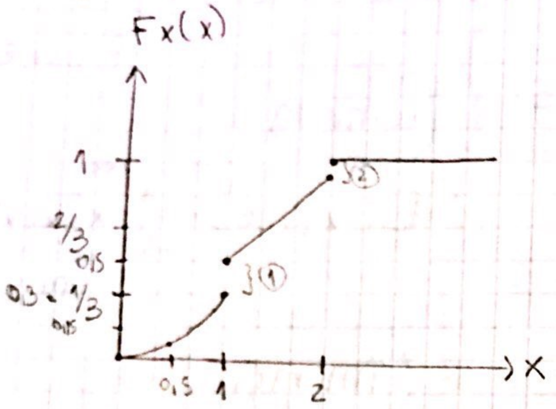
GUÍA 3

(17)

3.1 Sea X v.a con $F_X(x) = \frac{x^3}{3} \mathbb{1}\{0 \le x < 1\} + \frac{2x+1}{6} \mathbb{1}\{1 \le x < 2\} + \mathbb{1}\{x \ge 2\}$.



MIXTA

a) $E[X]$

b) $E[X \mid X<1]$ y $E[X \mid X \le 1]$.

a) $E[X] = \sum_{x\in A} x \overbrace{p(X=x)}^{\text{salto}} + \int_I x F_X'(x)\,dx$ (punto)

$I: I_1(0,1) + I_2(1,2)$

$A: 1 \to 1/6 \quad 2 \to 1/6$

$E[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \int_0^1 x \cdot x^2\,dx + \int_1^2 \frac{2}{6} x\,dx =$

$E[X] = \frac{1}{6} + \frac{2}{6} + \frac{x^4}{4}\Big|_0^1 + \frac{2}{6}\frac{x^2}{2}\Big|_1^2 = \frac{5}{4}$

b) $E[X \mid X<1] = \dfrac{E[X \mathbb{1}\{X<1\}]}{P(X<1)} =$

$= \dfrac{\int_0^1 x^3\,dx}{\underset{1/2}{F(X=1)} - \underset{1/6}{P(X=1)}} = \dfrac{1/4}{1/3} = 3/4$

ESPERANZA → valor esperado

$\boxed{\text{Discreta}}$: $A = \{x : p(X=x) > 0\}$ átomos

- $E[X] = \sum_{x\in A} x \cdot p_X(x)$
- $E[g(X)] = \sum_{x\in A} g(x)\, p_X(x)$

$g(x)$ es v.a.

$\boxed{\text{Continua}}$:

- $E[X] = \int_{-\infty}^{+\infty} x\, f_X(x)\,dx$
- $E[g(X)] = \int_{-\infty}^{+\infty} g(x)\, f_X(x)\,dx$

$\boxed{\text{Mixta}}$

- $E[X] = \sum_{x\in A} x\, p(X=x) + \int_I x\, F_X'(x)\,dx$

I ↓ ≠ intervalos

PROP

- $E[X]$ es lineal:
  - $E[X+Y] = E[X] + E[Y]$
  - $aE[X] = E[Xa]$
- $X \ge 0 \to E[X] \ge 0$
- $E[a] = a$
- $E[X+a] = E[X] + a$

$$E[X \mid X \leq 1] = \frac{E[X \, \mathbb{1}\{X \leq 1\}]}{P(X \leq 1)} =$$

$$= \frac{\int_0^1 x^3 \, dx + 1 \cdot 1/6}{F(1)} = \frac{1/4 + 1/6}{1/2} = \frac{5}{6}.$$

V.A truncada

discreta

$$E[X \mid X \in B] = \frac{E[X \, \mathbb{1}\{X \in B\}]}{P(X \in B)}$$

continua

$$E[X \mid X \in B] = \frac{\int_{-\infty}^{+\infty} x f_X(x) \mathbb{1}\{x \in B\} dx}{P(X \in B)}$$

mixta

Planteas esperanzas y prob y sacas según info/

3.2 Sea X una v.a com F. del 2.2.

a) $E[X] = \cancel{(-2) \cdot \frac{1}{3}} + \cancel{2 \cdot \frac{1}{3}} + \int_{-2}^{-1} 0 \, dx +$

$\int_{-1}^{1} 1/6 \, x \, dx = \frac{1}{6} \frac{x^2}{2} \Big|_{-1}^{1} = 0$

b) $E[[X] \mid |X| = 2] = \frac{E[X \, \mathbb{1}\{X = 2 \cup X = -2\}]}{P(X = 2 \cup X = -2)} = \underset{\text{lo q1 hay}}{-2 \cdot 1/3 + 2 \cdot \frac{1}{3}} = 0$

3.3 Pascal $\rightarrow$ $E[X] = \frac{k}{p}$

3.4 Sea $(\Omega, \mathcal{A}, P) \rightarrow \Omega = \{1, 2, 3, 4, 5, 6\}$

$\mathcal{A} = \{\emptyset, \{1,2\}, \{3,4\}, \{5,6\}, \{1,2,3,4\}, \{1,2,5,6\},$

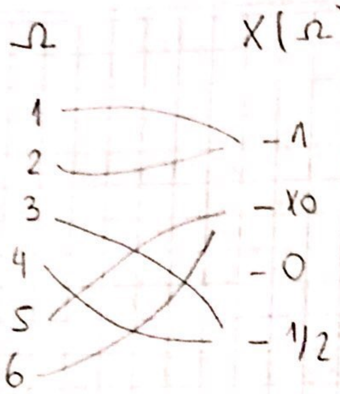
$\{3,4,5,6\}, \Omega\}$ y $P(\{1,2\}) = 1/6$

$P(\{3,4\}) = 1/3$

$P(\{5,6\}) = 1/2$

a) Definir X v.a / $X : \Omega \rightarrow \mathbb{R}$ / $\{-1, 1/2\} \subset X(\Omega)$ y $E[X] = 0$.

Scanned with CamScanner



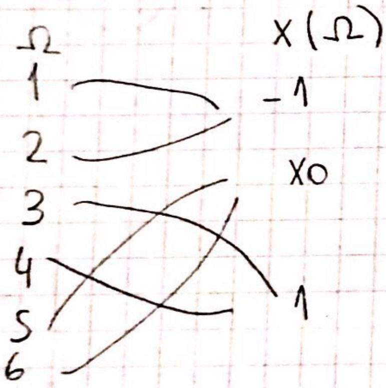
$E[X] = -1 \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{3} + X_0 \cdot \frac{1}{2} = 0$

$X_0 = 0$

$X = -1 \, \mathbb{1}\{w \in \{1,2\}\} + \frac{1}{2} \, \mathbb{1}\{w \in \{3,4\}\}$

b) $E[X \mid X > -1] = \dfrac{E[X \, \mathbb{1}\{X > -1\}]}{P(X > -1)} = \dfrac{1/6}{\underset{\downarrow\ 1}{F(-1)} - \underset{\downarrow\ 1/6}{P(X=-1)}} = 1/5$

c) $\{-1, 1\} \subset X(\Omega)$



$E[X] = -1 \cdot 1/6 + X_0 \cdot 1/2 + 1 \cdot 1/3 = 0$

$X_0 = -1/3$

$X = -1 \, \mathbb{1}\{w \in \{1,2\}\} + 1 \, \mathbb{1}\{w \in \{3,4\}\} + \frac{1}{3} \, \mathbb{1}\{w \in \{5,6\}\}.$

$E[X \mid X > -1] = 1/5$

DISCRETA

3.5 Cant moscas $\rightarrow$ Poison(10) $\rightarrow$ $\frac{10^x e^{-10}}{x!} = P_X(x)$

$E[X \mid X \leq 4] =$

$$= \frac{E[X \, 1\!\!1\{X \leq 4\}]}{P(X \leq 4)} = \frac{\sum_{i=0}^{4} x \cdot P_X}{\sum_{i=1}^{4} P_X(x)} =$$

POR SER DISCRETA SUMO C/ PROB PUNTUAL

$$= \frac{0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 \cdot P(4)}{P(1) + P(2) + P(3) + P(4)}$$

$$= \frac{\cancel{e^{-10}} \left(\frac{10^1}{1!} + \frac{10^2}{2!} \cdot 2 + \frac{10^3}{3!} \cdot 3 + \frac{10^4}{4!} \cdot 4 \right)}{\cancel{e^{-10}} \left(\frac{10^1}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!} + \frac{10^4}{4!} \right)} =$$

$$= \frac{6830}{3} \div \frac{1930}{3} = \boxed{3{,}539}$$

FÓRMULA DE PROB TOTAL

$E[X] = \sum_i P(X \in A_i) \cdot E[X \mid X \in A_i]$

$\downarrow$

PARTO a X en intervalos o conjuntos disjuntos

$E[T] = 3$ . . .

