

# Plan for Today

## Nonparametric RL

- “Nonparametric” function approximation
- Strong guarantees across:  
*Sample complexity, space complexity, storage complexity*

## Tree-Partitions

- Implement tree-based adaptive discretization from nonparametric RL algorithms
- Use ORSuite to test on “continuous Ambulance routing”

## Hindsight Learning

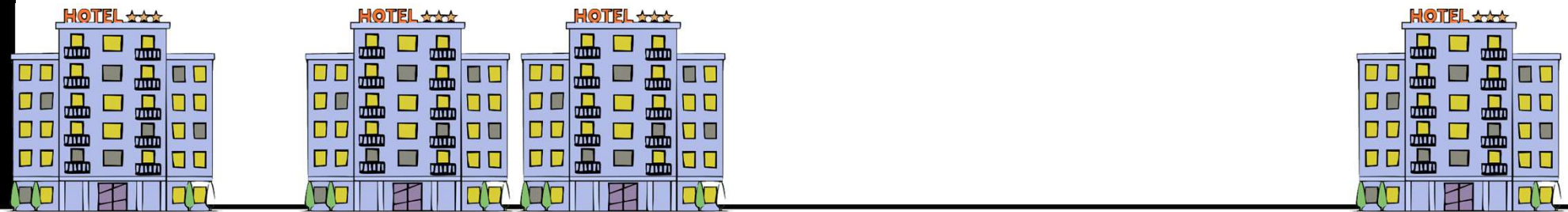
- Exogenous MDPs as model for OR problems
- Use of *Hindsight Planning* oracle for algorithm design
- Empirical results in VM allocation with Microsoft Azure

## Hindsight Planning for Exo-MDPs

- Use ORSuite model for revenue management and pricing (an example of an Exo-MDP)
- Implement Bayes Selector
- Use ORSuite to run simulations to compare performance against tabular algorithms

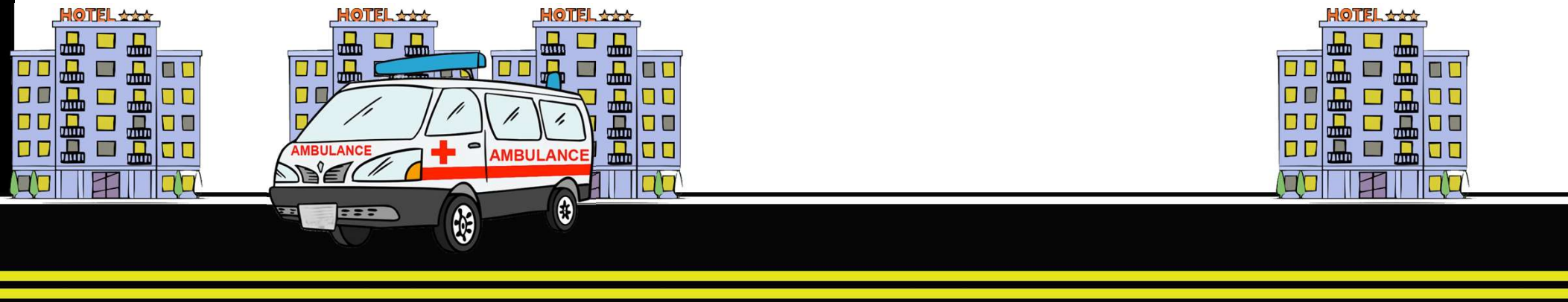
# Ambulance Routing

- Operator decides location to station ambulance, paying a transportation cost
- Random request realized, ambulance pays cost for travel delay to serve patient
- Goal: learn policy which minimizes costs w/o knowledge of arrival distribution



