

# Staring at Colorful Spectral Lines & Experimentally Calculating the Rydberg Constant

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Spectroscopy is arguably one of the most powerful tools physicists and astrophysicists contain in their back pockets, so-to-speak. We have the means to probe the farthest reaches of our universe; spectroscopy allows us to understand and appreciate matter close and far, identify substances based on characteristic atomic spectra. We venture into calculating and identifying several parameters associated with spectral lines for hydrogen and helium, then moving on to compare a helium-neon laser with its constituent components' spectra. We will use previously recorded data on undeflected angle, color, angle, and order of fringes for each experiment, and take several weighted averages to calculate estimates for each parameter. We find that for helium, the grating spacing  $d \approx 3.37 * 10^3 \text{ nm} \pm 2.23 [\text{nm} * \text{rad}]$ ; for hydrogen, we determine our experimental Rydberg constant for indices [1,2,4,7] to be  $R_H \approx 1.0991 * 10^7 \text{ m}^{-1} \pm 1.99 * 10^4 [\text{m}^{-1} * \text{rad}]$  with  $\tilde{\chi} = 1.34$  for an expected  $R_H = 1.0967 * 10^7 \text{ m}^{-1}$  [1]. We discuss the simultaneous observation of laser and neon lines, and determine that the neon is primarily responsible for the laser's rich, red color, with a weighted average wavelength  $\lambda \approx 630.3 \text{ nm} \pm 2.91 [\text{nm} * \text{rad}]$  for observed fringes.

## I. INTRODUCTION

Our main experiment consists of using a high-precision visible light spectrometer to confirm the expected energy level pattern of the hydrogen atom and thereby deduce a value for the Rydberg constant, an important value which determines the separation of energy levels for hydrogen. We carry out this experiment with the hope to demonstrate the sensitivity and power of spectroscopy as a method of analysis, possibly being one of the most important tools in the back pockets of physicists and astrophysicists.

Furthermore, we further our understanding of underlying concepts through data analysis and manipulation via. error propagation and statistical analysis, giving us a lens through which proper and legitimate conclusions can be made.

In this experiment, we will observe and measure wavelengths of radiation emitted from excited atoms in a dilute gas chamber. We will use a grating rather than a prism to isolate light into its component colors. At first, we will use a helium lamp with its known wavelength standard and use our data to calculate the spacing of the lines in our grating. Then, using our calculated value, we will take data for hydrogen, treating the hydrogen lines as unknown. The subsequent data will then be used to calculate the value of the Rydberg constant.

### A. Historical Background

Isaac Newton, at the early age of 23 in 1666, observed a "phenomena of colours" and coined the word *spectrum* after conducting an experiment in which he diffracted a

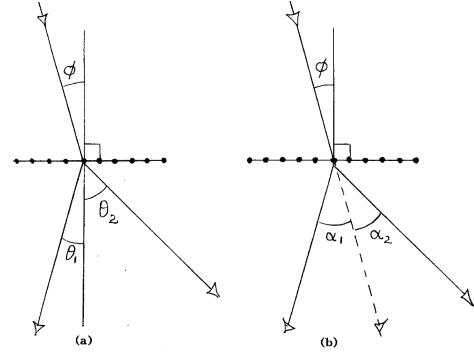


Figure 1. Light ray geometry with (a) diffraction and (b) measured angles displayed [1].

beam of light using a shard of glass [1]. There were a load of brilliant colors, spread from red to violet in the visible spectrum: he had recreated the rainbow in his laboratory.

Almost a whole century later, in 1752, Thomas Melvill made the first recorded observations of spectral lines by viewing burning spirits with various salts through a similar prism as Newton had [1].

However, most stunningly, Joseph Fraunhofer used the yellow sodium lines to measure the refractive index of certain glasses [1]. In 1814, this forefather of modern spectroscopy analyzed the spectrum of hundreds of dark absorption lines, among which are those now noted as Fraunhofer lines.

The field of optical spectroscopy has ever since flourished. Its instruments allow us to appreciate and understand matter close and far, stuff in the most remote reaches of our universe.