2° FORMA

puedo hacerlo con esa también.

$\downarrow$

Para no integrar

CONTINUA

3.6 exponencial media 3, $E[T \mid T \leq 2]$

$\downarrow$

$f_X = \frac{1}{3} e^{-1/3 x}$ $\quad 3 = 1/\lambda \rightarrow \lambda = 1/3$.

integro

$$E[T \mid T \leq 2] = \frac{E[T \, 1\!\!1\{T \leq 2\}]}{P(T \leq 2)} = \frac{\int_{-\infty}^{+\infty} x \cdot f(x)\, dx}{\int_0^2 f_X \, dx}$$

$F_X = 1 - e^{-1/3 x} \, 1\!\!1\{x > 0\}$

$$= \frac{\int_0^2 x \cdot \frac{1}{3} e^{-1/3 x} \, dx}{\int_0^2 \frac{1}{3} e^{-1/3 x} dx} = 0{,}89$$

$\nearrow 0{,}43$

$\searrow 0{,}48$.

continua

3.7 a) $f_X(x) = \frac{9!}{3!\,5!} x^3 (1-x)^5 \mathbb{1}\{0 < x < 1\}$

$$E[X] = \int_0^1 f(x)\, x\, dx = \frac{9!}{3!\,5!} \int_0^1 x^3 (1-x)^5\, dx$$

$$E[X] = 504 \int_0^1 x^4 - 5x^5 + 10x^6 - 10x^7 + 5x^8 - x^9 \, dx$$

$$E[X] = 504 \left(1/5 - 5/6 + 10/7 - 10/8 + 5/9 - 1/10 \right) = \boxed{\frac{2}{5}}$$

b) $f_X(x) = \frac{1}{6} x^3 \cdot e^{-x} \mathbb{1}\{x > 0\}$

integro...

$$E[X] = \int_0^\infty \left(\frac{1}{6} x^3 \cdot e^{-x}\right) \cdot x\, dx = \frac{1}{6} \cdot 24 = 4$$

otra forma $\rightarrow$ $\Gamma(4,1)$ $\rightarrow$ $E[X] = \frac{\nu}{\lambda} = 4$ ✓

veo que es una gama de

c) $E[X \mid X > 1/2] = \frac{E[X\, \mathbb{1}\{X > 1/2\}]}{P(X > 1/2)}$

$P(X > 1/2) \rightarrow 1 - P(X \leq 1/2) \rightarrow F(1/2) = \int_0^{1/2} f_X(x)\, dx$

$$E[X \mid X > 1/2) = \frac{\int_{1/2}^{+\infty} f_X \cdot x\, dx}{1 - \int_0^{1/2} f_X\, dx} = \frac{4}{1{,}99} = 4$$

gamma $\rightarrow \frac{\lambda}{(\nu-1)!} x^{\nu-1} e^{-\lambda x}$

$\nu - 1 = 4$ $\boxed{\nu = 4}$

$\boxed{\lambda = 1}$

38) Sea Z va una normal estándar, φ su función densidad, $z_0 > 0$

a) Hallar la media $Z \mid Z > z_0$ (en función de z_0).

$$\boxed{\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}}$$

función densidad de z

$$\boxed{E[Z \mid Z > z_0]} = \frac{E[Z \, 1\!\!1\{Z > z_0\}]}{P(Z > z_0)}$$

$$E[Z \mid Z > z_0] = \frac{\int_{z_0}^{+\infty} z \cdot \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{2}z^2}}{1 - \Phi(z_0)} = \boxed{\frac{\varphi(z_0)}{1 - \Phi(z_0)}}$$

b) Deducir que $(1 - \Phi(z_0)) \leq \frac{\varphi(z_0)}{z_0}$.

$$\frac{\varphi(z_0)}{1 - \Phi(z_0)} \geq z_0$$

$$\boxed{1 - \Phi(z_0) \leq \frac{\varphi(z_0)}{z_0}}$$

c) $$P(Z > 4) \leq \frac{\frac{1}{\sqrt{2\pi}} \cdot e^{-4^2/2}}{4} = 0{,}000033$$

$$\boxed{P(Z > 4) = \frac{\varphi(4)}{4}}$$ o

$$P(Z > 4) = 1 - \Phi(4) = 1 - 0{,}99997 = 0{,}0003.$$

$$\boxed{P(Z > 4) = 1 - \Phi(4)}$$

d) $X = \sigma Z + \mu$ $\quad x_0 > \mu$

$X \mid X > x_0 = ?$

$$\boxed{E[X \mid X > x_0]} = E[\sigma Z + \mu \mid \sigma Z + \mu > x_0] =$$

$$\sigma E\left[Z \mid Z > \frac{x_0 - \mu}{\sigma}\right] + \mu =$$

$$\boxed{\frac{\sigma \, \varphi\left(\frac{x_0 - \mu}{\sigma}\right)}{1 - \Phi\left(\frac{x_0 - \mu}{\sigma}\right)} + \mu =}$$

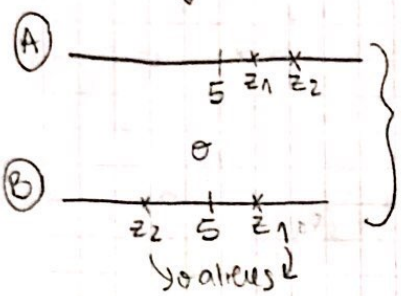
3.9 Sean Z_1 y Z_2 dos variables normales estándar (no necesariamente independientes)

Demostrar $P(\max(Z_1, Z_2) > 5) \leq \frac{2e^{-25/2}}{5\sqrt{2\pi}}$.

Si quiero q alguno >5 puede ocurrir

$$P(Z_1 > 5 \cup Z_2 > 5) \leq P(Z_1 > 5) + P(Z_2 > 5)$$

que alguno sea mayor q' 5.



UNIÓN

NO, pq ninguno es >5.

$$\leq \frac{\varphi(5)}{5} + \frac{\varphi(5)}{5} \leq$$

uso teoremas del 3.8

$$\leq \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{5^2}{2}}}{5} + \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{5^2}{2}}}{5}$$

$$P(A \cup B) = P(A) + P(B) - P(\overset{\circ}{A \cap B})$$

$$P(A \cup B) \leq P(A) + P(B)$$

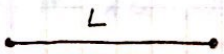
$$\leq \frac{2 \cdot e^{-25/2}}{5\sqrt{2\pi}}.$$

3.10 Se construye un círculo uniendo los extremos de un alambre.

a) $L \sim \exp(\lambda)$ $\quad \frac{1}{\lambda} = 60\text{cm} \rightarrow \lambda = 1/60$

Calcular la media del área del círculo.

$$\pi r^2$$



$2\pi r \rightarrow$ perímetro del círculo.

