Book of Carcione, third edition (2014):

$$\psi(t) = M_R \left[1 - \frac{1}{L} \sum_{l=1}^{L} \left(1 - \frac{\tau_{\epsilon l}}{\tau_{\sigma l}} \right) \exp(-t/\tau_{\sigma l}) \right] H(t). \tag{2.198}$$

The unrelaxed modulus is obtained for $t = 0$,

$$M_U = M_R \left[1 - \frac{1}{L} \sum_{l=1}^{L} \left(1 - \frac{\tau_{\epsilon l}}{\tau_{\sigma l}} \right) \right] = \frac{M_R}{L} \sum_{l=1}^{L} \frac{\tau_{\epsilon l}}{\tau_{\sigma l}}. \tag{2.199}$$

= Kr

$$\psi_{\mathcal{K}}(t) = \mathcal{K}_\infty \left[1 - \frac{1}{L_1} \sum_{l=1}^{L_1} \left(1 - \frac{\tau_{\epsilon l}^{(1)}}{\tau_{\sigma l}^{(1)}} \right) \exp(-t/\tau_{\sigma l}^{(1)}) \right] H(t), \tag{3.186}$$

$$\psi_{\mu}(t) = \mu_\infty \left[1 - \frac{1}{L_2} \sum_{l=1}^{L_2} \left(1 - \frac{\tau_{\epsilon l}^{(2)}}{\tau_{\sigma l}^{(2)}} \right) \exp(-t/\tau_{\sigma l}^{(2)}) \right] H(t), \tag{3.187}$$

where $\tau_{\epsilon l}^{(\nu)}$ and $\tau_{\sigma l}^{(\nu)}$ are relaxation times corresponding to dilatational ($\nu = 1$) and shear ($\nu = 2$) attenuation mechanisms. They satisfy the condition (2.169), $\tau_{\epsilon l}^{(\nu)} \geq \tau_{\sigma l}^{(\nu)}$, with the equal sign corresponding to the elastic case.

In terms of the Boltzmann operation (2.6), equation (3.143) reads

$$\sigma_{ij} = \psi_{\mathcal{K}} \odot \epsilon_{kk} \delta_{ij} + 2\psi_{\mu} \odot d_{ij}, \tag{3.188}$$

or

$$\sigma_{ij} = \mathcal{K}_U \left(\epsilon_{kk} + \sum_{l=1}^{L_1} e_l^{(1)} \right) \delta_{ij} + 2\mu_U \left(d_{ij} + \sum_{l=1}^{L_2} e_{ijl}^{(2)} \right), \tag{3.189}$$

where

= Kr

$$\mathcal{K}_U = \frac{\mathcal{K}_\infty}{L_1} \sum_{l=1}^{L_1} \frac{\tau_{\epsilon l}^{(1)}}{\tau_{\sigma l}^{(1)}}, \quad \mu_U = \frac{\mu_\infty}{L_2} \sum_{l=1}^{L_2} \frac{\tau_{\epsilon l}^{(2)}}{\tau_{\sigma l}^{(2)}} \tag{3.190}$$

As in the 1D case (see equation (2.329)), the memory variables satisfy

$$e_l^{(1)} = \varphi_{1l}(0)\epsilon_{kk} - \frac{e_l^{(1)}}{\tau_{\sigma l}^{(1)}}, \quad e_{ijl}^{(2)} = \varphi_{2l}(0) d_{ij} - \frac{e_{ijl}^{(2)}}{\tau_{\sigma l}^{(2)}}. \tag{3.194}$$

d / dt missing