HOJA Nº 1

FECHA

GUÍA 4

4.1

X V.A DISCRETA $\left\{ \frac{k}{8} : k = 0, \ldots, 1 \right\}$

$$P_X(x) = \frac{2}{9}x$$

(A) CALCULAR $P_Y(y)$

$$G(x) = 2X - 1 = Y \qquad X = \frac{1}{2}y + \frac{1}{2}$$

$$F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(G(x) \leq y) = \mathbb{P}(X \leq G^{-1}(y))$$

$$\mathbb{P}(X \leq G^{-1}(y)) = F_X(G^{-1}(y))$$

$$F_Y(y) = F_X(G^{-1}(y))$$

$$F_Y(y) = F_X\left(\frac{1}{2}y + \frac{1}{2}\right)$$

$$P_Y(y) = P_Y(Y = y) = P_Y(2X - 1 = y) =$$

$$P_Y\left(X = \frac{y+1}{2}\right) \qquad P_Y(y) = \sum P_X\left(\frac{y+1}{2}\right) = \frac{2}{9}\left(\frac{y+1}{2}\right)$$

$$P_Y(y) = \frac{y+1}{9}$$

PARA LOS PUNTOS

$$X = \frac{y+1}{2}$$

$X = 0 \rightarrow Y = -1$

$X = 1/8 \qquad Y = -3/4$

$X = 2/8 \qquad Y = -1/2$

⋮

NOTA

(B) $P_Y(y)$ $\quad Y = 128X^2$

$$P_Y(y) = \mathbb{P}(Y = y) = \mathbb{P}(128X^2 = y)$$
$$= \mathbb{P}\left(X^2 = \frac{y}{128}\right)$$
$$= \mathbb{P}\left(|X| = \frac{\sqrt{y}}{2\sqrt{8}}\right)$$
$$= \mathbb{P}\left(X = \frac{\sqrt{y}}{128} \wedge X = -\frac{\sqrt{y}}{128}\right)$$

PERO COMO LOS VALORES DE X SON >0 ME SACO LA PART NEGATIVA

$$\mathbb{P}\left(X = \frac{\sqrt{y}}{128}\right) \Rightarrow P_Y(y) = P_X\left(\frac{\sqrt{y}}{8\sqrt{2}}\right)$$

$$P_Y(y) = \frac{2}{9}\left(\frac{\sqrt{y}}{8\sqrt{2}}\right)$$

(C) $y = -64x^2 + 64x + 2$

$$y = \left(x - \frac{4+3\sqrt{2}}{8}\right)\left(x - \frac{4-3\sqrt{2}}{8}\right)$$

NOTA

4.2

$X \sim Poi(2)$

$$Y = \left|SEN\left(\frac{1}{2}\pi X\right)\right| = \begin{cases} 0 & X = 2N \\ 1 & X = 2N+1 \end{cases}$$

$$P_Y(y) = \mathbb{P}(Y = y) = \mathbb{P}(Y = 0) = \mathbb{P}(X = 2N)$$

$$P_X(x) = \frac{2^x e^{-2}}{x!}$$

$$\mathbb{P}(X = 2N) = \sum_{N=0}^{\infty} \frac{2^{2N} \cdot e^{-2}}{(2N)!} = e^{-2}\cosh(2)$$

$$\mathbb{P}(X = 2N+1) = \sum_{N=0}^{\infty} \frac{2^{2N+1}\, e^{-2}}{(2N+1)!} = e^{-2} SENh(2)$$

$$P_Y(y) = \begin{cases} e^{-2}\cosh(2) & \text{si } y = 0 \\ e^{-2} SENh(2) & \text{si } y = 1 \end{cases}$$

4.3

X V.A CONTINUA

$$f_X(x) = \frac{12x}{\pi^2(e^x+1)} \mathbb{1}\{x>0\}$$

(A) $Y = AX + B \longrightarrow X = \frac{Y-B}{A} = G^{-1}(Y)$

$$f_Y(y) = \frac{f_X(G^{-1}(y))}{G'(G^{-1}(y))}$$

$$f_Y(y) = \frac{f_X\left(\frac{y-B}{A}\right)}{A}$$

$$f_Y(y) = \frac{12\left(\frac{y-B}{A}\right)}{\pi^2\left(e^{\frac{y-B}{A}}+1\right)A} \mathbb{1}\left\{\frac{y-B}{A}>0\right\}$$

