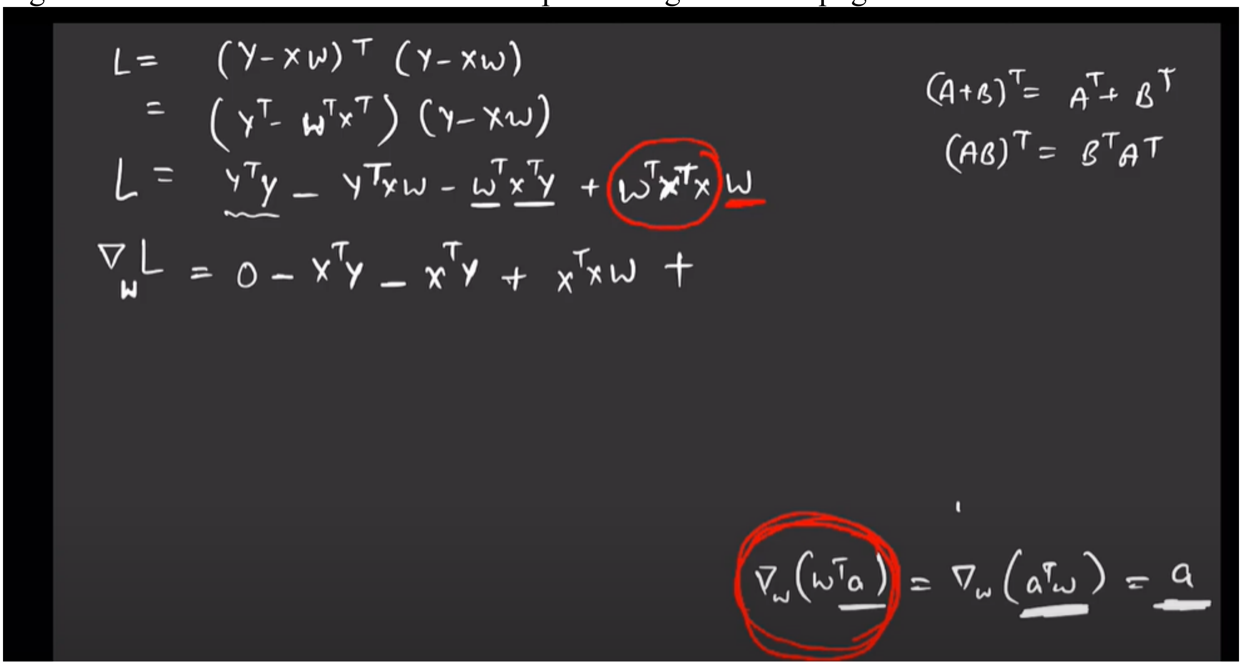


Figure 28:C:\Users\saktheeswaran\Desktop\linearregression\finalgf.png

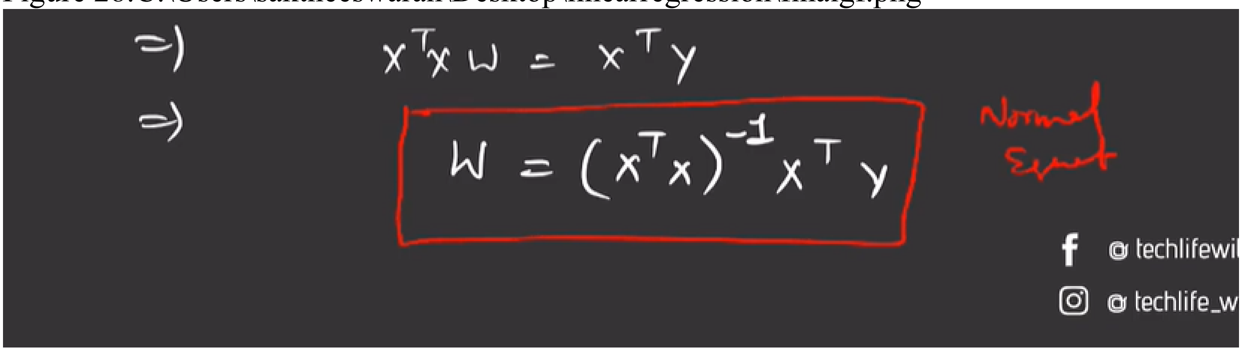


Figure 29:C:\Users\saktheeswaran\Desktop\linearregression\finalbefore.png

