CYN

GUÍA 5

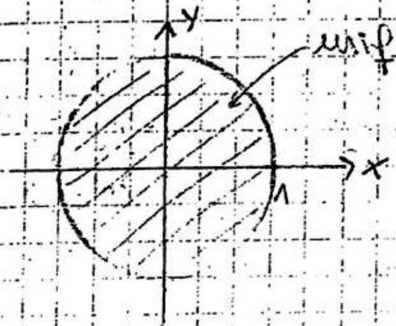
5.1) 4V, 3A, 3R   X = "cant de bolas verdes extraídas"

Y = "cant de bolas rojas extraídas"

$p_{Y|X=x}(y) = \frac{p_{XY}(x,y)}{p_X(x)}$ $\quad p_{XY}(x,y) = \frac{\binom{4}{x}\binom{3}{y}\binom{3}{3-x-y}}{\binom{10}{3}}$ $\quad p_X(x) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$\Rightarrow p_{Y|X=x}(y) = \frac{\binom{4}{x}\binom{3}{y}\binom{3}{3-x-y}}{\binom{4}{x}\binom{6}{3-x}} \Rightarrow p_{Y|X=x}(y) = \frac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}}$ $\quad x=0,1,2,3$; $y=0,1,2,3$

5.2) a) $f_{XY}(x,y) = \frac{1}{\pi}\mathbb{1}\{x^2+y^2\le 1\}$



$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi}\,dy = \frac{2\sqrt{1-x^2}}{\pi}$

$\Rightarrow f_X(x) = \frac{2}{\pi}\sqrt{1-x^2}\,\mathbb{1}\{-1\le x\le 1\}$

$f_{Y|X=x} = 2\sqrt{1-x^2}\{x^2+y^2\le 1, -1\le x\le 1\}$

b) $f_{XY}(x,y) = \frac{1}{2x+1} e^{-\left(2x+\frac{y}{4x+2}\right)}\mathbb{1}\{x>0, y>0\}$

$f_{XY}(x,y) = \frac{1}{2x+1} e^{-2x} e^{-\frac{y}{4x+2}} \frac{2}{2}\mathbb{1}\{x>0, y>0\} = \underbrace{2e^{-2x}\mathbb{1}\{}_{f_X(x)}$
$\underbrace{\frac{1}{2(2x+1)} e^{-\frac{y}{2(2x+1)}}\mathbb{1}\{y>0\}}_{f_{Y|X=x}(y)}$

$\Rightarrow f_X(x) = 2e^{-2x}\mathbb{1}\{x>0\}$, $X\sim exp(2)$

$\Rightarrow f_{Y|X=x}(y) = \frac{1}{2(2x+1)} e^{-\frac{y}{2(2x+1)}}\mathbb{1}\{y>0\}$; $Y|_{X=x} \sim exp(4x+2)$

c) $f_{XY}(x,y) = e^{-x}\mathbb{1}\{0<y<x\}$

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_{-\infty}^{\infty} e^{-x}\,dy = \int_0^x e^{-x}\,dy = (e^{-x}y)\Big|_0^x = x$

$\Rightarrow f_X(x) = xe^{-x}\mathbb{1}\{x>0\}$

$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} \Rightarrow f_{Y|X=x}(y) = \frac{1}{x}\mathbb{1}\{0<y<x\}$

d) $f_{XY}(x,y) = \frac{1}{6}x^4 y^3 e^{-xy}\ \mathbb{1}\{1<x<2,\ y>0\}$

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_{-\infty}^{\infty} \frac{1}{6}x^4 y^3 e^{-xy} = \Gamma(4,x) \Rightarrow \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = 1$

$f_X(x) = \mathbb{1}\{x>0\}$

$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} \Rightarrow f_{Y|X=x}(y) = \frac{1}{6}x^4 y^3 e^{-xy}\ \mathbb{1}\{y>0\},\ 1<x<2$

5.3) Y = " cantidad de litros de leche al comienzo del día "

X = " venta diaria "

$f_{XY}(x,y) = \frac{1}{2}\ \mathbb{1}\{0<x<y<2\}$

a) $f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy = \int_x^2 \frac{1}{2}dy = \left(\frac{y}{2}\right)\Big|_x^2 = 1 - \frac{x}{2}$

$f_X(x) = 1 - \frac{x}{2}\ \mathbb{1}\{0<x<2\}$

$\Rightarrow f_{Y|X=x}(y) = \frac{1}{2-x}\ \mathbb{1}\{x<y<2\},\ 0<x<2 \quad Y|X \sim U(x,2)$

$\Rightarrow f_{Y|X=1,5}(y) = 2\ \mathbb{1}\{1,5<y<2\} \quad Y|X=1,5 \sim U(1,5,\ 2)$

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_0^y \frac{1}{2}dx = \left(\frac{x}{2}\right)\Big|_0^y = \frac{y}{2}$

$\Rightarrow f_Y(y) = \frac{y}{2}\ \mathbb{1}\{0<y<2\}$

$f_{X|Y=y}(x) = \frac{1}{y}\ \mathbb{1}\{0<x<y\},\ 0<y<2 \quad X|Y \sim U(0,y)$

$f_{X|Y=0.8}(x) = \frac{5}{4}\ \mathbb{1}\{0<x<\frac{4}{5}\} \quad X|Y=\frac{4}{5} \sim U(0,\frac{4}{5})$

b) $P(1,75<Y<2 \mid X=1,5) = P(1,75 < Y|X=1,5 < 2) = F_{Y|X=1,5}(2) - F_{Y|X=1,5}(1,75) = 2.2 - 2.1,75 = \frac{1}{2}$

$P(0,5<X<0,75 \mid Y=0,8) = P(0,5 < X|Y=0,8 < 0,75) =$

$F_{X|Y=0,8}(0,75) - F_{X|Y=0,8}(0,5) = \frac{5}{4}\cdot\frac{3}{4} - \frac{5}{4}\cdot\frac{1}{2} = \frac{5}{16}$

c) No son indep porque $f_{Y|X=x}(y) \neq f_Y(y)$ y $f_{X|Y=y}(x) \neq f_X(x)$

5.4) $f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}(x^2 - \frac{6}{5}xy + y^2)}$ $\forall \{(x,y) \in \mathbb{R}^2\}$

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx \Rightarrow x^2 + y^2 - \frac{6}{5}xy + (\frac{3}{5}y)^2 - (\frac{3}{5}y)^2$

