Linear Fits of Mystery Impedances Lab

Introduction to Impedance Lab

1. We will analyze impedance vs frequency of several mystery boxes to determine their circuit components.

2. Notes:
   a. Measurement error is not the source of disagreement between this experiment's data and model, it's from the *approximate* circuit model

   b. Reading a dial or label is not a measurement (Multimeter: R, V, I, Oscilloscope: f)

   c. All circuits have some R, C, and L

6.2 Preliminary Experiment: Output Impedance

a. Review: Ohmic materials and their IV curves, and voltage/current of two resistors in series

b. Internal resistance: (figure 6.1)
   i. A 9V battery (say for a flashlight) supplies less than 9V, especially for small load resistance R_L.

   ii. If $R_L = \infty$, no current flows.
      1. This is equivalent to detaching the wires, "open circuit"
      2. Source output will be V_0.

   iii. If R_L is decreased to some finite value, current will begin to flow.
      1. The output voltage will be $V = V_0 - IR_0$ (using Kirchhoff's loop law)
      2. Exercise: Notice $I = V_0/(R_0 + R_L)$, or in terms of output voltage $I = V/R_L$ as expected for a source of voltage V with a load R_L.

6.2 Preliminary Experiment: Output Impedance

i. Source output characteristic: relation between voltage and output current
    1. Draw figure 6.2: linear $V(I) = V_0 - IR_0$ (slope and intercept)

ii. Limiting cases: open and short circuits
    i. Note: Low load impedances can cause large currents, internal heating which raises R_0, and consequently nonlinear behavior.

iii. Power: energy dissipated per second
    1. "Impedance matching": max power when load matches the source $R_L = R_0$
        a. At this R_L, $V = V_0/2$
    2. To minimize energy loss (i.e. voltmeter), we need $R_L \gg R_0$

6.2 Preliminary Experiment: Output Impedance

Preliminary Experiment (this goes in your report): Find the output (internal) resistance and open-circuit voltage of a source subcircuit (one of the battery powered boxes) by:

a. measuring its output voltage at 5 values of R_L (500 – 3000ohms)
b. plotting the IV curve (assume straight line)
c. and using the slope and intercept.
d. Video

Preliminary Experiment		
Box 2	V	Load Resistance R_L
	2.55	300
	5.22	1000
	5.99	1400
	6.74	2000
	7.09	2400
	7.46	3000
	9.5	10,000,000

6.4 Nonlinear Elements

Nonlinear elements: diodes are nonlinear circuit elements whose impedance may not vary with frequency but does with applied voltage.

Complex Impedance

How to use complex variables
to analyze linear AC circuits

6.5 How to Use Complex Variables to Analyze Linear AC Circuits

We will study some mystery circuit boxes by analyzing the voltage, and the phase between the current and voltage, as a function of the AC driving frequency.

i. The boxes may contain 1, 2, or 3 components (LRC) in series or parallel

ii. We will determine how they are connected, and the values of each component

Alternating Current in Complex Notation

- A resistor with an alternating current passing through it has voltage across it:

$$V = IR = RI_0\cos(\omega t)$$

- This is a real ($V \in \mathbb{R}$), measurable voltage that is in phase with the source.

- The voltage across a capacitor is $V_C = \frac{Q}{C} = \frac{1}{C}\int I dt$
- The voltage across an inductor is $V_L = L\frac{dI}{dt}$
- A phase is generated in the *C* and *L* cases, but not *R*.

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- A phase is generated in the *C* and *L* cases, but not *R*.

- Using complex numbers for voltage, current, and capacitive and inductive reactances (to get the net impedance) simplifies the calculations.
- If we invent an unphysical, mathematical imaginary part of the current $I_i = I_0 \sin(\omega t)$, we can write:

$$I = I_r + iI_i = I_0[\cos(\omega t) + i\sin(\omega t)] = I_0 e^{i\omega t}$$

Complex Reactances

- The voltage drop across a capacitor is then:

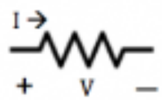
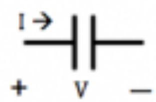
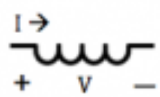
$$V = \frac{Q}{C} = \frac{1}{C}\int I dt = \frac{1}{C}\int (I_0 e^{i\omega t}) dt = \frac{I_0}{i\omega C} e^{i\omega t} = I\left(\frac{1}{i\omega C}\right) = IZ_C$$

- The **capacitive reactance $\boldsymbol{Z_C}$** is analogous to resistance but complex.

- Similarly for **inductive reactance $\boldsymbol{Z_L}$**, the voltage drop across an inductor is:

$$V = L\frac{dI}{dt} = L\frac{d}{dt} I_0 e^{i\omega t} = I(i\omega L) = IZ_L$$

Summary of Simple Circuit Elements' Behaviors

Circuit Element	**Symbol**	**Current-Voltage Relationship in Time**	Reactance or **Impedance**
Resistor		$V = IR$	R
Capacitor		$I = C\frac{dV}{dt}$	$\frac{1}{j\omega C}$
Inductor		$V = L\frac{dI}{dt}$	$j\omega L$

Combining Reactances into Total Impedance

- Reactances in series add: $Z = Z_1 + Z_2 + \cdots$

- Reactances in parallel add harmonically: $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots$

