# Heuristic Search
# (Informed Search)

# Heuristic Search

- In **uninformed search**, we don’t try to evaluate which of the nodes on the frontier/OPEN are most promising. We never “look-ahead” to the goal.

  E.g., in uniform cost search we always expand the cheapest path. We don’t consider the cost of getting to the goal from the end of the current path.

- Often we have some other knowledge about the merit of nodes, e.g., going the wrong direction in Romania.

# Heuristic Search

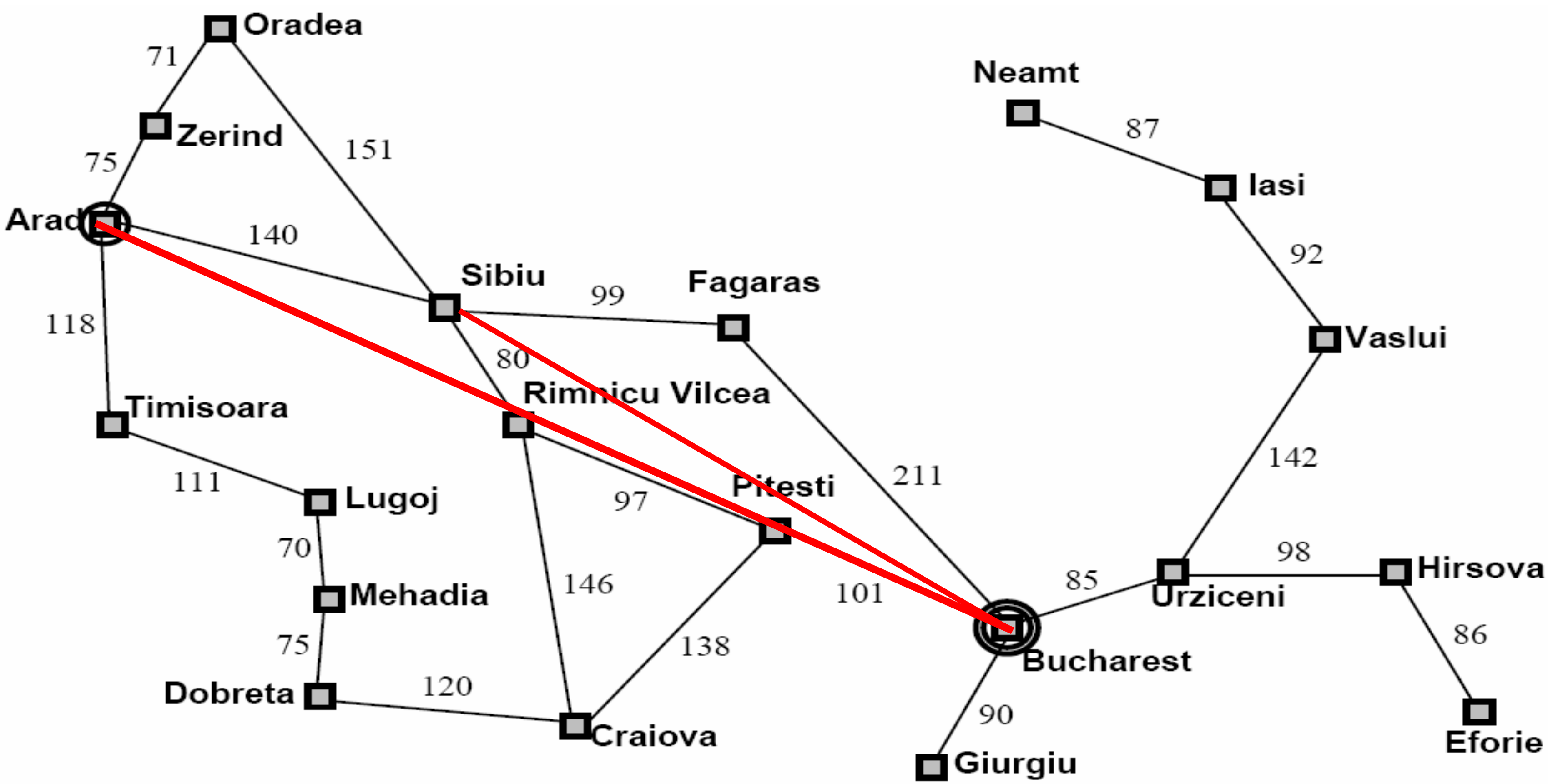
**Merit** of a frontier/OPEN node: different notions of merit.

- If we are concerned about the cost of the solution, we might want a notion of merit of how costly it is to get to the goal from that search node.
- If we are concerned about minimizing computation in search we might want to consider how easy it is to find the goal from that search node.
- We will focus on the "cost of solution" notion of merit.

# Heuristic Search

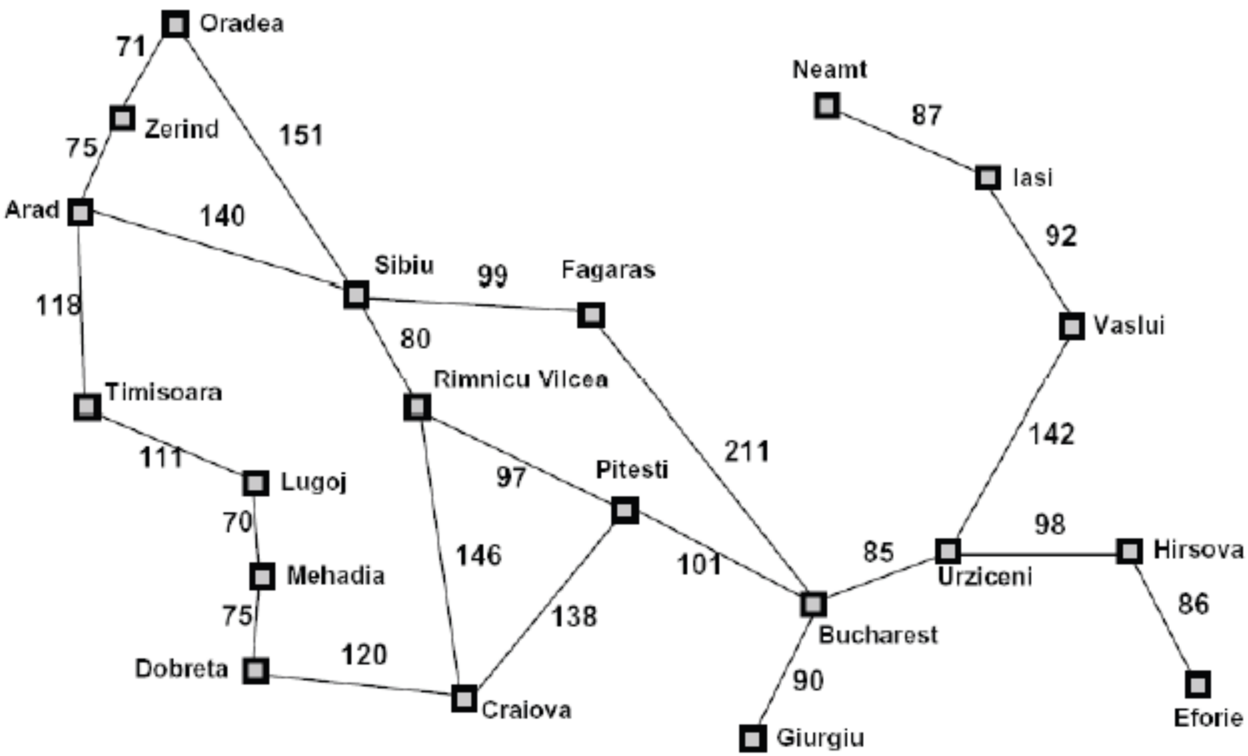
- The idea is to develop a domain specific heuristic function h(n).
- h(n) guesses the cost of getting to the goal from node n (the cost of completing the path that is captured bythe state of node n).
- There are different ways of guessing this cost in different domains. I.e., heuristics are domain specific.