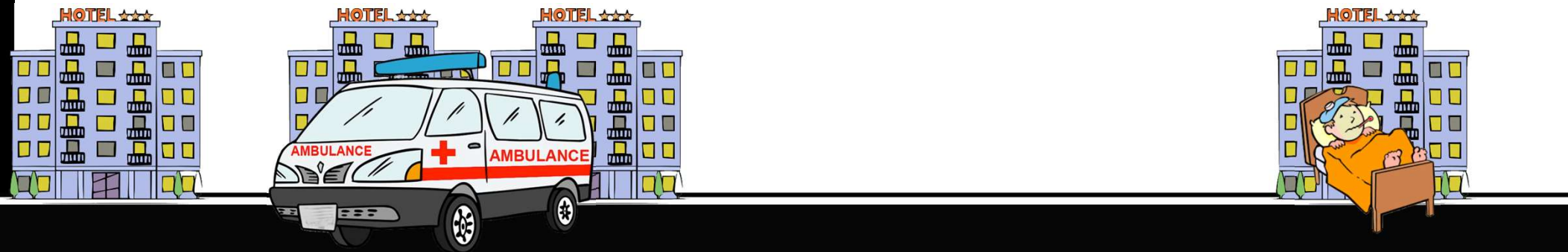
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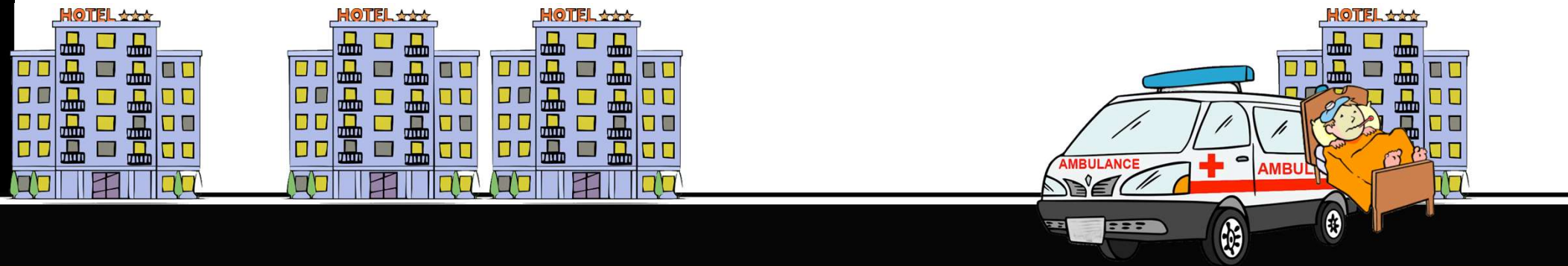
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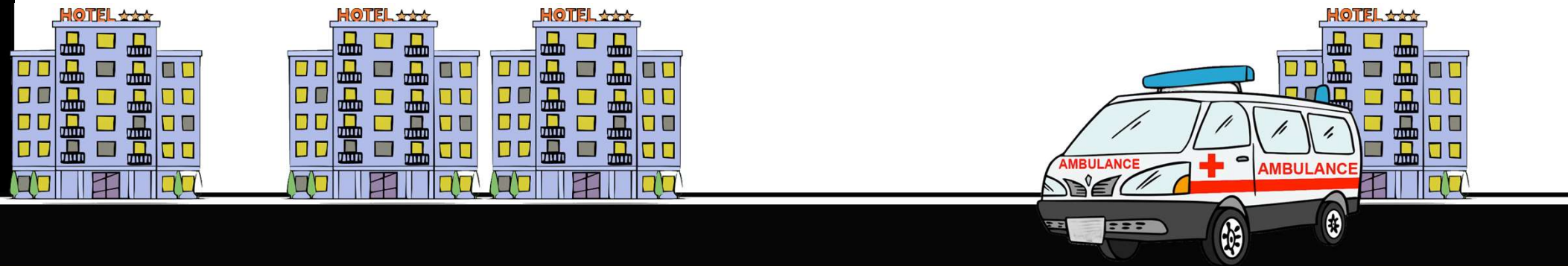
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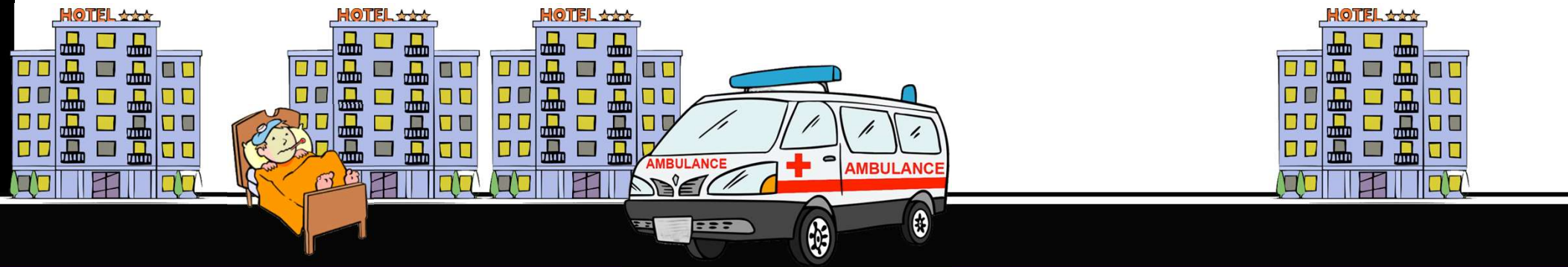
