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1 #include <bits/stdc++.h>
2 #define EPS 1e-10
3 #define equals(a, b) (fabs((a) - (b)) < EPS)
4 typedef long long ll;
5 #define PI 3.141592653589793238
6 using namespace std;
7
8 // COUNTER CLOCKWISE
9 static const int CCW_COUNTER_CLOCKWISE = 1;
10 static const int CCW_CLOCKWISE = -1;
11 static const int CCW_ONLINE_BACK = 2;
12 static const int CCW_ONLINE_FRONT = -2;
13 static const int CCW_ON_SEGMENT = 0;
14
15 //Intersect Circle & Circle
16 static const int ICC_SEPERATE = 4;
17 static const int ICC_CIRCUMSCRIBE = 3;
18 static const int ICC_INTERSECT = 2;
19 static const int ICC_INSCRIBE = 1;
20 static const int ICC_CONTAIN = 0;
21
22 struct Point{
23     double x,y;
24     Point(){}
25     Point(double x,double y) :x(x),y(y){}
26     Point operator+(Point p) {return Point(x+p.x,y+p.y);}
27     Point operator-(Point p) {return Point(x-p.x,y-p.y);}
28     Point operator*(double k){return Point(x*k,y*k);}
29     Point operator/(double k){return Point(x/k,y/k);}
30     double norm(){return x*x+y*y;}
31     double abs(){return sqrt(norm());}
32
33     bool operator < (const Point &p) const{
34         return x!=p.x?x<p.x:y<p.y;
35         //grid-point only
36         //return !equals(x,p.x)?x<p.x:!equals(y,p.y)?y<p.y:0;
37     }
38
39     bool operator == (const Point &p) const{
40         return fabs(x-p.x)<EPS && fabs(y-p.y)<EPS;
41     }
42 };
43
44 struct EndPoint{
45     Point p;
46     int seg,st;
47     EndPoint(){}
48     EndPoint(Point p,int seg,int st):p(p),seg(seg),st(st){}
49     bool operator<(const EndPoint &ep)const{
50         if(p.y==ep.p.y) return st<ep.st;
51         return p.y<ep.p.y;
52     }
53 };
54
55 istream &operator >> (istream &is,Point &p){
56     is>>p.x>>p.y;
57     return is;
58 }
59
60 ostream &operator << (ostream &os,Point p){

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61     os<<fixed<<setprecision(12)<<p.x<<" "<<p.y;
62     return os;
63 }
64
65 bool sort_x(Point a,Point b){
66     return a.x!=b.x?a.x<b.x:a.y<b.y;
67 }
68
69 bool sort_y(Point a,Point b){
70     return a.y!=b.y?a.y<b.y:a.x<b.x;
71 }
72
73 typedef Point Vector;
74 typedef vector<Point> Polygon;
75
76 istream &operator >> (istream &is,Polygon &p){
77     for(int i=0;i<(int)p.size();i++) is>>p[i];
78     return is;
79 }
80
81 struct Segment{
82     Point p1,p2;
83     Segment(){}
84     Segment(Point p1, Point p2):p1(p1),p2(p2){}
85 };
86 typedef Segment Line;
87
88 istream &operator >> (istream &is,Segment &s){
89     is>>s.p1>>s.p2;
90     return is;
91 }
92
93 struct Circle{
94     Point c;
95     double r;
96     Circle(){}
97     Circle(Point c,double r):c(c),r(r){}
98 };
99
100 istream &operator >> (istream &is,Circle &c){
101     is>>c.c>>c.r;
102     return is;
103 }
104
105 double norm(Vector a){
106     return a.x*a.x+a.y*a.y;
107 }
108 double abs(Vector a){
109     return sqrt(norm(a));
110 }
111 double dot(Vector a,Vector b){
112     return a.x*b.x+a.y*b.y;
113 }
114 double cross(Vector a,Vector b){
115     return a.x*b.y-a.y*b.x;
116 }
117
118 Point orth(Point p){return Point(-p.y,p.x);}
119
120 bool isOrthogonal(Vector a,Vector b){
