GUIA 1 (PARTE 1)

1.1) $\Omega = \{1,2,3,4,5,6\}$ → espacio muestral

a) menor $\mathcal{A}$ talque $\{1,2,3\} \in \mathcal{A}$

La menor algebra sería: $\{\Omega, \emptyset\}$, pero tengo que agregar el $\{123\}$ y su complemento, Y la UNIÓN!

$$\mathcal{A}_1 = \{\emptyset, \Omega, \{1,2,3\}, \{4,5,6\}\}$$

$(\emptyset \cup \Omega) = \Omega$ $(\{4,5,6\} \cup \Omega) = \Omega$

$(\{1,2,3\} \cup \{4,5,6\}) = \emptyset$

$(\{1,2,3\} \cup \Omega) = \Omega$

b) $\{1,2\}, \{3,4\}, \{5,6\}$

$$\mathcal{A}_2 = \{\emptyset, \Omega, \{1,2\}, \{3,4\}, \{5,6\}, \{3,4,5,6\}, \{1,2,5,6\}, \{1,2,3,4\}\}$$

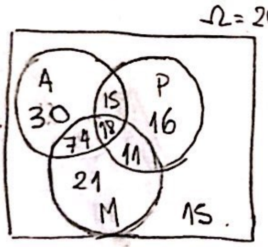
$\{1,2\} \cup \{3,4\} = \{1,2,3,4\}$ → CADA UNIÓN ya está expresada en el complemento

1.2) 200 ESTUDIANTES
- → 137 cursan Algebra II
- → 60 PROBA
- → 124 Materiales
- → 33 algebra y PROBA
- → 29 PROBA y materiales
- → 92 materiales y algebra
- → 18 las 3

$\Omega = 200$

UN estudiante al azar → ESPACIO EQUIPROBABLE → Laplace

$P = \frac{\text{FAVOR}}{\text{TOTALES}}$



a) curse algebra II ó PROBA (UNIÓN)

b) no curse A ni P → NO ALGEBRA Y NO PROBA

c) curse alguna de los 3

d) curse solo 1/3

e) no curse ninguna.

$\Omega = \{200 \text{ EST}\}$ $\#\Omega = 200$

A = curse Alg. M = curse Mat

P = curse Prob.

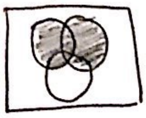
$\#A = 137$

$\#P = 60$

$\#M = 124$

a) $P(A \cup P) = P(A) + P(P) - P(A \cap P)$ PQ los conj no son disjuntos.

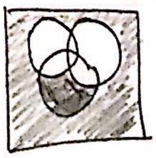
$$\boxed{P(A \cup P) = \frac{137}{200} + \frac{60}{200} - \frac{33}{200} = \frac{41}{50} = 0{,}82}$$



b) $P(\bar{A} \cap \bar{P}) = P(\overline{A \cup P}) = 1 - P(A \cup P) = 1 - (P(A) + P(P) - P(A \cap P)) = 1 - 0{,}82$

APLICO MORGAN

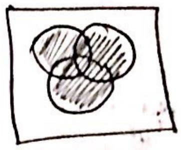
$$\boxed{P(\bar{A} \cap \bar{P}) = 0{,}18}$$



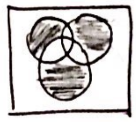
$$P(A \cup P \cup M) = P(A) + P(P) + P(M) - P(A \cap P) - P(A \cap M) - P(P \cap M) + P(A \cup P \cup M) =$$

$$= \frac{137}{200} + \frac{60}{200} + \frac{124}{200} - \frac{33}{200} - \frac{92}{200} - \frac{29}{200} + \frac{18}{200} \rightarrow$$

$$\left[P(A \cup P \cup M) = \frac{185}{200} = \frac{37}{40} = 0{,}925 \right]$$



d) Solo cualquiera de las 3 → algebra o mat o proba



$p(\text{solo algebra}) = \frac{30}{200} \rightarrow A \cap \bar{P} \cap \bar{M}$

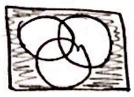
$P(\text{solo } P) = 16/200 \rightarrow P \cap \bar{A} \cap \bar{M}$

$P(\text{solo mat}) = 21/200 \rightarrow M \cap \bar{A} \cap \bar{P}$

la unión de esto.

$$\left[P\left[(A \cap \bar{P} \cap \bar{M}) \cup (P \cap \bar{A} \cap \bar{M}) \cup (M \cap \bar{A} \cap \bar{P}) \right] = \frac{67}{200} = 0{,}335. \right]$$

e) $\left[P(\bar{A} \cap \bar{P} \cap \bar{M}) = \frac{15}{200} = 0{,}075. \right]$



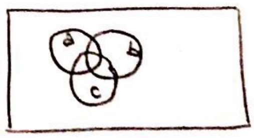
Ni A, Ni P, Ni Mat.

1.3) $\Omega = \{a, b, c\}$, $A \subset \Omega$, $P(A) = \sum_{w \in A} P(w)$ con $p(a) = \frac{1}{2}$ $p(b) = \frac{1}{3}$ $P(c) = \frac{1}{6}$

$A \subset \Omega \rightarrow$ A está incluido en Ω

8 subconjuntos de Ω.

Ω



$A: \{a, b, c\}$ $D: \{b\}$ $G: \{b, c\}$

$B: \{\emptyset\}$ $E: \{c\}$ $H: \{a, c\}$ — 8 subconjuntos.

$C: \{a\}$ $F: \{a, b\}$

$P(A) = \sum P(w) = p(a) + p(b) + p(c) = 1$

$\sum P(w)$ → elementos de a

$P(B) = P(\emptyset) = 0$ $P(G) = 1/3 + 1/6 = 1/2$

$P(C) = p(a) = 1/2$ $P(F) = 1/2 + 1/3 = 5/6$

$P(D) = p(b) = 1/3$ $P(H) = 1/2 + 1/6 = 2/3$

$P(E) = p(c) = 1/6$

DADO EQUILIBRADO → espacio equiprobable

↳ ARROJA 2 VECES $\Omega: \{(m,n) / m,n \in \{1,2,3,4,5,6\}\}$.