Combining Reactances into Total Impedance

- We can write the net complex impedance in polar form $Z = |Z|e^{i\theta}$
- The magnitude of the net complex impedance is:

$$|Z| = |Z_r + iZ_i| = \sqrt{Z_r^2 + Z_i^2}$$

Beware that $Z_{r,i}$ are the real or imaginary *parts* of the complex number Z.

$$|Z| = (Z^* Z)^{1/2}$$

works too (it's the same as above)

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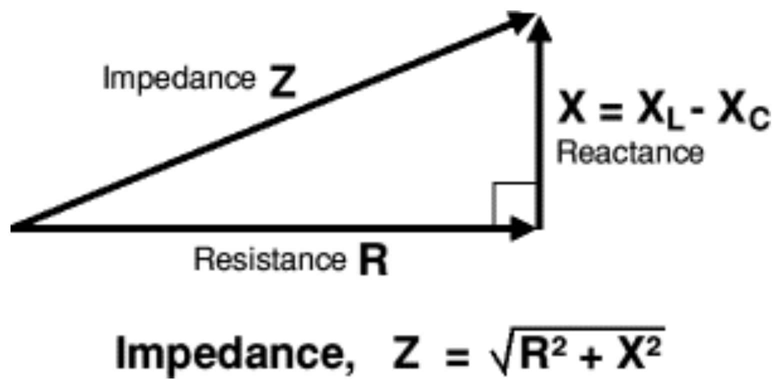
- The phase it introduces is: $\tan\theta = \frac{Z_i}{Z_r}$



Minus sign from $1/i$ in X_C

Example: Series RL Circuit (6.5.1 in manual, the exercises in 6.6 are homework)

Example: Resonant Series LRC Circuit (6.10 in manual)

Units

- Frequency f: measured in Hertz [Hz], what the oscilloscope reads
- Angular Frequency: $\omega = 2\pi f$, appears in oscillating voltage equation $V(t) = V_o \cos(\omega t + \theta)$

The Main Experiment

Study 3 mystery boxes:

a. By passing an alternating current through our mystery box and measuring the voltage and phase response as we vary the frequency, we can determine the impedance $|Z(\omega)|$ of the box.

b. If we can fit our data to a line, $y = mx + b$, then m and b tell us about our circuit components.
    a. You must hypothesize a number of possible combinations of circuit elements (LRC in series or parallel) until you get a linear fit
    b. Plot the raw data initially to determine whether you expect i.e. ω or $1/\omega$ dependence
    c. How will the resistance R will appear on a linear fit?
        a. What could it mean if R is found to be very small?

The Main Experiment

Study 3 mystery boxes:

a. Introduce figure 6.9, a circuit for measuring |Z|

   i. Channel 1 measures $V_1 = I|Z|$

   ii. Channel 2 measures $V_2 = IR_{decade}$

   iii. The currents are the same $I_1 = I_2$ (series), so $V_1 = \frac{V_2}{R}|Z|$, or $|Z| = \frac{V_1}{V_2}R$

   iv. We must record V_1, V_2, and R for each data point, and the phase between V_1 and V_2 is the phase between V_1 and I.

   v. The phase offset is measured in ms and translated to phase angle by (check units) $\phi = (\Delta t)f(360°)$

The Main Experiment

Study 3 mystery boxes:

Experiment: Pass an AC current through the black box, and measure output voltage and phase at various frequencies.

i. Record 20 data points between 10 Hz – 100 kHz in Excel

1. Measure the DC resistance of each box to rule out some initial guesses (ohmmeter)
    a. Inductors have a small DC resistance
    b. Capacitors have an infinite DC resistance

2. Take extra data points around resonances, non-linear regions

3. Pick R_{decade} so V_1 is within 10x V_2 to minimize stray pickup

4. We can use the measure function to obtain voltages, frequencies, and phases.

The Main Experiment

Study 3 mystery boxes:

Resonant circuits have both ω and $1/\omega$ dependence.

i. We can plot high and low frequencies (away from resonance, where both effects play a large role) separately as straight lines

ii. Resonant frequency is another probe of *L* and *C*: $\omega_0 = \frac{1}{\sqrt{LC}}$

iii. Width of resonance also gives info (frequency difference at phases $\pm 45^{\circ} = R/L$)

The Second Experiment

1. Secondary Experiment – Nonlinear Elements
    a. Diodes are nonohmic, IV curve not linear
    b. What to include in report? Procedure, circuit, drawings/photo, discussion?
    c. Display IV curve, I vertical, V horizontal (xy-mode?)
    d. Start zero amp, dot. Increase w, watch nonlinearity arise.
    e. The oscillating voltage source probes both directions of current in the diode
    f. Extra: see if the log of I varies linearly with V in the positive V region.
    g. Zener Diode: allows current when negatively biased beyond the Zener breakdown voltage

Uncertainties in Analysis of Impedance

a. Use error propagation formula $\sigma_Z = \sqrt{\left(\frac{\partial Z}{\partial A}\right)^2 \sigma_A^2 + \cdots}$

b. Assume frequencies contribute negligible error

c. The level of accuracy of the measuring devices introduce some error

   i. V_1 and V_2: The Techtronix oscilloscope user manual reports $\pm\left[3\%^* (\text{Voltage measured}) + 0.05\text{div}^* \left(\frac{\#V}{\text{div}}\right)\right]$

   ii. R: The multimeter's ohmmeter has uncertainties:

      1. $\pm 0.1\Omega$ for resistances between $1 - 10\Omega$

      2. $\pm 1\Omega$ for resistances on the order of 100Ω