L: perímetro

$$2\pi r = L$$

$$r = L/2\pi$$

$$\text{Área} = \pi \left(\frac{L}{2\pi}\right)^2$$

$$\boxed{A = \frac{L^2}{4\pi}}$$

$X = \text{Área} = g(L) \rightarrow$ el área me quedó como una función de la variable q' yo tenía.

continua

$$E[X] = E[g(L)] = \int_{-\infty}^{+\infty} g(l)\, f_L(l)\, dl = \Rightarrow \text{sigo}$$

siendo $f_L(l) = \lambda \cdot e^{-\lambda l}$ dl como el área es >0 → integro de 0 a ∞ → la expo te dice que el soporte es de 0 a +∞

$$E[g(L)] = \int_0^{\infty} \frac{l^2}{4\pi} \cdot \frac{1}{60}\, e^{-l/60}\, dl =$$

como es difícil de integrar busco otra forma

(1°) como la gamma y la expo tienen el MISMO SOPORTE → lo llevo a una función gamma. $\Gamma(\nu, \lambda)$

$$E[g(L)] = \frac{1}{60 \cdot 4\pi} \int_0^{\infty} l^2 \cdot e^{-l/60}\, dl$$

$$f_l(l) = \frac{\lambda^{\nu}}{\underbrace{(\nu-1)!}} l^{\nu-1} e^{-\lambda l}$$

$\boxed{\lambda = 1/60}$ $\quad \nu - 1 = 2.$ $\quad \boxed{\nu = 3.}$

agrego este término multiplicando y dividiendo

$$E[g(L)] = \frac{1}{60 \cdot 4\pi} \underbrace{\int_0^{\infty} l^2 \cdot e^{-l/60} \cdot \frac{\left(\frac{1}{60}\right)^3}{2!}\, dl}_{1} \cdot \frac{2!}{(1/60)^3}$$

$$E[g(L)] = \frac{1}{\cancel{60} \cdot 4\pi} \cdot 2! \, 60^{\cancel{3}\, 2} \quad (1) \quad = \boxed{572{,}96\, cm^2 = E[A]}$$

LA DENSIDAD INTEGRADA en el soporte = 1

$$\int_{sop} f_x(x)\, dx = 1$$

2° se que

$$VAR[L] = E[L^2] - (E[L])^2$$

$$E[L^2] = \underset{1/\lambda^2}{VAR[L]} + (\underset{1/60}{E[L]})^2$$

$$E[L^2] = 2 \cdot 60^2 \implies E[g(L)] = E\left[\frac{L^2}{4\pi}\right] = \frac{1}{4\pi} E[L^2]$$

$$\boxed{E[A] = 572{,}96\, cm^2}$$

b) si el $\acute{A} \sim Exp(\lambda)$ $E[A] = 15\ cm^2$

calculo perímetro → media $\frac{1}{\lambda} = 15\ cm^2$

$\acute{A} \sim Exp(1/15)$ $\lambda = 1/15$

$\acute{A} = \pi r^2$ Pongo a L en función de $\acute{A}$

$$E[g(a)] = \int_0^\infty g(a) \cdot f_a(a)\, dr \qquad g(r) = \underbrace{\sqrt{\frac{A}{\pi}}}_{\sqrt{A\,4\pi}} \cdot 2\pi = L \qquad L = \sqrt{\frac{A}{\pi}} \cdot 2\pi$$

siendo $f_r(r) = \lambda e^{-\lambda r}$

$$E[g(a)] = \int_0^\infty \sqrt{A \cdot 4\pi}\ \frac{1}{15} \cdot e^{-\frac{1}{15} a}\, da$$

$$= 2\sqrt{\pi} \int_0^\infty \sqrt{a} \cdot e^{-1/15\, a} \cdot \frac{1}{15}\, da$$

gamma $\frac{\lambda^v}{\Gamma(3/2)} \underbrace{x^{v-1}} \underbrace{e^{-\lambda x}}$ $\lambda = 1/15$, $x = a$

$\sqrt{a} = a^{v-1}$ $v - 1 = 1/2$ $\boxed{v = 3/2}$

$\Gamma(3/2) = \Gamma(1 + 1/2) =$

$1/2\, \Gamma(1/2) = 1/2 \sqrt{\pi}$ $\lambda^v = \left(\frac{1}{15}\right)^{3/2}$

$$= \frac{2\sqrt{\pi}\, \Gamma(3/2)}{15 (1/15)^{3/2}} \underbrace{\int_0^\infty \sqrt{a}\ e^{-a/15} (1/15)^{3/2} \frac{1}{\Gamma(3/2)}\, da}_{1}$$

$$E[g(a)] = \frac{2\sqrt{\pi}\ 1/2\, \pi}{(1/15)^{3/2} \cdot 15} = 21{,}6\ cm^2$$

$$\boxed{\begin{array}{l} \Gamma(T+1) = \tau\, \Gamma(T) \\ \Gamma(1/2) = \sqrt{\pi} \end{array}}$$

VARIANZA = como se dispersan los valores alrededor del promedio $E[x]$

+var : + alejados
-var : + cerca

$$VAR(x) = E[(x-\overbrace{\mu}^{E[x]})^2]$$

$$VAR(x) = E[x^2] - E^2[x]$$

$$E[x^2] = \int_{-\infty}^{+\infty} x^2 f_x(x)\,dx$$

DISCRETO

$$Var(x) = \sum_{x \in A_i} (x-\mu)^2 p_x(x)$$

CONTINUO

$$Var(x) = \int_{-\infty}^{+\infty} (x-\mu)^2 f_x(x)\,dx$$

PROP:

- $VAR(cte) = 0$
- $Var(x) \geq 0$
- $Var(x + cte) = Var(x)$
- $Var(ax) = a^2 \, var(x)$
- $Var(x+y) = Var(x) + Var(y) + 2cov(x,y)$
- $Var(x-y) = Var(x) + Var(y) - 2cov(x,y)$

$$COV[x,y] = E[x \cdot y] - E[x]E[y].$$

$$V[ax+by] = a^2 V[x] + b^2 V[y] + 2ab\,cov[x,y].$$

PROP:

- x e y son ind $\rightarrow cov(x,y) = 0$.
- $cov(ax,y) = a\,cov(x,y)$
- $cov(x,y) = cov(y,x)$
- $cov(x+y,z) = cov(x,z) + cov(y,z)$
- $cov(x,x) = E[x^2] - E^2[x] = VAR[x] \geq 0$

ESPERANZA DE VECTORES

DISCRETO

$$E[g(x,y)] = \sum_{(x,y) \in A} g(x,y)\, p_{xy}(x,y)$$

$$A = \{(x,y) \in \mathbb{R}^2 \,/\, P(X=x, Y=y) > 0\}$$

CONTINUO.

$$E[g(x,y)] = \iint_{\mathbb{R}^2} g(x,y)\, f_{xy}(x,y)\,dx\,dy$$

$E[x,y] = E[x]E[y]$ si son ind. $\not\Leftarrow$

$\downarrow$

sop $\neq$ rectangulo
no son ind.

ERA MIXTA

$$T^* = \begin{cases} T & \text{si } t < 1 \\ 1 & \text{si } T \geq 1 \end{cases} \qquad F_T(t) = \begin{cases} 1 - e^{-t} & \text{si } t > 0 \\ 0 & \text{si } t \leq 0 \end{cases}$$

3.11 Sea T^* v.a del 2.12. $\rightarrow T^* = \min(T, 1)$

T: duración en años del tiempo de trabajo sin fallos

$E[T^*]$ y var $[T^*]$

$$E[T^*] = \int_0^1 t^* e^{-t^*} dT^* + 1e^{-1} = 0{,}632$$

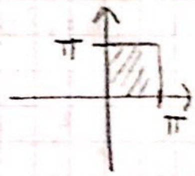
$$V(T^*) = E[T^{*2}] - E^2[T^*]$$

$$= \int_0^1 t^{*2} e^{-t^*} dT + 1e^{-1} - 0{,}632^2 = 0{,}129$$

→ CONTINUA

3.12 XY son va ind y UNIFORMES sobre $(0, \pi)$

$$E[\underbrace{X \operatorname{sen}(XY)}_{g(x,y)}] = \iint_{\mathbb{R}^2} g(x,y)\, f_{XY}(x,y)\, dx\, dy$$



como son ind

$$f_X(x) f_Y(y) = f(x,y)$$

$$\frac{1}{\pi} \cdot \frac{1}{\pi} = f(x,y)$$

$$\frac{1}{\pi^2} = f(x,y)$$

$$E[X \operatorname{sen}(x,y)] = E[X] \cdot E[\operatorname{sen}(x,y)] \quad \text{POR SER ind}$$

