DPSS-based Dictionary Design for Near-Field XL-MIMO Channel Estimation

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Slides & Codes: `https://github.com/scliubit/DPSS-Slides-Codes`

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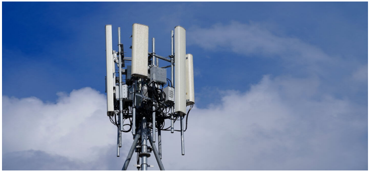
Outlines

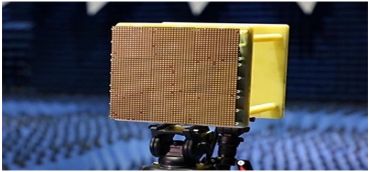
Outline

Introduction

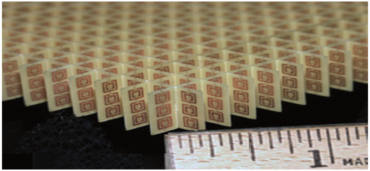
- XL-MIMO antenna arrays have been widely investigated in recent years.



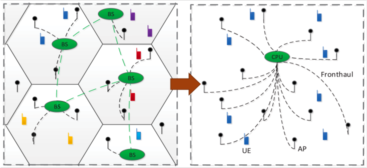
(a) Massive MIMO BS



(b) Recofigurable Intelligent Surface



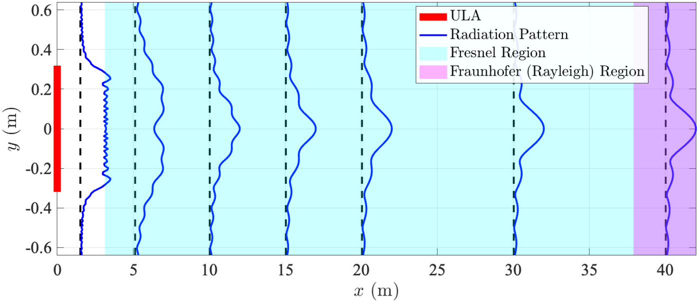
(c) Holographic MIMO



(d) Cell-free mMIMO

Figure 1: Widely investigated XL-MIMO related topics.

Introduction



- Near-field region, a.k.a., Fresnel region [1], is bounded as

$$\sqrt[3]{\frac{2D^4}{\boldsymbol{\lambda}}} \leq d_{\mathrm{NF}} \leq \frac{2D^2}{\boldsymbol{\lambda}}, \qquad D\uparrow \ \boldsymbol{\lambda}\downarrow \Rightarrow d_{\mathrm{NF}}\uparrow$$

- **XL-MIMO** with large aperture and massive elements **expands the NF region** $\Rightarrow$ NF effect becomes common.

[1] J. Goodman, *Introduction to Fourier Optics* (McGraw-Hill physical and quantum electronics series). W. H. Freeman, 2005, ISBN: 9780974707723.

Introduction

- In the **near-field** region, EM wavefront are not planar, but **spherical**.
- **Distance** r and **Angle** θ determine near-field channel together.

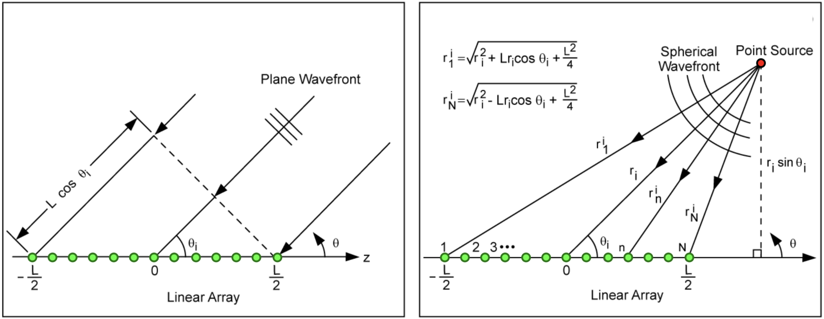


Figure 2: Wavefront illustration [2].

[2] A. Fenn, "Evaluation of adaptive phased array antenna, far-field nulling performance in the near-field region," *IEEE Trans. Antennas Propag.*, vol. 38, no. 2, pp. 173–185, 1990.

