Robotic control development with a dynamic and versatile plume simulation. Part I: Plume Model

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***Abstract*— In this paper, we present a model of the dynamics of an ocean plume suitable for evaluating the performance of robotic command and control algorithms. The key feature of our model is the ability to represent sparsity (in time and space) as a smoothly varying quantity. This approach provides a quantifiable, metric-based method for evaluating the relationship between the performance (and stability) of robotic control algorithms and the sparsity of plume dynamics. In Part II apply this model to the development and evaluation of multi-robot control approaches to characterize pollution plumes in a coastal environment.**

I. INTRODUCTION

The study of plume modelling originated by studying the dynamics of smoke rising from a chimney [2]. Over the 20^{th} century, research has expanded to study all kinds of plumes. This paper models the dynamics of oil spills, which is a growing concern with the increase in off-shore oil-rigs and global transport of oil. The results of an oil spill can be catastrophic. For damage mitigation and clean-up many research communities have turned their eye to the field of robotics.

The most critical part of robotic systems is the command and control algorithms, which determine how the robots manoeuvre around and in the oil spill and complete their objective. Many control algorithms are robust enough that when generated using a simplified world model they easily succeed in real application. Others are fragile and easily break if the model they are developed from is not perfect.

Our group has previously proposed a controller to use with multiple robots to track and monitor an oil spill [16]. The controller follows a modified gradient descent that moves the robot to a certain concentration threshold on the edge of the plume and then follows the edge around the plume. Unfortunately, the controller was designed to operate on an idealized plume and might not work when implemented and deployed on a real plume.

To test this, we have developed a plume simulation to capture the idealized plume dynamics, more realistic and erratic plume dynamics, and all of the variations in between. This paper will present a model that can capture both the time-averaged, idealized structure and the instantaneous, realistic structure of a dynamic plume. It will describe the two defining metrics of an instantaneous plume, the concentration sparsity and concentration peak/mean ratio, and show how to simulate the change from ideal plume to realistic plume by tuning just the sampling radius. This simulation will allow us to test the robustness of our controller and others like it. We can confirm that it works on the extremely idealized model, confirm it does not work on the most extreme of the realistic model, and by tuning certain parameters, we can find exactly what plume dynamics in between these two extremes will break our controller.

Section II-A of this paper will detail the plume characteristics most important for our use and briefly review the basic methods of modelling these characteristics. Section II-B will describe other plume models in prominent use and compare their usefulness with our objectives. Section III will detail our new model and explain the specific implementation. Section III-A will explain how the implemented method can capture both time-averaged and instantaneous characteristics of plumes and can be used to test robotic controllers.

A second paper will apply the model presented here to evaluate multi-robot command and control approaches. We will report on the application of this model to the development and evaluation of multi-robot control approaches to the problem of characterizing pollution plumes in a coastal environment.

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II. LITERATURE REVIEW

A. Fundamentals of Plume Modeling

Models have been developed for plumes in many different environments; atmosphere, water surface, deep water, and more. Over fifty different models specific to oil spills have been developed. Of these, a handful are still widely used [21]. This paper will focus on models for oil spills that model the dynamics of oil spills on the surface of the ocean. All the plume models are generally formulated in the same way, with the main components being advection and diffusion.

Advection is the process by which oil is carried by a current. Diffusion is the process where the material spreads outwards from areas of high concentration to low concentration due to turbulence. Other effects play a role, including evaporation, molecular diffusion, dispersion, emulsification, and interactions with ice and shore.

Evaporation is most dominate when the oil first reaches the surface and becomes less of an issue later when most of the materials have already evaporated. Emulsification is the breakdown of large globules of oil into smaller uniform globules. Molecular diffusion is diffusion as a result of molecular properties. Interaction with the shore and ice is relevant only when the oil spill is near the coast or in the Arctic. Many of these are not an issue as they either do not significantly effect the two metrics that classify an instantaneous plume, or are only relevant in specific situations, like when the oil in near floating ice.

Plumes are often generalized as having smooth gradients where the concentration follows a normal curve. This view is a time-averaged representation of the concentration and location, usually of three minutes or longer.

The Gaussian model, based on partial differential equations, is explicitly designed to model the time-averaged plume [7].

The defining equation for the Gaussian plume model is

$$\frac{\partial c(x,t)}{\partial t} + v^T(x,t) \cdot \nabla c(x,t) = k \nabla^2 c(x,t) \tag{1}$$

where v is the velocity vector of the environment flow, $\nabla c(x,t)$ is the concentration gradient, $\nabla^2 c(x,t)$ is the concentration divergence at point x and time t and D is a diffusion coefficient of the material.