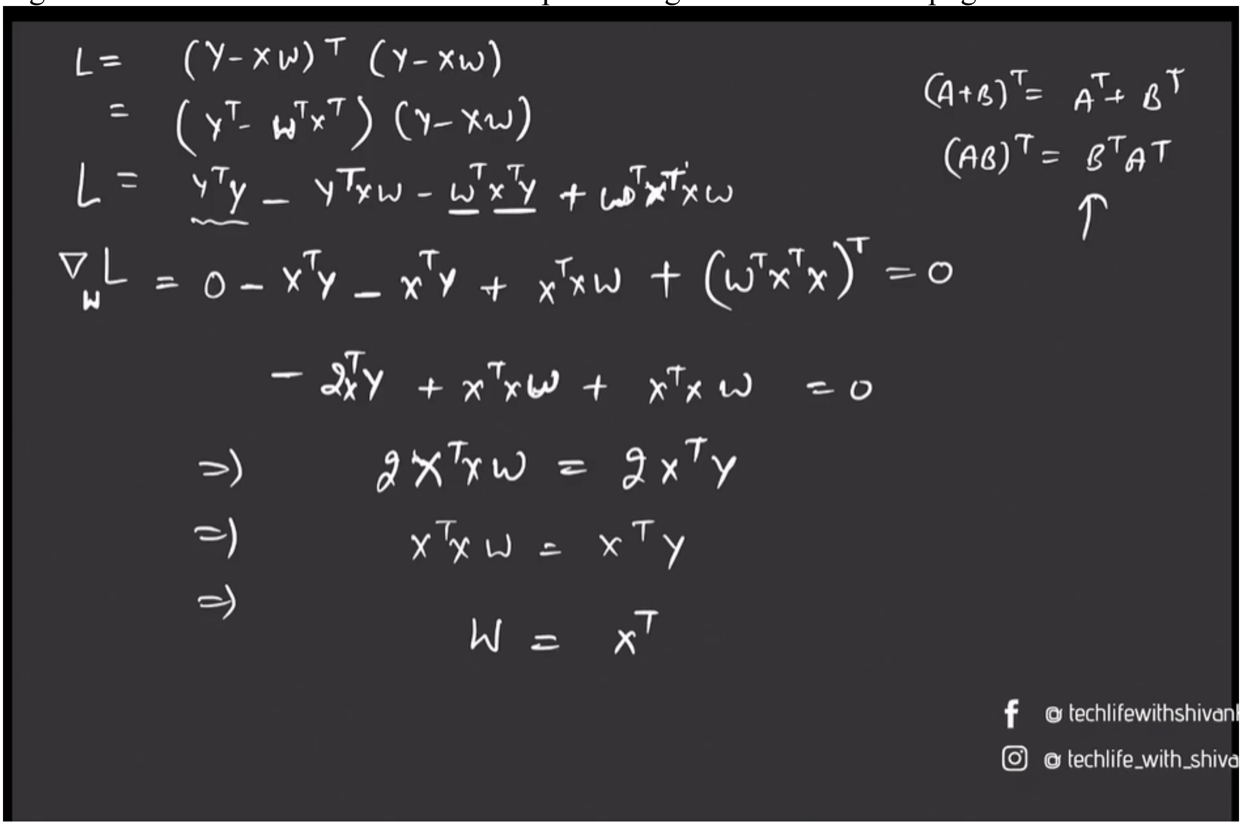


Figure 30:C:\Users\saktheeswaran\Desktop\linearregression\extraa22.png

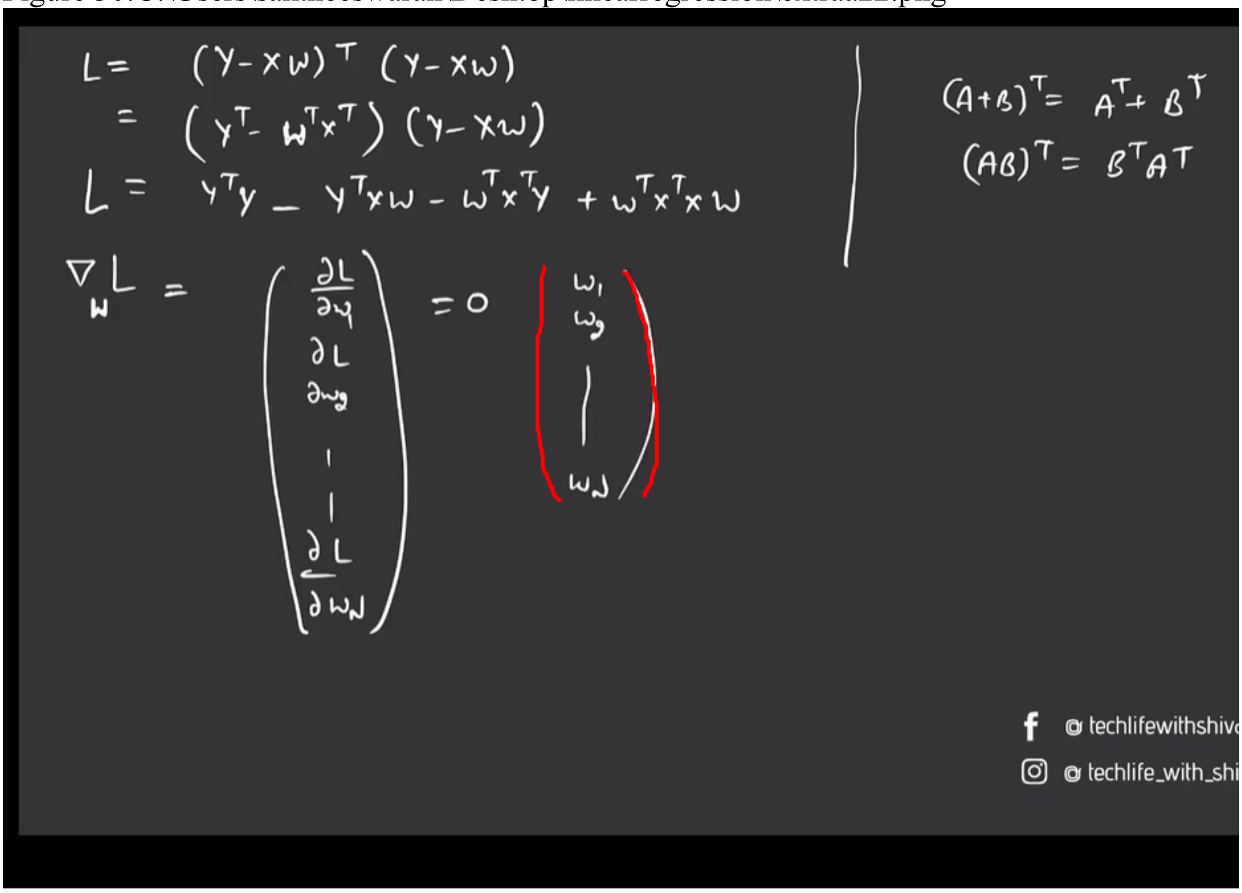


Figure 31:C:\Users\saktheeswaran\Desktop\linearregression\etragghexplain11.png