$\Omega: \{1,2,3,4,5,6\}^2$

$\mathcal{A} = 2^{\Omega}$ conj. partes.

a) SUMA SEA 7

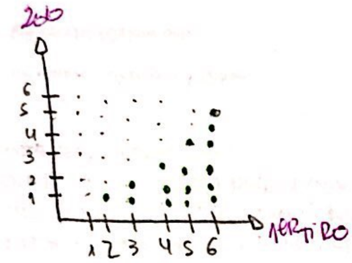
$\#\Omega = 36$

$\mathbb{P} = \frac{FAV}{POSIBLES}$

1-6
2-5
3-4
4-3
5-2
6-1

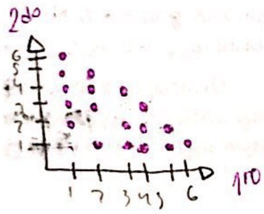
$\mathbb{P}(\text{suma sea } 7) = 6/36 = 0{,}167$

b)



$\mathbb{P}(1\text{ero} > 2\text{do}) = 15/36 = 0{,}42$

c) 2 RESULT ≠ Y NO SUMEN > 7 = C



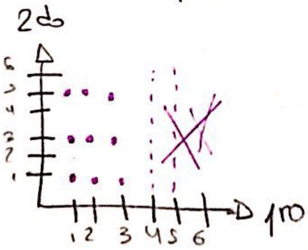
1-2
1-3 x2
1-4
1-5
1-6

2-3
2-4 x2
2-5

3-4 x2

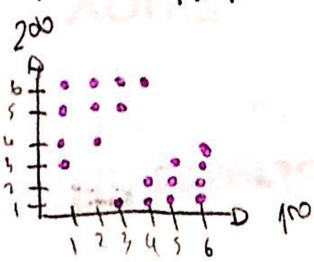
$P(C) = 18/36 = 0{,}5$.

d) 1er < 4 , 2do IMPAR = D



$P(D) = 9/36 = 0{,}25$

e) $|1\text{ro} - 2\text{do}| > 1 = E$



$P(E) = 20/36 = 0{,}56$

Guía Capít. 2

a

b

$R_1 R_2 R_3$ $R_4 R_5$ $B_1 B_2 B_3$

$R_6 R_7$ $B_4 B_5 B_6$

AZAR

$\Omega = \{ (m, m) \; / \; (m, m) \in \{a, b\} \}$

$\{1 2 3 4 5 6 7 8\} \times \{1, 2, 3, 4, 5\} =$

a) A: 2 ROJAS

$R_1 R_6 \quad R_2 R_6 \quad R_3 R_6 \quad R_4 R_6 \quad R_5 R_6$

$R_1 R_7 \quad R_2 R_7 \quad R_3 R_7 \quad R_4 R_7 \quad R_5 R_7$

$\# \Omega = 40$

$P(A) = \frac{10}{40} = 1/4$

b) B: mismo color

10 de ambos rojas

$B_1 <$

$B_2 < \quad + 9$

$B_3 <$

$P(B) = \frac{19}{40} = 0{,}475$.

c) $R_1 < \quad R_2 < \quad R_3 < \quad R_4 < \quad R_5 < \quad 3 \times 5 = 15$.

$B_1 < \quad B_2 < \quad B_3 < \quad 3 \times 2 = 6 \qquad 15 + 6 =$

$P(C) = \frac{21}{40} = 0{,}53$

OTRA FORMA

Leventos indep./

d) $8 \overset{\text{urna a}}{<} \times 3 = 24$

$P(d) = 24/40 = 3/5 = 0{,}6$.

1.6) 4 dados → NINGÚN 1

$\Omega = \{ (a, b, c, d) : \in \{1, 2, 3, 4, 5, 6\} \}$

$\# \Omega = 6^4 = 1296$

A: NO sale 1

$\underset{\text{no salga 1}}{P(A)} = 1 - \underset{\text{uno salga 1}}{P(\bar{A})} = 1 - (5/6)^4 \cong 0{,}52$ salga 1

0,48 no salga 1

Hay 0,04 más chances que salga uno en los 4 tirados.

a) $\Omega = \{X_1, X_2, X_3 \ldots X_n\}$

b) $P(A_1), P(A_8), P(A_{2016}) \to A_m$, en m sale el primer 6.

$P(A_1) \to$ en el 1er tiro salió el 6.

$P(A_1) = 1/6$

$P(A_8) = \frac{5^7 \cdot 1}{6^8} = 0{,}047$

$P(A_{2016}) = \frac{5^{2015} \cdot 1}{6^{2016}} \cong 0$

c) $B_m \to N > m \to$ no salió 6 en m tiros.

$P(B_1) = 5/6$

$P(B_8) = \frac{5^8}{6^8} = 0{,}23$

$P(B_{2016}) \cong 0$. Hay pocas prob que tirando 2016 veces ninguna salga 1 Lo m es muy grande.

d)

$P(B) = \lim_{m \to \infty} P(B_m) = \lim_{m \to \infty} \left(\frac{5}{6}\right)^m = 0.$

$B_{m+1} \subset B_m$

B_{m+1} está incluido en B_m

$P(B_{m+1}) = \left(\frac{5}{6}\right)^{m+1}$ $\quad P(B_m) = \left(\frac{5}{6}\right)^m$

$= \left(\frac{5}{6}\right)^m \cdot \frac{5}{6}^1 = A \cdot \underbrace{\frac{5}{6}}_{0{,}83} \quad = \left(\frac{5}{6}\right)^m = A \cdot 1.$

si $m = 3$

$P(B_4) = 0{,}48 < P(B_3) = 0{,}58$

si $P(B_{m+1}) < P(B_m)$ entonces.

$\boxed{P(B_{m+1}) \subset B_m}$

e) $B = \bigcap_{m=1}^{\infty} B_m \to$ Ax. continuidad $\lim_{m \to \infty} P(B_m) = \lim_{m \to \infty} \left(\frac{5}{6}\right)^m = 0.$

$\downarrow$

nunca sale un 6. $\to$ Probabilidad del vacío es cero.

Ω Ω 6

Lo puede ocurrir, pero no va a pasar.