# “As the crow flies” – Straight line heuristic




On the map, the numbers between cities represent the driving distance between cities on **potentially wiggly roads**, even though they are drawn as straight lines. Contrast this to the line-of-sight/``as the crow flies” distance which ignores wiggles in the road, cliffs, bridges, and assumes you can just drive in a straight line from one city to another.

# Example: Straight Line Distance




Straight−line distance to Bucharest

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Planning a path from Arad to Bucharest, we can utilize the straight line distance from each city to our goal as a heuristic/guess of the actual distance. This lets us plan our trip by picking cities at each time point that minimize the distance to our goal.

# Heuristic Search

- If $h(n_1) < h(n_2)$ this means that we guess that it is cheaper to get to the goal from $n_1$ than from $n_2$.

- We require that
  - **h(n) = 0** for every node n whose state satisfies the goal.
    - Zero cost of getting to a goal node from n.

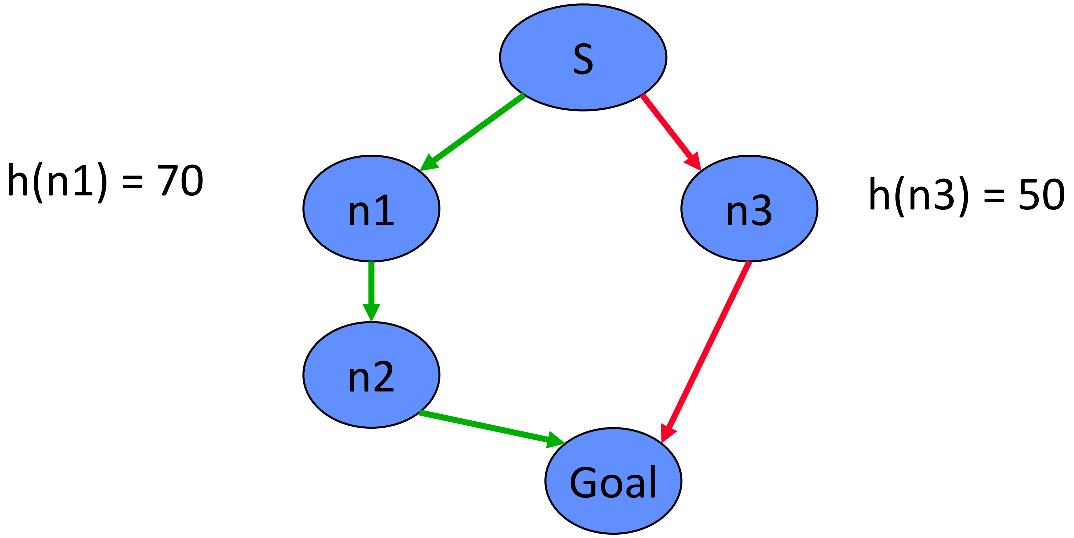
# Using only h(n): Greedy best-first search (Greedy BFS)

- We use h(n) to rank the nodes on the frontier/OPEN.
  - Always expand node with lowest h-value.
- We are greedily trying to achieve a low cost solution.

- However, this method **ignores the cost of getting to n**, so it can be lead astray exploring nodes that cost a lot to get to but seem to be close to the goal:

→ step cost = 10

→ step cost = 100




[S]
[n3,n1]
[Goal, n1]

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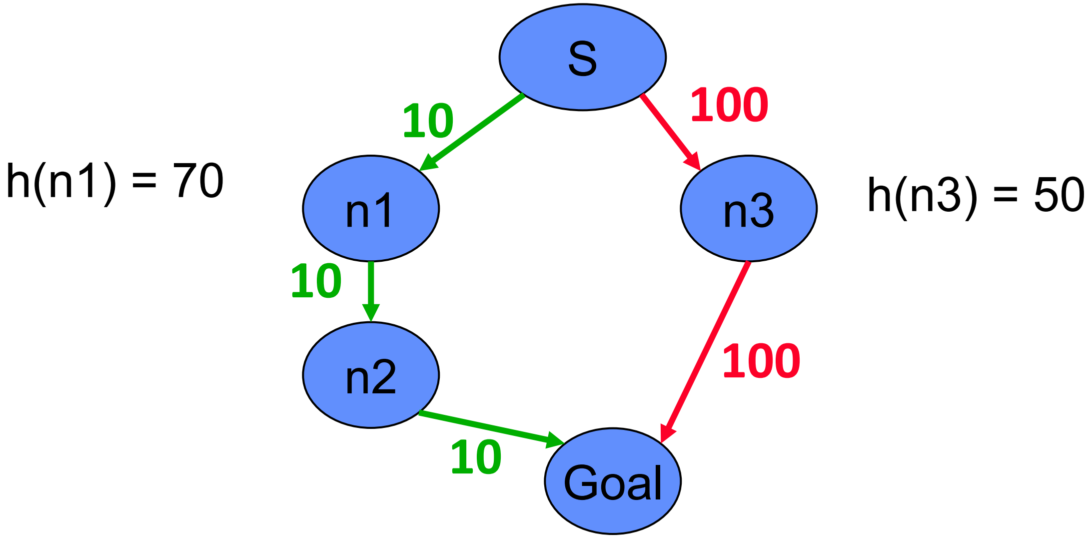
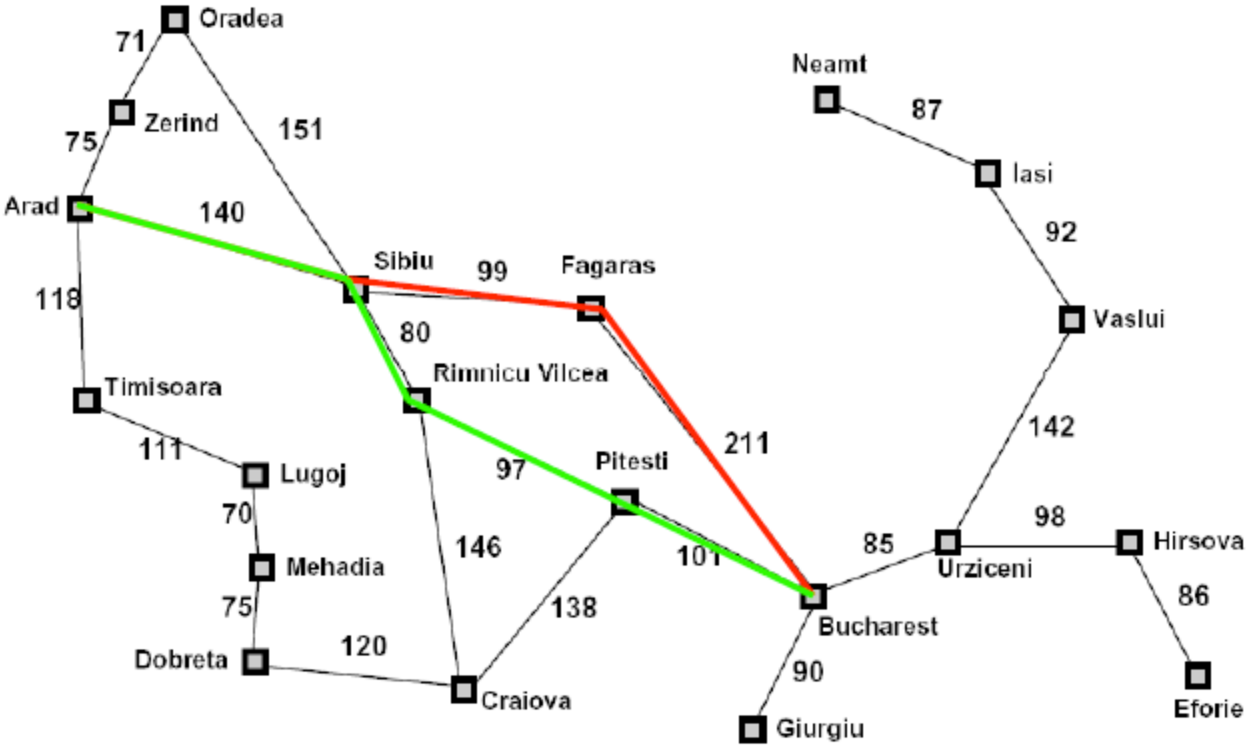