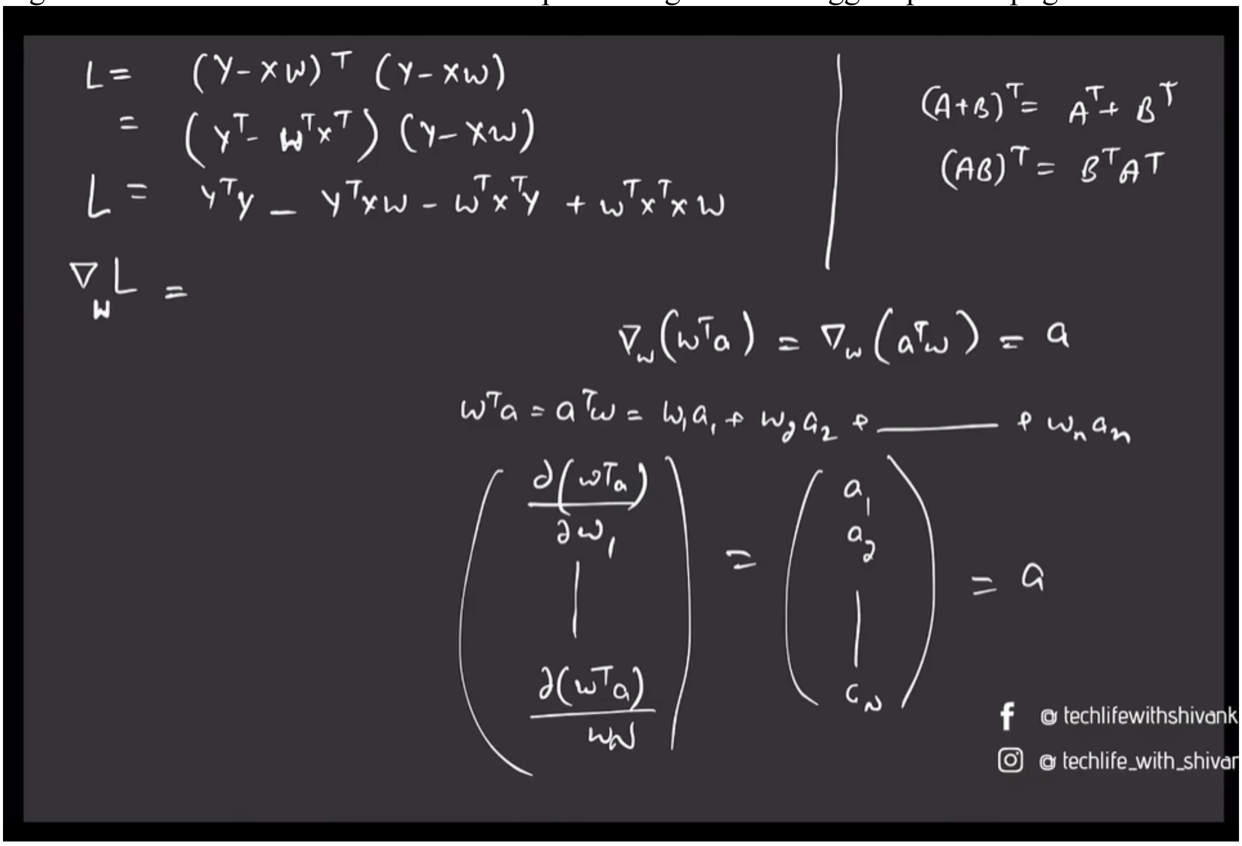


Figure 32:C:\Users\saktheeswaran\Desktop\linearregression\extra23344.png

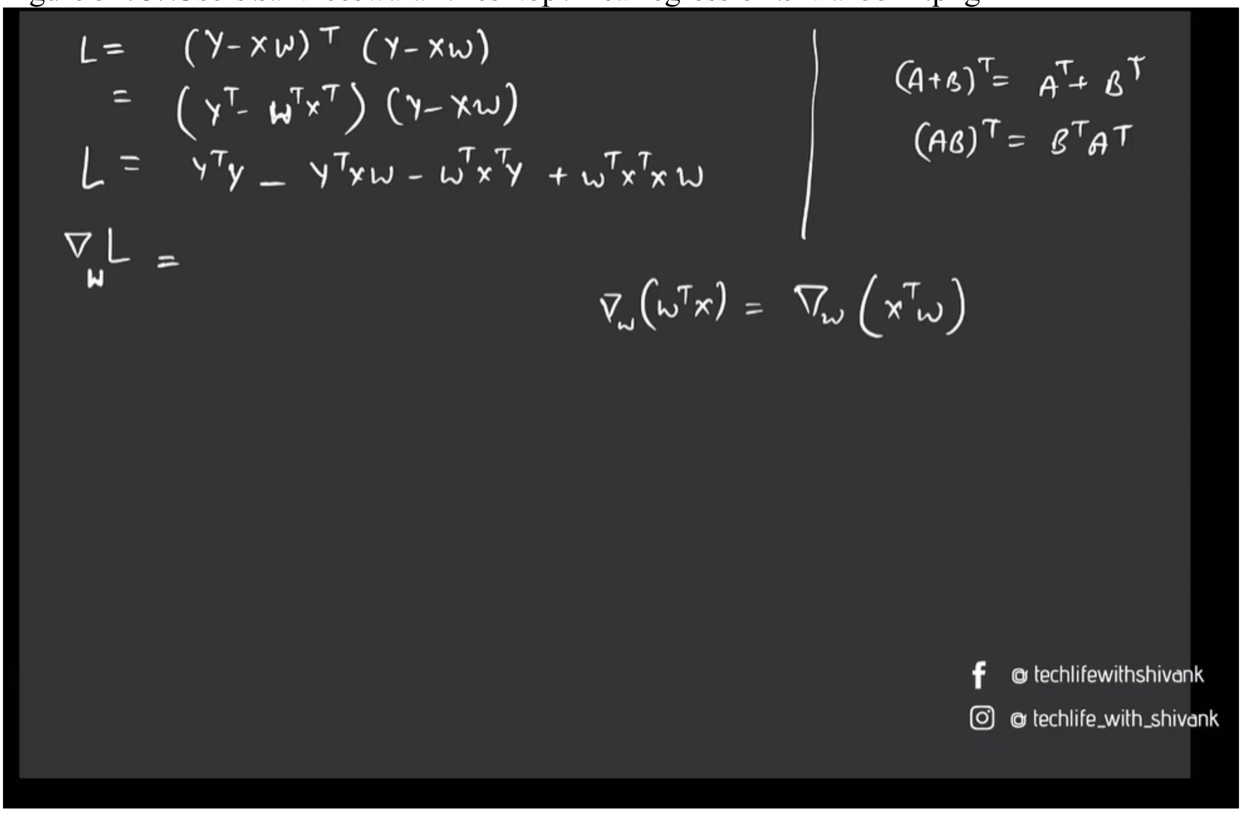


Figure 