Shepherding of Multiple Evaders with Single Pursuer

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***Abstract*— Shepherding is a very primitive activity practised to herd the livestock farm animals such as sheep, cows etc. to a particular destination. The driving agent in these cases are the shepherds as well as trained sheepdogs. These sheepdogs have this unique ability to herd large number of sheeps to the destination. For notational convenience, sheepdogs will be henceforth referred as pursuer while sheeps as evaders. This project work involves analysis of some simple situations of pursuer and evader. The entire task of driving the evaders to the destination can be divided into iterative steps of Aggregation and Driving. There is an added constraint that the pursuer needs to achieve the target by traversing minimum path. We show the various possible optimum trajectories of the pursuer and evaders in simple scenarios when number of evaders = 1,2,3.**

***Index Terms* - shepherding; pursuer; evader.**

I. Notation

Below are some common notations which will be used further in this report

N_e denotes the number of evaders

N_p denotes the number of pursuers (= 1 throughout the report) Let $p(t) \in R^2$ denote the position of the pursuer at time instant t

Let $e_i(t) \in R^2$ denote the position of the i^{th} evader at time instant t

$d(i,t) = e_i(t) - p(t)$ is the line of sight vector between i^{th} evader and pursuer

v_{emax} denotes maximum velocity of all evaders

v_{pmax} denotes maximum velocity of pursuer

II. Introduction

The task of the pursuer is to drive the evaders within an ϵ radius of a predefined fixed destination point $Z \in R^2$. This can be achieved with the help of the interaction forces between the agents (pursuer and evaders). There is a in-built assumption that each agent has complete information about the state of all other agents. The interaction forces can be mainly of the following categories.

1) Repulsion on evader due to the pursuer
2) Repulsion or Attraction between two evaders depending on the distance between them
3) Attraction force on an evader towards the centroid (local or all) of evaders.

The work in this project only considers the force of repulsion between the pursuer and evaders. The reason being, if the target is achievable under this condition, then the other two interaction forces described in 2 and 3 should make the task easier as the attraction between evaders and their centroid is only going to favour the aggregation as well as would make them less separable during driving mode.

The repulsion experienced by evader due to the pursuer should depend on the distance between them. If not, it would not be possible to aggregate them in the first place. The repulsion and hence the velocity of the evader should be such that when the pursuer is very far away from the evader, the interaction should be less compared to when the pursuer is nearer to the evader. This motivates us to consider inverse relation between distance between the pursuer and evader $\|d(i,t)\|$ and the velocity of evader.

$$\|v_e\|_2 \propto \frac{1}{\|d(i,t)\|} \tag{1}$$

$$\|v_e\|_2 \propto \frac{1}{\|d(i,t)\|^2} \tag{2}$$

where $\|v_e\|_2 = \|\dot{e}(t)\|$ denotes the speed of the evader

But because of the unbounded nature of these functions as $\|d(i,t)\| \rightarrow 0$, they cannot be used as such since the maximum velocity of both the evaders (v_{emax}) and the pursuer (v_{pmax}) is finite. An alternative way can be to use an exponentially decaying function.

$$\|v_e\|_2 \propto \exp(-k * d(i,t)) \tag{3}$$

where k is a scaling constant

The exponentially decaying function is always bounded and finite and nearly is equal to the inverse and inverse-square functions as $\|d(i,t)\| > 1$ as can be seen in Fig. 1

A. *Situation - 1 :* $N_e = 1$

Velocity of the i^{th} evader is given by

$$\dot{e}_i(t) = k_1 \exp(-k_2 * \|d(i,t)\|_2) u(d_{max} - \|d(i,t)\|) \frac{d(i,t)}{\|d(i,t)\|} \tag{4}$$

where $u(t)$ is the unit step function, k_1 and k_2 are constants and d_{max} is the maximum distance after which the repulsion between the pursuer and the evader vanishes

One Simple Strategy which the pursuer can follow to achieve the target of driving the evader within an ϵ radius of destination is detailed below