(Greedy BFS is
- Incomplete
- not optimal)

# Greedy best-first search example

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**When you're at Sibiu and contemplating whether to go to Fagaras or RV, the heuristic value of the successor nodes, i.e., the h value guess of the cost is: h(Fagaras) = 178 and h(RV) =193), so Fagaras looks like the better choice, but …**

**Actual Cost(Arad-Sibiu-RV-Pitesli-Bucharest): 140+80+97+101 = 140 + 278 = 418**

**Actual Cost (Arad-Sibiu-Fagaras-Bucharest): 140+99+211 = 140 + 310 = 450**

# A* search

- Take into account the cost of getting to the node as well as our estimate of the cost of getting to the goal from n.

- Define an **evaluation function f(n)**

  $f(n) = g(n) + h(n)$

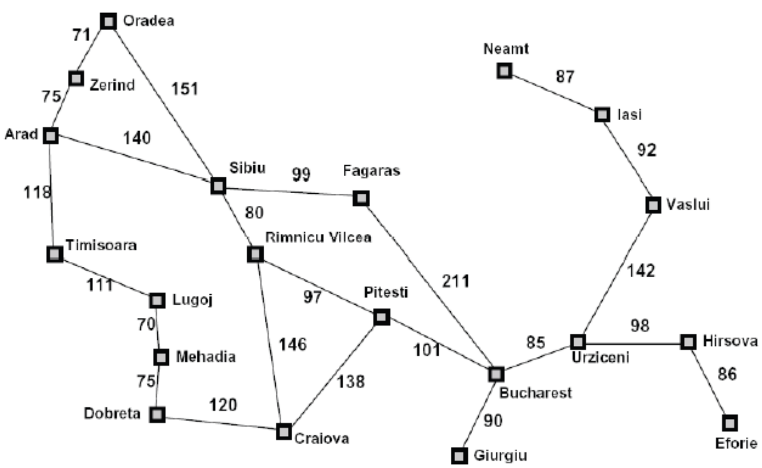
  - $g(n)$ is the cost of the path to node n
  - $h(n)$ is the heuristic estimate of the cost of getting to a goal node from n.

- Always expand the node with lowest f-value on the frontier.

- The f-value is an estimate of the cost of getting to the goal via this node (path).

# A* example

**f(n) = g(n) + h(n),**
**= actual cost to n + heuristic estimate of cost from n to the goal**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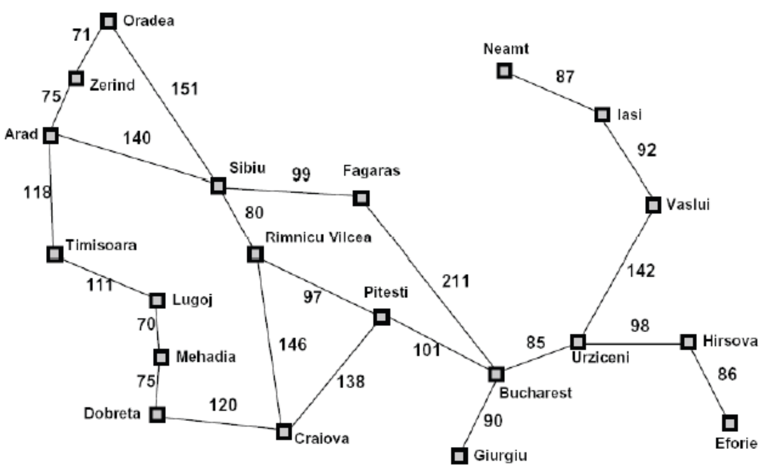


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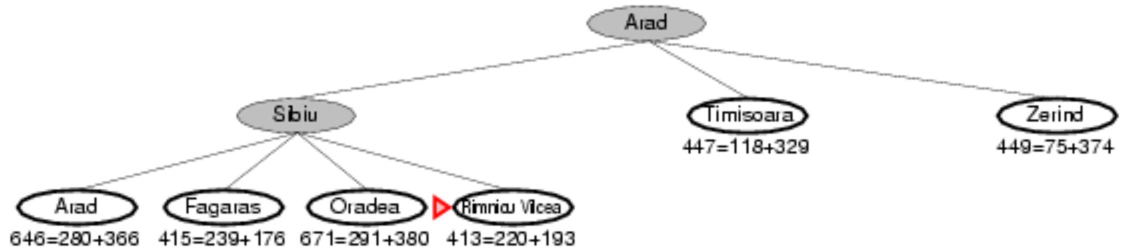


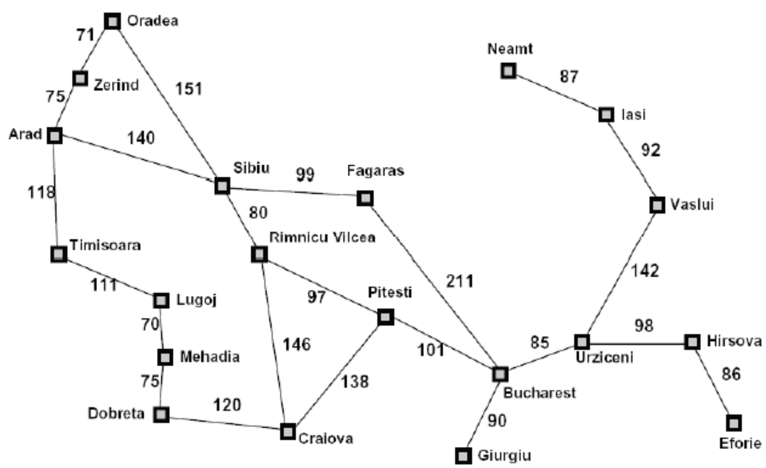


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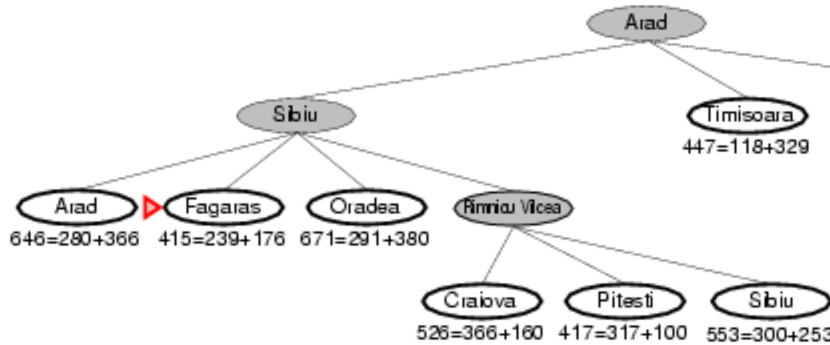


