**10.11** Se observará una muestra aleatoria $X_1, \ldots, X_n$ de una población cuya densidad $f(x)$ puede ser

$$f_0(x) = \frac{1}{50}\mathbf{1}\{0 < x < 50\} \quad \text{o} \quad f_1(x) = \frac{x}{1250}\mathbf{1}\{0 < x < 50\}.$$

(**a**) Hallar un test para $H_0 : f(x) = f_0(x)$ contra $H_1 : f(x) = f_1(x)$. ☞:*considerar la distribución de* $Y = -\log(X/50)$.

(**b**) Hallar la expresión de $\alpha$ y $\beta$ en función de n.

(a)

$F_Y(y)$ ?

$$F_Y(y) = \mathbb{P}(Y \le y) = \mathbb{P}\left(-\log\left(\frac{X}{50}\right) \le y\right)$$

$$= \mathbb{P}(X \ge 50e^{-y}) = 1 - \mathbb{P}(X < 50e^{-y})$$

$$= \begin{cases} 0 \text{ si } y < 0 \\ 1 - e^{-y} \text{ ó } 1 - e^{-2y} \text{ si } y \ge 0 \end{cases}$$

$$\int_0^{50e^{-y}} \frac{1}{50}\,dx = e^{-y}$$

$$\int_0^{50e^{-y}} \frac{x}{1250}\,dx = \frac{50^2 e^{-2y}}{2.1250}$$

$$\Longrightarrow Y \sim \mathcal{E}xp(\lambda), \text{ con } \lambda \in \{1,2\}$$

$$H_0 : \lambda = 1 \qquad\qquad H_1 : \lambda = 2$$

Hipotesis simples $\Rightarrow$ Criterio de máxima verosimilitud

$$\delta(\underline{y}) = \begin{cases} 1 & \text{si } \quad T > K_\alpha \\ 0 & \text{si } \quad T \leq K_\alpha \end{cases} \qquad T = \frac{f_{\lambda=2}(\underline{y})}{f_{\lambda=1}(\underline{y})}$$

$$f_\lambda(\underline{y}) = \prod_{i=1}^{n} \lambda e^{-\lambda y_i} = \lambda^n e^{-\lambda \sum_{i=1}^{n} y_i}$$

$$\frac{f_{\lambda=2}(\underline{y})}{f_{\lambda=1}(\underline{y})} = \frac{2^n e^{-2\sum y_i}}{1^n e^{-\sum y_i}} = 2^n e^{-\sum y_i} > K_\alpha$$

$$\Rightarrow e^{-\sum y_i} > \frac{K_\alpha}{2^n} \Rightarrow \sum y_i < \frac{\ln K_\alpha}{n \ln 2}$$

$$n \sum y_i < K'_\alpha$$

$$\Rightarrow \underbrace{\sum \underbrace{n Y_i}_{Exp(\frac{\lambda}{n})}}_{\Gamma(n, \frac{\lambda}{n})} < K'_\alpha \quad \text{o} \quad \underbrace{\sum Y_i}_{\Gamma(n,\lambda)} < K''_\alpha$$

cuantil

$$\text{Con } \alpha = \mathbb{P}_{\lambda_0}(\delta(\underline{Y}) = 1) \longrightarrow K''_\alpha = z_{\Gamma(n,\lambda_0),\alpha}$$

$$\Longrightarrow \boxed{\delta(\underline{Y}) = \mathbb{1}\left\{\sum_{i=1}^{n} Y_i < z_{\Gamma(n,\lambda_0),\alpha}\right\}}$$

(b)

$$\alpha = \mathbb{P}_{\lambda_0}(\delta(\underline{Y}) = 1) = \mathbb{P}\left(\sum Y_i < \overbrace{z_{\Gamma(n,\lambda_0),\alpha}}^{=t}\right)$$

$$\Longrightarrow \boxed{\alpha = 1 - \sum_{i=0}^{n-1} \frac{e^{-\lambda_0 t}(\lambda_0 t)^i}{i!}}$$