$\mathbb{1}\{y>B\}$

(B) $Y = -X^3$

$-Y = X^3$

$\sqrt[3]{-y} = X \rightarrow X = (-y)^{3/2} \quad X' = -\frac{3}{2}(-y)^{1/2}$

$F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(-X^3 \leq y) = \mathbb{P}(X \leq \sqrt{-y}^3)$

$F_X(\sqrt{-y^3}) = F_Y(y)$

$-3x^2$

$f_Y(y) = f_X(\sqrt[3]{-y}) \cdot \frac{1}{3}(-y)^{-2/3} \cancel{\cdot 1}$ $\quad g(x) = -x^3$

$g'(x) = -3x^2$

$= \frac{f_X(\sqrt[3]{-y})}{-3\cdot(-y)^{2/3}} \quad g'(g^{-1}(y)) \quad g^{-1}(y) = \sqrt[3]{-y^3}$

$g^{-1'} = -\frac{1}{3}y^{-2/3}$

NOTA

HOJA Nº 3

FECHA

(c) $y = x + x^{-1}$

$$y = x + \frac{1}{x}$$

$$y = \frac{x^2+1}{x} \longrightarrow x^2 - yx + 1 = 0$$

$$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$$

$$x_1 = \frac{y + \sqrt{y^2-4}}{2} \qquad x_2 = \frac{y - \sqrt{y^2-4}}{2}$$

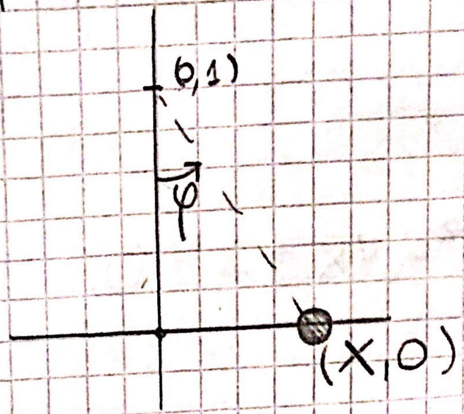
$$x_1 = \frac{1}{2}y + \frac{\sqrt{y^2-4}}{2} \qquad x_2 = \frac{1}{2}y - \frac{\sqrt{y^2-4}}{2}$$

$$x_1, x_2' = \frac{1}{2} \pm \frac{1}{4}\left(y^2-4\right)^{-1/2} \cdot 2y$$

$$f_Y(y) = \frac{f_X(x)}{G'(x)}\Bigg|_{x = G^{-1}(y)} = \frac{f_X\left(\overbrace{\frac{1}{2}y \pm \frac{\sqrt{y^2-4}}{2}}^{x_1 = G^{-1}(y)}\right)}{}$$

NOTA

4.4 $\varphi \sim \mathcal{U}(-\pi/2, \pi/2)$ $\qquad \varphi_x(x) = \frac{1}{\pi}$



$$X = \tan\varphi \longrightarrow \tan\varphi = \frac{O}{A} = \frac{X}{1}$$

$$\downarrow$$

$$\varphi = \arctan(X)$$

$$\varphi \sim \mathcal{U}(-\pi/2, \pi/2)$$

$$f_\varphi(\varphi) = \frac{1}{\pi}$$

$$F_X(x) = \mathbb{P}(X \le x) = \mathbb{P}(\tan\varphi \le x) =$$

$$\mathbb{P}(\varphi \le \arctan(x)) = F_\varphi(\arctan(x))$$

$$F_X(x) = F_\varphi(\arctan(x))$$

$$f_X(x) = f_\varphi(\arctan(x)) \cdot \frac{1}{1+x^2}$$

COMO NO ES SUAVE QUEDA ASÍ

$$f_X(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right) = \frac{1}{\pi(1+x^2)} = f_X(x)$$

HOJA Nº 4

FECHA

4.5

$\varphi \sim \mathcal{U}(-\pi, \pi)$

(A) HALLAR $f_C(C)$

$C = \cos\varphi$

$f_\varphi(\varphi) = \left\{ \frac{1}{2\pi} \; \mathbb{I}_\varphi \; -\pi < x < \pi \right\} =$

$F_C(C) = \mathbb{P}(C \leq C) = \mathbb{P}(\cos\varphi < C) =$

$\mathbb{P}(|\varphi| > \text{ARCCOS}(C)) = \mathbb{P}(-\text{ARCOS} < \varphi \leq \text{ARCOS}(C))$