$(x^2 - 2\frac{3}{5}xy + \frac{9}{25}y^2) + y^2 - \frac{9}{25}y^2$

$(x - \frac{3}{5}y)^2 + \frac{16}{25}y^2$

$f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}\left[(x-\frac{3}{5}y)^2 + \frac{16}{25}y^2\right]} =$

$\frac{5}{8\pi} e^{-\frac{1}{2}y^2} \cdot e^{-\frac{1}{2}\frac{(x-3y/5)^2}{(4/5)^2}}$

$\Rightarrow f_Y(y) = \frac{4\sqrt{2\pi}}{8\pi \cdot 5} e^{-\frac{1}{2}y^2} \underbrace{\int_{-\infty}^{\infty} e^{-\frac{1}{2}\frac{(x-\frac{3y}{5})^2}{(4/5)^2}} \frac{5}{4\sqrt{2\pi}}}_{\text{integral porque } N(\frac{3y}{5}; (\frac{4}{5})^2)} = \frac{\sqrt{2}}{2\sqrt{\pi}} e^{-\frac{1}{2}y^2}$

$\Rightarrow f_Y(y) = \frac{\sqrt{2}}{2\sqrt{\pi}} e^{-\frac{1}{2}y^2}$ $\forall \{y \in \mathbb{R}\}$

$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$, $f_X(x) = \frac{\sqrt{2}}{2\sqrt{\pi}} e^{-\frac{1}{2}x^2}$ $\forall \{x \in \mathbb{R}\}$

$f_{Y|X=x}(y) = \frac{\frac{5}{8\pi}\cancel{e^{-\frac{1}{2}x^2}} e^{-\frac{1}{2}\frac{(y-\frac{3x}{5})^2}{(4/5)^2}}}{\frac{\sqrt{2}}{2\sqrt{\pi}}\cancel{e^{-\frac{1}{2}x^2}}} = \frac{5}{4\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(y-\frac{3x}{5})^2}{(4/5)^2}\right]}$

$Y|X=x \sim N\left(\frac{3x}{5}; (\frac{4}{5})^2\right)$

X e Y no son indep porque $f_{Y|X=x}(y) \neq f_Y(y)$

b) $P(1 < XY < 5 \,|\, X=\sqrt{5}) = P(1 < Y|_{X=\sqrt{5}} < 5) = F_{Y|X=\sqrt{5}}(5) - F_{Y|X=\sqrt{5}}(1) = P\left(\frac{1 - 3\sqrt{5}/5}{4/5} < Z < \frac{5 - 3\sqrt{5}/5}{4/5}\right) = \phi\left(\frac{25 - 3\sqrt{5}}{4}\right) -$

$\phi\left(\frac{5 - 3\sqrt{5}}{4}\right) = \phi(4{,}573) - \phi(-0{,}427) = \phi(4{,}573) - (1 -$

$\phi(0{,}427)) = 1 - 1 + \phi(0{,}427) = \phi(0{,}427) =$

5.5) $X \sim U(3,4)$ $x \in (3,4)$ $Y|X=x \sim N(x, 1)$

$f_Y(y) = \frac{f_{XY}(x,y)}{f_{X|Y=y}}$

$$f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} \Rightarrow f_{XY}(x,y) = f_{Y|X=x}(y) \cdot f_X(x) =$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-x)^2} \mathbb{1}\{3<x<4\}$$

$$\Rightarrow f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y-x)^2}\,dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-y)^2}\,d$$

$$\Rightarrow f_Y(y) \sim N(x, 1)$$

$$\Rightarrow f_Y(5) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(5-x)^2}$$

$$P(X>3{,}5 \mid Y=5) = P(X|_{Y=5} > 3{,}5) = P(X>3{,}5) =$$

$$X|_{Y=y} = \mathbb{1}\{3<x<4\} \qquad = P(3{,}5<X<4) =$$

5.6) $T_1, T_2 \sim \exp(\lambda)$ independientes

$X_1 = T_1 + T_2$, $X_2 = T_1 - T_2$, $X_3 = \frac{T_1}{T_2}$

a) $f_{X_1X_2}(x_1,x_2) = ?$

$$f_{T_1T_2}(t_1,t_2) = \lambda^2 e^{-\lambda(t_1+t_2)} \mathbb{1}\{t_1>0, t_2>0\}$$

$(X_1, X_2) = (T_1+T_2, T_1-T_2)$ $\qquad X_1 = T_1 + T_2 \quad X_2 = T_1 - T_2$

$g(t_1,t_2) = (t_1+t_2, t_1-t_2) = (x_1,x_2)$ $\qquad X_1 = T_1 + T_1 - X_2 \quad T_2 = T_1 - X_2$

$T_2 = \frac{X_1}{2} - \frac{X_2}{2}$ $X_2 \Rightarrow T_2 = \frac{X_1 - X_2}{2}$ $\qquad X_1 + X_2 = 2T_1 \Rightarrow T_1 = \frac{X_1+X_2}{2}$

$$T_1 + T_2 = \frac{X_1+X_2+X_1-X_2}{2} = X_1$$

$g^{-1}(x_1,x_2) = (x_1, x_2)$

$$T_1 - T_2 = \frac{X_1+X_2-X_1+X_2}{2} = X_2$$

$$|Jac| = \begin{vmatrix} \frac{dt_1}{dx_1} & \frac{dt_1}{dx_2} \\ \frac{dt_2}{dx_1} & \frac{dt_2}{dx_2} \end{vmatrix} = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = \left| -\frac{1}{4} - \frac{1}{4} \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

$$f_{X_1X_2}(x_1,x_2) = f_{T_1T_2}(t_1,t_2)\Big|_{g^{-1}(x_1,x_2)} \cdot |Jac| = \frac{1}{2}\lambda^2 e^{-\lambda(x_1)} \mathbb{1}\{x_1>0\}$$

$\Rightarrow f\quad(x_1,x_2) = \frac{1}{2}\lambda^2 e^{-\lambda x_1} \mathbb{1}\{x_1>0, -x_1<x_2<x_1\}$ no son indep

b) $X_1 = T_1 + T_2$, $X_3 = \frac{T_1}{T_2}$

$(X_1, X_3) = (T_1 + T_2, T_1 / T_2)$

$g(t_1, t_2) = (t_1 + t_2, t_1 / t_2) = (x_1, x_3)$

$T_1 = X_1 - T_2$

$X_3 = \frac{X_1 - T_2}{T_2}$

$T_2 X_3 = X_1 - T_2$

$T_2 (X_3 + 1) = X_1$ $\qquad T_1 = X_1 - \frac{X_1}{X_3 + 1} = \frac{X_1(X_3+1) - X_1}{X_3 + 1}$

