BTP-1 : Shepherding of Multiple Evaders with Single Pursuer

Saqib Azim - 150070031

Advisor : Prof. Debraj Chakraborty

Electrical Engineering Department

Indian Institute of Technology, Bombay

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What is Shepherding ?

Figure: An example of a trained sheepdog herding the sheeps

A Modern way of Shepherding

Figure: Drones being used for shepherding

Source : `https://www.aljazeera.com/video/news/2017/03/zealand-sheep-farmers-drones-shepherd-170308065102076.html`

Applications of Shepherding in other domains

- Shepherding of large number of livestock farm animals using robots or drones
- Controlling the motion of crowd (for example, by a police or robots) to drive them towards a destination
- Collecting or Driving multiple robots using a controller robot

Notation

Various notations have been used in literature to denote the sheep and sheepdog

- Sheeps will be henceforth denoted as **Evader**
- Shepherd or Sheepdogs will be henceforth denoted as **Pursuer**

Below are some common notations which will be used further in this presentation

- N_e denotes the number of evaders
- N_p denotes the number of pursuers $(= 1)$
- Let $p(t) \in R^2$ denote the position of the pursuer at time instant t
- Let $e_i(t) \in R^2$ denote the position of the i^{th} evader at time instant t
- $d(i, t) = e_i(t) - p(t)$ is the line of sight vector between i^{th} evader and pursuer
- v_{emax} denotes maximum velocity of all evaders
- v_{pmax} denotes maximum velocity of pursuer

- Number of Pursuer $N_p = 1$
- Number of Evader $N_e \geq 1$

Task of the Pursuer

- N_e evaders located in a region
- The task of the pursuer is to drive the evaders within an ϵ radius of a predefined fixed destination point $Z \in R^2$ based on some interaction rules

Interaction Rules

1. **Repulsion on evader due to the pursuer**
2. Repulsion or Attraction between two evaders depending on the distance between them
3. Attraction force on an evader towards the centroid (local or all) of evaders.

Assumption : Each agent has complete information about the state of all other agents

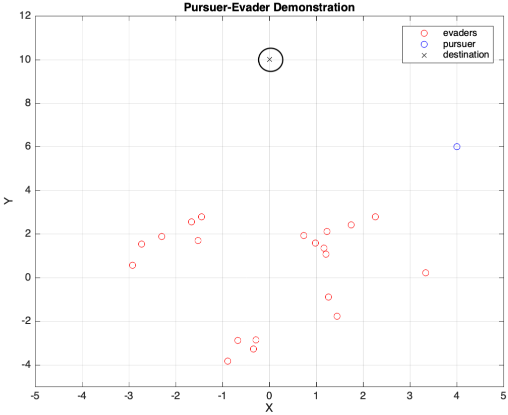

Figure: Pursuer Evader Problem Demonstration

How repulsion depends on distance ?

- Repulsion experienced by evader due to the pursuer should depend on the distance between them
- Inverse relation between distance $\|d(i,t)\|$ and the velocity of evader $\|v_e\|$

$$\|v_e\|_2 \propto \frac{1}{\|d(i,t)\|} \tag{1}$$

$$\|v_e\|_2 \propto \frac{1}{\|d(i,t)\|^2} \tag{2}$$

- Unbounded nature of above functions as $\|d(i,t)\| \to 0$
- Use an exponentially decaying function as it is always bounded and finite

$$\|v_e\|_2 \propto \exp(-k * d(i,t)) \tag{3}$$

How repulsion depends on distance ?

- Not much difference between the three curves once $d(i,t) \geq 1$ (approx)

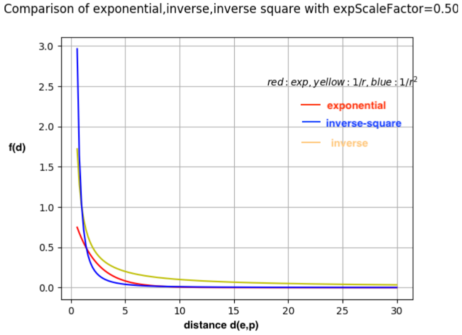

Figure: Comparison of exponential decaying, inverse and inverse-square speed relation with $\|d(i,t)\|$

Aim of the Project

- The entire task of driving all the evaders to the destination can be divided into iterative steps of Aggregation and Driving
- Analyze the trajectory taken by pursuer and evaders during aggregation and driving
- Develop a strategy for the shortest path the pursuer can take to achieve its goal

Velocity of the i^{th} evader is given by

$$\dot{e}_i(t) = k_1 \exp(-k_2 * \|d(i,t)\|_2) u(d_{max} - \|d(i,t)\|) \frac{d(i,t)}{\|d(i,t)\|} \tag{4}$$

$u(t)$ is the unit step function
k_1 and k_2 are constants
d_{max} is the maximum distance after which the repulsion between the pursuer and the evader vanishes

Situation-1 when $Ne = 1$

One Simple Strategy which the pursuer can follow to achieve the target of driving the evader within an ϵ radius of destination is detailed below