$F_C(C) = F_\varphi(-\text{ARCCOS}(C)) - F_\varphi(\text{ARCCOS}(X))$

$F_C(C) = \frac{\text{ARCCOS}(C) + \pi}{2\pi} - \frac{\text{ARCOS}(C) + \pi}{2\pi}$

$F_C(C) = -\frac{\text{ARCOS}(C)}{\pi} \qquad -\frac{1}{\pi} \cdot \frac{-1}{\sqrt{1-x^2}}$

(B) $\mathbb{P}(|C| < 0{,}5)$

$\mathbb{P}(-0{,}5 < C < 0{,}5) = \int_{-0{,}5}^{0{,}5} \frac{1}{\pi} \cdot \frac{1}{\sqrt{1-x^2}} \, dx = \frac{1}{3}$

NOTA

4.6

$X \sim \mathcal{U}(5, 50)$ X: "HORA DE LLEGADA"

Y: TIEMPO DE ESPERA

$$f_X(x) = \frac{1}{45} \mathbb{1}\{5 < x < 50\}$$

$$F_X(x) \int_{-\infty}^{+\infty} f_X(x)\,dx$$

$$Y = 15 - X\,\mathbb{1}\{5 < x < 15\} + 30 - X\mathbb{1}\{15 < x < 30\}$$
$$+ 45 - X\,\mathbb{1}\{30 < x < 45\} + 60 - X\,\mathbb{1}\{45 < x < 50\}$$

$\downarrow$

$G(X)$

$$F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(G(X) \leq y) =$$

NOTA

HOJA Nº 5

FECHA

4.7

$V_2 = Y$

$X \sim \mathcal{U}(180, 220)$ $\quad V_1 = X$

$$Y = G(X) = \frac{X-190}{20}\mathbb{1}\{190 \leq X \leq 210\} + \mathbb{1}\{210 < X\}$$

$$f_X(x) = \frac{1}{40} \qquad F_X(x) = \int_{180}^{x} \frac{1}{40}\,dx = \frac{1}{40}(x-180)$$

- $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}\left(\frac{X-190}{20} \leq y\right) =$

$\mathbb{P}(X \leq 20y + 190) = F_X(20y+190)$

- $F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(Y=1) + \mathbb{P}(Y<1)$

$$F_Y(y) = F_X(20y+190) + \int_{210}^{220} \frac{1}{40}\,dx$$

$$F_Y(y) = \overbrace{F_X(20y+190)}^{1/2y + 1/4}\mathbb{1}\{0 \leq y < 1\} + \frac{1}{4}\mathbb{1}\{y=1\}$$

NOTA

4.14

$X_1 \sim EXP(\lambda_1)$

$X_2 \sim EXP(\lambda_2)$

COMO SON INDEP $F_{X_1X_2} = F_{X_1} \cdot F_{X_2}$

$f_{X_1X_2} = \lambda_1 \lambda_2 \cdot e^{-(\lambda_1 x_1 + \lambda_2 x_2)}$

$U = MÍN(X_1, X_2)$

$V = MAX(X_1 X_2)$

$W = V - U$

$J = \mathbb{1}\{U = X_1\} + 2\,\mathbb{1}\{U = X_2\}$

(A) HALLAR $f_U(U)$

$$U = \begin{cases} X_1 & X_1 < X_2 \\ X_2 & X_1 > X_2 \end{cases}$$

$F_U(U) = \mathbb{P}(U \leq U) = \mathbb{P}(MÍN\{X_1, X_2\} \leq U, X_1 < X_2) + \mathbb{P}(MIN\{X_1, X_2\}, X_1 \geq X_2) = \mathbb{P}(X_1, X_1 < X_2) + \mathbb{P}(X_2, X_1 \geq X_2) =$

$$\iint\limits_{\substack{\forall x_1, x_2 \\ x_1 \leq u,\ x_1 < x_2}} F_{x_1x_2}\, dx_2 + \iint\limits_{\substack{\forall x_1, x_2 \\ x_2 \leq u\ \ x_1 \geq x_2}} F_{x_1x_2}\, dx_1$$

$$\int_0^u \int_{x_1}^{\infty} \lambda_1 \lambda_2 e^{-(\lambda_1 x_1 + \lambda_2 x_2)} dx_1 dx_2 + \int_0^u \int_{x_2}^{\infty} \lambda_1 \lambda_2 e^{-(\lambda_1 x_1 + \lambda_2 x_2)} dx_1 dx_2$$

