GUÍA 3

3.1 X V.A (MIXTA)

$$F_X(x) = \frac{x^3}{3}\mathbb{1}\{0 \le x < 1\} + \frac{2x+1}{6}\mathbb{1}\{1 \le x < 2\} + \mathbb{1}\{x \geq 2\}$$

(A) $\mathbb{E}[X] = \sum_{x \in D} x \cdot \mathbb{P}(X=x) + \int x\, F'(x)\, dx$

$f_X(x) = F_X'(x)$

$f_X(x) = \frac{3}{3}x^2\mathbb{1}\{0 \le x < 1\} + \frac{2}{6}\mathbb{1}\{1 \le x < 2\}$

$$\mathbb{E}[X] = \int_0^1 x(x^2)\,dx + \int_1^2 x\left(\frac{2}{6}\right)dx = \int_0^1 x^3\,dx + \int_1^2 \frac{2}{6}x\,dx$$

$$\mathbb{E}[X] = \int_0^1 x^3 dx + \int_1^2 \frac{2}{6}x\,dx + 1.\overbrace{\left(\frac{3}{6}-\frac{1}{3}\right)}^{\text{SALTO}} + 2.\overbrace{\left(1-\frac{5}{6}\right)}^{\text{SALTO}}$$

SALTO $= F^+(x) - F^-(x)$

$$\mathbb{E}[X] = \frac{1}{4} + \frac{1}{2} + \frac{1}{6} + 2.\frac{1}{6}$$

$$\mathbb{E}[X] = \frac{5}{4}$$

(B) $\mathbb{E}[X \mid X < 1]$ y $\mathbb{E}[X \mid X \le 1]$

$$\mathbb{E}[X \mid X<1] = \frac{\mathbb{E}[X\,\mathbb{1}\{X<1\}]}{\mathbb{P}(X<1)} = \frac{1/4}{F_X^-(1)} = \frac{1/4}{1/3}$$

NO INCLUYE EL SALTO

$\mathbb{E}[X \mid X<1] = 3/4$

XQ AHORA INCLUYE EL SALTO

$$\mathbb{E}[X \mid X \le 1] = \frac{\mathbb{E}[X\,\mathbb{1}\{X\le 1\}]}{\mathbb{P}(X \le 1)} = \frac{1/4 + 1/6}{1/3 + 1/6} = \frac{5}{6}$$ XQ ES <1

3.2

$$F_X(x) = \frac{1}{3}\mathbb{I}\{x=2\} + \frac{1}{6}x\,\mathbb{I}\{-1<x<1\} + \frac{2}{3}\mathbb{I}\{1<x<2\} + \mathbb{I}\{$$

$$f_X(x) = \frac{1}{6}\mathbb{I}\{-1<x<1\}$$

(A) $\mathbb{E}[X]$

$$\mathbb{E}[X] = \int_{-1}^{1} x\frac{1}{6}\,dx = \frac{x^2}{2}\cdot\frac{1}{6}\Big|_{-1}^{1} = \frac{1}{2}\cdot\frac{1}{6} - \frac{1}{2}\frac{1}{6} = 0$$

$\mathbb{E}[X] = 0$

(B) $\mathbb{E}[X \mid |X|=2]$

$$\mathbb{E}[X \mid X=2 \wedge X=-2] = \frac{\mathbb{E}[X]\,\mathbb{I}\{X=2 \wedge X=-2\}}{\mathbb{P}(|X|=2)}$$

$$\frac{\overset{2}{\widehat{X}}\cdot f_X(2) + \overset{-2}{\widehat{X}}\,f_X(-2)}{\mathbb{P}(|X|=2)} = 0$$

$\mathbb{E}[X \mid |X|=2] = 0$

3.4

$\Omega = \{1\ 2\ 3\ 4\ 5\ 6\}$ $A_i = \{\ldots\}$ $\mathbb{P}(\{1,2\}) = 1/6$

(A) DEFINIR X V.A $X:\Omega \to \mathbb{R}$ / • $\{-1, 1/2\} \subset X(w)$
• $\mathbb{E}[X] = 0$

$X(1) = X(2) = -1$ $\mathbb{P}(X=-1) = 1/6$

$X(3) = X(4) = 1/2$ $\mathbb{P}(X=1/2) = 1/3$

$X(5) = X(6) = X_0$ $\mathbb{P}(X=X_0) = 1/2$

NOTA

ENTONCES,

$E[X] = 0$

$$= -1 \cdot \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{3} + x_0 \cdot \frac{1}{2} = 0$$

$x_0 = 0 \Rightarrow X(5) = X(6) = 0$

$$E_\omega(X) = -1\, \mathbb{1}\{w \in \{1,2\}\} + \frac{1}{2}\, \mathbb{1}\{w \in \{3,4\}\}$$