$$E[X] = \frac{\pi}{2} \quad \rightarrow \text{POR SER unif}$$

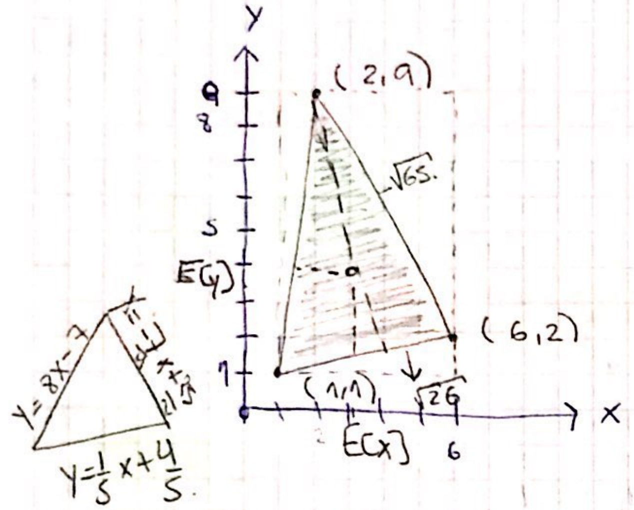
$$E[\operatorname{sen}(x,y)] = \int_0^{\pi}\int_0^{\pi} \operatorname{sen}(xy) \frac{1}{\pi^2}\, dx\, dy = \frac{1}{\pi^2} \int_0^{\pi} \left[-\cos(xy) \frac{1}{y} \Big|_0^{\pi} \right]$$

$$= -\frac{1}{\pi^2} \int_0^{\pi} \frac{\cos(\pi y)}{y} - \frac{1}{y}\, dy = 7{,}40 \times 10^{-3}$$

$$E[X \operatorname{sen}(x,y)] = \underline{7{,}40 \times 10^{-3}}\, \frac{\pi}{2} = 0{,}012.$$

3.13 Sea (X,Y) v.a $\sim U(\Delta)$ $\Delta: (1,1)(6,2)(2,9)$

$E[X]$ y $E[Y]$.



$$\boxed{E[X] = \iint_{\mathbb{R}^2 \to sop(x,y)} x \cdot f_{XY}\, dx\,dy = \int_{-\infty}^{+\infty} x f_X(x)\,dx}$$

$$\boxed{E[Y] = \iint_{\mathbb{R}^2 \to sop(x,y)} y\, f_{XY}\, dx\,dy = \int_{-\infty}^{+\infty} y f_Y(y)\,dy}$$

$$\boxed{f_{XY} = \frac{1}{\text{área}}}$$

$$\text{Área} = \frac{b \cdot h}{2} = \frac{39}{2}.$$

$$\boxed{f_{XY} = \frac{2}{39}}$$

$$h^2 = 1^2 + 5^2$$

$$\boxed{h = \sqrt{26} = b}$$

(1,1)
(2,9)

$1 = 0\,1 - 0$
$9 = 0\,2 - 0$

$$h'^2 = 7^2 + 4^2$$

$$h' = \sqrt{65}$$

$$E[X] = \int_1^2 \int_{\frac{1}{5}x+\frac{4}{5}}^{8x-7} x \cdot \frac{2}{39}\, dx\,dy + \int_2^6 \int_{1/5\,x+4/5}^{-7/4\,x+\frac{25}{2}} x\,2/39\, dx\,dy$$

$$E[X] = 3$$

$$h^2 = ALT^2 + \left(\frac{b}{2}\right)^2$$

$$65 = ALT^2 + \left(\frac{\sqrt{26}}{2}\right)^2$$

$$\sqrt{65 - \frac{26}{4}} = ALT$$

$7{,}65\,m = alt$

$$E[Y] = \int_1^2 \int_{1/5\,x+4/5}^{8x+7} \cancel{y}\; 2/39\, dx\,dy + \int_2^9 \int_{8x-7}^{-\frac{7}{4}x+\frac{25}{2}} y\, \frac{2}{39}\, dx\,dy =$$

$$E[Y] = \int_1^2 \frac{2}{39} \left.\frac{y^2}{2}\right|_{1/5\,x+4/5}^{8x+7} dx + \int_2^9 \frac{2}{39} \left.\frac{y^2}{2}\right|_{8x-7}^{-7/4\,x+25/2} = \ldots$$

etc

$V(x) = 1 \rightarrow$ var máxima

$V(x) = 0 \rightarrow$ var mínima

3.14 X va $\{2,3,4\}$ $P(X=x) = p_x$, $x \in \{2,3,4\}$

de media 3

a) $V[x] = \underbrace{E[x^2]}_{\downarrow} - \underbrace{E^2[x]}_{9}$

$$\boxed{\sum x^2 p_{x_i} = \underset{2^2}{x_2^2} \cdot p_2 + \underset{3^2}{x_3^2} p_3 + \underset{4^2}{x_4^2} p_4 =}$$

$V[x] = 1 = 4p_2 + 9p_3 + 16p_4 - 9$

y se que $p_2 + p_3 + p_4 = 1$

$p_2 = 1 - p_3 - p_4$

$E[x] = 2p_2 + 3p_3 + 4p_4 = 3$

$\vdots$ $\boxed{p_3 = 1 - 2p_4} \rightarrow \boxed{p_2 = p_4}$

$1 = 4p_4 + 9(1 - 2p_4) + 16p_4 - 9$

$10 = 4p_4 + 9 - 18p_4 + 16p_4$

$1 = 2p_4 \rightarrow \boxed{1/2 = p_4} \rightarrow p_2 = 1/2$ $p_3 = 0$

b) $V[X] = 0 \rightarrow$ $9 = 4p_4 + 9(1 - 2p_4) + 16p_4$

$\cancel{9} = 4p_4 + \cancel{9} - 18p_4 + 16p_4$

$\boxed{0 = p_4}$ $\boxed{p_2 = 0}$ $\boxed{p_3 = 1}$

3.15 Sea X uma V.A $\sim U(8,10)$

$E[a]=a$

a) $Y = 2(X-1) = 2X - 2$

$E[X] = \frac{8+10}{2}$

$E[Y] = E[2X-2] \underset{\text{PROP}}{=} 2E[X] - \underbrace{E[2]}_{2} =$

$\boxed{E[Y] = 18 - 2 = 16.}$

$V[Y] = E[Y^2] - E^2[Y]$

$V[2X-2] = E[(2X-2)^2] - \underbrace{E^2[2X-2]}_{16^2}$

$V[Y] = E[4X^2 - \underbrace{2(2X)(2)}_{8X} + 4] - 256$

$VAR[X] = E[X^2] - E^2[X]$

$\frac{(10-8)^2}{12} = E[X^2] - 81,$

$E[X^2] = 81 + \frac{1}{3}$

$E[X^2] = 244/3$

$V[Y] = 4E[X^2] - 8E[X] + E[4] - 256$

$V[Y] = 4 \cdot \frac{244}{3} - 8 \cdot 9 + 4 - 256.$

$\boxed{V[Y] = 1{,}33.}$

b) $Y = 2X^2 + 1$

$E[Y] = E[2X^2+1] = 2\overset{244/3}{E[X^2]} + \overset{1}{E[1]}.$

$\boxed{E[Y] = \frac{491}{3}}$

c) $Y = 2(X-1)(X-3) = (2X-2)(X-3) = 2X^2 \overbrace{-6X - 2X}^{-8X} + 6$

$= 2E[X^2] - 8E[X] + E[6] =$

$= 2 \cdot \frac{244}{3} - 8 \cdot 9 + 6$

$\boxed{E[Y] = \frac{290}{3}.}$

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Para hallar Emáx o mín, veo donde se anula y evaluo en ese punto

d) $E[(X-c)^2]$ → Hallar c para E mín.

$$E[(X-c)^2]=0$$

$$E[X^2-2Xc+c^2] = E[X^2]-2c\,E[X]+E[c^2]$$

$$0=\frac{244}{3}-2c\cdot 9+c^2$$

$$0=c^2-18c+\frac{244}{3} \quad \begin{cases} c_1 = 9 \\ c_2 = 9 \end{cases}$$

en $c=9$.

$$\boxed{E[(X-9)^2]=\frac{1}{3}}$$

↓ mínima.

en $c=8$ y $c=10$ } "los alrededores"

$E(X-8)^2 = 4/3$.