The Gaussian model is the oldest of the plume modeling techniques. It is used extensively to model atmospheric plumes. Because the Gaussian is a time-averaged model, it glazes over some of the more interesting characteristics a plume can have, specifically, the features of an instantaneous plume.

There are two main metrics used to quantify the instantaneous characteristics of a plume; the sparsity and the peak-to-mean ratio. The sparsity is defined as the percentage of sensor readings in a stationary point that read a value over a certain threshold as a plume migrates over that point.

Sparsity is calculated as the cardinality of the set of concentration readings at a point under a defined threshold, over the cardinality of the set of all samplings at the same point.

$$T_x = \{t \in S_x |\ t < \theta\} \tag{2}$$

$$\text{sparsity} := \frac{|T_x|}{|S_x|} \tag{3}$$

Here S_x is the set of all recent samples, T_x is the set of all these samples above a threshold θ.

The peak-to-mean ratio is defined as the ratio between the peak concentration value and the time-averaged mean of concentration at a stationary point as a plume migrates over that point,

$$\text{peak-to-mean ratio} := \max(S_x) / \frac{1}{|S_x|} \sum S_x. \tag{4}$$

Jones showed that the sparsity of plume at any given location is typically 80-90% [12]. That is, for 80% of the time, a stationary sensor will read zero (or below a sensing threshold) as the plume moves through the point. A study by Crimaldi et al found that peak concentration can be three orders of magnitude larger than the mean [4]. These metrics are shown qualitatively in Fig. 1. Notice the peak over 50 samples is 7, the mean is 2.34, and qualitatively, the majority of the readings are below the mean.

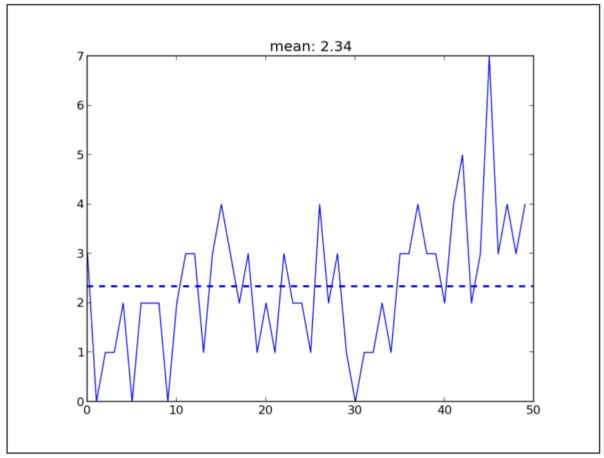

Fig. 1: The Simulation sparsity snapshot

Since the instantaneous concentrations are of great concern for robots navigating a plume, the Gaussian model is not suitable to test control algorithms that rely on fine scale use readings of a plume. The Lagrangian method is much more capable of representing the instantaneous dynamics described by Jones and Crimaldi.

The Lagrangian method is a particle based model that depicts the plume as an aggregate of many individual particles, each moving according to the advection laws and a random walk derived from the Gaussian equations. In the limit, as the number of particles approaches infinity and the time step approaches zero, the Lagrangian model approaches the Gaussian model, but when it is far from the limit, the Lagrangian model has the structure very similar to those metrics described by Jones and Crimaldi. This can allow a much more varied ability of the modeling.

The generic defining equation for the Lagrangian random walk method is

$$x(t+\Delta t) = x(t) + \text{adv}\,(t, x(t), \Delta t) + \text{diff}\,(t, x(t), \Delta t) \tag{5}$$

where Δt is the time step, $\text{adv}\,(t, x(t), \Delta t)$ is the advection component and $\text{diff}\,(t, x(t), \Delta t)$ is the diffusion component of the particle movement. Many different ways to model the advection and diffusion equations exist. Usually advection is determined by the velocity vector of the fluid field at location $x(t)$ and time t and the diffusion is a function of the diffusion coefficient for the material and a random variable. Many other higher order and situational effects can be modeled using the Lagrangian method by incorporating the their contribution to the movement, or removing particles periodically.

B. Related work

The Lagrangian model is now the dominate model used for studies of oil spill fate. Fate models focus on where the oil will eventually end up after a certain amount of time. These models use location specific environmental data for currents and wind. Many derivations of the Lagrangian model have been proposed and implemented [21].