1)

$\Omega = \{0,1,2,3,\ldots\}$; $A \subset \Omega$

$P(A) = \sum_{m \in A} P(m) = \frac{c}{m!}$. Hallar c y calcular $P(\{0,2,4,6,\ldots\})$

Yo sé que $P(\Omega) = 1$

$$\sum_{m=m_0}^{\infty} \cdot \frac{c}{m!} = 1$$

"x^m"

$$c \cdot \sum_{m=0}^{\infty} \frac{1}{m!} = 1$$

$$c \cdot e^1 = 1$$

$$\boxed{c = 1/e^1}$$

$m = 2k$

coseno hiperbólico

$$\sum_{m=0}^{\infty} \frac{c}{m!} = \sum_{k=0}^{\infty} \frac{e^{-1}}{(2k)!} = e^{-1} \cdot \cosh(1) = e^{-1}\left(\frac{e^1 + e^{-1}}{2}\right)$$

1.9)

$\mathbb{P} = \frac{1}{3 \cdot 2^{k-1}}$ con k= cantidad de partidos

Juan vs Pedro
ganador vs Maria

a)

$\Omega = \{JJ, PP, JMM \ldots\}$ resultados posibles.

A: gana Juan

JJ PMJJ
JMPJJ $(PMJ)^k J$
$(JMP)^k JJ$

$$A = \bigcup_{k=0}^{\infty} \{(JMP)^k JJ\} \cup \bigcup_{k=1}^{\infty} ((PMJ)^k J)$$

Todas las posibilidades gane Juan.

B: gana pedro

PP JMPP
$(PMJ)^k PP$ $(JMP)^k P$

$$B = \bigcup_{k=0}^{\infty} ((PMJ)^k PP) \cup \bigcup_{k=1}^{\infty} ((JMP)^k P)$$

C: gana Maria

JMM PMM
$(JMP)^k JMM$ $(PMJ)^k PMM$

$$C = \bigcup_{k=0}^{\infty} ((JMP)^k_J MM) \cup \bigcup_{k=0}^{\infty} ((PMJ)^k_P MM)$$

$\Omega = \{A \cup B \cup C\}$.

b)

COMO SON DISJUNTOS — partidos — partidos — POR GADDINUS

$$\boxed{P(A)} = P\left(\bigcup_{k=0}^{\infty} (JMP)^{k} JJ\right) + P\left(\bigcup_{k=1}^{\infty} (PMJ)^{k} J\right) =$$

with $3k+2$ above the first union and $3k+1$ above the second.

$$= \sum_{k=0}^{\infty} \frac{1}{3 \cdot 2^{(3k+2)-1}} + \sum_{k=1}^{\infty} \frac{1}{3 \cdot 2^{(3k+1)-1}} =$$

$$= \frac{1}{2 \cdot 3} \sum_{k=0}^{\infty} \left(\frac{1}{8}\right)^{k} + \frac{1}{3} \sum_{k=1}^{\infty} \left(\frac{1}{8}\right)^{k} \leftarrow \text{serie geométrica}$$

$$= \frac{1}{6}\left(\frac{1}{1-1/8}\right) + \frac{1}{3}\left(\frac{1/8}{1-1/8}\right) = \boxed{\frac{5}{21}}$$

[$P(B) = 5/21$ (es igual la forma)].

$$P(C) = \sum_{k=0}^{\infty} \frac{1}{3 \cdot 2^{\underbrace{(3k+3)-1}_{3k+2}}} + \sum_{k=0}^{\infty} \frac{1}{3 \cdot 2^{(3k+3)-1}}$$

$3k+3$

$$P(C) = \frac{1}{3} \cdot \left(\frac{1}{2}\right)^{2} \sum_{k=0}^{\infty} \left(\frac{1}{8}\right)^{k} + \frac{1}{12} \sum_{k=0}^{\infty} \left(\frac{1}{8}\right)^{k}$$

$$P(C) = \frac{1}{12} \cdot \left(\frac{1}{1-1/8}\right) + \frac{1}{12}\left(\frac{1}{1-1/8}\right)$$

$$\boxed{P(C) = 4/21}$$

c) $(JMP)^{k}$

$1 - 5/9$ $\qquad$ $4/21 + 10/21 = 14/21$

$$P\left(\text{Juan gane 1ro} \cap \text{no haya ganador}\right) = 1 - P\left(\text{Juan no gane 1ro} \cup \text{haya ganador}\right)$$

Hay intersec.

$$P(\sim) = 1 - \left(\frac{10}{9}\right) + P\left(\text{Juan no gane} \cap \text{Haya ganador}\right)$$

$\Rightarrow$.

$$P(\sim) = \frac{1}{9} + P\left(\begin{smallmatrix}\text{nou} \\ \text{rogove} \\ 1^{ro}\end{smallmatrix} \cap \begin{smallmatrix}\text{Hoya} \\ \text{ganador}\end{smallmatrix}\right) = 2/9$$

A: $(PMJ)^k JJ \to \frac{1}{3} \cdot \left(\frac{1/8}{1-1/8}\right) = 7/192$

B: $(PMJ)^{k-1} PP \to \frac{1}{3} \cdot \frac{1}{2} \left(\frac{1}{1-1/8}\right) = 4/21$

C: $(PMJ)^k PMM \to \frac{1}{3} \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{1-1/8}\right) = 2/21$

$\Big\}$ 0,33.

d) $1 - 14/21 = 7/21$

$$\boxed{P\left(\begin{smallmatrix}\text{juego} \\ \text{siempre}\end{smallmatrix}\right) = 7/21}$$

1.10) $[0,1] \to \mathbb{P}[0,1] = b-a$ longitud del intervalo

a) 0,314

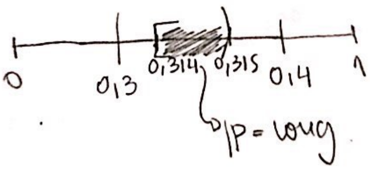
$\Omega = [0,1]$

$\mathcal{A} = 2^{[0,1]}$

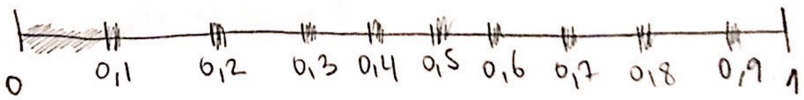
$$\mathbb{P}(0{,}314) = \mathbb{P}(a_1 = 3 \cap a_2 = 1 \cap a_3 = 4)$$

en que intervalo está?

$= 0{,}315 - 0{,}314 = 10^{-3}$



b)



A: el cero no aparece en los 1^{ros} 4. $\bar{A}$ = si aparece.

$$P(A) = 1 - P(\bar{A}) = 1 - (9^3 \cdot 10^{-4}) = 0{,}9271$$

c) $P(A) = 1 - P(\bar{A}) = 1 - \mathbb{P}\left(\bigcup_{n=1}^{\infty} A_n\right) = 1 - \sum_{n=1}^{\infty} \mathbb{P}(A_n) = 1 - \sum_{n=1}^{\infty} 9^{n-1} \cdot 10^{-n} =$

$$1 - 9^{-1} \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n = 1 - \frac{1}{9} \cdot \left(\frac{9/10}{1 - 9/10}\right) = 0$$

8	9		
4	5	6	7
0	1	2	3

AZAR CON REPOSICIÓN 5

a) $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ $\Omega = \{(a,b,c,d,e) : \{0123456789\}\}$.

