HOJA Nº 1

FECHA

GUÍA 5

5.1

URNA $\begin{cases} 4V \\ 3A \\ 3R \end{cases}$ EXTRAIGO 3

LO PUEDO HACER CON LA HIPERGEOM.

$$P(x,y) = \frac{\binom{\ }{x}\binom{\ }{y}\binom{\ }{-x-y}}{\binom{\ }{\ }}$$

X "CANT BOLAS VERDES"

Y : "CANT BOLAS ROJAS

$$P_{Y/X=x_0} = \frac{P_{XY}(x_0,Y)}{P_X(x_0)}$$

x/y	0	1	2	3	$P_X(x)$
0	1/120	3/40	3/40	1/120	1/6
1	1/10	3/10	1/10	0	5/10
2	3/20	3/20	0	0	6/20
3	1/30	0	0	0	1/30
$P_Y(y)$	7/24	21/40	7/40	1/120	1

- $P(x,y) = \dfrac{\binom{4}{x}\binom{3}{y}\binom{3}{3-x-y}}{\binom{10}{3}}$

- $P_X(X)$ = PUNTUAL , SALE DE TABLA

NOTA

5.2

(X,Y) VECTOR ALEATORIO CONTINUO

HALLAR $f_{Y|X=x}(x,y)$, $f_X(x)$

A-

$$(X,Y) \sim \mathcal{U}(x^2+y^2=1)$$

COMO ES UNIFORME $f_{XY}(x,y) = \frac{1}{\pi 1^2} = \frac{1}{\pi}$

$$f_X(x) = \int \frac{1}{\pi}\, dy$$

$$f_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi}\, dy = \frac{1}{\pi} y \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$f_X(x) = \frac{1}{\pi} \cdot 2\sqrt{1-x^2}\, \mathbb{1}\{-1<x<1\}$$

ENTONCES

$$f_{Y|X=x} = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{1}{\pi} \cdot \frac{\pi}{2\sqrt{1-x^2}}$$

$$f_{Y|X=x_0}(y) = \frac{1}{2\sqrt{1-x_0^2}}\, \mathbb{1}\{-1<y<1\}$$

B-

$$f_{XY}(x,y) = \frac{1}{2x+1} \cdot e^{-\left(2x - \frac{y}{4x+2}\right)}\, \mathbb{1}\{x>0\ y>0\}$$

$$\frac{1}{2x+1}\, e^{-2x + \frac{1}{4x+2} y}$$ → POR PROPIEDAD e^{A+B}

$e^A \cdot e^B$

TIENE LA PINTA DE DOS EXPONENCIALES

NOTA

HOJA N° 2

FECHA

$$f_X(x) = 2e^{-2x} \qquad X \sim EXP(2)$$

$$f_{Y|X=x} = \frac{1}{4x+2} e^{-\frac{1}{4x+2}} \sim EXP\left(\frac{1}{4x+2}\right)$$

SI LA MULTIPLICO, ME QUEDA LA CONJUNTA

$$2e^{-2x} \cdot \frac{1}{4x+2} e^{-\frac{1}{4x+2}}$$

$$\frac{\cancel{2}}{\cancel{2}(2x+1)} e^{-\left(2x+\frac{1}{4x+2}\right)}$$

C_

$$f_{XY}(x,y) = e^{-x} \mathbb{1}\{0<y<x\}$$

$$f_{Y|X=x} = \frac{1}{x} \sim \mathcal{U}(0<y<x)$$

$$f_X(x) = x e^{-x} \sim EXP(x) \; \mathbb{1}\{x>0\}$$

D_

$$f_{XY}(x,y) = \frac{1}{6} x^4 y^3 \cdot e^{-xy} \mathbb{1}\{1<x<2, y>0\}$$

$$f_{XY}(xy) = \frac{1}{6} x^3 y^3 \cdot x e^{-xy}$$

$$\frac{1}{6} x^4 \cdot y^3 \underbrace{e^{-xy}}$$

$$\frac{\lambda^{\gamma}}{\Gamma(\gamma)} y^{\gamma-1} \cdot e^{-\lambda y} \qquad \lambda = x$$

$$y^3 = y^{\gamma-1} \rightarrow \gamma = 4$$

$$x^4 = \checkmark$$

$$\Gamma(4) = 3! = 6$$

NOTA

$$f_{Y|X=x} = \frac{\lambda^{\gamma}}{\Gamma(\gamma)} \, y^{\gamma-1} \, e^{-\lambda y}$$

$$f_{Y|X=x} = \frac{x^4}{6} \, y^3 \cdot e^{-xy} \cdot \mathbb{1}\{y>0\}$$

$$f_X(x) = 1 \sim \mathcal{U}(1,2)$$

$$f_X(x) = \frac{1}{1} \, \mathbb{1}\{1<x<2\}$$

5.3

Y : 'MILES DE LITROS DE LECHE'

