

DPSS-based Dictionary Design for Near-Field XL-MIMO Channel Estimation

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Slides & Codes: <https://github.com/scliubit/DPSS-Slides-Codes>

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Outlines

1 Introduction

2 System Model

3 Proposed Dictionary

4 Simulations

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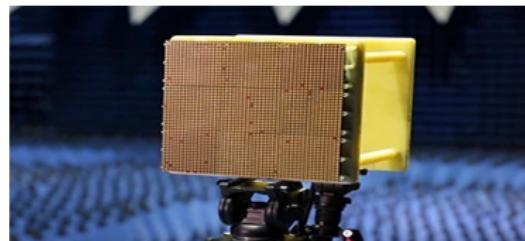
5 References

Introduction

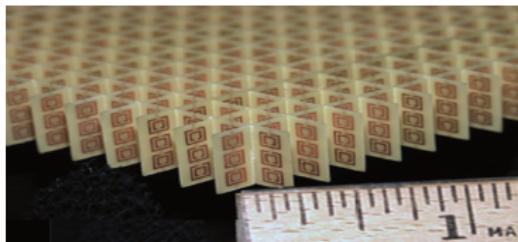
- XL-MIMO antenna arrays have been widely investigated in recent years.



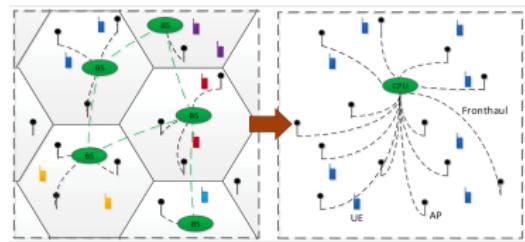
(a) Massive MIMO BS



(b) Reconfigurable Intelligent Surface



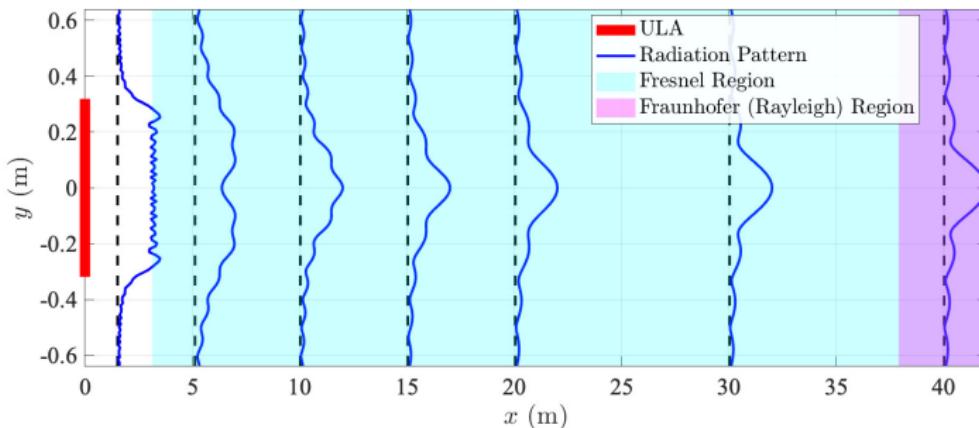
(c) Holographic MIMO



(d) Cell-free mMIMO

Figure 1: Widely investigated XL-MIMO related topics.

Introduction



- Near-field region, a.k.a., Fresnel region [1], is bounded as

$$\sqrt[3]{\frac{2D^4}{\lambda}} \leq d_{\text{NF}} \leq \frac{2D^2}{\lambda}, \quad D \uparrow \quad \lambda \downarrow \Rightarrow d_{\text{NF}} \uparrow$$

- **XL-MIMO** with large aperture and massive elements **expands the NF region** \Rightarrow NF effect becomes common.

[1] J. Goodman, *Introduction to Fourier Optics* (McGraw-Hill physical and quantum electronics series). W. H. Freeman, 2005, ISBN: 9780974707723.

Introduction

- In the **near-field** region, EM wavefront are not planar, but **spherical**.
- **Distance r** and **Angle θ** determine near-field channel together.