33:C:\Users\saktheeswaran\Desktop\linearregression\extraa22.png

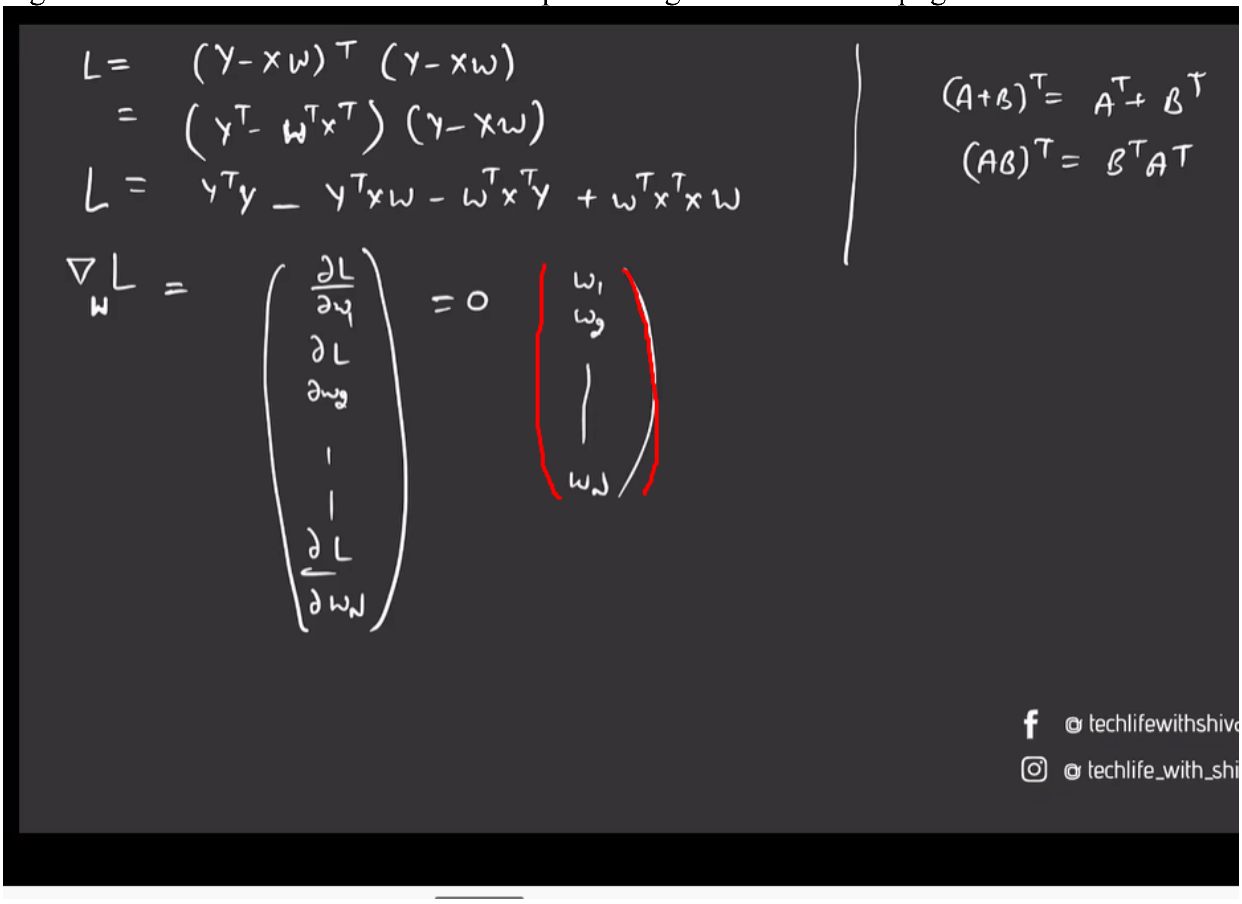


Figure 34:C:\Users\saktheeswaran\Desktop\linearregression\1u.png

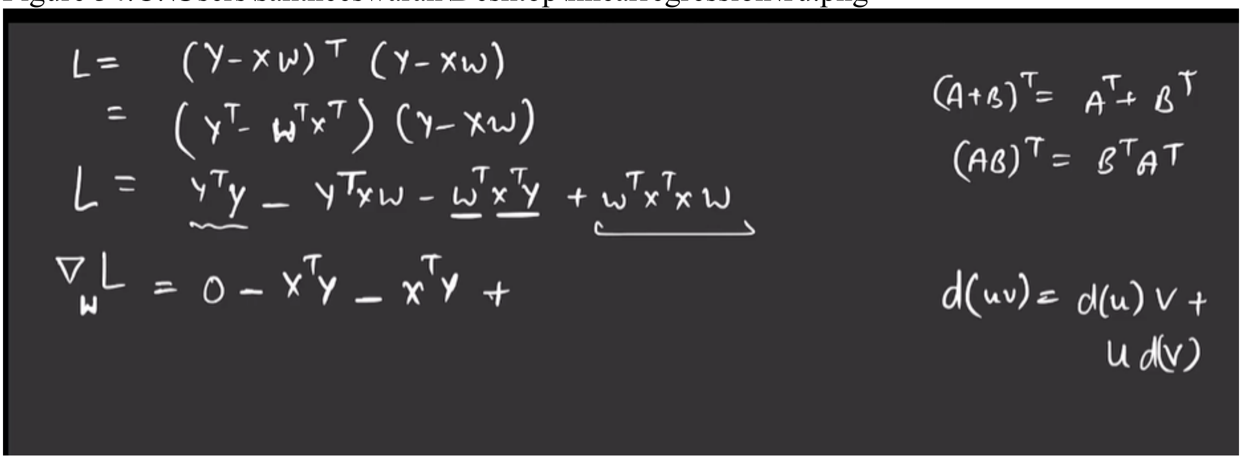


Figure 35:C:\Users\saktheeswaran\Desktop\linearregression\2u.png