$\Rightarrow T_2 = \frac{X_1}{X_3 + 1}$ $\qquad T_1 = \frac{X_1 X_3}{X_3 + 1}$ $\qquad T_1 + T_2 = \frac{X_1 + X_1 X_3}{X_3 + 1}$

$g^{-1}(x_1, x_3) = (x_1, x_3)$ $\qquad \frac{T_1}{T_2} = \frac{X_1 X_3}{X_3 + 1} \cdot \frac{X_3 + 1}{X_1} = X$

$$|Jac| = \begin{vmatrix} \frac{dt_1}{dx_1} & \frac{dt_1}{dx_3} \\ \frac{dt_2}{dx_1} & \frac{dt_2}{dx_3} \end{vmatrix} = \begin{vmatrix} \frac{x_3}{x_3+1} & \frac{x_1(x_3+1) - x_1 x_3}{(x_3+1)^2} \\ \frac{1}{x_3+1} & \frac{-x_1}{(x_3+1)^2} \end{vmatrix} = \left| \frac{-x_3 x_1}{(x_3+1)^3} - \frac{x_1(x_3+1) - x_1 x_3}{(x_3+1)^3} \right|$$

$$= \left| \frac{\cancel{-x_3 x_1} - x_3 x_1 - x_1 + \cancel{x_3 x_1}}{(x_3+1)^3} \right| = \left| \frac{-x_1(1 + x_3)}{(x_3+1)^3} \right| = \frac{x_1}{(x_3+1)^2}$$

$$f_{X_1 X_2}(x_1, x_2) = f_{T_1 T_2}(t_1, t_2) \Big|_{g^{-1}(x_1, x_2)} |Jac|$$

$$f_{X_1 X_2}(x_1, x_2) = \frac{x_1 \lambda^2 e^{-\lambda x_1}}{(x_3+1)^2} \mathbb{1}\{x_1 > 0, x_3 > 0\} = \lambda^2 x_1 e^{-\lambda x_1} \mathbb{1}\{x_1$$

$\frac{1}{(x_3+1)^2} \mathbb{1}\{x_3 > 0\}$ $\Rightarrow$ son independientes

c) $$f_{X_1 | X_2 = x_2}(x_1) = \frac{f_{X_1 X_2}(x_1, x_2)}{f_{X_2}(x_2)}$$

$$f_{X_2}(x_2) = \int_{-\infty}^{\infty} f_{X_1 X_2}(x_1, x_2)\, dx_1 = \int_{-\infty}^{\infty} \frac{1}{2} \lambda^2 e^{-\lambda x_1}\, dx_1 = \frac{1}{2} \lambda^2 \int_0^{\infty} e^{-\lambda x_1}\, dx$$

$$= \frac{1}{2}\lambda^2 \left(\frac{e^{-\lambda x_1}}{-\lambda} \right)\Big|_0^{\infty} = \frac{1}{2}\lambda \left[0 + 1 \right] = \frac{1}{2}\lambda \Rightarrow f_{X_2}(x_2) = \frac{1}{2}\lambda\, \mathbb{1}\{x_2 > 0\}$$

$$\Rightarrow f_{X_1 | X_2 = x_2}(x_1) = \frac{\frac{1}{2} \lambda^2 e^{-\lambda x_1}\, \mathbb{1}\{x_1 > 0, -x_1 < x_2 < x_1\}}{\frac{1}{2}\lambda\, \mathbb{1}\{x_2 > 0\}}$$

$$\Rightarrow f_{X_1 | X_2 = x_2}(x_1) = \lambda e^{-\lambda x_1} \mathbb{1}\{x_1 > 0\}, \ -x_1 < x_2 < x_1$$

$X_1 |_{X_2 = x_2} \sim exp(\lambda)$

$$f_{X_1|X_3=x_3}(x_1) = \lambda^2 x_1 e^{-\lambda x_1} \mathbb{1}\{x_1>0\} \sim \Gamma(2,\lambda)$$

$$\Rightarrow f_{X_1|X_3=1}(x_1) = \lambda^2 x_1 e^{-\lambda x_1} \mathbb{1}\{x_1>0\}$$

d) $P(X_1>1 | X_2=0) = P(X_1|_{X_2=0} > 1) = 1 - P(X_1|_{X_2=0} \leq 1) = 1 - F_{X_1|X_2=0}(1) = 1 - (1-e^{-\lambda}) = e^{-\lambda}$

$P(X_1>1|X_3=1) = P(X_1|_{X_3=1} > 1) = 1 - P(X_1|_{X_3=1} \leq 1) = 1 - F_{X_1|X_3=1}(1)$
$= 1 - \lambda e^{-\lambda}$

5.7) $T_1, T_2 \sim exp(\lambda)$ independientes

$S_1 = T_1$, $S_2 = T_1 + T_2$

a) $f_{T_1T_2}(t_1,t_2) = \lambda^2 e^{-\lambda(t_1+t_2)} \mathbb{1}\{t_1>0, t_2>0\}$

$(S_1, S_2) = (T_1, T_1+T_2)$ $\quad S_2 = S_1 + T_2 \Rightarrow T_2 = S_2 - S_1$, $T_1 = S_1$

$g(t_1,t_2) = (t_1, t_1+t_2) = (s_1, s_2)$ $\quad |Jac| = \begin{vmatrix} \frac{dt_1}{ds_1} & \frac{dt_1}{ds_2} \\ \frac{dt_2}{ds_1} & \frac{dt_2}{ds_2} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1$

$\Rightarrow g^{-1}(s_1,s_2) = (s_1, s_2 - s_1)$

$f_{S_1S_2}(s_1,s_2) = \lambda^2 e^{-\lambda s_2} \mathbb{1}\{s_1>0, s_2>s_1\}$ densidad conjunta

$$f_{S_1}(s_1) = \int_{-\infty}^{\infty} f_{S_1S_2}(s_1,s_2)\,ds_2 = \int_{-\infty}^{\infty} \lambda^2 e^{-\lambda s_2}\,ds_2 = \lambda^2 \int_0^{\infty} e^{-\lambda s_2}\,ds_2 = \lambda^2 \left(\frac{e^{-\lambda s_2}}{-\lambda}\right)\Big|_0^{\infty} =$$