$E(X-10)^2 = 4/3$.

e) Hallar a y b tq $aX+b$ tenga media 0 y desvio 1

$E[aX+b]=0$ $\qquad$ $V[aX+b]=1$

$aE[X]+E[b]=0$ $\qquad$ $V[aX+b]=E[(aX+b)^2]-\underbrace{E^2[aX+b]}_{0}$

$a\cdot 9+b=0$ $\qquad$ $V[aX+b]=E[a^2X^2+2aXb+b^2]-0$

$\boxed{b=-9a}$

$$a^2[E X^2]+2ab\,\underset{9}{E[X]}+b^2=1$$

$$a^2\,\frac{244}{3}+18ab+b^2=1$$

$$a^2\,\frac{244}{3}-162a^2+81a^2=1$$

$\boxed{a=\pm\sqrt{3}}$

$\boxed{b=\pm 9\sqrt{3}}$

$a=\sqrt{3}$ $\quad b=-9\sqrt{3}$

$a=-\sqrt{3}$ $\quad b=9\sqrt{3}$

3.16 $W = rV^2$ $r = cte.$

Voltaje $\sim$ normal

$E[W]$ con $\underline{r=3}$

media 6 y var 1.

$E[3V^2] = 3\,E[V^2]$

$\mu = 6$

$\sigma^2 = 1$

$Var[V] = E[V^2] - E^2[V]$

$1 + 36 = E[V^2]$

$37 = E[V^2]$

$E[3V^2] = 3 \cdot 37 = \boxed{111}$

3.17 X_1 y $X_2 \sim$ Bernoulli $Ber(p_1)$ y $Ber(p_2)$

a) $cov(X_1, X_2) = 0 \rightarrow$ Prob total.

$cov(X_1, X_2) = E[X_1, X_2] - E[X_1]E[X_2]$ al ser ind.

$E(xy) = E(x)E(y)$.

$= E[X_1]E[X_2] - E[X_1]E[X_2] = \underline{\underline{0}}$ ✓

b) si $p_1 = p_2 = 0{,}7$ $cov(X_1 X_2) = 0{,}1$ hallar $P(Y_1, Y_2)$

siendo $Y_1 = X_1(1 - X_2)$ $Y_2 = X_2(1 - X_1)$

del a). $cov(X_1, X_2) = E[X_1 X_2] - E[X_2]E[X_1]$.

$$cov(X_1, X_2) = \sum_{X_1=0}^{1} \sum_{X_2=0}^{1} X_1 X_2\, P_{X_1X_2}(X_1X_2) - P_1 P_2.$$

si $cov(X_1 X_2) = 0 \rightarrow P_{X_1X_2} = P_1 P_2$

sigo b

$$\rho(Y_1 Y_2) = \frac{cov(Y_1, Y_2)}{\sqrt{V(Y_1)}\sqrt{V(Y_2)}} =$$

$$\rho(Y_1, Y_2) = \frac{E[Y_1 Y_2] - E[Y_1]E[Y_2]}{\sqrt{V(Y_1)V(Y_2)}}$$

$$\rho(Y_1, Y_2) = \frac{cov(Y_1, Y_2)}{\sqrt{V(Y_1)\ V(Y_2)}}$$

↓ coeficiente de correlación

Y_2/Y_1	0	1	P_{Y2}
0	0,78	0,11	0,89
1	0,11	$\emptyset$	0,11
P_{Y1}	0,89	0,11	

$cov(Y_1 Y_2) = P_{Y_1 Y_2} - P_{Y1} P_{Y2}$

$\uparrow P_{X_1X_2} = P_1 P_2 + cov$.

$P_{Y_1Y_2}(0,0) = P_{X_1X_2}(0,0) + P_{X_1X_2}(1,1)$

$Y_1 = 0 \to X_1(1-X_2) \to X_1 = 0,\ X_2 = 0$

$Y_2 = 0 \to \ominus \to X_1 = 1,\ X_2 = 1$

$P_{Y_1Y_2}(0,0) = \overbrace{(1-0,7)}^{P_{X1}=0}\overbrace{(1-0,7)}^{P_{X2}=0} + 0,1 +$

$0,1 + \underbrace{0,7}_{P_{X_1}=1} \cdot \underbrace{0,7}_{P_{X_2}=1} = 0,78$.

X_2/X_1	0	1	P_{X2}
0	0,1		$1-P_2$
1			P_2
P_{X1}	$1-P_1$	P_1	1

$P_{Y_1Y_2}(1,0) = \overset{\uparrow \text{intersección}}{P_{X_1X_2}(1,0)} =$

$= P_{X1=1} \cdot P_{X2=0} - 0,1$

$= 0,11$

$cov(Y_1Y_2) = P_{X_1X_2}(X_1X_2) - P_1P_2 = 0$

$P_{X_1X_2} = P_{1_{X_1}} P_{2_{X_2}}$

$cov(Y_1Y_2) = \overbrace{P_{Y_1Y_2}}^{\overset{E[Y_1Y_2]\uparrow}{0}} - \overbrace{P_{Y1}P_{Y2}}^{0,11 \cdot 0,11} = -0,0121$

$Y_1 \sim B(0,11)$

$Y_2 \sim B(0,11)$

$$\rho(Y_1 Y_2) = \frac{-0,0121}{\sqrt{0,11}\sqrt{0,89}} = \checkmark$$

$\Rightarrow$ (en clase)

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$\boxed{\sqrt{V(y_1)} = \sigma_{y1}}$

otra forma.

$$\rho(y_1 y_2) = \frac{cov(y_1 y_2)}{\sigma_{y_1}\sigma_{y_2}} = \frac{E[y_1 y_2] - E[y_1]E[y_2]}{\sqrt{E[y_1^2]-E^2[y_1]}\sqrt{E[y_2^2]-E^2[y_2]}}$$

$E[y_1] = E[x_1] - E[x_1 x_2] = 0,7 - 0,49 = 0,21$

$E[y_2] = E[x_2] - E[x_1 x_2] = 0,7 - 0,49 = 0,21$

$cov(x_1 x_2) = 0,1 \qquad V[x_1] = 0,21 = V[x_2]$

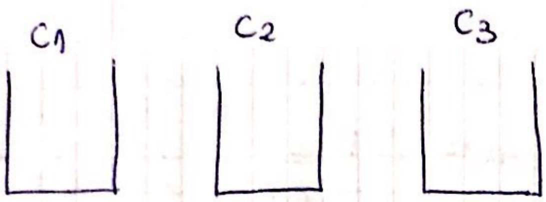
$\downarrow$

$P(1-P)$

$E[y_1 y_2] = P_{y_1} P_{y_2} = 0$

NO//

3.18.



X_i : cantidad de bolas en C_i $\quad X_i = 0, 1, 2, 3$

N : cantidad de urnas con alguna bola. $N = 1, 2, 3$

a)

N / X_1	0	1	2	3	$P_N(n)$
1	2/27	0	0	1/27	3/27
2	6/27	6/27	6/27	0	18/27
3	0	6/27	0	0	6/27
$P_{X_1}(x)$	8/27	12/27	6/27	1/27	1

$\#\Omega = 3^3 = 27$

→ todas en una urna

→ todas en 2 urnas

→ todas en 3 urnas

$P(X_i, N) =$

$P(0,1) = 2/27$ $\quad P(1,1) = 0$ $\quad P(2,1) =$

2 en X_1
1 todo

0 bolas en X_1
3 en → todas X_2
o
todas X_3

$P(0,2) = 6/27$

0 en X_1
2 urnas →

X_1	X_2	X_3
0	$b_1 b_2$	b_3
0	$b_1 b_3$	b_2
0	$b_2 b_3$	b_1

$\times 2$

$P(2,2) =$

X_1	X_2	X_3
$b_1 b_2$	b_3	
$b_1 b_2$		b_3

$\} \times 3$

$P(1,2) =$

1 bola X_1
2 contigo

X_1	X_2	X_3
b_1	$b_2 b_3$	0
b_2	$b_1 b_3$	0
b_3	$b_2 b_1$	0

$\} \times 2$

$P(3,2) = 0$

$P(1,3) = 6/27$

X_1	X_2	X_3
b_1	b_2	b_3

$\times 6$

prob total

$$E[N] = \sum N \cdot P_N = \overset{N}{1} \cdot \frac{3}{27} + \overset{N}{2} \cdot \frac{18}{27} + \overset{N}{3} \cdot \frac{6}{27} = \boxed{\frac{19}{9}}$$

$X = 1 \to$ todos en 1.

