EE698G - Probabilistic Mobile Robotics Assignment

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1

$< p_i, p_j > = 0 \; \forall \; i \neq j$ & $1 \leqslant i, j \leqslant n$ and $p_i \neq 0 \; \forall \; 1 \leqslant i \leqslant n$
Let us assume that $\{p_1, p_2, ..., p_n\}$ is linearly dependent.
$\therefore \quad \sum_{i=1}^{n} c_i p_i = 0$ & $\{c_1, c_2, ..., c_n\} \neq \{0, 0, ..., 0\}$.
$\implies c_j \neq 0$ for some j such that $1 \leqslant j \leqslant n$.
$\implies c_j p_j = -\sum_{i \neq j, 1 \leqslant i \leqslant n} c_i p_i$
$\implies < p_j, c_j p_j > = < p_j, -\sum_{i \neq j, 1 \leqslant i \leqslant n} c_i p_i >$
$\implies c_j < p_j, p_j > = -\sum_{i \neq j, 1 \leqslant i \leqslant n} c_i < p_j, p_i >$

$$\implies c_j < p_j, p_j > = 0 \tag{1}$$

Now, $p_i \neq 0 \; \forall \; 1 \leqslant i \leqslant n$
$\therefore < p_j, p_j > = k \neq 0$ where $k \in R$.
Also, $c_j \neq 0$

$$\implies c_j < p_j, p_j > \neq 0. \tag{2}$$

$\therefore$ We have a contradiction from (1) & (2).
Hence, $\{p_1, p_2, ..., p_n\}$ is linearly independent.

2

Let $H = \begin{bmatrix} H_A \\ H_B \\ H_C \end{bmatrix}$ be the real heights and $\hat{H} = \begin{bmatrix} \hat{H}_A \\ \hat{H}_B \\ \hat{H}_C \end{bmatrix}$ be the estimated heights of the buildings A, B & C.

Expressing the the data collected by the sensor in the form $A\hat{H} + e = b$, where e is the error vector, we have :

Now, $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{H}_A \\ \hat{H}_B \\ \hat{H}_C \end{bmatrix} + e = \begin{bmatrix} -14.22 \\ -23.55 \\ -9.5 \\ 24.64 \\ 38.8 \\ 48.3 \end{bmatrix}$ using the data provided by the instrument.

On minimizing LSE, we have $\hat{H} = (A^T A)^{-1} A^T b = \begin{bmatrix} 24.65 \\ 38.82 \\ 48.27 \end{bmatrix}$

[Solved using GNU Octave 4.0.0; File : '/code/question2.m']

3

The $\mathscr{L}agrangian$ is given by:
$\mathscr{L} = x^T Q x - \lambda(x^T x - I)$ & $\lambda \neq 0$.
Maximizing $x^T Q x$, we have : $\frac{\partial \mathscr{L}}{\partial x} = 0$
$\implies Qx = \lambda x$
$\implies (Q - \lambda I)x = 0$
$\implies \|Q - \lambda I\| = 0$
Also,

$$x^T Q x = x^T \lambda x = \lambda x^T x = \lambda \tag{3}$$

Hence, x is the eigenvector corresponding to the maximum eigenvalue of Q.
The eigenvalues of Q are 1 ± 0.5.
Choosing λ as 1.5, we get $Qx = \lambda x$

Taking $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad Q \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1.5x$, we have :

$x_1 + 0.5x_2 = 1.5x_1 \implies x_1 = x_2$ & $0.5x_1 + x_2 = 1.5x_2 \implies x_1 = x_2$

Ensuring $\|x\| = 1 \implies x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ or $\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$ are values of x for which the maximum value of $x^T Q x$ occurs for $\|x\| = 1$.
The maximum value of $x^T Q x$ is 1.5 [from (3)].

4

4.1 (a)

(i)

We have, $e^{x^2} > 0 \; \forall \; x$ as $e^{-t} > 0 \; \forall \in R$.
Also, $\sigma > 0 \implies \frac{1}{\sigma\sqrt{2\pi}} > 0$
$\implies f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} > 0 \; \forall \; x \in R$

(ii)

f_X is also clearly continuous.