General NOAA Operational Modeling Environment (GNOME) is a widely accepted tool produced and verified by NOAA [1]. This software tool uses historical weather data to model the fate of the oil spill. GNOME can be very useful for simulating the transport and fate of oil spills in specific places around the world. It comes equipped with historical current and wind data and can simulate oil spills of many different kinds of oil. The main concern with the GNOME software model is that the step size is on a scale much to large to accurately reflect the instantaneous dynamics of a plume. GNOME has a minimum resolution of step-size of 15 minutes. A European model MEDSLICK provides detailed simulation of oil spills very similar to GNOME [5] [6]. It suffers from the same problem as GNOME of using a time and space scale much too large for testing of robotic response.

Researchers have tackled the problem of understanding and containing oil-spills from a number of different prespectives. In the robotics communty, researchers study source localization, edge tracking, and autonomous clean-up, with single and multiple robots. Those who study plume source localization generally base their approach on biomimetic techniques, trying to understand and emulate how animals find plume sources [9] [17] [22]. Other strategies focus on the problem by looking at optimal ways to layout robotic sensor networks for spill tracking and monitoring of the plume edge [10] [16] [25] [20]. Some strategies specifically focus on robotic clean-up and how to generate complete coverings of the dynamic plume [11] [24], and spill finding [18]. Additionally, many of the techniques have been developed for individual and multiple robots [10] [16] [18] [24].

A list of some plume modeling strategies employed by roboticists are as follows. Marjovi and Marques use a Gaussian plume model for development of their algorithms for swarm odor plume finding [18]. Sahyoun et al also use a Gaussian plume model to develop algorithms for plume tracking with mobile sensors [20]. Attempts to capture the instantaneous structure of plume behavior has been made by Farrell et al [9]. Farrell at al developed a simulation that uses a variation of the Lagrangian method [8]. This model is used by Li et al to test source localization algorithms based on moth behavior [9] [17]. Zarzhitsky et al also use the model developed by Farrell to simulate swarm chemical plume tracing [8] [24] . Farrell's simulation is suited for capturing the instantaneous dynamics of an odor plume. Farrrell designed a simulation to test behavioral based control strategies that emulate how moths find the source of an odor plume [8]. This model is not applicable to oil spills because it does not model large-scale turbulence and thus presents a largely laminar plume. Though it emulates the metrics described by Jones and Crimaldi, it is incapable of also emulating the idealized plume described by the Gaussian equations.

III. IMPLEMENTATION

Many models for the advection and diffusion component of the Lagrangian method exist. In this paper we use an adaptation of the equations derived by Chao and verified experimentally by Nagheeby et al [3] [19]. This model accounts for many effects on the plume besides advection and diffusion, such as evaporation, dissolution, shoreline and ice interaction, and emulsification. All of these but advection and diffusion can be ignored for this simulation as they do not play a significant role in the time scale we are interested in or do not significantly effect the specific plume characteristics we are interested in.

The x and y components of the advection and diffusion components are as follows. Eq. (6) shows the equations that fill (5) used in this simulation,

$$\mathrm{adv}_x(x,t) = k_t u_x + k_w w_x \tag{6}$$
$$\mathrm{adv}_y(x,t) = k_t u_y + k_w w_y \tag{7}$$
$$\mathrm{diff}_x(x,t) = R_1 \left(12D\Delta t\right)^{1/2} \cos(2\pi R_3) \tag{8}$$
$$\mathrm{diff}_y(x,t) = R_3 \left(12D\Delta t\right)^{1/2} \sin(2\pi R_4) \tag{9}$$

where u and w are the current and wind vectors respectively, k_t and k_w are constants used to tune the impact that current and wind have. These are usually set to 1 and 0.03 respectively. R represents a uniform random variable on the interval $[0,1]$. Each R_i is unique. D is the diffusion coefficient d usually between $0.01m^3/s$ and $0.5m^2/s$.

Fig. 2 shows an example of the simulated plume flow. This density histogram shows the results of the plume begin emitted from a source located at $(12,26)$ at times $2,5,8$, and 11 hours after it's start.

