GUÍA 2

2.1

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$A = \{\emptyset, \{1,3,5\}, \{2,4,6\}, \Omega\}$

FUNCIÓN $X: \Omega \to \mathbb{R}$ ES $\underbrace{\text{V.A}}$

↳ CADA $w \in \Omega$ SE LE ASIGNA UN NUMERO $\in \mathbb{R}$.

$X(w) = x \quad (x \in \mathbb{R})$

(A) $X(w) = w$

$w \in \Omega = \{1,2,3,4,5,6\}$

NO PORQUE LA FUNCIÓN VA DE $X: \Omega \to \Omega$

(B) $X(w) = 1\{w \text{ ES PAR}\}$

SÍ, PORQUE $X: \Omega \to \mathbb{R}$ Y ADEMAS $\{\text{ES PAR}\} \in A$

(C) $X(w) = 1\{w \in \{1,4\}\}$

NO, PORQUE AUNQUE $X: \Omega \to \mathbb{R}$, $\{(1,4)\} \notin A$

2.2 $X \to F_X(x) = \mathbb{P}(X \leq x)$

(A)

(B) UTILIZANDO LAS PROPIEDADES DE $F_X(x)$

$\mathbb{P}(-2 < X \leq \quad) = F_X(2^+) - F(-2^+)$

$= 1 - 1/3 = 2/3$

$\mathbb{P}(-2 \leq X \leq 2) = F_X(2^+) - F_X(-2^-)$

$= 1 - 0 = 1$

$\mathbb{P}(-2 \leq X < 2) = F_X(2^-) - F_X(-2^-)$

$= 2/3 - 0 = 2/3$

$\mathbb{P}(-2 < X < 2) = F_X(2^-) - F_X(-2^+)$

$= 2/3 - 1/3 = 1/3$

(C)

$\mathbb{P}(X \in (-2, -1)) = \mathbb{P}(-2 < X < -1)$

$= F_X(-1^-) - F_X(-2^+)$

$= 1/3 - 0 = 1/3$

$\mathbb{P}(|X| \leq 1) = \mathbb{P}(-1 \leq X \leq 1)$

$= F_X(1^+) - F_X(-1^-)$

$= 2/3 - 1/3 = 1/3$

$\mathbb{P}(X \in (1, 2)) = \mathbb{P}(1 < X < 2)$

$= F_X(2^-) - F_X(1^+)$

$= 2/3 - 2/3 = 0$

NOTA

HOJA Nº 2

FECHA

(D) $\mathbb{P}(X \leq 1{,}5 \mid X < 2)$ VARIABLE TRUNCADA

$$\mathbb{P}(X \leq 1{,}5 \mid X<2) = \frac{\mathbb{P}(X \leq 1{,}5 \wedge X < 2)}{\mathbb{P}(X<2)} =$$

$$\frac{\mathbb{P}(X \leq 1{,}5)}{\mathbb{P}(X<2)} = \frac{F_X(1{,}5)^+}{F_X(2)^-} = \frac{2/3}{2/3} = 1$$

$$\mathbb{P}(X \leq 1{,}5 \mid X \leq 2) = \frac{F_X(1{,}5)^+}{F_X(2)^+} = \frac{2/3}{1} = 2/3$$

(E) $\mathbb{P}(X = -2 \mid |X| = 2)$

$$\mathbb{P}(X=-2 \mid |X|=2) = \frac{F_X(-2)^+}{\mathbb{P}(-2 \leq X \leq 2)} = \frac{F_X(-2)^+}{F_X(2)^+ - F_X(-2)^-}$$

$$= \frac{1/3}{1-0} = 1/3$$

2.3

URNA → 3V + 5R

(A) 4 EXTRACCIONES CON REPO → ENSAYOS FIJOS
↓
OBSERVO CUANTOS EXITOS

POR V.A BINOMIAL

$$P_X(K) = P^K (1-P)^{N-K} \binom{N}{K}$$
↓
EXITOS

P = SALGA VERDE = $\frac{3}{8}$

N = 4

K = EXITOS

$P_X(0) = \left(\frac{5}{8}\right)^4 =$ $\quad P_X(2) = \left(\frac{3}{8}\right)^2 \cdot \left(\frac{5}{8}\right)^2 \cdot 6 =$

$P_X(1) = \frac{3}{8} \cdot \left(\frac{5}{8}\right)^3 \cdot 4 =$ $\quad P_X(3) = \left(\frac{3}{8}\right)^3 \cdot \left(\frac{5}{8}\right)^1 \cdot 4 =$

NOTA

CASOS FAVORABLES / CASOS POSIBLES

(B) 4 EXTRACCIONES SIN REPO $\rightarrow$ FIJO ENSAYOS

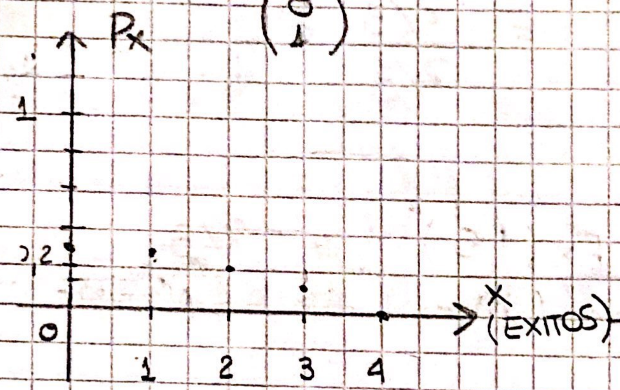
$$\mathbb{P}_X(k) = p^k (1-p)^{N-k} \binom{N}{k}$$