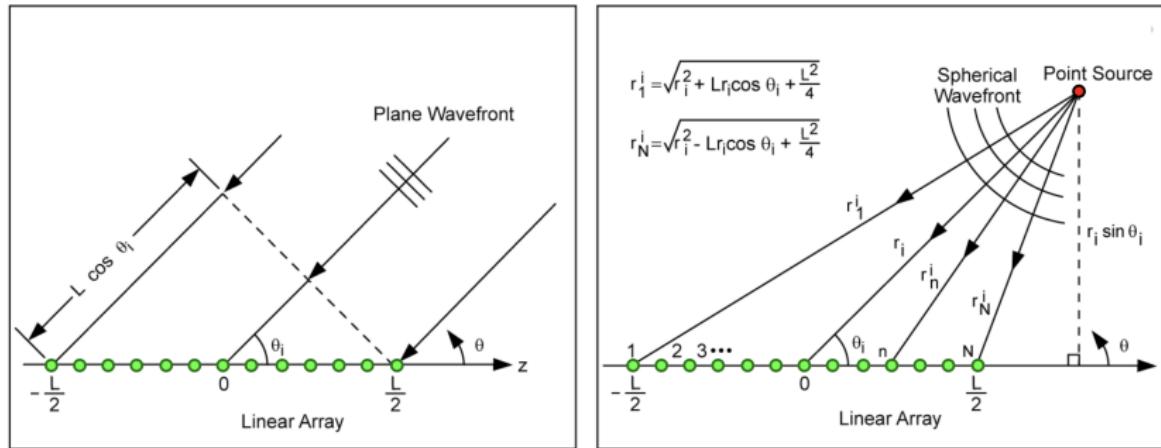


Figure 2: Wavefront illustration [2].

[2] A. Fenn, "Evaluation of adaptive phased array antenna, far-field nulling performance in the near-field region," *IEEE Trans. Antennas Propag.*, vol. 38, no. 2, pp. 173–185, 1990.

Introduction

- Proliferation of antennas (in XL-MIMO) calls for **hybrid analog-digital** architecture
 - ✓ Cost
 - ✓ Hardware complexity
 - ✗ Limited number of baseband samples per slot

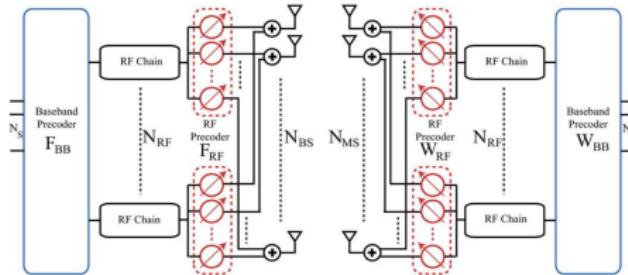


Figure 3: Hybrid analog-digital architecture [3]

- Compressive sensing (CS)-based algorithms proposed for channel estimation (CE) in hybrid MIMO architecture

[3] A. Alkhateeb, O. El Ayach, et al., "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831–846, 2014.

Introduction

Challenges for CS-based channel estimation in XL-MIMO systems

- **Sparsity structure changed.**

- ▶ New sparse representation matrix required.

- **Number of sparse supports increased.**

- ▶ More sampling and algorithm iterations.

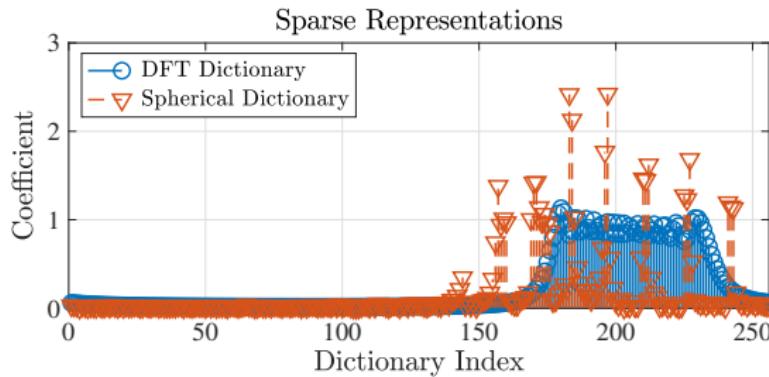


Figure 4: Sparse representations of near-field channel matrix

Previous Works

Overview:

- Uniform sampling on $1/r$ for reduced correlation [4].
- Hierarchical dictionary: Upper-layer for location search, lower-layer for fine-tuning [5].
- Spatial-Chirp beam [6].
- Distance-parameterized dictionary learning [7].

[4] M. Cui and L. Dai, "Channel estimation for extremely large-scale MIMO: Far-field or near-field?" *IEEE Trans. Commun.*, vol. 70, no. 4, pp. 2663–2677, 2022.