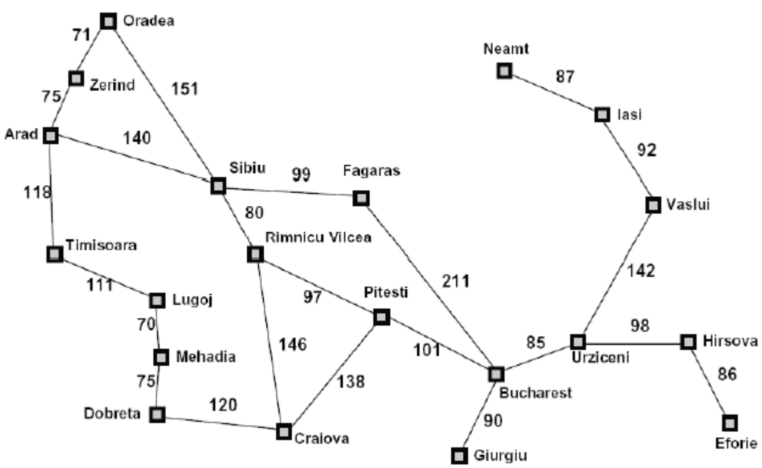


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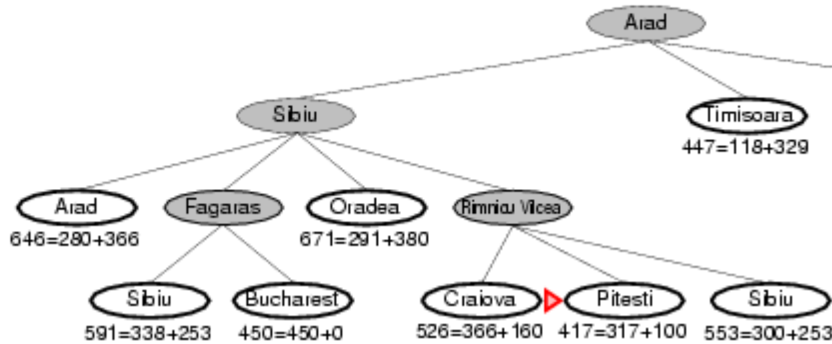


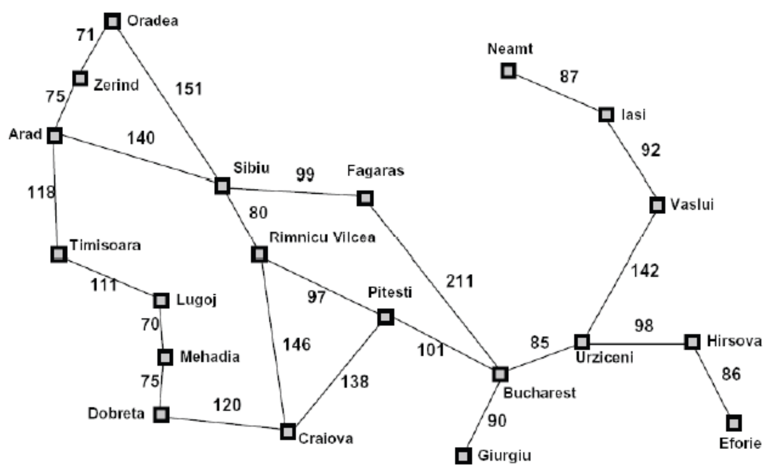


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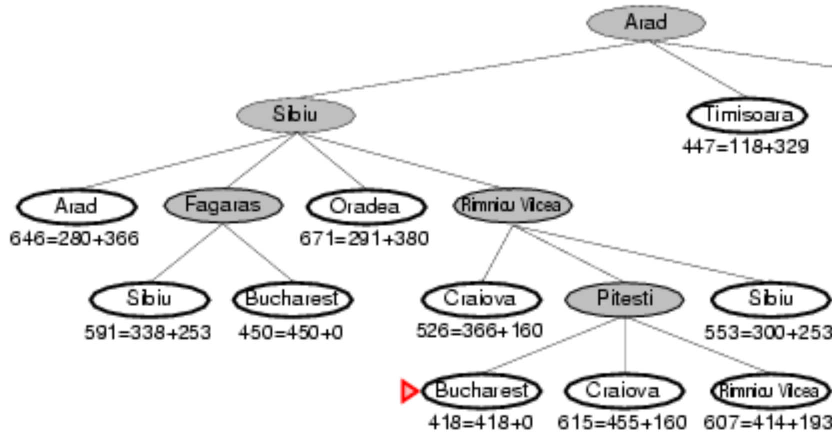


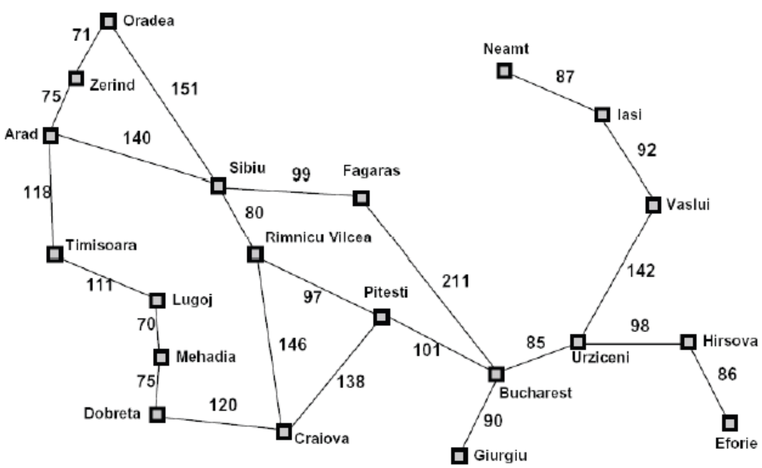


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- Always expand the node with lowest f-value on the frontier.

- The f-value is an estimate of the cost of getting to the goal via this node (path).

# Conditions on h(n)

- We want to analyze the behavior of the resultant search.
  - Completeness, time and space, optimality?

- To obtain such results we must put some further conditions on the heuristic function h(n) and the search space.

# Conditions on h(n): Admissible

- We always assume that $c(n1 \rightarrow n2) \geq \varepsilon > 0$. The cost of any transition is greater than zero and can't be arbitrarily small.
- Let h*(n) be the **cost of an optimal path** from n to a goal node (∞ if there is no path). Then an admissible heuristic satisfies the condition

  $h(n) \leq h^*(n)$

  admissible heuristic h always underestimates the true cost to reach the goal. i.e., it is optimistic ☺
- Hence
  - $h(g) = 0$, for any goal note, g
  - $h^*(n) = \infty$ if there is not path from n to a goal node

# Consistency (aka monotonicity)

- Is a stronger condition than $h(n) \leq h^*(n)$.

- A monotone/consistent heuristic satisfies the triangle inequality (for all nodes n1,n2):

$$h(n1) \leq c(n1 \rightarrow n2) + h(n2)$$

- Note that there might be more than one transition (action) between n1 and n2, the inequality must hold for all of them.

