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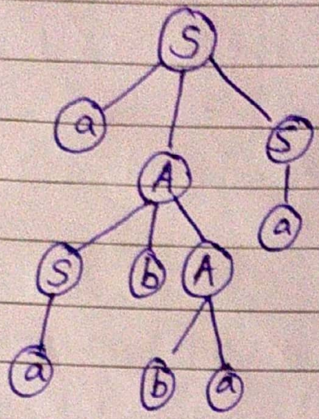
1. $S \rightarrow aAS \mid a$

$A \rightarrow SbA \mid SS \mid ba$

By LMD →

$S \rightarrow aAS \rightarrow aSbAS \rightarrow aabAS \rightarrow aabbaS \rightarrow aabbaa.$

Derivation Tree →



2. $S \rightarrow 0B \mid 1A$

$A \rightarrow 0 \mid 0S \mid 1AA$

$B \rightarrow 1 \mid 1S \mid 0BB$

String = 00110101

(a) LMD →

$S \rightarrow 0B \rightarrow 00BB \rightarrow 001B \rightarrow 0011S \rightarrow 00110B \rightarrow 001101S \rightarrow 0011010B \rightarrow 00110101$

(b) RMD →

$S \rightarrow 0B \rightarrow 00BB \rightarrow 00B1S \rightarrow 00B10B \rightarrow 00B101S \rightarrow 00B1010B \rightarrow 00B10101 \rightarrow 00110101$

(c) Derivation Tree →

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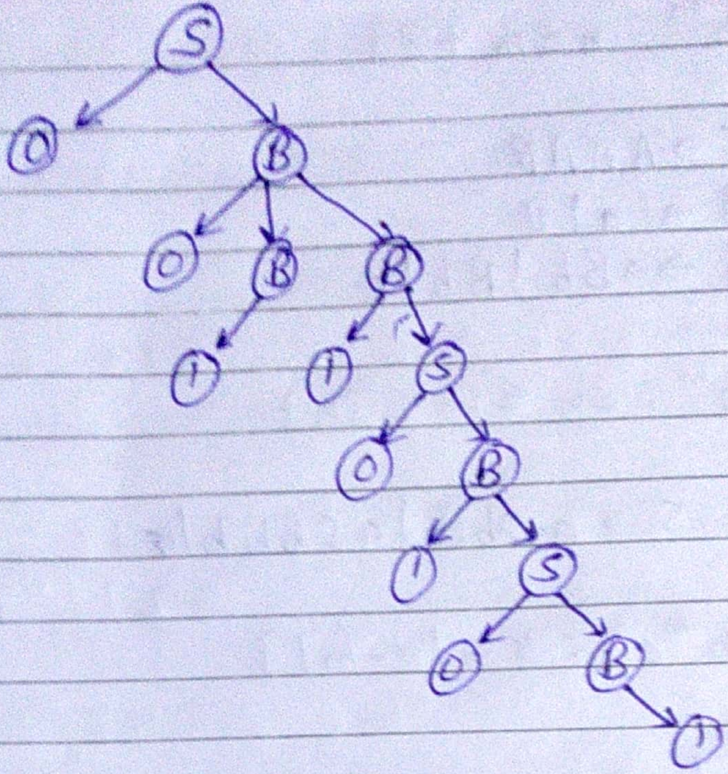

3.

(i) $L = (aaa^* + b)$

$= \{b, aa, aaa, aaaa, \dots\}$

$G(L) = S \rightarrow aaA \mid b$

$A \rightarrow aA \mid \varepsilon$

(ii) $L = \{a^n b^n : n \geq 1\}$

$G(L) = S \rightarrow aSb \mid ab$

(iii) $L = \{a^n b^{n+1} : n \geq 2\}$

$G(L) = S \rightarrow aSb \mid abb$

(iv) $L = ww^R : w \in \{a, b\}^*$

$G(L) = S \rightarrow aSa \mid bSb \mid aa \mid bb \mid \varepsilon$

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(v) $L = \{a^n b^m : n \leq m + 3\}$

$G(L) = S \rightarrow AAAB$
$A \rightarrow a \mid \varepsilon$
$B \rightarrow aBb \mid Bb \mid \varepsilon$