(B) $E[X \mid X > -1]$

$$\frac{E[X\, \mathbb{1}\{X > -1\}]}{\mathbb{P}(X > -1)} = \frac{\frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{2}}{\frac{1}{3} + \frac{1}{6}} = \frac{1}{5}$$

$\downarrow$ ó $1 - \mathbb{P}(X \leq -1)$

(C)

3.5 $X \sim Poi(10) \Rightarrow \mathbb{E}[X] = 10$

$$\mathbb{E}[X \mid X \leq 4] = \frac{\mathbb{E}[X]\,\mathbb{I}\{X \leq 4\}}{\mathbb{P}(X \leq 4)}$$

$$P_X(x) = \frac{\mu^x e^{-\mu}}{x!} = \frac{10^x \cdot e^{-10}}{x!}$$

$$\mathbb{E}[X]\,\mathbb{I}\{X \leq 4\} = 1(10e^{-10}) + 2\left(\frac{10^2 e^{-10}}{2}\right) + 3\left(\frac{10^3 e^{-10}}{6}\right) = + 4\left(\frac{10^4 e^{-10}}{24}\right)$$

$\mathbb{E}[X] = 0{,}103$

$$\mathbb{P}(X \leq 4) = e^{-10}\left(10 + \frac{10^2}{2} + \frac{10^3}{6} + \frac{10^4}{24}\right) = 0{,}029$$

$\mathbb{E}[X \mid X \leq 4] = 3{,}55$

3.6

$T \sim EXP(\lambda) \Rightarrow \mathbb{E}[X] = 1/\lambda = 3 \rightarrow \lambda = 1/3$

$f_T(T) = 3 \cdot e^{-1/3 T}$ $\qquad F_T(T) = 1 - e^{-1/3 T}$

$$\mathbb{E}[T \mid T \leq 2] = \frac{\mathbb{E}[T]\,\mathbb{I}\{T \leq 2\}}{\mathbb{P}(T \leq 2)}$$

$$\mathbb{E}[T] = \int_0^2 T \cdot \frac{1}{3} e^{-1/3 T}\, dT = \int_0^2 \frac{1}{3} T \cdot e^{-1/3 T} =$$

$\mathbb{E}[T] = 0{,}43$

$\mathbb{P}(T \leq 2) = F(2) = 1 - e^{-1/3 \cdot 2} = 0{,}49$

$\mathbb{E}[X \mid X \leq 4] = 0{,}87$

NOTA

3.7 X VA

(A)

$$F_X(x) = \frac{9!}{3!\,5!}\, x^3 (1-x)^5 \, \mathbb{1}\{0 < x < 1\}$$

$9 \cdot 8!$

$$\rightarrow \binom{8}{3} p^3 (1-p)^{8-3}$$

¡NO ES BINOMIAL XQ ES CONTINUA!

$$\mathbb{E}[X] = 9\int_0^1 x \cdot \binom{8}{3} x^3 (1-x)^5\, dx$$

$$\mathbb{E}[X] = 9\binom{8}{3}\int_0^1 x^4 \cdot (1-x)^5\, dx = 9\binom{8}{3} \cdot 7{,}94 \times 10^{-4}$$

$$\mathbb{E}[X] = 0{,}400$$

(B)

$$F_X(x) = \frac{1}{6} x^3 e^{-x}\, \mathbb{1}\{x > 0\} \rightarrow \lambda = 1$$

$$\gamma =$$

$$x^{\gamma - 1} = x^3$$

$$\gamma - 1 = 3$$

$$\gamma = 4$$

$$\sim \text{GAMMA}(\ \ ,\ 1)$$

$$\mathbb{E}[X] = \gamma/\lambda = 4$$

(C) $\mathbb{E}[X \mid X > 1/2]$

$$\mathbb{E}[X \mid X > 1/2] = \frac{\mathbb{E}[X\, \mathbb{1}\{X > 1/2\}]}{\mathbb{P}(X > 1/2)} = \frac{\int_{1/2}^{+\infty} x \cdot \frac{1}{6} x^3 e^{-x}}{1 - \mathbb{P}(X \leq 1/2)}$$

$$\mathbb{E}[X] = \frac{1}{6}\int_{1/2}^{+\infty} x^4 e^{-x}\, dx = 3{,}99$$

$$1 - \mathbb{P}(X \leq 1/2) = 1 - \int_0^{1/2} \frac{1}{6} x^3 e^{-x} = 0{,}99$$

$$\mathbb{E}[X \mid X > 1/2] = 4{,}03$$

3.8 $Z \sim N(0,1) \Rightarrow \varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

