Planar Surface Description, Sensor Bounds Detection, and Global and Local coordinate conversion:

Planar surface description:-

The local coordinate system is described in terms of three sets of parameters:the origin of the local coordinate system in the Global coordinate system; the angles the local x-axis makes with the three global axes; the angles the local y-axis makes with the Global axes.

The two axes' basis vectors, and the origin are used to calculate the equation of the planar surface in the form ax + by + cz = d.

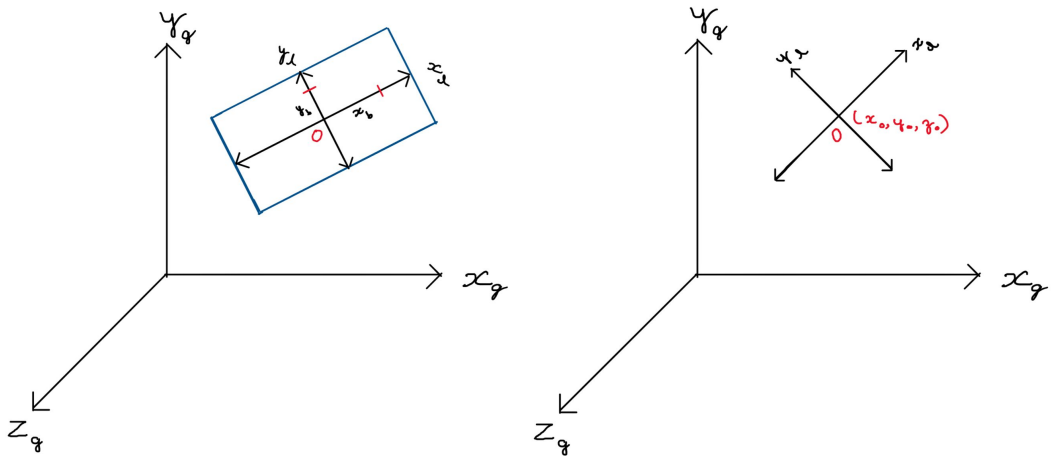

Fig. 1 - Description of the planar surface and local coordinate system

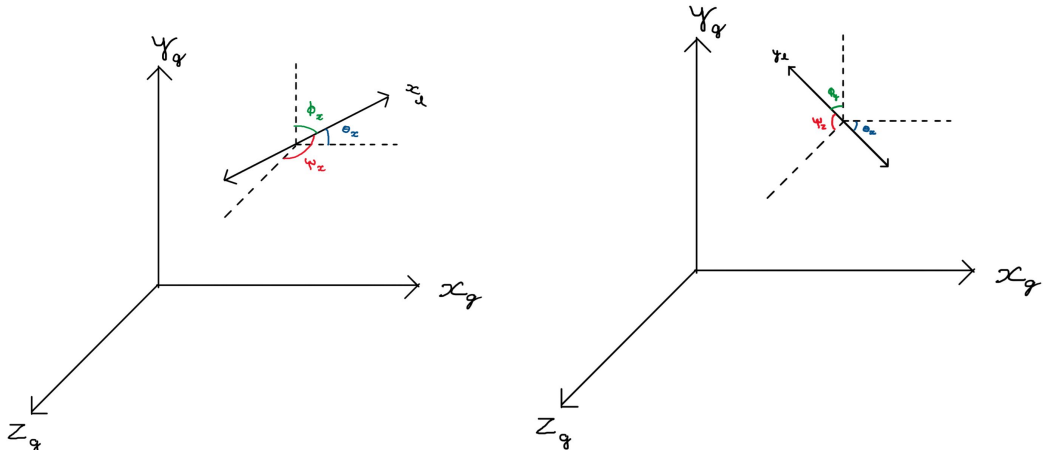

Fig. 2 - Local axes is defined in terms of its angles with the three global axes.

A point in the Global coordinate system is defined as (x_g,y_g,z_g) and in the local coordinate system is defined as (x_l,y_l).

Sensor bound detection:-

The bounds for the rectangular sensor are stored as distances from the origin on either side.

Global to local coordinate conversion:-

The conversion is carried out by a composite transform. First, is a translation, which accounts for the change in origin. The second, is a rotation. The product of these two transforms gives us an Affine transform, which is stored for future use.

$$M_{32} = \begin{bmatrix} \cos\theta_x & \cos\varphi_x & \cos\psi_x & 0 \\ \cos\theta_y & \cos\varphi_y & \cos\psi_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 3 - Affine transform matrix to convert global to local coordinate system

$$\text{Local coordinates} = M_{32}\, X \begin{bmatrix} x_g \\ y_g \\ z_g \\ 1 \end{bmatrix}$$

Local to Global coordinate conversion:-

The local x and y coordinates are projected onto to global x and y axes, respectively. These Global coordinates are used to calculate the z coordinate using the equation of the plane, ax + by + cz = d. Finally, a translation is used to account for the change in origin.

These three steps are represented as three matrices, which are combined into a single affine transformation matrix, which is stored for future use.

$$M_{23} = \begin{bmatrix} 1 & 0 & 0 & x_t \\ 0 & 1 & 0 & y_t \\ 0 & 0 & 1 & z_t \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -a/c & -b/c & d & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_x & 0 & 0 & 0 \\ 0 & \cos\varphi_y & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 4 - Affine transform matrix to convert local to global coordinate system

$$\text{Local coordinates} = M_{23} \, X \begin{bmatrix} x_l \\ y_l \\ 1 \\ 1 \end{bmatrix}$$