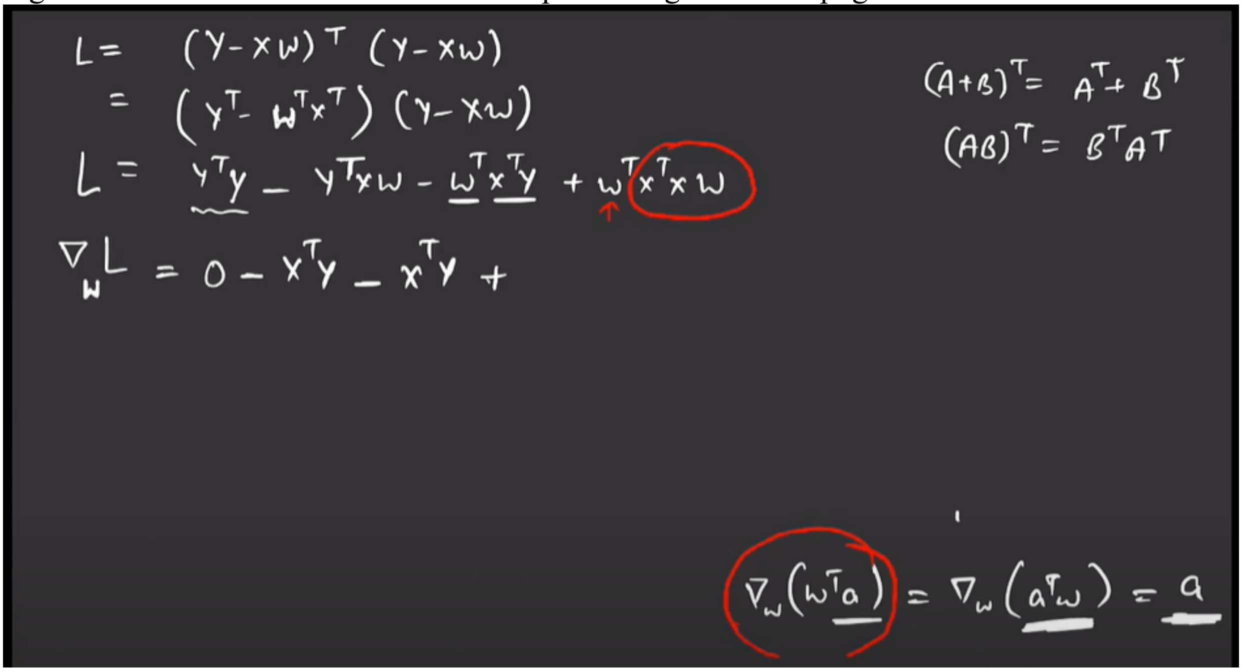


Figure 36:C:\Users\saktheeswaran\Desktop\linearregression\3v.png

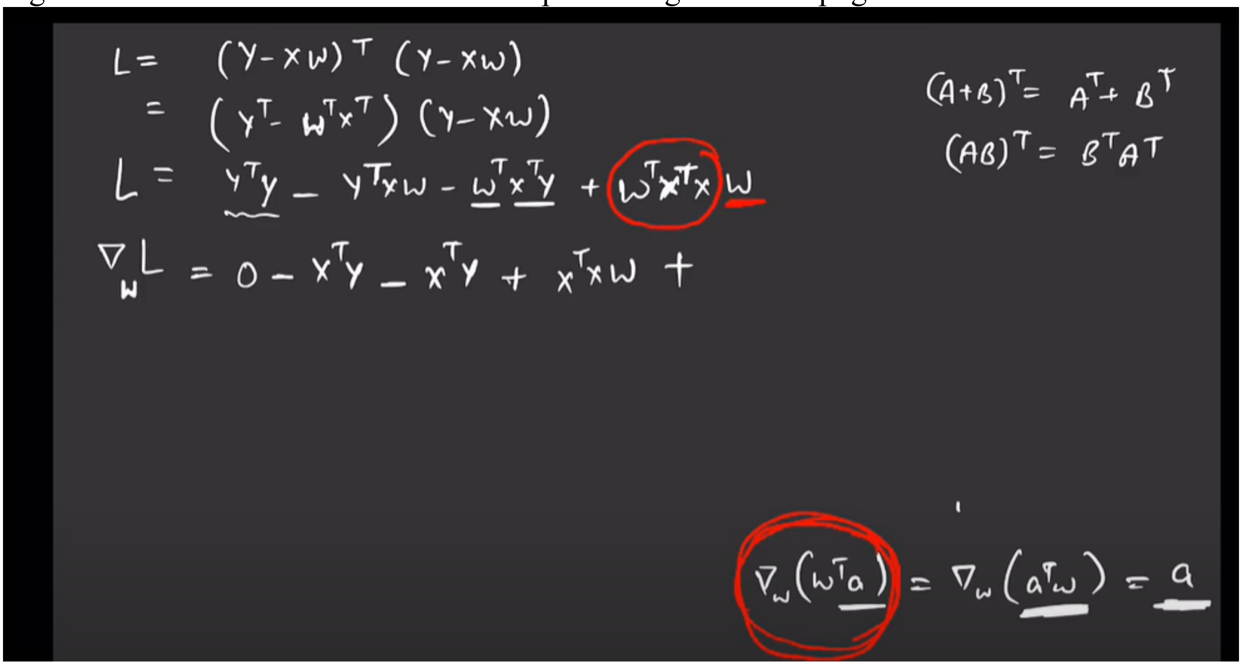


Figure 37:C:\Users\saktheeswaran\Desktop\linearregression\4v.png