(vi) $L = \{a^n b^m : 2n \leq m \leq 3n\}$

$G(L) = S \rightarrow aSbb \mid aSbbb \mid \varepsilon$

(vii) $L = \{a^n b^m c^k : k = |n - m|\}$

$k = |n - m|$
$k = n - m$ for $n \geq m$ or $k = m - n$ for $m \geq n$
$n = k + m$ or $m = k + n$

$L = L_1 \cup L_2$ where,
$L_1 = \{a^n b^m c^k : n = k + m\}$ & $L_2 = \{a^n b^m c^k : m = k + n\}$

$G(L_1) = S_1 \rightarrow T_1 \mid T_2$
$T_1 \rightarrow aT_1 b \mid \varepsilon$
$T_2 \rightarrow aT_2 c \mid T_1 \mid \varepsilon$

$G(L_2) = S_2 \rightarrow T_3 \mid T_4$
$T_3 \rightarrow aT_3 b \mid \varepsilon$
$T_4 \rightarrow bT_4 c \mid \varepsilon$

$G(L) = S \rightarrow S_1 \mid S_2$
$S_1 \rightarrow T_1 \mid T_2$
$T_1 \rightarrow aT_1 b \mid \varepsilon$
$T_2 \rightarrow aT_2 c \mid T_1 \mid \varepsilon$
$S_2 \rightarrow T_3 \mid T_4$
$T_3 \rightarrow aT_3 b \mid \varepsilon$
$T_4 \rightarrow bT_4 c \mid \varepsilon$

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4. $S \rightarrow aSbS \mid bSaS \mid \varepsilon$

Using Left Most Derivative

$LMD_1 \rightarrow S \rightarrow aSbS \rightarrow abSaSbS \rightarrow abaSbS \rightarrow ababS$
$\rightarrow abab$

$LMD_2 \rightarrow S \rightarrow aSbS \rightarrow abS \rightarrow abaSbS \rightarrow ababS \rightarrow abab$

We have have two LMD for same string
Hence grammar is a ambiguous

5. $S \rightarrow AB \mid aaB$,
$A \rightarrow a \mid Aa$,
$B \rightarrow b$

$LMD_1 \rightarrow S \rightarrow AB \rightarrow AaB \rightarrow aaB \rightarrow aab$
$LMD_2 \rightarrow S \rightarrow aaB \rightarrow aab$

Hence given grammar is ambiguous.

6. A regular grammar is also a context free grammar as regular grammar is a subset of context free grammar. So a regular grammar may or may not be ambiguous depending upon the language given in a grammar but if a grammar is having only regular languages it can never be ambiguous or inherently ambiguous.

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7. $E \rightarrow E+E$

$E \rightarrow E*E$

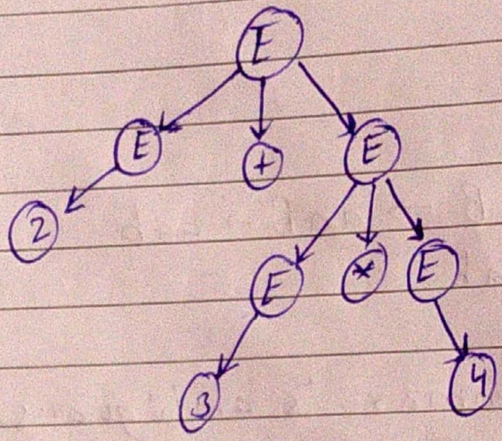
$E \rightarrow id(2|3|4)$

Input $\rightarrow 2+3*4$

LMD $\rightarrow E \rightarrow E+E \rightarrow 2+E \rightarrow 2+E*E \rightarrow 2+3*E \rightarrow 2+3*4$

RMD $\rightarrow E \rightarrow E+E \rightarrow E+E*E \rightarrow E+E*4 \rightarrow E+3*4 \rightarrow 2+3*4$

Derivation Tree $\rightarrow$



8.

