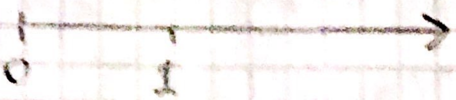
GUÍA - 7

7.1

$\{N(T), T \geq 0\}$ PROCESO DE CONTEO $\sim$ Poi (2)

$N(A,B) = N(B) - N(A)$ INCREMENTO DEL PROCESO.

A.



HAY CERO MARCAS

$N(T) \sim Poi(2T) \quad \mu = 2 \quad T = 1$

$$\mathbb{P}(\underbrace{N(1)}_{X} = 0)$$

$$P_X(0) = \frac{2^0 e^{-2}}{0!} = e^{-2}$$

B. $\mathbb{P}(N(1,2) = 1)$

$N(1,2) = N(1)$

$\mathbb{P}(N(1) = 1) \quad N(1) \sim Poi(2.1)$

$$P_X(1) = \frac{2^1 \; e^{-2}}{1!} = 2e^{-2}$$

C. $\mathbb{P}(N(1) = 0, N(1,2) = 1, N(2,4) = 2)$

$\mathbb{P}(N(1) = 0 \;\; N(1) = 1 \;\; N(2) = 2)$

2.2

$$P_X(2) = \frac{(2.1)^2 \; e^{-2}}{2!} = 2e^{-2}$$

$N(2) \sim P(4)$

$2T$

$\mathbb{P}(N(1)=0,\ N(1)=1)\ N(2)=2) =$ COMO SON INDEP

$\mathbb{P}(N(1)=0)\ \mathbb{P}(N(1)=1)\ \mathbb{P}(N(2)=2)$

$$e^{-2} \cdot 2e^{-2} \cdot 8e^{-4} = 16e^{-8}$$

D- $COV(N(1,3), N(2,4)) =$

$$COV\left[(N(1,2)+N(2,3)), (COV(N(2,3)), N(3,4))\right]$$

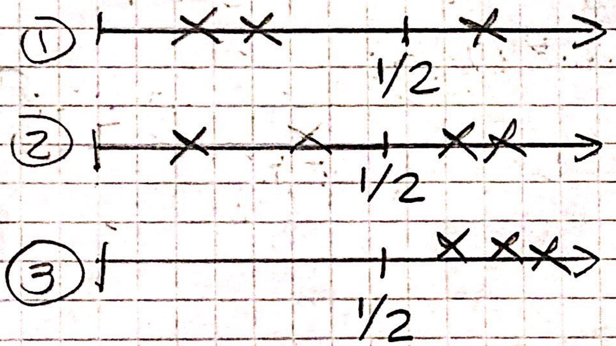
$$COV(N(1,2), N(2,4)) + COV(N(1,2), N(3,4))$$

$$COV(N(2,3), N(2,3)) = VA(N(2,3))$$

$$VA(N(1)) = \mu \cdot T = 2$$

S_3 TIEMPO DE ESPERA HASTA EXITO 3.

E-



LA SUMA ES $\mathbb{P}(S_3 > 1/2)$

① $N(1/2)=2$ + ② $N(1/2)=1$ + $N(1/2)=0$

$$e^{-1} + e^{-1} + \frac{1}{2}e^{-1} = \frac{5}{2}e^{-1}$$

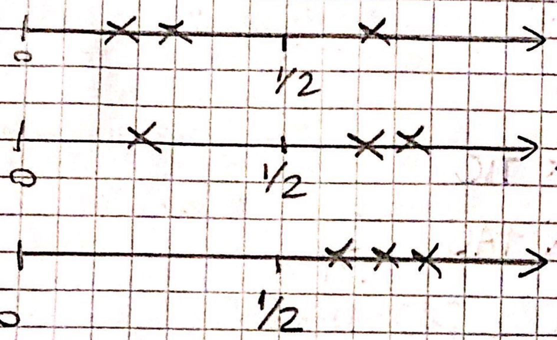
7.2

RECORDAR QUE $\sim Poi(2T)$

X = "# ACCIDENTES"

$X \sim Poi(\lambda)$ $\quad \lambda = \dfrac{2 \text{ ACCID}}{1 \text{ hs}}$

A- $\mathbb{P}(X > 1/2)$



$N(1/2) = 2$

$N(1/2) = 1$

$N(1/2) = 0$

$\mathbb{P}(N(1/2) > 3)$ LA SUMA.