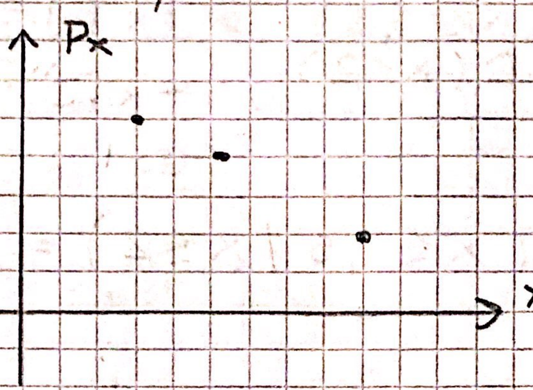
N = 4 , K =

P = SALGA VERDE

$$P_X(1) = \frac{\binom{5}{3}\binom{3}{1}}{\binom{8}{4}} \qquad P_X(3) = \frac{\binom{5}{1}\binom{3}{3}}{\binom{8}{4}}$$

$$P_X(2) = \frac{\binom{5}{2}\binom{3}{2}}{\binom{8}{4}} \qquad P_X(4) = 0 \qquad P_X(0) = \frac{\binom{5}{4}\binom{3}{0}}{\binom{8}{4}}$$





2.4

MONEDA $\rightarrow$ $\mathbb{P}$(CARA) = 5/8

N_k: 'CANT DE LANZAMIENTOS HASTA k-ESIMA CARA'

PRIMER EXITO

POR VARIABLE ALEATORIA PASCAL

$$\mathbb{P}_N(k) = \binom{N-1}{k-1} p^k (1-p)^{N-k}$$

k = SALGA CARA

N = ENSAYOS

$$\mathbb{P}_N(k) = \binom{N-1}{k-1} \left(\frac{5}{8}\right)^k \left(\frac{3}{8}\right)^{N-k}$$

$\downarrow$ LANZAMIENTOS

(B) $\mathbb{P}(N_1 \text{ SEA PAR})$ $(k = 1)$

$$\mathbb{P}(N_1 \text{ SEA PAR}) = \sum_{N=0}^{\infty} \left(\frac{5}{8}\right)^k \left(\frac{3}{8}\right)^{2N+1} = \frac{5}{8} \sum_{N=0}^{\infty} \left(\frac{3}{8}\right)^{2N} \left(\frac{3}{8}\right)$$

$$= \frac{5}{8} \cdot \frac{3}{8} \sum_{N=0}^{\infty} \left(\left(\frac{3}{8}\right)^2\right)^N = \frac{5}{8} \cdot \frac{3}{8} \sum_{N=0}^{\infty} \left(\frac{9}{64}\right)^N = \frac{15}{64} \left(\frac{1}{1 - 9/64}\right) = \frac{3}{11}$$

NOTA

SERIE GEOMETRICA

(<1)

↑ V. TRUNCADA

(C) $\mathbb{P}(N_1 = 3)$ y $\mathbb{P}(N_2 = 5 \mid N_1 = 2)$

$$\mathbb{P}(N_1 = 3) = \binom{3-1}{1-1}\left(\frac{5}{8}\right)\left(\frac{3}{8}\right)^2 = \binom{2}{0}\frac{5}{8}\cdot\frac{9}{64}$$

- $\mathbb{P}(N_1 = 3) = \frac{45}{512} = 0{,}087$

- $\mathbb{P}(N_2 = 5) = \binom{5-1}{2-1}\left(\frac{5}{8}\right)^2\left(\frac{3}{8}\right)^{5-2} = \binom{4}{1}\left(\frac{25}{64}\right)\left(\frac{27}{512}\right)$

$\mathbb{P}(N_2 = 5 \mid N_1 = 2)$

$\underbrace{0 \quad 1}_{\text{YA PASO}} \mid 0 \quad 0 \quad 1$

↳ $\mathbb{P}(N_1 = 3) \cong 0{,}087$

(D) $\mathbb{P}(N_1 > 3)$ $\mathbb{P}(N_1 > 5 \mid N_1 > 2)$

- $\mathbb{P}(N_1 > 3) = \left(1 - \frac{5}{8}\right)^3 = 0{,}053$

0 0 0 . . . LO QUE SEA

$\mathbb{P}(N_1 > 5 \mid N_1 > 2)$

$0 \quad 0 \mid \underbrace{0 \quad 0 \quad 0}_{\mathbb{P}(N_1 > 3) = 0{,}053}$ . . LO QUE SEA

2.8

VARIABLE ALEATORIA T_k (TIEMPO EN EMITIR/S)

K PARTICULAS EMITIDAS

$$f_{T_k}(T) = \frac{(1/2)^k}{(k-1)!} T^{k-1} e$$

¡PERDIDA DE MEMORIA!

$$\mathbb{P}(\overset{T}{X} > \overset{p+t}{S+T} \mid X > S) = \mathbb{P}(X > T)$$