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## B. Geometry of Spectral Lines

In order to determine spectral wavelengths using the spectrometer, we must first determine a value for  $d$ , the average spacing between rulings on our diffraction grating. Although the manufacturer provides a nominal grating constant printed on the grating itself, it is crucial to calibrate an experimental value using some set of spectral wavelengths which are already known: for example, helium is a perfect candidate. Helium's calibration spectrum becomes what we use to deduce  $d$ 's value.

So, for each experiment we record the angle of the undeflected light, as well as the angles of diffracted lines. Admiring Fig. (1) provided by our lab manual, we see that for a given  $m$ , there will be two diffracted beams, with angles  $\theta_1$  and  $\theta_2$  with respect to the grating normal. These angles must satisfy the grating diffraction equation

$$\begin{aligned} \frac{m\lambda}{d} &= \sin \theta_1 + \sin \phi \\ \frac{m\lambda}{d} &= \sin \theta_2 + \sin \phi \end{aligned} \quad (1)$$

where  $m$ , the order of the spectrum, is always a positive integer. The spectrometer allows us to measure  $\alpha_1$  and  $\alpha_2$ , which are the angles of the diffracted rays of light with respect to the undiffracted beam. If we were to have perfect alignment of our instrumentation to  $\phi = 0$ , the diffracted rays would be symmetric, causing  $\theta_1 = \theta_2 = \alpha_1 = \alpha_2$ . However, in the more realistic case where  $\phi \neq 0$ , we can show the following:

$$\frac{m\lambda}{d} = \left[ 1 + \left( \frac{\sin \Delta}{\cos \Delta - \cos \theta} \right)^2 \right]^{-\frac{1}{2}} \sin \theta \quad (2)$$

Here, we define  $\theta = \frac{1}{2}(\alpha_1 + \alpha_2)$  and  $\Delta = \frac{1}{2}(\alpha_1 - \alpha_2)$ . Furthermore, we show that if  $\phi$  is sufficiently small (less than a couple arc minutes), then  $\Delta$  becomes extremely small, on order  $\phi^2$ . In this scenario, these terms become negligible and our diffraction equation reduces to something more familiar:

$$\frac{m\lambda}{d} = \sin \theta \quad (3)$$

## C. Rydberg Determination

If an atom is given a sufficient amount of some characteristic energy, whether by colliding with another atom or by absorbing radiation, it may then emit radiation whose wavelengths are individual to the type of atom. Thus, an atomic spectrum is really a kind of signature from which an atom can later be identified. We can also deduce important information on the physical structure of the atom, all thanks to the central idea of discrete energy levels within atoms. Any atom in some energy state  $E_1$  can make a transition to a lower state with some lower

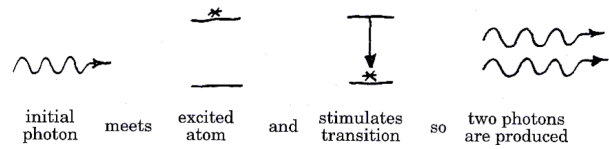


Figure 2. Illustration of the LASER phenomenon [1].

energy  $E_2$  by emitting a photon whose energy is  $E_1 - E_2$ . We can then relate the energy of this outgoing packet to its frequency  $\nu$  and wavelength  $\lambda$  by the Planck relation

$$E_1 - E_2 = h\nu = \frac{hc}{\lambda} \quad (4)$$

assuming  $h$  is Planck's constant and  $c$  is the speed of light. The inverse of this phenomenon may also occur, in that an atom absorbs a photon whose energy equals the difference in energy between two states and is thereby raised, or excited, from a lower energy state to a higher energy state.

Let us consider the hydrogen atom, the simplest of all atoms, consisting of a single proton and electron only. Quantum theory describes the energy levels  $E_n$  for the hydrogen atom as

$$E_n = -\frac{\mu e^4}{8\epsilon_0^2 h^2} \frac{1}{n^2} \quad (5)$$

where  $\mu$ , the reduced mass, we define as  $\frac{1}{\mu} \equiv \frac{1}{m} + \frac{1}{M}$  ( $m$  is the mass of the electron and  $M$  is the mass of the proton),  $\epsilon_0$  is the permittivity of free space,  $e$  is the charge on the electron, and  $n$  is the principle quantum number.

