Primer parcial

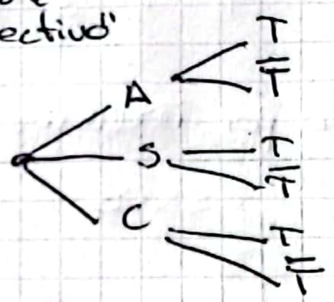
① defino los eventos

T = "Franco llego a tiempo"

A = "Franco eligio auto"

S = "franco — sobte"

C = "franco — colectivo"



$\mathbb{P}(\bar{T}) = 0,2$

$\mathbb{P}(C) = 0,3$

$\mathbb{P}(S) = 0,5$

dedozco.

$\mathbb{P}(T|S) = 0,9 \longrightarrow \mathbb{P}(\bar{T}|S) = 1 - \mathbb{P}(T|S) = 1 - 0,9 = 0,1$

$\mathbb{P}(T|C) = 0,7 \longrightarrow \mathbb{P}(\bar{T}|C) = 1 - \mathbb{P}(T|C) = 1 - 0,7 = 0,3.$

$\mathbb{P}(\bar{T}|A) = ?$ $\qquad \mathbb{P}(A) = 1 - (\mathbb{P}(C) + \mathbb{P}(S)) = 1 - 0,8 = 0,2$

$$\mathbb{P}(\bar{T}) \overset{\text{prob total}}{=} \mathbb{P}(\bar{T}|A)\,\mathbb{P}(A) + \mathbb{P}(\bar{T}|S)\,\mathbb{P}(S) + \mathbb{P}(\bar{T}|C)\,\mathbb{P}(C).$$

despejo y calculo

$$\mathbb{P}(\bar{T}|A) = \frac{\mathbb{P}(\bar{T}) - \mathbb{P}(\bar{T}|S)\,\mathbb{P}(S) - \mathbb{P}(\bar{T}|C)\,\mathbb{P}(C)}{\mathbb{P}(A)} = \frac{0,2 - (0,1 \cdot 0,5) - (0,3 \cdot 0,3)}{0,2}$$

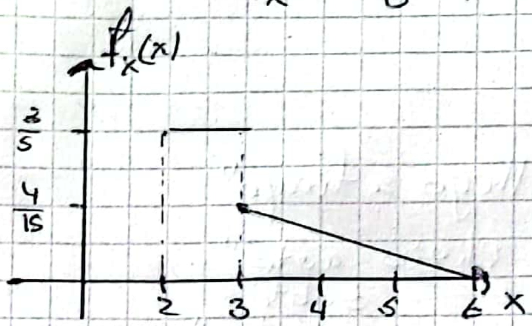
$$\boxed{\mathbb{P}(\bar{T}|A) = \frac{0,06}{0,2} = \frac{3}{10}}$$

Escaneado con CamScanner

Primer parcial

(2)

$$f_X(x) = \frac{3}{5}\,\mathbb{1}\{2 \le x < 3\} + \frac{24-4x}{45}\,\mathbb{1}\{3 \le x < 6\}$$



$$f_X(x) = \begin{cases} \frac{3}{5} & 2 \le x < 3 \\ \frac{24-4x}{45} & 3 \le x < 6 \\ 0 & \text{otro.} \end{cases}$$

Calculo por intervalos

(*) si $2 \le x < 3$

$$F_X(x) = \int_2^x \frac{3}{5}\,dx = \frac{3}{5}(x-2).$$

(**) si $3 \le x < 6$

wolfram alpha

$$F_X(x) = \int_2^3 \frac{3}{5}\,dx + \int_3^x \frac{24-4x}{45}\,dx = -\frac{2x^2}{45} + \frac{8x}{15} - \frac{3}{5}$$

$$\boxed{F_X(x) = \begin{cases} 0 & x < 2 \\ \frac{3}{5}(x-2) & 2 \le x < 3 \\ -\frac{2x^2}{45} + \frac{8x}{15} - \frac{3}{5} & 3 \le x < 6 \\ 1 & x > 6. \end{cases}}$$

