Thu Jan 20

- suppose now we are given a DT DE

$$x^+ = F(x,u),\ x \in \mathbb{R}^n,\ u \in \mathbb{R}^m$$

and we wish to choose inputs over time $u : [0,t] \to \mathbb{R}^m$ to minimize cost

$$c(x,u) = \underbrace{\ell(t, x(t))}_{\text{"final" cost}} + \sum_{s=0}^{t-1} \underbrace{\mathcal{L}(s, x(s), u(s))}_{\text{"running" cost}}$$

given $\tilde{\mathcal{L}}(s, x(s+1), x(s), u(s))$

define $\mathcal{L}(s, x(s), u(s)) = \tilde{\mathcal{L}}(s, \underbrace{F(x(s), u(s))}_{= x(s+1)}, x(s), u(s))$

$$v_s^*(x(s)) = \min_{u(s) \in \mathbb{R}^m} \left[\mathcal{L}(s, x|_{[s,t]}, u^*|_{[s+1,t]}, u(s)) + v_{s+1}^*(x(s+1)) \right]$$

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ex: suppose 2nd-order approx to c at $u_0 \in \mathbb{R}^m$ is exact, i.e.:

$$(c(u) - c(u_0)) = b^T (u - u_0) + \tfrac{1}{2}(u - u_0)^T C (u - u_0) :$$

$C \in \mathbb{R}^{m \times m}$ / $\neq c : \mathbb{R}^m \to \mathbb{R}$

$\to$ determine necessary conditions on $b^T \in \mathbb{R}^{1 \times m}$, $C^T = C$ for $u_0 \in \mathbb{R}^m$ to be local min

$\to$ determine sufficient cond's on spec $D^2 c(v)$ for $u_0 \in \mathbb{R}^m$ to be strict local min

$\to$ if strict local min, solve for u_0

– necessary that $Dc(u_0) = b^T + (u_0 - u_0)^T C = b^T = 0$

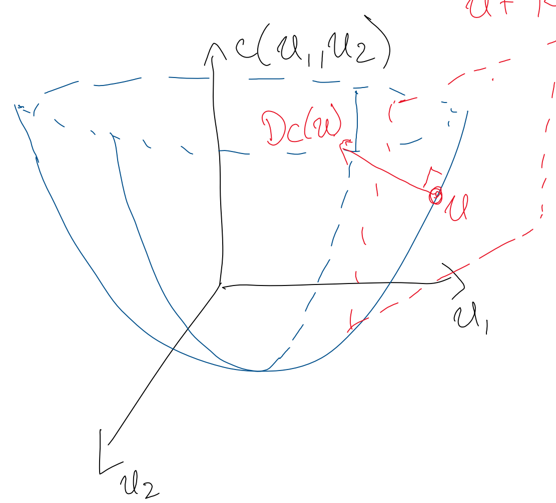
and $D^2 c(u_0) = C \geq 0$

$c : \mathbb{R}^m \to \mathbb{R}$

$Dc(u) \in \mathbb{R}^{1 \times m}$

$\parallel$

$[\partial_{u_1} c(u), \ldots, \partial_{u_m} c(u)] \in \mathbb{R}^{1 \times m}$



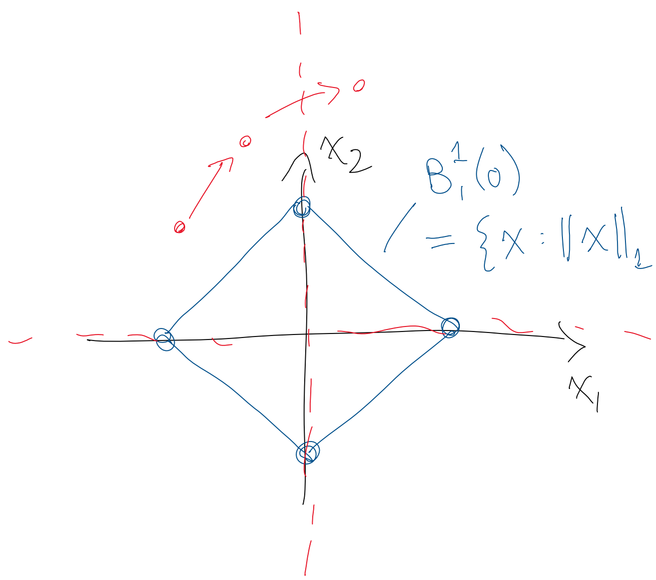
- DT linear quadratic regulation (DT-LQR)

$$\min_u C_\tau(x,u) \quad \text{s.t.} \quad x_{t+1} = A_t x_t + B_t u_t$$

$$C_\tau(x,u) = \frac{1}{2} x_t^T P_t x_t + \frac{1}{2}\sum_{s=\tau}^{t-1} x_s^T Q_s x_s + u_s^T R_s u_s = \|x\|_2^2 + \|u\|_2^2$$

why 2? what about 1? or ∞?

$$\|x\|_2^2 = x^T \begin{bmatrix} Q_0 & & & 0 \\ & Q_1 & & \\ & & \ddots & \\ & & Q_{t-1} & \\ 0 & & & P \end{bmatrix} x$$



$B_1^1(0) = \{x : \|x\|_1 = 1\}$