Using Eq. (4) and Eq. (5) we can encapsulate an expression for  $\lambda$  which corresponds to the entire emission spectrum of hydrogen, and so all possible transitions between energy levels as well:

$$\begin{aligned} \frac{1}{\lambda} &= \frac{E_{n_1} - E_{n_2}}{hc} = \frac{\mu e^4}{8\epsilon_0^2 h^3 c} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \\ \frac{1}{\lambda} &= R_H \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \end{aligned} \quad (6)$$

Here,  $n_1$  and  $n_2$  represent the principal quantum numbers of the initial and final states, respectively.  $R_H$ , the Rydberg constant with subscript H (for hydrogen) has an accepted experimental value of  $10967758m^{-1}$ , with an astoundingly low error of only  $1m^{-1}$  [1].

## D. Spectra of He-Ne LASER

The characteristically blinding-red light emitted by a helium-neon LASER (Light Amplification by Stimulated Emission of Radiation) stems from a spectral transition between two highly energized states in the neon atom.

In a typical spectral emission, each atom radiates independently with a *spontaneous* lifetime on the order of  $10^{-8}$  seconds [1]. More complex atomic structures have metastable, excited energy levels with much longer lifetimes. Transitions occurring from such metastable states are called *forbidden* transitions. If radiation present with the frequency corresponding to such a transition, it can actually stimulate more forbidden transitions to occur through stimulated emission of radiation (hence, LASER). Fig. 2 illustrates this process.

In a laser, some working substance is placed between two parallel mirrors to continuously reflect light back and forth between, forming a standing wave pattern. This light stimulates emission from the atoms in our working substance, causing a cascade effect and allowing for the extraction of a fraction of the energy from the atoms in the form of an external beam [1]. The interplay between excitation and emission of atoms provides a continuous and sufficient supply of atoms in the excited state.

For our helium-neon laser experiment, radiationless transfer of energy from the helium to neon is allowed by them nearly sharing energy levels for one of their excited states [1].

### E. Error Propagation & Statistical Methods

For this spectroscopy laboratory assignment, we are handed data from previous students' work. So, we really have no say in the accuracy of our data-taking. We will begin by optimistically assuming these previous students practiced good scientific etiquette. In order to conduct the majority of our calculations, we must assume that our  $\Delta$  is significantly small, minuscule enough for Eq. (3) to become completely applicable. Unfortunately, after looking at the data we have been given and computing values for  $\Delta$ , we note that our results are affected by this error non-trivially. Yet, we continue the lab assignment with the reduction in complexity by stating that  $\phi$  is sufficiently small.

In order to propagate error, we utilize the general formula for error propagation, given the term we wish to isolate an uncertainty for  $f(x_1, \dots, x_N)$ :

$$\sigma_{x_i} = \sqrt{\sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \sigma_{x,i} \right)^2} \quad (7)$$

We start with  $\sigma_\alpha$ , the error in the alpha measurements:

$$\begin{aligned} \sigma_{\alpha_1} = \sigma_{\alpha_2} &= \sqrt{(\sigma_{\text{vernier}})^2 + (\sigma_\Delta)^2} \\ \sigma_{\text{vernier}} &= 1' \\ \sigma_\Delta &= \Delta = \frac{1}{2}(\alpha_1 - \alpha_2) \end{aligned} \quad (8)$$

All terms with units degrees must be converted into radians to ensure a legitimate, unit-full uncertainty. In

reality, each uncertainty defined in the paper of length contains units of  $[\text{length} * \text{rad}]$ .  $\sigma_{\text{vernier}}$  is defined to be a single arc-minute, as described by the manufacturers of our spectrometer. Then, we continue to propagate the error into  $\theta$

$$\begin{aligned} \sigma_\theta &= \sqrt{\left( \frac{\sigma_\alpha}{2} \right)^2 + \left( \frac{\sigma_\alpha}{2} \right)^2} \\ \sigma_\theta &= \frac{\sigma_\alpha}{\sqrt{2}} \end{aligned} \quad (9)$$

where we defined theta as  $\theta = \frac{1}{2}(\alpha_1 + \alpha_2)$ , and would then move on to the next variable dependent on previously calculated uncertainties.