[5] J. Chen, F. Gao, et al., "Hierarchical codebook design for near-field mmwave MIMO communications systems," *IEEE Wireless Commun. Lett.*, vol. 12, no. 11, pp. 1926–1930, 2023.

[6] X. Shi, J. Wang, et al., "Spatial-chirp codebook-based hierarchical beam training for extremely large-scale massive MIMO," *IEEE Trans. Wireless Commun.*, pp. 1–1, 2023.

[7] X. Zhang, H. Zhang, and Y. C. Eldar, "Near-field sparse channel representation and estimation in 6G wireless communications," *IEEE Trans. Commun.*, vol. 72, no. 1, pp. 450–464, 2024.

Previous Works

Remaining Issues:

- Strict Orthogonality.
 - ▶ High **mutual correlation** jeopardizes convergence by degraded condition number [8].

- Dictionary size.
 - ▶ Additional DoF leads to quadratically increased dictionary size.

$$\mathbf{A}_{r\theta} = \mathbf{A}_r \otimes \mathbf{A}_\theta,$$

which leads to higher storage requirements.

- ▶ Complexity increased in dictionary matching.

[8] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

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System Model

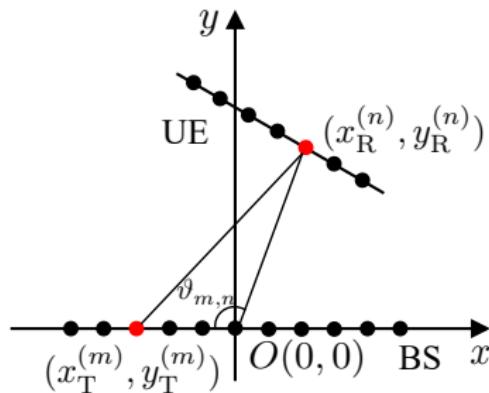


Figure 5: The considered near-field transmission scenario. $\mathbf{r}_T^{(m)} = (x_T^{(m)}, y_T^{(m)})$, and $\mathbf{r}_R^{(n)} = (x_R^{(n)}, y_R^{(n)})$

■ Near-field steering vector

$$g(\mathbf{r}_T, \mathbf{r}_R) = \frac{e^{-jk\|\mathbf{r}\|}}{\|\mathbf{r}\|} = \frac{e^{-jk\sqrt{\|\mathbf{r}_T^{(m)}\|^2 + \|\mathbf{r}_R^{(n)}\|^2 - 2\|\mathbf{r}_T^{(m)}\|\|\mathbf{r}_R^{(n)}\|\cos(\vartheta_{m,n})}}}{\|\mathbf{r}_T^{(m)} - \mathbf{r}_R^{(n)}\|}. \quad (1)$$

where $\mathbf{r} = \mathbf{r}_T - \mathbf{r}_R$

Channel Model

- Line-of-Sight (LoS) paths are more likely to occur in the near field [9], Rician model would be fair.
- LoS component

$$\mathbf{H}_{\text{LoS}}[:, m] = \mathbf{g}_R \left(\mathbf{r}_T^{(m)} \right) = \left[g(\mathbf{r}_T^{(m)}, \mathbf{r}_R^{(1)}), \dots, g(\mathbf{r}_T^{(m)}, \mathbf{r}_R^{(N_R)}) \right]^T, \quad (2)$$

- Overall Rician channel model

$$\mathbf{H} = \sqrt{\frac{K}{1+K}} \mathbf{H}_{\text{LoS}} + \sqrt{\frac{1}{1+K}} \mathbf{H}_{\text{NLoS}}, \quad (3)$$

[9] 3GPP, "Study on channel model for frequencies from 0.5 to 100 GHz," Tech. Rep., 2023, Release 17.