$\#\Omega = 10^5$

1. A = Las 5 sean iguales.

Regla del producto →

$\underset{\substack{\downarrow \\ \text{Tengo la opción de elegir cualquiera.}}}{\underline{10}}\ \underbrace{\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}}_{\text{Solo puedo elegir la que tenía.}}$. $\#A = 10.1.1.1.1 = 10$

$P(A) = \frac{\#A}{\#\Omega} = \frac{10}{10^5}$.

$\boxed{P(A) = \frac{1}{10^4}}$

2. 1, 3, 5, 7, 9.

$\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}$ $\boxed{P(B) = \frac{1}{10^5}}$

3. $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$ $\#C = 5! = 120$.

$\boxed{P(C) = \frac{120}{10^5} = 0{,}0012}$

4. $\#D = 10.9.8.7.6 = 30240$

$\boxed{P(D) = 0{,}3024}$

b) SIN REPO $\#\Omega = 10.9.8.7.6$.

$\#\Omega = 30240$

1. evento imposible
$P(A) = 0$

2. 1.1.1.1 1 $P(B) = \frac{1}{30240}$

3. $P(C) = \frac{120}{30240} = \frac{1}{252}$

4. $\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}$ $P(D) = 1$

$♀ + 29\ p = 30$ azar

a) CON REPO PERO SIN CONTAR A LA encargada

$\#\Omega = 29^{12}$

$\#A = 29$

PORQUE DE LOS 29 QUE PODRÍAN SER, SOLO 12 VECES SE CONTÓ el rumor

$P(A) = \dfrac{29 \cdot 28^{11}}{29^{12}} = 0{,}6797$

b) $P(B) = \dfrac{29!}{29^{12}\ 17!} = \dfrac{29\ 28 \cdot 27\ 26\ 25 \cdot 24\ 23 \cdot 22 \cdot 21\ 20\ 19\ 18.}{29^{12}} = 0{,}07$

1.15) 13 piratas y 3 puertos P, M, G azar

a) $4 \to P$, $4 \to M$, $5 \to G$

$\# \Omega = \{$cada pirata elije los 3 puertos$\} = 3^{13}$

$\underline{3}\ \underline{3}\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_$

$\#A = \underbrace{\frac{13}{4} \cdot \frac{12}{3} \cdot \frac{11}{2} \cdot \frac{10}{1}}_{P}\ \underbrace{\frac{9}{4}\ \frac{8}{3}\ \frac{7}{2} \cdot \frac{6}{1}}_{M} \cdot \underbrace{\frac{5}{5} \cdot \frac{4}{4}\ \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}}_{G}$

$\#A = \dfrac{13!}{4!\,4!\,5!} = 90090$ $\qquad P(A) = \dfrac{90090}{3^{15}} = 0{,}0565.$

b) $6 \to P$, $6 \to M$, $1 \to G$. UNIÓN $\oplus$ $6 \to P$, $7 \to M$ } unión DISJUNTA.

$P(B) = P(B_1 \cup B_2) = P(B_1) + P(B_2)$

$\#B_1 = \dfrac{13!}{6!\,6!\,1!}$

$P(B) = \dfrac{\#B_1}{\#\Omega} + \dfrac{\#B_2}{\#\Omega} = 7{,}53 \times 10^{-3} + 1{,}07 \times 10^{-3} = 8{,}6 \times 10^{-3}$

$\#B_2 = \dfrac{13!}{6!\,7!\,0!}$

c) 5 PoGoM
4 PoGoM
4 PoGoM

$\#\Omega = 3^{13}$

$\#C = \binom{13}{5}\binom{8}{4}\binom{4}{4} 3! = 540540$

tengo 3 opciones más

$\boxed{P(C) = \frac{540540}{3^{13}} = 0{,}34}$

d) ¿

1.16) 7 gatos y 5 cajas $\boxed{\text{EQUIPROBABLE}}$

indistinguibles → "ÚNICA FORMA de ORDENARLOS"

5 cajas y 7 gatos

11010110101 → voy a tener 4 ceros
y 7 unos.

$\#\Omega = \frac{11!}{7!\,4!}$

a) 1ra caja → 2 gatos y última cero

$\underbrace{11\,0}_{\text{quiero}}\ \underbrace{\_\ 0\ \_\ 0\ \_}_{\text{5 gatos como quiera}}\ \underbrace{0}_{\text{NADA}}$

quiero → lo que varía –

$\#A = \binom{7}{2} = \frac{(5+2)!}{2!\,5!}$ $\quad P(a) = \frac{\#A}{\#\Omega} = \frac{7!}{5!\,2!}\cdot\frac{4!\,7!}{11!}$

$\boxed{P(a) = 0{,}06}$

cuanta caja contenga más de 3 rojos.

a) _ 0 _ 0 _ 0 1111 0 _
b) _ 0 _ 0 _ 0 11111 0 _
c) _ 0 _ 0 _ 0 111111 0 _
d) _ 0 _ 0 _ 0 1111111 0 _

SUMO TODO

$\#\Omega = \frac{11!}{4!\,7!}$

a) #A → 3 1's, 3 0's $\#A = \frac{6!}{3!\,3!}$.

b) #B → 2 1's, 3 0's. $\#B = \frac{5!}{3!\,2!}$

c) $\#C = \frac{4!}{3!\,1!}$ d) $\#D = \frac{3!}{3!\,0!} = 1$

$$P(b) = \frac{\frac{6!}{3!\,3!} + \frac{5!}{3!\,2!} + \frac{4!}{3!\,1!} + \frac{3!}{3!\,0!}}{\frac{11!}{4!\,7!}}$$

720, 36

$$P(b) = \frac{20 + 10 + 4 + 1}{330} = \frac{7}{66}$$

$$\boxed{P(b) = 0{,}106}$$

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1.17

25 piezas → k defectuosos

5 al azar → si hay al menos 1 defect se rechaza

a) $K=3$ → hay 3 defectuosos → ¿ P: pase inspección?

total → 25 piezas < 3 defectuosos, 22 bien } elijo 5

Para que pase tiene que haber 0 defectuosos ↓ si hay 1 chau.

de 22 no def elijo 5

de 3 def. elijo 0.

$$P = \frac{\binom{22}{5}\binom{3}{0}}{\binom{25}{5}} =$$

de 25 saco 5

22 ✓ → x

3 ✓ → y

$$P = \frac{\frac{22!}{5!\,17!} \cdot \frac{3!}{3!\,0!}}{\frac{25!}{5!\,20!}} = \frac{26334 \cdot 1}{53130} = 0{,}496$$

b) ¿P(K)?

$$P = \frac{\binom{25-k}{5}\binom{k}{0}}{\binom{25}{5}} \rightarrow \text{"pase inspección"}$$

c) más de 5 defectuosos → acepte una partida con + 5 defectuosos

1.18

A, T, C, G → 4 letras

a) "A precede a T" y "C precede a G" son ind.