Similarly, we find  $\sigma_d$ , the uncertainty in the grating spacing through

$$\sigma_d = \sqrt{\left( -\frac{m\lambda \cos \theta}{\sin^2 \theta} \sigma_\theta \right)^2} \quad (10)$$

where  $\theta$  and  $\sigma_\theta$  are defined in radians,  $m$  is the order of the observed fringe, and  $\lambda$  is the wavelength of light. Then, we obtain an uncertainty for  $\lambda$  in order to carry through calculations for the hydrogen experiment:

$$\sigma_\lambda = \sqrt{\left( \frac{\sin \theta}{m} \sigma_{d_w} \right)^2 + \left( \frac{d_w \cos \theta}{m} \sigma_\theta \right)^2} \quad (11)$$

Here,  $d_w$  represents the weighted average value for the grating spacing. Finally, we can show that the uncertainty for our experimental Rydberg constant is

$$\sigma_{R_H} = \frac{\sigma_\lambda}{\lambda^2 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)} \quad (12)$$

where  $n_f$  and  $n_i$  are the final or terminal and initial energy levels, respectively.

Eq. (7) applies to any function of multiple variables. So, we apply this idea to each of our calculated values during this laboratory experiment, and find the uncertainty associated with each.

More importantly, we employ a series of weighted averages to obtain accurate estimations of our values for the grating spacing,  $d$ , our Rydberg constant,  $R_H$ , as well as the wavelength for our Helium-Neon laser,  $\lambda$ .

In general, for a weighted average, the weighted value, weight parameter, and deviation of the weighted average are defined as

$$x_w = \frac{\sum_{i=1}^N x_i w_i}{\sum_{i=1}^N w_i}, \quad w_i = \frac{1}{\sigma_{x,i}^2} \quad (13)$$

$$\sigma_{\bar{x}, wav} = \frac{1}{\sqrt{\sum_{i=1}^N w_i}} \quad (14)$$

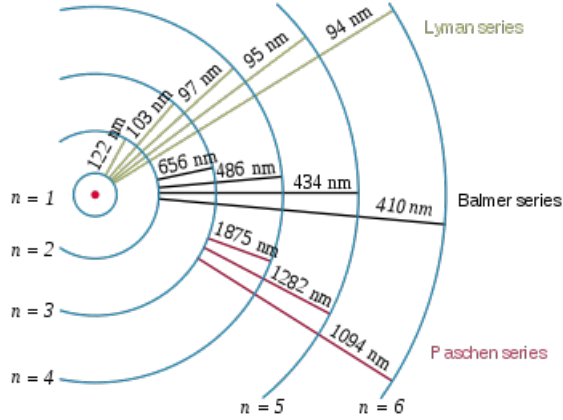


Figure 3. Hydrogen energy transitions and resulting wavelengths. Energy levels are not to scale [2].

where  $x_w$  is the weighted average of some parameter  $x$ ,  $w_i$  is the weighting on the average calculation, and  $\sigma_{\bar{x},w,w}$  represents the uncertainty on our new, weighted parameter of  $x$ .

## II. APPARATUS AND PROCEDURE

Rather than using a prism, we will be using a grating to disperse light into its component colors. Interference drives the inner workings of the grating, and its index of refraction varies with frequency. We employ the use of a Spencer spectrometer to analyze light through the instruments adjustable slits, collimator, and cross-hair-fitted eyepiece.

Again, we were not present inside the laboratory to physically use the equipment; rather, we were handed data and told to analyze assuming good practices were taken. Previous students recorded angle measurements using a vernier scale, observed color by eye, and noted the order of each fringe by careful examination.

### A. Helium Experiment

For the first part of the laboratory assignment, we take a helium lamp as a known wavelength with negligible uncertainty and use data taken from that lamp to ascertain a value for  $d$ , the grating constant. We identify the wavelengths of eight helium spectral lines, and use a weighted average to calculate the grating constant and its associated error.