$\lambda[0-(-1)] = \lambda$

$\Rightarrow f_{S_1}(s_1) = \lambda \mathbb{1}\{s_1 > 0\}$

$$f_{S_2}(s_2) = \int_{-\infty}^{\infty} f_{S_1S_2}(s_1,s_2)\,ds_1 = \int_0^{s_2} \lambda^2 e^{-\lambda s_2}\,ds_1 = \lambda^2 e^{-\lambda s_2} s_2$$

$\Rightarrow f_{S_2}(s_2) = \lambda^2 s_2 e^{-\lambda s_2} \mathbb{1}\{s_2>0\}$ $\quad S_2 \sim \Gamma(2,\lambda)$

no son independientes

b) $P(S_1 \in [1/2; 1] | S_2 = 2)$

$$f_{S_1|S_2=s_2}(s_1) = \frac{f_{S_1S_2}(s_1,s_2)}{f_{S_2}(s_2)} = \frac{\cancel{\lambda^2} \cancel{e^{-\lambda s_2}} \mathbb{1}\{s_2>0, s_2>s_1\}}{s_2 \cancel{\lambda^2} \cancel{e^{-\lambda s_2}} \mathbb{1}\{s_2>0\}} = \frac{1}{s_2}\mathbb{1}\{s_2>s_1\}$$

$P(1/2 < S_1|_{S_2=2} < 1) = \frac{1}{2} - \frac{0.5}{2} = \frac{1}{4}$

5.8) T_i = "tiempo de llegada de i" $i = D, R$

$f_{XY}(x,y) = \frac{1}{35} e^{-\left(\frac{x}{5}+\frac{y}{7}\right)} \mathbb{1}\{x>0, y>0\}$

$t_0 = 0 \quad T_D = X \qquad P(X<Y \mid W=5) = ?$

$T_R = Y$

$U = \min\{X,Y\}$; $V = \max\{X,Y\}$; $W = \max\{X,Y\} - \min\{X,Y\}$

$\Rightarrow W = V - U$

$f_U(u) = (\lambda_x + \lambda_y) e^{-(\lambda_x+\lambda_y)u}$ (por 4.14)

$f_X(x) = \frac{1}{5} e^{-\frac{x}{5}} \mathbb{1}\{x>0\} \quad f_Y(y) = \frac{1}{7} e^{-\frac{y}{7}} \mathbb{1}\{y>0\}$

$f_W(w) = \frac{\lambda x}{\lambda_x+\lambda_y} f_Y(w) + \frac{\lambda y}{\lambda_y+\lambda_x} f_X(w)$ (por 4.14)

$\Rightarrow P(X<Y \mid W=5) \underset{\text{B. y mezcla}}{=} \dfrac{f_{W|X<Y}(5) \; P(X<Y)}{f_W(5)}$ $\underbrace{\frac{1/5}{1/5+1/7}}$

$f_{W|X<Y}(w) = \frac{\lambda x}{\lambda x+\lambda y} f_Y(w) \Rightarrow f_{W|X<Y}(5) = \frac{7}{12} \frac{1}{7} e^{-\frac{5}{7}} = \frac{1}{12} e^{-\frac{5}{7}}$

$P(X<Y) = \frac{\lambda x}{\lambda x+\lambda y} = \frac{7}{12}$

$f_W(5) = \frac{7}{12} \frac{1}{7} e^{-5/7} + \frac{5}{12} \frac{1}{5} e^{-5/5} = \frac{1}{12} e^{-5/7} + \frac{1}{12} e^{-1}$

$\Rightarrow P(X<Y \mid W=5) = \dfrac{\frac{1}{12} e^{-\frac{5}{7}} \cdot \frac{7}{12}}{\frac{1}{12} e^{-5/7} + \frac{1}{12} e^{-1}} = \dfrac{\cancel{\frac{1}{12} e^{-\frac{5}{7}}} \; \frac{7}{12}}{\cancel{\frac{1}{12} e^{-5/7}} \left(1 + e^{-2/7}\right)}$

$= \frac{7}{12} \frac{1}{(1+e^{-2/7})} \approx 0{,}333$

5.9) $X = S + N$

S es discreta, equiprob sobre $\{0{,}1 ; 0{,}2 ; 0{,}3\}$

$N \sim N(0,1)$ S y N independientes

$P(S=0{,}2 \mid X=0{,}87)$

$X|_{S=0,1} \sim N(0,1;1)$; $X|_{S=0,2} \sim N(0,2;1)$; $X|_{S=0,3} \sim N(0$

$$P(S=0,2 \mid X=0,87) \overset{\text{BP.M}}{=} \frac{f_{X|S=0,2}(0,87)\, P(S=0,2)}{f_X(0,87)}$$

$$f_X(0,87) = f_{X|S=0,2}(0,87)P(S=0,2) + f_{X|S=0,1}(0,87)P(S=0,1) + f_{X|S=0,3}(0,87)P(S=0,3)$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0,87-0,2)^2} \cdot \frac{1}{3} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0,87-0,1)^2} \cdot \frac{1}{3} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0,87-0,3)^2} \cdot \frac{1}{3}$$

$$= \frac{1}{3\sqrt{2\pi}} \left(e^{-\frac{1}{2}\left(\frac{67}{100}\right)^2} + e^{-\frac{1}{2}\left(\frac{77}{100}\right)^2} + e^{-\frac{1}{2}\left(\frac{57}{100}\right)^2} \right) \cong 0,318$$

$$f_{X|S=0,2}(0,87)\,P(S=0,2) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{67}{100}\right)^2} \approx 0,1062$$

$$\Rightarrow P(S=0,2 \mid X=0,87) = \frac{0,1062}{0,318} \approx 0,3339$$

5.10) P("fallados") $= \frac{1}{4}$ P("detect fallados") $= \frac{4}{5}$

X = "cantidad de VoboCops fallados en 6 R-C"

$X \sim Bi(6, 1/4)$

Y = "cantidad detectados en 6 R-C"

$Y \sim Bi(6, 4/5)$

a) $\varphi(x) = E[Y \mid X=x]$

$Y|_{X=x}$ = "cant de detectados en x fallados"

