(a) Mostrar que la familia de distribuciones $\mathcal{N}(\mu, \sigma^2)$ puede expresarse en la forma

$$f_{\theta}(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \exp\left(\frac{\mu}{\sigma^2}x - \frac{1}{2\sigma^2}x^2\right),$$

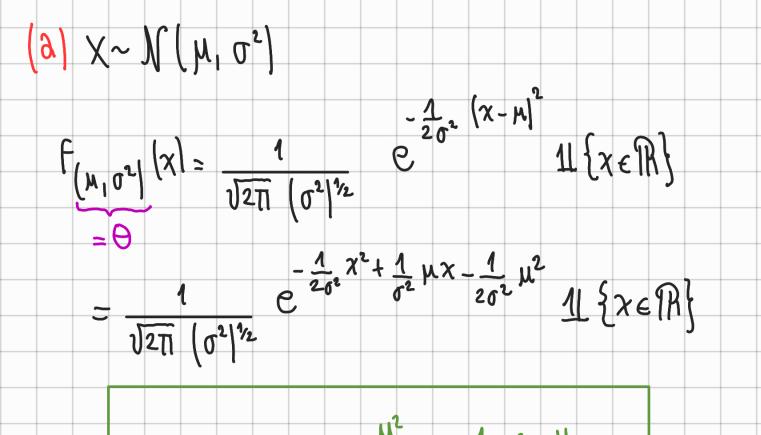
donde $\theta = (\mu, \sigma^2)$.

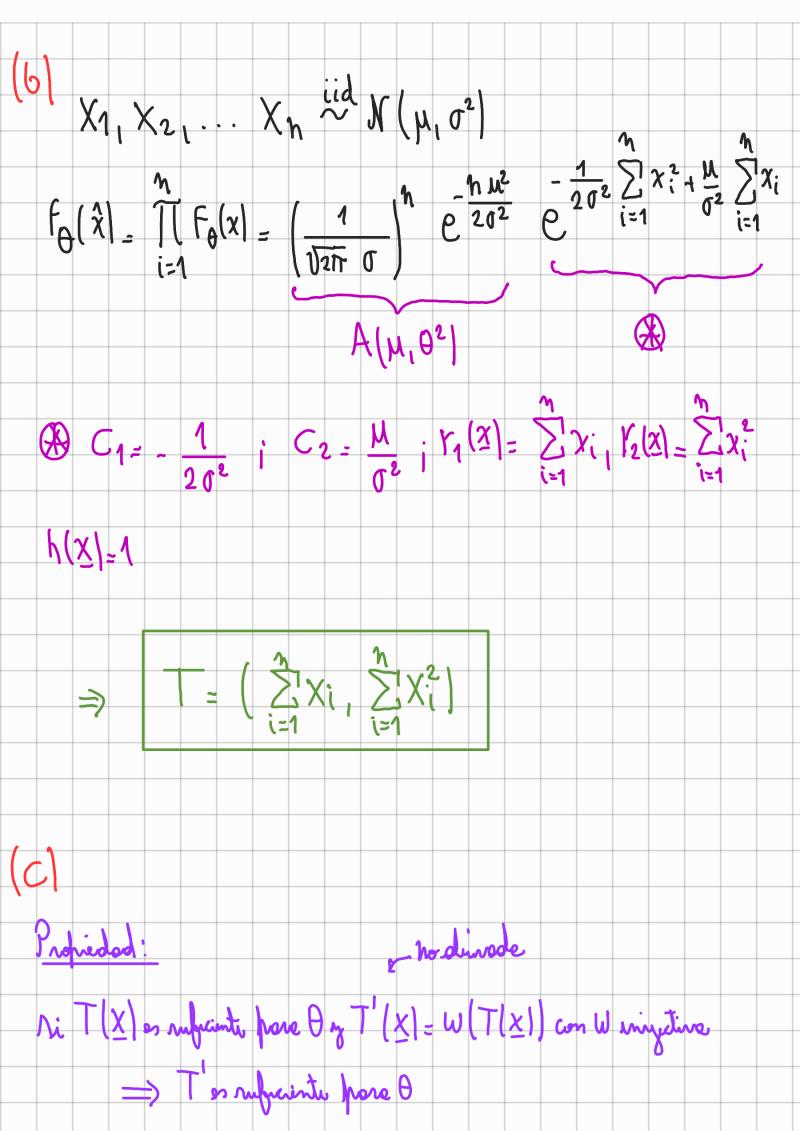
- (b) Sea X_1, \ldots, X_n una muestra aleatoria de tamaño n de la distribución $\mathcal{N}(\mu, \sigma^2)$. Hallar la expresión de la densidad conjunta y mostrar que $T = \left(\sum_{i=1}^n X_i, \sum_{i=1}^n X_i^2\right)$ es un estadístico suficiente para θ .
- (c) Sea $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$. Mostrar que $\sum_{i=1}^n (X_i \bar{X})^2 = \sum_{i=1}^n X_i^2 n\bar{X}^2$ y deducir que $T' = (\bar{X}, S^2)$, donde

$$S^2 := \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2,$$

es un estadístico suficiente para θ .