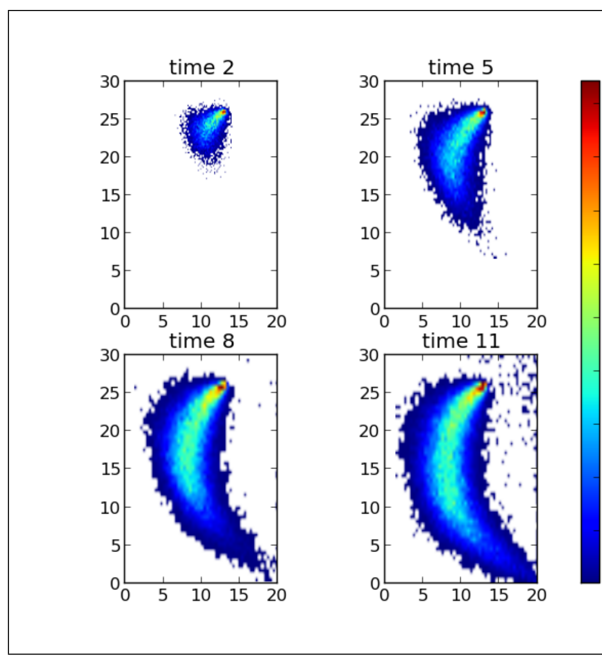

Fig. 2: 2D histogram of plume concentration at 2, 5, 8, and 11 hours after release

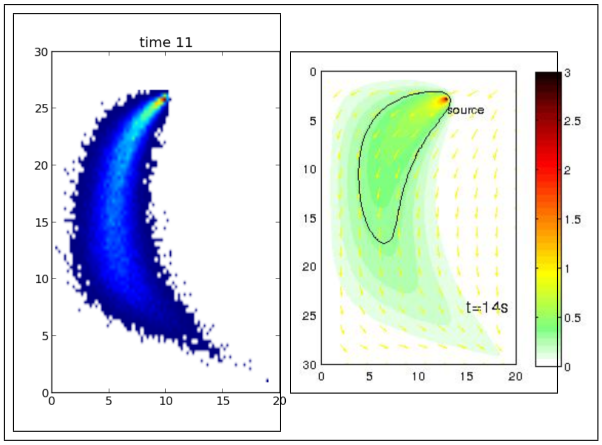

Fig. 3: Lagrangian model compared with the Gaussian model

Calculating the concentration is done by counting all of the particles within a certain radius from the point in question,

$$c(x,t) = (\text{card}\,\{(m|\forall m \in P, m \in B_x)\})\,n \qquad (10)$$

where the concentration at location x and time t is the size of the subset of all particles, P inside a ball centered at x with raidus r, $B_{(x,r)}$. The count is then multiplied by a normalizing factor n :

$$n = \sigma/A. \qquad (11)$$

based on the area of the ball, A, and the density of each particle, σ.

A. Parameter Tuning

We consider the time-averaged plume and the instantaneous plume the two extremes of what our plume model should be capable of simulating. On one hand, the smooth gradients are completely idealized; On the other hand, the instantaneous plume is extremely sporadic and irregular. The simplicity of the Lagrangian method modeling the plume as composite of many individual particles allows the model to reach near the Gaussian time averaged plume and near the instantaneous plume, and everywhere in between. Also, because the Lagrangian model uses a moving frame of reference and thus only calculates the concentration of a point based on the particles that are nearest that point, no interpolation and smoothing between gridded blocks occurs in the simulation. This allows for the simulation to produce extremely varying fluctuations in concentration readings over both space and time [4] [12] [23] .

In the limit as the time step approaches zero and the number of particles approaches infinity, the model approaches the Gaussian model.

Since this is not feasible in simulation to turn the number of particles up high enough, nor the time step low enough to mimic the Gaussian, we use just the size of the sampling radius of concentration to adjust the sparsity and peak/mean ratio. The range of sampling radius is chosen with only the resulting values of the two metrics in mind. Generally, a sampling radius between 0.01m and 1 m give the desired range of sparsity and peak-to-mean ratios. We calculate the peak-to-mean ratio of the plume by taking samplings every time-step for at least 100 time-steps. The mean is found by averaging all the time steps and the peak is taken as the maximum concentration reading of all time steps.

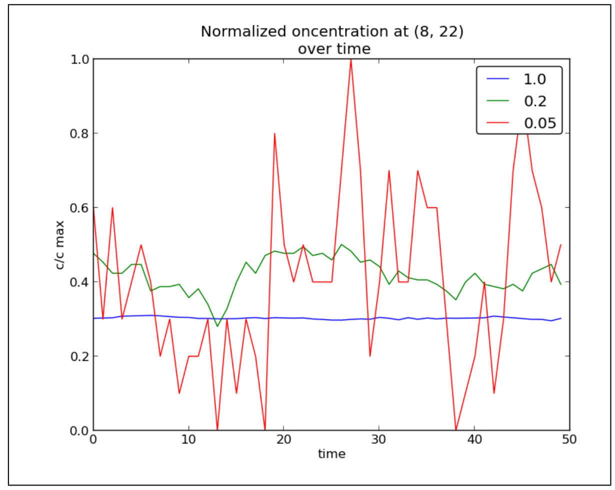

Fig. 4: Concentration profile at location (8,22) over time.