X : " # LECHE VENDIDA'

A-

$$f_{XY} = 0,5 \, \mathbb{1}\{0<X<Y<2\}$$

$$f_X(x) = \int_x^2 0,5 \, dy = 0,5 y \Big|_2^x = 0,5(2-x)$$

$$f_X(x) = 1 - \frac{1}{2}x \quad \mathbb{1}\{0<x<2\}$$

$$f_{Y|X=1,5}(y) = \frac{0,5}{1-\frac{1}{2}x} = \frac{0,5}{1-\frac{1}{2}1,5} = 2 \, \mathbb{1}\{\overset{1,5}{\cancel{x}}<y<2\}$$

$$f_{X|Y=0,8}(x) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

$$f_Y(y) = \int_0^y 0,5 \, dx = 0,5 y$$

$$f_{X|Y=0,8}(x) = \frac{0,5}{0,5y} = \frac{0,5}{0,5 \cdot 0,8} = 1,25 \, \mathbb{1}\{0<x<\overset{0,8}{y}\}$$

NOTA

B.-

$\mathbb{P}(1{,}75 < Y < 2 \mid X = 1{,}5)$

$$\mathbb{P} = \frac{\mathbb{P}(1{,}75 < Y < 2 \cap X = 1{,}5)}{\mathbb{P}(X = 1{,}5)}$$ COMO YA TENGO LA MARGINAL CALCULO DIRECTO

• $\mathbb{P}(1{,}75 < Y < 2 \cap X = 1{,}5) = \int_{1,75}^{2} 2\, \mathbb{1}\{1{,}5 < y < 2\}$

$= 0{,}5$

$\mathbb{P}(0{,}5 < X < 0{,}75 \mid Y = 0{,}8) = \int_{0,5}^{0,75} \frac{5}{4}\, \mathbb{1}\{0 < x < 0{,}8\}$

$= 0{,}3125$

C.- X e Y SON INDEP?

$f_{XY} = f_X \cdot f_Y$

$0{,}5 \neq (1 - \frac{1}{2}x)(0{,}5y)$ NO SON INDEPENDIENTES

5.4

$$f_{XY} = \frac{5}{8\pi} e^{-\frac{25}{32}(x^2 - \frac{6}{5}xy + y^2)}$$ → COMPLETO CUADRADOS PARA BUSCAR LLEVARLO A UNA NORMAL

$$y^2 + x^2 - \frac{6}{5}xy = \underbrace{y^2 - 2\left(\frac{3}{5}x\right)y + \left(\frac{3}{5}x\right)^2}_{\left(y - \frac{3}{5}x\right)^2} \underbrace{- \left(\frac{3}{5}x\right)^2 + x^2}_{\frac{16x^2}{25}}$$

$$f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}\left[\left(y - \frac{3}{5}x\right)^2 + \frac{16}{25}x^2\right]}$$

$$= \frac{5}{8\pi} e^{-\frac{25}{32}\left(y - \frac{3}{5}x\right)^2} e^{-\frac{25}{32}\cdot\frac{16}{25}x^2}$$

$$= \frac{5}{8\pi} \cdot e^{-\frac{1}{2}\frac{\left(y - \frac{3}{5}x\right)^2}{\left(\frac{32}{25}\cdot\frac{1}{2}\right)}} e^{-\frac{1}{2}x^2}$$

$$= \underbrace{\left(\frac{1}{\sqrt{2\pi}\cdot 4/5} e^{-\frac{1}{2}\frac{\left(y - \frac{3}{5}x\right)^2}{(4/5)^2}}\right)}_{f_{Y|X=x}(y)} \underbrace{\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\right)}_{f_X(x)} \quad \overbrace{2\pi \cdot \frac{4}{5} \cdot \frac{5}{8\pi}}^{1}$$

SI NECESITARA $f_{X|Y=y}$ ← $f_{Y|X=x}(y)$

COMPLETO CUADRADOS PARA X.

$$\mathbb{P}(1 < XY < 5 \mid X = \sqrt{5}) = \mathbb{P}\left(\frac{1}{\sqrt{5}} < Y < \frac{5}{\sqrt{5}} \mid X = \sqrt{5}\right)$$

$$= \mathbb{P}\left(\frac{1}{\sqrt{5}} < Y|_{X=\sqrt{5}} < \frac{5}{\sqrt{5}}\right)$$