(iii)

$$
\begin{aligned}
&Let\ \ J = \int_{-\infty}^{\infty} f_X dx. \\
&\quad = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx. \\
&Let\ \ t = \frac{x-\mu}{\sqrt{2}\sigma}. \\
&\implies dx = \sqrt{2}\sigma dt \\
&\implies J = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt \\
&Let\ \ I = \int_{-\infty}^{\infty} e^{-t^2} dt \\
&\implies I^2 = \int_{-\infty}^{\infty} e^{-t_1^2} dt_1 \int_{-\infty}^{\infty} e^{-t_2^2} dt_2 \\
&\implies \quad = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(t_1^2+t_2^2)} dt_1 dt_2
\end{aligned}
$$

$$Taking\ \ t_1^2 + t_2^2 = r^2,$$

$$dt_1 dt_2 = r d\theta dr$$

$$\implies I^2 = \int_0^{2\pi} (\int_{-\infty}^{\infty} r e^{-r^2} dr) d\theta$$

$$Let\ \ r^2 = \lambda$$

$$\implies\ 2rdr = d\lambda \implies\ rdr = \frac{d\lambda}{2}$$

$$
\begin{aligned}
&\implies I^2 = \frac{1}{2} \int_0^{2\pi} (\int_0^{\infty} e^{\lambda} d\lambda) d\theta \\
&\qquad = \pi \\
&\implies I = \sqrt{\pi} \\
&\implies J = \frac{1}{\sqrt{\pi}} \sqrt{\pi} \\
&\implies \int_{-\infty}^{\infty} f_X dx = 1
\end{aligned}
$$

(i), (ii) & (iii) $\implies$ f_X is a valid p.d.f.

4.2 (b)

$f_X(\mu + x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$
$f_X(\mu - x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$
Hence, $f_X(\mu + x) = f_X(\mu - x)$
$\therefore f_X$ is symmetric about μ.

4.3 (c)

$$mean,\ E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$
$$= \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx$$

$$Let\ \ t = \frac{x-\mu}{\sqrt{2}\sigma}.$$
$$\implies x = \sqrt{2}\sigma t + \mu$$
$$\implies dx = \sqrt{2}\sigma dt$$

$$\therefore\ E[X] = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} (\sqrt{2}\sigma t + \mu) e^{-t^2} dt$$
$$= \int_{-\infty}^{\infty} \frac{te^{-t^2}}{\sqrt{\pi}} dt + \int_{-\infty}^{\infty} \frac{\mu e^{-t^2}}{\sqrt{\pi}} dt$$

$$Now,\ \int_{-\infty}^{\infty} \frac{te^{-t^2}}{\sqrt{\pi}} dt = 0 \qquad \because\ it\ is\ an\ odd\ function$$
$$\&\ \int_{-\infty}^{\infty} \frac{\mu e^{-t^2}}{\sqrt{\pi}} = \mu \qquad [Using\ result\ of\ 4.1(a)(iii)]$$
$$\therefore\ mean,\quad E[X] = \mu$$

$$variance,\ \Sigma_X = E[X^2] - E[X]^2$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} x^2 e^{-(\frac{(x-\mu)^2}{2\sigma^2})} dx$$

$$Let\ \ t = \frac{x-\mu}{\sqrt{2}\sigma}.$$

$$\implies x = \sqrt{2}\sigma t + \mu$$

$$\implies dx = \sqrt{2}\sigma dt$$

$$\therefore\ E[X^2] = \int_{-\infty}^{\infty} \frac{(\sqrt{2}\sigma t + \mu)^2}{\sqrt{\pi}} e^{-t^2} dt$$

$$= \int_{-\infty}^{\infty} \frac{2\sigma^2 t^2}{\sqrt{\pi}} e^{-t^2} dt + \int_{-\infty}^{\infty} \frac{\mu^2}{\sqrt{\pi}} e^{-t^2} dt + \int_{-\infty}^{\infty} \frac{2\sqrt{2}\sigma t}{\sqrt{\pi}} e^{-t^2} dt$$

$$Now,\ \int_{-\infty}^{\infty} \frac{2\sqrt{2}\sigma t}{\sqrt{\pi}} e^{-t^2} dt = 0 \qquad \because\ it\ is\ an\ odd\ function$$

$$\&\ \int_{-\infty}^{\infty} \frac{\mu^2}{\sqrt{\pi}} e^{-t^2} = \mu^2 \qquad [Using\ result\ of\ 4.1(a)(iii)]$$