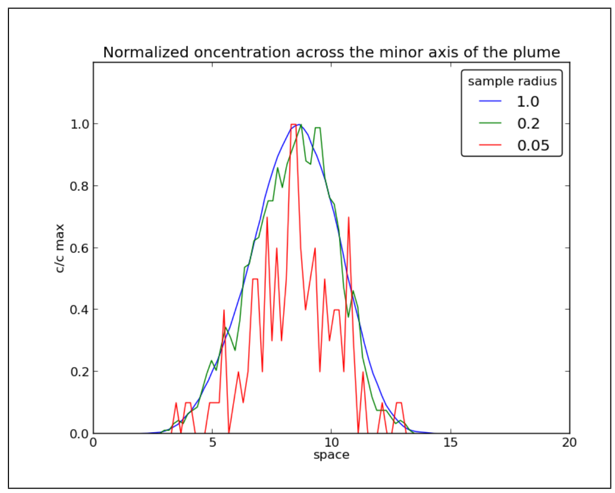

Fig. 5: Concentration profile across the minor axis of the plume 6 meters down stream from the source

Fig. 3 shows a concentration map of the Lagrangian particle plume generated in this study side-by-side the Gaussian model originally used to develop the controller under study. This shows qualitatively how similar the two models can be from a macro perspective.

Fig. 2 shows the concentration profile of the plume using three different radius tunings over a minor axis of the plume 6 meters down stream from the source. When the radius is at 1 m, the concentration profile nearly follows a normal curve, like what we would expect in the limit. Then when the sampling radius is set to 0.05 m the concentration profile is very irregular. Though it generally follows a bell curve, it has a very large variation. The middle line at 0.2 m shows a point between the two extremes.

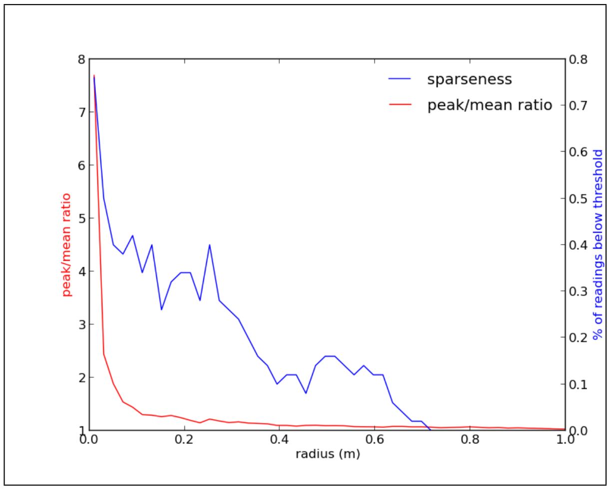

Fig. 6: Sparsity and Peak-to-mean radius as a function of the sampling radius

Fig. 5 shows the concentration profile at a location downstream from the source of the plume. When the sampling radius is chosen to be 1 m, the concentration is nearly constant. Further, when the sampling radius is very low, 0.05 m, the concentration profile is very similar to how Jones and Crimaldi described the structure of an instantaneous plume [4] [12]. This graph shows that far from the limit the simulation represents the metrics similar to the experimental sampling represented in Fig. 1

Fig. 6 shows the sparsity and peak to mean ratio as the sampling radius moves from 1 to 0. This shows how tuning the radius effects the sparsity and the peak/mean ratio. Additionally, by adjusting the threshold value, we can bring these two curves closer together. Note the use of two y axes.

IV. Results and Conclusions

This conclusion will be transformed to presenting the results of using the controller on the tuned up and tuned down plume.

- We test the controller designed in [**?**] on the plume model.
- We see that when the sampling radius is 0.5, the robot behaves very similarly to how it was designed to on the Gaussian plume.
- Then, we see that when run using a sampling radius of 0.05, the robot does not circle the perimeter of the plume.
- We can see qualitatively that the controller does not work well when the model is not ideal or very close to ideal.

This paper presents a model for oil spills suitable for the development of robotic command and control algorithms. This model can be tuned to represent the theoretical and idealized image of a plume, and can also represent the extreme characteristics that in an instantaneous plume. By developing and testing control algorithms on a continuous variation of models, control algorithm developers can create a more robust controller. Many control algorithms are developed for situations that are very specific and are then tested in simulation on these very specific situation. In application, an oil spill can have a wide range of characteristics. It is important to know which conditions and kinds of plumes the controller will work for. This simulation allows developers to test controllers on many varying know under which conditions and which kinds of plumes the controller will function.

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