(A) $\mathbb{E}[Z \mid Z > z_0]$

$$\mathbb{E}[Z \mid Z > z_0] = \frac{\mathbb{E}[Z]\, \mathbb{1}\{Z > z_0\}}{\mathbb{P}(Z > z_0)} =$$

$$\mathbb{E}[Z \mid Z > z_0] = \frac{\mathbb{E}[Z]\, \mathbb{1}\{Z > z_0\}}{1 - \mathbb{P}(Z \leq z_0)}$$

$$\mathbb{E}[Z]\, \mathbb{1}\{Z > z_0\} = \int_{z_0}^{+\infty} z\, \varphi(z)\, dz$$

$$= \int_{z_0}^{+\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \frac{1}{\sqrt{2\pi}} \int_{z_0}^{+\infty} z\, e^{-z^2/2}$$

CAMBIO DE VARIABLES

$$u = -\frac{z^2}{2} \quad du = \frac{-\cancel{2}z}{\cancel{2}} dz$$

$$\frac{1}{\sqrt{2\pi}} \int_{z_0}^{+\infty} \cancel{z}\, e^u \frac{du}{-\cancel{z}} = \frac{1}{\sqrt{2\pi}} \int_{z_0}^{+\infty} -e^u dz$$

$$\frac{1}{\sqrt{2\pi}} \left(-e^u\right)\Big|_{z_0}^{+\infty} = \frac{-1}{\sqrt{2\pi}} \left(\cancel{e^{+\infty}}^{\,0} - e^{z_0}\right)$$

$$\mathbb{E}[Z \mid Z > z_0] = \frac{1/\sqrt{2\pi}\; e^{z_0^2/2}}{1 - \Phi(z_0)}$$

$$\mathbb{E}[Z \mid Z > z_0] = \frac{\varphi(z_0)}{1 - \Phi(z_0)}$$

$\varphi(z_0) \to f$

$\Phi(z_0) \to F$

NOTA

HOJA Nº 4

FECHA

3-10

TENGO UN CIRCULO UNIENDO LOS EXTREMOS DE UN ALAMBRE

(A) $\mathbb{E}$[AREA DEL CIRULO)

LARGO $L \sim EXP(\lambda)$ $\mathbb{E}[X] = 1/\lambda = 60\,cm$

$1/\lambda = 60$

$\lambda = 1/60$

$A(R) = \pi R^2 \rightarrow$ LA QUIERO DEPENDIENDO DE MI V.A :

$P(R) = 2\pi R$

$L = 2\pi R \rightarrow R = \frac{L}{2\pi}$

$$A(L) = \pi\left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4}$$

$$E[X] = \mathbb{E}[A(L)] = \int_{-\infty}^{+\infty} A(L) \cdot f_L(L)\, dL$$

$$\mathbb{E}[A(L)] = \int_0^{+\infty} \frac{L^2}{4\pi} \cdot \frac{1}{60} e^{-1/60 L}\, dL$$

$$\mathbb{E}[A(L)] = \frac{1}{4\pi \cdot 60} \int_0^{+\infty} L^2 \cdot e^{-1/60 L}\, dL$$

$$\mathbb{E}[A(L)] = e^{-1/60 L}\left(\frac{L^2}{-1/60} - \frac{2L}{(-1/60)^2} + \frac{2}{(-1/60)^3}\right)\Bigg|_0^{\infty} = 0 - \left[1\left(\frac{2}{-1/60}\right)^3\right.$$

NOTA $\mathbb{E}[A(L)] = 572{,}9\ cm^2$

PERO TMB SE PUEDE RESOLVER COMO:

$$VAR(L) = \mathbb{E}[L^2] - \mathbb{E}[L]^2$$

$$\frac{1}{\lambda^2} = \mathbb{E}[L^2] - \left(\frac{1}{\lambda}\right)^2$$

$$\mathbb{E}[L^2] = \frac{1}{(1/60)^2} + \left(\frac{1}{1/60}\right)^2 = 2.60^2$$

$$\mathbb{E}[A(L)] = \mathbb{E}\left[\frac{L^2}{4\pi}\right] = \frac{1}{4\pi}\,\mathbb{E}[L^2] = \frac{1}{\cancel{4}\pi} \cdot \cancel{2} \cdot 60^2$$

$$\mathbb{E}[A(L)] = 572,9 \text{ cm}^2$$

(B) $A \sim EXP(\lambda)$ $\qquad \mathbb{E}[X] = \frac{1}{\lambda} = 15 \text{ cm}^2$

$\mathbb{E}[P(R)]$ AHORA MI V.A ES A $\qquad \lambda = \frac{1}{15}$

- $A = \pi R^2 \rightarrow R^2 = \frac{A}{\pi} \rightarrow R = \frac{\sqrt{A}}{\sqrt{\pi}}$
- PERIMETRO $= 2\pi R$