$$\therefore\ E[X^2] = \int_{-\infty}^{\infty} \frac{2\sigma^2 t^2}{\sqrt{\pi}} e^{-t^2} dt + \mu^2$$

$$Let\ \ I = \int_{-\infty}^{\infty} t^2 e^{-t^2} dt$$

$$\implies I^2 = \int_{-\infty}^{\infty} t_1^2 e^{-t_1^2} dt_1 \int_{-\infty}^{\infty} t_2^2 e^{-t_2^2} dt_2$$

$$= \int_{-\infty}^{\infty} t_1^2 t_2^2 e^{-t_1^2} e^{-t_2^2} dt_1 dt_2$$

$$Taking\ \ t_1^2 + t_2^2 = r^2,$$

$$dt_1 dt_2 = r d\theta dr$$

$$\implies I^2 = \int_0^{2\pi} (\int_0^{\infty} r^5 e^{-r^2} dr) sin^4\theta cos^4\theta d\theta$$

$$Let\ \ J = \int_0^{\infty} r^5 e^{-r^2} dr\ \ \&\ \ K = \int_0^{2\pi} sin^4\theta cos^4\theta d\theta$$

$$Let\ \ r^2 = \lambda$$

$$\implies 2rdr = d\lambda \implies rdr = \frac{d\lambda}{2}$$

$$\implies J = \frac{1}{2} \int_0^{\infty} t^2 e^{-t} dt$$

$$= \frac{1}{2} \int_0^{\infty} 2te^{-t} dt \qquad [By\ using\ Integration\ by\ parts]$$

$$= \frac{2}{2} \int_0^{\infty} e^{-t} dt = 1 \qquad [By\ using\ Integration\ by\ parts\ again]$$

$$Now,\ \ K = \int_0^{2\pi} sin^4\theta cos^4\theta d\theta$$

$$= \frac{1}{4} \int_0^{2\pi} sin^2(2\theta) d\theta$$

$$= \frac{1}{4} \int_0^{2\pi} \frac{1 - cos(4\theta)}{2} d\theta$$

$$= \frac{\pi}{4}$$

$$
\begin{aligned}
Now,\ I^2 &= JK \\
&= \frac{\pi}{4} \\
\implies I &= \frac{\sqrt{\pi}}{2} \\
\therefore\ E[X^2] &= \frac{2\sigma^2}{\sqrt{\pi}} I + \mu^2 \\
&= \frac{2\sigma^2}{\sqrt{\pi}} \frac{\sqrt{\pi}}{2} + \mu^2 \\
&= \sigma^2 + \mu^2 \\
Hence,\ variance,\ \Sigma_X &= E[X^2] - E[X]^2 \\
&= \sigma^2 + \mu^2 - \mu^2 \\
&= \sigma^2
\end{aligned}
$$

5

We know that :

If $g : S_X \to R$ is strictly monotone with inverse function $g^{-1}(y)$ such that $\frac{dg^{-1}(y)}{dy}$ is continuous. Let $g(S_X) = \{g(x)\ :\ x\ \in\ S_X\}$. Then, $Y = g(X)$ is an absolutely continuous random variable with p.d.f :

$$f_Y(y) = f_X(g^{-1}(y))|\tfrac{dg^{-1}(y)}{dy}|I_{g(S_X)}(y)$$

where, for a set A, $I_A(\cdot)$ denotes its indicator function, i.e., $I_A = \begin{cases} 1,\ if\ y\ \in\ A \\ 0,\ otherwise \end{cases}$

Also, $\because$ X and Y have uniform distribution U[0, 1], $f_X(x) = f_Y(y) = 1$ for $x \in$ [0, 1] and for $y \in$ [0, 1] respectively. Both $f_X(x)$ & $f_Y(y)$ are equal to 0 outside of [0, 1].

