Assignment - 1

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***Abstract*—This document contains the solution to Exercise 3.17 of Oppenheimer.**

Problem 1. Consider an LTI system with input x[n] and output y[n] that satisfies the difference equation

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = x[n] - x[n-1] \quad (1)$$

Determine all possible values for the system's impulse response $h[n]$ at n = 0.

Solution: We solve this problem by finding the system function $H(z)$ of the system, and then looking at the different impulse responses which can result from our choice of the ROC.

Taking the z-transform the difference equation, we get

$$Y(z)(1 - \frac{5}{2}z^{-1} + z^{-2}) = X(z)(1 - z^{-1}), \quad (2)$$

and thus

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}} \quad (3)$$

$$= \frac{1 - z^{-1}}{(1 - 2z^{-1})(1 - \frac{1}{2}z^{-1})} \quad (4)$$

$$= \frac{\frac{2}{3}}{1 - 2z^{-1}} + \frac{\frac{1}{3}}{1 - \frac{1}{2}z^{-1}} \quad (5)$$

If the ROC is

(a) $|z| < \frac{1}{2}$:

$$h[n] = -\frac{2}{3}2^n u[-n-1] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[-n-1] \quad (6)$$

$$\implies h[0] = 0. \quad (7)$$

(b) $\frac{1}{2} < |z| < 2$:

$$h[n] = -\frac{2}{3}2^n u[-n-1] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \quad (8)$$

$$\implies h[0] = \frac{1}{3}. \quad (9)$$

(c) $|z| > 2$:

$$h[n] = \frac{2}{3}2^n u[n] + \frac{1}{3}\left(\frac{1}{2}\right)^n u[n] \quad (10)$$

$$\implies h[0] = 1. \quad (11)$$

(d) $|z| > 2or|z| < \frac{1}{2}$:

$$h[n] = \frac{2}{3}2^n u[n] - \frac{1}{3}\left(\frac{1}{2}\right)^n u[n-1] \quad (12)$$

$$\implies h[0] = \frac{2}{3}. \quad (13)$$