Introduction

- Proliferation of antennas (in XL-MIMO) calls for **hybrid analog-digital** architecture
  - ✓ Cost
  - ✓ Hardware complexity
  - ✗ Limited number of baseband samples per slot

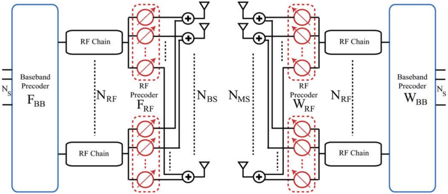


Figure 3: Hybrid analog-digital architecture [3]

- Compressive sensing (CS)-based algorithms proposed for channel estimation (CE) in hybrid MIMO architecture

[3] A. Alkhateeb, O. El Ayach, *et al.*, "Channel estimation and hybrid precoding for millimeter wave cellular systems," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831–846, 2014.

Introduction

Challenges for CS-based channel estimation in XL-MIMO systems

- **Sparsity structure changed**.
  - New sparse representation matrix required.
- Number of **sparse supports increased**.
  - More sampling and algorithm iterations.

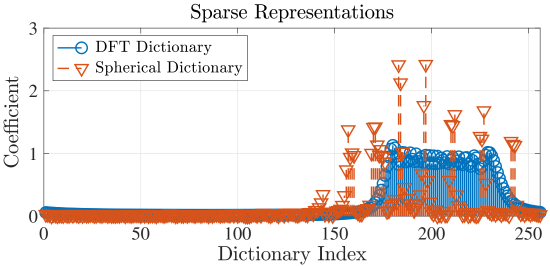


Figure 4: Sparse representations of near-field channel matrix

Previous Works

Overview:

- Uniform sampling on $1/r$ for reduced correlation [4].
- Hierarchical dictionary: Upper-layer for location search, lower-layer for fine-tuning [5].
- Spatial-Chirp beam [6].
- Distance-parameterized dictionary learning [7].

[4] M. Cui and L. Dai, "Channel estimation for extremely large-scale MIMO: Far-field or near-field?" *IEEE Trans. Commun.*, vol. 70, no. 4, pp. 2663–2677, 2022.

[5] J. Chen, F. Gao, *et al.*, "Hierarchical codebook design for near-field mmwave MIMO communications systems," *IEEE Wireless Commun. Lett.*, vol. 12, no. 11, pp. 1926–1930, 2023.

[6] X. Shi, J. Wang, *et al.*, "Spatial-chirp codebook-based hierarchical beam training for extremely large-scale massive MIMO," *IEEE Trans. Wireless Commun.*, pp. 1–1, 2023.

[7] X. Zhang, H. Zhang, and Y. C. Eldar, "Near-field sparse channel representation and estimation in 6G wireless communications," *IEEE Trans. Commun.*, vol. 72, no. 1, pp. 450–464, 2024.

Previous Works

Remaining Issues:

- Strict Orthogonality.
  - High **mutual correlation** jeopardizes convergence by degraded condition number [8].
- Dictionary size.
  - Additional DoF leads to quadratically increased dictionary size.

$$\boldsymbol{A}_{\boldsymbol{r}\boldsymbol{\theta}} = \boldsymbol{A}_{\boldsymbol{r}} \otimes \boldsymbol{A}_{\boldsymbol{\theta}},$$

which leads to higher storage requirements.
  - Complexity increased in dictionary matching.

[8] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

Outline

System Model

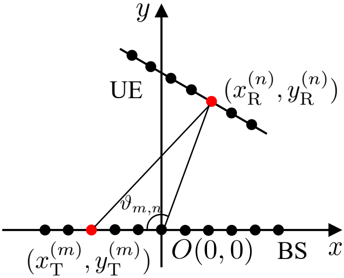


Figure 5: The considered near-field transmission scenario. $\mathbf{r}_{\mathrm{T}}^{(m)} = (x_{\mathrm{T}}^{(m)}, y_{\mathrm{T}}^{(m)})$, and $\mathbf{r}_{\mathrm{R}}^{(n)} = (x_{\mathrm{R}}^{(n)}, y_{\mathrm{R}}^{(n)})$