$\hookrightarrow P(A) = 2\pi \cdot \frac{\sqrt{A}}{\sqrt{\pi}}$

$$\mathbb{E}[P(A)] = \int_{-\infty}^{+\infty} 2\pi \cdot \frac{\sqrt{A}}{\sqrt{\pi}} \cdot \frac{1}{15} \cdot e^{-1/15 A}\, dA$$

$$\mathbb{E}[P(A)] = \frac{2\pi}{\sqrt{\pi} \cdot 15} \int_0^{+\infty} \sqrt{A} \cdot e^{-1/15 A}\, dA$$

$\sqrt{\pi}$

ESTA INTEGRAL ES DIFICIL DE RESOLVER Y NO SALE POR LA VARIANCIA, ENTONCES TRATO DE RESOLVER POR $\gamma(\gamma, \lambda)$

NOTA

$$f_T(T) = \frac{\lambda^{\gamma} x^{\gamma-1}}{\Gamma(\gamma)} e^{-\lambda T}$$

$$\frac{2\pi}{15} \sqrt{A} \cdot e^{-1/15 A}$$

$$\underbrace{2\pi \sqrt{A} \; \frac{1}{15}}\; \underbrace{e^{-1/15 A}}_{e^{-\lambda T} \to \lambda = 1/15}$$

$$A^{1/2} = A^{\gamma-1}$$

$$1/2 = \gamma - 1$$

$$\gamma = 3/2$$

$$\Gamma(3/2) = \Gamma(1 + 1/2)$$

$$\lambda^{\nu} = \left(1/15\right)^{3/2}$$

$$\frac{2\pi}{15} \int_0^{+\infty} \frac{\left(\frac{1}{15}\right)^{3/2} A^{1/2}}{\Gamma(3/2)} \cdot e^{-\frac{1}{15}A} \, dA = \int_{sop} f_A(a)\, da = 1$$

$$= \frac{2\pi}{\sqrt{\pi}\,15} \int_0^{+\infty} \sqrt{a}\, e^{-1/15 a}\, da =$$

$$f_S(s) = \frac{\lambda^{\nu} S^{\nu-1}}{\Gamma(\nu)} e^{-\lambda s}$$

$\lambda = \frac{1}{15}$

$\nu = 3/2$

$$\Gamma(3/2) = \frac{1}{2}\Gamma(1/2) = \frac{1}{2}\sqrt{\pi}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\; 2\pi}{\sqrt{\pi}\; 15 \left(\frac{1}{15}\right)^{3/2}} \underbrace{\int_0^{+\infty} \frac{a^{1/2} \left(\frac{1}{15}\right)^{3/2}}{\frac{1}{2}\sqrt{\pi}} e^{-1/15 a}\, da}_{1}$$

$T \sim EXP(1)$

3.11

$$T^* = MIN(T,1) = \begin{cases} T & t<1 \\ 1 & t>1 \end{cases}$$

ENTONCES $\left.\begin{array}{ll} T<1 & T^*=T \\ T\geq 1 & T^*=1 \end{array}\right\} F_{T^*}(T) = \begin{cases} F_T(T) & 0<T<1 \\ 1 & T>1 \end{cases}$

$$\mathbb{E}[T^*] = \int_{-\infty}^{+\infty} F_T(T) \cdot T^* \, dT$$

$$\mathbb{E}[T^*] = \int_0^1 e^{-T^*} T^* \, dT^* + \underbrace{1 \cdot e^{-1}}_{F^+(1) - F^-(1)} = 0{,}632$$

$F^+(1) - F^-(1) \rightarrow 1 - (1 - e^{-1})$

COMO $G(x)$

$$VAR[T^*] = \mathbb{E}[T^{*2}] - \mathbb{E}[T^*]^2$$

$$VAR[T^*] = \int_0^1 T^{*2} \cdot e^{-T^*} + e^{-1}) - (0{,}632)^2$$

$$VAR[T^*] = 0{,}129$$

3.12

X e Y SON INDEPENDIENTES $\sim \mathcal{U}(0,\pi)$

CALCULAR $\mathbb{E}[X \cdot SEN(XY)]$ $\rightarrow G(x)$

COMO SON $\mathcal{U} \sim (0,\pi)$ $\quad F_X(x) = \frac{1}{\pi} \mathbb{1}\{0<x<\pi\}$