5.5

$X \sim \mathcal{U}(3,4)$

$f_X(x) = 1 \, \mathbb{1}\{3<x<4\}$

$f_{Y|X=x} \sim N(x, 1)$

$x \in (3,4)$

A-

CALCULAR $f_Y(5)$

$f_{XY} = f_X(x) \cdot f_{Y|X=x}$

$$f_{XY} = \frac{1}{\sqrt{2\pi}\,x} e^{-\frac{(y-x)^2}{2}} \mathbb{1}\{3<x<4\}$$

$$f_Y(y) = \int_0^{+\infty} f_{XY}\, dx = \int_3^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}$$

$\downarrow$ xq $x \in (3,4)$

$$f_Y(5) = \int_3^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{(5-x)^2}{2}} = \mathbb{P}(3<W<4) = -$$

$W \sim N(5,1)$

$$\phi\left(\frac{3-5}{1}\right) - \phi\left(\frac{4-5}{1}\right)$$

B- $\mathbb{P}(X > 3{,}5 \mid Y=5) = \mathbb{P}(X|_{Y=5} > 3{,}5)$

$= 0{,}136$

$$f_{X|Y=y_0} = \frac{f_{XY}(x, y_0)}{f_Y(y_0)}$$

$0{,}02275 - 0{,}159$

$$f_{X|Y=5} = \frac{1}{\sqrt{2\pi}\,x} e^{-\frac{(y-x)^2}{2}} \cdot \frac{1}{0{,}136}$$

$\phi(-2) - \phi(-1)$

$$\mathbb{P}(X|Y=5 > 3{,}5) = 1 - \int_0^{3{,}5} \cdots =$$

$\phi(-z) = 1 - \phi(z)$

HOJA Nº 4

FECHA

5.7

$T_1 \sim EXP(\lambda)$

$T_2 \sim EXP(\lambda)$

T_1 Y T_2 SON INDEPENDIENTES

$S_1 = T_1 \rightarrow S_1 \sim EXP(\lambda)$

$S_2 = T_1 + T_2$

A - HALLAR $f_{S_1 S_2}$

$$f_{T_1 T_2} = f_{T_1} \cdot f_{T_2}$$
$$= \lambda e^{-\lambda T_1} \cdot \lambda e^{-\lambda T_2}$$
$$= \lambda^2 e^{-\lambda (T_1 + T_2)}$$

$$(S_1, S_2) = g(T_1, T_2) = (T_1, T_1 + T_2)$$

$$f_{S_1 S_2}(s_1, s_2) = \left. \frac{f_{T_1 T_2}(t_1, t_2)}{\left| \det \frac{\partial g(t_1, t_2)}{\partial (t_1, t_2)} \right|} \right|_{(t_1, t_2) = g^{-1}(s_1, s_2)}$$

$$\begin{cases} S_1 = T_1 \\ S_2 = T_1 + T_2 \end{cases} \Longrightarrow \begin{cases} T_1 = S_1 \\ T_2 = S_2 - S_1 \end{cases}$$

$$g^{-1}(s_1, s_2) = (s_1, s_2 - s_1)$$

$$f_{S_1 S_2}(s_1, s_2) = \left. f_{T_1 T_2}(t_1, t_2) \right|_{g^{-1}(s_1, s_2)} \cdot \left| \det \frac{\partial g^{-1}(s_1, s_2)}{\partial (s_1, s_2)} \right|$$

NOTA

5.8

DRACULA 0:00 + X
RANFIELD 0:00 + Y } SON INDEP

$$f_{XY} = \frac{1}{35} e^{-\left(\frac{x}{5} + \frac{y}{7}\right)}$$

$$f_X(x) = \frac{1}{5} e^{-\frac{1}{5}x} \sim EXP(1/5)$$

$$f_Y(y) = \frac{1}{7} e^{-\frac{1}{7}y} \sim EXP(1/7)$$

PRIMERO EN LLEGAR TUVO QUE ESPERAR 5 MIN

LO QUE ME PIDEN ES COMO 4.14 (B)

$$J = 1 \, \mathbb{1}\{\underbrace{U = X}_{\text{PRIMERO DRACULA}}\} + 2\{\underbrace{U = Y}_{\text{PRIMERO RANFIELD}}\}$$

W = TIEMPO DE ESPERA → (W = 5)
= MAX − MIN

$$\mathbb{P}(J = 1 \mid W = 5) = \frac{\overbrace{f_{W|J=1}(5)\cdot \mathbb{P}(J=1)}^{\text{LO TENGO}}}{f_{W|J=1}(5)\cdot \mathbb{P}(J=1) + f_{W|J=2}(5)\cdot \mathbb{P}(J=2)}$$