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```
121     return equals(dot(a,b),0.0);
122 }
123
124 bool isOrthogonal(Point a1,Point a2,Point b1,Point b2){
125     return isOrthogonal(a1-a2,b1-b2);
126 }
127
128 bool isOrthogonal(Segment s1,Segment s2){
129     return equals(dot(s1.p2-s1.p1,s2.p2-s2.p1),0.0);
130 }
131
132 bool isParallel(Vector a,Vector b){
133     return equals(cross(a,b),0.0);
134 }
135
136 bool isParallel(Point a1,Point a2,Point b1,Point b2){
137     return isParallel(a1-a2,b1-b2);
138 }
139
140 bool isParallel(Segment s1,Segment s2){
141     return equals(cross(s1.p2-s1.p1,s2.p2-s2.p1),0.0);
142 }
143
144 Point project(Segment s,Point p){
145     Vector base=s.p2-s.p1;
146     double r=dot(p-s.p1,base)/norm(base);
147     return s.p1+base*r;
148 }
149
150 Point reflect(Segment s,Point p){
151     return p+(project(s,p)-p)*2.0;
152 }
153
154 double arg(Vector p){
155     return atan2(p.y,p.x);
156 }
157
158 Vector polar(double a,double r){
159     return Point(cos(r)*a,sin(r)*a);
160 }
161
162 int ccw(Point p0,Point p1,Point p2);
163 bool intersectSS(Point p1,Point p2,Point p3,Point p4);
164 bool intersectSS(Segment s1,Segment s2);
165 bool intersectPS(Polygon p,Segment l);
166 int intersectCC(Circle c1,Circle c2);
167 bool intersectSC(Segment s,Circle c);
168 double getDistanceLP(Line l,Point p);
169 double getDistanceSP(Segment s,Point p);
170 double getDistanceSS(Segment s1,Segment s2);
171 Point getCrossPointSS(Segment s1,Segment s2);
172 Point getCrossPointLL(Line l1,Line l2);
173 Polygon getCrossPointCL(Circle c,Line l);
174 Polygon getCrossPointCC(Circle c1,Circle c2);
175 int contains(Polygon g,Point p);
176 Polygon andrewScan(Polygon s);
177 Polygon convex_hull(Polygon ps);
178 double diameter(Polygon s);
179 bool isConvex(Polygon p);
180 double area(Polygon s);
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181 Polygon convexCut(Polygon p,Line l);
182 Line bisector(Point p1,Point p2);
183 Vector translate(Vector v,double theta);
184 vector<Line> corner(Line l1,Line l2);
185 vector< vector<int> >
186 segmentArrangement(vector<Segment> &ss, Polygon &ps);
187
188 int ccw(Point p0,Point p1,Point p2){
189     Vector a = p1-p0;
190     Vector b = p2-p0;
191     if(cross(a,b) > EPS) return CCW_COUNTER_CLOCKWISE;
192     if(cross(a,b) < -EPS) return CCW_CLOCKWISE;
193     if(dot(a,b) < -EPS) return CCW_ONLINE_BACK;
194     if(a.norm()<b.norm()) return CCW_ONLINE_FRONT;
195     return CCW_ON_SEGMENT;
196 }
197
198 bool intersectSS(Point p1,Point p2,Point p3,Point p4){
199     return (ccw(p1,p2,p3)*ccw(p1,p2,p4) <= 0 &&
200         ccw(p3,p4,p1)*ccw(p3,p4,p2) <= 0 );
201 }
202
203 bool intersectSS(Segment s1,Segment s2){
204     return intersectSS(s1.p1,s1.p2,s2.p1,s2.p2);
205 }
206
207 bool intersectPS(Polygon p,Segment l){
208     int n=p.size();
209     for(int i=0;i<n;i++){
210         if(intersectSS(Segment(p[i],p[(i+1)%n]),l)) return 1;
211     }
212     return 0;
213 }
214
215 int intersectCC(Circle c1,Circle c2){
216     if(c1.r<c2.r) swap(c1,c2);
217     double d=abs(c1.c-c2.c);
218     double r=c1.r+c2.r;
219     if(equals(d,r)) return ICC_CIRCUMSCRIBE;