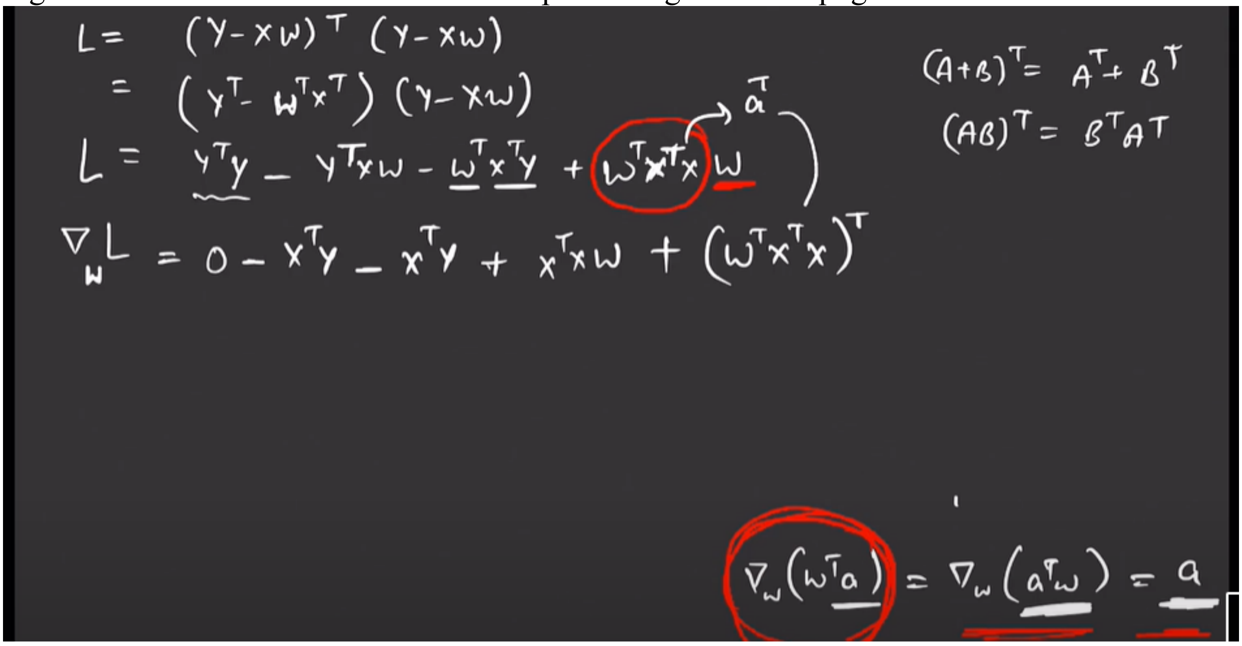


Figure 38:C:\Users\saktheeswaran\Desktop\linearregression\5v.png

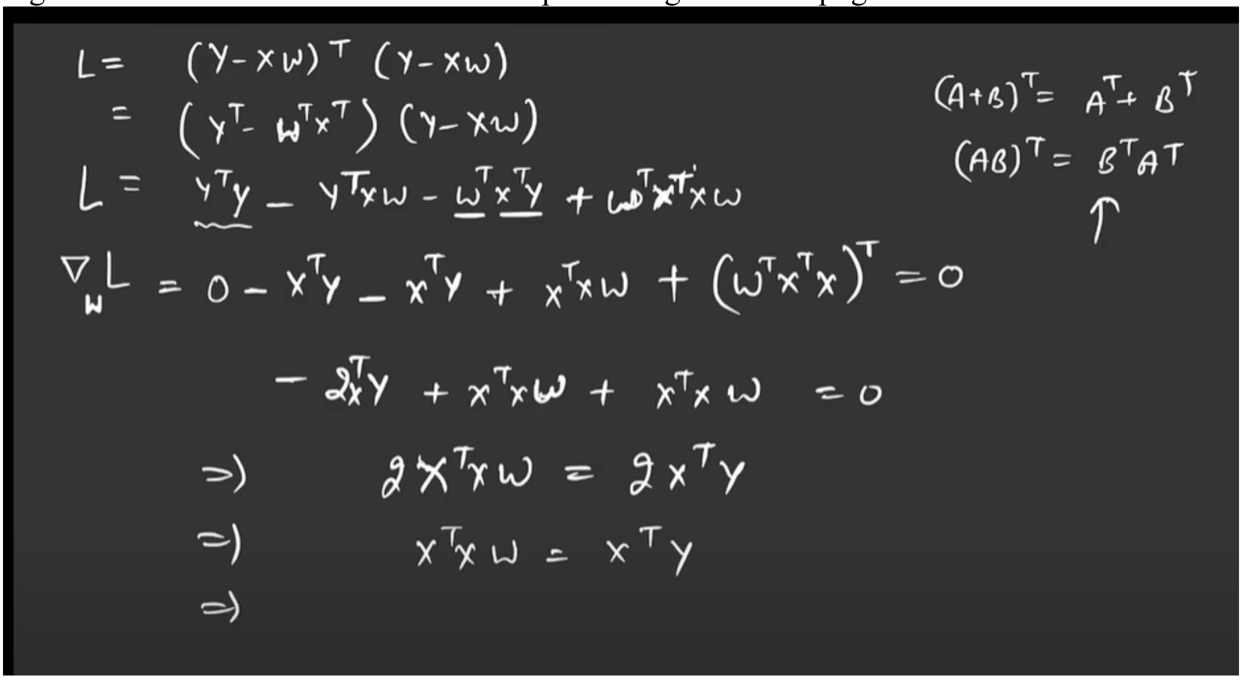


Figure 39:C:\Users\saktheeswaran\Desktop\linearregression\6vvv.png