$Y|_{X=x} \sim Bi(x, 4/5)$

$$E[Y \mid X=x] = \frac{4x}{5}$$

b) $X|_{Y=y}$ = "cantidad de fallados dado que se detectaron y"

$X|_{Y=y}$ = y + "cantidad de fallados no detectados en 6-y R-C"

$Z|_{Y=y}$ = "cant de fallados no detectados en 6-y R-C"

$$Z\big|_{Y=y} \sim Bi\left(6-y\,;\,P(\text{fallo no det})\right)$$

$$P(\text{fallo y no det}) = P(\text{No det}\,|\,\text{Fallo})\cdot P(\text{fallo}) = \frac{1}{5}\cdot\frac{1}{4} = \frac{1}{20}$$

$$\Rightarrow Z\big|_{Y=y} \sim Bi\left(6-y\,;\,1/20\right)$$

$$\Rightarrow X\big|_{Y=y} = y + Z\big|_{Y=y}$$

$$E[X|Y=y] = y + \frac{(6-y)}{20} = \frac{19y}{20} + \frac{3}{10}$$

5.11) $(\Omega, \mathcal{A}, P)$ / $\Omega = \{1, 2, \dots, 11, 12\}$ $\mathcal{A} = 2^{\Omega}$ $P(\{\omega\}) = \frac{1}{1[illegible]}$

$\forall \omega \in \Omega$ $X: \Omega \to \mathbb{R}$ / $X(\omega) = \mathbb{1}\{\omega \in \{1,2\}\} + 2\,\mathbb{1}\{\omega \in \{3,4,5,6[illegible]$

$+ 3\,\mathbb{1}\{\omega \in \{7, 8, \dots, 12\}\}$

a) $Y(\omega) = \omega$

X	$P_X(x)$
1	2/12
2	4/12
3	6/12

$Y \backslash X$	1	2	3	$P_Y(y)$
1	1/12	0	0	1/12
2	1/12	0	0	1/12
3	0	1/12	0	1/12
4	0	1/12	0	1/12
5	0	1/12	0	1/12
6	0	1/12	0	1/12
7	0	0	1/12	1/12
8	0	0	1/12	1/12
9	0	0	1/12	1/12
10	0	0	1/12	1/12
11	0	0	1/12	1/12
12	0	0	1/12	1/12
$P_X(x)$	2/12	4/12	6/12	

$$P(Y|X=x) = \frac{P_{XY}(x,y)}{P_X(x)}$$

$$P(Y|X=1) = \frac{1/12}{2/12} = \frac{1}{2}$$

$Y\|_{X=2}$	$P(Y\|X=2)$
1	0
2	0
3	1/4
4	1/4
5	1/4
6	1/4
7	0
8	0
9	0
10	0
11	0
12	0

$Y\|_{X=3}$	$P(Y\|X=3)$
1	0
2	0
3	0
4	0
5	0
6	0
7	1/6
8	1/6
9	1/6
10	1/6
11	1/6
12	1/6

$Y\|_{X=1}$	$P(Y$[illegible]
1	1[illegible]
2	1[illegible]
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

$$E[Y|X=x] = \frac{1}{2}\mathbb{1}\{x=1\} + \frac{1}{4}\mathbb{1}\{x=2\} + \frac{1}{6}\mathbb{1}\{x=3\}$$

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5.12) X = " cant de resultados pares en 36 tiros " $X \sim Bi(3...$

Y = " cantidad de resultados impares en 36 tiros "

$Y|_{X=x} = 36 - x \Rightarrow E[Y|X] = 36 - X$

~~$E[Y|X] = E[36 - X] = 36 - E[X] = 36 - \frac{36}{2} = 18$~~

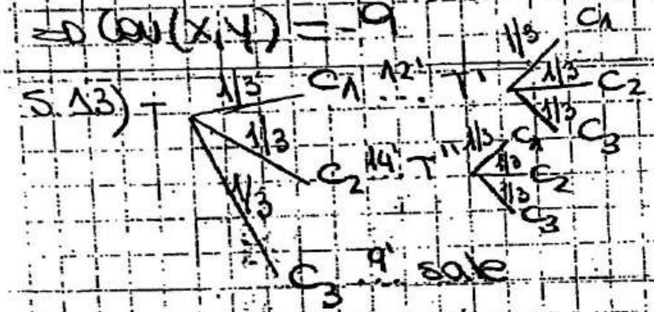
$Cov(X,Y) = ?$ Como $E[Y|X]$ es una recta $\Rightarrow$ es la que mejor aproxima a Y

$$36 - X = \frac{Cov(X,Y)}{Var(X)}[E[X] - X] + E[Y]$$

$$\Rightarrow -1 = \frac{Cov(X,Y)}{Var(X)} \Rightarrow -Var(X) = Cov(X,Y)$$

$$\Rightarrow Cov(X,Y) = -9$$

5.13)



T' y T'' distribuyen como T

$T|C_1 = 12' + T'$

$T|C_2 = 14' + T''$

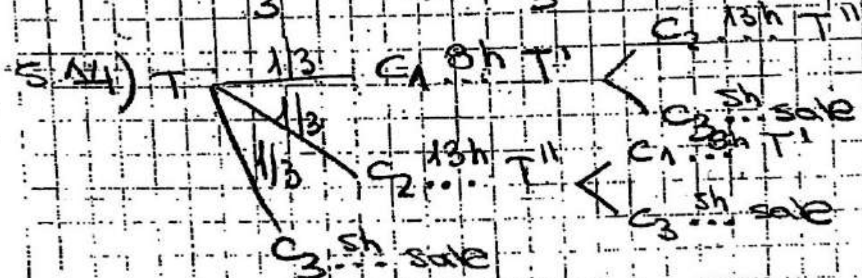
$T|C_3 = 9'$

$$E[T] = E[T|C_1]P(C_1) + E[T|C_2]P(C_2) + E[T|C_3]P(C_3)$$

$$= E[12' + T]\frac{1}{3} + E[14' + T]\frac{1}{3} + E[9'] = \frac{12'}{3} + \frac{1}{3}E[T] + \frac{14'}{3} + \frac{1}{3}E[T] + 9' = \frac{2}{3}E[T] + \frac{35'}{3} \Rightarrow$$

$$E[T] - \frac{2}{3}E[T] = \frac{35'}{3} \Rightarrow E[T] = 35'$$

5.14)