$F_Y(y) = \frac{1}{\pi} \mathbb{1}\{0<y<\pi\}$

Y COMO SON INDEPENDIENTES

$$F_{XY}(x,y) = F_X(x) \cdot F_Y(y)$$

NOTA

HOJA Nº 6

FECHA

$$F_{XY}(x,y) = \frac{1}{\pi^2} \mathbb{1}\{0<x<\pi,\ 0<y<\pi\}$$

$$\mathbb{E}[G(X)] = \int_0^\pi \int_0^\pi G(x)\, F_{XY}(x,y)\, dx\, dy$$

$$\mathbb{E}[G(X)] = \int_0^\pi \int_0^\pi x \operatorname{SEN}(xy) \cdot \frac{1}{\pi^2}\, dx\, dy$$

$$\mathbb{E}[G(X)] = \frac{1}{\pi^2} \int_0^\pi \int_0^\pi x \cdot \operatorname{SEN}(xy)\, dx\, dy$$

$$\mathbb{E}[G(X)] = \frac{1}{\pi^2} \int_0^\pi \cancel{x} \cdot \left(\frac{-\cos(xy)}{\cancel{x}} \right) \Big|_0^\pi dy$$

$$= -\frac{1}{\pi^2} \int_0^\pi \cos(\pi y) - 1\, dy$$

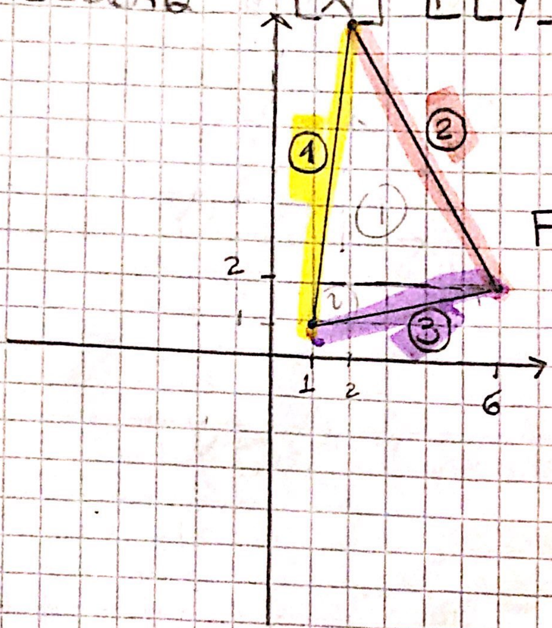
$$\mathbb{E}[x \operatorname{SEN}(xy)] = 0{,}332$$

3.13

(X,Y) V.A $\sim \mathcal{U}$ (TRIANGULO)

(1 1) (6 2) (2 9)

CALCULAR $E[X]$ $E[Y]$



$(X,Y) \sim \mathcal{U}$ (

$F_{XY}(x,y) = \dfrac{1}{AREA}$

•) PARA LOS LIMITES DE LAS INTEGRALES TRABAJO CON LAS RECTAS!

1 (1,1) (2,9)

$M = \dfrac{9-1}{2-1} = 8$ $\quad Y = MX + B$

$1 = 8.1 + B \rightarrow B = -7$

$\boxed{Y = 8X - 7}$ $\quad X = \dfrac{1}{8}Y - \dfrac{7}{8}$

2 (6,2) (2,9)

$M = \dfrac{9-2}{2-6} = \dfrac{7}{-4}$ $\quad Y = MX + B$

$2 = -\dfrac{7}{4} \cdot 6 + B \quad B = \dfrac{25}{2}$

$\boxed{Y = -\dfrac{7}{4}X + \dfrac{25}{2}}$

$X = -\dfrac{4}{7}Y - \dfrac{50}{7}$

3 $\boxed{Y = \dfrac{1}{5}X + \dfrac{4}{5}}$

$X = 5Y - 4$

NOTA

$$\mathbb{E}[X] = \int_{-\infty}^{+\infty} x \; f_X(x)\, dx$$

$$\mathbb{E}[Y] = \int_{-\infty}^{+\infty} y \; f_Y(y)\, dy$$

$$f_X(x) = \int_{-\infty}^{+\infty} F_{XY}(x,y)\, dy$$

$$f_Y(y) = \int_{-\infty}^{+\infty} F_{XY}(x,y)\, dx$$

$$f_{XY}(x,y) \sim \mathcal{U}\left((1\,1)(6\,2)(2\,9)\right)$$

$$F_{XY}(x,y) = \frac{1}{\text{AREA}} \longrightarrow \text{BUSCO CON LA INTEGRAL}$$

AREA

① $$\int_{L_1}^{L_2}\int_{2}^{9} dy\,dx = \int_{2}^{9} -\frac{4}{7}y + \frac{50}{7} - \left(\frac{1}{8}y + \frac{7}{8}\right) dy$$

$$\int_{2}^{9} -\frac{39}{56}y + \frac{351}{56}\, dy = -\frac{39}{56}\int_{2}^{9} y + \frac{351}{56}\, dy = \frac{273}{16}$$

