guía (4)

OBS

$P(|x| \leq y) = P(-y < x \leq y)$
$= F_x(y) - F_x(-y)$

Función de V.A. "cambio de variable"

- sea X una V.A / $X: \Omega \to \mathbb{R}$

$Im(x) = R(x) = X(\Omega) \subset \mathbb{R}$

- sea $g: D \subset \mathbb{R} \to \mathbb{R}$ tal que $R(x) \subset D$.

$y = g(x)$.

IMP: ver el soporte de la densidad q' te den de dato.

* método de eventos equivalentes

- $F_Y(y) = P(Y \leq y) = P(g(x) \leq y) = P(X \leq g^{-1}(y))$

$$F_Y(y) = F_X(g^{-1}(y))$$

$g^{-1}(y) = g(x)$

si quiero $f_Y(y) \to$ derivo

teorema

↳ si X va abs continua, y $Y = g(X)$ con g con derivada no nula → Y abs cont.

$$f_Y(y) = \frac{f_X(x)}{|g'(x)|}\Big|_{x=g^{-1}(y)}$$

g debe ser inyectiva (estricta creciente o decreciente).

Si es inyectiva localmente → parto g en funciones → $f_Y(y) = \sum_{i=1}^{k} \frac{f_X(x)}{|g'(x)|}\Big|_{g^{-1}(y)=x}$

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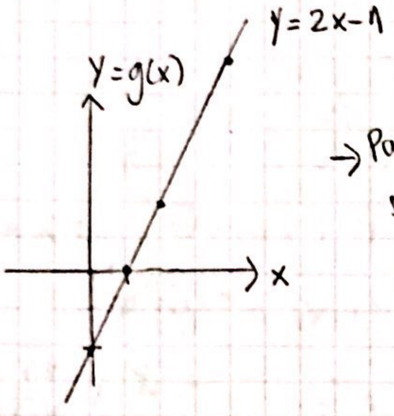
GUÍA 4

(4.1) X, v.a discreta valores $\left\{ \frac{k}{8} : k = 0,1, \dots 8 \right\}$

$\mathbb{A}_X = R(X) = \left\{ \frac{k}{8} : k = 0, \; ,8 \right\}$

$P_X(x) = \frac{1}{36} x$.

a) $Y = 2X - 1$



→ Para c/ x hay un y

X. va **DISCRETA**

$R(X)$ → rango X: conj de átomos

$Y = g(X)$ v.a discreta

$R(Y) = \{ y = g(x), \; x \in R(X) \}$

$\boxed{P_Y(y)} = P(Y=y) = P(g(X)=y) =$

$\sum\limits_{\forall x / y = g(x)} P_X(x) =$

$g(x) = 2x - 1$

$P_Y(2x-1=y) = \sum P_X\left(\frac{y+1}{2} = x\right)$

lo pongo en función de x, que es la que tengo el dato

$\mathbb{A}_Y = g(\mathbb{A}_X) = \left\{ 2 \cdot \frac{k}{8} - 1, \; k = 0, \dots, 8 \right\}$

$2x - 1 = y$
$x = \frac{k}{8}$

$= \{ -1, -3/4, \dots, 1 \}$

los valores q' toma Y

si x tiene un solo valor no hace falta $\sum$

ej: hago $\sum$ cuando $y = 1 \to y = x^2 \to x = 1, x = -1$

$P_Y(y) = \frac{1}{36} \left(\frac{y+1}{2}\right)$

$$P_Y(y) = \begin{cases} 1/36 & y = 1 \to k = 8 \\ 0 & y = -1 \to k = 0 \\ 1/288 & y = -3/4 \to k = 1 \\ 1/144 & y = -1/2 \to k = 2 \\ 1/96 & y = -1/4 \to k = 3 \\ 1/72 & y = 0 \to k = 4 \\ 5/288 & y = 1/4 \to k = 5 \\ 3/144 & y = 1/2 \to k = 6 \\ 7/288 & y = 3/4 \to k = 7 \end{cases}$$

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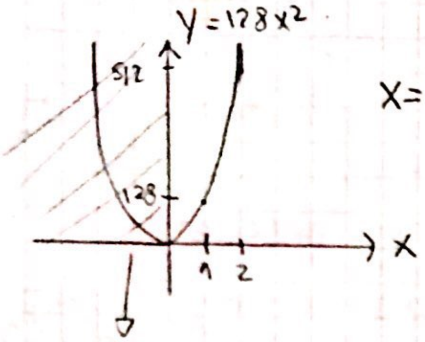
b) $Y = 128\,x^2$



$X = \frac{k}{8}$

Para c/ Y hay 2 X → solo tomo der, sigue siendo inyec.

$X = \left\{ 0, \frac{1}{8}, \frac{2}{8}, \dots, 1 \right\}$

$Y = \{0, 2, 8, 18, 32, 50, 72, 98, 128\}$

como me da los valores de $X > 0$, solo tomo lado derecho

$P(Y=y) = P(Y = 128x^2) = \sum P_X\left(x = \pm\sqrt{\frac{y}{128}}\right)$

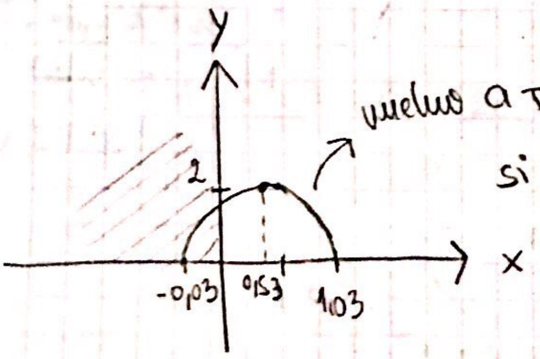
$= \sum \frac{1}{36}\left(\mp \frac{\sqrt{y}}{128}\right) = P\left(x = \cancel{-\frac{\sqrt{y}}{\sqrt{128}}} \cup x = +\frac{\sqrt{y}}{\sqrt{128}}\right)$

descarto

$= P\left(\frac{\sqrt{y}}{\sqrt{128}}\right)$

$$P(Y=y) = \begin{cases} 0 & y = 0 \\ 1/288 & y = 2 \\ 1/144 & y = 8 \\ 1/96 & y = 18 \\ 1/72 & y = 32 \\ 5/288 & y = 50 \\ 1/48 & y = 72 \\ 7/288 & y = 98 \\ 1/36 & y = 128 \end{cases}$$

c) $Y = -64x^2 + 64x + 2 \rightarrow x = \frac{k}{8} \rightarrow 0 < k < 1$



vuelvo a tomar la parte >0, pero ace si en algunos partes hay 2 valores de X para cada y.

max

$x = 0$, $y = 2$	$x = \frac{1}{8}$, $y = 9$	$x = \frac{2}{8}$, $y = 14$	$x = \frac{3}{8}$, $y = 17$	$x = \frac{4}{8}$, $y = 18$
$x = \frac{5}{8}$, $y = 17$	$x = \frac{6}{8}$, $y = 14$	$x = \frac{7}{8}$, $y = 9$	$x = \frac{8}{8}$, $y = 2$	

$A_y = \{2, 9, 14, 17, 18\}$

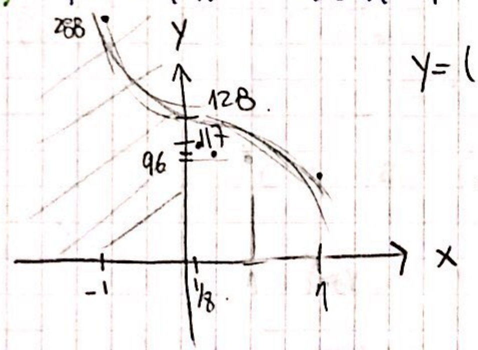
↳ los q' se repiten una vez más.