$T|C_1 = 8h + T'$

$T|C_2 = 13h + T''$

$T|C_3 = 5h$

$T'|C_2 = 13h + T''$

$T'|C_3 = 5h$

$T''|C_1 = 8h + T'$

$T''|C_3 = 5h$

$E[T] = E[T|C_1]P(C_1) + E[T|C_2]P(C_2) + E[T|C_3]P(C_3) =$
$(8h + E[T']) \frac{1}{3} + (13h + E[T'']) \frac{1}{3} + E[5h] \frac{1}{3} =$
$\frac{8h}{3} + \frac{1}{3}E[T'] + \frac{13h}{3} + \frac{1}{3}E[T''] + \frac{5h}{3} = \frac{26}{3}h + \frac{E[T']}{3}$
$\frac{E[T'']}{3} - *$

$E[T'] = E[T'|C_2]P(C_2) + E[T'|C_3]P(C_3) = 9h + \frac{1}{2}E[$
$E[T''] = E[T''|C_1]P(C_1) + E[T''|C_3]P(C_3) = \frac{13h}{2} + \frac{1}{2}E[$
$= \frac{13h}{2} + \frac{1}{2}\left[9h + \frac{1}{2}E[T'']\right] = 11h + \frac{1}{4}E[T'']$

$\frac{3}{4}E[T''] = 11h \Rightarrow E[T''] = \frac{44}{3}h$

$E[T'] = 9h + \frac{1}{2}\frac{44h}{3} = \frac{49h}{3}$

$* \; E[T] = \frac{26h}{3} + \frac{49}{9}h + \frac{44}{9}h \Rightarrow E[T] = 19h$

S.15) $X \sim N(0,1)$
$E[Y|X] = X^2$ $\quad \underline{Cov(X,Y) = Cov(X, E(Y|X))}$ N.E. impar
$E[X^3] = 0$
$Cov(X,Y) = ?$
$Cov(X,Y) = Cov(X, E(Y|X)) = E[X E[Y|X]] - E[X]E[E[Y|X]] =$
$\underbrace{E[X^3]}_{=0} - \underbrace{E[X]}_{=0}E[X^2] \Rightarrow Cov(X,Y) = 0$

S.16) $X \sim exp(1/2)$ $\quad E[Y|X] = X \quad , \quad Var[Y|X] = X$
$Var[Y] = Var[E[Y|X]] + E[Var[Y|X]] = Var[X] + E[X]$
$= 4 + 2 = 6 \Rightarrow Var[Y] = 6$

5.17) a) $\varphi(x) = E[Y|X=x]$, $\phi(x) = Var[Y|X=x]$

$$P_{Y|X=x}(y) = \frac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}} \qquad Y|X=x \sim Hip(N=6, n=3-x, \ldots$$

$$E[Y|X=x] = \frac{(3-x)\cancel{3}}{\cancel{6}\,2} = \frac{3}{2} - \frac{x}{2}$$

$$Var[Y|X=x] = \frac{(3-x)3}{6}\,\frac{(6-3)}{6}\,\frac{(6-(3-x))}{(6-1)} = \frac{(9-3x)3(3+x)}{36(5)}$$

$$= \frac{(9-3x)(9+3x)}{180} = \frac{81+27x-27x-9x^2}{180} = \frac{81-9x^2}{180}$$

$$Var[Y|X=x] = \frac{9-x^2}{20}$$

b) $E[Y|X] = \frac{3}{2} - \frac{X}{2}$

$$Var[Y|X] = \frac{9}{20} - \frac{X^2}{20}$$

5.18) a) $f_{XY}(x,y) = \frac{1}{2x+1} e^{-(2x+\frac{y}{4x+2})} \mathbb{1}\{x>0, y>0\}$

1) $f_{Y|X=x}(y) = \frac{1}{2(2x+1)} e^{-\frac{y}{2(2x+1)}} \mathbb{1}\{y \geq 0\}$ $\quad Y|X=x \sim exp\left(\frac{1}{4x+2}\right)$

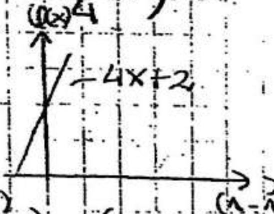
2) $\varphi(x) = E[Y|X=x] = 4x+2$

3) $\varphi(X) = E[Y|X] = 4X+2$

4) $F_{E[Y|X]}(q) = P(Q \leq q) = P(4X+2 \leq q) = P\left(X \leq \frac{q-2}{4}\right) =$

$$1 - e^{-2\left(\frac{q-2}{4}\right)} = 1 - e^{-\frac{q}{2}} e^{1} = 1 - e^{\left(1-\frac{q}{2}\right)}$$

$$\Rightarrow F_{E[Y|X]}(q) = 1 - e^{\left(1-\frac{q}{2}\right)} \mathbb{1}\{q > 0\}$$



5) $P(1 < E[Y|X] < 2) = F_Q(2) - F_Q(1) = \left(1 - e^{(1-1)}\right) - \left(1 - e^{(1-1/2)}\right)$

$$= -1 + e^{1/2} \approx 0{,}6487$$

6) $\phi(x) = Var[Y|X=x] = (4x+2)^2$

7) $Var[Y|X] = (4X+2)^2$

8) $F_{Var[Y|X]}(c) = P(C \leq c) = P\left((4X+2)^2 \leq c\right) = P\left(|4X+2| \leq \sqrt{c}\right) =$

$$P\left(-\sqrt{c} \leq 4X+2 \leq \sqrt{c}\right) = P\left(\frac{-\sqrt{c}-2}{4} \leq X \leq \frac{\sqrt{c}-2}{4}\right) =$$

$$F_X\left(\frac{\sqrt{c}-2}{4}\right) - F_X\left(\frac{-\sqrt{c}-2}{4}\right) = \left(1 - e^{-2\left(\frac{\sqrt{c}-2}{4}\right)}\right) - \left(1 - e^{-2\left(\frac{-\sqrt{c}}{4}\right.}\right.$$

$$= -e^{\left(\frac{\sqrt{c}-2}{2}\right)} + e^{-\left(\frac{-\sqrt{c}-2}{2}\right)} = -e^{-\frac{\sqrt{c}}{2}} e^{1} + e^{\frac{\sqrt{c}}{2}} e^{1} =$$

$$e^{1}\left(-e^{-\frac{\sqrt{c}}{2}} + e^{\frac{\sqrt{c}}{2}}\right)$$

9) $P(Var(Y|X) > 1) = P((4X+2)^2 > 1) = P(|4X+2| > 1) =$

$P(-1 < 4X+2 < 1) = P\left(\frac{-1-2}{4} < X < \frac{1-2}{4}\right) = P\left(-\frac{3}{4} < X < -\frac{1}{4}\right)$