$\underbrace{\text{precede a T}}_{A}$ $\underbrace{\text{precede a G}}_{B}$

que este ... outros

Para que sean ind

CG...

ATCG	CGAT
ATGC	CGTA
CG AT	ATCG
GCAT	TACG

$$P(A \cap B) = P(A)\,P(B)$$

$P(A) = \frac{12}{24} = 1/2$

Prob de que AT

$\boxed{\#\Omega = 24} = 6 + 6 + 6 + 6$

ATCG	TACG	CATG
ATGC	TAGC	CAGT
ACGT	TCGA	CTAG
ACTG	TCAG	CTGA
AGTC	TGAC	CGAT
AGCT	TGCA	CGTA

GACT
GATC
GTCA
GTAC
GCAT
GCTA

$P(B) = \frac{12}{24} = 1/2$.

$P(A)P(B) = 1/4$

coinciden

$P(A \cap B)$ → A vaya antes de T y podemos C antes de G

$P(A \cap B) = 6/24 = 1/4$

$\boxed{\text{son indep} ✓}$

b) "AT" y "CG" no son ind

↳ C ↳ D

$P(C) = 4/24$ $P(D) = 4/24$ $P(C)P(D) = 1/36$

$P(C \cap D) = 2/24 = 1/12$ — $\neq$

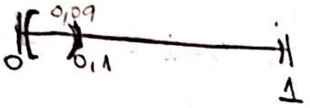
1.19) $(0,1) \to$ azar

$P(a,b) = b-a$ ↙ long del intervalo.

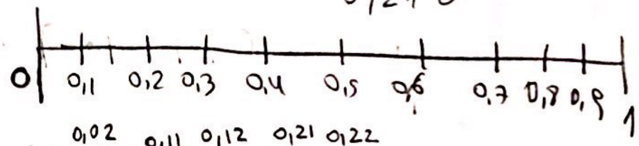
a) "el 1ro es cero" y "el 2do es 1" seguido

$A: 0,0\ldots$

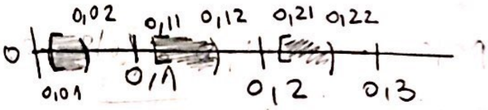
$0,00\ldots$



$B: 0,\_1\ldots$ 0,010
0,110
0,210



$P(A) = 0{,}09$

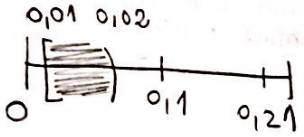


$0,01\ldots$

$0{,}02 - 0{,}01 = \underline{0{,}01} \to$ esto 9 veces

$P(C) = P(A) \cap P(B)$

$P(B) = 0{,}09$



$0{,}02 - 0{,}01 = 0{,}01$

P. 17

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(1.20)

2 veces $\longrightarrow \#\Omega = 36$

a) A: "1er resultado par"
B: " 2do resultado par"
C: "suma resultados es par"

$\left.\right\}$ A, B, C son ind de a dos, pero no general

DADO 1

DADO 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

1 tiro → 6 result posibles → 3 par

$P(A) = 1/2$

$P(B) = 1/2$

$P(C) = 18/36 = 1/2$

$P(A)\,P(B)\,P(C) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 1/8$

ambos par suma par

$P(A \cap B \cap C) = 9/36 = 1/4 \neq$

ambos par

$P(A \cap B) = 9/36 = 1/4 = P(A)P(B) = 1/4$ ✓

$P(A \cap C) = 9/36 = 1/4$ ✓

1ro par suma par

$P(B \cap C) = 9/36 = 1/4$ ✓

c) A: 1er resultado es 1, 2 ó 3 C: suma es 9
B: 1er resultado es 3, 4 ó 5

$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$ → no son indep

1er 3. $P(A \cap B) = 6/36 = 1/6$ $P(A) . P(B) = 1/4$

$P(A) = 18/36$, $P(B) = 18/36$

$\neq$

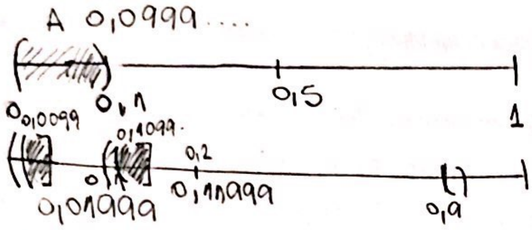
19) $(0,1) \to$ azar

a) A: "el primer dígito es cero" — 0,0....
B: "el segundo dígito es 1" — 0,01...
$\overline{B}$

} son independientes

$\boxed{P(A \cap B) = P(A)P(B)}$

$\mathbb{P}(\text{int}) =$ distancia

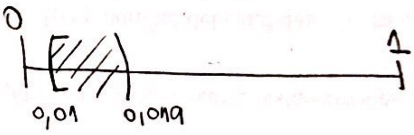


$P(A) = P((0, 0{,}1)) = 0{,}1$

$P(B) = (0{,}01) \times 10 = 0{,}1$

1 2 3 4 5 6 7 8 9 10
0 0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9

0,01 lo que haya $\leftarrow$ $P(A \cap B) = \frac{1}{100}$ ✓



$P(A)P(B) = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$

~~0,009~~ 0,010 0,012 0,014 0,016 0,017 ~~0,02~~
0,011 0,013 0,015 0,018 0,019

son ind.

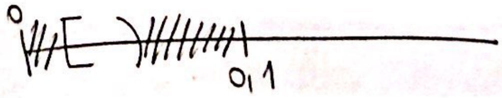
b) Si A y B son ind $\longrightarrow$ A y $\overline{B}$ lo son.