For each lamp setup, students record the angle of undeflected light (essential for checking alignment), as well as the angles of the diffracted lines. Given these values, Eq. (3) may be used to determine a value for  $d$  for each pair of  $\alpha_1$ ,  $\alpha_2$  in our observations.

### B. Hydrogen Experiment

For the second portion of the lab, we take our weighted grating constant and obtain values for the wavelength of eight hydrogen lines using recorded angles. With each calculated wavelength, we provide some principle quantum number assigned based on known transition energies for hydrogen's Balmer series (Fig. 3).

We choose to select four data pairs based on our Balmer series fits to ensure the most accurate calculation of subsequent values. The hydrogen data would then be used to calculate an experimental value for the Rydberg constant with some associated uncertainty. For each of the hydrogen lines we have, we can now determine  $R_H$  assuming the theory of Eq. (6) is accurate.

|   | $m$ | $\lambda[nm]$ | $\theta$ | $d[nm]$  | $\delta_d$ |
|---|-----|---------------|----------|----------|------------|
| 0 | 1   | 668.0         | 11.50°   | 3.35e+03 | 3.39       |
| 1 | 1   | 588.0         | 10.00°   | 3.39e+03 | 3.95       |
| 2 | 1   | 501.0         | 8.99°    | 3.21e+03 | 52.7       |
| 3 | 1   | 492.0         | 8.11°    | 3.49e+03 | 118        |
| 4 | 1   | 447.0         | 7.93°    | 3.24e+03 | 76.1       |
| 5 | 1   | 388.0         | 6.86°    | 3.25e+03 | 120        |
| 6 | 2   | 588.0         | 20.29°   | 3.39e+03 | 4.91       |
| 7 | 2   | 501.0         | 17.34°   | 3.36e+03 | 11.5       |

Figure 4. Helium Experiment: order of fringe observed  $m$ , calculated wavelength  $\lambda$  in nanometers, angle  $\theta$  in degrees, grating spacing  $d$  in nanometers, uncertainty in  $d$   $\delta_d$  in  $[nm * rad]$  due to nature of angles and error propagation. Color indicates magnitude of uncertainty.

|                         | $d_w[nm]$ | $\delta_{d_w}$ |
|-------------------------|-----------|----------------|
| <b>Weighted Average</b> | 3.37e+03  | 2.23           |

Figure 5. Helium's weighted average for the grating spacing  $d_w$  in nanometers and its uncertainty  $\delta_{d_w}$  in  $[nm * rad]$ .

### C. Helium-Neon LASER Experiment

For the last part of our experiment, we utilize data recorded for a helium-neon laser setup and calculated the

weighted average of its wavelength with associated uncertainty. We later discuss whether helium or neon is mainly responsible for the laser's color. This is accomplished by scattering some of the laser light into the entrance slit of our spectrometer, and determining whether the laser radiation coincides with any lines observed in the neon or helium spectrum.

### III. RESULTS AND DISCUSSION

#### A. Helium Experiment

Using the lab manual provided, we first identify eight of the helium spectral lines as seen in Fig. 4. All values of undeflected angle, color, incident angle, and order are recorded and preserved in the appendix section IV, Fig. 11. Utilizing the application of Eq.'s (13) and (14), we calculate the weighted average of  $d$ , the grating spacing for the helium experiment. For the setup described in our error propagation & statistical methods section IE, we experimentally conclude in Fig. 5 that  $d \approx 3370nm \pm 2.23[nm * rad]$ .

Fig. 4 illustrates the absurdly large uncertainty in  $d$ , dependent on  $\Delta$ , which seems to have varied throughout the duration of the experiment. This parameter, directly associated with the calibration and alignment of the instrumentation, would ideally be constant and near zero in value. However, this is not the case, and so we will tend to see larger errors than normal given that proper procedure was not followed in the lab.

#### B. Hydrogen Experiment

Again, previous students have recorded the central undeflected angle, color, angle, and order of each set of data as seen for the hydrogen experimental values in the appendix section IV, Fig. 12.