$F_X(-1/4) - F_X(-3/4) = \left(1 - e^{-2\left(\frac{1}{4}\right)}\right) - \left(1 - e^{-2\left(\frac{3}{4}\right)}\right) =$

10) $Var(Y) = E(Var(Y|X)) + Var(E(Y|X)) = E((4X+2)^2) + Var(4X+2)$

$E(16X^2 + 16X + 4) + 16Var(X) = 16E(X^2) + 16E(X) + 4 + 16V$

$= 16\frac{3}{4} + 16\frac{1}{2} + 4 + 16\frac{1}{4} = 28 \Rightarrow Var[Y] = 28$

S 19) $f_{XY}(x,y) = \frac{13}{48\pi}\exp\left(-\frac{169}{288}\left(\frac{(x-1)^2}{4} - \frac{5(x-1)y}{13} + y^2\right)\right)$

a) $E[Y|X]$ $\quad f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$

$f_X(x) = \int_{-\infty}^{\infty} f_{xy}\, dy$

$f_{XY}(x,y) = \frac{13}{48\pi}\exp\left(-\frac{169}{288}\left(\left(y - \frac{5}{26}(x-1)\right)^2 + \frac{(x-1)^2}{4} - \frac{25}{676}(x-1)^2\right)\right)$

$= \frac{13}{48\pi} e^{-\frac{1}{8}(x-1)^2} e^{-\frac{1}{2}\left(\frac{\left(y-\frac{5(x-1)}{26}\right)^2}{(12/13)^2}\right)}$ $\quad \frac{1}{2}\left(\frac{169}{144}\right.$

$\Rightarrow f_X(x) = \int_{-\infty}^{\infty} \frac{13}{48\pi} e^{-\frac{1}{8}(x-1)^2} e^{-\frac{1}{2}\left(\frac{(y-5(x-1)/26)^2}{(12/13)^2}\right)} dy = \frac{e^{-\frac{1}{8}(x-1)^2}}{4\pi}\sqrt{2}$

$$\underbrace{\int_{-\infty}^{\infty} \frac{13}{12}\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{(y-5(x-1)/26)^2}{(12/13)^2}\right)} dy}_{\longrightarrow 1 \,\sim N\left(\frac{5(x-1)}{26};\left(\frac{12}{13}\right)^2\right)} \Rightarrow f_X(x) = \frac{\sqrt{2}}{4\sqrt{\pi}} e^{-\frac{1}{8}(x-1)^2}$$

$$\Rightarrow f_{Y|X=x}(y) = \frac{\frac{13}{48\pi}\cancel{e^{-\frac{1}{8}(x-1)^2}} e^{-\frac{1}{2}(\cdots)}}{\frac{\sqrt{2}}{4\sqrt{\pi}}\cancel{e^{-\frac{1}{8}(x-1)^2}}} = \frac{13\sqrt{2}}{24\sqrt{\pi}} e^{-\frac{1}{2}\left(\frac{\left(y-\frac{5(x-1)}{26}\right)^2}{(12/13)^2}\right)} =$$

$$\frac{13}{12\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\left(y-\frac{5(x-1)}{26}\right)^2}{(12/13)^2}\right)} \Rightarrow Y|X=x \sim N\left(\frac{5(x-1)}{26};\left(\frac{12}{13}\right)^2\right)$$

$$E[Y|X=x] = \frac{5(x-1)}{26} \Rightarrow E[Y|X] = \frac{5(X-1)}{26}$$

b) $\hat{Y} = E[Y|X] = \frac{cov(X,Y)}{Var(X)}(X - E[X]) - E[Y]$

$\sigma_X^2 = 4$, $\sigma_Y^2 = 1$, $\mu_X = 1$, $\mu_Y = 0$ $\quad \frac{\cancel{2}\rho}{\cancel{\sigma_X\sigma_Y}} = \frac{5}{13}$

$\Rightarrow \rho = \frac{5}{13} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \Rightarrow \frac{5}{13} = \frac{cov(X,Y)}{2.1} \Rightarrow cov(X,Y) = \frac{10}{13}$

$\Rightarrow \hat{Y} = \frac{10/13}{4}\left[X - 1\right] = \frac{5}{26}(X-1)$

5.20) $(X,Y) \sim U(\{(x,y)\in\mathbb{R}^2 : 0<x<\frac{3}{4}, 0<y<\frac{3}{4}\} \cup \{(x,y)\in\mathbb{R}^2 : \frac{3}{4}<x<1, \frac{3}{4}<y<1\})$

$E[Y|X]$, $Var[Y|X]$, $E[Y|X] = Q$

$E[Y|X=x] = \frac{3}{8} 1\{0<x<\frac{3}{4}\} + \frac{7}{8} 1\{\frac{3}{4}<x<1\}$

$E[Y|X] = \frac{3}{8} 1\{0<X<\frac{3}{4}\} + \frac{7}{8} 1\{\frac{3}{4}<X<1\}$

$P(3/8) = \frac{\frac{3}{4}\frac{3}{8}}{\frac{3}{4}\frac{3}{4} + (1-\frac{3}{4})(1-\frac{3}{4})} = \frac{9/32}{\frac{9}{16}+\frac{1}{16}} = \frac{9}{20}$

$P(7/8) = \frac{\frac{1}{4}\frac{1}{8}}{\frac{10}{16}} = \frac{1}{20}$

$\Rightarrow F_Q(q) = \frac{9}{20} 1\{0<X<\frac{3}{4}\} + \frac{1}{20} 1\{\frac{3}{4}<X<1\}$

5.21) L = "longitud de los rollos de tela" $L \sim U(20,30)$ metros

a) N = "cantidad rollos producidos hasta el 1ro que mide al menos 28 metros" $N \sim Geo(1/5)$

$E[N] = 5$

b) W = "cantidad total de tela producida"