- Note that **monotonicity implies admissibility**.
  - (forall n1, n2) $h(n1) \leq c(n1 \rightarrow n2) + h(n2)$ ➔ (forall n) $h(n) \leq h^*(n)$

# Intuition behind admissibility

$h(n) \leq h^*(n)$ means that the search won't miss any promising paths.

- If it really is cheap to get to a goal via n (i.e., both $g(n)$ and $h^*(n)$ are low), then $f(n) = g(n) + h(n)$ will also be low, and the search won't ignore n in favour of more expensive options.

- This can be formalized to show that admissibility implies optimality.

# Intuition behind monotonicity

$h(n1) \leq c(n1 \rightarrow n2) + h(n2)$

- This says something similar, but in addition one won't be "locally" mislead. See next example.

# Consistency → Admissible

- **Assume consistency:** $h(n1) \leq c(n1 \rightarrow n2) + h(n2)$
  **Prove admissible:** $h(n) \leq h^*(n)$

**Proof:**

**If** no path exists from n to a goal then $h^*(n) = \infty$ and $h(n) \leq h^*(n)$

**Else** let $n \rightarrow n1 \rightarrow \ldots \rightarrow n^*$ be an OPTIMAL path from n to a goal.

Note the cost of this path is $h^*(n)$, and each subpath ($ni \rightarrow \ldots \rightarrow n^*$) has cost equal to $h^*(ni)$.

Prove $h(n) \leq h^*(n)$ by induction on the length of this optimal path.

**Base Case: $n = n^*$** [optimal path length = 0]

By our conditions on h, $h(n) = 0 \leq h(n^*) = 0$

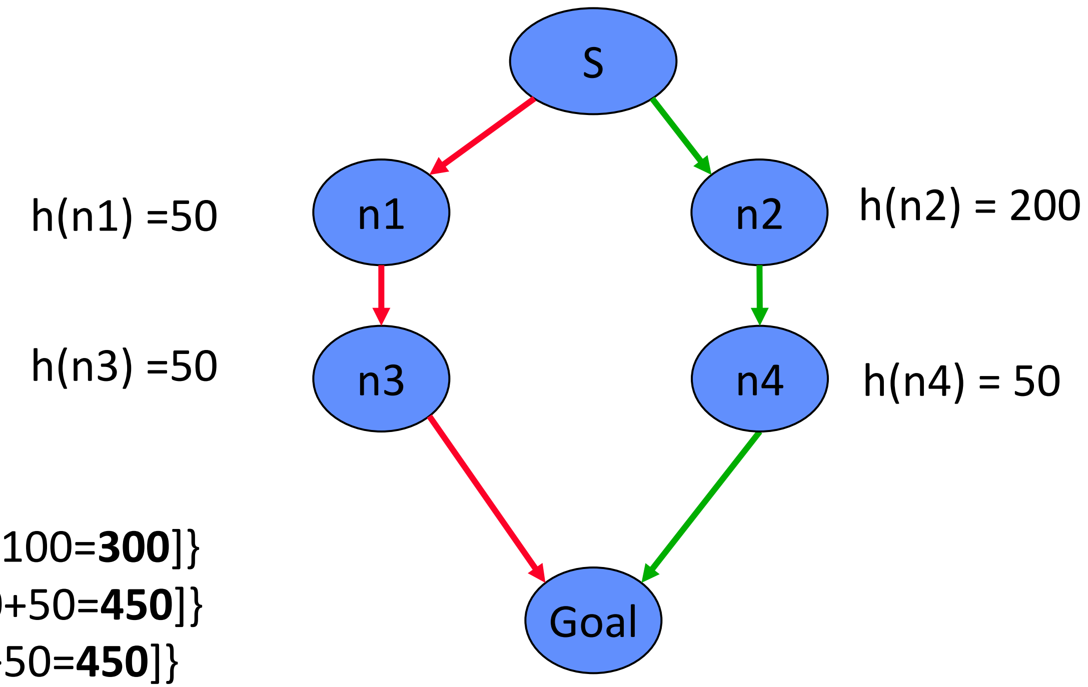
**Induction Hypothesis: $h(n1) \leq h^*(n1)$**

$$
\begin{aligned}
h(n) &\leq c(n \rightarrow n1) + h(n1) && \text{[consistency]} \\
&\leq c(n \rightarrow n1) + h^*(n1) && \text{[defn } h^*\text{]} \\
&= h^*(n)
\end{aligned}
$$

# Example: admissible but nonmonotonic

The following *h* is **not consistent (i.e., not monotone)** since *h(n2)>c(n2→n4)+h(n4). But it is* ***admissible****.*

→ step cost = 200
→ step cost = 100




**g(n)+h(n)=f(n)**

{S} → {n1 [200+50=**250**], n2 [200+100=**300**]}
→ {n2 [100+200=**300**], n3 [400+50=**450**]}
→ {n4 [200+50=**250**], n3 [400+50=**450**]}
→ {goal [300+0=**300**], n3 [400+50=**450**]}

We **do find** the optimal path as the heuristic is still admissible. **But** we are mislead into ignoring n2 until after we expand n1.

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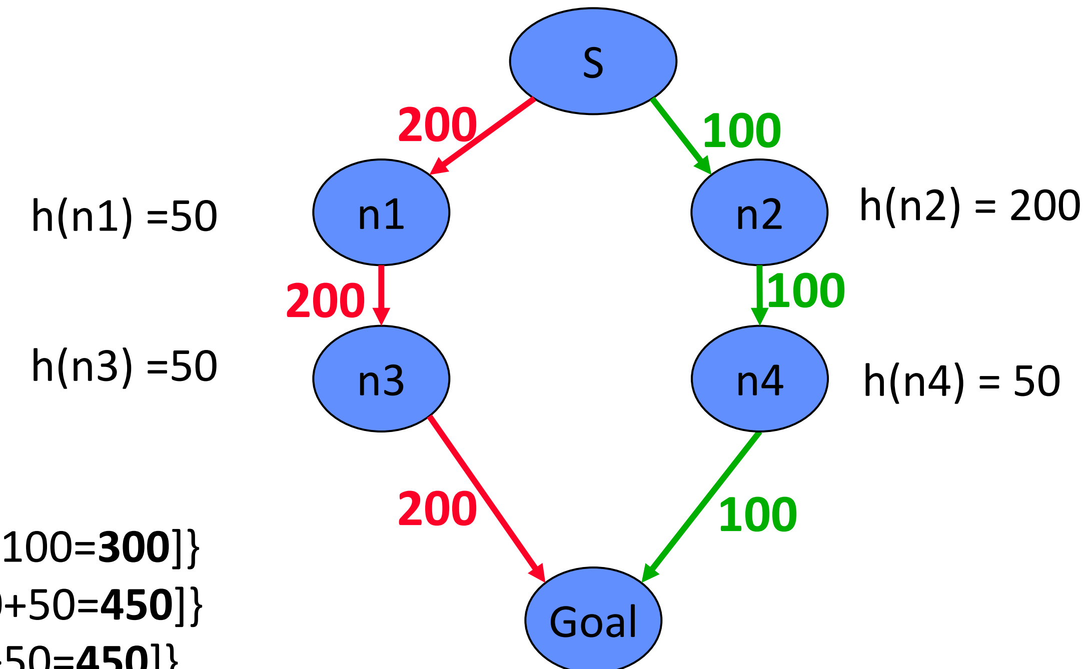


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We **do find** the optimal path as the heuristic is still admissible. **But** we are mislead into ignoring n2 until after we expand n1.