$\vdots$

$$VAR[N] = E[N^2] - E^2[N] = \boxed{\frac{26}{81}}$$

$$E[N^2] = 1^2 \cdot \frac{3}{27} + 4 \cdot \frac{18}{27} + 9 \cdot \frac{6}{27} = 43/9$$

$\text{COV}(N, X_1) = E[N, X_1] - E[N]\,E[X_1] =$

$\boxed{\text{COV}(N, X_1) = 0.}$

$E[N X_1] = \sum N . X_1 \cdot P_{NX} = 2 \cdot 1 \cdot \frac{6}{27} + 3 \cdot 1 \cdot \frac{6}{27} + 2 \cdot 2 \cdot \frac{6}{27} +$

$3 \cdot 1 \cdot \frac{1}{27} = 19/9.$

$E[X_1] = \cancel{X} \cdot P_X = 1 \cdot \frac{12}{27} + 2 \cdot \frac{6}{27} + \frac{1}{27} 3 + 0 \cdot \frac{8}{27} = 1.$

$E[N X_1] = \sum N X_1 \cdot \underbrace{P_{NX}}_{\text{conjunta}}$

$E[X_1] = \sum X_1 \underbrace{P_{X_1}}_{\text{marginal}}$ $\quad E[N] = \sum N \cdot \underbrace{P_N}_{\text{marginal}}$

b) Si fueran ind. $P(x_i N_i) = P(x_i)\,P(N_i)$

$P(X_1 = 0, N = 1) = P(X_1 = 0)\; P(N = 1)$

$\frac{2}{27} \neq 0 \cdot \frac{1}{27}$ Puede ocurrir.

$\downarrow \quad \downarrow$

$\frac{8}{27} \quad 3/27 \Rightarrow$ compr. COV $(X, N) = 0$ y no son ind.

c) $\text{COV}(X_i, X_j)$ $\quad 1 \leq i \leq j \leq 3$

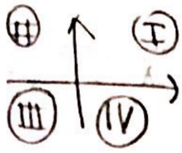
$\text{COV}(X_1, X_1) = \text{VAR}(X_1)$ $\quad 1 \leq i \leq 3$

$= E[X_1^2] - E^2[X_1].$ $\quad 1 \leq j \leq 3$

11	21	31
12	22	32
13	23	33

$\Rightarrow$ muy largo.

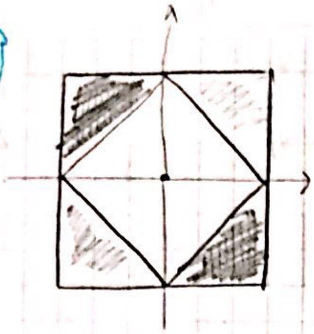
$E[X_1 X_2] = \sum X_1 X_2 \cdot P_{X_1 X_2} \ldots$



La cov > 0 mientras + puntos halla en I y III

3.19

a)



○ $f_{XY} = 1$

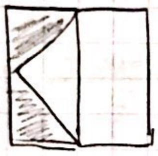
◎ $f_{XY} = 1/2$

● $f_{XY} = 3/2$

$$\operatorname{cov}(X,Y) = E[XY] - E[X]E[Y]$$

$\operatorname{cov}(X,Y) < 0$ pq la densidad de mayor valor está en los cuad II y IV q' suman

b)



$\operatorname{cov}(X,Y) = 0$ pq se compensan al estar en cuad II y III

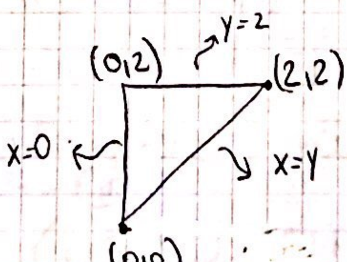
c) cuadrante I > 0 y III

II < 0 IV

PERO $|\operatorname{cov}_2(X,Y)| < |\operatorname{cov}_1(X,Y)|$

tiene nomás altos.

3.20



ÁREA $\triangle = 2$.

$$\boxed{f_{XY} = \frac{1}{\text{área}}}$$

X,Y vector aleat. $\sim \mathcal{U}(\Delta)$

continua

a) $\operatorname{cov}(X,Y) = E[XY] - E[X]E[Y] = 1/9$

$$E[X \cdot Y] = \iint_{\mathbb{R}^2} xy\, f_{XY} = \int_x^2 \int_0^2 \frac{xy}{2}\, dx\, dy = 1$$

$$E[X] = \int_0^2 \int_x^2 x \cdot \frac{1}{2}\, dx\, dy = \frac{2}{3} \qquad E[Y] = \int_0^2 \int_x^2 y\, \frac{1}{2}\, dx\, dy = 4/3$$

b) $VAR[X+Y] = E[(X+Y)^2] - E^2[X+Y] =$

$x^2 + 2xy + y^2$

$VAR[X+Y] = E[X^2] + 2E[XY] + E[Y^2] - E^2[X+Y] =$

no lo tengo

$E[X^2] = \int_0^2 \int_x^2 \frac{x^2}{2}\, dx\,dy = \frac{2}{3}$

$E[Y^2] = \int_0^2 \int_x^2 \frac{y^2}{2}\, dx\,dy = 2$

$VAR[X+Y] = VAR[X] + VAR[Y] + 2COV[XY]$

$VAR[X] = E[X^2] - E^2[X] = \frac{2}{3} - \frac{4}{9} = \frac{2}{9}$

$VAR[Y] = 2 - \frac{16}{9} = \frac{2}{9}$

$VAR[X+Y] = 2/9 + 2/9 + 2 \cdot 1/9 = \frac{2}{3}$

c) $COV(3X - Y + 2, X + Y) = \boxed{\frac{2}{3}}$

$COV(3X - Y + 2, X) + COV(3X - Y + 2, Y)$ $\boxed{COV(2, X) = 0.}$

I II

I $COV(3X, X) - COV(Y, X) + COV(2, X)$

$\to 0$

$3\,COV(X, X) - COV(X, Y) = 3V(X) - COV(X, Y) = 3 \cdot \frac{2}{9} - \frac{1}{9} = \frac{5}{9}$

II $COV(3X, Y) - COV(Y, Y) + COV(2, Y) =$

0

$3\,COV(X, Y) - V(Y) = 3 \cdot 1/9 - 2/9 = 1/9$

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Recta de Regresión

(X,Y) vector aleatorio. "conocido" la v.a X; se obtiene info sobre Y. "estimación"

$\hat{y} = ax + b$ tal que $E[(Y-\hat{Y})^2]$ sea mínima

Recta Regresión

ESTIMACION de Y en f. de X ←
$$\boxed{\hat{Y}(X) = E[Y] + \frac{cov(X,Y)}{V(X)}(X - E[X])}$$

⇓ $E[X]$ y $E[Y]$ ∈ recta.reg.

Es la mejor estimación lineal de Y dada X.

3.21 Juan y María

X: tiempo de reacción de Juan
Y: tiempo de reacción de María

$X,Y \sim U(0,2)$ son ind.