② $$\int_{L_1}^{L_3}\int_{1}^{2} 1\, dy\,dx = \int_{1}^{2} 5y - 4 - \left(\frac{1}{8}y + \frac{7}{8}\right) dy$$

$$\int_{1}^{2} \frac{39}{8}y - \frac{39}{8}\, dy = \frac{39}{16}$$

$$\text{AREA} = 19{,}5\ \text{cm}^2$$

$f_{XY}(xy) = \frac{1}{19,5}$

$V(X) = 1$ MAX
$V(X) = 0$ MÍN

$$\mathbb{E}[X] = \iint_{SOP(x,y)} x \, F_{XY} \, dx dy = 3$$

$$\mathbb{E}[Y] = \iint_{SOP(x,y)} y \, F_{X}(x,y) \, dy =$$

3.14

X V.A A VALORES $\{2,3,4\}$ / $\mathbb{P}(X=x) = P_x$

$\mathbb{E}[X] = 3$

(A) $VAR[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$ MAXIMA

$VAR[X] = \mathbb{E}[X^2] - 9$

$1 = x_2^2 P_2 + x_3^2 P_3 + x_4^2 P_4 - 9$

(1) $1 = 4P_2 + 9P_3^2 + 16P_4^2 - 9$

(2) $\mathbb{E}[X] = 3$

$4P_2 + 9P_3 + 16P_4 = 3$

(3) $P_2 + P_3 + P_4 = 1$

$P_2 = 1/2 \quad P_3 = 0$

$P_4 = 1/2$

(B) IDEM QUE ARRIBA PERO $VAR[X] = 0$

NOTA

HOJA Nº 8

FECHA

3.15

X V.A $\sim \mathcal{U}(8,10)$

$X \sim \mathcal{U}(8,10) \quad f_X(x) = \frac{1}{2} \mathbb{1}\{8 < x < 10\}$

$$\mathbb{E}[2X-2] = \int_8^{10} 2x-2 \cdot \frac{1}{2}\, dx = \int_8^{10} x-1\, dx$$

$$\mathbb{E}[2X-2] = \frac{x^2}{2} - x \Big|_8^{10} = 40 - 24 = 16$$

$$VAR[X] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$\mathbb{E}[X^2] = \int_8^{10} (2x-2)^2 \cdot \frac{1}{2}\, dx = \int_8^{10} 4x^2 - 8x + 4 \cdot \frac{1}{2}\, dx$$

$$= \int_8^{10} 2x^2 - 4x + 2\, dx = \frac{2x^3}{3} - 2x^2 + 2x \Big|_8^{10} = \frac{1460}{3} - \frac{688}{3}$$

$$VAR[X] = 1,33$$

(B) y (C) IDEM

$$\mathbb{E}[2(X-1)] = 2 . \mathbb{E}[X-1]$$
$$= 2(\mathbb{E}[X] - \mathbb{E}[1])$$

(D) y (E)

3.16

$W = RV^2 \rightarrow W = 3V^2 \qquad V \sim N(6,1)$

CALCULAR $\mathbb{E}[W] = \mathbb{E}[3V^2] = 3\,\mathbb{E}[V^2]$

$V \sim N(6,1)$

$VAR(V) = 1$

$\sigma^2 = 1$

$\underline{\underline{\sigma = 1}}$

$VAR[V] = \mathbb{E}[V^2] - \mathbb{E}[V]^2$

$1 = \mathbb{E}[V^2] - 36$

$\mathbb{E}[V^2] = 37$

$\underline{\mathbb{E}[W] = 111}$

3.17

(A) $X_1 \sim BER(P_1)$

$X_2 \sim BER(P_2)$

$COV(X_1, X_2) = 0 \Rightarrow X_1$ e X_2 SON INDEP

$COV(X_1, X_2) = = \mathbb{E}[X_1 X_2] - \mathbb{E}[X_1]\,\mathbb{E}[X_2]$

$COV(X_1, X_2) = \mathbb{E}[X_1 X_2] - \mathbb{E}[X_1]\,\mathbb{E}[X_2]$

$COV(X_1\, X_2) = \mathbb{E}[X_1 X_2] - P_1 \cdot P_2$

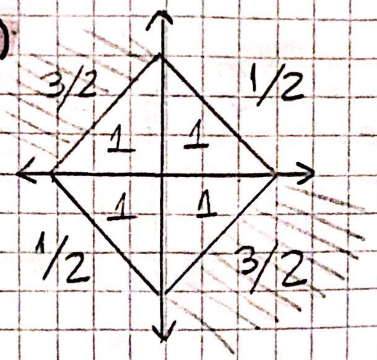
Si X_1 e X_2 SON INDEP $\Rightarrow \mathbb{E}[X_1 X_2] = \mathbb{E}[X_1]\,\mathbb{E}[X_2]$

? $\mathbb{E}[X_1 X_2] =$

NOTA

3.19

(A)