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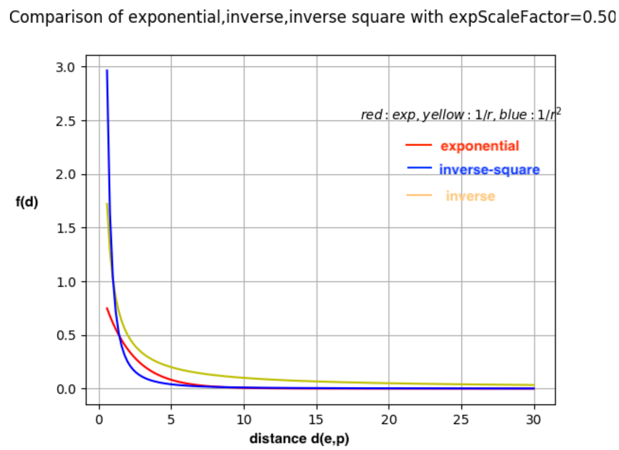

Figure 1: Comparison of exponential decaying, inverse and inverse-square speed relation with $\|d(i,t)\|$

- The pursuer aim is to be in a straight line with the evader and the destination point
- For that, it first moves in a straight line with constant speed opposite to evader until $\|d(i,t)\| > d_{max}$
- Then it follows a circular trajectory with the evader as center until $p(t), e_i(t)$and$Z$ are in a straight line
- Finally the pursuer drives the evader towards the destination

Pursuer velocity for the circular trajectory with evader position $e_i(t)$ as center and $\|d(i,t)\|$ as radius are given by equations 5, 6 and 7

$$\|v_p\| = k_3 \tag{5}$$

$$\dot{p}_x(t) = \frac{k_3}{\sqrt{1+\frac{(p_x(t)-e_x(t))^2}{(p_y(t)-e_y(t))^2}}}[u(p_x(t)-e_x(t))-u(e_x(t)-p_x(t))] \tag{6}$$

$$\dot{p}_y(t) = \frac{k_3}{\sqrt{1+\frac{(p_y(t)-e_y(t))^2}{(p_x(t)-e_x(t))^2}}}[u(p_x(t)-e_x(t))-u(e_x(t)-p_x(t))] \tag{7}$$

where $u(t)$ is the unit step function, k_3 is a constant

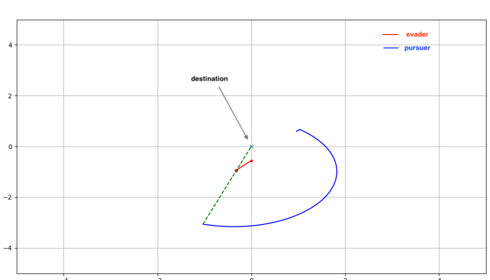

Figure 2: The pursuer and evader initially move in opposite directions and once $\|d(i,t)\| > d_{max}$, the repulsion on evader vanishes and evader follows a circular trajectory with evader as the center until pursuer comes along a straight line with evader and the destination (Z)

B. Situation - 2 : $N_e = 1$

Velocity of the i^{th} evader is given by

$$\dot{e}_i(t) = k_1 \exp(-k_2\|d(i,t)\|)\frac{d(i,t)}{\|d(i,t)\|} \tag{8}$$

where k_1 and k_2 are constants.
The only difference between the evader velocity defined in situation-1 and situation-2 is because of the unit step function. Thus, in this case, the repulsion on evader never vanishes. Unlike in situation-1, the pursuer does not need to move opposite to evader upto d_{max}. In this case, one strategy can be to directly circle around the evader $(e_i(t))$ until the pursuer, evader and destination come in straight line. Note that in this case, the circular motion would not be perfect as the center $(e_i(t))$ is continuously changing.

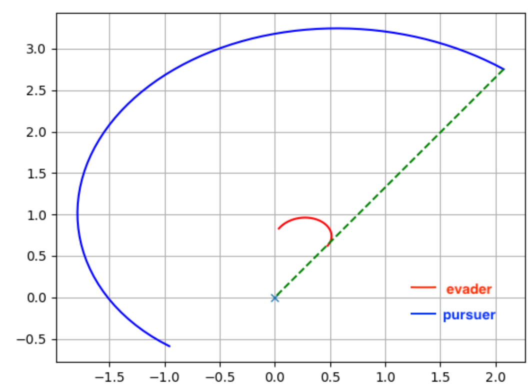

Figure 3: Following a circular trajectory with evader as center may achieve the target

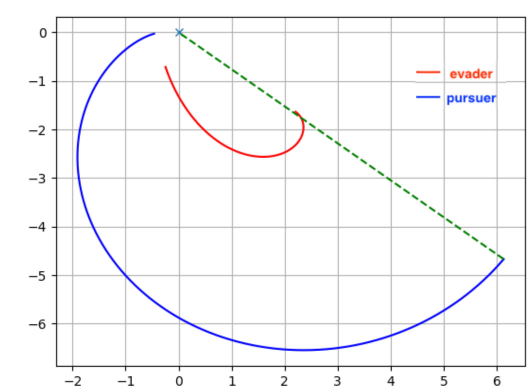

Figure 4: Following a circular trajectory with evader as center may achieve the target

C. Situation - 2 : $N_e = 2$

In case of two evaders, with the velocity of the i^{th} governed by equation 8, one possible strategy for the pursuer can be to circle around the centroid of the evaders, but in some cases it may lead to separation of evaders rather than aggregation as shown in Fig. 5.