$$P_Y(y) = \frac{1}{36}(x) \qquad X = \frac{\left(-64 + \sqrt{64^2 + 4 \cdot 64(2-y)}\right)}{2(-64)}$$

$$P_Y(y) = \begin{cases} P_X(0) + P_X(1) & Y = 2 \\ P_X(1/8) + P_X(7/8) & Y = 9 \\ P_X(2/8) + P_X(6/8) & Y = 14 \\ P_X(3/8) + P_X(5/8) & Y = 17 \\ P_X(4/8) & Y = 18. \end{cases}$$

$$\underset{\substack{x=0 \\ y=2}}{P_X(0)} + \underset{\substack{x=1 \\ y=9}}{P_X(1)} = \frac{1}{36} \cdot 0 + \frac{1}{36} \cdot \frac{1}{8} = \frac{1}{288} \ \ldots$$ sigo con los cuentos

d) $Y = 64x^2 - 96x + 128 \qquad X = \left(0, \frac{1}{8}, \frac{2}{8}, \ldots, \frac{8}{8}\right)$



$Y = (8x-6)^2 + 92$

$\boxed{1X \ \forall \ Y.}$

$X = \frac{1}{8} \rightarrow Y = 117.$

$$P_Y(y) = P\left(X = \left[\sqrt{Y-92} + 6\right] \cdot \frac{1}{8}\right)$$

$$P_Y(y) = \frac{1}{36}(x)$$

$$P_Y(y) = \begin{cases} 1/128 & Y = 117 \rightarrow x = 1/8 \\ 1/24 & Y = 128 \rightarrow x = 0. \\ 1/36 & Y = 96 \rightarrow x = 1 \end{cases} \ldots$$

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discreta

4.2 $X \sim P(2)$ $\quad Y = \left|\operatorname{sen}\left(\frac{1}{2}\pi x\right)\right| = \begin{cases} 0 & x = \underbrace{2m}_{\text{par}} \\ 1 & x = \underbrace{2m+1}_{\text{impar}} \end{cases}$

$$\text{Poison} = \frac{(\mu^x e^{-\mu})}{x!}$$

↓ tengo 2 opciones
↳ par o impar.

$$P_Y(y=0) = P(x=2m) = \sum_{m=0}^{\infty} P(x=2m) = \sum_{m=0}^{\infty} \frac{2^{2m} e^{-2}}{(2m)!}$$

$$= e^{-2}\cosh(2)$$

$$P(y=1) = P(x=2m+1) = \sum_{m=0}^{\infty} P(x=2m+1) =$$

$$\sum_{m=0}^{\infty} \frac{2^{2m+1} e^{-2}}{(2m+1)!} = e^{-2}\operatorname{senh}(2)$$

$$P_Y(y) = \begin{cases} e^{-2}\cosh(2) & \text{si } y=0 \\ e^{-2}\operatorname{senh}(2) & \text{si } y=1 \end{cases}$$

---

si X. va abs continua.

ej: si $Y = X^2$.

$$F_Y(y) = P(Y \leq y) = P(g(x) \leq y) =$$

$$= P(x^2 \leq y) = P(x \leq \sqrt{y}) =$$

con $1 \leq y \leq 5$.

$$= F_X(\sqrt{y}) =$$

si quiero densidad derivo $\quad$ o $\quad f_Y(y) = \frac{f_X(x)}{g'(x)}$

con $x = g^{-1}(y)$

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(4.3) Va. continua

$$f_X(x) = \frac{12x}{\pi^2 (e^x + 1)} \, \mathbb{1}\{x > 0\}$$

OBS:

$$y = ax + b$$

$$f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right)$$

a) $y = ax + b \quad (a \neq 0,\ b \in \mathbb{R})$

si usara el de equivalentes → muy complicado → $F_X = \int_{\mathbb{R}^2} f_X \, dx \ldots$

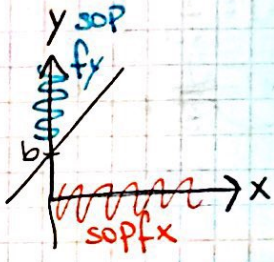
$$f_Y(y) = \frac{f_X(x)}{g'(x)} = \frac{1}{a} f_X\left(\frac{y-b}{a}\right)$$

OBS:

$$f_Y(y) = \left.\frac{f_X(x)}{g'(x)}\right|_{x = g^{-1}(y)} = \left. f_X(x)\right|_{x=g^{-1}(y)} \cdot \left(\frac{dg^{-1}(y)}{dy}\right)$$

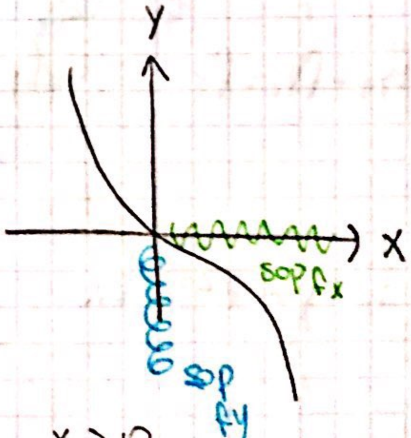
$$y = g(x) = ax + b$$

$$g'(x) = a$$



$$f_Y(y) = \frac{1}{a}\left[\frac{12\left(\frac{y-b}{a}\right)}{\pi^2\left(e^{\left(\frac{y-b}{a}\right)} + 1\right)}\right] \mathbb{1}\Big\{\underbrace{\frac{y-b}{a} > 0}_{\boxed{y > b}}\Big\}$$

b) $y = -x^3 \rightarrow x = \sqrt[3]{-y}$



$$f_Y(y) = \frac{1}{-3(-y)^{2/3}}\left[\frac{12(\sqrt[3]{-y})}{\pi^2\left(e^{(\sqrt[3]{-y})} + 1\right)}\right] \mathbb{1}\{y < 0\}$$

$g(x) = -x^3$

$g'(x) = -3x^2$
$= -3(-y)^{2/3}$

OBS!

si $x \in \mathbb{R}^2$, tendría que partir a y en 2

pq $x < 0 \rightarrow \sqrt[3]{y}$

$x > 0 \rightarrow \sqrt[3]{-y}$

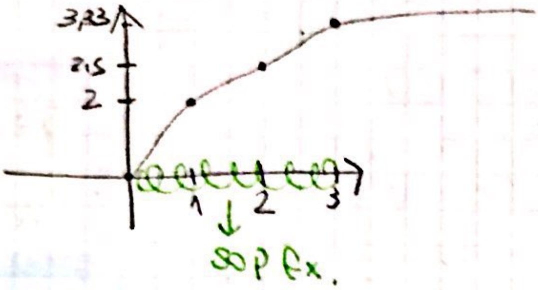
$x > 0$

$\sqrt[3]{-y} > 0$

$-y > 0$

$\boxed{y < 0}$

c) $y = x + x^{-1} = x + \frac{1}{x} = \frac{x^2+1}{x} =$



$xy = x^2 + 1$

$0 = x^2 - xy + 1$

pongo a x en función de y

$x_1 = \frac{y + \sqrt{y^2-4}}{2}$

$x_2 = \frac{y - \sqrt{y^2-4}}{2}$

$y' = g'(x) = \frac{dy(x)}{dx}$

$x = g^{-1}(y)$

$x' = g^{-1\prime}(y) = \frac{dx(y)}{dy}$

$g'(x) = \frac{1}{g^{-1\prime}(y)}$

$x = \frac{y}{2} \pm \frac{\sqrt{y^2-4}}{2}$

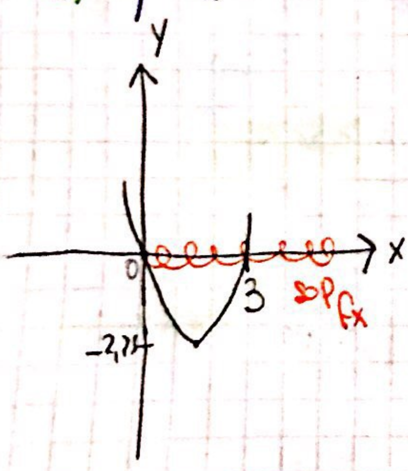
$\frac{dx(y)}{dy} = \frac{1}{2} \pm \frac{1}{4}(y^2-4)^{-1/2} \cdot 2y$

porque tengo 2 derivadas

↑

$f_{y_1} + f_{y_2} = f_y$. $\left[f_y(y) = f_x \cdot \frac{dx(y)}{dy} \right]$ como siempre...

d) $y = x^2 - 3x$ $\quad 0 = x^2 - 3x - y$ $\quad \frac{-b \pm \sqrt{b^2-4ac}}{2a}$



$\frac{3 \pm \sqrt{9 - 4(-y)}}{2} = \frac{3 \pm \sqrt{9+4y}}{2} \begin{cases} x_1 \\ x_2 \end{cases}$

$g_1' = \ldots \quad g_2' = \ldots$

$f_y = f_{y_1} + f_{y_2}$

con g_1 f_{x_1} — con g_2' f_{x_2}.