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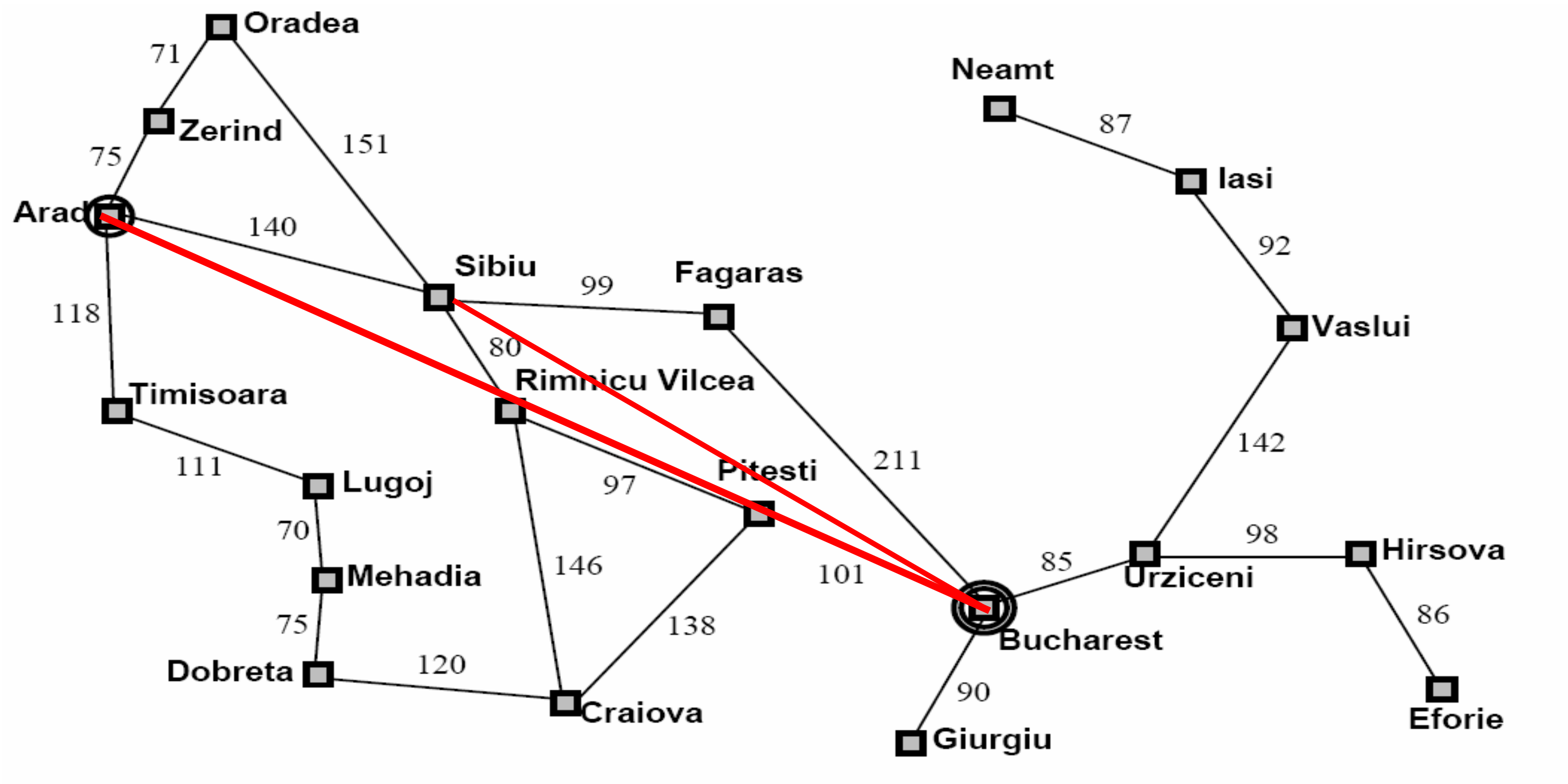

- Most admissible heuristics are also monotone.

(Indeed it's hard to find an admissible heuristic that is not monotone!)

# Consequences of monotonicity

1. The f-values of nodes along a path must be non-decreasing.

Let <Start→ n1→ n2…→ nk> be a path.

We claim that

$f(ni) \leq f(ni+1)$

Proof:

$$
\begin{aligned}
f(ni) &= c(Start\rightarrow \ldots\rightarrow ni) + h(ni) \\
&\leq c(Start\rightarrow \ldots\rightarrow ni) + c(ni\rightarrow ni+1) + h(ni+1) \quad \text{[monotonicity]} \\
&= c(Start\rightarrow \ldots\rightarrow ni\rightarrow ni+1) + h(ni+1) \\
&= g(ni+1) + h(ni+1) \\
&= f(ni+1).
\end{aligned}
$$

# Consequences of monotonicity

2. If n2 is expanded after n1, then $f(n1) \leq f(n2)$
   (the f-value increases monotonically)

**Proof (2 cases):**

- If n2 was on the frontier/OPEN when n1 was expanded,
  then $f(n1) \leq f(n2)$ otherwise we would have expanded n2.
- If n2 was added to the frontier/OPEN after n1's expansion, then let n be an ancestor of n2 that was present when n1 was being expanded (this could be n1 itself). We have $f(n1) \leq f(n)$ since A* chose n1 while n was present in the frontier/OPEN. Also, since n is along the path to n2, by property (1) we have $f(n) \leq f(n2)$. So, we have $f(n1) \leq f(n2)$.

---------------------------

1) The f-values of nodes along a path must be non-decreasing.
2) If n2 is expanded after n1, then $f(n1) \leq f(n2)$

# Consequences of monotonicity

**Corollary:** the sequence of f-values of the nodes expanded by A* is non-decreasing. I.e, If n2 is expanded **after** (not necessarily immediately after) n1, then f(n1) ≤ f(n2)

(the f-value of expanded nodes is **monotonic** non-decreasing)

Proof:

- If n2 was on frontier/OPEN when n1 was expanded,

  then f(n1) ≤ f(n2) otherwise we would have expanded n2.
- If n2 was added to frontier/OPEN after n1's expansion, then let n be an ancestor of n2 that was present when n1 was being expanded (this could be n1 itself). We have f(n1) ≤ f(n) since A* chose n1 while n was present on frontier/OPEN. Also, since n is along the path to n2, by property (1) we have f(n)≤f(n2). So, we have f(n1) ≤ f(n2).

# Consequences of monotonicity

3. When n is expanded every path with lower f-value has already been expanded.

- **Proof:** Assume <u>by contradiction</u> that there exists a path <Start, n0, n1, ni-1, ni, ni+1, …, nk> with f(nk) < f(n) and ni is its last expanded node.
  - ni+1 must be on the frontier/OPEN while n is expanded, so

    a) by (1) f(ni+1) ≤ f(nk) since they lie along the same path.

    b) since f(nk) < f(n) (given) so we have f(ni+1) < f(n) (from a)

    c) by (2) f(n) ≤ f(ni+1) because n is expanded before ni+1.
  - Contradiction from b&c!

---------------------------------------------------

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2) If n2 is expanded after n1, then f(n1) ≤ f(n2)
3) When n is expanded every path with lower f-value has already been expanded.