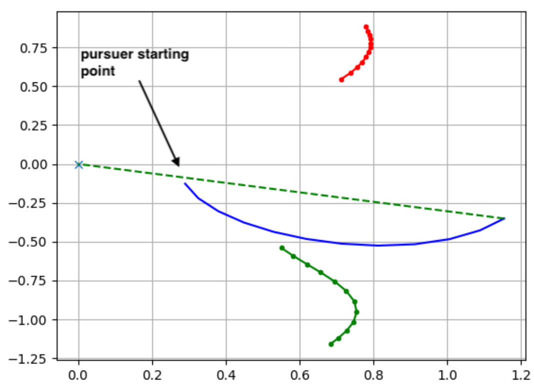

Figure 5: The pursuer moves in a circle with centroid of two agents as center but it may result in diverging behaviour

D. *Situation - 3 : $N_e > 1$*

The situations described above are very limited as well as their approach to their solution such as circling or moving in an ellipse etc, are too specific which can achieve the target in one scenario but can fail in others. Hence there is a need to come up with a general approach which is reliable as well as scalable with the number of evaders N_e.
Shepherding of large number of evaders can be achieved by following an iterative approach in which the pursuer first collects (all or in batches) the evaders and then drives them until the farthest evader's distance from the centroid exceeds some threshold.

Algorithm
Until the target is achieved
repeat

1) Aggregate the evaders within an ϵ radius of their centroid
2) Drive the centroid towards the destination until the distance of farthest evader from the centroid exceeds some threshold (say $d_{farthest}$)

E. *Aggregation or Collection*

As the name suggests, aggregate all the evaders within an ϵ radius of their centroid. From now, the pursuer's task is not just to achieve the target but also minimize the path-length while doing so. The numerical optimization was done using fmincon tool in matlab which has a couple of built-in local optimization algorithms. The optimization problem is described below in detail.

$$\text{minimize} \int_0^T \dot{p}(t)^T \dot{p}(t) dt \tag{9}$$

based on the following constraints

$$\|(e_i(T) - \frac{1}{N_e}\sum_{j=1}^{N_e} e_j(T)\|_2^2 \leq \epsilon \quad \forall i = 1, 2.....N_e \tag{10}$$

$$\dot{e}_i(t) = v_{emax} \exp(-k\|d(i,t)\|_2) \frac{(1 + cos\theta_{i,t})}{2} \frac{d(i,t)}{\|d(i,t)\|} \tag{11}$$

$\forall i = 1, 2....N_e \quad \forall 0 < t < T$

$$\|\dot{p}(t)\|_2 \leq v_{pmax} \quad \forall 0 < t < T \tag{12}$$

where in the above equations $cos\theta_{i,t} = \frac{\dot{p}(t).d(i,t)}{\|\dot{p}(t)\|\|d(i,t)\|}$ represents approximately the angle which the velocity vector of pursuer makes with $d(i,t)$, T denotes the time when all evaders have aggregated within ϵ radius of their centroid, k is a scaling constant

The objective function is not exactly equal to the path length of pursuer but is proportional to it. The first inequality constraint puts the restriction that all the evaders must be inside an ϵ radius of their centroid at time T. The second equality constraint describes the evader velocity as a function of $d(i,t)$ and $\theta_{i,t}$. The third inequality constraint puts an upper bound on the maximum velocity of the pursuer.