OBS

Ángulo de x y tango θ x

X es mi Y de siempre

θ es mi X de siempre

(4.4) $X \sim U(-\pi/2, \pi/2)$ $\quad f_X(x)$

$(X,0)$ = punto inter entre vertical $y=0$ y el eje x

$$Tg\,\theta = \frac{OP = x}{ady \cong 1} \rightarrow \boxed{X = Tg\,\theta}$$

$f_\theta = \frac{1}{\pi}$

$$\underline{F_X(x)} = P(X \le x) = P(Tg\,\theta \le x) = P(\theta \le arctg\, x)$$

$$= \underline{F_\theta(arctg\, x)}$$

integro f_θ

$$\left[F_\theta(\theta) = \frac{\theta + \frac{\pi}{2}}{\frac{\pi}{2} + \frac{\pi}{2}} = \frac{\theta + \frac{\pi}{2}}{\pi} = \frac{\theta}{\pi} + \frac{1}{2}\right]$$

$f_\theta = \frac{1}{b-a}$ $\qquad \theta = arctg(x)$ $\qquad \frac{d\theta}{dx} = \frac{1}{1+x^2}$

$$F_\theta(arctg(x)) = \frac{1}{2} + \frac{arctg\, x}{\pi} = F_X(x)$$

derivo

$$\boxed{f_X(x) = F_X'(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right)}$$

(4.5) La fase ϕ de un generador eléctrico, va $\sim U(-\pi,\pi)$

a) $C = \cos\phi$ $\qquad (arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$ $\qquad \phi \sim U(-\pi,\pi)$

tabla

integro

$$f_\phi(\varphi) = \frac{1}{2\pi}\, \mathbb{1}\{-\pi < \varphi < \pi\}$$

$$F_\phi(\varphi) = \frac{\varphi + \pi}{2\pi}\, \mathbb{1}\{-\pi < \varphi < \pi\} + \mathbb{1}\{\varphi \ge \pi\}$$

teoría

$$F_C(c) = P(\cos\theta \le c) = P(|\phi| \ge arccos\, c) =$$

$$P(\arccos(c) \le \underset{\text{"}\varphi\text{"}}{\Phi} \le -\arccos(c)) =$$

$$F_\Theta(-\arccos(c)) - F_\Theta(\arccos c) =$$

$$-\frac{\arccos(c) + \pi}{2\pi} - \frac{\arccos c + \pi}{2\pi} = -\frac{\arccos(c)}{\pi}$$

reemplazo en la F. integrada.

$$\left[F_C(c) = -\frac{\arccos(c)}{\pi} \, \mathbb{1}\{-1 < c < 1\}\right].$$

derivo ↓ derivo.

$$\left[f_C(c) = \frac{1}{\pi\sqrt{1-c^2}} \, \mathbb{1}\{-1 < c < 1\}\right].$$

b) $P(|c| < 0.5) = P(-0{,}5 < c < 0{,}5) =$

$F_C(0{,}5) - F_C(-0{,}5)$ = evaluo en c= ...

$$= -\frac{\pi}{3\pi} - \frac{-2\pi}{3\pi} = \boxed{\frac{1}{3}}.$$

OBS

$$\boxed{\begin{array}{l} P(|c| < x) = P(-x < c < x) = \\ F_C(x) - F_C(-x) = \end{array}}$$

4.6 Lucas llega (7:05 , 7:50)

$L \sim U(7{:}05,\ 7{,}50)$

Subte $\sim$ (0, 15) → a partir de las 6.

6:00 , 6:15, 6:30, 6:45, 7:00, 7:15, 7:30, 7:45, 8:00

Lucas

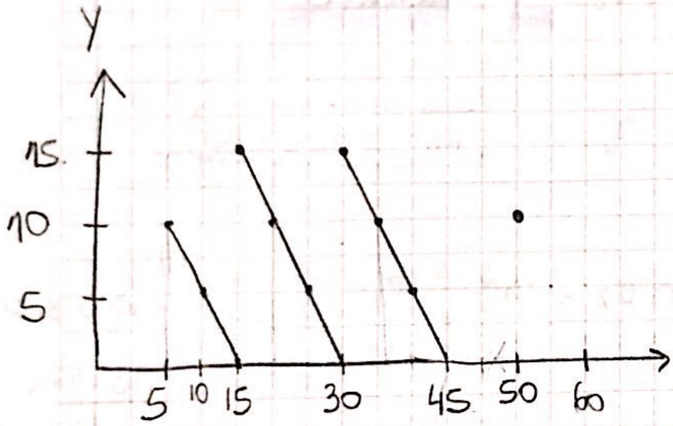
↓

en el tiempo que Lucas esta llegan 3 subtes

X: hora de llegada de lucas

$X \sim U(5, 50)$, min.

Y: tiempo que espera



$$f_Y = \frac{f_X}{|g'(x)|}\Bigg|_{y=g(x)} = \frac{1}{45}\,\mathbb{1}\{\text{intervalo}\}$$

$Y = 15 - X$

$X = 15 - Y$

$g'(x) = -1$

$|g'(x)| = 1$

$$f_X(x) = \frac{1}{45}\,\mathbb{1}\{5 < x < 50\}$$

$y = 15 - x$

$$y = g(x) = \begin{cases} 15 - x & \overbrace{7{:}05}^{x=5} \le x \le \overbrace{7{:}15}^{x=15} \to 0 \le y \le 10 \\ 30 - x & 7{:}15 < x \le 7{:}30 \to 0 \le y \le 15 \\ 45 - x & 7{:}30 < x \le 7{:}45 \to 0 \le y \le 15 \\ 60 - x & 7{:}45 < x \le 7{:}50 \to 10 \le y \le 15 \end{cases}$$

del grafico

$\frac{1}{45}\, 0 \le y \le 10 + \frac{1}{45}\, 10 \le y \le 15$. (por aparte)

$$f_Y(y) = \frac{1}{45}\mathbb{1}\{0 \le y \le 10\} + \frac{1}{45}\mathbb{1}\{0 \le y \le 15\} + \frac{1}{45}\mathbb{1}\{0 \le y \le 15\} + \frac{1}{45}\mathbb{1}\{10 \le y \le 15\}$$

$$\left[f_Y(y) = \frac{1}{15}\mathbb{1}\{0 \le y \le 10\} + \frac{1}{15}\mathbb{1}\{10 \le y \le 15\} \right]$$

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→ carpeta (p.65)

tabla

(4.7) $V_1 \sim U(180, 220)$ ← $f(v_1) = \frac{1}{40} \mathbb{1}\{180 \le v_1 \le 220\}$

uso solo de la derivada pq es mixta.

$V_2 = g(v_1) = \frac{v_1 - 190}{20} \mathbb{1}\{190 \le v_1 \le 210\} + \mathbb{1}\{210 < v_1\}$.

$V_2 = g(v_1)$

$$g(v_1) = \begin{cases} \frac{v_1 - 190}{20} & 190 \le v_1 \le 210 \to g_1 \\ 1 & 210 < v \to g_2 \end{cases}$$

integro $f(v_1)$ → $F(v_1) = \int_{180}^{v_1} f_{v_1}\, dv_1 = \frac{1}{40} v \Big|_{180}^{v_1} =$

$F(v_1) = \frac{v_1 - 180}{40} \mathbb{1}\{180 \le v_1 \le 220\}$.

↓ hay tramos q'no hay densidad o $g'(x)=0$. no sirve.