(i) $S \rightarrow AB,$

$A \rightarrow a,$

$B \rightarrow b,$

$B \rightarrow C,$

$E \rightarrow c$

$V_1' = \{A, B, E\}$ $\quad A \rightarrow a, B \rightarrow b, E \rightarrow c$

$V_2' = \{A, B, E, S\}$ $\quad S \rightarrow AB$

$V_3' = \{A, B, E, S\}$

$= V_2'$

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We get,

$V' = \{S, A, B, E\}$

$P' = S \to AB,\ A \to a,\ B \to b \ \& \ E \to c$

After phase 1 →

$G' \to S \to AB,\ A \to a,\ B \to b \ \& \ E \to c$

$V_1'' = \{S\}$

$V_2'' = \{S, A, B\}$ $\quad S \to AB$

$V_3'' = \{S, A, B\}$ $\quad A \to a,\ B \to b$

$= V_2''$

We get,

$V'' = \{S, A, B\}$

$P'' = S \to AB,\ A \to a,\ B \to b$

(ii) $S \to AB \mid CA$

$B \to BC \mid AB$

$A \to a$

$C \to aB \mid b$

Phase 1 $\quad V_1' = \{A, C\}$

$V_2' = \{A, C, S\}$

$V_3' = \{A, C, S\}$

$= V_2'$

$S' = \{\{A, C, S\}, \{a, b\}, P', \{S\}\}$

where $P' = S \to CA,\ A \to a,\ C \to b$

Phase 2 $\quad V_1'' = \{S\}$

$V_2'' = \{S, A, C\}$

$V_3'' = \{S, A, C\} = V''$

$S_2'' = \{\{S, A, C\}, \{a, b\}, P'', \{S\}\}$

where $P'' = S \to CA,\ A \to a,\ C \to b$

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(iii) $S \to aAa,$
$A \to Sb | bCC | DaA,$
$C \to abb | DD,$
$E \to aC,$
$D \to aDA$

Phase 1 $V_1' = \{C\}$
$V_2' = \{C, A, E\}$
$V_3' = \{S, C, A, E\}$
$V_4' = \{S, C, A, E\} = V_3'$
$G' = \{\{S, C, A, E\}, \{a, b\}, P', \{S\}\}$
where $P' = S \to aAa, A \to Sb | bCC, C \to abb, E \to aC$

Phase 2 $V_1'' = \{S\}$
$V_2'' = \{S, A\}$
$V_3'' = \{S, A, C\}$
$V_4'' = \{S, A, C\} = V_3''$

$G'' = \{\{S, A, C\}, \{a, b\}, P'', \{S\}\}$
where $P'' = \{S \to aAa, A \to Sb | bCC, C \to abb$

9.
(i) $S \to ASB | \varepsilon$
$A \to aAS | a$
$B = bb | A | Sbs$

Remove ε production $\to$
$S \to ASB$
$A \to aAS | a$
$B \to bb | A | Sbs$

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Remove Unit production →

$S \to ASB$
$A \to aAS \mid a$
$B \to bb \mid SbS$

Remove Useless symbols

There is no ~~useles~~ useless symbols

$S \to ASB$ Let $P \to AS$
$\Rightarrow S \to PB$

$A \to aAS$ Let $Q \to aA$
$\Rightarrow A \to QS$ Let $X \to a$
$Q \to XA$

$B \to bb$
$\Rightarrow B \to YY$ Let $Y \to b$

$B \to SbS$ Let $R \to SY$
$B \to SYS$
$\Rightarrow B \to RS$

So given grammar into CNF is

$S \to PB$, $P \to AS$, $A \to QS$, $Q \to aA$, $X \to a$, $Q \to XA$,
$B \to YY$, $Y \to b$, $B \to RS$, $R \to SY$

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(ii) $S \to 0A0 \mid 1B1 \mid BB$