III. Experimental Results and Observations

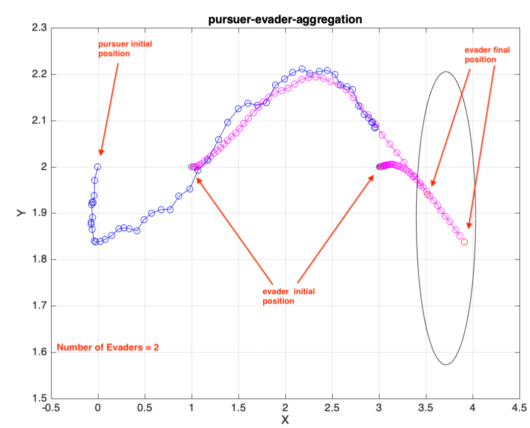

Figure 6: pursuer-evader aggregation

In most cases, the fmincon optimization algorithm either gives an infeasible solution or a local minimum in which some constraints are not satisfied. Some of the reasons for this might probably be because of improper parameter tuning such as maximum function evaluations, maximum iterations, tolerances on objective function and constraints and most importantly the starting point which plays the most important role in reaching local optimum solution. Though the results are shown for only $N_e \leq 3$, one important pattern can be seen in all these above figures that the pursuer first approaches one of the nearest evaders and tries to bring all of them in an approximate line.
These results can help to formulate a feedback control law for the pursuer.

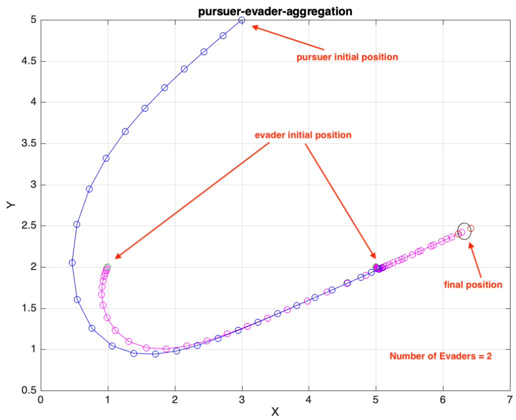

Figure 7: pursuer-evader aggregation

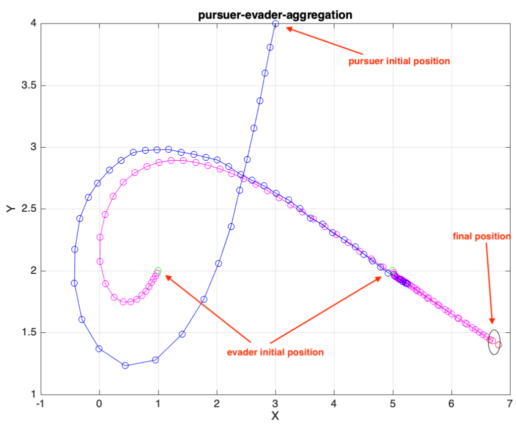

Figure 8: pursuer-evader aggregation

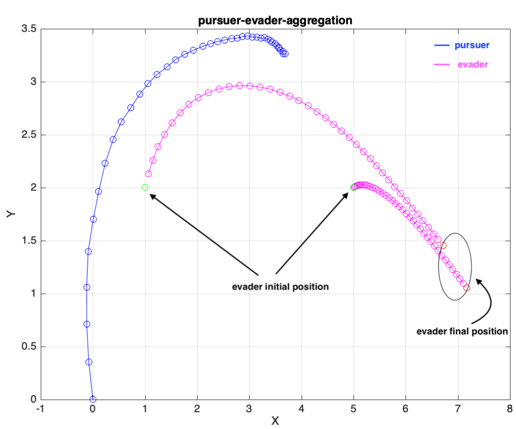

Figure 9: pursuer-evader aggregation

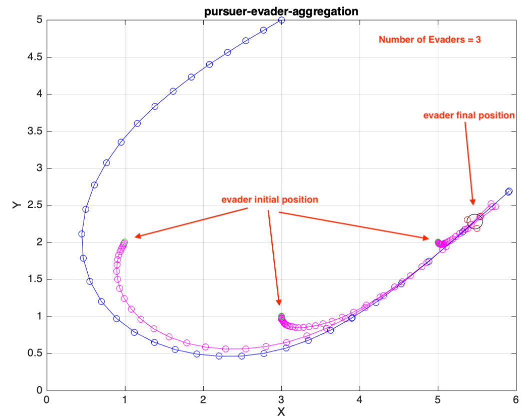

Figure 10: pursuer-evader aggregation

IV. Future Steps

1) Try to formulate a feedback control law for the pursuer based on the observations or develop an iterative solution to achieve the same
2) Shift to some other powerful optimization tool such as GPOPS to analyze the results for large number of evaders (say > 10)
3) Once the aggregation is achieved, focus on driving the herd of evaders