$[F_{V_2}(v_2) = P(V_2 \le v_2) = P(g v_1 \le v_2) = P(v_1 \le g^{-1}(v_2))$

$= F_{V_1}(g^{-1}(v_2))]$ → TEOREMA

$g(v_1) = v_2$.

$g_1(v_1) = \frac{v_1 - 190}{20} = v_2$ $\boxed{190 \le v_1 \le 210}$.

↓

$V_1 = 20 v_2 + 190$ $190 \le 20v_2 + 190 \le 210$

$0 \le v_2 \le \frac{210 - 190}{20}$

$\boxed{0 \le v_2 \le 1}$

$210 \le v_1 \le 220$ → $g_2(v_1) = 1 = v_2$ $210 < v_1$

$\boxed{v_2 > 1}$ ← sigo esa analogía.

$\boxed{g_1^{-1}(v_2) = \cancel{g_1}(v_1)}$

$v_1 = 20v_2 + 190$.

$F_{V_2}(v_2) = \underbrace{F_{V_1}}_{\frac{v_1 - 180}{40}}(g_1^{-1}(v_2)) + F_{V_1}(g_2^{-2}(v_2)) =$

$F_{V_2}(v_2) = \frac{20v_2 + 190 - 180}{40} \mathbb{1}\{v_2 \in (0,1)\} +$

$\mathbb{1}\{v_2 > 1\}$.

$\boxed{g_1(v_1) = V_2}$

(4.8) X: duración llamada v.a. $1/\lambda = E = 8$ min.

$X \sim \exp(1/8)$ $\lambda = 1/8$.

$$f_X(x) = \frac{1}{8} e^{-1/8 x} \mathbb{1}(x>0)$$

se factura 1 pulso c/ 2 min

cantidad de c facturados por llamada ← $P(C=c)$ C = "cant de pulsos facturados por llamada"

$$C \begin{cases} 1 & 0 \le X < 2 \\ 2 & 2 \le X < 4 \\ 3 & 4 \le X < 6 \end{cases}$$

pongo a X en función de c

eventos equivalentes

C $\boxed{2c-2 \le x \le 2c}$

$$P(2c-2 \le x \le 2c) = F_X(2c) - F_X(2c-2) =$$

$$= \int_{2c-2}^{2c} f_X \, dx = \int_{2c-2}^{2c} \frac{1}{8} e^{-1/8 x} \, dx =$$

$$= \frac{1}{8} \left. \frac{e^{-\frac{1}{8}x}}{-1/8} \right|_{2c-2}^{2c} = -e^{-\frac{x}{8}} \Big|_{2c-2}^{2c} =$$

$$= \boxed{-e^{-\frac{2c}{8}} + e^{-\frac{2c+2}{8}}}$$

CREO

$$\int e^{-1/8 x} dx = \frac{e^{-1/8 x}}{-1/8}$$

Recuerdo (guía 2)

$$P(a \le x \le b) = F_X(b) - F_X(a) =$$

$$= \int_a^b f_X \, dx$$

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Para Vectores aleatorios (2 variables)

antes: x y $g(x)=y$

(x,y) V.A $\quad g: D \subset \mathbb{R}^2 \to \mathbb{R} \quad R(x,y) \subset D$

Se define $\boxed{z = g(x,y)}$

$$F_z(z) = P(z \leq z) = P(g(x,y) \leq z) = P(g(x,y) \in (-\infty, z))$$

$$= P((x,y) \in g^{-1}((-\infty, z)))$$

↳ integro f_{xy} en el área.

ya por eventos equivalentes ↑, suele ser muy complicado.

JACOBIANO

(x,y) V.A abs continua.

$g: D \subset \mathbb{R}^2 \to \mathbb{R}^2 \quad sop(f_{xy}) \subset D$

$$g(x,y) = (g_1(x,y), g_2(x,y))$$

Defino $\quad (u,v) = (\underbrace{g_1(x,y)}_{u}, \underbrace{g_2(x,y)}_{v}) = g(x,y)$

Para hallar $f(u,v)$

$$f_{uv}(u,v) = f_{xy}(X(u,v), Y(u,v)) \cdot \left| \frac{d(x,y)}{d(u,v)} \right| \quad \begin{vmatrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{vmatrix}$$

o

$$f_{uv}(u,v) = \frac{f_{xy}(X(u,v), Y(u,v))}{\left| \frac{d(u,v)}{d(x,y)} \right|} \quad \begin{vmatrix} \frac{du}{dx} & \frac{du}{dy} \\ \frac{dv}{dx} & \frac{dv}{dy} \end{vmatrix}$$

donde $(X(u,v), Y(u,v)) = g^{-1}(u,v)$

Nota: ver soporte de u, v.

si son inyectivas localmente (como antes)

$$f_{UV}(u,v) = \sum_{i=1}^{m} f_{XY}(x(u,v), y(u,v)) \left| \frac{d(x,y)}{d(u,v)} \right| \mathbb{1}\{u,v \in D\}$$

↓ densidad por partes

$g^{-1}(u,v) = (x(u,v), y(u,v))$

Sop(x,y) : valores q' puede tomar el vector aleatorio en $\mathbb{R}^2$.

Si $Y = g(x)$

g estrictamente creciente → $F_Y(y) = F_X(g^{-1}(y))$

g estrictamente decreciente → $F_Y(y) = 1 - F_X(g^{-1}(y))$

- $U = \min(x,y)$ → x si $x \le y$; → y si $y \le x$.
- $V = \max(x,y)$ → x si $x \ge y$; → y si $y \ge x$.

49 a) $U = X \quad V = X + Y$

$$\left| \frac{d(u,v)}{d(x,y)} \right| = \begin{vmatrix} \frac{du}{dx} & \frac{du}{dy} \\ \frac{dv}{dx} & \frac{dv}{dy} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

x y → u v

$(-2,-2) = (-2,-4)$
$(-2,-1) = (-2,-3)$
$(-2,1) = (-2,-1)$
$(-2,2) = (-2,0)$
$(-1,-2) = (-1,-3)$
$(-1,-1) = (-1,-2)$
$(-1,1) = (-1,0)$
$(-1,2) = (-1,1)$
$(1,-2) = (1,-1)$
$(1,-1) = (1,0)$
$(1,1) = (1,2)$
$(1,2) = (1,3)$

nueva tabla

V

u/v	-4	-3	-2	-1	0	1	2	3	4
-2	1/16	1/16	0	1/8	1/8	0	0	0	0
-1	0	1/16	1/16	0	1/8	1/16	0	0	0
1	0	0	0	0	0	0	1/16	1/16	0
2	0	0	0	0	0	0	0	1/16	1/6.

U

$(2,-2) = (2,0)$
$(2,-1) = (2,1)$
$(2,1) = (2,3)$
$(2,2) = (2,4)$
xy uv.

$$\boxed{f_{UV} = \frac{f_{XY}}{J} = \frac{f_{XY}}{1} = f_{XY}}$$

b) $U = \min(x,y)$ $\quad V = \max(x,y)$

X Y	U V
$(-2,-2)$	$(-2,-2)$
$(-2,-1)$	$(-2,-1)$
$(-2,1)$	$(-2,1)$
$(-2,2)$	$(-2,2)$

V

U/V	-2	-1	1	2
-2	1/16	1/16	1/8	1/8
-1	0	1/16	1/8	1/8
1	0	0		
2	0	0		

U

$\begin{pmatrix} -1,-2 \\ -1,-1 \\ -1,1 \\ -1,2 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ -1 & -1 \\ -1 & 1 \\ -1 & 2 \end{pmatrix}$

1 −2 $\quad$ → −2 −1 seguin...

c) $U = x^2 + y^2 \quad V = \dfrac{y}{x}$ $\quad J = \begin{vmatrix} 2x & 2y \\ -\frac{1}{x^2}y & \frac{1}{x} \end{vmatrix}$

mismo q' antes

4.11 $Z_1; Z_2 \sim (0,1)$ $\quad Z_1$ y Z_2 ind. $\quad \mu=0 \quad \sigma=1$

a) f_{UV}, f_U, f_V $\quad$ 1) $U = Z_1 + Z_2 \quad V = Z_1 - Z_2$

ind

$f_{Z_1} = \dfrac{1}{2\pi} e^{-z_1^2/2}$

$f_{Z_1Z_2} = f_{Z_1} f_{Z_2}$. $\quad f_{Z_2} = \dfrac{1}{2\pi} e^{-z_2^2/2}$

POR SER INDEP.