220     if(d>r) return ICC_SEPERATE;
221     if(equals(d+c2.r,c1.r)) return ICC_INSCRIBE;
222     if(d+c2.r<c1.r) return ICC_CONTAIN;
223     return ICC_INTERSECT;
224 }
225
226 bool intersectSC(Segment s,Circle c){
227     return getDistanceSP(s,c.c)<=c.r;
228 }
229
230 int intersectCS(Circle c,Segment s){
231     if(norm(project(s,c.c)-c.c)-c.r*c.r>EPS) return 0;
232     double d1=abs(c.c-s.p1),d2=abs(c.c-s.p2);
233     if(d1<c.r+EPS&& d2<c.r+EPS) return 0;
234     if((d1<c.r-EPS&& d2>c.r+EPS)|| (d1>c.r+EPS&& d2<c.r-EPS)) return 1;
235     Point h=project(s,c.c);
236     if(dot(s.p1-h,s.p2-h)<0) return 2;
237     return 0;
238 }
239
240 double getDistanceLP(Line l,Point p){
241     return abs(cross(l.p2-l.p1,p-l.p1)/abs(l.p2-l.p1));

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241 }
242
243 double getDistanceSP(Segment s,Point p){
244     if(dot(s.p2-s.p1,p-s.p1) < 0.0 ) return abs(p-s.p1);
245     if(dot(s.p1-s.p2,p-s.p2) < 0.0 ) return abs(p-s.p2);
246     return getDistanceLP(s,p);
247 }
248
249 double getDistanceSS(Segment s1,Segment s2){
250     if(intersectSS(s1,s2)) return 0.0;
251     return min(min(getDistanceSP(s1,s2.p1),getDistanceSP(s1,s2.p2)),
252               min(getDistanceSP(s2,s1.p1),getDistanceSP(s2,s1.p2)));
253 }
254
255 Point getCrossPointSS(Segment s1,Segment s2){
256     for(int k=0;k<2;k++){
257         if(getDistanceSP(s1,s2.p1)<EPS) return s2.p1;
258         if(getDistanceSP(s1,s2.p2)<EPS) return s2.p2;
259         swap(s1,s2);
260     }
261     Vector base=s2.p2-s2.p1;
262     double d1=abs(cross(base,s1.p1-s2.p1));
263     double d2=abs(cross(base,s1.p2-s2.p1));
264     double t=d1/(d1+d2);
265     return s1.p1+(s1.p2-s1.p1)*t;
266 }
267
268 Point getCrossPointLL(Line l1,Line l2){
269     double a=cross(l1.p2-l1.p1,l2.p2-l2.p1);
270     double b=cross(l1.p2-l1.p1,l1.p2-l2.p1);
271     if(abs(a)<EPS&&abs(b)<EPS) return l2.p1;
272     return l2.p1+(l2.p2-l2.p1)*(b/a);
273 }
274
275 Polygon getCrossPointCL(Circle c,Line l){
276     Polygon ps;
277     Point pr=project(l,c.c);
278     Vector e=(l.p2-l.p1)/abs(l.p2-l.p1);
279     if(equals(getDistanceLP(l,c.c),c.r)){
280         ps.emplace_back(pr);
281         return ps;
282     }
283     double base=sqrt(c.r*c.r-norm(pr-c.c));
284     ps.emplace_back(pr+e*base);
285     ps.emplace_back(pr-e*base);
286     return ps;
287 }
288
289 Polygon getCrossPointCS(Circle c,Segment s){
290     Line l(s);
291     Polygon res=getCrossPointCL(c,l);
292     if(intersectCS(c,s)==2) return res;
293     if(res.size()>1u){
294         if(dot(l.p1-res[0],l.p2-res[0])>0) swap(res[0],res[1]);
295         res.pop_back();
296     }
297     return res;
298 }
299
300

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301 Polygon getCrossPointCC(Circle c1,Circle c2){
302     Polygon p(2);
303     double d=abs(c1.c-c2.c);
304     double a=acos((c1.r*c1.r+d*d-c2.r*c2.r)/(2*c1.r*d));
305     double t=arg(c2.c-c1.c);
306     p[0]=c1.c+polar(c1.r,t+a);
307     p[1]=c1.c+polar(c1.r,t-a);
308     return p;
309 }
310
311 // IN:2 ON:1 OUT:0
312 int contains(Polygon g,Point p){
313     int n=g.size();
314     bool x=false;