- The pursuer aim is to be in a straight line with the evader and the destination point
- For that, it first moves in a straight line with constant speed opposite to evader until $\|d(i,t)\| > d_{max}$
- Then it follows a circular trajectory with the evader as center until $p(t), e_i(t) \text{and} Z$ are in a straight line
- Finally the pursuer drives the evader towards the destination

Situation-1 when $Ne = 1$

Pursuer velocity for the circular trajectory with evader position $e_i(t)$ as center and $\|d(i,t)\|$ as radius are given by below equations

$$\|v_p\| = k_3 \tag{5}$$

$$\dot{p}_x(t) = \frac{k_3}{\sqrt{1 + \frac{(p_x(t)-e_x(t))^2}{(p_y(t)-e_y(t))^2}}}[u(p_x(t) - e_x(t)) - u(e_x(t) - p_x(t))] \tag{6}$$

$$\dot{p}_y(t) = \frac{k_3}{\sqrt{1 + \frac{(p_y(t)-e_y(t))^2}{(p_x(t)-e_x(t))^2}}}[u(p_x(t) - e_x(t)) - u(e_x(t) - p_x(t))] \tag{7}$$

where $u(t)$ is the unit step function, k_3 is a constant

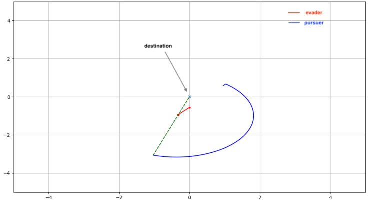

Figure: The pursuer and evader initially move in opposite directions and once $\|d(i,t)\| > d_{max}$, the repulsion on evader vanishes and evader follows a circular trajectory with evader as the center

Evader Velocity

$$\dot{e}_i(t) = k_1 \exp(-k_2\|d(i,t)\|)\frac{d(i,t)}{\|d(i,t)\|} \tag{8}$$

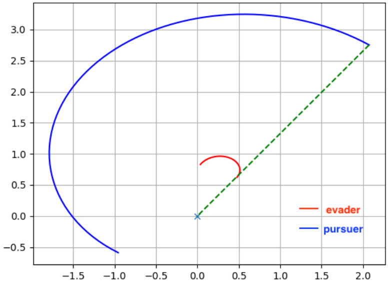

Figure: Following a circular trajectory with evader as center may achieve the target

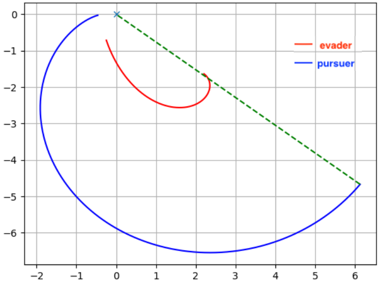

Figure: Following a circular trajectory with evader as center may achieve the target

One possible strategy for the pursuer can be to circle around the centroid of the evaders, but in some cases it may lead to separation of evaders rather than aggregation

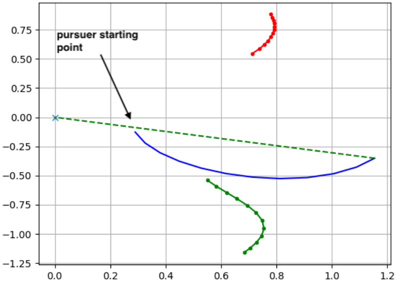

Figure: The pursuer moves in a circle with centroid of two agents as center but it may result in diverging behaviour

Situation-3 when $Ne > 1$

The situations described above are :

- Limited
- Unreliable
- Non-scalable

Shepherding of large number of evaders (say 2-20) can be achieved by following an iterative approach

Algorithm

Until the target is achieved
repeat{

1. Aggregate the evaders within an ϵ radius of their centroid
2. Drive the centroid towards the destination until the distance of farthest evader from the centroid exceeds some threshold (say $d_{farthest}$)

}

Objective

1. Aggregate all the evaders within an ϵ radius of their centroid
2. Minimize the path length of the pursuer

Used numerical optimization using fmincon tool in matlab which has a couple of built-in local optimization algorithms.

Mathematical Formulation

Objective Function

$$\text{minimize} \int_0^T \dot{p}(t)^T \dot{p}(t) dt \tag{9}$$

Constraints

no square in equation 10

$$\|(e_i(T) - \frac{1}{N_e}\sum_{j=1}^{N_e} e_j(T)\|_2^2 \leq \epsilon \quad \forall i = 1, 2.....N_e \tag{10}$$

$$\dot{e}_i(t) = v_{emax} \exp(-k\|d(i,t)\|_2)\frac{(1 + cos\theta_{i,t})}{2}\frac{d(i,t)}{\|d(i,t)\|} \tag{11}$$

$\forall i = 1, 2....N_e \quad \forall 0 < t < T$

$$\|\dot{p}(t)\|_2 \leq v_{pmax} \quad \forall 0 < t < T \tag{12}$$

$cos\theta_{i,t} = \frac{\dot{p}(t).d(i,t)}{\|\dot{p}(t)\|\|d(i,t)\|}$ represents approximately the angle which the velocity vector of pursuer makes with $d(i,t)$