Problem Formulation

- Pilot training for hybrid analog-digital array at slot t

$$\begin{aligned}\mathbf{y}^{(t)} &= \left(\mathbf{W}_{\text{RF}}^{(t)} \mathbf{W}_{\text{BB}}^{(t)} \right)^H \left(\mathbf{H} \mathbf{F}_{\text{RF}}^{(t)} \mathbf{F}_{\text{BB}}^{(t)} \mathbf{s}^{(t)} + \mathbf{n}^{(t)} \right) \\ &= \left((\mathbf{f}^{(t)})^T \otimes \mathbf{W}^{(t)} \right) \text{vec}(\mathbf{H}) + \tilde{\mathbf{n}}^{(t)},\end{aligned}\tag{4}$$

- Stacking τ measurements ($\tau < N_{\text{T}}N_{\text{R}}$, under-determined)

$$\mathbf{y} = \Phi \mathbf{h} + \tilde{\mathbf{n}} = \Phi \Psi \tilde{\mathbf{h}} + \tilde{\mathbf{n}},\tag{5}$$

where Ψ is the sparsification basis, and $\tilde{\mathbf{h}}$ is the sparse support.

- Compressed reconstruction problem

$$\min_{\tilde{\mathbf{h}}} \|\tilde{\mathbf{h}}\|_0, \quad \text{s.t. } \|\Phi \Psi \tilde{\mathbf{h}} - \mathbf{y}\|_2 \leq \varepsilon,\tag{6}$$

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Dictionary Design

- Orthogonality and sparsest representation \Rightarrow Singular/Eigen Value Decomposition (SVD/EVD)
- For general representation, consider auto-correlation

$$\begin{aligned}\mathbf{R}_T &= \mathbb{E} [\mathbf{H}^H \mathbf{H}] \\ &= \gamma K \mathbf{H}_{\text{LoS}}^H \mathbf{H}_{\text{LoS}} + \gamma \mathbf{I},\end{aligned}\tag{7}$$

- Each element in the auto-correlation matrix

$$\begin{aligned}\mathbf{R}_T[m', m] &= \gamma \mathbf{1}_{m,m'} + \gamma K \mathbf{g}_R^H(\mathbf{r}_T^{(m)}) \mathbf{g}_R(\mathbf{r}_T^{(m)}) \\ &= \gamma \mathbf{1}_{m,m'} + \gamma K \sum_{n=1}^{N_R} \frac{e^{-j\kappa \|\mathbf{r}_T^{(m)} - \mathbf{r}_R^{(n)}\|}}{\|\mathbf{r}_T^{(m)} - \mathbf{r}_R^{(n)}\|} \times \frac{e^{j\kappa \|\mathbf{r}_T^{(m')} - \mathbf{r}_R^{(n)}\|}}{\|\mathbf{r}_T^{(m')} - \mathbf{r}_R^{(n)}\|},\end{aligned}\tag{8}$$

with indicator $\mathbf{1}_{m,m'} = 1$ for $m = m'$, and 0 otherwise.

Dictionary Design

■ Introducing paraxial approximation [10]

$$\begin{aligned} \mathbf{R}_T[m', m] &\approx \gamma \mathbb{1}_{m,m'} + \frac{\gamma K}{r_0^2} \sum_{n=1}^{N_R} e^{-j\kappa \frac{(x_T^{(m)} - x_R^{(n)})^2 - (x_T^{(m')})^2}{2y_0}} \\ &\triangleq \gamma \mathbb{1}_{m,m'} + \gamma K e^{j\kappa \frac{(x_T^{(m')})^2 - (x_T^{(m)})^2}{2y_0}} \mathbf{R}'_T[m', m], \end{aligned} \quad (9)$$

■ The auto-correlation matrix is represented by

- ▶ Indicator function $\mathbb{1}_{m,m'}$
- ▶ Phase compensation $\exp\left(j\kappa \frac{(x_T^{(m')})^2 - (x_T^{(m)})^2}{2y_0}\right)$
- ▶ Common remainder $\mathbf{R}'_T[m', m]$

[10] D. A. B. Miller, "Communicating with waves between volumes: Evaluating orthogonal spatial channels and limits on coupling strengths," *Appl. Opt.*, vol. 39, no. 11, pp. 1681–1699, Apr. 2000.

Remark

Remark on Auto-Correlation \mathbf{R}_T

The phase term $\exp\left(j\kappa \frac{(x_T^{(m')})^2 - (x_T^{(m)})^2}{2y_0}\right)$ includes location information, which indicates that \mathbf{R}_T may not have a universal SVD/EVD.

Compensating such phase term requires **localization**, which will be discussed in the full-length version [11].