- Near-field *steering vector*

$$g(\mathbf{r}_{\mathrm{T}}, \mathbf{r}_{\mathrm{R}}) = \frac{e^{-j\kappa\|\mathbf{r}\|}}{\|\mathbf{r}\|} = \frac{e^{-jk\sqrt{\|\mathbf{r}_{\mathrm{T}}^{(m)}\|^2 + \|\mathbf{r}_{\mathrm{R}}^{(n)}\|^2 - 2\|\mathbf{r}_{\mathrm{T}}^{(m)}\|\|\mathbf{r}_{\mathrm{R}}^{(n)}\|\cos(\vartheta_{m,n})}}}{\|\mathbf{r}_{\mathrm{T}}^{(m)} - \mathbf{r}_{\mathrm{R}}^{(n)}\|}. \quad (1)$$

where $\mathbf{r} = \mathbf{r}_{\mathrm{T}} - \mathbf{r}_{\mathrm{R}}$

Channel Model

- Line-of-Sight (LoS) paths are more likely to occur in the near field [9], Rician model would be fair.

- LoS component

$$\mathbf{H}_{\mathrm{LoS}}[:,m] = \mathbf{g}_{\mathrm{R}}\left(\mathbf{r}_{\mathrm{T}}^{(m)}\right) = \left[g(\mathbf{r}_{\mathrm{T}}^{(m)}, \mathbf{r}_{\mathrm{R}}^{(1)}), \cdots, g(\mathbf{r}_{\mathrm{T}}^{(m)}, \mathbf{r}_{\mathrm{R}}^{(N_{\mathrm{R}})})\right]^{T}, \quad (2)$$

- Overall Rician channel model

$$\mathbf{H} = \sqrt{\frac{K}{1+K}}\mathbf{H}_{\mathrm{LoS}} + \sqrt{\frac{1}{1+K}}\mathbf{H}_{\mathrm{NLoS}}, \quad (3)$$

[9] 3GPP, "Study on channel model for frequencies from 0.5 to 100 GHz," Tech. Rep., 2023, Release 17.

Problem Formulation

- Pilot training for hybrid analog-digital array at slot t

$$\begin{aligned}\mathbf{y}^{(t)} &= \left(\mathbf{W}_{\mathrm{RF}}^{(t)}\mathbf{W}_{\mathrm{BB}}^{(t)}\right)^{H}\left(\mathbf{H}\mathbf{F}_{\mathrm{RF}}^{(t)}\mathbf{F}_{\mathrm{BB}}^{(t)}\mathbf{s}^{(t)} + \mathbf{n}^{(t)}\right) \\ &= \left((\mathbf{f}^{(t)})^{T} \otimes \mathbf{W}^{(t)}\right)\mathrm{vec}(\mathbf{H}) + \tilde{\mathbf{n}}^{(t)},\end{aligned} \tag{4}$$

- Stacking τ measurements ($\tau < N_{\mathrm{T}}N_{\mathrm{R}}$, under-determined)

$$\mathbf{y} = \mathbf{\Phi}\mathbf{h} + \tilde{\mathbf{n}} = \mathbf{\Phi}\mathbf{\Psi}\tilde{\mathbf{h}} + \tilde{\mathbf{n}}, \tag{5}$$

where $\mathbf{\Psi}$ is the sparsification basis, and $\tilde{\mathbf{h}}$ is the sparse support.

- Compressed reconstruction problem

$$\min_{\tilde{\mathbf{h}}} \|\tilde{\mathbf{h}}\|_0, \qquad \text{s.t. } \|\mathbf{\Phi}\mathbf{\Psi}\tilde{\mathbf{h}} - \mathbf{y}\|_2 \leq \varepsilon, \tag{6}$$

Outline

Dictionary Design

- Orthogonality and sparsest representation $\Rightarrow$ Singular/Eigen Value Decomposition (SVD/EVD)
- For general representation, consider auto-correlation

$$\begin{aligned} \mathbf{R}_{\mathrm{T}} &= \mathbb{E}\left[\mathbf{H}^{H}\mathbf{H}\right] \\ &= \gamma K \mathbf{H}_{\mathrm{LoS}}^{H}\mathbf{H}_{\mathrm{LoS}} + \gamma \mathbf{I}, \end{aligned} \tag{7}$$