T denotes the time when all evaders have aggregated within ϵ radius of their centroid

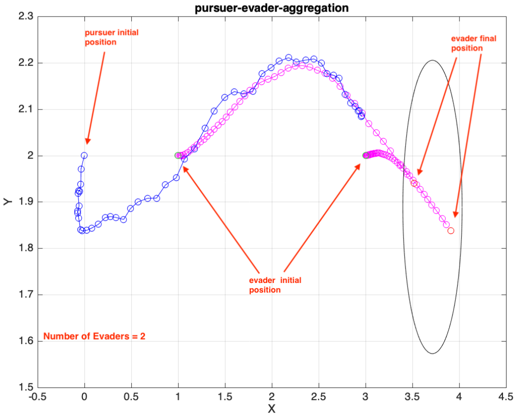

Figure: pursuer-evader aggregation

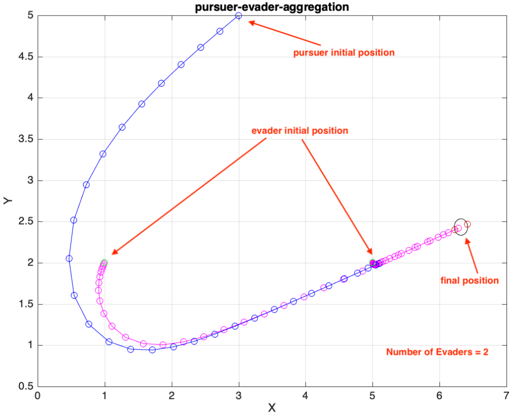

Figure: pursuer-evader aggregation

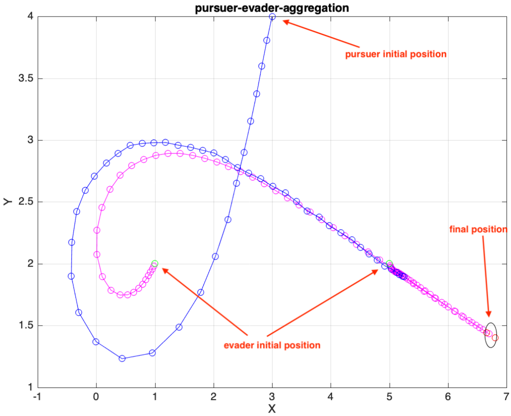

Figure: pursuer-evader aggregation

Results and Plots

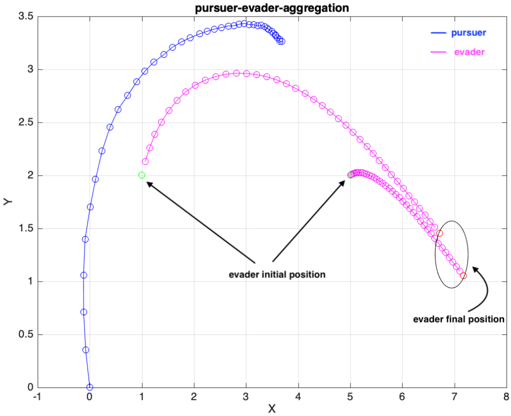

Figure: pursuer-evader aggregation

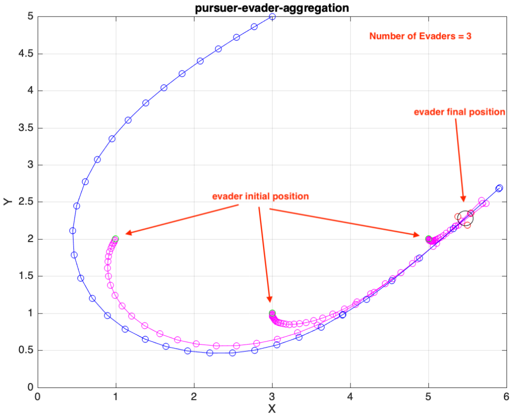

Figure: pursuer-evader aggregation

Observations

- In most cases, the fmincon optimization algorithm either gives an infeasible solution or a local minimum in which some constraints are not satisfied
- Possible Causes : improper parameter tuning such as maximum function evaluations, tolerances on objective function and constraints, tolerance on step size values
- Starting point plays the most important role in reaching local optimum solution

- One important pattern can be seen in all the above figures that the pursuer first approaches one of the nearest evaders
- Tries to bring all of the evaders in an approximate line

Future Steps

1. Try to formulate a feedback control law for the pursuer based on the observations or develop an iterative solution to achieve the same
2. Shift to some other powerful optimization tool such as GPOPS to analyze the results for large number of evaders (say > 10)
3. Once the aggregation is achieved, focus on driving the herd of evaders

1. I would like to thank Prof. Debraj for constantly pushing me and giving the necessary directions every week
2. I would also like to thank Aditya Choudhary for all the help throughout the project

Thank You :)