SOLO PARA EXPONENCIALES Y GEOMETRICAS

(A) $P(T_1 > 3)$ $\quad P(T_1 > 5 \mid T$

$$f_{T_1}(t) = \frac{1}{2} e^{-1/2 t} \longrightarrow V$$

ENTONCES YO SE QUE $F_{T_1}(t) = 1 - e^{-1/2 t}$

$$\mathbb{P}(T_1 > 3) = 1 - \mathbb{P}(T_1 \leq 3) = 1 - \mathbb{P}(T_1 \leq 3)$$

$$0 \text{ PORQUE } \mathbb{1}\{T>0\} = \int_{-\infty}^{3} f_{T_k}(T)\, dT = F_1(3) - \underbrace{F_1(0)}_{1-1=0}$$

$$\mathbb{P}(T_1 \leq 3) = F_1(3) = 1 - e^{-3/2} \cong 0{,}776$$

$$\mathbb{P}(T_1 > 3) = 1 - 0{,}776 = 0{,}224$$

$\mathbb{P}(T_1 > 5 \mid T_1 > 2) \rightarrow$ VARIABLE TRUNCADA

$$P(T_1 > 5 \mid T_1 > 2) = \frac{P(T_1 > 5 \wedge T_1 > 2)}{P(T_1 > 2)} = \frac{P(T_1 > 5)}{P(T_1 > 2)}$$

$$\frac{1 - \mathbb{P}(T_1 \leq 5)}{1 - \mathbb{P}(T_1 \leq 2)} = \frac{1 - F_1(5)}{1 - F_1(2)} = \frac{e^{-5/2}}{e^{-2/2}} = 0{,}223$$

(B) $\mathbb{P}(T_3 > 3)$ y $\mathbb{P}(T_3 > 5 \mid T_3 > 2)$

$$\mathbb{P}(T_3 > 3) = 1 - \mathbb{P}(T_3 \leq 3)$$

$$\mathbb{P}(T_3 \leq 3) = \int_{-\infty}^{3} f_{T_k}(T)\, dT = F_3(3)$$

LA F GRANDE ACUMULA

$$f_{T_k}(T) = \frac{(1/2)^3}{2!} 3^2 \cdot e^{-3/2} \rightarrow \left(-\tfrac{1}{2} \cdot 3\right)$$

$$f_{T_k}(T) = \frac{1}{16} T^2 e^{-T/2} \longrightarrow \text{ES COMO } \Gamma$$

$$F_T(T) = 1 - e^{-\lambda T} \sum_{k=0}^{m-1} \frac{(\lambda T)^k}{k!}$$

$$F_3(3) = 1 - e^{-1/2 \cdot 3} \sum_{k=0}^{2} \frac{(3/2)^k}{k!}$$

$$= 1 - e^{-3/2} \cdot \left(\frac{1}{1} + \frac{(3/2)^1}{1!} + \frac{(3/2)^2}{2!} \right)$$

$$= 1 - e^{-3/2} \left(1 + \frac{3}{2} + \frac{9}{8} \right)$$

$$= 0{,}81$$

$\mathbb{P}(T_3 > 5 \mid$

2.9

V.A $Z \rightarrow \sim N(0,1)$ $\quad \underset{F}{\Phi} Z = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$

(A) $\Phi(z) = 0, 1, 2, 3 \ldots$ SALE DE TABLA DE DISTRIBUCIÓN NORMAL

(B) $\Phi(z_\alpha) = \alpha \rightarrow$ CUANTIL $\rightarrow$ TMB TABLA

BUSCO QUE Z CUMPLE CON α

NOTA

2.5

N: PARTICULAS α EMITADAS POR SEGUNDO

$$N \sim Pois(\mu) \quad \text{DONDE } \mu = \frac{1}{2}$$

$$P_x(x) = \frac{\mu^x e^{-\mu}}{x!}$$

(A) $\mathbb{P}(N>3)$

$$\mathbb{P}(N>3) = 1 - \mathbb{P}(N \leq 3)$$
$$= 1 - (\mathbb{P}(N=3) + \mathbb{P}(N=2) + \mathbb{P}(N=1) + \mathbb{P}(N=0))$$

$$\left.\begin{array}{l} P_x(3) = \dfrac{\frac{1}{2}^3 \cdot e^{-1/2}}{3!} = \dfrac{1/8 \cdot e^{-1/2}}{6} = 0{,}0126 \\ P_x(2) = \dfrac{\frac{1}{2}^2 \cdot e^{-1/2}}{2!} = 0{,}0758 \\ P_x(1) = \dfrac{1/2 \, e^{-1/2}}{1} = 0{,}3032 \end{array}\right\} 0{,}9981$$

$$P_x(0) = 1 \cdot e^{-1/2} = 0{,}6065$$

$$\mathbb{P}(N>3) = 1{,}86 \times 10^{-3}$$

(B) $\mathbb{P}(2N+1)$

$$\mathbb{P}(2N+1) = \sum_{N=0}^{\infty} \frac{\left(\frac{1}{2}\right)^{2N+1} e^{-1/2}}{(2N+1)!} \rightarrow \text{BUSCO SERIE GEOMETRICA}$$

$$= e^{-1/2} \sum \frac{(1/2)^{2N+1}}{(2N+1)!} = 0{,}32$$

$\downarrow$

COMO RESUELVO?