(a)

$Z_1 := log(\frac{1}{X})$
Let $g_1(x) = log(\frac{1}{x})$
Now, g_1 is both continuous and strictly monotonous for [0, 1]

$$
\begin{aligned}
Also,\ z_1(x) &= g_1(x) && for\ \ x \in [0,\ 1] \\
Clearly,\ z_1(x) &= g_1(x) \in [0,\ \infty) && for\ \ x \in [0,\ 1] \\
\therefore\ f_{Z_1}(z_1) &= f_X(x)|(\frac{dg_1(x)}{dx})^{-1}| && for\ \ x \in [0,\ 1] \\
&= x && for\ \ x \in [0,\ 1] \\
Now,\ z_1 &= log(\frac{1}{x}) \\
\implies \frac{1}{x} &= e^{z_1} \\
\implies x &= e^{-z_1} \\
Hence,\ f_{Z_1}(z_1) &= e^{-z_1} && for\ \ z_1 \in [0, \infty) \\
Also,\ f_{Z_1}(z_1) &= 0 && if\ \ z_1 < 0
\end{aligned}
$$

(b)

$Z_2 := e^X$

Let $g_2(x) = e^x$

Now, g_2 is both continuous and strictly monotonous for [0, 1].

$$
\begin{aligned}
Also,\ z_2(x) &= g_2(x) && for\ \ x \in [0,\ \ 1] \\
Clearly,\ z_2(x) &= g_2(x) \in [1,\ \ e] && for\ \ x \in [0,\ \ 1] \\
\therefore\ f_{Z_2}(z_2) &= f_X(x)|(\frac{dg_2(x)}{dx})^{-1}| && for\ \ x \in [0,\ \ 1] \\
&= e^x && for\ \ x \in [0,\ \ 1] \\
Now,\ z_2 &= e^{-x} \\
\implies\ x &= log(z_2) \\
Hence,\ f_{Z_2}(z_2) &= e^{-log(z_2)} && for\ \ x \in [0,\ \ 1] \\
&= \frac{1}{z_2} && for\ \ z_2 \in [1,\ \ e) \\
Also,\ f_{Z_2}(z_2) &= 0 && if\ \ z_2 \notin [1,\ \ e)
\end{aligned}
$$

(c)

Let F_{Z_3} be the distribution function of Z_3

$$
\begin{aligned}
joint\ p.d.f\ of\ X,\ Y;\ f_{X,Y}(x,y) &= f_X(x)f_Y(y) && \because\ X,\ Y\ are\ i.i.d \\
&= \begin{cases} 1\ if\ 0 \le x,y \le 1 \\ 0\ otherwise \end{cases} \\
Also,\ F_{Z_3}(z) &= \int_{-\infty}^{z-t_1}(\int_{-\infty}^{\infty} f_{X,Y}(t_1,t_2)dt_1)dt_2 \\
F_{Z_3}(z) &= 0 && if\ z \le 0 \\
We\ have,\ F_{Z_3}(z) &= \int_0^z(\int_0^{z-t_1} 1\ dt_2)dt_1 && if\ 0\ < z \le 1 \\
&= \int_0^z (z-t_1)dt_1 && if\ 0\ < z \le 1 \\
&= \frac{z^2}{2} && if\ 0\ < z \le 1 \\
Now,\ P(\{Z_3 > z\}) &= \int_{z-1}^1(\int_{z-t_1}^1 1\ dt_2)dt_1 && if\ 1\ < z \le 2 \\
&= \int_{z-1}^1 (1-z+t_1)dt_1 && if\ 1\ < z \le 2 \\
&= (2-z)(1-z) + \frac{1-(z-1)^2}{2} && if\ 1\ < z \le 2 \\
&= 2-3z+z^2+\frac{2z-z^2}{2} && if\ 1\ < z \le 2 \\
&= 2-2z+\frac{z^2}{2} && if\ 1\ < z \le 2 \\
We\ know\ that,\ F_{Z_3}(z) &= 1 - P(\{Z > z\}) \\
&= 1-(2-2z+\frac{z^2}{2}) && if\ 1\ < z \le 2 \\
&= -1+2z-\frac{z^2}{2} && if\ 1\ < z \le 2 \\
Also,\ F_{Z_3}(z) &= 1 && if\ z \ge 2 \\
\because\ p.d.f\ of\ Z_3,\ f_{Z_3}(z) &= \frac{dF_{Z_3}(z)}{dz},\ we\ have: \\
f_{Z_3}(z) &= \begin{cases} z & if\ 0\ < z \le 1 \\ 2-z & if\ 1\ < z \le 2 \\ 0 & otherwise \end{cases}
\end{aligned}
$$

6

(a)

$\because\ P(B \cap C) > 0,\ P(B) > 0 \,\&\, P(C) > 0$

$$
\begin{aligned}
Now,\ P(A \cap B|C) &= \frac{P(A \cap B \cap C)}{P(C)} \\
P(A|B \cap C) &= \frac{P(A \cap B \cap C)}{P(B \cap C)} \\
\&\ P(B|C) &= \frac{P(B \cap C)}{P(C)} \\
\therefore\ P(A|B \cap C)P(B|C) &= \frac{P(A \cap B \cap C)}{P(C)}
\end{aligned}
$$

Hence, proved.