315     for(int i=0;i<n;i++){
316         Point a=g[i]-p,b=g[(i+1)%n]-p;
317         if(fabs(cross(a,b)) < EPS && dot(a,b) < EPS) return 1;
318         if(a.y>b.y) swap(a,b);
319         if(a.y < EPS && EPS < b.y && cross(a,b) > EPS ) x = !x;
320     }
321     return (x?2:0);
322 }
323
324 Polygon andrewScan(Polygon s){
325     Polygon u,l;
326     if(s.size()<3) return s;
327     sort(s.begin(),s.end());
328     u.push_back(s[0]);
329     u.push_back(s[1]);
330     l.push_back(s[s.size()-1]);
331     l.push_back(s[s.size()-2]);
332     for(int i=2;i<(int)s.size();i++){
333         for(int n=u.size();n>=2&&ccw(u[n-2],u[n-1],s[i])!=CCW_CLOCKWISE;n--){
334             u.pop_back();
335         }
336         u.push_back(s[i]);
337     }
338     for(int i=s.size()-3;i>=0;i--){
339         for(int n=l.size();n>=2&&ccw(l[n-2],l[n-1],s[i])!=CCW_CLOCKWISE;n--){
340             l.pop_back();
341         }
342         l.push_back(s[i]);
343     }
344     reverse(l.begin(),l.end());
345     for(int i=u.size()-2;i>=1;i--) l.push_back(u[i]);
346     return l;
347 }
348
349 Polygon convex_hull(Polygon ps){
350     int n=ps.size();
351     sort(ps.begin(),ps.end(),sort_y);
352     int k=0;
353     Polygon qs(n*2);
354     for(int i=0;i<n;i++){
355         while(k>1&&cross(qs[k-1]-qs[k-2],ps[i]-qs[k-1])<0) k--;
356         qs[k++]=ps[i];
357     }
358     for(int i=n-2,t=k;i>=0;i--){
359         while(k>t&&cross(qs[k-1]-qs[k-2],ps[i]-qs[k-1])<0) k--;
360         qs[k++]=ps[i];

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361     }
362     qs.resize(k-1);
363     return qs;
364 }
365
366 double diameter(Polygon s){
367     Polygon p=s;
368     int n=p.size();
369     if(n==2) return abs(p[0]-p[1]);
370     int i=0,j=0;
371     for(int k=0;k<n;k++){
372         if(p[i]<p[k]) i=k;
373         if(!(p[j]<p[k])) j=k;
374     }
375     double res=0;
376     int si=i,sj=j;
377     while(i!=sj||j!=si){
378         res=max(res,abs(p[i]-p[j]));
379         if(cross(p[(i+1)%n]-p[i],p[(j+1)%n]-p[j])<0.0){
380             i=(i+1)%n;
381         }else{
382             j=(j+1)%n;
383         }
384     }
385     return res;
386 }
387
388 bool isConvex(Polygon p){
389     bool f=1;
390     int n=p.size();
391     for(int i=0;i<n;i++){
392         int t=ccw(p[(i+n-1)%n],p[i],p[(i+1)%n]);
393         f&=t!=CCW_CLOCKWISE;
394     }
395     return f;
396 }
397
398 double area(Polygon s){
399     double res=0;
400     for(int i=0;i<(int)s.size();i++){
401         res+=cross(s[i],s[(i+1)%s.size()])/2.0;
402     }
403     return res;
404 }
405
406 double area(Circle c1,Circle c2){
407     double d=abs(c1.c-c2.c);
408     if(c1.r+c2.r<=d+EPS) return 0;
409     if(d<=abs(c1.r-c2.r)){
410         double r=min(c1.r,c2.r);
411         return PI*r*r;
412     }
413     double rc=(d*d+c1.r*c1.r-c2.r*c2.r)/(2*d);
414     double th=acos(rc/c1.r);
415     double ph=acos((d-rc)/c2.r);
416     return c1.r*c1.r*th+c2.r*c2.r*ph-d*c1.r*sin(th);
417 }
418
419 Polygon convexCut(Polygon p,Line l){
420     Polygon q;

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421 for(int i=0;i<(int)p.size();i++){
422     Point a=p[i],b=p[(i+1)%p.size()];