# Consequences of monotonicity

4. With a monotone heuristic, the first time A* expands a state, it has found the minimum cost path to that state.

- Let **PATH1 = <Start, s0, s1, …, sk, s>** be **the first** path to a state s found. We have f(path1) = c(PATH1) + h(s).
- Let **PATH2 = <Start, t0, t1, …, tj, s>** be another path to s found later. we have f(path2) = c(PATH2) + h(s).
  - Note h(s) is dependent only on the state s (terminal state of the path) it does not depend on how we got to s.
- By the corollar, f(path1) ≤ f(path2)
- hence: c(PATH1) ≤ c(PATH2)

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- Let **PATH2 = <Start, m0,m1, …, mj, n>** be another path to n found later. we have f(path2) = c(PATH2) + h(n).
- By property (3) and its corollary, f(path1) ≤ f(path2)
- hence: c(PATH1) ≤ c(PATH2)

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# Consequences of monotonicity

**Complete.**

- Yes, consider a least cost path to a goal node
  - SolutionPath = <Start→ n1→ …→ G> with cost c(SolutionPath)
  - Since each action has a cost ≥ ε > 0, there are only a finite number of paths that have cost ≤ c(SolutionPath).
  - All of these paths must be explored before any path of cost > c(SolutionPath).
  - So eventually SolutionPath, or some equal cost path to a goal must be expanded.

**Time and Space complexity.**

- When h(n) = 0, for all n, h is monotone.    (a very *un*informative heuristic!!!)
  - A* becomes uniform-cost search!
- It can be shown that when h(n) > 0 for some n, the number of nodes expanded can be no larger than uniform-cost.
- Hence the same bounds as uniform-cost apply. (These are worst case bounds). Still exponential unless we have a very good *h*!
- In real world problems, we run out of time and memory! IDA* can sometimes be used to address memory  issues, but IDA* isn't very good whenmany cycles are present.

# Consequences of monotonicity

Optimality

- Yes, by (4) the first path to a goal node must be optimal.

> 4. With a monotone heuristic, the first time A* expands a state, it has found the minimum cost path to that state.

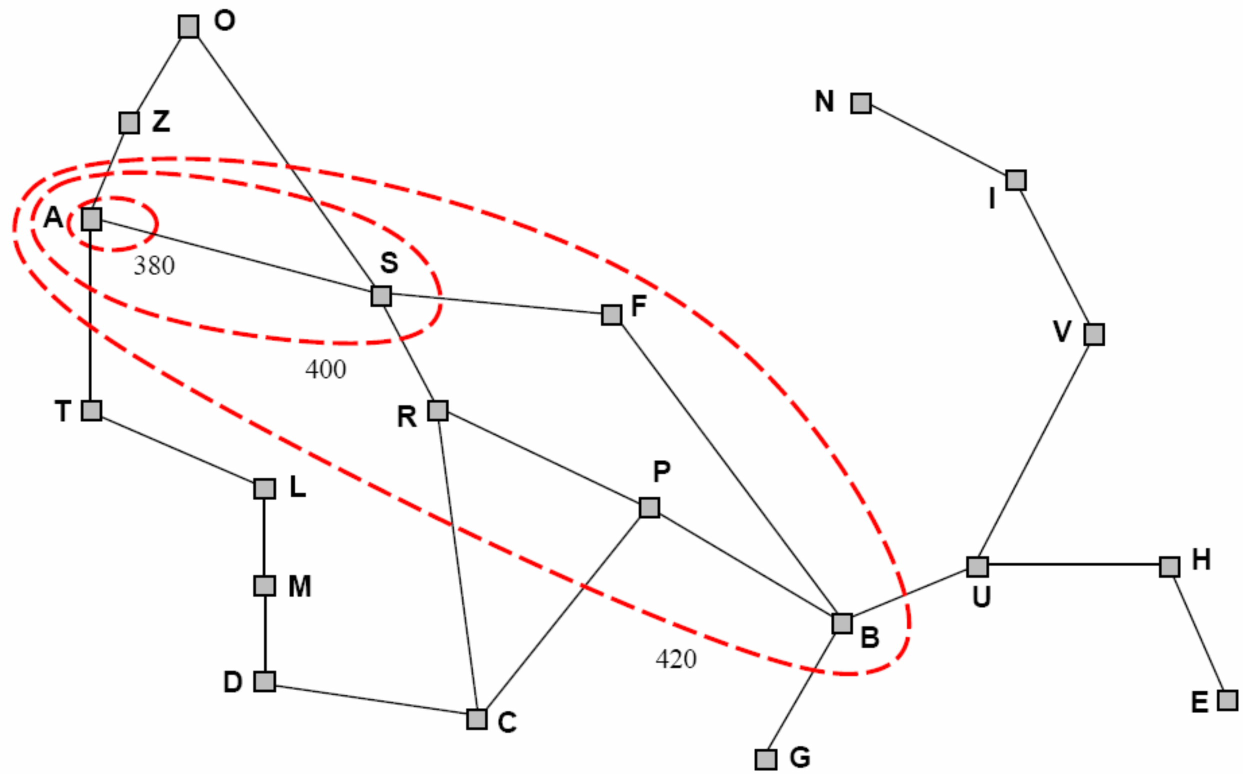
Cycle Checking

- We can use a simple implementation of cycle checking (multiple path checking)---just reject all search nodes visiting a state already visited by a previously expanded node. By property (4) we need keep only the first path to a node, rejecting all subsequent paths.

# Search generated by monotonicity

Gradually adds "$f$-contours" of nodes (cf. breadth-first adds layers)

**Inside each counter, the f values are less than or equal to counter value!**




- **For uniform cost search, bands are *"circular".***
- **With more accurate heuristics, bands stretch out more toward the goal.**

# Admissibility without monotonicity

**When "h" is admissible but not monotonic.**

- Time and Space complexity remain the same. Completeness holds.
- Optimality still holds (without cycle checking), but need a different argument: don't know that paths are explored in order of cost.

Proof (by contradiction) of optimality (without cycle checking) :

- Assume the goal path <S,…,G> found by A* has cost bigger than the optimal cost: i.e. $C^*(G) < f(G)$.
- There must exists a node $n$ in the optimal path that is still in the frontier.
- We have: $f(n)=g(n)+h(n) \leq g(n)+h^*(n) = C^*(G) < f(G)$
- Therefore, f(n) must have been selected before G by A*. contradiction!

# Admissibility without monotonicity

**What about Cycle Checking?**

- No longer guaranteed we have found an optimal path to a node ***the first time*** we visit it.

- So, cycle checking might not preserve optimality.
  - To fix this: for previously visited nodes, must remember cost of previous path. If new path is cheaper must explore again.

# Admissibility without monotonicity

## What about Cycle Checking?

- No longer guaranteed we have found an optimal path to a node ***the first time*** we visit it.
- So, cycle checking might not preserve optimality.
  - To fix this: for previously visited nodes, must remember cost of previous path. If new path is cheaper must explore again.
- contours of monotonic heuristics don’t hold.

# Space Problems with A*

- A* has the same potential space problems as BFS or UCS

- IDA* - Iterative Deepening A* is similar to Iterative Deepening Search and similarly addresses space issues.

# IDA* - Iterative Deepening A*

Objective: reduce memory requirements for A*

- Like iterative deepening, but now the "cutoff" is the f-value (g+h) rather than the depth
- At each iteration, the cutoff value is the smallest f-value of any node that exceeded the cutoff on the previous iteration
- Avoids overhead associated with keeping a sorted queue of nodes
- Two new parameters:
  - curBound (any node with a bigger f-value is discarded)
  - smallestNotExplored (the smallest f-value for discarded nodes in a round) when frontier/OPEN becomes empty, the search starts a new round with this bound
    - Easier to expand all nodes with f-value EQUAL to the f-limit. This way we can compute "smallestNotExplored" more easily.