NOTA

(C)

$$\mathbb{P}(-0{,}43 < z < 1.32) = \int_{-0{,}43}^{1.32} \varphi(z)\,dz$$

$$= \varphi(1{,}32) - \varphi(-0{,}43)$$

PROPIEDAD XQ NO TENGO LOS (-)

$$= \varphi(1{,}32) - \varphi(1 - \varphi(0{,}43))$$

$$= 0{,}573$$

ANALOGO CON LOS OTROS CASOS

(D)

$$\mathbb{P}(z < A) = 0{,}05$$

$$\mathbb{P}(z < A) = \varphi(A) = 0{,}05 \rightarrow$$ SE QUE A ES (-)

$$= \varphi(A) = 1 - 0{,}95 = 0{,}05$$

TENGO QUE $\varphi(-A) + \varphi(A) = 1$

$$\varphi(-A) + 0{,}05 = 1$$

$$\varphi(-A) = 0{,}95$$

$$-A = 1{,}64485$$

$$A = -1{,}64485$$

ANALOGO LOS DEMÁS

2.11

ESTO SIGNIFICA QUE HAY SALTOS DE $1/n$

FUNCIÓN DE DISTRIBU. EMPIRICA

METODO PARA ESTIMAR LA $F_X(x)$ DE UNA V.A A PARTIR DE UN CONJUNTO DE MUESTRAS.

$$F_X(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}\{x_i \leq x\}$$

$$= \frac{\#(i=1,2,\dots n / x_i \leq x)}{n}$$

(A)

ORDENO LOS VALORES DE MENOR A MAYOR $\longrightarrow x$

Y DESP ESTIMO SU $F_X(X)$ CON LA EMPIRICA $\uparrow y = F_X(x)$

EJ

CON 7,63 $\longrightarrow \frac{1}{24} \cdot 3 = \frac{3}{24} \longrightarrow \frac{\text{CASOS POSIBLES}}{\text{CASOS FAVORA}}$

ESTOY EN LA MUESTRA 3

$$\mathbb{P}(X > 9{,}5) = 1 - \mathbb{P}(X \leq 9{,}5)$$

$$= 1 - F(9{,}5)$$

$$= 1 - \left(\frac{18}{24}\right)$$

SALE DE MIRAR GRAFICO

$\downarrow$ CAE JUSTO EN LA MISMA RAYA QUE 9,44

(B)

TENGO 5 INTERVALOS

$I_1 = 7{,}1 - 7{,}85 \rightarrow F = \frac{1}{24} \cdot 2$

$I_2 = 8{,}35 - 9{,}65 = \frac{1}{24}$

$I_3 =$

$I_4 =$

$I_5 -$

$\rightarrow$ Y HAGO HISTOGRAMA.

FUNCIÓN HISTOGRAMA

$$P_i = \frac{1}{n}\sum_{j=1}^{n} \mathbb{1}\{A_{i-1} < x < A_i\}$$

$\downarrow$

CUANTAS MUESTRAS CAEN EN EL INTERVALO

NOTA

2.12

$T \to$ V.A $\quad T \sim EXP(1)$

$$F_T(T) = 1 - e^{-T} \, \mathbb{1}\{T>0\}$$

(A) HALLAR $f_{T^*}(T)$ $\qquad T^* = MIN(T, 1)$

$$T^* = \begin{cases} T & T<1 \\ 1 & T>1 \end{cases}$$

ENTONCES

$T<1 \quad T^* = T \longrightarrow \quad F_{T^*}(T) = 1 - e^{-T}$

$T \geq 1 \quad T^* = 1$

PARA $0<T<1$

$T = T^*$

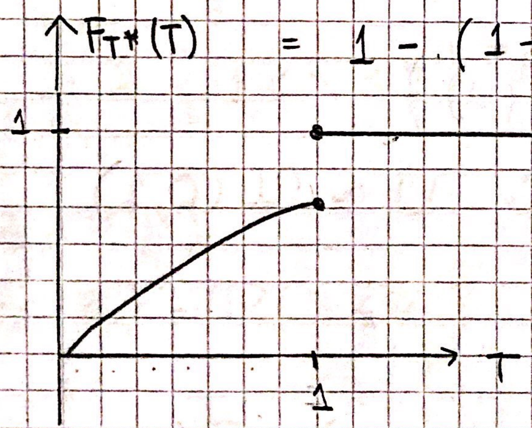
$$\mathbb{P}(T^* < T) = \mathbb{P}(T<T) = F_T(T) = F_{T^*}(T)$$

PARA $T \geq 1$

$\cancel{xa} < 1$ ES $F_{T^*} = F_T$

$$\mathbb{P}(T^* \geq 1) = 1 - \mathbb{P}(T^* < 1) = 1 - F_T(1)$$

$$= 1 - (1 - e^{-1}) = e^{-1} = 0{,}367$$



2.13

LO QUE ACUMULO

$X \to$ V.A CONTINUA CON $F_X(x) = \mathbb{P}(X \leq x)$ (E. CREC).

(A)

$T = G(X)$ COMO G ES ESTRICTAMENTE CRECIENTE Y CONTINUA $\exists\, G^{-1}: \text{IMG}(G) \to \mathbb{R}$

$F_T(t) = \mathbb{P}(T < t) = \mathbb{P}(G(X) < t)$

APLICO INVERSA DE AMBOS LADOS

$F_T(t) = \mathbb{P}(T < t) = \mathbb{P}(X \leq G^{-1}(t))$

$$\boxed{F_T(t) = F_X(G^{-1}(t))}$$

$$\begin{cases} T = G(X) \\ G^{-1}(T) = G^{-1}(G(X)) \\ G^{-1}(T) = X \end{cases}$$

(B)

$U = F_X(X)$

BUSCO EN LOS 3 CASOS QUE DE ESTE LADO QUEDE X.