[11] S. Liu, X. Yu, et al., *Sensing-enhanced channel estimation for near-field XL-MIMO systems*, 2024. arXiv: 2403.11809 [cs.IT]

Dictionary Design

- The common remainder in (9) can be further derived as

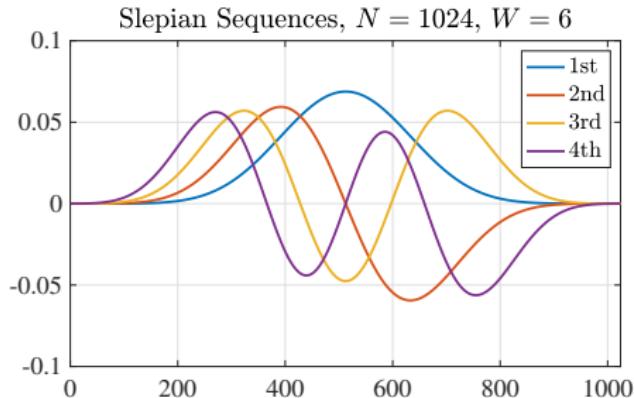
$$\begin{aligned}\mathbf{R}'_{\mathrm{T}}[m', m] &= \frac{1}{r_0^2} \sum_{n=1}^{N_{\mathrm{R}}} e^{j\kappa \frac{x_{\mathrm{R}}^{(n)} (x_{\mathrm{T}}^{(m)} - x_{\mathrm{T}}^{(m')})}{y_0}} \\ &\propto \text{sinc} \left(2W \left(x_{\mathrm{T}}^{(m)} - x_{\mathrm{T}}^{(m')} \right) \right),\end{aligned}\tag{10}$$

where $W = \kappa L_{\mathrm{R}} / (4\pi y_0)$.

- \mathbf{R}'_{T} is a **Toepplitz Sinc** matrix.

Dictionary Design

- The Eigenvectors of $\{\mathbf{v}_m\}_{m=1}^M$ of auto-correlation matrix \mathbf{R}'_T are known as *discrete prolate spheroidal sequence* (DPSS) or *Slepian sequence* within frequency W [12].



- Estimating the auto-correlation requires **numerous** samples. The **closed-form** solution here make it **efficient** to generate the dictionary.

[12] D. Slepian, "Estimation of signal parameters in the presence of noise," *IRE Trans. Inf. Theory*, vol. 3, no. 3, pp. 68–89, 1954.

Channel Estimation

- Coarse Localization
 - ▶ Coarsely estimate the location of UE.
- Build dictionary and estimate channel
 - ▶ Design dictionary according to **Algorithm 1**
 - ▶ Estimate channel matrix using OMP.

Algorithm 1: Proposed Dictionary Design Algorithm

Require: Estimated coordinate (\hat{x}_i, \hat{y}_i) and the numbers of antennas N_T and N_R .

Ensure: The DPSS-based eigen-dictionary Ψ_e .

- 1: Estimate the compensation matrix $\hat{\mathbf{D}}_T$ and $\hat{\mathbf{D}}_R$ according to (??).
 - 2: Calculate the frequency $\hat{W} = \kappa L_R / (4\pi\hat{y}_i)$.
 - 3: Generate \mathbf{R}_T and \mathbf{R}_R with DPSS according to (10).
 - 4: Perform EVD for $\mathbf{R}_T = \mathbf{V}\Lambda\mathbf{V}^{-1}$ and $\mathbf{R}_R = \mathbf{U}\Lambda'\mathbf{U}^{-1}$.
 - 5: Compensate phase shift $\mathbf{V}^c = \hat{\mathbf{D}}_T \mathbf{V}$, $\mathbf{U}^c = \hat{\mathbf{D}}_R \mathbf{U}$.
 - 6: Return eigen-dictionary $\Psi_e = (\mathbf{V}^c)^* \otimes \mathbf{U}^c$.
-

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Simulation Setup

- Evaluation criteria normalized mean square error (NMSE)

$$\text{NMSE} \left(\hat{\mathbf{H}}, \mathbf{H} \right) = \mathbb{E} \left[\| \hat{\mathbf{H}} - \mathbf{H} \|_F^2 / \| \mathbf{H} \|_F^2 \right], \quad (11)$$