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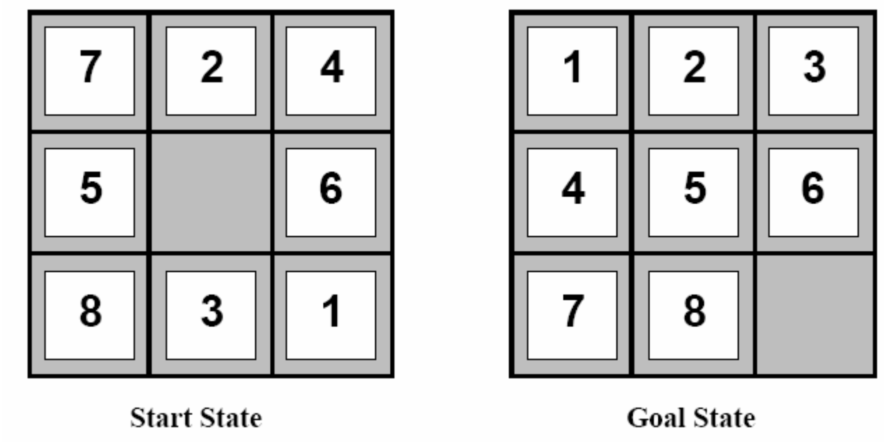
# Constructing Heuristics

# Building Heuristics: Relaxed Problem

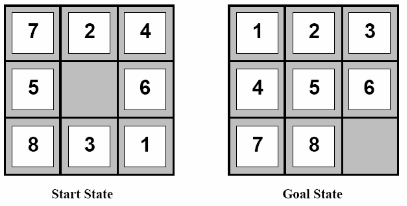
- One useful technique is to consider an **easier problem**, and let h(n) be the cost of reaching the goal in the easier problem.

**8-Puzzle**




- Can move a tile from square A to B if
  - A is adjacent (left, right, above, below) to B
  - and B is blank

# Building Heuristics: Relaxed Problem




8-Puzzle moves (continued)

- Can move a tile from square A to B if
  - A is adjacent (left, right, above, below) to B
  - and B is blank

- Can **relax some of these conditions**
  1. can move from A to B if A is adjacent to B (ignore whether or not position is blank)
  2. can move from A to B if B is blank (ignore adjacency)
  3. can move from A to B (ignore both conditions).

# Building Heuristics: Relaxed Problem

- **#3** *"can move from A to B (ignore both conditions)".*

  **leads to the misplaced tiles heuristic.**
  - To solve the puzzle, we need to move each tile into its final position.
  - Number of moves = number of misplaced tiles.
  - Clearly h(n) = number of misplaced tiles ≤ the h*(n) the cost of an optimal sequence of moves from n.

- **#1** ***"****can move from A to B if A is adjacent to B (ignore whether or not position is blank)"*

  **leads to the manhattan distance heuristic**.
  - To solve the puzzle we need to slide each tile into its final position.
  - We can move vertically or horizontally.
  - Number of moves = sum over all of the tiles of the number of vertical and horizontal slides we need to move that tile into place.
  - Again h(n) = sum of the manhattan distances ≤ h*(n)
    - in a real solution we need to move each tile at least that far and we can only move one tile at a time.

# Building Heuristics: Relaxed Problem

The **optimal** cost to nodes in the relaxed problem is an admissible heuristic for the original problem!

Proof Idea: the optimal solution in the original problem is a solution for relaxed problem, therefore it must be at least as expensive as the optimal solution in the relaxed problem.

So admissible heuristics can sometimes be constructed by finding a relaxation whose optimal solution can be **easily computed**.

# Building Heuristics: Relaxed Problem

The optimal cost to nodes in the relaxed problem is an admissible heuristic for the original problem!

**Proof**: the optimal solution in the original problem is a (*not necessarily optimal*) solution for relaxed problem, therefore it must be at least as expensive as the optimal solution in the relaxed problem.

# Building Heuristics: Relaxed Problem

Comparison of IDS and A* (average total nodes expanded ):

| Depth | IDS | A*(Misplaced) **h1** | A*(Manhattan) **h2** |
|---|---:|---:|---:|
| 10 | 47,127 | 93 | 39 |
| 14 | 3,473,941 | 539 | 113 |
| 24 | --- | 39,135 | 1,641 |

Let h1=Misplaced, h2=Manhattan

- Does h2 **always** expand fewer nodes than h1?
  - Yes! Note that **h2 dominates h1**, i.e. for all n: $h1(n) \leq h2(n)$. From this you can prove h2 is faster than h1.
  - Therefore, among several admissible heuristic the one with highest value expands the fewest nodes. Is it the fastest?

# Building Heuristics: Pattern databases.

- Admissible heuristics can also be derived from solution to **subproblems:** **Each state is mapped into a partial specification, e.g. in 15-puzzle only** ***position of specific tiles matters.***

- Here are goals for two sub-problems (called Corner and Fringe) of 15-puzzle.
- Note the location of BLANK!

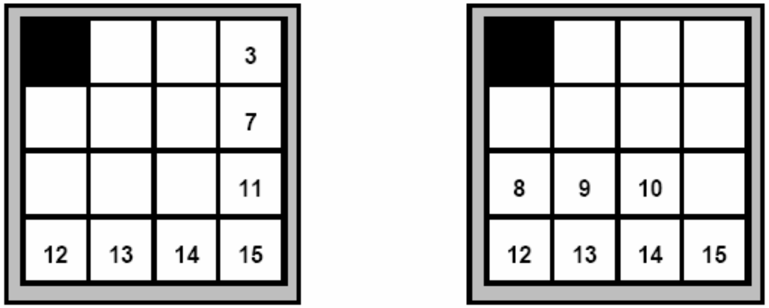



**Fig. 2.** The Fringe and Corner Target Patterns.

- By searching backwards from these goal states, we can compute the distance of any configuration of these tiles to their goal locations. We are ignoring the identity of the other tiles.
- For any state n, the number of moves required to get these tiles into place form a lower bound on the cost of getting to the goal from n.

# Building Heuristics: Pattern databases.

- These configurations are stored in a database, along with the number of moves required to move the tiles into place.
- The maximum number of moves taken over all of the databases can be used as a heuristic.
- On the 15-puzzle
  - The fringe data base yields about a 345 fold decrease in the search tree size.
  - The corner data base yields about 437 fold decrease.
- Sometimes disjoint patterns can be found, then the number of moves can be added rather than taking the max (if we only count moves of the target tiles).

# Local Search

- So far, we keep the paths to the goal.
- For some problems (like 8-queens) we don't care about the path, we **only care about the solution.** Many real problem like Scheduling, IC design, and network optimizations are of this form.
- Local search algorithms operate using a single current state and generally move to neighbors of that state.
- There is an objective function that tells the value of each state. The goal has the highest value (global maximum).
- Algorithms like Hill Climbing try to move to a neighbour with the highest value.
- Danger of being stuck in a local maximum. So some randomness is added to "shake" out of local maxima.
- Simulated Annealing: Instead of the best move, take a random move and if it improves the situation then always accept, otherwise accept with a probability <1.
- [If interested read these two algorithms from the R&N book].