(b)

Let ε be the random experiment of the casting of a red die and green die.
Let A be the event that an even number appears on the red die,
B be the event that an odd number appears on the green die,
and C be the event that sum of the numbers on the dice is 7.
Clearly, events A and B are independent events.
Using relative frequency model for assigning probability,

$$
\begin{aligned}
P(A \cap B|C) &= \frac{3}{6} = 0.5 \quad [even\ number\ appears\ on\ red\ die \\
&\qquad\qquad\quad and\, odd\ number\ appears\ on\ green\ die, \\
&\qquad\qquad\quad given\ that\ the\ sum\ is\ 7] \\
P(A|C) &= \frac{3}{6} = 0.5 \quad [even\ number\ appears\ on\ red\ die \\
&\qquad\qquad\quad given\ that\ the\ sum\ is\ 7] \\
P(B|C) &= \frac{3}{6} = 0.5 \quad [odd\ number\ appears\ on\ green\ die \\
&\qquad\qquad\quad given\ that\ the\ sum\ is\ 7]
\end{aligned}
$$

$$\therefore\ P(A|C)P(B|C) = 0.5^2 = 0.25$$

$But,\ \ P(A \cap B|C) = 0.5 \neq P(A|C)P(B|C) = 0.25$
Hence, the premise that $P(A \cap B|C) = P(A|C)P(B|C)$ if A and B are independent events is false.

7

We are given $X = \{\vec{x_1}, \vec{x_2}, ..., \vec{x_n}\}$, $\vec{x_i} \in R^d \; \forall \; 1 \leq i \leq n$

Let $\vec{a} \in R^d$ such that $\|\vec{a}\| = 1$ and variance of $P = \{p_1, p_2, \ldots, p_n\}$ is maximum, where p_i is the projection of $\vec{x_i}$ along $\vec{a}$.

Now, variance of P, V = $\frac{1}{n}\sum_{i=1}^{n} \|p_i - \bar{p}\|^2$, where $\bar{p}$ is the mean of P.

$$
\begin{aligned}
p_i &= \vec{a}^T\vec{x_i} \\
\implies \bar{p} &= \frac{1}{n}\sum_{1}^{n} p_i \\
&= \frac{1}{n}\sum_{1}^{n} \vec{a}^T\vec{x_i} \\
&= \vec{a}^T\frac{1}{n}\sum_{1}^{n} \vec{x_i} \\
&= \vec{a}^T\vec{\mu}_X \;\; where \; \vec{\mu}_X \; is \; the \; mean \; of \; X \\
Also, \;\; V &= \frac{1}{n}\sum_{i=1}^{n} \|p_i - \bar{p}\|^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} (\vec{a}^T\vec{x_i} - \bar{p})^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} (\vec{a}^T\vec{x_i} - \vec{a}^T\vec{\mu}_X)^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} (\vec{a}^T(\vec{x_i} - \vec{\mu}_X))^2 \\
&= \frac{1}{n}\sum_{i=1}^{n} (\vec{a}^T(\vec{x_i} - \vec{\mu}_X))(\vec{a}^T(\vec{x_i} - \vec{\mu}_X))^T \\
&= \frac{1}{n}\sum_{i=1}^{n} \vec{a}^T(\vec{x_i} - \vec{\mu}_X)(\vec{x_i} - \vec{\mu}_X)^T\vec{a} \\
&= \frac{1}{n}\sum_{i=1}^{n} \vec{a}^T(\vec{x_i} - \vec{\mu}_X)(\vec{x_i} - \vec{\mu}_X)^T\vec{a} \\
&= \vec{a}^T(\frac{1}{n}\sum_{i=1}^{n} (\vec{x_i} - \vec{\mu}_X)(\vec{x_i} - \vec{\mu}_X)^T)\vec{a} \\
&= \vec{a}^TQ\vec{a}, \;\; where \; Q \; is \; the \; covariance \; matrix \; of \; X.
\end{aligned}
$$

As $\|\vec{a}\| = 1$, the $\mathscr{L}agrangian$ is given by:

$\mathscr{L} = \vec{a}^TQ\vec{a} - \lambda(\vec{a}^T\vec{a} - I)$ & $\lambda \neq 0$.