- $f_c = 28$ GHz
- $N_T = 192, N_R = 4$
- Near field range [1 m, 20 m]
- Rician factor $K = 13$ dB
- Compression Ratio (CR)

$$\mu = \frac{\tau}{N_T N_R} \in \{0.25, 0.4, 0.6\} \quad (12)$$

Numerical Results

■ Overview

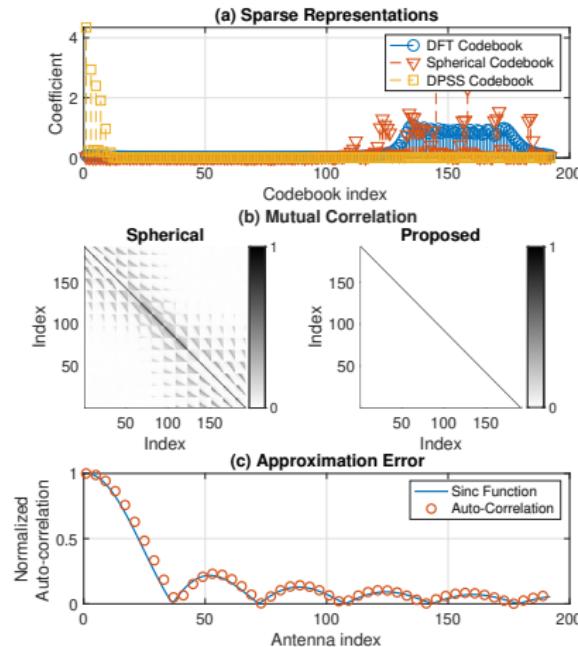


Figure 6: (a) The sparse representations of near-field channels under different dictionary, (b) the mutual correlation matrix $\Psi^H \Psi$ of the spherical wave dictionary and the proposed dictionary, (c) the approximation error in (10).

Numerical Results

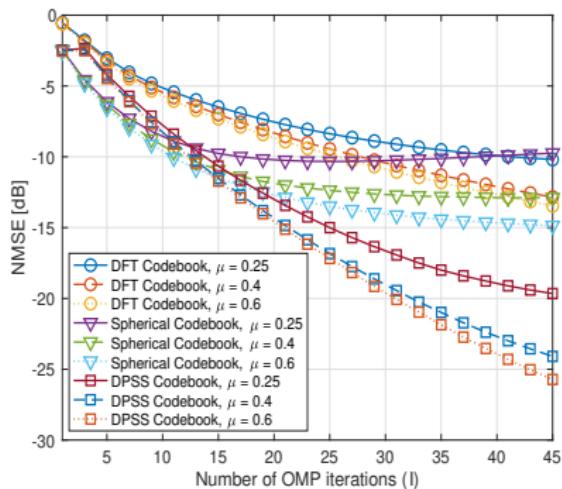
Near-field CE NMSE error versus

(a) CR μ and number of iterations I .

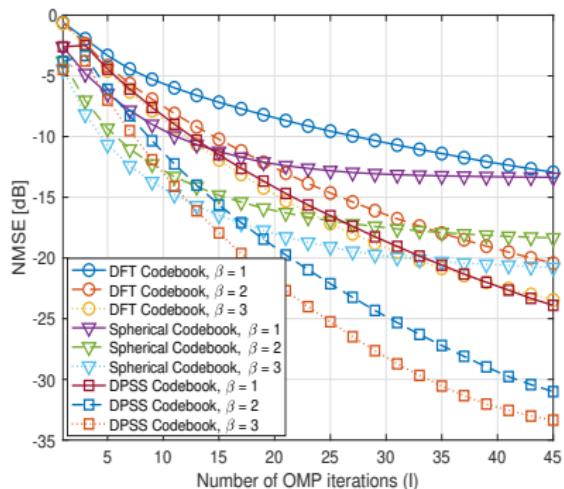
s.t. $\beta = 1$.

(b) number of iterations I and dictionary oversampling rate β .

s.t. $\mu = 0.4$.



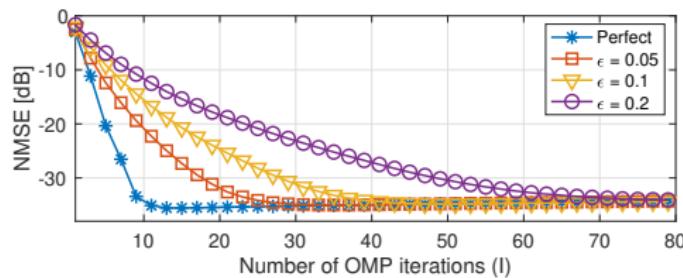
(a) CR [μ]



(b) Oversampling Rate [β]

Numerical Results

- Convergence against localization errors ϵ .
- Minimum required dictionary size for target NMSE



Dictionary	Target NMSE			
	-15 dB	-20 dB	-25 dB	-30 dB
DFT	768	855	1,150	N/A
Spherical[4]	1,150	3,072	15,552	N/A
Proposed	768	768	768	768

Summary

- Proposed a DPSS-based eigen-dictionary for near-field XL-MIMO CE.
 - ▶ Mutual **orthogonality** achieved among codewords
 - ▶ **Storage** requirements efficiently relaxed.

