Empirical Analysis of M/M/1 Queuing System

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Abstract

This paper aims to prove Little's theorem. It will do so by calculating theoretical average wait times in an M/M/1 queuing system and comparing them to results gained from a discrete simulator.

1 Background

To calculate the number of packets in a system (r):

$$r = \frac{\rho}{1 - \rho}$$

To calculate server utilization::

$$\rho = \frac{\Lambda}{\mu}$$

Finally, to calculate theoretical average wait time in a system, Little's theorem is used:

$$T_r = \frac{r}{\Lambda}$$

The results from the above formula were compared against the discrete simulator results.

2 Solution

2.1 Simulator

The discrete simulator was written in Java using the Uncommons Maths API for Exponential Distribution.

2.2 Measures

The experiments were done by varying Λ and setting μ to 1. Λ represents the packet arrival rate into the system, and μ represents the server time. Λ is the independent variable in this experiment because the validity of Little's theorem is being tested, which uses Λ.

2.3 Simulation Setup

Λ was expressed in packets/sec. The simulator was run with an ending time of 500,000 seconds to ensure accurate results.

3 Results

The discrete simulation results were closely aligned with the theoretical results with almost no bias. This was due to the high amount of time that the simulator was run for. Figure 1 shows the results of comparing the discrete event simulator results to the theoretical ones.

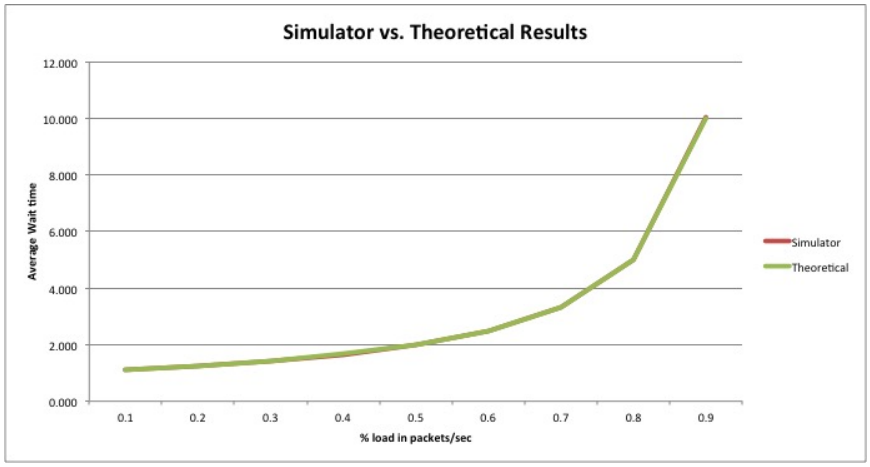

Figure 1: Graph showing simulator results and theoretical results.

As the results are very closely aligned, a table of the results is included below for Λ in the range of 0.1-0.9. The average waiting times (T_r) are rounded to three decimal places.

Λ	T_r simulated	T_r theoretical
0.1	1.109	1.111
0.2	1.251	1.250
0.2	1.424	1.429
0.2	1.663	1.667
0.2	1.999	2.000
0.2	2.498	2.500
0.2	3.335	3.333
0.2	5.011	5.000
0.2	10.020	10.000

Table 1: Simulator results for Λ between 0.1 and 0.9.

4 Conclusion

Little's Theorem was proved empirically to hold. The simulator results aligned very closely with the theoretical ones while varying Λ.