$\boxed{f_{Z_1Z_2} = f_{Z_1} f_{Z_2}}$

$Z_2 = U - Z_1$

$V = Z_1 - U + Z_1$

$V = 2Z_1 - U$

$\boxed{Z_1 = \dfrac{V+U}{2}}$

$J = \begin{vmatrix} \frac{dU}{Z_1} & \frac{dU}{Z_2} \\ \frac{dV}{Z_1} & \frac{dV}{dZ_2} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} =$

$= -1 - 1 = -2$

$|J| = 2$

$\boxed{Z_2 = \dfrac{-V+U}{2}}$

$f_{UV} = \dfrac{f_{Z_1Z_2}\left(z_1 = \frac{U}{2} + \frac{V}{2},\ z_2 = \frac{-V+U}{2}\right)}{2}$

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$$f_{z_1 z_2} = \frac{1}{2\pi} e^{-\frac{1}{2}(z_1^2 + z_2^2)} \quad 1\!\!1\{(z_1,z_2) \in \mathbb{R}^2\}$$

$$\boxed{f_{UV} = f_U \cdot f_V}$$

$$\left(-\frac{v}{2}\right)^2 + 2\left(-\frac{v}{2}\right)\left(\frac{u}{2}\right) + \left(\frac{u}{2}\right)^2$$

$$\left(\frac{u}{2}\right)^2 - 2\left(\frac{u}{2}\right)\left(-\frac{v}{2}\right) + \left(-\frac{v}{2}\right)^2$$

$$f_{UV} = \frac{1}{4\pi} e^{-\frac{1}{2}\left(\left(\frac{+v+u}{2}\right)^2 + \left(\frac{u}{2} - \frac{v}{2}\right)^2\right)}$$

$$f_U = \frac{1}{2\pi} e^{-1/2\left(\frac{u}{2}\right)^2} \quad f_V = \frac{1}{2\pi} e^{-1/2(v)^2}$$

CHEQUEAR cuentas igual

$\boxed{U \sim N(0,2)}$ $\boxed{V \sim N(0,1)}$

2) $U = \cos\theta\, z_1 - \operatorname{sen}\theta\, z_2$

$z_1 \sim N(0, \cos^2\theta)$ $V = \operatorname{sen}\theta\, z_1 + \cos\theta\, z_2$

$z_2 \sim N(0, \operatorname{sen}^2\theta)$

misma idea

$\hookrightarrow$ $\boxed{U \sim N(0,1) \quad V \sim N(0,1)}$

3) $U = z_1^2 + z_2^2$ $\quad V = \frac{z_2}{z_1}$ tamb.

$\boxed{U \sim exp(1/2)}$ $\boxed{V \sim \text{cauchy}(0,1)}$

b) son ind, porque si uno es mín, depende del otro.

c) $P(z_1^2 + z_2^2 > 4) = P(U > 4) = 1 - \underbrace{P(U \leq 4)}_{F_U(4)} =$

$$1 - e^{-4/2} \cong 0{,}86$$

$P(z_2 > \sqrt{3} z_1) = P\left(\frac{z_1}{z_2} > \sqrt{3}\right) = P(V > \sqrt{3}) =$

$$1 - P(V \leq \sqrt{3}) = 1 - F_V(\sqrt{3}) =$$

IMP:

aveces al hacer cambio de variables, quedan mismas distribuciones, aveces cambian (analizarlo)

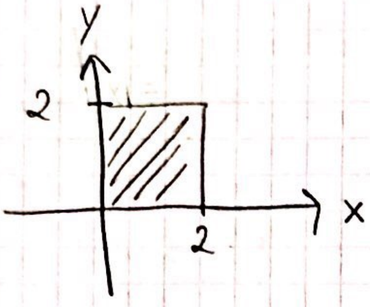
4.12 ↙ Mirar 4.10 carpeta.

$X_1, X_2 \sim (0,2)$ ↗ ≠ estándar $\mu = 0$ $\sigma = 2$

independientes

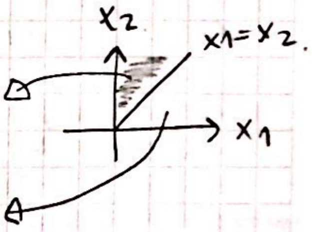
$U = \min(X_1, X_2)$ $V = \max(X_1, X_2)$

a) f_{UV}



$f_{X_1X_2} = \frac{1}{\text{área}} \mathbb{1}\{x_1,x_2 \in (0,2)^2\}$

área = 4



$$(U,V) = \begin{cases} (X_1, X_2) & \text{si } X_1 < X_2 \\ (X_2, X_1) & \text{si } X_1 > X_2 \end{cases}$$

$F_{UV}(u,v) = P((U,V) \leq (u,v)) = P(\min(x_1x_2), \max(x_1x_2) \leq u, \max)$

$= P(U \leq u_1, V \leq v_1, \underset{\downarrow \text{ min.}}{X_1 < X_2}) + P(U \leq u_1, V < v_1, X_2 \leq X_1$

$$F_{UV}(u,v) = \begin{cases} 0 & u < 0, v < 0 \\ \frac{1}{2} v \cdot u - \frac{1}{4} u^2 & (u,v) \in (0,2) \\ 1 & u > 2, v > 2 \end{cases}$$ → no entendí mucho

$|J| = 1$ en ambos casos

MEJOR ↘

$f_{UV} = \sum_{1}^{2} f_{XY} |J| =$

caso 1: $u = x$ $v = y$

caso 2: $u = y$ $v = x$

$$f_{UV} = \frac{1}{\sqrt{2\pi} \cdot 2} e^{-\frac{(u)^2}{2 \cdot 4}} + \frac{1}{\sqrt{2\pi} \cdot 2} e^{-\frac{v^2}{8}} \mathbb{1}\{(u,v) \in (0,2)\}.$$

b) $W = V - U = \max - \min$

Solo para exponenciales

TEOREMA

4.14

- $X \sim exp(\lambda_1)$ • $Y \sim exp(\lambda_2)$ • X, Y ind
- $U = min(X, Y)$ • $W = V - U$
- $V = máx(X, Y)$ • $J = 1 \, 1\{U = X\} + 2 \, 1\{U = Y\}$

a) densidad de U.

$$F_U(u) = 1 - e^{-(\lambda_1 + \lambda_2) u}$$

derivo

$$f_U(u) = (\lambda_1 + \lambda_2) e^{-(\lambda_1 + \lambda_2) u} \quad 1\{u > 0\}.$$

$$U \sim exp(\lambda_1 + \lambda_2)$$

EL MÍNIMO ENTRE EXPONENCIALES INDEPENDIENTES, sigue siendo una exponencial de parametro $\lambda_1 + \lambda_2$

b) Función PROB de J.

$$P(J = 1) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \rightarrow \text{el mínimo es } X$$

$$P(J = 2) = 1 - \frac{\lambda_1}{\lambda_1 + \lambda_2} \rightarrow \text{el mínimo es } Y$$

c) densidad de $W = V - U$

$F_W(0) = 0$

$F_W(\infty) = 1$

$$F_W(w) = 1 - \frac{\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_1 w} - \frac{\lambda_1}{\lambda_1 + \lambda_2} e^{-\lambda_2 w}$$

$$f_W(w) = \underbrace{\frac{\lambda_2}{\lambda_1 + \lambda_2}}_{P} \underbrace{\lambda_1 e^{-\lambda_1 w}}_{\substack{f_X(w) \\ \text{densida } x}} + \underbrace{\frac{\lambda_1}{\lambda_1 \lambda_2}}_{1-P} \underbrace{\lambda_2 e^{-\lambda_2 w}}_{\substack{f_Y(w) \\ \text{densida } y}}$$