- Each element in the auto-correlation matrix

$$\begin{aligned} \mathbf{R}_{\mathrm{T}}[m', m] &= \gamma \mathbb{1}_{m,m'} + \gamma K \mathbf{g}_{\mathrm{R}}^{H}(\mathbf{r}_{\mathrm{T}}^{(m)})\mathbf{g}_{\mathrm{R}}(\mathbf{r}_{\mathrm{T}}^{(m)}) \\ &= \gamma \mathbb{1}_{m,m'} + \gamma K \sum_{n=1}^{N_{\mathrm{R}}} \frac{e^{-j\kappa\|\mathbf{r}_{\mathrm{T}}^{(m)} - \mathbf{r}_{\mathrm{R}}^{(n)}\|}}{\|\mathbf{r}_{\mathrm{T}}^{(m)} - \mathbf{r}_{\mathrm{R}}^{(n)}\|} \times \frac{e^{j\kappa\|\mathbf{r}_{\mathrm{T}}^{(m')} - \mathbf{r}_{\mathrm{R}}^{(n)}\|}}{\|\mathbf{r}_{\mathrm{T}}^{(m')} - \mathbf{r}_{\mathrm{R}}^{(n)}\|}, \end{aligned} \tag{8}$$

with indicator $\mathbb{1}_{m,m'} = 1$ for $m = m'$, and 0 otherwise.

Dictionary Design

- Introducing paraxial approximation [10]

$$
\begin{aligned}
\mathbf{R}_{\mathrm{T}}[m', m] &\approx \gamma \mathbb{1}_{m,m'} + \frac{\gamma K}{r_0^2} \sum_{n=1}^{N_{\mathrm{R}}} e^{-j\kappa \frac{\left(x_{\mathrm{T}}^{(m)} - x_{\mathrm{R}}^{(n)}\right)^2 - \left(x_{\mathrm{T}}^{(m')} - x_{\mathrm{R}}^{(n)}\right)^2}{2y_0}} \\
&\triangleq \gamma \mathbb{1}_{m,m'} + \gamma K e^{j\kappa \frac{(x_{\mathrm{T}}^{(m')})^2 - (x_{\mathrm{T}}^{(m)})^2}{2y_0}} \mathbf{R}'_{\mathrm{T}}[m', m],
\end{aligned}
\tag{9}
$$

- The auto-correlation matrix is represented by
  - Indicator function $\mathbb{1}_{m,m'}$
  - Phase compensation $\exp\left(j\kappa \frac{(x_{\mathrm{T}}^{(m')})^2 - (x_{\mathrm{T}}^{(m)})^2}{2y_0}\right)$
  - Common remainder $\mathbf{R}'_{\mathrm{T}}[m', m]$

[10] D. A. B. Miller, "Communicating with waves between volumes: Evaluating orthogonal spatial channels and limits on coupling strengths," *Appl. Opt.*, vol. 39, no. 11, pp. 1681–1699, Apr. 2000.

Remark

Remark on Auto-Correlation $\mathbf{R}_{\mathrm{T}}$

The phase term $\exp\left(j\kappa\frac{(x_{\mathrm{T}}^{(m')})^2-(x_{\mathrm{T}}^{(m)})^2}{2y_0}\right)$ includes location information, which indicates that $\mathbf{R}_{\mathrm{T}}$ may not have a universal SVD/EVD.

Compensating such phase term requires **localization**, which will be discussed in the full-length version [11].