BAYES PARA MEZCLAS

NOTA

HOJA Nº 5

FECHA

NOTA

5.9

MEZCLA ↑ MEZCLADORA ↑

RECEPTOR $X = S + N$

S ↓ DISCRETA $\{0.1 - 0.2 - 0.3\}$ $\mathbb{P}(\downarrow) = \frac{1}{3}$

N ↳ CONTINUA $N \sim N(0,1)$

CALCULAR $\mathbb{P}(S = 0{,}2 \mid X = 0{,}87)$

↓ NO PUEDO RESOLVER XQ NO PUEDO EVALUAR UNA CONTINUA EN UN PUNTO

BAYES PARA MEZCLA DE CONTINUAS → f_X EN CADA PUNTO

$$\mathbb{P}(M = i \mid X_M = X) = \frac{f_{X_i}(X) \cdot \mathbb{P}(M = i)}{\sum_{j=1}^{m} f_{X_j} \cdot \mathbb{P}(M = j)}$$

$$\left.\begin{aligned} X_1 &= 0{,}1 + N \\ X_2 &= 0{,}2 + N \\ X_3 &= 0{,}3 + N \end{aligned}\right\}$$

NECESITO SUS $f_X - f_{X_1}\ f_{X_2}\ f_{X_3}$

↓ COMO SON UN CAMBIO DE VARIABLE LINEAL $Y = AX + B$

$X_i = 1N + S_i$

USO:

$$f_{X_i} = \frac{f_X\left(\frac{X - B}{A}\right)}{|A|}$$

$A = 1$

$B = S_i$

$$f_{X_i} = f_X(X - S_i)$$

↑ $\sim N(0,1)$ TABLA.

$\mathbb{P}(M = i) = 1/3$ (EQUIPROBABLE)

$$\mathbb{P}(S=0{,}2 \mid X=0{,}87) = \frac{\overset{f_X(0{,}87-0{,}2)}{f_{X_2}(0{,}87)}\cdot \mathbb{P}(S=0{,}2)}{f_{X_1}(0{,}87)\,\mathbb{P}(S=0{,}1) + f_{X_2}(0{,}87)\,\mathbb{P}(S=0{,}2) + f_{X_3}(0{,}87)\,\mathbb{P}(S=0{,}3)}$$

$$\mathbb{P}(S=0{,}2 \mid X=0{,}87) = \frac{0{,}749 \cdot 1/3}{0{,}761\cdot \frac{1}{3} + 0{,}749\cdot\frac{1}{3} + 0{,}716\cdot\frac{1}{3}}$$

$$= 0{,}336$$

5.10

CORPORACION $\rightarrow$ 6 ROBOTS

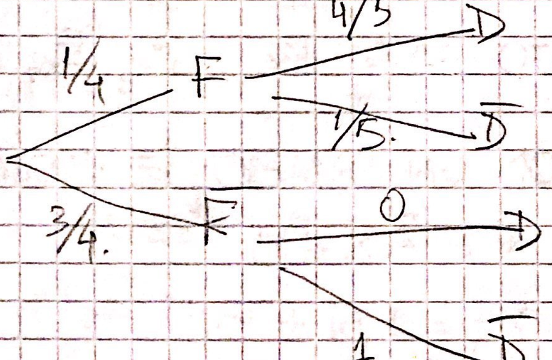
$\downarrow$ PROBABILIDAD DE (C/U) FALLA $= 1/4$

PRUEBA $\rightarrow$ SI FALLA $\rightarrow 4/5$

X: # ROBOTS FALLADOS $\sim Bi(6, 1/4)$

Y: # DETECTADA DE ROBOTS FALLADOS

A- $\varphi(x) = \mathbb{E}[Y \mid X = x_0]$



$$Y \mid X = x_0 \sim Bi\left(x_0, \frac{4}{5}\right) \rightarrow \mathbb{E}[Y \mid X = x_0] = \frac{4}{5}x_0$$

$$X \mid Y = y \sim y + \underset{\sim Bin(6-y,\ \mathbb{P}(F \mid \bar{D}))}{\underset{\downarrow}{M}} \rightarrow \mathbb{E}[X \mid Y = y_0] = 6 - y \cdot \mathbb{P}(i)$$

NOTA

HOJA N°

FECHA

5.12

ARROJO UN DADO 36 VECES

X: # PARES

Y: # IMPARES $\sim$ BER (

HALLAR $\mathbb{E}[Y|X]$ y $COV(X,Y)$

$$X + Y = 36$$

$$X \sim BIN(36, P_X) = BIN(36, 1/2)$$

$$Y \sim BIN(36, P_Y) = BIN(36, 1/2)$$

$$X + Y = 36 \begin{cases} X = 36 - Y \\ Y = 36 - X \end{cases}$$

$$\mathbb{E}[Y|X] = \mathbb{E}[36 - X | X] = 36 - X$$

↓ PROPIEDAD

X ESTA FIJO → QUEDA CTE

NOTA