B-

$N(1,2) = N(1) = 1$

$$\mathbb{P}(N(1) = 1) = \frac{(1.2)^1 \, e^{-(1.2)}}{1!} = 2e^{-2}$$

7.3

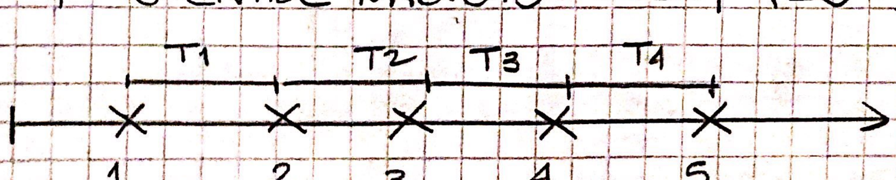
$X(T)$: "# PARTICULAS EMITIDAS"

$X \sim Poi(10T)$ $\quad \lambda = \dfrac{10 \text{ PART}}{1 \text{ hs}}$ $\qquad 2 \text{ MIN} = \dfrac{1}{30}$

$\downarrow$

EMISOR

T: t ENTRE MARCAS $\sim EXP\left(10 \dfrac{\text{PAR}}{\text{hs}}\right)$



$\mathbb{P}\left(T_1 > \frac{1}{30}, T_2 > \frac{1}{30}, T_3 > \frac{1}{30}, T_4 > \frac{1}{30}\right)$

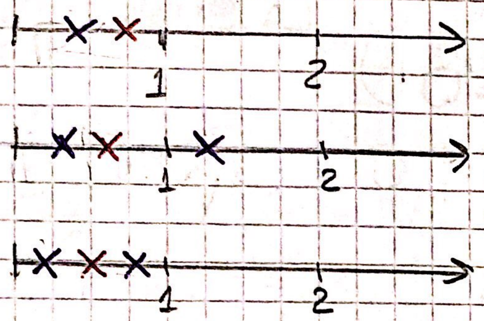
COMO SON INTERVALOS ≠ → SON DISJUNTOS

$$5 \cdot \mathbb{P}\left(T > \frac{1}{30}\right) = 5 \cdot \left(1 - \mathbb{P}\left(T \leq \frac{1}{30}\right)\right)$$

$$= 5e^{-10\frac{1}{30}} = 0,2636$$

7.4

$X \sim Poi(2T)$ $\lambda = \frac{2 \text{ PART}}{1 \text{ SEG}}$



× TIC

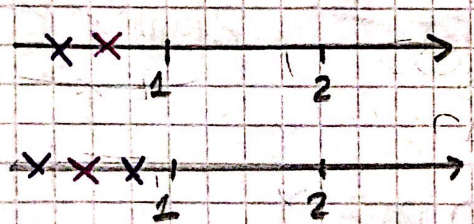
× TAC

3 CASOS POSIBLES

$$\mathbb{P}(N(1,2) = 0 \mid N(0,1) = 1)$$

$$\mathbb{P}(N(0,1) = 2, N(2,1) = 0) + \mathbb{P}(N(0,1) = 2, N(1,2) = 1)$$
$$+ \mathbb{P}(N(0,1) = 3, N(1,2) = 0)$$

SOLO LOS CASOS EN LOS QUE $N(0,1) = 1$



$$\mathbb{P}(N(0,1) = 2) + \mathbb{P}(N(0,1) = 3)$$

$$= 0,2077$$

NOTA

HOJA N°

FECHA

7.5

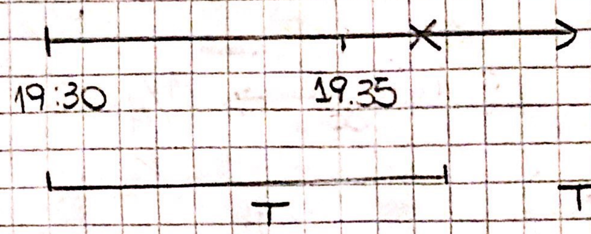
COLECTIVO LLEGA $\sim$ POI $\left(\frac{12\,C}{hs}\right)$

A-

1 | 60 MIN | 1 hs

5 MIN | X

$\downarrow$ $1/12$



$T \sim EXP(12)$

$$\mathbb{P}\left(T > \frac{1}{12}\right) = 1 - \mathbb{P}\left(T \leq \frac{1}{12}\right)$$
$$= 1 - (1 - e^{-1})$$
$$= e^{-1}$$

B- ES LO MISMO QUE (A)

C- SIGUE SIENDO LO MISMO PORQUE NO INTERESA EL T INICIAL SE CUENTA 5 DESDE AHÍ.