We calculate the  $\lambda$  value for every hydrogen spectral line observed during this experiment, using the grating constant previously found during the helium experiment.

Fig. 6 depicts each predicted wavelength, uncertainty, and associated Balmer series principal quantum number for each pair of values.

We go on to calculate  $\delta_{balmer}$ , a paired deviation from the expected Balmer transition wavelength, in order to identify which entries in our table are most accurately representative of the hydrogen atomic spectrum. Upon observation, we find that indices 1,2,4, and 7 have the best Balmer transition series fits for principle quantum numbers 4,3,5 and 5 respectively (Fig. 6).

Through a weighted average, we use the unweighted Rydberg constants in Fig. 7 to obtain the two, weighted Rydberg constants seen in Fig. 8. Including all indices or pairs of data,  $R_H \approx 1.0954 * 10^7 m^{-1} \pm 5.98 * 10^3 [m^{-1} * rad]$  where  $\tilde{\chi}^2 = 5.42$ ; if we truncate our data to only the hydrogen lines which we feel confident in classifying as

|   | $\lambda[nm]$ | $\delta_\lambda$ | $n_i$ | $\delta_{balmer}$<br>[nm] |
|---|---------------|------------------|-------|---------------------------|
| 0 | 660.4         | 0.48             | 3     | 4.37                      |
| 1 | 485.2         | 1.13             | 4     | 0.82                      |
| 2 | 655.1         | 2.46             | 3     | 0.91                      |
| 3 | 483.4         | 0.46             | 4     | 2.58                      |
| 4 | 432.9         | 2.35             | 5     | 1.10                      |
| 5 | 650.3         | 5.16             | 3     | 5.67                      |
| 6 | 478.7         | 6.84             | 4     | 7.31                      |
| 7 | 435.0         | 3.58             | 5     | 0.98                      |

Figure 6. Hydrogen Experiment:  $\lambda$  in  $nm$  and its uncertainty  $[nm * rad]$ , the principal quantum number  $n_i$  for each atom's initial state and associated uncertainty between observed and expected wavelengths using Balmer series  $\delta_{balmer}$ .

|   | $R_H[m^{-1}]$ | $\delta_{R_H}$ |
|---|---------------|----------------|
| 0 | 1.0903e+07    | 7.85e+03       |
| 1 | 1.0993e+07    | 2.56e+04       |
| 2 | 1.0991e+07    | 4.12e+04       |
| 3 | 1.1032e+07    | 1.05e+04       |
| 4 | 1.1000e+07    | 5.98e+04       |
| 5 | 1.1071e+07    | 8.79e+04       |
| 6 | 1.1142e+07    | 1.59e+05       |
| 7 | 1.0948e+07    | 9.02e+04       |

Figure 7. hydrogen's experimental Rydberg constant values in  $m^{-1}$  with uncertainty  $\delta_{R_H}$  in  $[m^{-1} * rad]$ .

legitimate ([1,2,4,7]),  $R_H \approx 1.0991 * 10^7 m^{-1} \pm 1.99 * 10^4 [m^{-1} * rad]$  with  $\tilde{\chi}^2 = 1.34$ . Our chi-square goodness of fit test has been applied with an expected Rydberg constant for hydrogen  $R_H = 1.0967 * 10^7 m^{-1}$  as given by the lab manual [1]. For this reason, we choose to believe in the selective weighted average for our experimental Rydberg constant over the other.

|                                      | $R_{H_w}[m^{-1}]$ | $\delta_{R_{H_w}}$ | $\tilde{\chi}^2$ |
|--------------------------------------|-------------------|--------------------|------------------|
| [:] <b>Weighted Average</b>          | 1.0954e+07        | 5.98e+03           | 5.42             |
| [1, 2, 4, 7] <b>Weighted Average</b> | 1.0991e+07        | 1.99e+04           | 1.34             |

Figure 8. Hydrogen’s final, weighted averages for our experimental Rydberg constants. [:] includes all eight spectral lines, while [1,2,4,7] selects those with the lowest  $\delta_{balmer}$ , and are most promisingly legitimate hydrogen spectra. Expected Rydberg constant for hydrogen  $R_H = 1.0967 * 10^7 m^{-1}$  [1].