$P(A \cap \overline{B}) = P(A)\,P(\overline{B})$ $\qquad P(\overline{B}) = 9/10$

$P(A \cap \overline{B}) = \frac{9}{100}$
$\downarrow$
1ro cero
2do no uno

$P(A)\,P(\overline{B}) = \frac{1}{10} \cdot \frac{9}{10} = \frac{9}{100}$ ✓

son ind.



$0{,}1 - \frac{1}{100} = \frac{9}{100}$.

21) azar $\to (0,1)$

$[0{,}2 \;,\; 0{,}3)$

a) "1er dígito es 2": $A_1 \to P(A_1) = \frac{1}{10}$

"2do es 3": $A_2 \to P(A_2) = \frac{1}{10}$

"3ro es 5": $A_3 \to P(A_3) = \frac{1}{10}$

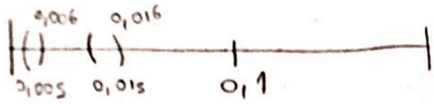
"4to es 8" y $A_4 \to P(A_4) = \frac{1}{100}$

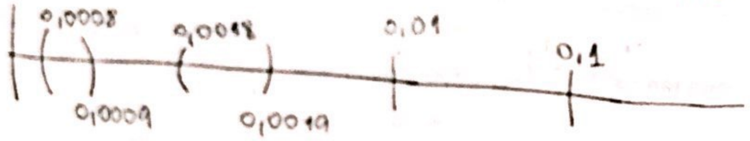
[El 1 aparece alguna vez desp del 10 mo dígito]

$[0{,}03 \;,\; 0{,}04] \times 10$

$[0{,}005 \;,\; 0{,}006) \times 100$

$0{,}0008 \;,\; 0{,}0009$





$\underbrace{10 \qquad\qquad 10}$

$100 \times \underset{(0,1)\ \text{a1}}{10} = 1000$

$(0{,}0009 - 0{,}0008) \times 1000 =$

$P(4^{to} \text{ es } 8) = \frac{1}{10}$

$P(\frac{1^{ro}}{}) = 0{,}0008000001 \to$ 1 PROB POR c/ intervalo.

$P(A_4) = \frac{1}{100}$.

$[0{,}23 \;,\; 0{,}24)$

$0{,}235$

$0{,}2308$

$\uparrow$

$P(A_1 \cap A_2) = P(A_1)P(A_2) = 1/100$ ✓

$0{,}025 \leftarrow P(A_2 \cap A_3) = P(A_2)P(A_3) = 1/100$

$P(A_3 \cap A_4) = P(A_3)P(A_4) = 1/1000$

$P(A_2 \cap A_4) = P(A_2)P(A_4) = 1/1000$

$\hookrightarrow 0{,}0308$

$P(A_1 \cap A_2 \cap A_3) = \frac{1}{1000}$

$P(A_1 \cap A_2 \cap A_4) = 1/10^4$

$P(A_2 \cap A_3 \cap A_4) = 1/10^4$

$P(A_1 \cap A_3 \cap A_4) = 1/10^4$

$P(A_4 \cap A_1) = P(A_4)P(A_1) = 1/1000$

$P(A_3 \cap A_1) = P(A_3)P(A_1) = 1/100$.

SON IND.

$P(A_1 \cap A_2 \cap A_3 \cap A_4) = 1/10^5$ ✓

$0{,}2358000001$

"el 1ro no es 2" $\rightarrow P(\bar{A}_1) = 1 - P(A_1) = 9/10$.

"el 2do es 3 y 3ro 5" $\rightarrow P(\underbrace{A_2 \cap A_3}_{A_U}) = \frac{1}{100}$

$$P(\bar{A}_1 \cap (A_2 \cap A_3)) = \frac{9}{1000} = P(\bar{A}_1)\, P(A_2 \cap A_3)$$

son ind.

0,135) [NO 0,12) 0,13

(0,136 - 0,135) → por 9. → porque el de (0,2) → (0,3)

c) "el 1ro es 1 o 3ro no es 5" $P(\underbrace{A_1 \cup \bar{A}_3}_{B}) = \frac{1}{10} \cdot \frac{9}{10} = \frac{9}{100}$.

"el 2do es 3 y 4to A4" $P(\underbrace{A_2 \cap A_4}_{C}) = 1/1000$

$$P(B \cap C) = \frac{9}{10000}$$

$0,13\underbrace{\ }_{x9}8\,000001$

1.22

300 componentes individuales

300 proovedores

$P(\text{no defecto}) = 0,9$

$P(\text{defecto}) = 0,01$ o menor.

$P(d) = 0 \rightarrow$ aceptable.

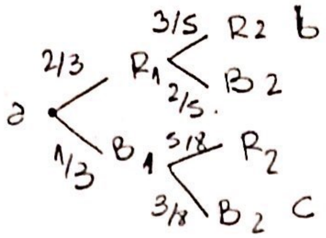
a) $P(A) = 1 - \underbrace{0{,}01}_{\text{PROB defecto}}{}^{300} \cong 1$. A: 300 piezas sin defecto

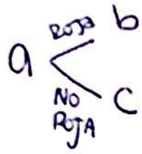
b) 98% aceptable $\rightarrow 0,98 =$ aceptable

$$P(A) = 0,98 = 1 - \overbrace{x^{300}}^{\text{Prob defecto}} \rightarrow x \cong 0,987$$

3

a	b	c
2R 1B	3R 2B	5R 3B





$R_i, i = 1,2 \rightarrow i$ fue roja

$B_i, i = 1,2 \rightarrow i$ fue blanca (no roja).

a) $P(B_1) = 1/3$

b) $P(B_2) = \overbrace{\frac{2}{3} \cdot \frac{2}{5}}^{R_1 \cap B_2} + \overbrace{\frac{1}{3} \cdot \frac{3}{8}}^{B_1 \cap B_2} = \frac{47}{120}$

c) Primera B si 2da roja

$$P(B_1 | R_2) = \frac{P(B_1 \cap R_2)}{P(R_2)} = \frac{\frac{1}{3} \cdot \frac{5}{8}}{\frac{2}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{5}{8}} = \frac{5/24}{73/120}$$

$$P(B_1 | R_2) = \frac{25}{73}$$

d) alguna sea roja : $R_N \longrightarrow \begin{array}{c|c} R_1 R_2 & B_1 R_2 \\ \hline R_1 B_2 & \end{array} \longrightarrow \boxed{R_1 \cup R_2}$ otra forma.