$$\lambda_1 \cancel{\lambda_2} \int_0^u \left. \frac{-e^{-(\lambda_1 x_1 + \lambda_2 x_2)}}{\cancel{\lambda_2}} \right|_{x_1}^{\infty} dx_1 + \cancel{\lambda_1} \lambda_2 \int_0^u \left. \frac{-e^{-(\lambda_1 x_1 + \lambda_2 x_2)}}{\cancel{\lambda_1}} \right|_{x_2}^{\infty} d$$

$$\lambda_1 \int_0^u 0 + e^{-(\lambda_1+\lambda_2)x_1}\, dx_1 = \lambda_1 \cdot \left[\frac{-e^{-(\lambda_1+\lambda_2)x_1}}{\lambda_1+\lambda_2} \right|_0^u$$

$$\frac{\lambda_1}{\lambda_1+\lambda_2}\left[-e^{-(\lambda_1+\lambda_2)u} + 1 \right]$$

$$F_U(u) = \frac{\lambda_1}{\lambda_1+\lambda_2}\left[1 - e^{-(\lambda_1+\lambda_2)u}\right] + \frac{\lambda_2}{\lambda_1+\lambda_2}\left[1 - e^{-(\lambda_1+\lambda_2)u}\right]$$

SACO FACTOR COMÚN LOS [ ]

$$F_U(u) = 1 - e^{-(\lambda_1+\lambda_2)u} \underbrace{\left[\frac{\lambda_1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\lambda_1+\lambda_2}\right]}_{1} \quad (u \geq 0)$$

SI ES MENOR NO CUMPLE CON EL SOPORTE

$$F_U(u) = 1 - e^{-(\lambda_1+\lambda_2)u}$$

ENTONCES $\boxed{U \sim \text{EXP}(\lambda_1+\lambda_2)}$

SE DA CUANDO

$U = \text{MÍN}\{X_1, X_2\}$ SIENDO

$X_1 \sim \text{EXP}(\lambda_1)$ Y $X_2 \sim \text{EXP}(\lambda_2)$

Y X_1, X_2 INDEPENDIENTES

(B) HALLAR $P_J(j)$

$$J = \mathbb{1}\{U = X_1\} + 2\,\mathbb{1}\{U = X_2\} = \begin{cases} 1 & X_1 < X_2 \\ 2 & X_2 \leq X_1 \end{cases}$$

$$\mathbb{P}(J=1) = \mathbb{P}(X_2 > X_1) = \int_0^{+\infty}\int_{x_1}^{+\infty} f_{X_1X_2}(x_1 x_2)\, dx_1 dx_2$$

$$= \int_0^{+\infty}\int_{x_1}^{+\infty} \lambda_1\lambda_2\, e^{-(\lambda_1 x_1 + \lambda_2 x_2)}\, dx_2 dx_1 = \lambda_1\lambda_2 \int_0^{+\infty} \frac{-e^{-(\lambda_1 x_1+\lambda_2 x_2)}}{\lambda_2}\Bigg|_{x_1}^{+\infty}$$

NOTA

HOJA N° 7

FECHA

$$= \lambda_1 \int_0^{+\infty} - - e^{-(\lambda_1+\lambda_2)x_1} dx_1 = \lambda_1 \int_0^{+\infty} e^{-(\lambda_1+\lambda_2)x_1} dx_1$$

$$\lambda_1 \left[- \frac{e^{-(\lambda_1+\lambda_2)x_1}}{\lambda_1+\lambda_2} \right]_0^{+\infty} = \lambda_1 \left[0 + \frac{1}{\lambda_1+\lambda_2} \right]$$

$$\boxed{\mathbb{P}(J=1) = \frac{\lambda_1}{\lambda_1+\lambda_2}} \longrightarrow X_1 \text{ SEA EL MÍN}$$

$\rightarrow$ X_2 SEA EL MÍN

$$\mathbb{P}(J=2) = 1 - \mathbb{P}(J<2) = 1 - \mathbb{P}(J=1)$$

$$\mathbb{P}(J=2) = 1 - \frac{\lambda_1}{\lambda_1+\lambda_2} = \boxed{\frac{\lambda_2}{\lambda_1+\lambda_2} = \mathbb{P}(J=2)}$$