El promedio entre densidad x e y = densidad máx − mín

d) MOSTRAR U y J son independientes

$U \to$ CONTINUA

$J \to$ DISCRETA

$$\left[P(U > u, J = 1) = \underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}}_{P(J=1)} \underbrace{e^{-(\lambda_1+\lambda_2)u}}_{P(U>u)} = P(J=1)\,P(U>u) \right]$$

U y J son independientes

e) U y W son independientes

$\underbrace{\text{U y W}}_{\text{ambas continuas}}$

$$f_{UW}(u,w) = f_U(u)\, f_W(w)$$

$$\left[f_U(u) = (\lambda_1+\lambda_2)\, e^{-(\lambda_1+\lambda_2)u} \right]$$

$$\left[f_W(w) = \frac{\lambda_2\lambda_1}{\lambda_1+\lambda_2} e^{-\lambda w} + \frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2} e^{-\lambda_2 w} \right]$$

$$f_{UV}(u,v) = \frac{f_{XY}}{|J_1|} + \frac{f_{XY}}{|J_2|}$$

$\downarrow$ si

$$f_{XY} = \frac{1}{2} \{0<x<1,\ 0<y<1\} +$$

2 intervalos

$$2\left\{ \tfrac{1}{2} < x < 1,\ \tfrac{1}{2} < y < 1 \right)$$

Pág 69 clase

J y P independientes.

4.15 $X = J \sim exp\left(\frac{5}{60}\right)$ J: tiempo entre llamadas de Juan.

$Y = P \sim exp\left(\frac{10}{60}\right)$ $\lambda_1 = \frac{5}{60}$. $\lambda_2 = \frac{10}{60}$. λ_1, λ_2 en minutos

- antes de los 5 min no se atienden llamadas
- desde la última llamada, dejan pasar 5 minutos

Arrancan 10:00.

$J = \begin{cases} 1 & \text{si 1er llamada Juan } (U = X) \\ 2 & \text{si 1r llamada Pedro } (U = Y) \end{cases}$

OBS.

$\lambda = 5/60$
$U < 5$
o
$\lambda = 5$
$U < \frac{5}{60}$
xHORA ...

10:00 10:05 ... depende cuando llamen

no atienden

$\underline{\underline{U}}$ = 1er llamada → mínimo

a) $P(U<5) = 1 - P(U \geq 5) = 1 - F_U(5) =$

POR EL 4.14 $U \sim exp(\lambda_1 + \lambda_2)$

$= 1 - e^{-(\lambda_1 + \lambda_2)u}$

$= 1 - e^{-\left(\frac{15}{60}\right) \cdot 5}$

$\boxed{P(U<5) \cong 0{,}71}$

1er llamada es de Juan

b) $P(J=1) = \frac{\lambda_1}{\lambda_1 + \lambda_2} = \frac{1}{3}$

c) $P(U<5 \mid J=1) = \frac{P(J=1) \cap \mathbb{P}(U<5)}{P(J=1)} =$

son indep.

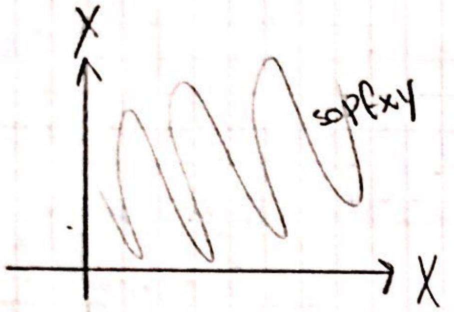
$= \frac{\cancel{P(J=1)} P(U<5)}{\cancel{P(J=1)}} \cong 0{,}71$

4.16

X, Y ind y exp $\lambda > 0$ → veo en tabla que su soporte es de $[0, +\infty]$.

$U = X+Y \qquad V = \frac{X}{X+Y}$

a)



$\frac{du}{dx} = 1 \quad \frac{du}{dy} = 1$

$\frac{dv}{dx} = \frac{1(\cancel{x+y}) - \cancel{x}(1)}{(x+y)^2}$

$$\left|\frac{d(u,v)}{d(x,y)}\right| = \begin{vmatrix} 1 & 1 \\ \frac{y}{(x+y)^2} & \frac{-x}{(x+y^2)} \end{vmatrix}$$

$\frac{dv}{dy} = \frac{-x(1)}{(x+y)^2}$

por ser ind
↓
$f_{xy} = f_x f_y$

$$\left| -\frac{x}{(x+y^2)} - \frac{y}{(x+y)^2} \right| = \left| \frac{-(x+y)}{(x+y)^2} \right| = \frac{x+y}{(x+y)^2}$$

$\det J = \boxed{\frac{1}{x+y}}$ → como no se anula ↓ g es biyectiva ↓ $\exists g^{-1}$

$$f_{UV} = \frac{f_{XY}(x,y)}{\left|\frac{d(u,v)}{d(x,y)}\right|}$$

$X = U - Y \qquad V = \frac{U-Y}{U \cancel{-Y} \cancel{+Y}} \Rightarrow 1$

$X = U - U(1-V)$

$X = \cancel{U} - \cancel{U} + UV$

$\boxed{X = UV}$

$V \cdot U - U = -Y$

$U - VU = Y$

$\boxed{U(1-V) = Y.}$

$$f_{UV} = \frac{\lambda e^{-\lambda x} \cdot \lambda e^{-\lambda y}}{\frac{1}{x+y}} = \frac{\lambda^2 e^{-\lambda(x+y)}}{1/x+y} \; \mathbb{1}\{x>0, y>0\}$$

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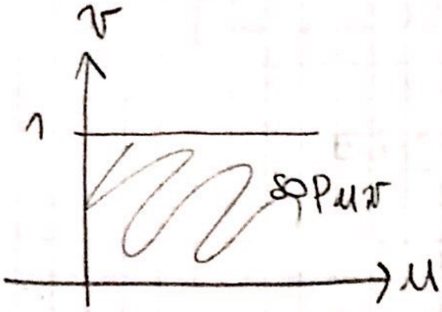
$x>0 \to uv>0$

$v>0$ o $u>0$



$y = u(1-v) > 0$

$1 - v > 0$

$\boxed{1>v}$

$\boxed{0<v<1}$

$\boxed{u>0}$

↓ nuevos parametros.

$$f_{UV}(u,v) = \frac{\lambda^2 e^{-\lambda(\overbrace{uv + u(1-v)}^{u})}}{\frac{1}{(\underbrace{uv + u(1-v)}_{u})}} =$$

$$\boxed{f_{UV}(u,v) = u.\lambda^2 e^{-\lambda u}} \quad \begin{array}{l} 0<v<1 \\ u>0 \end{array}$$

como el soporte es un región que no depende que no sean ind.

Para las marginales integro sobre el soporte
↓
$f_U \to$ sobre v como varia
$f_V \to$ sobre u.

↓ ya q' la de f_{UV} viendo el soporte

$$f_U(u) = \int_0^1 \lambda^2 u e^{-\lambda u} \, dv$$

$$f_U(u) = \lambda^2 u e^{-\lambda u} \quad \boxed{u>0}$$

$$f_V(v) = \int_0^{+\infty} \lambda^2 u e^{-\lambda u} \, du$$

↳ por el soporte puedo decir que se parece
↓
uniforme
$[a,b]$
↓
$V \sim U(0,1)$

se parece a una gamma → $r=2$, $\Gamma(2,\lambda)$ → $U \sim \Gamma(2,\lambda)$

↓ por el soporte

b) $f_{UV} = f_U \cdot f_V$ ✓ son ind

4.17 (X, Y) punto aleatorio $\sim N(0,1)$, X e Y ind.

$$f_{XY} = \frac{1}{2\pi} e^{-1/2 (x^2+y^2)} \mathbb{1}\{(x,y) \in \mathbb{R}^2\}$$

$\downarrow$

$f_X \cdot f_Y$ $\quad\quad X = \rho \cos\theta \quad Y = \rho \operatorname{sen}\theta$

$$\text{Jac} \begin{vmatrix} \frac{dx}{d\rho} & \frac{dx}{d\theta} \\ \frac{dy}{d\rho} & \frac{dy}{d\theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \rho \\ \operatorname{sen}\theta & \rho \end{vmatrix} = \rho$$

$$f_{\rho\theta}(\rho,\theta) = f_{XY}(xy) \cdot |\text{Jac}|$$

$$= \frac{1}{2\pi} e^{-\frac{1}{2}(\rho^2)} \cdot \rho \ \mathbb{1}\{\rho > 0, \theta (0, 2\pi))$$

$$\boxed{f_{\rho\theta}(\rho\theta) = \frac{\rho}{2\pi} e^{-1/2(\rho^2)} \ \mathbb{1}\{\rho > 0, \theta [0, 2\pi]}$$

$$f_\rho(\rho) = \int_0^{2\pi} f_{xy}\, d\theta = \rho e^{-1/2(\rho^2)} \mathbb{1}\{\rho > 0\}.$$

$$f_\theta(\theta) = \int_0^{+\infty} f_{xy}\, d\rho = \int_0^{+\infty} \frac{\rho}{2\pi} e^{-1/2(\rho^2)}\, d\rho$$

como el sopor te de θ es $[0, 2\pi)$ $\longrightarrow$ uniforme.