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- [8] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.
- [9] 3GPP, "Study on channel model for frequencies from 0.5 to 100 GHz," Tech. Rep., 2023, Release 17.
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- [11] S. Liu, X. Yu, Z. Gao, J. Xu, D. W. K. Ng, and S. Cui, *Sensing-enhanced channel estimation for near-field XL-MIMO systems*, 2024. arXiv: 2403.11809 [cs.IT].
- [12] D. Slepian, "Estimation of signal parameters in the presence of noise," *IRE Trans. Inf. Theory*, vol. 3, no. 3, pp. 68–89, 1954.

Full-length Version

Full-length version available at arXiv

[Ref] S.Liu, X. Yu, Z. Gao, D. W. K. Ng, and S. Cui, “**Sensing-Enhanced Channel Estimation for Near-Field XL-MIMO Systems**”, arXiv preprint. [Online] Available: <https://arxiv.org/abs/2403.11809>

- Details on how to obtain location coordinates without computation-intensive MUSIC algorithms.
- Generalized multi-path channel model

Thanks for your attention!

Q & A

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Full-length version: <https://arxiv.org/abs/2403.11809>



How to Obtain the Locations of UE/Scatterers

- One single iteration using a spherical dictionary (adopted here).

$$i = \operatorname{argmax}_j \|(\Psi_p^H) [j, :] \Phi^H \mathbf{y}\|^2, \quad (13)$$

- Other low-complexity algorithms (e.g., **Time Inversion Algorithm** proposed in journal version)

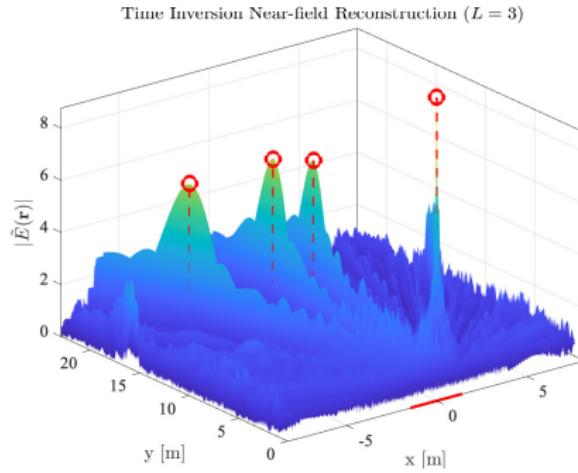


Figure 8: A demonstration of the proposed localization method with $N_{\text{BS}} = 512$ antenna elements and $L = 3$ NLoS paths. $f_c = 28$ GHz.

Connection with MUSIC Algorithm

Motivation: EVD based methods

MUSIC

- Measurement

$$\mathbf{Y} = \mathbf{AX} + \mathbf{N}$$

- Auto-correlation

$$\mathbf{R}_Y = \mathbf{A}\mathbf{R}_X\mathbf{A}^H + \sigma_n^2\mathbf{I}$$

- EVD

$$\mathbf{R}_Y = \mathbf{U}^{-1} (\mathbf{\Lambda} + \sigma_n^2 \mathbf{I}) \mathbf{U}$$

- Noise subspace orthogonality...

Proposed

- Channel Model

$$\mathbf{H} = \sqrt{\gamma K} \mathbf{H}_{\text{LoS}} + \sqrt{\gamma} \mathbf{H}_{\text{NLoS}}$$

- Auto-correlation

$$\mathbf{R}_H = \gamma K \mathbf{R} + \gamma \mathbf{I}$$

- EVD

$$\mathbf{R}_H = \gamma \mathbf{D}^{-1} \mathbf{U}^{-1} (K \mathbf{\Lambda} + \mathbf{I}) \mathbf{U} \mathbf{D}$$

- Estimate \mathbf{D} , obtain \mathbf{U} , with

$$\mathbf{R}' = \mathbf{U}^{-1} \mathbf{\Lambda} \mathbf{U}$$

as a Toeplitz Sinc matrix.