$P(R_N) = P(R_1 \cap R_2) \cup P(R_1 \cap B_2) \cup P(B_1 R_2) \rightarrow$ como son disjuntos la suma es la unión.

$$P(R_N) = \frac{2}{3} \cdot \frac{3}{5} + \frac{2}{3} \cdot \frac{2}{5} + \frac{1}{3} \cdot \frac{5}{8} = 7/8$$

e)

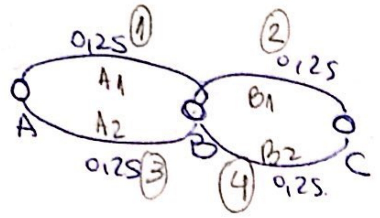
200 R 100 B	150 R 100 B	125 R 75 B.

antes

2 ✓ 1	3 ✓ 2	5 ✓ 3

$\rightarrow$ serán los mismos resultados porque respecto con el total están en la misma proporción.

(14)

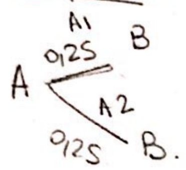


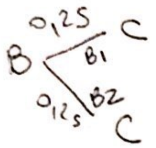
0,25 → bloqueado

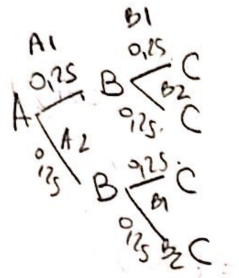
0,75 → ✓

Bloqueado.

B→C sabiendo que no hay conexión de A|C.







A=abierto

Ā = bloqueado.

$$P(BC\,|\,\overline{AC}) = \frac{P(BC \cap \overline{AC})}{P(\overline{AC})} = ?$$

caminos de $\overline{AC}$ → x4 porque vario cierro 3 y 1 abierto nunca vas a llegar.

$$\left.\begin{array}{l} 4\,(A_1\,\overline{B_1}\,\overline{B_2}\,\overline{A_2}) \\ 2\,(\overline{A_1}\,\overline{A_2}\,B_1B_2) \\ \overline{A_1}\,\overline{A_2}\,\overline{B_1}\,\overline{B_2} \end{array}\right\} \quad 4\cdot\left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right)^3 + 2\left(\tfrac{1}{4}\right)^2\left(\tfrac{3}{4}\right)^2 + \left(\tfrac{1}{4}\right)^4 = \frac{31}{256}$$

$\overline{A_1}\,\overline{A_2}\,B_1B_2$

$\overline{B_1}\,\overline{B_2}\,A_1A_2$

$$P(BC \cap \overline{AC}) = P(\overline{A_1} \cap \overline{A_2} \cap B_1 \cap B_2) + P(\overline{A_1} \cap \overline{A_2} \cap \overline{B_1} \cap B_2)\cdot 2$$

$$= (1/4)^2(3/4)^2 + (1/4)^3(3/4) = 15/256$$

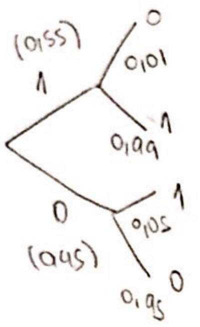
PR $\overline{B_2} \cap B_1$

$$\boxed{P(BC\,|\,\overline{AC}) = 15/31 \cong 0{,}484}$$

1.25 0 y 1

emita

$P(1) = 0{,}55$
$P(0) = 0{,}45$.
$P(0|0) = 0{,}95$.
$P(1|1) = 0{,}99$



a) recibe $P(1) = 0{,}55 \cdot 0{,}99 + 0{,}45 \cdot 0{,}05$

$P(1) = 0{,}567$

b) recibió / emite

$P(1|1) = \dfrac{P(1 \cap 1)}{P(1)} = 0{,}96$.

$P(1) \approx 0{,}57$

$P(1 \cap 1) = 0{,}54$.

1.26

E: emite 1 $\bar{E}$: emite 0.
R: recibe 1 $\bar{R}$: recibe 0.

$P(\bar{E}) = 0{,}05$ $P(E) = 0{,}95$.

$P(\bar{R}|\bar{E}) = 0{,}9 \rightarrow P(R|\bar{E}) = 0{,}1$

¿ $P(R|E)$? $\rightarrow P(\bar{E}|\bar{R}) = 0{,}99$.

$\downarrow$

$P(E|\bar{R}) = 0{,}01$

$$\left[P(R|E) = \frac{P(R \cap E)}{P(E)} \right]$$

$P(E|\bar{R}) = \dfrac{P(E \cap \bar{R})}{P(\bar{R})}$ $P(R|\bar{E}) = \dfrac{P(E \cap \bar{R})}{P(\bar{E})}$

$P(E|\bar{R}) \cdot P(\bar{R}) = P(E \cap \bar{R})$ III

0,01 0,5 5×10^{-3}

$P(E|\bar{R})\, P(\bar{R}) = P(\bar{E})\, P(R|\bar{E})$.

0,01 0,05 0,1

I $P(\bar{R}) = 0{,}5$ $P(R) = 0{,}5$.

II $P(R|E) + P(\bar{R}|E) = 1$

$P(\bar{R}|E) = \dfrac{P(\bar{R} \cap E)}{P(E)}$

$P(R|E) + 5{,}26 \times 10^{-3} = 1$

$\boxed{P(R|E) = 0{,}9947}$.

27

equilibrada

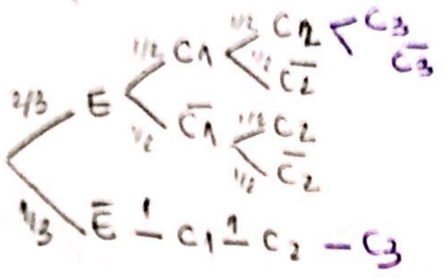
2C (1) | 1C 1S (2) | 1C 1S (3)

S = sello
C = cara

E = equilibrada

a)

$$P(C_2|C_1) = \frac{P(C_1 \cap C_2)}{P(C_1)}$$



$$P(C_1) = \tfrac{2}{3} \cdot \tfrac{1}{2} + \tfrac{1}{3} \cdot 1 = \tfrac{2}{3}$$

$$P(C_1 \cap C_2) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot 1 \cdot 1 = 0{,}5$$

$$\boxed{P(C_2|C_1) = 3/4}$$

b)

