Thu Jan 27

given $\min_{u} c(x,u)$ s.t. $x(s+1) = F(s, x(s), u(s))$

(DT-OCP) $c(x,u) = \ell(t, x(t)) + \sum_{s=0}^{t-1} \mathcal{L}(s, x(s), u(s))$

there are 2 main interpretations / strategies

→ show that (DT-OCP) can be rewritten as

$\min_{\bar{u} \in \mathbb{R}^{\bar{m}}} \bar{c}(\bar{x}, \bar{u})$ s.t. $\bar{f}(\bar{x}, \bar{u}) = 0$

(i.e. determine $\bar{u}, \bar{x}, \bar{c}, \bar{f}$)

– recalling that $(u : [0,t] \to \mathbb{R}^m) \in (\mathbb{R}^m)^{[0,t-1]}$,

let $\bar{u} \in \mathbb{R}^{tm}$ be $[\bar{u}]_s = u(s)$,

and similarly $[\bar{x}]_s = x^u(s)$

where x^u is the trj generated by u

– then $\bar{c}(\bar{x}, \bar{u}) = c(x,u)$,

$\bar{f}(\bar{x}, \bar{u}) = 0 \iff x(s+1) = F(s, x(s), u(s))$

$$\bar{c}(\bar{x}, \bar{u}) = c\left(\varphi_x^{-1}(\bar{x}), \varphi_u^{-1}(\bar{u})\right)$$

there is a "natural" correspondence between discrete-time signals

$z : [0,t) \to \mathbb{R}^d$

and "stacked"/block vectors

$$\bar{z} = \begin{bmatrix} z(0) \\ z(1) \\ z(2) \\ \vdots \\ z(t-1) \end{bmatrix} \in \mathbb{R}^{t \cdot d}$$

$$\breve{z} = [z(0), z(1), \cdots, z(t-1)] \in \mathbb{R}^{d \times t}$$

$$\{z : [0,t) \to \mathbb{R}^d\} = Z \underset{\varphi^{-1}}{\overset{\varphi}{\rightleftarrows}} \bar{Z} = \{\bar{z} \in \mathbb{R}^{t \cdot d}\}$$

1°. dynamic programming $\xleftrightarrow[\text{problems / strategies}]{\text{spectrum of}}$ 2°. trajectory optimization

interp: output will be an optimal control policy $u^* : [0,t) \times X \to \mathcal{U}$ that tells us the optimal action $u^*(\tau, x(\tau))$ for every time τ and state $x(\tau)$

- output will be an optimal control signal $u^*_{x_0} : [0,t) \to \mathcal{U}$ that tells us the optimal action $u^*_{x_0}(\tau)$ for every time τ from initial state $x(0) = x_0$

$u(\tau, x(\tau))$ for every
time τ and state $x(\tau)$

strat: solve a sequence of parametrized NLP backward in time

time τ from initial state $x(0) = x_0$

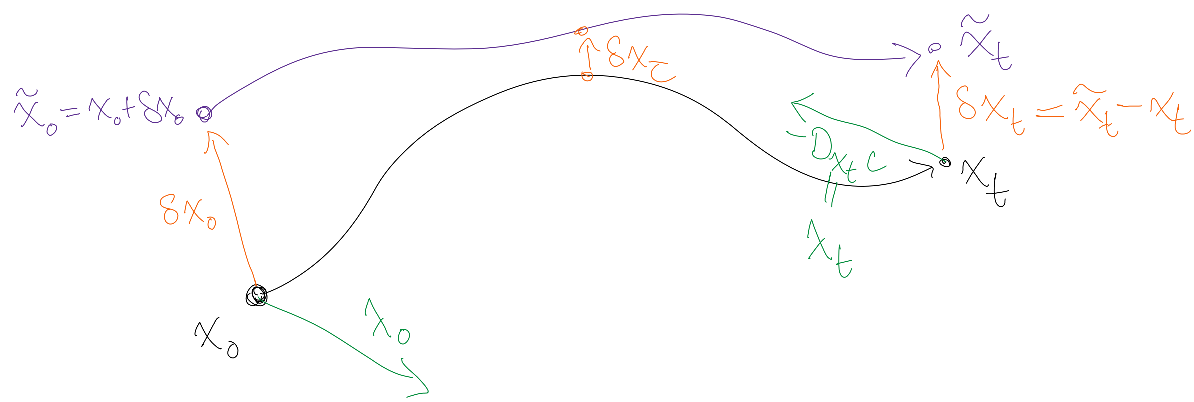
∘ solve one NLP "forward" in time

* since these are working from same underlying problem, they're consistent.

$$u^*(\tau, x^*(\tau)) = u^*_{x_0}(\tau)$$

assuming $x^*(0) = x_0$ and $u^*_{x_0}$ is applied until τ

$$\dot{x} / x^+ = f(x)$$



$$* \quad \delta \dot{x}_\tau / \delta x^+_\tau \simeq A(\tau) \cdot \delta x_\tau, \quad A(\tau) = D_x f(x(\tau))$$

$$\Rightarrow \quad \delta x_\tau = \Phi(\tau, 0)\, \delta x_0$$

$$\hookrightarrow \quad \dot{\lambda}_\tau / \lambda^-_\tau = \lambda_\tau \cdot A(\tau)$$