Maximizing $\vec{a}^TQ\vec{a}$, we have : $\frac{\partial\mathscr{L}}{\partial\vec{a}} = 0$

$\implies \; Q\vec{a} = \lambda\vec{a}$

$\implies \; (Q - \lambda I)\vec{a} = 0$

$\implies \; \|Q - \lambda I\| = 0$

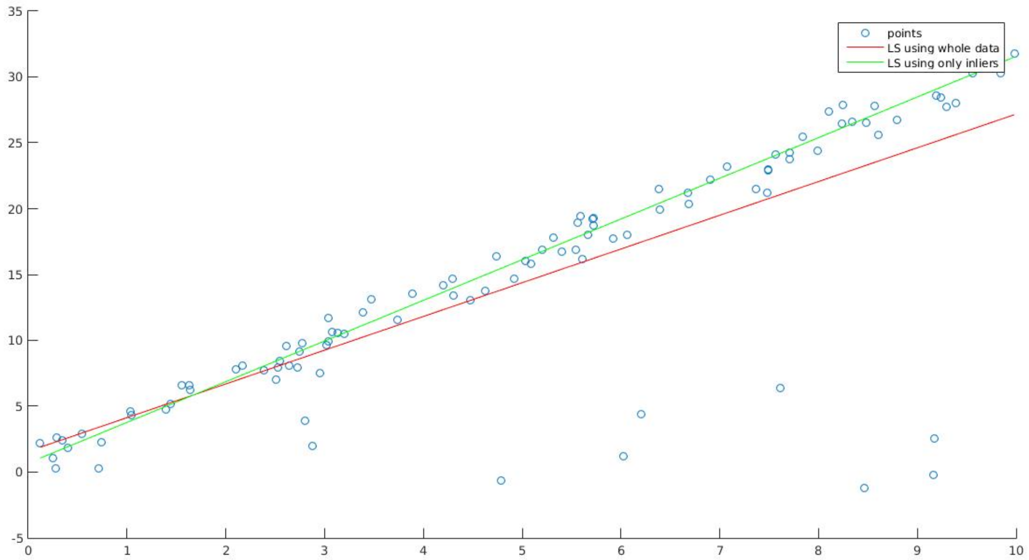
Also,

$$\vec{a}^TQ\vec{a} = \vec{a}^T\lambda\vec{a} = \lambda\vec{a}^T\vec{a} = \lambda$$

Hence, $\vec{a}$ is the eigenvector corresponding to the maximum eigenvalue of Q which is the sample covariance matrix.
$\therefore$ The direction of maximum variance is given by the eigenvector corresponding to the maximum eigenvalue of the sample covariance matrix of the data.
Hence, proved.
$\because Q = (\frac{1}{n}\sum_{i=1}^{n}(\vec{x_i} - \vec{\mu}_X)(\vec{x_i} - \vec{\mu}_X)^T)$, it is a $d \times d$ matrix as x_i & $\vec{\mu}_X$ are d dimensional vectors.
The number of principal components can be taken as n where $1 \leq n \leq d$, $n \in N$.