D-

LLEGADA 19:00 + U $\qquad U \sim \mathcal{U}(0,60)$

$$f_U(u) = \frac{1}{60}\,\mathbb{I}\{0 < u < 60\}$$

7.6

EL PESO DE BOLSAS = X $\qquad X \sim EXP(1/3)$

SE AGREGA HASTA 5 kg.

$$\mathbb{P}(S_N > 7 \mid S_N > 5) = \mathbb{P}(5 + Y > 7) = \mathbb{P}(Y > 2)$$

$$\mathbb{P}(Y > 2) = 1 - \mathbb{P}(Y \leq 2) = e^{-2/3}$$

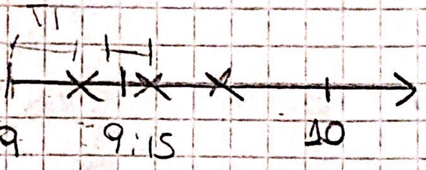
B- TODAS LAS BOLSAS TIENEN LA MISMA MEDIA = 3

NOTA

7.7

X = '# LLAMADAS QUE ARRIBAN'

$X \sim Poi(4)$ $\qquad \lambda = \frac{4 \text{ LLAMADAS}}{1 \text{ hs}}$



T = 'TIEMPO ENTRE LLAMADAS'.

$T \sim EXP(4)$

A -

15 MIN $\longrightarrow$ 1/4 hs

$\mathbb{P}(N(1/4) \geq 1 \mid N(0,1) = 3)$

$\mathbb{P}(N(1/4) \geq 1 \mid N(1) = 3) =$

↑ FAVORABLE   NO VIOLETA

$$\frac{\mathbb{P}(N(1/4) \geq 1) \cdot \mathbb{P}(N(1/4, 1) \leq 2)}{\mathbb{P}(N(1) = 3)}$$

SEPARO EL INTERVALO PARA QUE SEAN INDEP

$$\left.\begin{array}{l} X = (N(1/4) \mid N(1) = 3) \\ X \sim Bin\left(3, \frac{1/4}{1}\right) \end{array}\right\} \text{PROPIEDAD!!}$$

XQ ES UNIFORME

$\mathbb{P}(X \geq 1) = 1 - \mathbb{P}(X < 1) = 1 - P_X(0)$

$= 1 - \binom{3}{0}\left(\frac{1}{4}\right)^0 (1 - 1/4)^3 = 0,58$

B- $\mathbb{P}(N(0, \frac{1}{2}) \geq 2 \mid N(0,1) = 3)$

$$\frac{\mathbb{P}(N(1/2) \geq 2) \cdot \mathbb{P}(N(1) \leq 1}{\mathbb{P}(N(1) = 3)}$$

$N(1/2) \mid N(1) = 3 = Y \qquad Y \sim Bin(3, \frac{1}{2})$

NOTA

$\mathbb{P}(Y \geq 2) = 1 - \mathbb{P}(Y < 2) = 1/2$

FECHA

7.8

X: "# DE PERSONAS QUE LLEGAN A LA ESTACIÓN"

$X \sim Poi(50)$ $\quad \lambda = \frac{50\,P}{1\,MIN}$

- T: "# TOTAL DE TIEMPO QUE ESPERARON LOS P"
- $T = \sum_{1}^{N(0,15)} T_i$ SIENDO T_i LO QUE ESPERO CADA PASAJERO.