$F_U(u) = \mathbb{P}(U \leq u) = \mathbb{P}(F_X(X) \leq u)$

$F_U(u) = \mathbb{P}(F_X^{-1}(F_X(X)) \leq F_X^{-1}(u))$

$F_U(u) = \mathbb{P}(X \leq F_X^{-1}(u))$

$\quad '' \quad = F_X(F_X^{-1}(u))$

$\boxed{F_U(u) = u} \quad \therefore\ U \sim \mathcal{U}(0,1)$

(C) $V = G^{-1}(F_X(X))$

$F_V(v) = \mathbb{P}(V < v) = \mathbb{P}(G^{-1}(F_X(X)) < v) = \mathbb{P}(F_X(X) < G(v))$

$F_V(v) = \mathbb{P}(F_X(X) < G(v)) = \mathbb{P}(X \leq F_X^{-1}(G(v)))$

NOTA

$F_V(v) = F_X(F_X^{-1}(G(v))) = \boxed{F_V(v) = G(v)}$

HOJA Nº 7

FECHA

2.14

$U = F_X(x) = 1 - e^{-\lambda(x)}$

POR 2.13 SABEMOS QUE $F_U(U) = U$

$X \sim EXP(2/hs)$

3 MUESTRAS
$\left.\begin{array}{l} 105 \text{ MÍN} \\ 6 \text{ MÍN} \\ 28 \text{ MÍN} \end{array}\right\}$ METO EN $F_X(x)$ Y LISTO

$U \sim U(0,1)$ $F_U(U) = X$

$X \sim U(0,1)$

2.15

$U \sim \mathcal{U}(0,1)$

HALLAR H / $H(U) = Z$

$F_Z(Z) = \frac{1}{3} \mathbb{1}\{-2 \leq x < -1\} + \frac{1}{6}x + \frac{1}{2}\mathbb{1}\{-1 < x < 1\}$

$+ \frac{2}{3}\mathbb{1}\{1 < x < 2\} + 1\,\mathbb{1}\{x \geq 2\}$

$Z = H(U)$

$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(H(U) \leq z)$

$F_Z(z) = F(U \leq H^{-1}(z))$

$F_Z(z) = F_U(H^{-1}(z))$

$F_Z(z) = F_U(H^{-1}(z)) \rightarrow$ CONOZCO

$F_U - U \sim \mathcal{U}$

NOTA

2.16

X V.A $\quad X \sim EXP(1/2)$

HALLAR H / $H(X) = Z \qquad F_Z(Z) = \left\{ \right.$

$$F_X(x) = 1 - e^{-1/2 x}$$

$$F_Z(z) = \mathbb{P}(Z \leq z) = \mathbb{P}(H(x) < z)$$

$$F_Z(z) = \mathbb{P}(X < H^{-1}(z))$$

$$F_Z(z) = F_X(H^{-1}(z))$$

$$\left(F_X^{-1}(F_Z(z))\right)^{-1} = H^{-1}(z)$$

$$F_X(F_Z(z)) = H(z) = \left(H^{-1}(z)\right)^{-1}$$

$$X \xrightarrow{F_X(X)} U \xrightarrow{F_Z^{-1}(U)} Z$$

$$Z = F_Z^{-1}(F_X(X)) = F_Z^{-1}(U)$$

NOTA

HOJA Nº 8

FECHA

2.17

T V.A → TIEMPO HASTA 1° FALLA

$$\lambda(T) = \beta/\alpha \left(T/\alpha\right)^{\beta-1} \mathbb{I}\{T>0\}$$ ↳ FUNCIÓN INTENSIDAD DE FALLAS

$$F_T(t) = 1 - e^{-\int_0^t \lambda(s)ds}$$ → USADO X EL ABUSO DE NOTACIÓN

(A)

$$F_T(t) = 1 - e^{-\beta/\alpha \int_0^t \left(T/\alpha\right)^{\beta-1} dT}$$

$$\int_0^t \left(\frac{T}{\alpha}\right)^{\beta-1} = \left(\frac{1}{\alpha}\right)^{\beta-1} \cdot \left(T\right)^{\beta-1}$$

$$-\beta/\alpha \cdot \left(\frac{1}{\alpha}\right)^{\beta-1} \int_0^t T^{\beta-1} dT$$

$$= \frac{T^\beta}{\beta}$$

$$F_T(t) = 1 - e^{\overbrace{-\beta/\alpha \left(1/\alpha\right)^{\beta-1}}^{\beta/\alpha^\beta} \cdot \frac{T^\beta}{\beta}}$$

$$F_T(t) = 1 - e^{-\frac{T^\beta}{\alpha^\beta}} = 1 - e^{-\left(T/\alpha\right)^\beta}$$

$$f_T(t) = F_T'(t)$$

$$= -e^{-(T/\alpha)^\beta} \cdot \lambda(t)$$

(B)

- $_1$ $\beta = 1$
- $_2$ $\beta > 1$
- $_3$ $\beta < 1$
- $_4$ $\beta < 1$
- $_5$ $\beta = 1$
- $_6$ $\beta = 1$