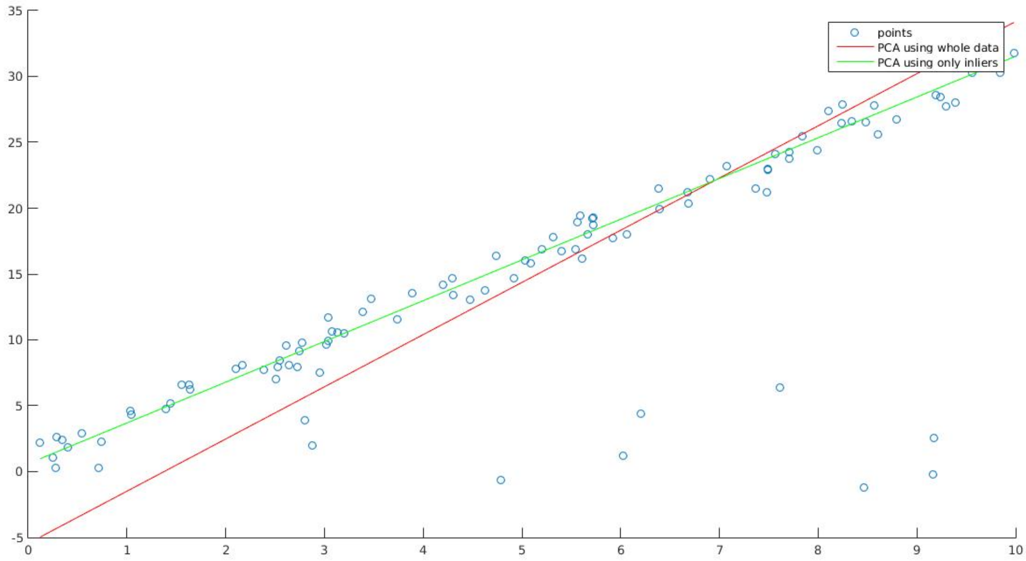
8

8.1 (a)



The line obtained through LSE after running RANSAC algorithm to obtain the inliers set is better than the the line obtained by simply applying LSE over the test data; since on observation, the RANSAC line is closer to more points than the basic line estimate is.

8.2 (b)

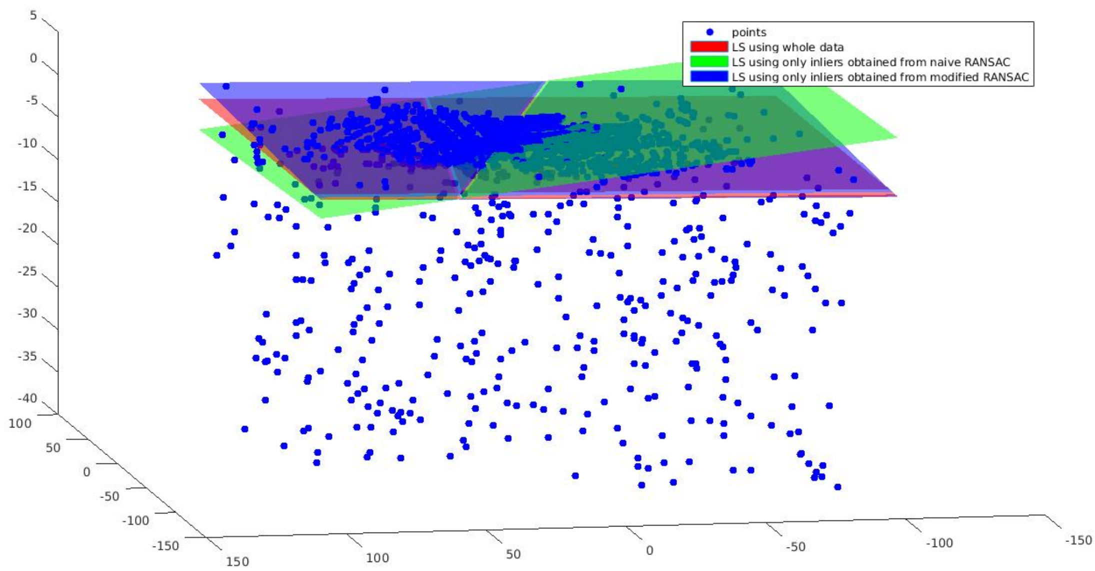


The line obtained through PCA after running RANSAC algorithm to obtain the inliers set is better than the the line obtained by simply applying PCA over the test data; since on observation, the RANSAC line is closer to more points than the basic line estimate is.

Remark:

From the above two experiments, it can be inferred that applying RANSAC algorithm over sample data to remove outliers is an effective way to reduce skew in estimates stemming from extreme errors.

9



I implemented two versions of RANSAC :

(i) Naive Method

In this method, only 3 points are chosen during the trial stage of the RANSAC algorithm. On using this implementation, highly skewed planes were often appearing even on setting the 'inlier set probability' parameter to 0.99. This artifact can be attributed to large variations among the inlier points along the direction perpendicular to the actual plane.

(ii) Modified Method

To account for the problem described above, a modified RANSAC implementation was developed, in which, 's' (= 25) points are taken while fitting a model during a trial. I observed that on choosing 25 points, the implementation regularly generates a plane which on observation appears to be a good fit for the provided data points in a reasonable amount of time.