$E[W] = E[E[W|N]]$

$W|N=n = \sum_{i=1}^{n-1} L_i | L_i < 28 + L_n | L_n \geq 28$

$E[W|N=n] = E\left[\sum_{i=1}^{n-1} L_i | L_i<28 + L_n | L_n \geq 28\right] = (n-1) E[\underbrace{L_i 1\{L_i<28\}}_{U(20,28)}] + E[\underbrace{L_n 1\{L_n \geq 28\}}_{U(28,30)}] = (n-1)\,24 + 29 = n24 + 5$

$E[W|N] = 24N + 5$

$E[W] = E[24N+5] = 5 + 24E[N] = 5 + 24 \times 5 = 125$

$\Rightarrow E[W] = 125$

c) Y = "cantidad de tela de los rollos que irán a stock"

$E[Y] = E[E[Y|N]]$

$E[Y|N=n] = E\left[\sum_{i=1}^{n-1} L_i \mid L_i < 28\right] = (n-1)24 = n24 - 24$

$E[Y|N] = N24 - 24$

$E[Y] = E[N24 - 24] = E[N]24 - 24 = 120 - 24 = 96$

$\Rightarrow E[Y] = 96$

5.22) X = "cantidad de impactos por segundo" $X \sim Po(2)$

Y = "desplazamiento en el impacto i"

$-1 \xleftarrow{2/5} 0 \xrightarrow{3/5} 1$

$p_X(x) = \frac{2^x}{x!} e^{-2}, x \in \mathbb{N}_0$

Y_i	P
-1	2/5
1	3/5

F = "posición final de la partícula"

$F = \sum_{i=1}^{X} Y_i \Rightarrow E[F] = E[E[F|X]]$

$F|_{X=x} = \sum_{i=1}^{x} Y_i$

$E[F|X=x] = E\left[\sum_{i=1}^{x} Y_i\right] = xE[Y_i]$

$E[Y_i] = E[Y|D]P(D) + E[Y|I]P(I) = 1 \cdot \frac{3}{5} - 1 \cdot \frac{2}{5} = \frac{1}{5}$

$\Rightarrow E[F|X=x] = x\frac{1}{5} \Rightarrow E[F|X] = \frac{X}{5}$

$E[F] = E\left[\frac{X}{5}\right] = \frac{1}{5}E[X] = \frac{2}{5}$

otra forma:

D = "cant de mov a la derecha"

I = "cant de mov a la izquierda"

$F = I - D \qquad I \sim Bi(x, \frac{3}{5}) \qquad D \sim Bi(x-I, 2/5)$

$F|_{X=x} = I - (x-I) = 2I - x$

$E[F|X=x] = E[2I - x] = 2E[I] - E[x] = 2\frac{3}{5}x - E[x] = \frac{6}{5}x - E[x]$

$E[F|X] = \frac{6}{5}X - E[X] = \frac{6}{5}X - 2$

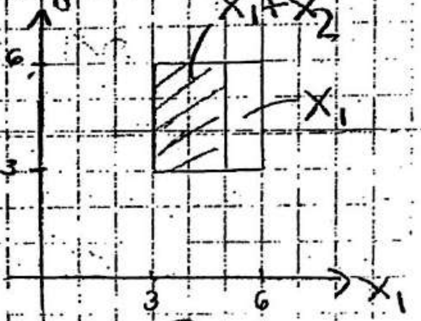
$E[F] = E\left[\frac{6}{5}X - 2\right] = \frac{6}{5}2 - 2 = \frac{2}{5}$

5.23) X = "peso de una bolsa de naranja"

$X_i \sim U(3,6)$ kilos

P = "peso final de la balanza"

$P = \begin{cases} X_1 & si\ X_1 > 5 \\ X_1 + X_2 & si\ X_1 \leq 5 \end{cases}$



$E[P] = E[\underbrace{X_1 | X_1 > 5}_{U(5;6)}] P(X_1 > 5) + E[\underbrace{X_1}_{U(3,5)} + \underbrace{X_2}_{U(3,6)} | X_1 \leq 5] \Rightarrow$

$P = g(x_1, x_2) \Rightarrow E[P] = E[g(x_1,x_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1,x_2) f_{X_1X_2} dx_1 dx_2$

$= \int_3^6\int_3^5 (x_1+x_2)\frac{1}{9} dx_1 dx_2 + \int_3^6\int_5^6 x_1 \frac{1}{9} dx_1 dx_2 = (1)\ \frac{1}{9}\int_3^6 \left(\frac{x_1^2}{2} + x_2 x_1\right)\Big|_3^5 dx_2$

$= \frac{1}{9}\int_3^6 8 + 2x_2\, dx_2 = \frac{1}{9}\left(8x_2 + x_2^2\right)\Big|_3^6 = \frac{17}{3}$

$\frac{17}{3} + (2) \qquad (2) = \int_3^6 \left(\frac{x_1^2}{18}\right)\Big|_5^6 dx_2 = \int_3^6 \frac{11}{18} dx_2 = \left(\frac{11}{18}x_2\right)\Big|_3^6 = \frac{11}{6}$

$\Rightarrow E[P] = \frac{17}{3} + \frac{11}{6} = \frac{15}{2} = 7,5$

$Var[P] = E[P^2] - E[P]^2, \quad E[P^2] = \int_3^6\int_3^5 (x_1+x_2)^2 \frac{1}{9} dx_1 dx_2 + \int_3^6\int_5^6 x_1^2 \frac{1}{9}$

$= 63$

$\Rightarrow Var[P] = 63 - \left(\frac{15}{2}\right)^2 = \frac{27}{4}$

5.24) $E[X] = E[E[X|S]]$

$E[X|S=s] = s \Rightarrow E[X|S] = S$

$\Rightarrow E[X] = E[S] = 0{,}1 \cdot \frac{1}{3} + 0{,}2 \cdot \frac{1}{3} + 0{,}3 \cdot \frac{1}{3}$

ή $E[X] = E[X|S=0{,}1]\,P(S=0{,}1) + E[X|S=0{,}2]\,P(S=0$
$+ E[X|S=0{,}3]\,P(S=0{,}3)$

$\Rightarrow E[X] = \frac{1}{5}$

$Var[X] = E[Var[X|S]] + Var[E[X|S]] = E[1] + Var$
$= 1 + Var[S] = 1 + (0{,}1)^2 \frac{1}{3} + (0{,}2)^2 \frac{1}{3} + (0{,}3)^2 \frac{1}{3} = \frac{151}{150}$

$\Rightarrow Var[X] = \frac{151}{150}$