9.4 Mostrar que las siguientes familias de distribuciones son familias exponenciales a 1 parámetro: (**a**) Bernoulli(p); (**b**) Pascal($4, p$); (**c**) Poisson(λ); (**d**) Exponencial(λ).

(a)

$X \sim Ber(p)$

$$f_p(x) = p^x (1-p)^{1-x} \mathbb{1}\{x \in \{0,1\}\}$$

$$= (1-p) \left(\frac{p}{1-p}\right)^x \mathbb{1}\{x \in \{0,1\}\}$$

$$= \underbrace{(1-p)}_{A(p)} e^{x \ln\left(\frac{p}{1-p}\right)} \underbrace{\mathbb{1}\{x \in \{0,1\}\}}_{h(x)}$$

$$C(p) = \ln\left(\frac{p}{1-p}\right)$$

$$r(x) = x$$

$$\Longrightarrow \boxed{\text{Es familia Exponencial}}$$

(b)

$X \sim Par(4, \theta)$

$$f_\theta = \frac{\theta 4^\theta}{x^{\theta+1}} \mathbb{1}\{x>4\}, \quad \theta > 0$$

$$= \underbrace{\theta 4^\theta}_{A(\theta)} e^{-(\theta+1)\ln x} \underbrace{\mathbb{1}\{x>4\}}_{h(x)}$$

$C(\theta) = -\theta - 1$

$r(x) = \ln x$

$\Longrightarrow$ $\boxed{\text{Es familia Exponencial}}$

(d)

$$X \sim \mathcal{E}(\lambda) \Rightarrow f_\lambda(x) = \underbrace{\lambda}_{A(\lambda)} e^{-\lambda x} \overbrace{\mathbb{1}\{x>0\}}^{h(x)}, \quad K=1$$

$C(\lambda) = -\lambda$, $r_1(x) = x$ ó $C(\lambda) = \lambda$, $r_1(x) = -x$ ó $C(\lambda) = 2\lambda$, $r_1(x) = \frac{-x}{2}$

$\Longrightarrow$ $\boxed{\text{Es familia Exponencial}}$

(c)

$X \sim \mathcal{P}oi(\mu)$

$$f_\mu(x) = \frac{\mu^x}{x!} e^{-\mu} \mathbb{1}\{x \in \mathbb{N}_0\} = \underbrace{\frac{1}{x!} \mathbb{1}\{x \in \mathbb{N}_0\}}_{h(x)} e^{\overbrace{x \log \mu}} \underbrace{e^{-\mu}}_{A(\mu)}$$

$c(\mu) = \log \mu$

$r(x) = x$

$\Longrightarrow$ $\boxed{\text{Es familia Exponencial}}$