(d) Hallar el estimador de máxima verosimilitud para θ basado en la muestra aleatoria X_1, \ldots, X_n .





$$T(X) = \left(\sum_{i}^{n} X_{i}, \sum_{i}^{n} X_{i}^{2}\right)$$

$$T(X) = \left(\sum_{i}^{n} X_{i}, \sum_{i}^{n} X_{i}^{2} + n\left(\frac{\sum_{i}^{n}}{n}\right)^{2}\right)$$

$$V_{n} = T = \left(\frac{\alpha}{n}, \frac{b - \alpha^{2} / n}{n - 1}\right) = \omega(a_{1}b) = (c, d)$$

$$\begin{cases} c = \frac{\alpha}{n} \\ d = \frac{b - \alpha^{2} / n}{n - 1} \end{cases}$$

$$A_{n} = \frac{b - \alpha^{2} / n}{n - 1}$$

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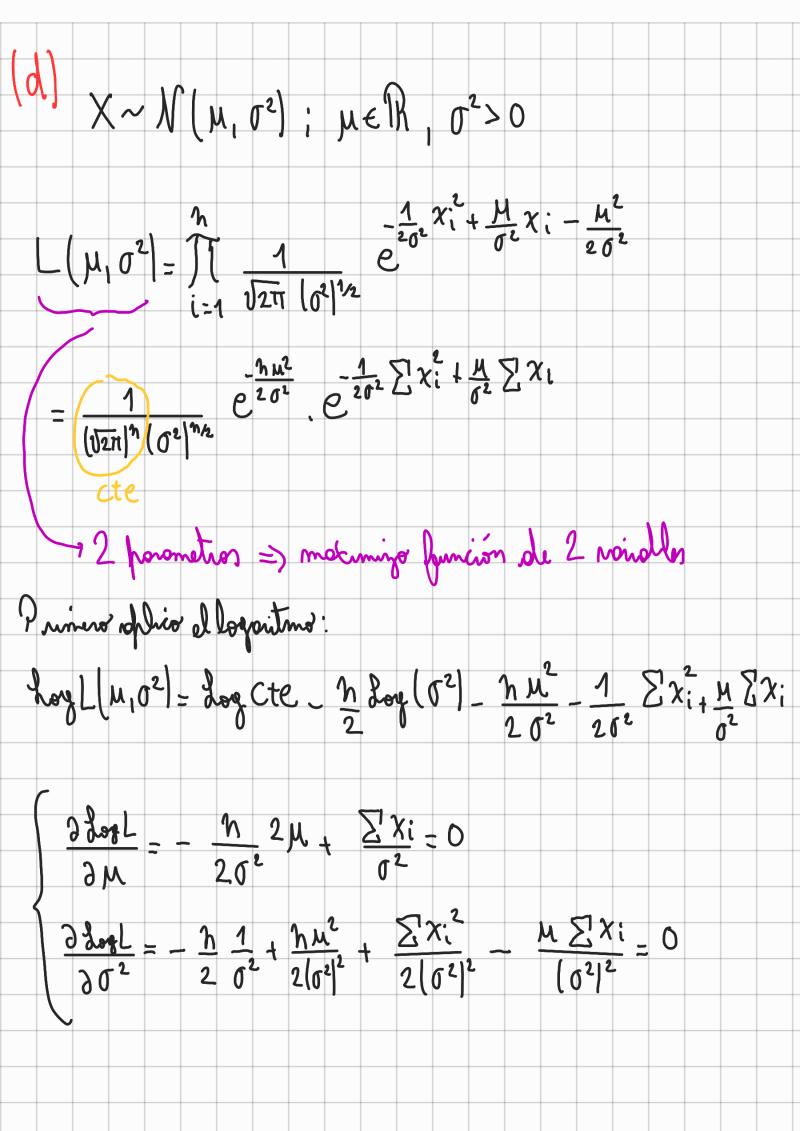
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De la puniera acusión
$$\longrightarrow$$
 $\mu = \frac{\sum x_i}{n}$

Obra roy a la regunda

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