423     if(ccw(l.p1,l.p2,a)!=-1) q.push_back(a);
424     if(ccw(l.p1,l.p2,a)*ccw(l.p1,l.p2,b)<0)
425         q.push_back(getCrossPointLL(Line(a,b),l));
426 }
427 return q;
428 }
429
430 Line bisector(Point p1,Point p2){
431     Circle c1=Circle(p1,abs(p1-p2)),c2=Circle(p2,abs(p1-p2));
432     Polygon p=getCrossPointCC(c1,c2);
433     if(cross(p2-p1,p[0]-p1)>0) swap(p[0],p[1]);
434     return Line(p[0],p[1]);
435 }
436
437 Vector translate(Vector v,double theta){
438     Vector res;
439     res.x=cos(theta)*v.x-sin(theta)*v.y;
440     res.y=sin(theta)*v.x+cos(theta)*v.y;
441     return res;
442 }
443
444 vector<Line> corner(Line l1,Line l2){
445     vector<Line> res;
446     if(isParallel(l1,l2)){
447         double d=getDistanceLP(l1,l2.p1)/2.0;
448         Vector v1=l1.p2-l1.p1;
449         v1=v1/v1.abs()*d;
450         Point p=l2.p1+translate(v1,90.0*(PI/180.0));
451         double d1=getDistanceLP(l1,p);
452         double d2=getDistanceLP(l2,p);
453         if(abs(d1-d2)>d){
454             p=l2.p1+translate(v1,-90.0*(PI/180.0));
455         }
456         res.push_back(Line(p,p+v1));
457     }else{
458         Point p=getCrossPointLL(l1,l2);
459         Vector v1=l1.p2-l1.p1,v2=l2.p2-l2.p1;
460         v1=v1/v1.abs();
461         v2=v2/v2.abs();
462         res.push_back(Line(p,p+(v1+v2)));
463         res.push_back(Line(p,p+translate(v1+v2,90.0*(PI/180.0))));
464     }
465     return res;
466 }
467
468 Polygon tangent(Circle c1,Point p2){
469     Circle c2=Circle(p2,sqrt(norm(c1.c-p2)-c1.r*c1.r));
470     Polygon p=getCrossPointCC(c1,c2);
471     sort(p.begin(),p.end());
472     return p;
473 }
474
475 vector<Line> tangent(Circle c1,Circle c2){
476     vector<Line> ls;
477     if(c1.r<c2.r) swap(c1,c2);
478     double g=norm(c1.c-c2.c);
479     if(equals(g,0)) return ls;
480     Point u=(c2.c-c1.c)/sqrt(g);

```



```

481 Point v=orth(u);
482 for(int s=1;s>=-1;s-=2){
483     double h=(c1.r+s*c2.r)/sqrt(g);
484     if(equals(1-h*h,0)){
485         ls.emplace_back(c1.c+u*c1.r,c1.c+(u+v)*c1.r);
486     }else if(1-h*h>0){
487         Point uu=u*h,vv=v*sqrt(1-h*h);
488         ls.emplace_back(c1.c+(uu+vv)*c1.r,c2.c-(uu+vv)*c2.r*s);
489         ls.emplace_back(c1.c+(uu-vv)*c1.r,c2.c-(uu-vv)*c2.r*s);
490     }
491 }
492
493 return ls;
494 }
495
496 double closest_pair(Polygon &a,int l=0,int r=-1){
497     if(r<0){
498         r=a.size();
499         sort(a.begin(),a.end(),sort_x);
500     }
501     if(r-l<=1) return abs(a[0]-a[1]);
502     int m=(l+r)>>1;
503     double x=a[m].x;
504     double d=min(closest_pair(a,l,m),closest_pair(a,m,r));
505     inplace_merge(a.begin()+l,a.begin()+m,a.begin()+r,sort_y);
506
507     Polygon b;
508     for(int i=l;i<r;i++){
509         if(fabs(a[i].x-x)>=d) continue;
510         for(int j=0;j<(int)b.size();j++){
511             double dy=a[i].y-next(b.rbegin(),j)->y;
512             if(dy>=d) break;
513             d=min(d,abs(a[i]-*next(b.rbegin(),j)));
514         }
515         b.emplace_back(a[i]);
516     }
517     return d;
518 }
519
520 vector<vector<int> >
521 segmentArrangement(vector<Segment> &ss, Polygon &ps){
522     int n=ss.size();