para corroborar integro f_X en todo su dominio y deberia dar 1

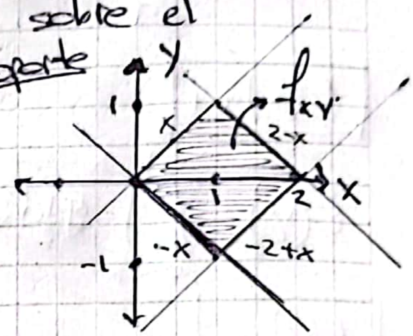
$$\int_2^3 \frac{3}{5}\,dx + \int_3^6 \frac{24-4x}{45}\,dx = 1 \checkmark$$

Segundo parcial

① → guía 3

(X,Y) v.a con distribución uniforme sobre el cuadrado $(0,0),(1,1),(0,2),(-1,1)$

soporte



Al tener distribución uniforme su función de densidad conjunta queda.

$$f_{XY}(x,y) = \frac{1}{\text{Area}} \mathbb{1}\{(x,y)\in \text{sop}\}$$

Area = 4 . $\overset{\text{Areas}}{\text{triangulos}}$ Area triangulo = $\frac{1 . 1}{2} = \frac{1}{2}$

Area = $\frac{4.}{2} = 2$.

$$f_{XY} = \frac{1}{2} \mathbb{1}\{(x,y)\in \text{sop}\}$$

$$\text{cov}(3X-Y, X-1) \underset{\text{Prop}}{=} \overset{*}{\text{cov}(3X, X-1)} - \overset{**}{\text{cov}(Y, X-1)}$$

(*) $\text{cov}(3X, X-1) = 3\,\text{cov}(XX) - 3\,\cancel{\text{cov}(X,-1)}^{0} = 3\,\text{cov}(X X) = 3\,\text{var}(X)$

$= 3\,\text{var}(X) = 3\left(E(X^2) - E^2(X)\right)$

(**) $\text{cov}(Y, X-1) = \text{cov}(Y X) - \cancel{\text{cov}(Y,-1)}^{0} = \text{cov}(YX)$.

$= \text{cov}(YX) = E(YX) - E(Y)E(X)$.

$\Rightarrow$ (***) $\text{cov}(3X-Y, X-1) = 3\left(E(X^2) - E^2(X)\right) - \left(E(XY) - E(Y)E(X)\right)$

como me quedaba solo
Calcular las esperanzas
las obtengo y resuelvo.

$$E(X) = \iint_{sop} x \, f_{XY}(xy)\, dy\, dx = \int_0^1 \int_{-x}^{x} x \tfrac{1}{2}\, dy\, dx + \int_1^2 \int_{x-2}^{-x+2} x \tfrac{1}{2}\, dy\, dx = 1.$$

$$\boxed{E(X) = 1}$$

$$E(Y) = \int_0^1 \int_{-x}^{x} y \tfrac{1}{2}\, dy\, dx + \int_1^2 \int_{x-2}^{-x+2} y \tfrac{1}{2}\, dy\, dx = 0$$

$$\boxed{E(Y) = 0.}$$

$$E(X^2) = \int_0^1 \int_{-x}^{x} \frac{x^2}{2}\, dy\, dx + \int_1^2 \int_{x-2}^{-x+2} \frac{x^2}{2}\, dy\, dx = \frac{7}{6}.$$

$$\boxed{E(X^2) = \frac{7}{6}}$$

$$E(XY) = \int_0^1 \int_{-x}^{x} xy \tfrac{1}{2}\, dy\, dx + \int_1^2 \int_{x-2}^{-x+2} xy \tfrac{1}{2}\, dy\, dx = 0.$$

$$\boxed{E(XY) = 0}$$

reemplazo y resuelvo (***)

$$\Rightarrow cov(3X-Y, X-1) = 3(E(X^2) - E^2(X)) - (\overset{0}{E(XY)} - E(X)\overset{0}{E(Y)}).$$

$$= 3\left(\frac{7}{6} - 1\right) - 0 = \frac{1}{2}$$

$$\boxed{cov(3X-Y, X-1) = \frac{1}{2}}$$