NOTA

2.18

$T \to$ INTENSIDAD DE FALLAS

$$F_T(t) = \left(1 - e^{-\sqrt{t/60}}\right) \mathbb{1}\{T>0\}$$

PRODUCTO $\to$ $\leq$ 30 DÍAS

$$\mathbb{P}(T<60 \mid T>30) = \frac{\mathbb{P}(T<60 \cap T>30)}{\mathbb{P}(T>30)}$$

$$= \frac{\mathbb{P}(30<T<60)}{\mathbb{P}(T>30)} = \frac{F(60) - F(30)}{1 - F(30)} =$$

$$= \frac{0,632 - 0,507}{1 - 0,507} = 0,254$$

2.19

$$X \to f_X(x) = \frac{2x}{225}$$

$$F_X(x) = \int_0^{15} \frac{2x}{225}\,dx = \frac{1}{225}\int_0^{15} 2x\,dx$$

$$F_X(x) = \frac{1}{225} \cdot \frac{2x^2}{2} = \frac{x^2}{225}\ \mathbb{1}\{0<x\leq 15\}$$

DESCARTE $(3 > X > 12)$

(A) A : ' ARANDELAS NO DESCARTADAS '

$(3 < X < 12)$

$$f_{X|A} = \frac{2x/225\ \mathbb{1}\{3<x<12\}}{\mathbb{P}(A)} \quad \mathbb{P}(A) \longrightarrow \mathbb{P}(3<X<12)$$

NOTA

$$f_{X|A} = \frac{2x/225 \ \mathbb{1}\{3<x<12\}}{F_X(12) - F_X(3)} = \frac{''}{\frac{16}{25} - \frac{1}{25}}$$

$$f_{X|A} = \frac{2x}{225} \cdot \frac{5}{3} \ \mathbb{1}\{3<x<12\}$$

$$f_{X|A} = \frac{10}{675} x \ \mathbb{1}\{3<x<12\}$$

(B) A: 'ARANDELAS DESCARTADAS'

$(3>x>12)$

$$f_{X|A} = \frac{2x/225 \ \mathbb{1}\{3>x>12\}}{\mathbb{P}(A)}$$

$$f_{X|A} = \frac{2x/25 \ \mathbb{1}\{3>x>12\}}{(F(3)-F(0)) + (F(15)-F(12))}$$

ó

$\left(1 - \frac{3}{5}\right) \rightarrow$ CASO CONTRARIO A (A)

$$f_{X|A}(x) = \frac{\cancel{2}x}{225} \cdot \frac{5}{\cancel{2}} \ \mathbb{1}\{3>x>12\}$$

$$f_{X|A}(x) \ \frac{x}{45} \ \mathbb{1}\{3>x>12\}$$

2.20 X EN DECIMETROS

$$f_X(x) = \frac{2}{7}\mathbb{1}\{0 \le x < 2\} + \frac{10-2x}{21}\mathbb{1}\{2 \le x < 5\}$$

$$F_X(x) = \int_0^x \frac{2}{7}\,dx = \frac{2}{7}x\,\mathbb{1}\{0 \le x < 2\}$$

$$F_X(x) = \int_0^x \frac{10}{21} - \frac{2x}{21}\,dx = \frac{10}{21}x - \frac{x^2}{21}\,\mathbb{1}\{2 \le x < 5\}$$

$$F_X(x) = \frac{2}{7}x\,\mathbb{1}\{0 \le x < 2\} + \frac{1}{21}x(10-x)\,\mathbb{1}\{2 \le x < 5\}$$

A': MENORES QUE 3 dm

$$f_{X|A} = \frac{f_{X|A}\,\mathbb{1}\{x<3\}}{\mathbb{P}(A)} \longrightarrow$$ COMO ESTA CORTADA NO PUEDO HACER $F(3) - F(0)$

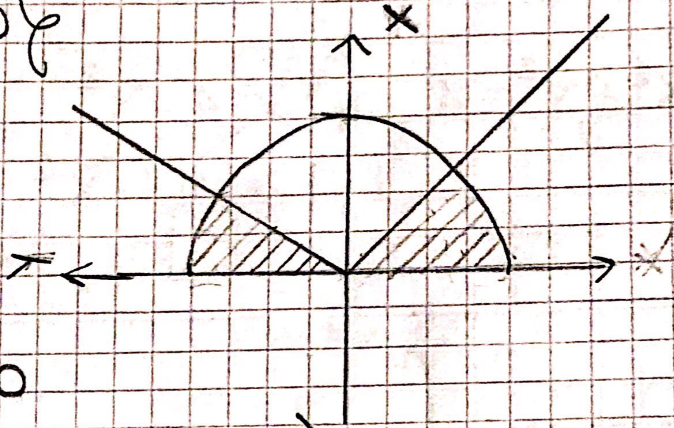
$$\mathbb{P}(A) = \left(\int_2^3 \frac{10-2x}{21}\,dx = 1 - \frac{16}{21}\right) + \int_0^2 \frac{2}{7}\,dx = \frac{4}{7}$$

$$\mathbb{P}(A) = 0{,}81$$

$$f_{X|A} = \frac{6}{17}\mathbb{1}\{0 \le x < 2\} + \frac{10-2x}{17}\mathbb{1}\{2 \le x < 3\}$$