[11] S. Liu, X. Yu, *et al.*, *Sensing-enhanced channel estimation for near-field XL-MIMO systems*, 2024. arXiv: 2403.11809 [cs.IT]

Dictionary Design

- The common remainder in (9) can be further derived as

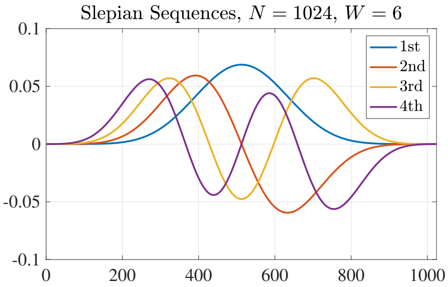
$$\begin{aligned} \mathbf{R}'_{\mathrm{T}}[m', m] &= \frac{1}{r_0^2} \sum_{n=1}^{N_{\mathrm{R}}} e^{j\kappa \frac{x_{\mathrm{R}}^{(n)} \left(x_{\mathrm{T}}^{(m)} - x_{\mathrm{T}}^{(m')}\right)}{y_0}} \\ &\propto \mathrm{sinc}\left(2W\left(x_{\mathrm{T}}^{(m)} - x_{\mathrm{T}}^{(m')}\right)\right), \end{aligned} \tag{10}$$

where $W = \kappa L_{\mathrm{R}}/(4\pi y_0)$.

- $\mathbf{R}'_{\mathrm{T}}$ is a **Toeplitz Sinc** matrix.

Dictionary Design

- The Eigenvectors of $\{\mathbf{v}_m\}_{m=1}^{M}$ of auto-correlation matrix $\mathbf{R}'_{\mathrm{T}}$ are known as *discrete prolate spheroidal sequence* (DPSS) or *Slepian sequence* within frequency W [12].



- Estimating the auto-correlation requires **numerous** samples. The **closed-form** solution here make it **efficient** to generate the dictionary.

[12] D. Slepian, "Estimation of signal parameters in the presence of noise," *IRE Trans. Inf. Theory*, vol. 3, no. 3, pp. 68–89, 1954.

Channel Estimation

- Coarse Localization
  - Coarsely estimate the location of UE.
- Build dictionary and estimate channel
  - Design dictionary according to **Algorithm** 1
  - Estimate channel matrix using OMP.

Algorithm 1: Proposed Dictionary Design Algorithm

Require: Estimated coordinate $(\hat{x}_i, \hat{y}_i)$ and the numbers of antennas N_{T} and N_{R}.

Ensure: The DPSS-based eigen-dictionary $\boldsymbol{\Psi}_e$.

1: Estimate the compensation matrix $\hat{\mathbf{D}}_{\mathrm{T}}$ and $\hat{\mathbf{D}}_{\mathrm{R}}$ according to (**??**).

2: Calculate the frequency $\hat{W} = \kappa L_{\mathrm{R}}/(4\pi\hat{y}_i)$.

3: Generate $\mathbf{R}_{\mathrm{T}}$ and $\mathbf{R}_{\mathrm{R}}$ with DPSS according to (10).

4: Perform EVD for $\mathbf{R}_{\mathrm{T}} = \mathbf{V}\boldsymbol{\Lambda}\mathbf{V}^{-1}$ and $\mathbf{R}_{\mathrm{R}} = \mathbf{U}\boldsymbol{\Lambda}'\mathbf{U}^{-1}$.

5: Compensate phase shift $\mathbf{V}^c = \hat{\mathbf{D}}_{\mathrm{T}}\mathbf{V}$, $\mathbf{U}^c = \hat{\mathbf{D}}_{\mathrm{R}}\mathbf{U}$.

6: Return eigen-dictionary $\boldsymbol{\Psi}_e = (\mathbf{V}^c)^* \otimes \mathbf{U}^c$.

Outline

Simulation Setup

- Evaluation criteria normalized mean square error (NMSE)

$$\mathrm{NMSE}\left(\hat{\mathbf{H}}, \mathbf{H}\right) = \mathbb{E}\left[\|\hat{\mathbf{H}} - \mathbf{H}\|_F^2 / \|\mathbf{H}\|_F^2\right], \tag{11}$$

- $f_c = 28$ GHz
- $N_{\mathrm{T}} = 192,\ N_{\mathrm{R}} = 4$
- Near field range $[1\ \mathrm{m}, 20\ \mathrm{m}]$
- Rician factor $K = 13$ dB
- Compression Ratio (CR)

$$\mu = \frac{\tau}{N_{\mathrm{T}} N_{\mathrm{R}}} \in \{0.25, 0.4, 0.6\} \tag{12}$$