ENTONCES, SEAN $X_k \sim EXP(\lambda_k)$, $k=1,2,\dots m$

$$\mathbb{P}(X_m = \text{MÍN}(X_k)) = \frac{\lambda_m}{\sum_1^m \lambda_k}$$

(C) $f_W(w)$

$W = V - U$

$W = \text{MAX}\{X_1, X_2\} - \text{MÍN}\{X_1, X_2\} = |X_1 - X_2|$

$$|X_1 - X_2| = \begin{cases} X_1 - X_2 & X_1 \geq X_2 \\ X_2 - X_1 & X_2 > X_1 \end{cases}$$

$$F_W(w) = \mathbb{P}(W \leq w) = \mathbb{P}(|X_1 - X_2| \leq w, X_2 > X_1) + \mathbb{P}(|X_1 - X_2| \leq w, X_1 \geq X_2)$$

NOTA

$$\mathbb{P}(x_1 < X_2 \leq W,\ x_1 < x_2) + \mathbb{P}(x_1 \geq X_2 \leq W,\ x_1 \geq x_2)$$

$$\mathbb{P}(X_1 < X_2 \leq X_1 + W) + \mathbb{P}(X_2 \leq X_1 \leq X_2 + W)$$

$$\mathbb{P}(x_1 < X_2 \leq X_1 + W) = \int_0^{+\infty}\int_{x_1}^{x_1+w} \lambda_1 \lambda_2 e^{-(\lambda_1 x_1 + \lambda_2 x_2)}\, dx_2\, dx_1$$

⋮

$$\mathbb{P}(X_1 < X_2 \leq X_1 + W) = \frac{\lambda_1}{\lambda_1 + \lambda_2}\left(1 - e^{-\lambda_2 w}\right) \mathbb{1}\{w > 0\}$$

CUANDO $(W > 0)$ SI ES MENOR $\mathbb{P} = 0$

$$\mathbb{P}(X_2 < X_1 < X_2 + W) = \frac{\lambda_2}{\lambda_1 + \lambda_2}\left(1 - e^{-\lambda w}\right) \mathbb{1}\{w > 0\}$$

$$f_W(w) = \underbrace{\frac{\lambda_2}{\lambda_1 + \lambda_2}}_{\mathbb{P}(J=1)} \cdot \underbrace{\lambda_1 e^{-\lambda_1 w}}_{f_X(w)} + \underbrace{\frac{\lambda_1}{\lambda_1 + \lambda_2}}_{1-\mathbb{P}(J=1)} \cdot \underbrace{\lambda_2 e^{-\lambda_2 w}}_{f_Y(w)}$$

$\downarrow \mathbb{P}(W < w / J=1)$ $\qquad \hookrightarrow \mathbb{P}(W \leq w / J=2)$

INTEGRO

OBS! W ES UNA MEZCLA DE X_1 Y X_2

$W / X_1 < X_2 \sim EXP(\lambda_2) \qquad W / X_2 \leq X_1 \sim EXP(\lambda_1)$

(D) MOSTRAR QUE U Y J / U Y W SON INDEPENDIENTES

DEMOSTRADO EN EJS MANU!!

$$F_W(w) = \frac{\lambda_2}{\lambda_1 + \lambda_2}\left(1 - e^{-\lambda_1 T}\right) + \frac{\lambda_1}{\lambda_2 + \lambda_1}\left(1 - e^{-\lambda_2 T}\right) \mathbb{1}\{T > 0\}$$

NOTA

4.15

$J \sim EXP\left(\frac{5}{60}\right) \qquad P \sim EXP\left(\frac{10}{60}\right)$

J y P SON INDEPENDIENTES

NO ATIENDEN → 5 MIN DESDE INICIO DEL JUEGO
↘ ULTIMA INTERRUPCION

COMIENZAN A LAS 10:

(A)

PRIMER LLAMADO $= U = \text{MÍN}\{J, P\}$

$\mathbb{P}(U<5) = F_U(5) = 1 - e^{-(1/12 + 1/6)\cdot 5}$

$= 0,713$

(B) $\mathbb{P}(U = J)$

$\mathbb{P}(J = \text{MIN}(J,P)) = \frac{\lambda_J}{\lambda_J + \lambda_P} = \frac{1}{3}$

(C) $\mathbb{P}(U<5 \mid J=1)$

SON INDEP
(DEM 4.14)

$\mathbb{P}(U<5 \mid J=1) = \frac{\mathbb{P}(U<5) \cap \mathbb{P}(J=1)}{\mathbb{P}(J=1)}$

(D)

NOTA