$\downarrow$

$$f_\theta(\theta) = \frac{1}{2\pi - 0} = \frac{1}{2\pi}$$

Corroboro q' sean ind.

NOTA

4.19 X, Y va $\sim$ Unif inde. ↗ discreta = tiene átomos.

$X \sim U(1, 36)$

$Y \sim U(1, 4)$

$P_x(x) = \frac{1}{m} \to$ m siempre 36.

$\hookrightarrow \mathcal{U}(m)$

$[1, 36]$

$\underbrace{[1, m]}_{\text{Soporte}}$

$Z = g(x, y) = X + Y \qquad P(W = w) = P(X + Y = w)$

$$P(X=1) = P(X=2) = \ldots = P(X=36) = \frac{1}{36}$$

$$P(Y=1) = \ldots = P(Y=4) = \frac{1}{4}$$

$$P(W=w) = \begin{cases} P(W=2) = P(X=1, Y=1) \overset{\text{por ind}}{=} \frac{1}{4} \cdot \frac{1}{36} = \frac{1}{144} \\ P(W=3) = P(X=1, Y=2) + P(X=2, Y=1) = \frac{1}{144} + \frac{1}{144} = \frac{2}{144} \\ \vdots \\ \text{y así con todos los variables} \\ P(W=m) = 1/36 \end{cases}$$

4.20 $L \sim \text{Poisson}(2)$ $\quad$ L y M ind. ↶ DISCRETAS

$M \sim \text{Poisson}(8)$

IMP

M+L = m	
L = 0	L = 1
M = m	M = m - 1

a) $$P_{M+L}(m) = P(M+L = m) \overset{\text{POR ind}}{=} \sum_{l=0}^{m} P(L = l, M = m - l) =$$

$$= \sum_{l=0}^{m} P(L = l) \cdot P(M = m - l) =$$

$$= \sum_{l=0}^{m} \frac{e^{-2}\, 2^{l}}{l!} \; \frac{e^{-8}\, 8^{(m-l)}}{(m-l)!} \Rightarrow$$

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$$e^{-(10)} \sum_{l=0}^{m} \frac{2^{l}\, 8^{m-l}}{l!\,(m-l)!} \frac{m!}{m!}$$

$M+L=m$
$L=l$
$M=m-l$

$$\boxed{M+L \sim Po(\underbrace{\mu_M+\mu_L}_{10})}$$

Binomio de Newton

$$(a+b)^2 = \sum_{i=0}^{m} \binom{m}{i} a^i b^{m-i}$$

$$\left.\begin{array}{l} M \sim Po(\mu_1) \\ L \sim Po(\mu_2) \end{array}\right\} M+L \sim Po(\mu_1+\mu_2)$$

b) $P(M \mid (L+M)=10) = P(M=m \mid L+M=10)$

$$= \frac{\sum_{m=0}^{10} P(M=m \mid L=10-m)}{P(L+M=10)}$$

$M=m=m-l$
$L=m-m$

$$= \frac{\dfrac{e^{-8}\,8^{m}}{m!} \cdot \dfrac{e^{-2}\,2^{(10-m)}}{(10-m)!}}{\dfrac{e^{-10}\,10^{10}}{10!}} = \frac{10!\,8^{m}\,2^{10-m}}{m!\,(10-m)!\,10^{10}}$$

$$= \frac{10!}{m!(10-m)!} \left(\frac{8}{10}\right)^{m} \left(\frac{2}{10}\right)^{10-m}$$

gral: $M|_{L+M=10} \sim B\binom{m=10}{p=8/10}$

$$\boxed{M|_{L+M=m} \sim B\left(m, \frac{\mu_M}{\mu_M+\mu_L}\right)}$$

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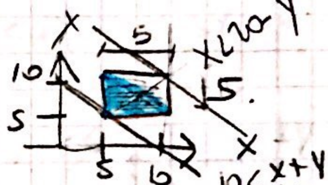
↑ pag 72

c) $P(M>2 \mid L+M=10) = \sum_{i=3}^{10} P_{M|L+M=10}(i)$

$1 - \sum_{i=0}^{2} P_{M|L+M=10}) = 1 - \left[\binom{10}{0}\left(\frac{8}{10}\right)^0\left(\frac{2}{10}\right)^{10} + \right.$

$\left.\binom{10}{1}\left(\frac{8}{10}\right)^1\left(\frac{2}{10}\right)^9 + \binom{10}{2}\left(\frac{8}{10}\right)^2\left(\frac{2}{10}\right)^8\right] =$

4.21 X, Y ind $\sim U(5,10)$



$z = y + x$.

X: tiempo 1° proceso Y: tiempo 2do proceso

a) $z = x + y$ $\quad F_z(z) = P(z \le z) = P(x+y \le z)$

$f_{XY} = \frac{1}{5 \times 5} = 1/25 = \begin{cases} 0 & \text{si } z < 10 \\ \iint \blacktriangle & \text{si } 10 \le z \le 15 \\ \iint \square - \triangledown & \text{si } 15 \le z \le 20 \\ 1 & \text{si } z \ge 20 \end{cases}$

x mayor

↓ cte sale afuera y integro el área.

$X+Y$ ↓ MIN = 10 MAX = 20

$\blacktriangle = \frac{bh/2}{25} = \frac{(z-10)^2}{50}$

$\triangledown = \frac{(20-z)^2}{50}$

y mayor

$z = x+y$

$10 < z < 20$.

$10 < x+y$

$10-y < x$

$x < 20-y$

$F_z(z) = \begin{cases} 0 & \text{si } z < 10 \\ \frac{(z-10)^2}{50} & \text{si } 10 \le z \le 15 \\ \frac{(20-z)^2}{50} & \text{si } 15 \le z \le 20 \\ 1 & \text{si } z \ge 20 \end{cases}$

derivo

$f_z(z) = \frac{z}{25} - \frac{2}{25} \mathbb{1}\{10 \le z < 15\} + \frac{4}{5} - \frac{z}{25} \mathbb{1}\{15 \le z < 20\}$

$+ \mathbb{1}\{z < 20\}$

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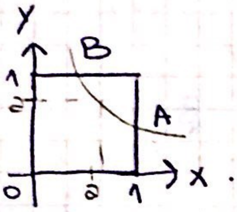
b) el objeto se produzca en menos de 16 min

$X+Y < 16$.

$$P(X+Y \le 16) = \underset{\text{cumple en } 15<z<20}{F_Z(\overset{z=16}{16})} = -\frac{z^2}{50} + \frac{4x\,z}{5} - 7 = \frac{17}{25} = 0{,}68$$

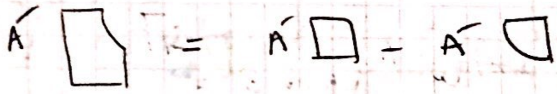
4.22



$A, B \sim U(0,1)$ ind.

a) $f_R \Rightarrow R = A \times B$

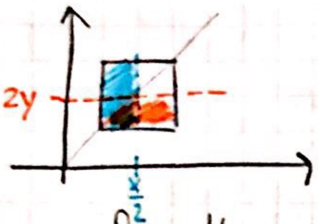
$$F_R(r) = P(R \le r) = P(A \times B \le r)$$



mi va.
↳ $A = X \cdot L$

(4.24) X e Y, va ind $\sim U(5,10)$

a) $U = \frac{X}{2} \mathbb{1}\{X \leq Y\} + 2Y \mathbb{1}\{X > Y\}$ → densidad de U.