$<$ ← W: tiempo de reacción del ganador. $W = \min\{(X,Y)\}$ $\begin{cases} X & \text{si } X<Y \\ Y & \text{si } X \geq Y \end{cases}$

$>$ ← L: tiempo de reacción del perdedor.

a) Recta de $[L|W]$ "L dado W"

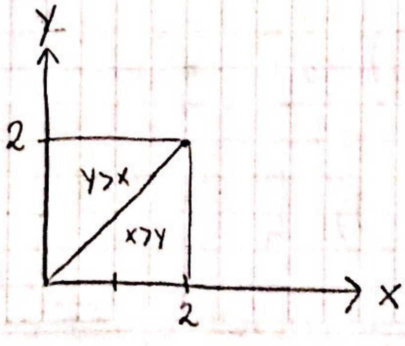
$L = aW + b$

$L = \max\{(X,Y)\}$ $\begin{cases} X \text{ si } X>Y \\ Y \text{ si } X \leq Y \end{cases}$

$$\hat{L}(W) = E[L] + \frac{cov(W,L)}{V(W)}(W - E[W])$$

$$\hat{L}(W) = E[h(X,Y)] + \frac{cov[g,h]}{V(g(X,Y))}(\underbrace{g(X,Y)}_{W} - E[g])$$

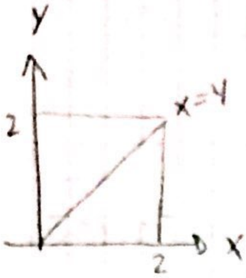
llamo $W = g(X,Y)$, $L = h(X,Y)$.



$$f_{XY}(x,y) = \frac{1}{\text{área}} = \frac{1}{4}\, \mathbb{1}\{(x,y) \in [0,2]^2\}$$

$$E[g(x,y)] = \iint_{\mathbb{R}^2} g(x,y) \cdot f_{XY} \, dx\, dy$$

↳ como tengo una función partida dividida en dos integrales



$$\int_0^2 \int_x^2 x \, \tfrac{1}{4} \, dy\, dx \, \mathbb{1}\{x<y\} + \int_0^2 \int_0^x y \, \tfrac{1}{4} \, dy\, dx \, \mathbb{1}\{x \geq y\}$$

$g(x,y)$
↳ x $x<y$
↳ y $x \geq y$

$$\int_0^2 \frac{x}{4} \cdot y \Big|_x^2 dx + \frac{1}{4} \int_0^2 \frac{y^2}{2} \Big|_0^x dx$$

$$\int_0^2 \frac{x}{4}(2-x) \, dx + \frac{1}{4} \int_0^2 \frac{x^2}{2} \, dx =$$

$$\frac{1}{2} \cdot \frac{x^2}{2} - \frac{x^3}{3} \cdot \frac{1}{4} \Big|_0^2 + \frac{1}{4} \left(\frac{x^3}{3.2} \right) \Big|_0^2 =$$

$$\frac{4}{4} - \frac{8}{12} + \frac{1}{4} \left(\frac{8}{6} \right) = \boxed{\frac{2}{3} = E[g(x,y)]} \to E[W]$$

$$E[h(x,y)] = \int_0^2 \int_0^x x \, \tfrac{1}{4} \, dy\, dx \, \mathbb{1}\{x \geq y\} + \int_0^2 \int_x^2 y \, \tfrac{1}{4} \, dy\, dx \, \mathbb{1}\{x<y\}$$

$h(x,y)$
↳ x $x \geq y$
↳ y $x<y$

$$E[h(x,y)] = \frac{1}{4} \int_0^2 x^2 \, dx + \frac{1}{4} \int_0^2 \frac{y^2}{2} \Big|_x^2 dx \longrightarrow \frac{4}{2} - \frac{x^2}{2}$$

$$E[h(x,y)] = \frac{1}{4} \frac{x^3}{3} \Big|_0^2 + \frac{1}{4} \left[2x - \frac{x^3}{6} \Big|_0^2 \right]$$

$$= \frac{1}{4}\left(\frac{8}{3}\right) + \frac{1}{4}\left(4 - \frac{8}{6}\right) = \frac{2}{3} + \frac{2}{3} = \boxed{\frac{4}{3} = E[L]}$$

$$\mathrm{Var}(W) = E[W^2] - E^2[W] = \frac{1}{3} - \left(\frac{2}{3}\right)^2 = \boxed{\frac{2}{9}}$$

$$E[W^2] = \int_0^2 \int_x^2 \frac{x^2}{4} \, dy\, dx + \int_0^2 \int_0^x \frac{y^2}{4} \, dy\, dx = \quad \to \frac{y^3}{3}\Big|_0^x$$

$$E[W^2] = \int_0^2 \frac{x^2}{4}(2-x) \, dx + \frac{1}{4} \int_0^2 \frac{x^3}{3} \, dx = \frac{1}{4}\left(\frac{2x^3}{3} - \frac{x^4}{4} \Big|_0^2 \right) + \frac{1}{12} \cdot \frac{x^4}{4} \Big|_0^2$$

$$\frac{1}{4}\left(\frac{16}{3}-\frac{16}{4}\right)+\frac{1}{4}\left(\frac{16}{12}\right)=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

$$\operatorname{cov}(g,h)=E[gh]-E[h]E[g]=1-\left(\frac{4}{3}\cdot\frac{2}{3}\right)=\frac{1}{9}$$

$$E[gh]=E[X\cdot Y]=\int_0^2\int_0^2\frac{xy}{4}\,dx\,dy=\frac{1}{4}\int_0^2\frac{x^2}{2}\Big|_0^2\,y\,dy$$

$$=\frac{1}{4}\left(\frac{4}{2}\right)\frac{y^2}{2}\Big|_0^2=\frac{1}{4}\cdot 2\cdot 2=1$$

si $x<y$

$gh=xy$ — vale xy $\forall$ soporte.

si $x\geq y$

$gh=xy$

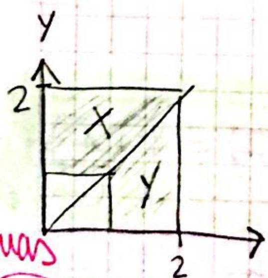
$$\hat{L}(w)=\frac{4}{3}+\frac{\frac{1}{9}}{2/9}\left(w-\frac{2}{3}\right)=\frac{4}{3}+\frac{1}{2}\left(w-\frac{2}{3}\right)$$

$\frac{1}{2}\left(-\frac{2}{3}\right)=-\frac{1}{3}$

$$\boxed{\hat{L}(w)=\frac{1}{2}w+1}$$

b)

$$E[W\mid L>1]=\frac{E[W\,\mathbb{1}\{L>1\}]}{P(L>1)}=\frac{\int_1^2\int_0^y\frac{x}{4}+\int_1^2\int_0^x\frac{y}{4}\,dy\,dx}{1-\underbrace{P(L\leq 1)}_{F_{XY}(1)}}$$



en continuas

$P(L>1)=1-P(L\leq 1)$

$P(L\leq 1)=\iint_{\text{Área total } L\leq 1}\underbrace{f_{xy}}_{1/\text{Área total}}\,dx\,dy$

$$\iint_A\frac{1}{\text{Área}}\,dx\,dy=\frac{1}{4}\,\underbrace{\text{Área}(L\leq 1)}_{=1}=\frac{1}{4}$$

$$1-\frac{1}{4}=3/4$$

$$\frac{7/24+7/24}{3/4}\longrightarrow\boxed{E[W\mid L>1]=7/9}$$

3.22 guía 5

$$f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}\left(x^2 - \frac{6}{5}xy + y^2\right)}$$

$$= \frac{1}{\pi \frac{8}{5}} e^{-\frac{25}{32}\left(\left(y - \frac{3}{5}x\right)^2 + \frac{16}{25}x^2\right)}$$

↳ factorizo para buscar 2 normales

$$= \frac{1}{\sqrt{\pi}\,\frac{8}{5}} e^{-\frac{\left(y - \frac{3}{5}x\right)^2}{32/25}} \cdot \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}x^2} =$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}\,\frac{4}{5}} e^{-\frac{1}{2}\frac{\left(y - 3/5\,x\right)^2}{16/25}}}_{f_{Y|X=x}} \cdot \underbrace{\frac{1}{\sqrt{2\pi}} e^{-1/2\,x^2}}_{f_X}$$

$f_{Y|X=x}$ ↓ $X \sim N\left(\frac{3}{5}x, \frac{16}{25}\right)$ ↳ no estandar

f_X ↓ $X \sim N(0,1)$ → estandar

$$E[Y|X] = \boxed{\frac{3}{5}X = \varphi(X)}$$

como mi función φ (regresión) es lineal, esa ya es la ~~función~~ Recta de Regresión