$$\boxed{T = \sum_{1}^{N(15)} T_i}$$ PROCESO DE POISON COMPUESTO

S_i = INSTANTE DE ARRIBO DE CADA PASAJERO

$$T = \sum_{1}^{N(15)} (15 - S_i) \qquad T_i = 15 - S_i$$

$$T_i = 15 - S_i \mid N(0,15) = n \sim \mathcal{U}(0,15)$$

$$\mathbb{E}[T] = \mathbb{E}[\underbrace{\mathbb{E}[T \mid N(0,15)]}\,]$$

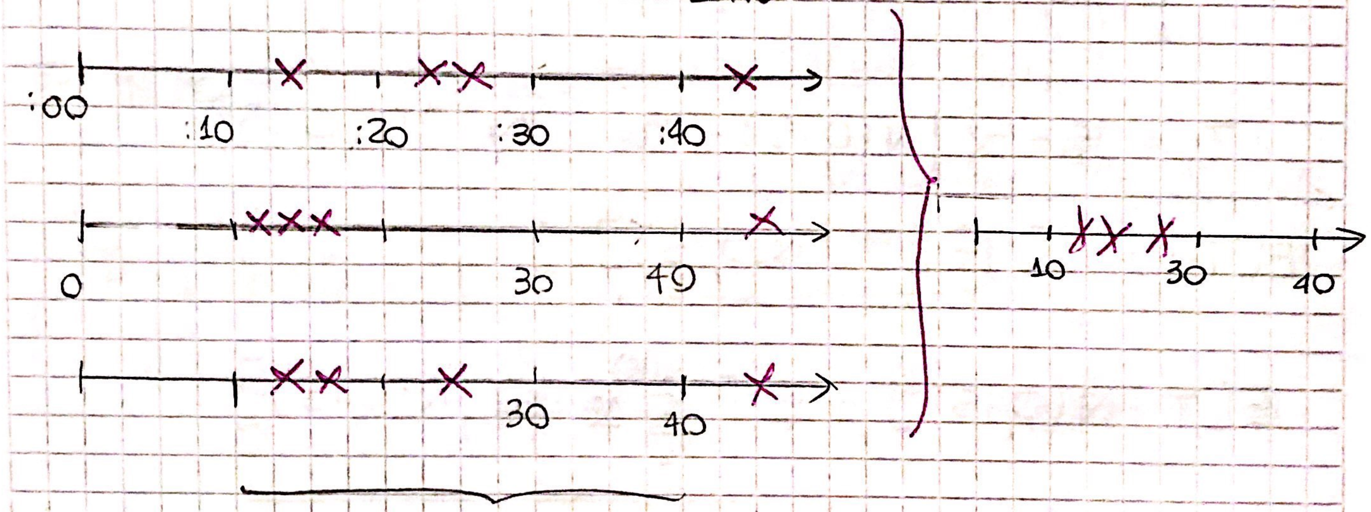
$$\mathbb{E}[T \mid N(0,15)] = \mathbb{E}\underbrace{\left[\sum_{1}^{N(15)} 15 - S_i \,\middle|\, N(0,15) = n\right]}_{\mathcal{U}(0,15)}$$

NOTA

7.9

X = '# EMISION DE PARTICULAS'

$X \sim Poi(\lambda)$ CON $\lambda = \frac{6p}{1hs}$



EN VERDAD YO SE QUE DE 10 - 40 HAY 3

$\mathbb{P}(N(0,10) = 0 \cap N(30,40) = 0 \mid N(0,30) = 3)$

$\mathbb{P}(N(0,10) = 0 \cap N(10,30) = 3 \cap N(30,40) = 0 \mid N(0,30) = 3$

HOJA N°

FECHA

$$\mathbb{P}(N(0,10)=0 \cap N(30,40)=0 \mid N(0,30)=3)$$

$\downarrow$

$$\underbrace{\mathbb{P}(N(0,10)=0 \mid N(0,30)=3)}_{\text{NO ES INDEP, ENTONCES LO HAGO}} \cap \underbrace{(N(30,40)=0 \mid N(0,30)=3)}_{\text{YA ES INDEP ENTONCES}}$$

$$\frac{\mathbb{P}(N(0,10)=0)\,\mathbb{P}(N(10,30)=3)}{\mathbb{P}(N(0,30)=3)} \cdot \mathbb{P}(N(30,40)=0)$$

(10 - 0) INTERVALO EXITO

$$Z \sim BIN\left(3, \frac{10}{30}\right)$$

TOTAL

$$\mathbb{P}(Z=0) \cdot \mathbb{P}(N(10)=0)$$

$\hookrightarrow \sim Poi\left(10 \cdot \frac{1}{10}\right)$ $\frac{6}{1ns}$

$$\mathbb{P}(Z=3) \cdot \mathbb{P}(N(10)=0) \qquad Z \sim BIN\left(3, \frac{20}{30}\right)$$

$$\mathbb{P}(\text{FINAL}) = \frac{8}{27} \cdot e^{-1} = 0{,}109$$

NOTA

7.10

X = FALLAS DEL ALAMBRE

$X \sim \text{POI}\left(\frac{1F}{20m}\right)$

A-

MAQUINA DETECTA → UNA FALLA → $P_F = 0{,}75$

↳ CORTA EL ALAMBRE EN LA 1 FALLA

CORTA

NO CORTA

$\Pi_1 = \{\text{CORTA}\} \sim \text{POI}\left(0{,}75 \cdot \frac{1}{20}\right)$

$\Pi_2 = \{\text{NO CORTA}\} \sim \text{POI}\left(0{,}25 \cdot \frac{1}{20}\right)$

LONGITUD → METROS HASTA EL PRIMER CORTE

$$\text{LONGITUD} \sim \text{EXP}\left(0{,}75 \, \frac{1}{20}\right)$$

$$E[L] = \frac{1}{0{,}75} \cdot 20 = \frac{80}{3}$$

$$\text{VAR}(L) = \frac{1}{\lambda^2} = \frac{6400}{9}$$

B-

$E[\text{FALLAS EN UN ROLLO}] = E[\Pi_2$

NOTA