2.22

(X,Y) UN PUNTO CON DISTRIBUCIÓN UNIFORME EN $\{x^2+y^2 \leq 4,\ x \geq 0\}$



(A) $\mathbb{P}(|Y| < X)$

$\mathbb{P}(-X < Y < X)$

COMO ES UNIFORME PUEDO USAR LAPLACE $\dfrac{\text{AREA (FAVORABLES)}}{\text{AREA (POSIBLES)}}$

$$A.P = \frac{\pi R^2}{2} = \frac{\pi \cdot 4}{2} = 2\pi$$

$A.F =$

$X = R\cos\theta$

$Y = R\,\text{SEN}\,\theta$

$$\int_0^{\pi/4}\int_0^2 R\,dR\,d\theta + \int_{3\pi/4}^{\pi}\int_0^2 R\,dR\,d\theta$$

$A.F = \pi$

$$\mathbb{P}(|Y| < X) = \frac{\pi}{2\pi} = 1/2$$

(B) DENSIDADES MARGINALES

- $F_X(x) = \int_{-\infty}^{+\infty} F_{XY}(x,y)\,dy$
- $F_Y(y) = \int_{-\infty}^{+\infty} F_{XY}(x,y)\,dx$

$$F_{XY} = \frac{1}{AREA(\Omega)} \mathbb{1}\{(x,y) \in (\Omega)\} \rightarrow \text{XQ ES UNIFORME}$$

$$F_{XY} = \frac{1}{2\pi} \mathbb{1}\{(x,y) \in x^2+y^2 \leq 4 \wedge x \geq 0\}$$

$$x^2+y^2=4 \qquad y^2 = 4-x^2$$

$$|y| = \sqrt{4-x^2} \qquad y = \sqrt{4-x^2}$$

$$y = -\sqrt{4-x^2}$$

$$F_X(x) = \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{1}{2\pi}\, dy = \frac{1}{2\pi}\left(\sqrt{4-x^2}+\sqrt{4-x^2}\right)$$

$$F_X(x) = \frac{1}{\cancel{2}\pi}\cancel{2}\sqrt{4-x^2} =$$

- $F_X(x) = \frac{1}{\pi}\sqrt{4-x^2}\ \mathbb{1}\{0 \leq x \leq 2\}$

$$F_Y(y) = \int_0^{\sqrt{4-y^2}} \frac{1}{2\pi}\, dx = \frac{1}{2\pi}\sqrt{4-y^2}$$

- $F_Y(y) = \frac{1}{2\pi}\sqrt{4-y^2}\ \mathbb{1}\{-2 < y < 2\}$

(c)

(1) $SOP(F_X) = [0,2]$

$SOP(F_Y) = [-2,2]$ $\Big\}$ $[0,2].[-2,2] = [0,4]$

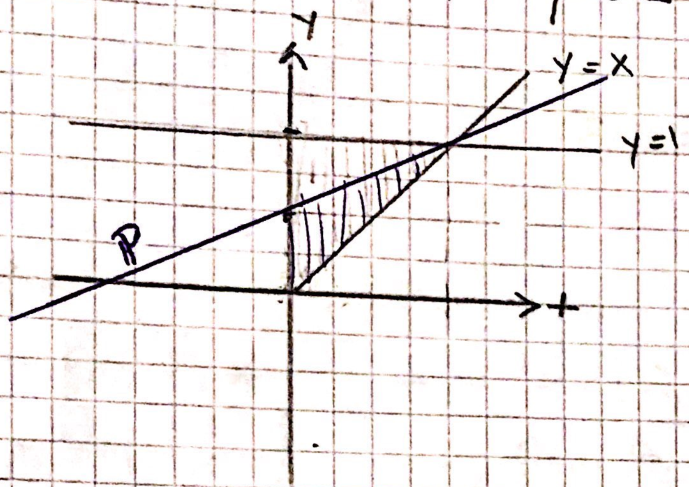
$SOP(F_{XY}) = D \neq [0,4]$ NO SON INDEP.

NOTA

2.23

X e Y V.A

$$F_{XY}(x,y) = 8xy \, \mathbb{1}\{0 \le x \le y \le 1\}$$



(A) $\mathbb{P}(X+1 > 2Y) = \mathbb{P}\left(\frac{X+1}{2} > Y\right) = \left(\frac{1}{2}X + \frac{1}{2} > Y\right)$

$$\mathbb{P}\left(Y < \frac{1}{2}X + \frac{1}{2}\right) = \int_0^1 \int_x^{\frac{x+1}{2}} 8xy \, dy \, dx =$$

$$\int_0^1 \frac{8}{2} x y^2 \Big|_x^{\frac{x+1}{2}} dx = \int_0^1 4x\left(\left(\frac{x+1}{2}\right)^2 - x^2\right) dx$$

$$\int 4x\left(\frac{x^2+2x+1}{4} - x^2\right) dx = x^3 + 2x^2 + x - 4x^3$$

$$\int_0^1 -3x^3 + 2x^2 + x \, dx = -\frac{3}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 \Big|_0^1$$

$$\mathbb{P}\left(Y < \frac{1}{2}X + \frac{1}{2}\right) = \frac{5}{12}$$

(B) • $F_X(x) = \int_x^1 8xy \, dy$ • $F_Y(y) = \int_0^y 8xy \, dx$

NOTA

DIBUJO ORIGINAL LOS LIMITES.

2.24

2.25

ROLLOS DE TELA X: FALLAS TEJIDO $\sim Poi(2)$

Y: FALLAS TEÑIDO $\sim Poi(4)$

X e Y SON INDEP.

