Homework 7: Graph Traversal and Components

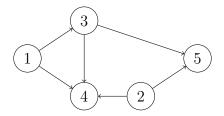
Due: Friday, March 27 at 11:59 pm on Canvas

Concepts: graphs, graph traversal, random graphs

Theory

1. (6 points) A topological ordering (also called topological sort) of a directed graph G = (V, A) is an ordering of the nodes in G such that for all directed arcs (u, v), u comes before v in the ordering. An example is given below where the node label is the ordering. Explain how you could either find a valid topological ordering or determine that no such ordering exists. Include pseudocode for your algorithm and be sure to explain the logic behind your ordering.

Hint: It may help to know that a topological ordering exists if and only if there is not a cycle in G (think about why this is true).



Practice

2. (5 points) Write a function that finds the number of components in an undirected graph G = (V, E) and write a corresponding test function. **Hint:** Think about using depth-first or breadth-first search to find a single component.

For your graph representation, I recommend that you use the library networkx, but you should not rely on any networkx algorithms for this implementation. A snippet of code using this library is shown below.

import networkx as nx

```
G = nx.Graph()
G.add_node(1) # adds node with label 1
G.add_node(2) # adds node with label 2
G.add_edge(1,2) # adds an undirected edge between nodes 1 and 2
G.edges() # returns an iterator over all edges
G.nodes() # returns an iterator over all nodes
G.neighbors(1) # returns the neighbors of node 1
```

3. (5 points) A random binomial graph G(n, p) is a graph on n nodes such that for every pair of nodes (i, j) there exists an edge between i and j with probability p. For $n = 5, 6, \ldots, 50$, find the smallest probability p (that is a multiple of 0.01) such that ten generated binomial random graphs G(n, p) each consists of a single component. Plot this probability as a function of n and comment on the results.

To create the random graphs, it may be useful to use np.random.uniform.