$f_{XY} = 1/25$.

$F_U(u) = P(U \leq u) = P(U \leq u, X \leq Y) +$

$P(U \leq u, X > Y) =$

$\mathbb{P}\left(\frac{X}{2} \leq u, X \leq Y\right) + P(2Y \leq u, X > Y)$

veo mi nuevo soporte

$\frac{X}{2} = u \qquad X \leq Y$

$X = 2u \rightarrow \quad u = [5/2, 10/2]$.

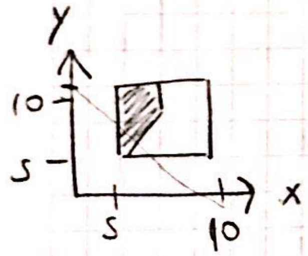
$u[2{,}5, 5]$.

$\uparrow$

$\begin{cases} \frac{X}{2} = u \\ 2Y = u. \end{cases}$

$\downarrow$

$[10, 20]$.



$F_U(u) \begin{cases} 0 & \text{si } u \leq 2{,}5. \\ \square & \text{si } 2{,}5 \leq u \leq 5 \\ \square & \text{si } 10 \leq u \leq 20. \\ \text{cte} & \text{si } 5 \leq u \leq 10. \\ 1 & \text{si } u \geq 20. \end{cases}$

derivo hallo $f_U(u)$.

b) $V = \mathbb{1}\{X + Y \leq 10\}$

$V = 0$ si $X + Y > 10$

$V = 1$ si $X + Y \leq 10$.

$P(V = v) = \begin{cases} P(V=0) = P(X+Y > 10) = 1/2 \\ P(V=1) = P(X+Y \leq 10) = 1/2 \end{cases}$

4.25 $f_X(x) = 2x \, \mathbb{1}\{0 \le x \le 1\}$

b) $f_Y(y) = (2-2y) \, \mathbb{1}\{0 \le y \le 1\}$

$F(u) = F(X+Y)$

$$F_Z(3) = P(X+Y \le 3) = \int_{-\infty}^{+\infty} \int_{-\infty}^{3-x} f_{XY}(x,y)\,dy\,dx$$

$$f_Z = \int_{-\infty}^{\infty} f_{XY}(x, z-x)\,dx = \int_{-\infty}^{+\infty} f_X(x)\, f_Y(z-x)\,dx$$

$$= \int_{-\infty}^{\infty} 2x\,(2-2(z-x)) = \int_{0}^{1} 4x(1-z) + 4x^2\,dx$$

$$= 2x^2(1-z) - 2x^2 \Big|_0^1 = 1-2z$$

$$\boxed{f_Z(z) = 1-2z \, \mathbb{1}\{z = x+y\}}$$

4.24

TEORIA

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4.24 c) Hallar la función densidad de $X_1 + Y_1$ para $(X_1, Y_1) = (X,Y) | X-Y<0$

TEORÍA.

$$f_{X|X\in B} = \frac{f_X}{P(X\in B)} \mathbb{1}\{X\in B\}$$

$$P(X\in B) = \int_{X\in B} f_X(x)\,dx$$

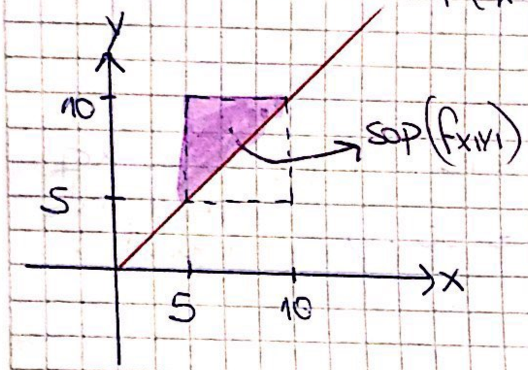
llamo $Z = X_1 + Y_1$

↳ quiero la función densidad de Z

$$f_{X_1Y_1}(x,y) = \frac{f_{XY}(x,y)}{P(X<Y)} \mathbb{1}\{x<y\}$$

$x-y<0$

$x<y$



$$P(X<Y) = \iint_{\triangle} f_{XY}(x,y)\,dx\,dy$$

$$P(X<Y) = \iint_{\triangle} \frac{1}{25}\,dx\,dy = \frac{1}{25}\text{ área } \triangle$$

altura 5.

base 5

$$\frac{5\times 5}{2} = \frac{25}{2}.$$

$$\boxed{P(X<Y) = \frac{1}{25}\cdot\frac{25}{2} = \frac{1}{2}}$$

al ser uniforme.

$X \sim U(5,10)$

$Y \sim U(5,10)$ } indep.

$f_{XY} = f_X f_Y = \ldots$

$= \frac{1}{5}\cdot\frac{1}{5} = \frac{1}{25}$ ↘

$f_{XY} = \frac{1}{\text{área}\ \square} = \frac{1}{25}.$ → same

$$\boxed{f_{X_1Y_1}(x,y) = \frac{2}{25}\,\mathbb{1}\{5<x<10,\ 5<y<10,\ x<y\}}$$

PERO yo estoy hablando de Z

$$Z = X_1 + Y_1.$$

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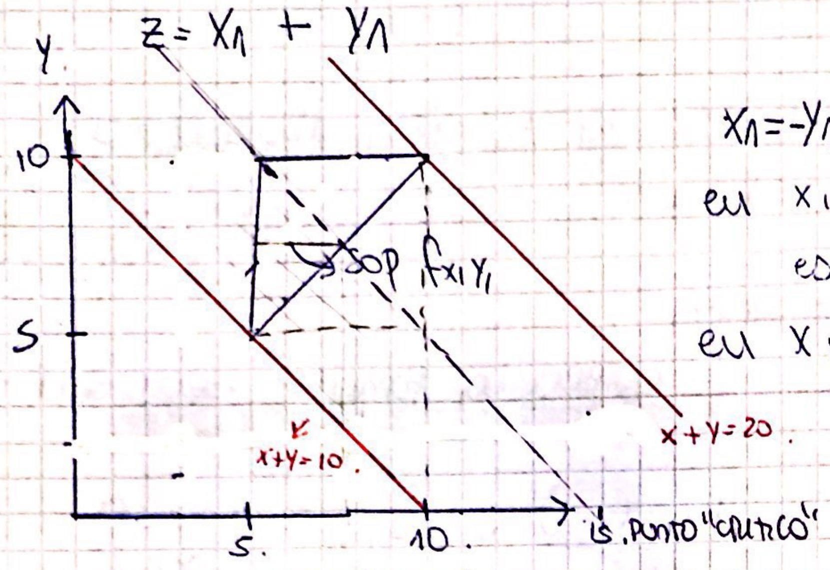

$x_1 = -y_1 + z$

en $x_1 = 5$ $y_1 = 5 \to z = 10$.

es el mínimo

↳ 0

en $x_1 = 10$ $y_1 = 10$

$z = 20$.

↓

máximo.

↳ 1.

voy a ir barriendo las zonas.

$$F_Z(z) = \begin{cases} 0 & z \leq 10. \\ 1 & 10 < z \leq 15. \\ 2 & 15 < z \leq 20. \\ 1 & z > 20. \end{cases}$$

los divido xq no varía todo igual.

1 $10 < z \leq 15$ $P(M \leq z)$

$$\frac{(m-10)^2}{25}$$

$$F_{M|M\in B}(m) = \int \frac{f_{x_1 y_1}}{P(x \in B)} \, dt$$

$$= \frac{2}{25} \cdot \frac{1}{2}.$$

2 $1 - \dfrac{(20-m)^2}{25}.$