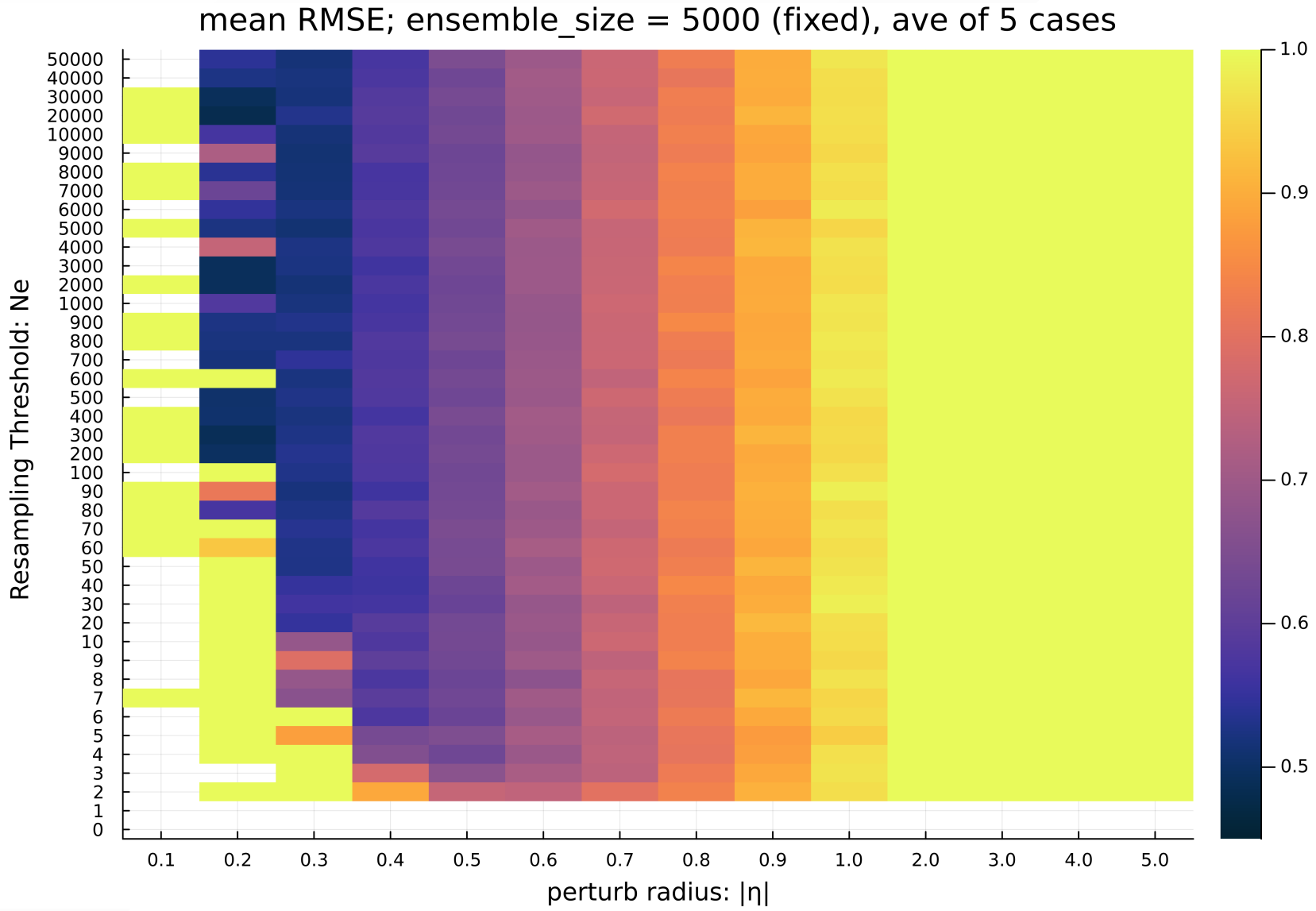
10月31日 進捗報告

課題2: TLM, ADJ の実装 (風速、Lorenz-63)