(A) P(ROLLO NO TENGA FALLAS)

$$P_X(x) = \frac{\mu^x e^{-\mu}}{x!}$$

(A) P(NO TENGA FALLAS)

$$P_X(x) = \frac{2^0 \cdot e^{-2}}{0!} = e^{-2} = 0{,}14$$

$$P_Y(y) = \frac{4^0 \cdot e^{-4}}{0!} = e^{-4} = 0{,}018$$

$$\mathbb{P} = 2{,}52 \times 10^{-3}$$

(B) P(TELA TENGA EXACTAMENTE UNA FALLA

$$P_X(0) = \frac{2^0 \cdot e^{-2}}{0!} = e^{-2} =$$

$$P_Y(1) = \frac{4 \cdot e^{-4}}{1!} = 4e^{-4} =$$

$$4e^{-6}$$

$$+$$

ó (+)

$$P_X(1) = \frac{2^1 \cdot e^{-2}}{1!} = 2e^{-2} =$$

$$P_Y(0) = \frac{4^0 \cdot e^{-4}}{0!} = e^{-4} =$$

$$2e^{-6}$$

$$\mathbb{P} = 6e^{-6} = 0{,}015$$

NOTA

(C)

$$\mathbb{P}(X=1 \mid X+Y=1) =$$

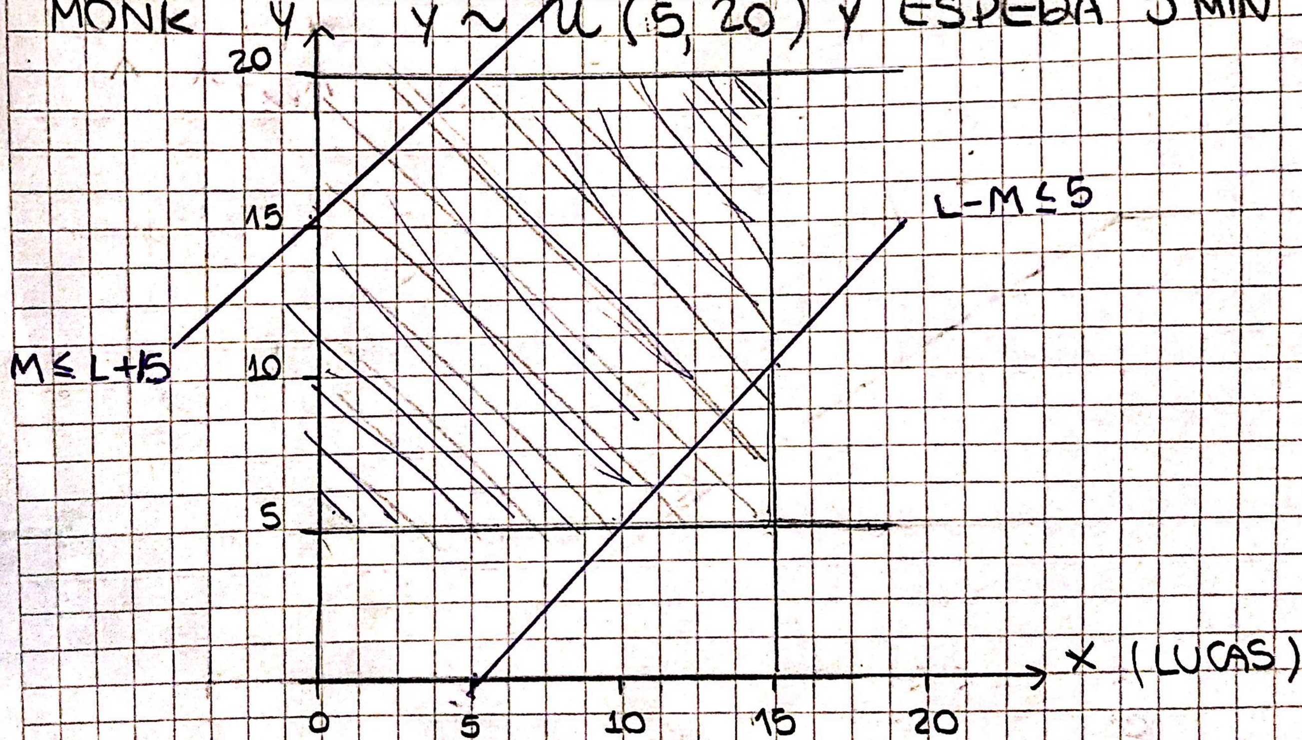
$$\frac{\mathbb{P}(X=1 \cap X+Y=1)}{\mathbb{P}(X+Y=1)} = \frac{\mathbb{P}(X=1)\,\mathbb{P}(Y=0)}{\mathbb{P}(X+Y=1)}$$

$$= \frac{2e^{-2} \cdot e^{-4}}{6e^{-6}} = \frac{1}{3}$$

2.26

LUCAS : X $\quad X \sim \mathcal{U}(0,15)$ Y ESPERA 15 MIN

MONK $\quad$ Y $\quad Y \sim \mathcal{U}(5,20)$ Y ESPERA 5 MIN



CALCULO EL AREA DE LOS △ Y LOS RESTO

$$1 - \left(\frac{1}{225} \cdot 25\right) = 0{,}88$$

AMBOS TRIANGULOS SON =
FORMAN UN CUADRADO