### C. Helium-Neon LASER Experiment

|          | $\lambda[nm]$ | $\delta_\lambda$ |
|----------|---------------|------------------|
| <b>0</b> | 628.7         | <b>10.2</b>      |
| <b>1</b> | 638.1         | <b>4.84</b>      |
| <b>2</b> | 625.6         | <b>3.89</b>      |

Figure 9. He-Ne LASER Experiment: observed wavelengths  $\lambda$  in  $nm$ , uncertainty  $\delta_\lambda$  in  $[nm * rad]$ .

|                         | $\lambda_w$<br>[nm] | $\delta_{\lambda_w}$ |
|-------------------------|---------------------|----------------------|
| <b>Weighted Average</b> | 630.3               | 2.91                 |

Figure 10. He-Ne laser’s weighted average  $\lambda_w$  in  $nm$  with deviation  $\delta_{\lambda_w}$  in  $[nm * rad]$ .

We apply Eq. (3) with values taken from the appendix section IV, Fig. 13, and our weighted value for the grating spacing to calculate a list of wavelengths for each set of values as seen in Fig. 9. We obtain uncertainty in  $\lambda$  using Eq. (11), and duly note which deviations are largest in the figure. Then in Fig. 10, utilizing the general formula for calculating a weighted average,

we estimate our helium-neon laser’s wavelength to be  $\lambda \approx 630.3[nm] \pm 2.91[nm * rad]$ .

Let’s address whether we predict the helium or neon to be responsible for the laser’s color. Since our weighted wavelength for our laser setup came out to  $\lambda \approx 630nm$ , disregarding uncertainty for a moment, we see in Fig. 14 that this vibrant-red color only comes about in spectral lines from the neon atom (none of the hydrogen spectrum lines are even close to the laser’s perceived light wavelength). For the observed value stated previously and an expected value of  $\lambda = 6328\text{\AA} = 632.8nm$ , the characteristic red light emitted by the laser has no apparent overlap with helium. Fig. 14 illustrates the many, gathered spectral lines of neon’s atomic signature, which all seem to fit our individually recorded  $\lambda$  terms.

## IV. CONCLUSIONS

In this paper, we have achieved the following: (1) for the helium experiment, we have identified the wavelengths of eight helium lines, recorded the undeflected angle, color, angle and order for a set of measurements (Fig. 11), and used a weighted average to calculate the grating spacing  $d$  and its uncertainty (Fig. 4) yielding  $d \approx 3.37 * 10^3 nm \pm 2.23[nm * rad]$ ; (2) for the hydrogen experiment, we recorded the central undeflected angle, color, angle, and order of each (Fig. 12), calculate the wavelengths of all hydrogen lines using the  $d$  we found (Fig. 6, 7), determine the best associated principal quantum numbers for each line using simple error analysis, and then finally tackle a weighted average and its error to calculate an experimental Rydberg constant and its associated uncertainty (Fig. 8),  $R_H \approx 1.0991 * 10^7 m^{-1} \pm 1.99 * 10^4[m^{-1} * rad]$ , with  $\tilde{\chi} = 1.34$  (this particular value for indices [1,2,4,7] was given priority over the other value, given the error in their Balmer series fits); (3) for the helium-neon laser experiment, we determined their wavelengths to be quite close in value to many of neon’s atomic spectra (Fig. 14) with an estimated weighted average  $\lambda \approx 630.3nm \pm 2.91[nm * rad]$ , suggesting that neon is primarily responsible for the laser’s rich, red color. Greater data taking must be conducted to cross-check and reaffirm our results.

## ACKNOWLEDGMENTS

SE is a part of the PHYS 133, Intermediate Laboratory course. None of this information is presented for journal publication, only for writing improvement.