0500-32-7354,　佐藤 匠

SIR 法のパラメータ


mean RMSE; ensemble_size = 5000 (fixed), ave of 5 cases
Resampling Threshold: Ne
50000
40000
30000
20000
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
4.0
5.0
perturb radius: |η|
1.0
0.9
0.8
0.7
0.6
0.5

RK4(Lorenz-96) | TLM, ADJ

例えば重み $\vec{k}_3$ を求めるためには $\vec{k}_2, \vec{k}_1$ までの情報を F_{L96} に入力する必要がある。

$$
\begin{aligned}
\mathcal{M}(t,\cdot) &= \mathrm{RK4}(F_{\mathrm{L96}}(t,\cdot)) \\
&= \mathrm{RK4}_{[\Delta\vec{x}]}(\mathrm{RK4}_{[4]}(F_{\mathrm{L96}}(t, \mathrm{RK4}_{[3\leftarrow 1]}(F_{\mathrm{L96}}(t,\cdot))))) \\
&= \cdots \\
&= \mathrm{RK4}_{[\Delta\vec{x}]}(\mathrm{RK4}_{[4]}(F_{\mathrm{L96}}(t, \mathrm{RK4}_{[3]}(F_{\mathrm{L96}}(t, \mathrm{RK4}_{[2]}(F_{\mathrm{L96}}(t, \mathrm{RK4}_{[1]}(F_{\mathrm{L96}}(t,\cdot)))))))))
\end{aligned}
$$

であるから、以下のような行列の積で表せる。

$$
\mathbf{M} = \mathbf{L}_{\mathrm{RK4}[\Delta\vec{x}]} \cdot \mathbf{L}_{\mathrm{RK4}[4]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[3]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[2]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[1]} \cdot \mathbf{L}_{\mathrm{L96}}
$$

$$
\begin{aligned}
\mathbf{M}^T &= (\mathbf{L}_{\mathrm{RK4}[\Delta\vec{x}]} \cdot \mathbf{L}_{\mathrm{RK4}[4]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[3]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[2]} \cdot \mathbf{L}_{\mathrm{L96}} \cdot \mathbf{L}_{\mathrm{RK4}[1]} \cdot \mathbf{L}_{\mathrm{L96}})^T \\
&= \mathbf{L}^T_{\mathrm{L96}} \cdot \mathbf{L}^T_{\mathrm{RK4}[1]} \cdot \mathbf{L}^T_{\mathrm{L96}} \cdot \mathbf{L}^T_{\mathrm{RK4}[2]} \cdot \mathbf{L}^T_{\mathrm{L96}} \cdot \mathbf{L}^T_{\mathrm{RK4}[3]} \cdot \mathbf{L}^T_{\mathrm{L96}} \cdot \mathbf{L}^T_{\mathrm{RK4}[4]} \cdot \mathbf{L}^T_{\mathrm{RK4}[\Delta\vec{x}]}
\end{aligned}
$$

RK4(Lorenz-96) | Lorenz-96 モデルの TLM・ADJ

Lorenz-96モデルの接線形演算子 $\mathbf{L}$ = モデルのヤコビ行列 $\partial F_{\mathrm{L96}}/\partial \boldsymbol{x}$:

$$\delta\left(\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}\right) = \begin{pmatrix} \delta\dot{x} \\ \delta\dot{y} \\ \delta\dot{z} \end{pmatrix} \approx \begin{pmatrix} \frac{\mathrm{d}F^x_{\mathrm{L96}}}{\mathrm{d}x} & \frac{\mathrm{d}F^x_{\mathrm{L96}}}{\mathrm{d}y} & \frac{\mathrm{d}F^x_{\mathrm{L96}}}{\mathrm{d}z} \\ \frac{\mathrm{d}F^y_{\mathrm{L96}}}{\mathrm{d}z} & \frac{\mathrm{d}F^y_{\mathrm{L96}}}{\mathrm{d}y} & \frac{\mathrm{d}F^y_{\mathrm{L96}}}{\mathrm{d}z} \\ \frac{\mathrm{d}F^z_{\mathrm{L96}}}{\mathrm{d}x} & \frac{\mathrm{d}F^z_{\mathrm{L96}}}{\mathrm{d}y} & \frac{\mathrm{d}F^z_{\mathrm{L96}}}{\mathrm{d}z} \end{pmatrix} \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix} = \mathbf{L}_{\mathrm{L96}}\,\delta\vec{x}$$

$$L_{jk} = \frac{\partial}{\partial x_k}\left(\frac{dx_j}{dt}\right) = \begin{cases} -x_{j-1} & (k = j-2) \\ x_{j+1} - x_{j-2} & (k = j-1) \\ -1 & (k = j) \\ x_{j-1} & (k = j+1) \\ 0 & (\mathrm{Otherwise}) \end{cases}, \quad (\mathbf{L}^T)_{jk} = L_{kj}.$$

RK4(Lorenz-96) | L96 の TLM・ADJ 動作テスト

$$\|\mathbf{L}\,\delta\vec{x}\|^2 = \langle \mathbf{L}\,\delta\vec{x},\ \mathbf{L}\,\delta\vec{x}\rangle = \langle \delta\vec{x},\ \mathbf{L}^T\mathbf{L}\,\delta\vec{x}\rangle$$

$$\delta\vec{y} = F_{\mathrm{L96}}(\vec{x}+\delta\vec{x}) - F_{\mathrm{L96}}(\vec{x}) \approx \mathbf{L}\,\delta\vec{x}.$$

```
# Lorenz-96 TLM and ADJ
dV_true = L96(X0 + dX0, 0.0) - L96(X0, 0.0)
Lx = l96_tlm(X0, dX0)
LTLx = l96_adj(X0, Lx)

println(stderr, "Lorenz96: dV·dV ~= Lx·Lx == x·LTLx")
println(stderr, "$(dV_true · dV_true) ~= $(Lx · Lx) == $(dX0 · LTLx)")
```

```
実行結果 >>> Lorenz96: dV·dV      ~= Lx·Lx                  == x·LTLx
             0.005153768926428285 ~= 0.005161167043242083 == 0.005161167043242082
```

※ F によらない

RK4(Lorenz-96) | TLM・ADJ 動作テスト

$$\left\|\mathbf{M}\,\delta\vec{x}_i\right\|^2 = \left\langle \mathbf{M}\,\delta\vec{x}_i,\ \mathbf{M}\,\delta\vec{x}_i \right\rangle = \left\langle \delta\vec{x}_i,\ \mathbf{M}^T\mathbf{M}\,\delta\vec{x}_i \right\rangle$$

$$\delta\vec{x}_{i+1} = \mathcal{M}(\vec{x}_i + \delta\vec{x}) - \mathcal{M}(\vec{x}_i) \approx \mathbf{M}\,\delta\vec{x}_i.$$

```
    # Model TLM and ADJ
    dnX_true = Lorenz96.step(X0 + dX0, 0.0, lorenz_parameter) - Lorenz96.step(X0, 0.0, lorenz_parameter)
    Mx = Lorenz96.TangentLinearCode(0.0, X0, dX0, lorenz_parameter)
    MTMx = Lorenz96.AdjointCode(0.0, X0, Mx, lorenz_parameter)

    println(stderr, "Model: dnX·dnX ~= Mx·Mx == x·MTMx")
    println(stderr, "$(dnX_true · dnX_true) ~= $(Mx · Mx) == $(dX0 · MTMx)")
```

```
実行結果 >>> Model: dnX·dnX          ~= Mx·Mx                        == x·MTMx
             0.0038632772810651525 ~= 0.0038632773615402682 == 0.0038632773615402 69
```