$COV(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$

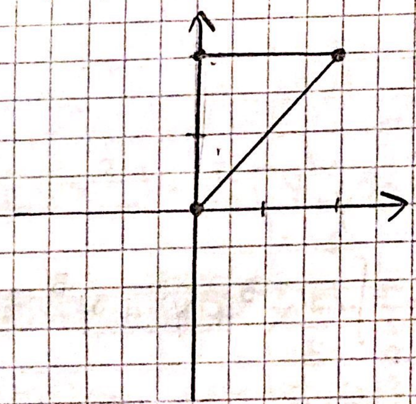
$COV(XY) < 0$

(B) $COV(XY) = 0$

(C) $COV(XY) > 0$

3.20

(X,Y) V.A $\sim \mathcal{U}\ (0,0)\ (2\ 2)\ (0\ 2)$



$$F_{xy}(XY) = \frac{1}{\frac{B.H}{2}} = \frac{1}{2}$$

(A) $COV(X\ Y)$

$$COV(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$$

$$\mathbb{E}[X] = \iint F_{xy}\,dx\,dy = \int_0^y \int_0^2 x\,\frac{1}{2}\,dy\,dx$$

$$\mathbb{E}[X] = \frac{1}{2}\int_0^2 \frac{x^2}{2}\Big|_0^y = \int_0^2 \frac{1}{4}y^2\,dy = \frac{1}{4}\,\frac{y^3}{3}\Big|_0^2 = \frac{1}{4}\cdot\frac{8}{3}$$

$$\mathbb{E}[X] = 2/3$$

$$\mathbb{E}[Y] = \int_0^{2}\int_0^{y} y\,\frac{1}{2}\,dx\,dy = \int_0^{2} \frac{1}{2} y x \Big|_0^{y} dy = \int_0^{2} \frac{1}{2} y^2\,dy$$

$$\mathbb{E}[Y] = \frac{1}{2}\frac{y^3}{3}\Big|_0^{2} = \frac{1}{2}\frac{8}{3} = \frac{4}{3}$$

$$\mathbb{E}[XY] = \int_0^{2}\int_0^{y} xy\,\frac{1}{2}\,dx\,dy = \int_0^{2} \frac{x^2}{2}\cdot\frac{1}{2}\Big|_0^{y} = \int_0^{2} \frac{y^2}{4} y\,dy$$

$$\int_0^{2} \frac{y^3}{4}\,dy = \frac{1}{4}\frac{y^4}{4}\Big|_0^{2} = \frac{16}{16} = 1$$

$$\operatorname{COV}(XY) = 1 - \frac{4}{3}\cdot\frac{2}{3} = \frac{-1}{9}$$

(B) VAR $(X+Y) =$ $\qquad (X+Y) = Z$

$$\operatorname{VAR}(X+Y) = \mathbb{E}[Z^2] - \mathbb{E}[Z]^2$$

$$(X+Y)^2 = X^2 + 2XY + Y^2$$

$$\mathbb{E}[Z^2] = \int_0^{2}\int_0^{y} x^2 + 2xy + y^2 \cdot \frac{1}{2}\,dx\,dy$$

$$= \int_0^{2} \left(\frac{x^3}{3} + 2\frac{x^2}{2}y + y^2 x\right)\frac{1}{2}\,dy = \frac{1}{2}\int_0^{2} \frac{1}{3}x^3 + y^3 + y^3\,dy$$

$$\frac{1}{2}\int_0^{2} \frac{7}{3} y^3\,dy = \frac{1}{2}\cdot\frac{7}{12} y^4 \Big|_0^{2} = \frac{7}{24}\cdot 16 = \frac{28}{3}$$

$$\mathbb{E}[Z^2] = \frac{4}{3} + \frac{16}{3} = \frac{10}{3}$$

$$\mathbb{E}[Z] = \frac{1}{2}\int_0^{2}\int_0^{y} x + y\,dx\,dy = \frac{1}{2}\int_0^{2} \frac{x^2}{2} + yx \Big|_0^{y} dy$$

$$\int_0^{2} \frac{y^2}{4} + \frac{y^2}{2}\,dy = \frac{3}{4} y^2\,dy = \frac{3}{4}\frac{y^3}{3}\Big|_0^{2} = 2$$

NOTA

$VAR(X+Y) = \frac{2}{3}$

$VAR(X+Y) = \mathbb{E}[X^2 + 2XY + Y^2] - \mathbb{E}[X+Y]^2$

$= \mathbb{E}[X^2] + 2\mathbb{E}[XY] + \mathbb{E}[Y^2] - (\mathbb{E}[X] + \mathbb{E}[Y])^2$