523     for(int i=0;i<n;i++){
524         ps.emplace_back(ss[i].p1);
525         ps.emplace_back(ss[i].p2);
526         for(int j=i+1;j<n;j++)
527             if(intersectSS(ss[i],ss[j]))
528                 ps.emplace_back(getCrossPointSS(ss[i],ss[j]));
529     }
530     sort(ps.begin(),ps.end());
531     ps.erase(unique(ps.begin(),ps.end()),ps.end());
532
533     vector<vector<int> > G(ps.size());
534     for(int i=0;i<n;i++){
535         vector<pair<double,int> > ls;
536         for(int j=0;j<(int)ps.size();j++)
537             if(getDistanceSP(ss[i],ps[j])<EPS)
538                 ls.emplace_back(make_pair(norm(ss[i].p1-ps[j]),j));
539
540         sort(ls.begin(),ls.end());

```

```

541     for(int j=0;j+1<(int)ls.size();j++){
542         int a=ls[j].second,b=ls[j+1].second;
543         G[a].emplace_back(b);
544         G[b].emplace_back(a);
545     }
546 }
547 for(auto &v:G){
548     sort(v.begin(),v.end());
549     v.erase(unique(v.begin(),v.end()),v.end());
550 }
551 return G;
552 }
553
554 int manhattanIntersection(vector<Segment> ss,const int INF){
555     const int BTM = 0;
556     const int LFT = 1;
557     const int RGH = 2;
558     const int TOP = 3;
559
560     int n=ss.size();
561     vector<EndPoint> ep;
562     for(int i=0;i<n;i++){
563         if(ss[i].p1.y==ss[i].p2.y){
564             if(ss[i].p1.x>ss[i].p2.x) swap(ss[i].p1,ss[i].p2);
565             ep.emplace_back(ss[i].p1,i,LFT);
566             ep.emplace_back(ss[i].p2,i,RGH);
567         }else{
568             if(ss[i].p1.y>ss[i].p2.y) swap(ss[i].p1,ss[i].p2);
569             ep.emplace_back(ss[i].p1,i,BTM);
570             ep.emplace_back(ss[i].p2,i,TOP);
571         }
572     }
573     sort(ep.begin(),ep.end());
574
575     set<int> bt;
576     bt.insert(INF);
577
578     int cnt=0;
579     for(int i=0;i<n*2;i++){
580         if(ep[i].st==TOP){
581             bt.erase(ep[i].p.x);
582         }else if(ep[i].st==BTM){
583             bt.emplace(ep[i].p.x);
584         }else if(ep[i].st==LFT){
585             auto b=bt.lower_bound(ss[ep[i].seg].p1.x);
586             auto e=bt.upper_bound(ss[ep[i].seg].p2.x);
587             cnt+=distance(b,e);
588         }
589     }
590
591     return cnt;
592 }
593
594 double area(Polygon ps,Circle c){
595     if(ps.size()<3u) return 0;
596     function<double(Circle, Point, Point)> dfs=
597         [&](Circle c,Point a,Point b){
598             Vector va=c.c-a,vb=c.c-b;
599             double f=cross(va,vb),res=0;
600             if(equals(f,0.0)) return res;

```

```
601     if(max(abs(va),abs(vb))<c.r+EPS) return f;
602     Vector d(dot(va,vb),cross(va,vb));
603     if(getDistanceSP(Segment(a,b),c.c)>c.r-EPS)
604         return c.r*c.r*atan2(d.y,d.x);
605     auto u=getCrossPointCS(c,Segment(a,b));
606     if(u.empty()) return res;
607     if(u.size()>1u&&dot(u[1]-u[0],a-u[0])>0) swap(u[0],u[1]);
608     u.emplace(u.begin(),a);
609     u.emplace_back(b);
610     for(int i=1;i<(int)u.size();i++)
611         res+=dfs(c,u[i-1],u[i]);
612     return res;
613 };
614 double res=0;
615 for(int i=0;i<(int)ps.size();i++)
616     res+=dfs(c,ps[i],ps[(i+1)%ps.size()]);
617 return res/2;
618 }
```