Numerical Results

Overview

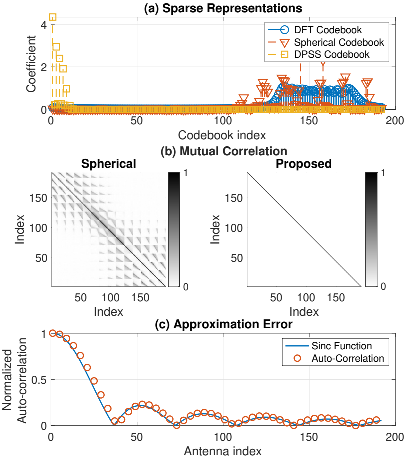


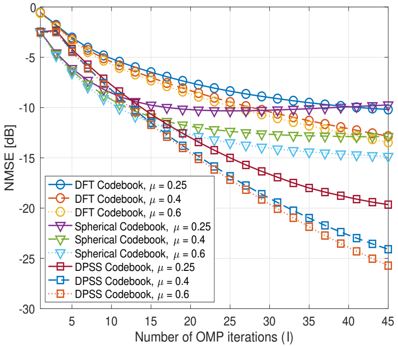
Figure 6: (a) The sparse representations of near-field channels under different dictionary, (b) the mutual correlation matrix $\mathbf{\Psi}^H\mathbf{\Psi}$ of the spherical wave dictionary and the proposed dictionary, (c) the approximation error in (10).

Numerical Results

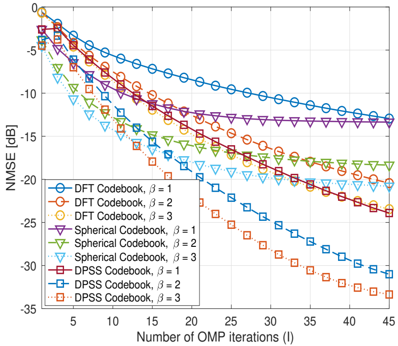
Near-field CE NMSE error versus

(a) CR μ and number of iterations I. s.t. $\beta = 1$.

(b) number of iterations I and dictionary oversampling rate β. s.t. $\mu = 0.4$.



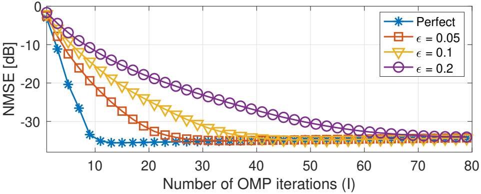
(a) CR $[\mu]$



(b) Oversampling Rate $[\beta]$

Numerical Results

- **Convergence** against localization errors ϵ.
- **Minimum required** dictionary size for target NMSE



Dictionary	Target NMSE			
	-15 dB	-20 dB	-25 dB	-30 dB
DFT	768	855	$1,150$	N/A
Spherical[4]	$1,150$	$3,072$	$15,552$	N/A
Proposed	**768**	**768**	**768**	**768**

Summary

- Proposed a DPSS-based eigen-dictionary for near-field XL-MIMO CE.
  - Mutual **orthogonality** achieved among codewords
  - **Storage** requirements efficiently relaxed.

Outline

[1] J. Goodman, *Introduction to Fourier Optics* (McGraw-Hill physical and quantum electronics series). W. H. Freeman, 2005, ISBN: 9780974707723.

[2] A. Fenn, "Evaluation of adaptive phased array antenna, far-field nulling performance in the near-field region," *IEEE Trans. Antennas Propag.*, vol. 38, no. 2, pp. 173–185, 1990.

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[5] J. Chen, F. Gao, M. Jian, and W. Yuan, "Hierarchical codebook design for near-field mmwave MIMO communications systems," *IEEE Wireless Commun. Lett.*, vol. 12, no. 11, pp. 1926–1930, 2023.

[6] X. Shi, J. Wang, Z. Sun, and J. Song, "Spatial-chirp codebook-based hierarchical beam training for extremely large-scale massive MIMO," *IEEE Trans. Wireless Commun.*, pp. 1–1, 2023.

