Universal Quantification Example

$\mathcal{D}$ = {bob, jack, fred}
$\mathcal{I}$[Happy] = {bob, jack, fred}
$\mathfrak{I}$ |= ∀x Happy(x)?

1. μ[x] = bob. $\mathfrak{I}$, μ |= Happy(x) iff $||x||_{\mathfrak{I}, \mu} \in \mathcal{I}$[Happy] iff bob $\in \mathcal{I}$[Happy]

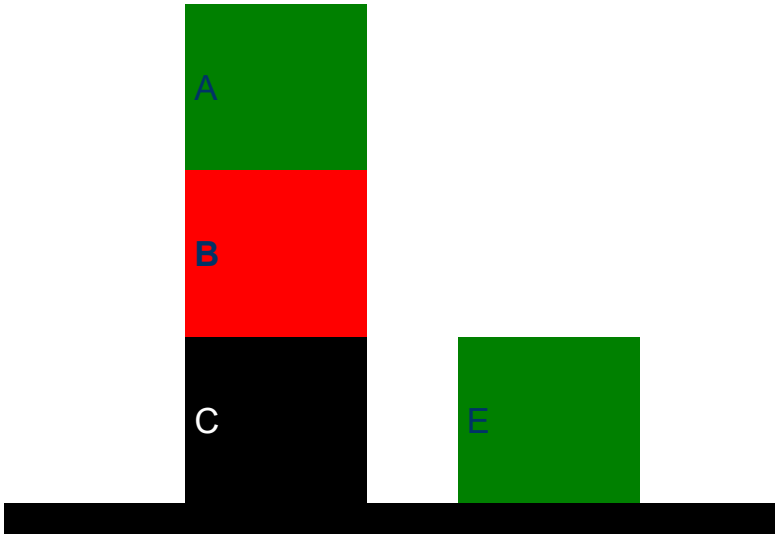
2. μ'[x] = jack. $\mathfrak{I}$, μ' |= Happy(x) iff $||x||_{\mathfrak{I}, \mu'} \in \mathcal{I}$[Happy] iff jack $\in \mathcal{I}$[Happy]

3. μ''[x] = fred. $\mathfrak{I}$, μ'' |= Happy(x) iff $||x||_{\mathfrak{I}, \mu''} \in \mathcal{I}$ [Happy] iff fred $\in \mathcal{I}$ [Happy]

Therefore $\mathfrak{I}$ |= ∀x Happy(x)

Example: Interpretation

Environment



- Language (Syntax)

Constants: a,b,c,e

Functions:

None

Predicates:

On: binary

Above: binary

Clear: unary

OnTable: unary

Example: Interpretation

Language (syntax)

Constants: a,b,c,e

Predicates:

- On: binary
- Above: binary
- Clear: unary
- OnTable: unary

An interpretation $\mathfrak{I}$ (semantics)

$\mathcal{D} = \{A, B, C, E\}$

$\mathcal{I}(a) = A$, $\mathcal{I}(b) = B$,

$\mathcal{I}(c) = C$, $\mathcal{I}(e) = E$.

$\mathcal{I}(on) = \{(A,B),(B,C)\}$

$\mathcal{I}(Above) =$

$\{(A,B),(B,C),(A,C)\}$

$\mathcal{I}(Clear) = \{A,E\}$

$\mathcal{I}(OnTable) = \{C,E\}$

Think of this as a possible way the world could be

Example: Interpretation

Interpretation $\mathfrak{I}$

$\mathcal{D}$ = {A, B, C, E}

$\mathcal{I}$(a) = A, $\mathcal{I}$(b) = B,

$\mathcal{I}$(c) = C, $\mathcal{I}$(e) = E.

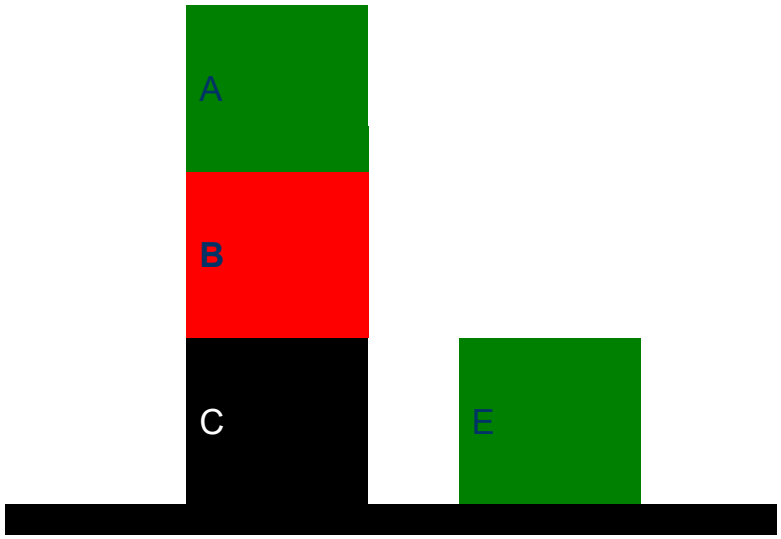
$\mathcal{I}$(On) = {(A,B),(B,C)}

$\mathcal{I}$(Above) = {(A,B),(B,C),(A,C)}

$\mathcal{I}$(Clear)={A,E}

$\mathcal{I}$(OnTable)={C,E}

Environment



Are these formulas true/false in this interpretation?

Interpretation $\mathfrak{I}$

D = {A, B, C, E}

$\mathcal{I}$(a) = A, $\mathcal{I}$(b) = B, $\mathcal{I}$(c) = C, $\mathcal{I}$(e) = E.

$\mathcal{I}$(On) = {(A,B),(B,C)}

$\mathcal{I}$(Above) = {(A,B),(B,C),(A,C)}

$\mathcal{I}$(Clear)={A,E}

$\mathcal{I}$(OnTable)={C,E}

$\forall x,y.\ On(x,y) \supset Above(x,y)$

x=A, y=B √

x=C, y=A √

... √

$\forall x,y.\ Above(x,y) \supset On(x,y)$

x=A, y=B √

x=A, y=C ×

Are these formulas true/false in this interpretation?

Interpretation $\mathfrak{I}$

$\mathcal{D}$ = {A, B, C, E}

$\mathcal{I}$(a) = A, $\mathcal{I}$(b) = B, $\mathcal{I}$(c) = C, $\mathcal{I}$(e) = E.

$\mathcal{I}$(On) = {(A,B),(B,C)}

$\mathcal{I}$(Above) = {(A,B),(B,C),(A,C)}

$\mathcal{I}$(Clear)={A,E}

$\mathcal{I}$(OnTable)={C,E}

$\forall x \exists y$ (Clear(x) $\vee$ On(y,x))

x=A √
x=C, x=B √
... √

$\exists y \forall x$(Clear(x) $\vee$ On(y,x))

y=A ? No! (x=C)
y=C? No! (x=B)
y=E? No! (x=B)
y=B ? No! (x=B)

KB - has many models

A Knowledge base (KB)

1. On(b,c)
2. Clear(e)