$A \to C$

$B \to S \mid A$

$C \to S \mid \varepsilon$

Remove ε-productions $\to$

$S \to 00 \mid 0A0 \mid 11 \mid 1B1 \mid B \mid BB$

$A \to C$

$B \to S \mid A$

$C \to S$

Remove unit productions $\to$

$S \to 00 \mid 0A0 \mid 1B1 \mid BB$

$A \to 00 \mid 0A0 \mid 1B1 \mid BB$

$B \to 00 \mid 0A0 \mid 1B1 \mid BB$

$C \to 00 \mid 0A0 \mid 1B1 \mid BB$

Remove Useless Symbols $\to$

Remove, A, B, C $\to$

$S \to 00 \mid 0S0 \mid 1S1 \mid SS$

$S \to 00$ Let $P \to 0$

$\Rightarrow S \to PP$

$S \to 0S0$

$S \to PPS$ Let $Q \to PS$

$\Rightarrow S \to PQ$

$S \to 1S1$ Let $R \to 1$

$S \to RSR$

$\Rightarrow S \to RT$ Let $T \to SR$

$\Rightarrow S \to SS$

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CNF grammar is →

~~$S \to AA \mid A$~~

$S \to PP \mid PQ \mid RT \mid SS$

$P \to 0$

$Q \to PS$

$R \to 1$

$T \to SR$

(iii) $S \to AAA \mid B,$

$A \to aA \mid B,$

$B \to \varepsilon$

Remove ε-productions →

$S \to AAA \mid B \mid \varepsilon$

$A \to aA \mid B \mid \varepsilon$

$\downarrow$

$S \to AA \mid A \mid AAA \mid B \mid \varepsilon$

$A \to aA \mid a \mid B$

$\downarrow$

$S \to AA \mid A \mid AAA \mid B$

$A \to aA \mid a \mid B$

Remove unit productions →

$S \to AAA \mid AA \mid aA \mid a$

$A \to aA \mid a$

CNF → $S \to S_1A \mid AA \mid A_1A \mid a$

$S_1 \to AA$

$A_1 \to a$

$A \to A_1A \mid a$

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(iv) $S \to aAa \mid bBb \mid \varepsilon$
$A \to C \mid a$
$B \to C \mid b$
$C \to CDE \mid \varepsilon$
$D \to B \mid A \mid ab$

Removing ε-production →

$S \to aAa \mid bBb$
$A \to C \mid a \mid \varepsilon$
$B \to C \mid b \mid \varepsilon$
$C \to CDE \mid DE$
$D \to B \mid A \mid ab$

↓

$S \to aAa \mid aa \mid bBb \mid bb$
$A \to C \mid a$
$B \to C \mid b$
$C \to CDE \mid DE$
$D \to A \mid B \mid ab \mid \varepsilon$

↓

$S \to aAa \mid bBb \mid aa \mid bb$
$A \to C \mid a$
$B \to C \mid b$
$C \to CDE \mid DE \mid CE \mid E$
$D \to A \mid B \mid ab$

Removing Unit Production →

$S \to aAa \mid bBb \mid aa \mid bb$
$A \to CDE \mid DE \mid CE \mid a$
$B \to CDE \mid DE \mid CE \mid b$
$C \to CDE \mid DE \mid CE$

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$D \to CDE \mid DE \mid CE \mid a \mid b \mid ab$

Removing useless production :-

Since E is no variable with any production so all productions with E are useless.

$S \to aAa \mid bBb \mid aa \mid bb$
$A \to a$
$B \to b$
$D \to a \mid b \mid ab$

D cannot be reached from S hence it is also useless

$S \to aAa \mid bBb \mid aa \mid bb$
$A \to a$
$B \to b$

Converting to CNF -

$S \to S_1 A_1 \mid S_2 B_1 \mid A_1 A_1 \mid B_1 B_1$,
$S_1 \to A_1 A$,
$S_2 \to B_2 B$
$A_1 \to a$
$B_1 \to b$

10. (i) $S \to aSb \mid bSa \mid a \mid b$

GNF Conversion

$S \to aSB \mid bSA \mid a \mid b$
$B \to b$
$A \to a$

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(ii) $S \to aSb|ab$

GNF $\to$

$S \to aSB|aB$

$B \to a$

(iii) $S \to ab|aS|aaS$

GNF $\to$

$S \to aB|aS|aAS$

$B \to b$

$A \to a$

(iv) $S \to ABb|a$

$A \to aaA|B$

$B \to bAB$

Removing unit production:-

$S \to ABb|a$

$A \to aaA|bAB.$

$B \to bAB$

GNF $\to$

$S \to aA_1ABB_1 | bABABB_1|a$

$A \to aA_1A|bAB$

$B \to bAB$

$A_1 \to a$

$B_1 \to b$