[7] X. Zhang, H. Zhang, and Y. C. Eldar, "Near-field sparse channel representation and estimation in 6G wireless communications," *IEEE Trans. Commun.*, vol. 72, no. 1, pp. 450–464, 2024.

[8] D. L. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, 2006.

[9] 3GPP, "Study on channel model for frequencies from 0.5 to 100 GHz," Tech. Rep., 2023, Release 17.

[10] D. A. B. Miller, "Communicating with waves between volumes: Evaluating orthogonal spatial channels and limits on coupling strengths," *Appl. Opt.*, vol. 39, no. 11, pp. 1681–1699, Apr. 2000.

[11] S. Liu, X. Yu, Z. Gao, J. Xu, D. W. K. Ng, and S. Cui, *Sensing-enhanced channel estimation for near-field XL-MIMO systems*, 2024. arXiv: `2403.11809 [cs.IT]`.

[12] D. Slepian, "Estimation of signal parameters in the presence of noise," *IRE Trans. Inf. Theory*, vol. 3, no. 3, pp. 68–89, 1954.

Full-length Version

Full-length version available at arXiv

[Ref] S.Liu, X. Yu, Z. Gao, D. W. K. Ng, and S. Cui, "Sensing-Enhanced Channel Estimation for Near-Field XL-MIMO Systems", arXiv preprint. [Online] Avaiable: https://arxiv.org/abs/2403.11809

- Details on how to obtain location coordinates without computation-intensive MUSIC algorithms.
- Generalized multi-path channel model

Thanks for your attention!
Q & A

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How to Obtain the Locations of UE/Scatterers

- One single iteration using a spherical dictionary (adopted here).

$$i = \underset{j}{\operatorname{argmax}} \left\| \left(\mathbf{\Psi}_p^H\right)[j,:]\mathbf{\Phi}^H\mathbf{y}\right\|^2, \tag{13}$$

- Other low-complexity algorithms (e.g., **Time Inversion Algorithm** proposed in journal version)

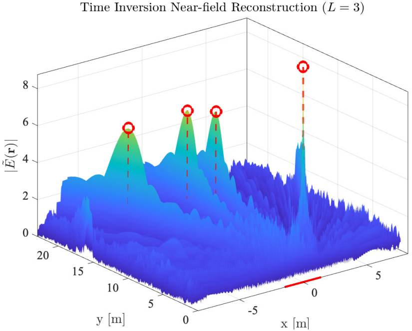

Figure 8: A demonstration of the proposed localization method with $N_{\mathrm{BS}} = 512$ antenna elements and $L = 3$ NLoS paths. $f_c = 28$ GHz.

Connection with MUSIC Algorithm

Motivation: EVD based methods

MUSIC

- Measurement

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{N}$$

- Auto-correlation

$$\boldsymbol{R}_Y = \boldsymbol{A}\boldsymbol{R}_X\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I}$$

- EVD

$$\boldsymbol{R}_Y = \boldsymbol{U}^{-1}\left(\boldsymbol{\Lambda} + \sigma_n^2\boldsymbol{I}\right)\boldsymbol{U}$$

- Noise subspace orthogonality...

Proposed

- Channel Model

$$\boldsymbol{H} = \sqrt{\gamma K}\boldsymbol{H}_{\mathrm{LoS}} + \sqrt{\gamma}\boldsymbol{H}_{\mathrm{NLoS}}$$

- Auto-correlation

$$\boldsymbol{R}_H = \gamma K\boldsymbol{R} + \gamma\boldsymbol{I}$$

- EVD

$$\boldsymbol{R}_H = \gamma\boldsymbol{D}^{-1}\boldsymbol{U}^{-1}\left(K\boldsymbol{\Lambda} + \boldsymbol{I}\right)\boldsymbol{U}\boldsymbol{D}$$

- Estimate $\boldsymbol{D}$, obtain $\boldsymbol{U}$, with

$$\boldsymbol{R}' = \boldsymbol{U}^{-1}\boldsymbol{\Lambda}\boldsymbol{U}$$

as a Toeplitz Sinc matrix.