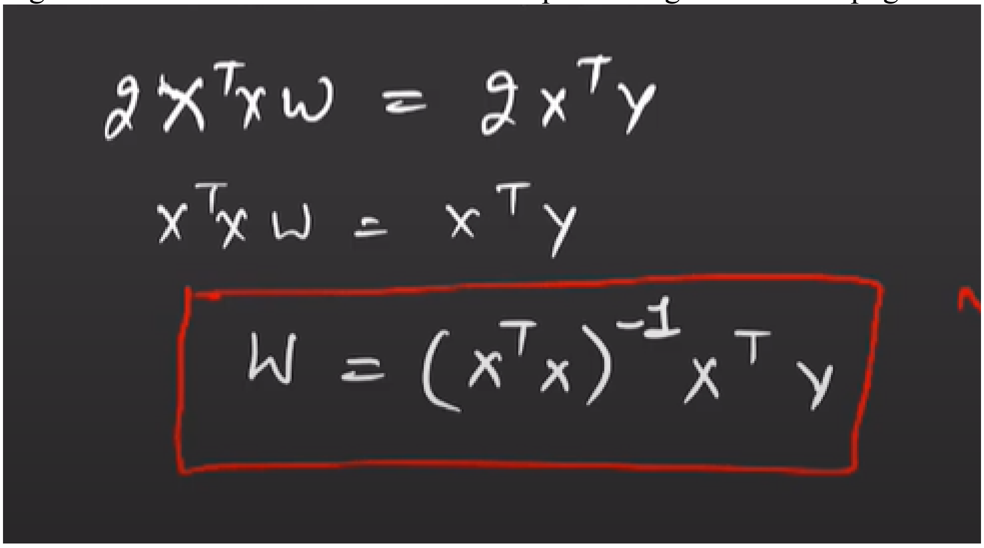


Figure 40:C:\Users\saktheeswaran\Desktop\linearregression\extraa22.png

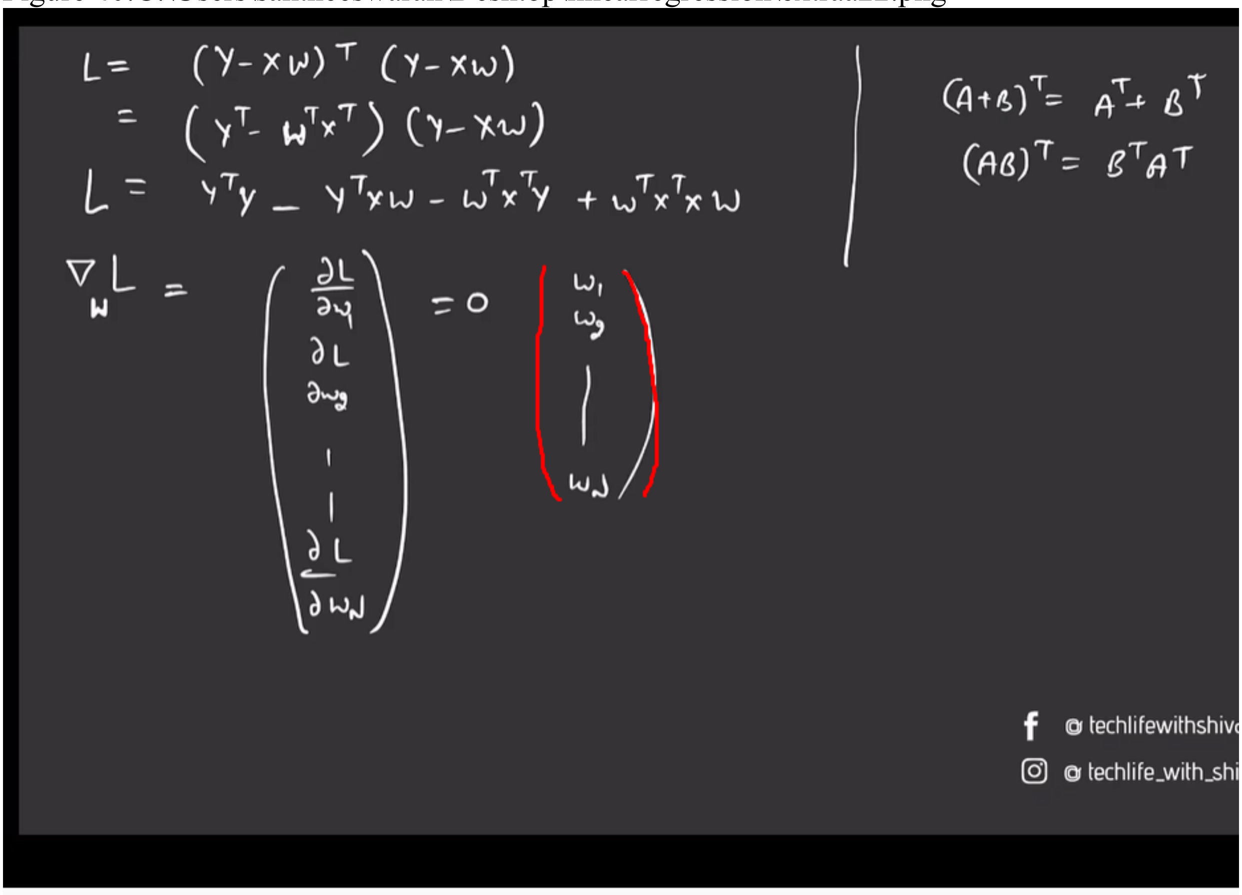


Figure 41:C:\Users\saktheeswaran\Desktop\matrix-calculus.png