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# APPENDIX

Table 1: Helium Spectra Collected Data

| <b>m</b> | <b><math>\alpha_1</math></b> | <b><math>\alpha_2</math></b> | <b>color</b> | <b><math>\lambda</math></b> |
|----------|------------------------------|------------------------------|--------------|-----------------------------|
| 1        | 11.50°                       | 11.50°                       | red          | 668.0 nm                    |
| 1        | 10.00°                       | 10.00°                       | yellow       | 588.0 nm                    |
| 1        | 8.78°                        | 9.20°                        | green        | 501.0 nm                    |
| 1        | 7.72°                        | 8.50°                        | cyan         | 492.0 nm                    |
| 1        | 7.67°                        | 8.20°                        | blue         | 447.0 nm                    |
| 1        | 6.50°                        | 7.22°                        | violet       | 388.0 nm                    |
| 2        | 20.25°                       | 20.33°                       | yellow       | 588.0 nm                    |
| 2        | 17.25°                       | 17.42°                       | green        | 501.0 nm                    |

Figure 11. Previously taken data for helium experiment.

Table 4: Hydrogen Spectra Collected Data

| <b>m</b> | <b><math>\alpha_1</math></b> | <b><math>\alpha_2</math></b> | <b>color</b> |
|----------|------------------------------|------------------------------|--------------|
| 3        | 36.00°                       | 36.00°                       | red          |
| 3        | 25.67°                       | 25.50°                       | cyan         |
| 2        | 23.00°                       | 22.75°                       | red          |
| 2        | 16.67°                       | 16.67°                       | cyan         |
| 2        | 15.00°                       | 14.77°                       | violet       |
| 1        | 11.25°                       | 11.00°                       | red          |
| 1        | 8.33°                        | 8.00°                        | cyan         |
| 1        | 7.50°                        | 7.33°                        | violet       |

Figure 12. Previously taken data for hydrogen experiment.

Table 7: He-Ne Laser and Ne Spectra Overlap

| <b>m</b> | <b><math>\alpha_1</math></b> | <b><math>\alpha_2</math></b> | <b><math>\theta</math></b> | <b>color</b> | <b><math>\lambda</math></b> |
|----------|------------------------------|------------------------------|----------------------------|--------------|-----------------------------|
| 1        | 10.50°                       | 11.00°                       | 10.75°                     | red          | 628.06 nm                   |
| 2        | 22.00°                       | 22.50°                       | 22.25°                     | red          | 637.49 nm                   |
| 3        | 33.50°                       | 34.17°                       | 33.84°                     | red          | 624.96 nm                   |

Figure 13. Previously taken data for He-Ne LASER experiment.

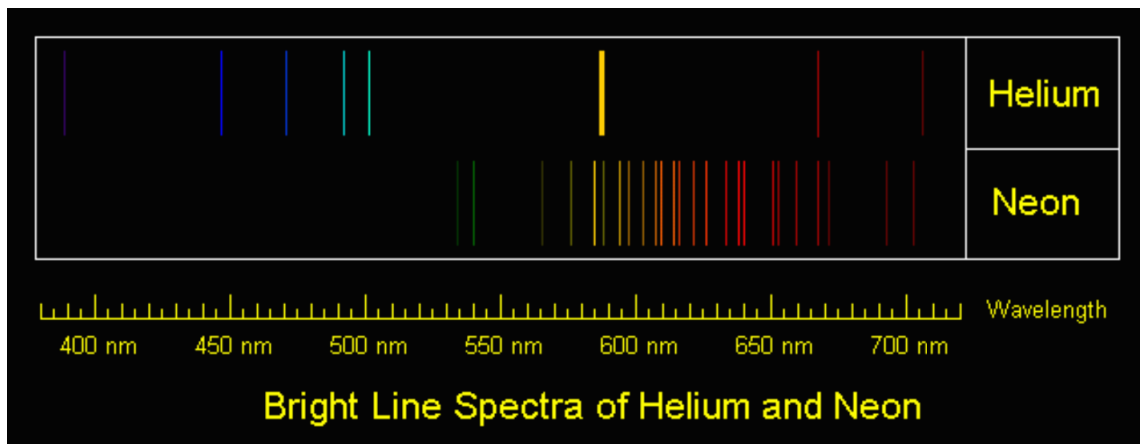


Figure 14. Comparison of spectra for helium and neon visualized [3].