(c) $COV(\overbrace{3X - Y + 2}^{X_1}, \overbrace{X + Y}^{X_2})$

$COV(X_1 X_2) = \mathbb{E}[X_1 X_2] - \mathbb{E}[X_1]\mathbb{E}[X_2]$

$= \mathbb{E}[(3X - Y + 2)(X+Y)] - \mathbb{E}[3X - Y + 2] \cdot \mathbb{E}[X+Y]$

$3X^2 - YX + 2X + 3XY - Y^2 + 2Y$

$3X^2 - Y^2 + 3XY - XY + 2X + 2Y$

$\mathbb{E}[3X^2 - Y^2 + 3XY - XY + 2X + 2Y] - 3\mathbb{E}[X] - \mathbb{E}[Y] + \mathbb{E}[2] \cdot \mathbb{E}[X] + \mathbb{E}[Y]$

$3\mathbb{E}[X^2] - \mathbb{E}[Y^2] + 2\mathbb{E}[XY] + 2\mathbb{E}[X] + 2\mathbb{E}[Y]$

TENGO TODAS DEL EJ (B)

$\mathbb{E}[X^2] = \frac{2}{3}$; $\mathbb{E}[Y^2] = 2$; $\mathbb{E}[XY] = 1$ $\mathbb{E}[X] = \frac{2}{3}$

$\mathbb{E}[Y] = 4/3$

$COV(X_1 X_2) = \cancel{3}\frac{2}{\cancel{3}} - 2 + 2 . \frac{2}{3} + 2\frac{2}{3} + 2\frac{4}{3} - ((\cancel{3}\frac{2}{\cancel{3}} - \frac{4}{3} + 2)(2))$

$COV(X_1 X_2) = 1$

NOTA

ES MEJOR HACER

$$COV(3X - Y + 2, X + Y)$$

$$=$$

$$COV(3X - Y + 2, X) + COV(3X - Y + 2, Y)$$

$$\left[COV(3X, X) - COV(Y, X) + \cancel{COV(2, X)}^{0}\right] + \left[COV(3X, Y) - COV(Y, Y) + \cancel{COV(2, Y)}_{0}\right]$$

$$\left[3\,\overbrace{COV(X, X)}^{VAR} - COV(Y, X)\right] + \left[3\,COV(X, Y) - \overbrace{COV(Y, Y)}^{VAR}\right]$$

$$3\,VA(X) - COV(Y, X) + 3\,COV(X, Y) - VA(Y)$$

$$3\,VA(X) + 2\,COV(X, Y) - VA(Y)$$

$$3\left(\frac{2}{3} - \frac{4}{9}\right) + 2 \cdot \frac{1}{9} - \left(2 - \frac{16}{9}\right) = \frac{2}{3} + \frac{2}{9} - \frac{2}{9}$$

$$COV(3X - Y + 2, X + Y) = \frac{2}{3}$$

3.22

$$F_{xy} = \frac{5}{8\pi} e^{-\frac{25}{32}\left(x^2 - \frac{6}{5}xy + y^2\right)} \sim N($$

RECTA DE REGRESIÓN DE Y SOBRE X

$$Y = \frac{COV(XY)}{VA(X)} \cdot (X - \mathbb{E}[X]) + \mathbb{E}[Y]$$

$$COV(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$$

$$VA(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$Y = \frac{\mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]}{\mathbb{E}[X^2] - \mathbb{E}[X]^2}\,(X - \mathbb{E}[X]) + \mathbb{E}[Y]$$

3.23

X V.A $\mathbb{E}[X] = 15.$

$\mathbb{P}(X \geqslant 60) \leqslant 0{,}25$

$\mathbb{P}(X \geqslant A) \leqslant \dfrac{\mathbb{E}[X]}{A}$

$\mathbb{P}(X \geqslant 60) \leqslant \dfrac{15}{60}$

$\mathbb{P}(X \geqslant 60) \leqslant 0{,}25$ DESIGUALDAD DE MARKOV

3.24

X (V.A) $\mathbb{E}[X] = \overset{\mu}{10}$ $VA(X) = 15$

$= \sigma_X^2 \Rightarrow \sigma_X = 3{,}87$

$\mathbb{P}(5 < X < 15) \geqslant 0{,}4$

DESIGUALDAD DE CHEBYCHOV

$\mathbb{P}(|X - \mu| > \varepsilon) \leqslant \dfrac{VA(X)}{\varepsilon^2} = \dfrac{\sigma^2}{\varepsilon^2}$

$\mathbb{P}(|X - 10| \geqslant 5) \leqslant \dfrac{15}{25}$

$\mathbb{P}(-5 \geqslant X - 10 \geqslant 5)$

$\mathbb{P}(5 \geqslant X \geqslant 15) \leqslant 0{,}6$

$\mathbb{P}(5 \geqslant X \geqslant 15) > 1 - 0{,}6$

$\mathbb{P}(5 \geqslant X \geqslant 15) \geqslant 0{,}4$ ✓