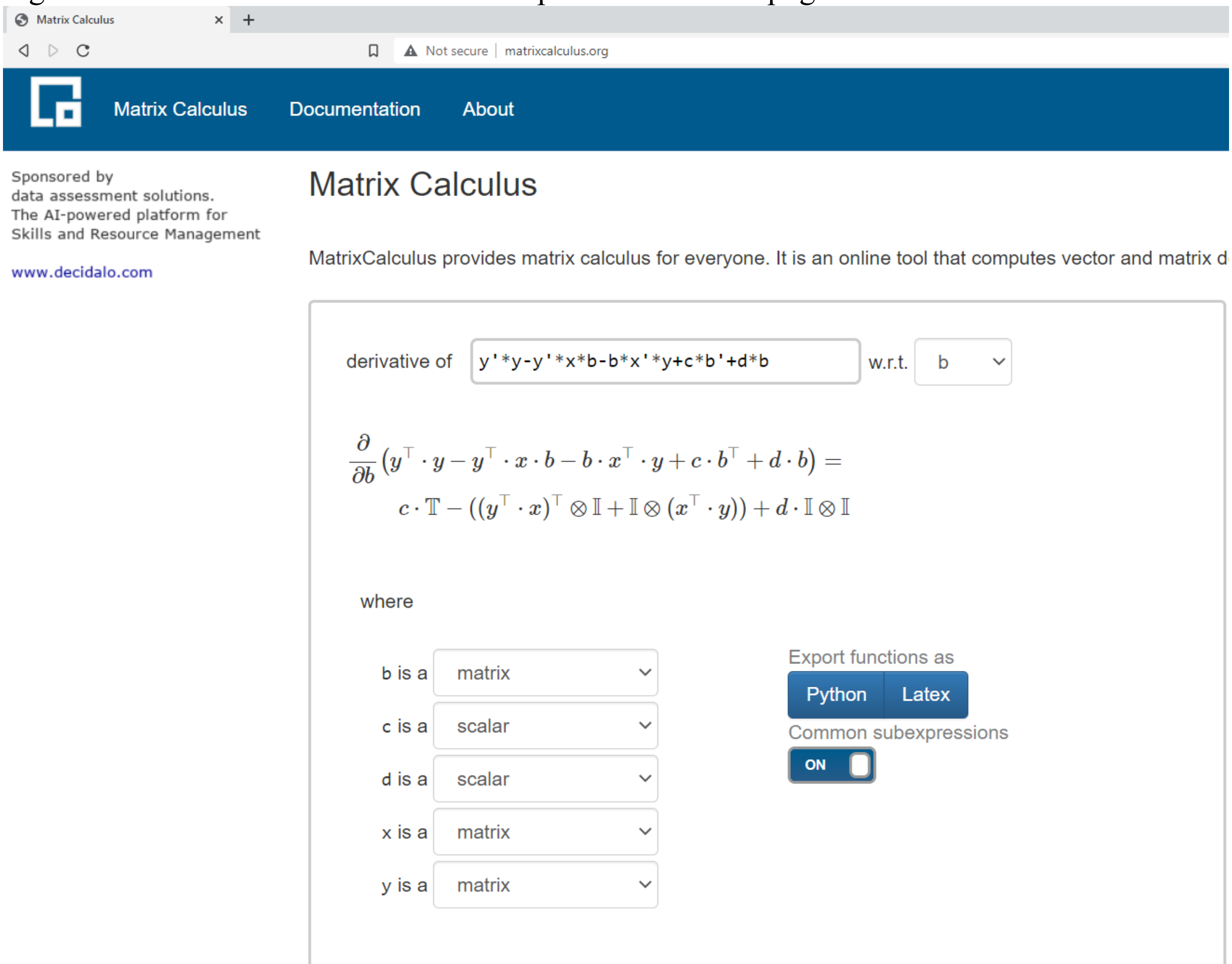
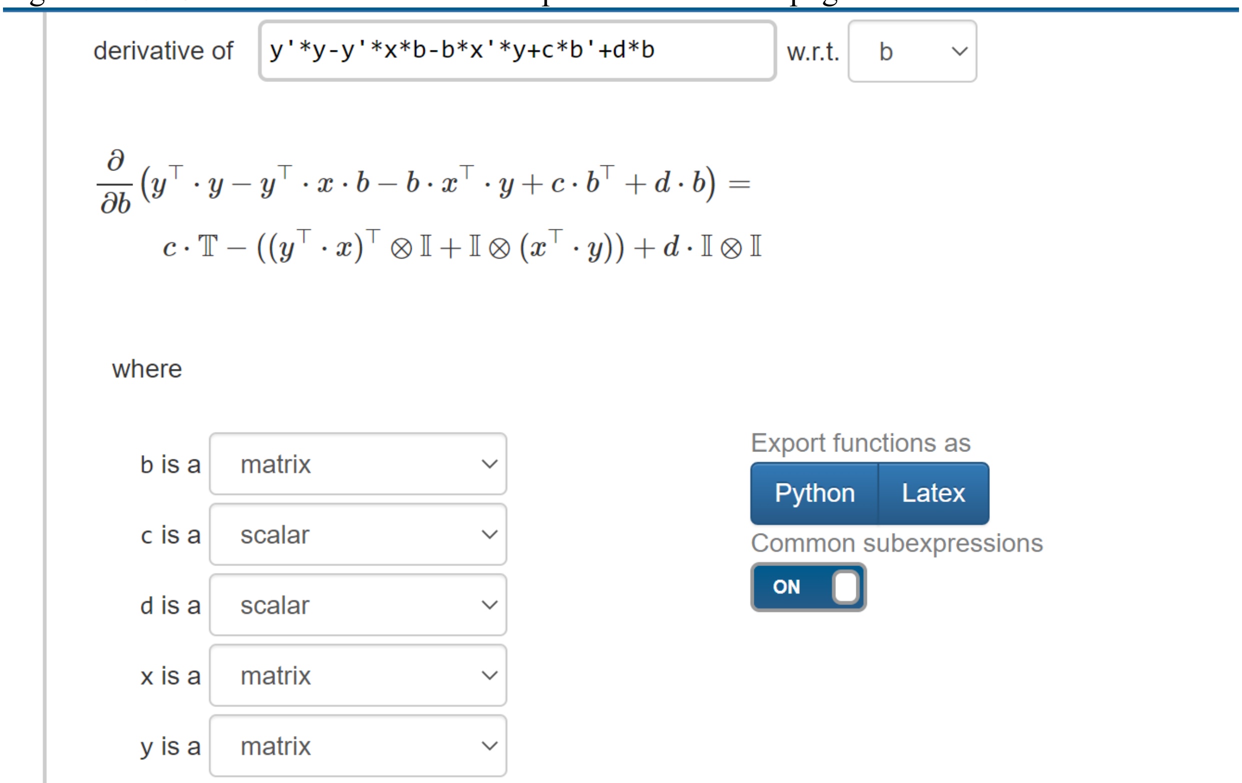


Figure 42:C:\Users\saktheeswaran\Desktop\matrix-calculusd.png



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