3.23

X va ≥ 0, $E[X] = 15$

$P(X \geq 60) \leq 0.25$

Por Markov

$$P(X > a) \leq \frac{E[X]}{a}$$

$$P(X \geq 60) \leq \frac{15}{60}$$

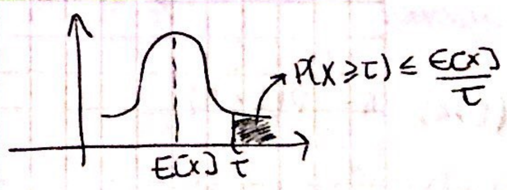
$$P(X \geq 60) \leq 0{,}25 \checkmark$$

Desigualdad de MARKOV

Sea $X \geq 0$, con $E[X]$ finita

$\forall \tau > 0$,

$$\mathbb{P}(X \geq \tau) \leq \frac{E[X]}{\tau}$$



3.24 X v.a $E[X] = 10$ $V(X) = 15$.

$$P(5 < X < 15) \geq 0.4$$

Por chebyshev

$$|x-10| \begin{cases} \to x-10 \\ \to -x+10 \end{cases}$$

$$P(|X-\mu| > \varepsilon) \leq \frac{V(x)}{\varepsilon^2}$$

$x - 10 \leq 5$.
$\boxed{x \leq 15}$

$$P(|x-10| \leq 5) \geq 1 - \underbrace{\frac{V(x)}{\varepsilon^2}}_{} \quad \frac{15}{5^2}$$

$-x+10 \leq 5$.

$\boxed{5 \leq x}$

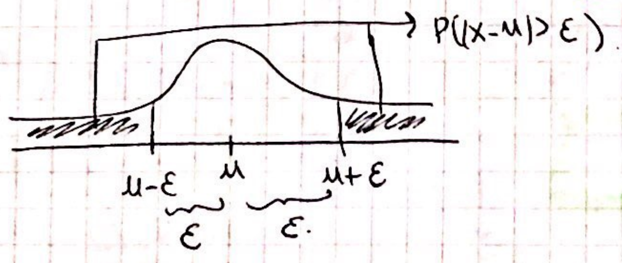
$$\boxed{P(5 \leq x \leq 15) \geq 0,4}$$

Desigualdad de
CHEVICHEB

sea X. v.a cualquiera, con $E[X]$ y $V(X)$ finitos.

$E[X] = \mu$ $V(x) = \sigma^2$ $\forall \varepsilon > 0$

$$\left.\begin{array}{l} P(|x-\mu| > \varepsilon) \leq \dfrac{V(x)}{\varepsilon^2} \\ P(|x-\mu| \leq \varepsilon) \geq 1 - \dfrac{V(x)}{\varepsilon^2} \end{array}\right\} \text{equivalentes.}$$



3.25 X: Peso paquete de cafe $X \sim \text{normal}(500g, \sigma)$

¿Cómo deben ser los valores de σ, para tener una seg del 99 % de que el peso promedio de 100 paquetes no se desvié en mas de 10g de 500g?

$6/\ P(\underbrace{490 < \bar{X} < 510}) = \underbrace{0{,}99}$

como varia X: peso

99% → seguridad

NORMAL ~ (500, σ)

$E[\bar{X}] = \frac{1}{100} \underbrace{\sum_{i=1}^{100} E[x_i]}_{500 \times 100} = 500$

siendo

$\underbrace{\bar{X} = \frac{\sum_{i=1}^{100} x_i}{100}}$

peso promedio de 100 paquetes

$\sigma = V[\bar{X}] = \frac{1}{100^2} \sum_{i=1}^{100} V(x_i) = \frac{\sigma^2}{100}$

$\boxed{Z = \frac{X - \mu}{\sigma}}$ → para estandarizar

$Z = \frac{X - 500}{\sigma^2/100} \rightarrow X = Z \cdot \frac{\sigma^2}{100} + 500$

$P\left(\frac{490 - 500}{\frac{\sigma^2}{100}} < Z < \frac{510 - 500}{\sigma^2/100}\right) = 0{,}99$

$P\left(-\frac{1000}{\sigma^2} < Z < \frac{1000}{\sigma^2}\right) = 2\,\Phi\left(\frac{1000}{\sigma^2}\right) - 1 \cong 0{,}99$

guía 8.

$\Phi\left(\frac{100}{\sigma^2}\right) = 0{,}995$

$\frac{100}{\sigma^2} = \Phi^{-1}(995)$

$\boxed{\sigma \approx 6{,}23}$

pero con dia en mal estado.

$490 < x < 510 \rightarrow |x - 500| \leq 10$

si no

Chebichev

$X - 500 \leq 10$

$x \leq 510$

$-X + 500 \leq 10$

$490 \leq X$

$\mathbb{P}(|x - 500| \leq 10) \geq \overbrace{0{,}99}^{1 - \frac{V(x)}{\varepsilon^2}}$

$\mathbb{P}(|x - 500| \geq 10) \leq \underbrace{1 - 0{,}99}$

$\frac{V(x)}{\varepsilon^2} = \frac{\sigma}{\varepsilon^2} \leq 0{,}01$

$1 - \frac{V(x)}{\varepsilon^2} = a$

$1 - a = \left(\frac{V(x)}{\varepsilon^2}\right)$

$a = 0{,}99$

$\boxed{\sigma \leq 10}$ ← $\frac{\frac{\sigma^2}{100}}{10^2} \leq 0{,}01$

3.26 Fumadores en una población

$E[X] = P$

$V[X] = P(1-P)$

$0 < P < 1$

$P(X=1) = P$

$P(X=0) = 1-P$

$X_i = \begin{cases} 1 & \text{si } \text{♀} \text{ fuma} \\ 0 & \text{si } \text{♀} \text{ no fuma} \end{cases} \rightarrow$ 2 tomos 1 o 0 Bernoulli.

$X_i \sim Be(p) \rightarrow$ prob fumador.

Def

$\boxed{Y = \frac{\sum x_i}{m}} \rightarrow$ proporción de los que fuman en muestra m

¿ valor de m, para que la proporción de fumadores no difiera de 0,01, con prob mayor o igual a 0,95?

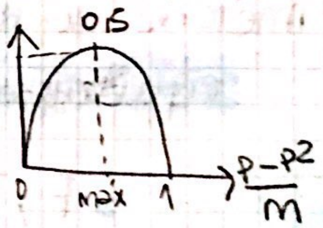
BUSCO P $\rightarrow V(y) = \frac{P - P^2}{m} \rightarrow$ parabola

Y = proporción de fumadores

BUSCO su máximo $\boxed{P = 0{,}5}$ → valor máx de P

p = prob de fumador

$VAR(Y) \leq \frac{0{,}5 - 0{,}5^2}{\underbrace{m}_{\text{máx } VAR(X)}}$



$E[Y] = \frac{1}{m} \sum E[x_i] = \frac{1}{m} \cdot m \cdot p = P$

$V[Y] = \frac{1}{m^2} \sum V[x_i] = \frac{1}{m^2}\, m\,(p(1-p)) = \frac{P(1-P)}{m}$

IMP

$Q = \frac{P(1-P)}{m} = \frac{P-P^2}{m}$

$V[Y]$

$\frac{dQ}{dP} = 0 \rightarrow$ máx

si es una parab. $\frac{1-0}{2} = 0{,}5$

Chebychev.

$P(|Y - p| \leq 0{,}01) \geq 0{,}95 \rightarrow 1 - \frac{V(Y)}{\varepsilon^2}$

$1 - \frac{0{,}5 - 0{,}5^2}{\underbrace{\varepsilon^2}_{0{,}01} \cdot m} \geq 0{,}95$

$\frac{1}{20} \geq \frac{1/4}{0{,}01^2 \, m}$

$\boxed{m \geq 50.000}$

PROPORCIONES

aplica ∀ V.A

Y: proporción de X sobre total

$V[X] = P(1-P)$

$E[X] = P$

$Y = \frac{\sum x_i}{m} \rightarrow$ total

$V[Y] = \frac{1}{m^2} \sum_{i=1}^{m} V[x_i] = \frac{1}{m^2} \cdot m\, V[x]$

$E[Y] = \frac{1}{m} \sum_{i=1}^{m} E[x_i] = \frac{1}{m} (m \cdot p)$