$P(C_1) = 2/3$

$$P(E|C_1) = \frac{P(E \cap C_1)}{P(C_1)} = \frac{1/3}{2/3}$$

$$\boxed{P(E|C_1) = 1/2} \qquad P(\bar{E}|C_1) = 1/2$$

c)

$$P(E|C_1 \cap C_2) = \frac{P(E \cap (C_1 \cap C_2))}{P(C_1 \cap C_2)} = \frac{2/3 \cdot 1/2 \cdot 1/2}{2/3 \cdot 1/2 \cdot 1/2 + 1/3}$$

$$\boxed{P(E|C_1 \cap C_2) = \frac{1/6}{1/2} = 1/3}$$

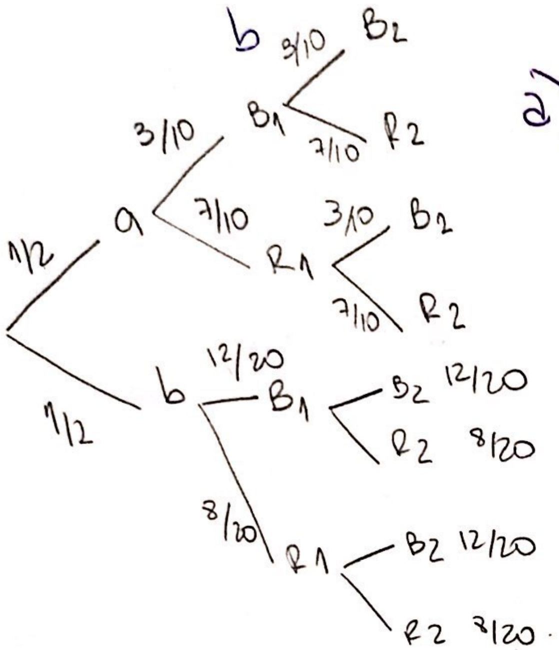
d)

$$\boxed{P(E|C_1 \cap C_2 \cap C_3) = \frac{2/3 \cdot 1/2 \cdot 1/2 \cdot 1/2}{1/12 + 1/3} = \frac{1}{5}}$$

28

2 con repo

3B	12 B
7 R	8 R
a	b



a) $P(B_2|B_1) = \dfrac{P(B_2 \cap B_1)}{P(B_1)}$

$P(B_1) = \frac{1}{2} \cdot \frac{3}{10} + \frac{1}{2} \cdot \frac{12}{20} = \frac{9}{20}$

$P(B_2 \cap B_1) = \frac{1}{2} \cdot \frac{3}{10} \cdot \frac{3}{10} + \frac{1}{2} \cdot \frac{12}{20} \cdot \frac{12}{20}$

$= 9/40$.

$P(B_2|B_1) = 1/2$.

b) "primera blanca", "segunda blanca"

$P(B_1 \cap B_2) = P(B_1)\, P(B_2)$

$9/40 = 9/20 \cdot 9/20$.

$\boxed{9/40 \neq 81/400}$ → NO SON IND.

$P(B_2) = \frac{1}{2} \cdot \frac{3}{10} \cdot \frac{3}{10} + \frac{1}{2} \cdot \frac{7}{10} \cdot \frac{3}{10} + \frac{1}{2} \cdot \frac{12}{20} \cdot \frac{12}{20} + \frac{1}{2} \cdot \frac{8}{20} \cdot \frac{12}{20} =$

$= \frac{9}{200} + \frac{21}{200} + \frac{9}{50} + \frac{3}{25} = 9/20$.

a $\qquad$ b

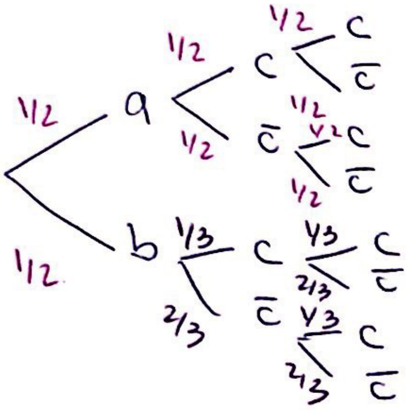
$P(C) = 1/2 \qquad P(C) = 1/3$

$P(\overline{C}) = 1/2 \qquad P(\overline{C}) = 2/3$

A_1 = sale cara en el 1er tiro

A_2 = " " 2do tiro.

B = se elige la moneda a



Dado.

a1) ¿ B, A_1 y A_2 son ind?

a2) ¿ A_1 y A_2 son ind?

a1) $$P(A_1 \cap A_2 | B) = \frac{P(A_1 \cap A_2)}{P(B)} = \frac{1/8}{1/2} = 1/4$$

$P(B) = 1/2$

$$P(A_1 \cap A_2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(A_1 | B) = \frac{P(A_1 \cap B)}{P(B)} = 1/4 \div 1/2$$

$$P(A_2 | B) = \frac{\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{1/2} = 1/2$$

$$P(A_2 \cap A_1 | B) = P(A_1 | B) \, P(A_2 | B)$$

$$1/4 = 1/2 \cdot 1/2 \checkmark$$ son ind A_1 y A_2.

a2)

$$P(A_1 \cap A_2) = P(A_1) \, P(A_2)$$

$$\frac{13}{72} \neq \frac{25}{144}$$ NO SON IND.

$$P(A_1) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} = \frac{5}{12}$$

$$P(A_2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{3} = \frac{5}{12}$$

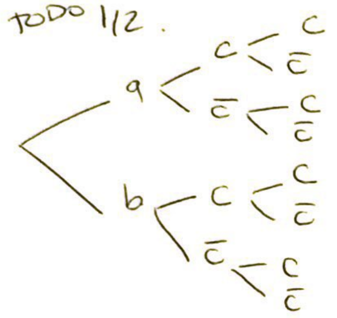
$$P(A_1 \cap A_2) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = 13/72$$

) A_1 = sale cara 1ro   B = sale 1 ceca.   moneda equilibrada.
A_2 = sale 2do.

¿A_1 y A_2 son ind?

¿Dado B, A_1 y A_2 lo son?

TODO 1/2.



$$P(A_1) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$$

$$P(A_2) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 1/2.$$

$$P(A_1 \cap A_2) = \frac{1}{8} + \frac{1}{8} = 1/4$$

son ind ✓

$$\underbrace{P(A_1 \cap A_2 | B)}_{= 0} = \underbrace{P(A_1|B)\, P(A_2|B)}_{\neq 0}$$

no son ind.