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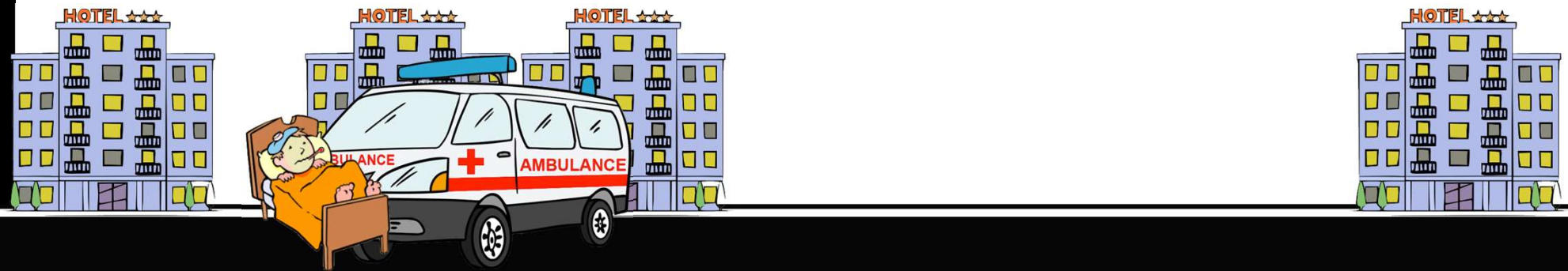
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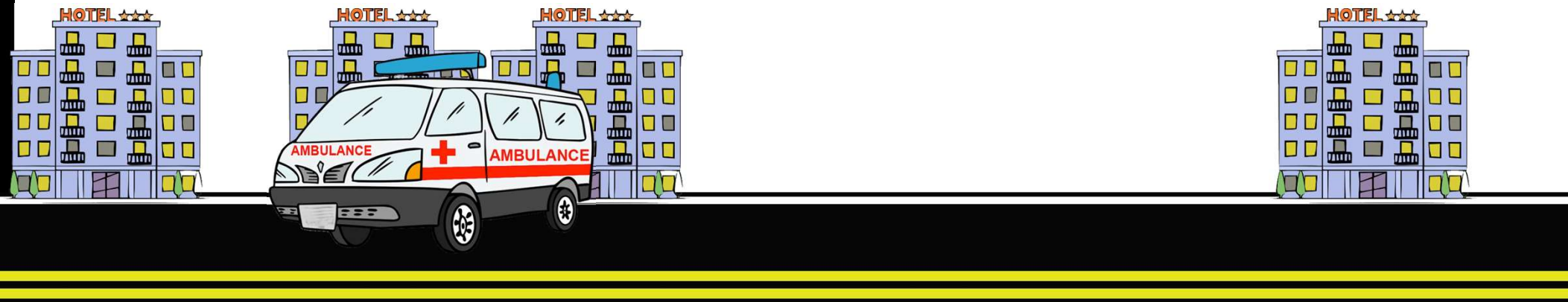
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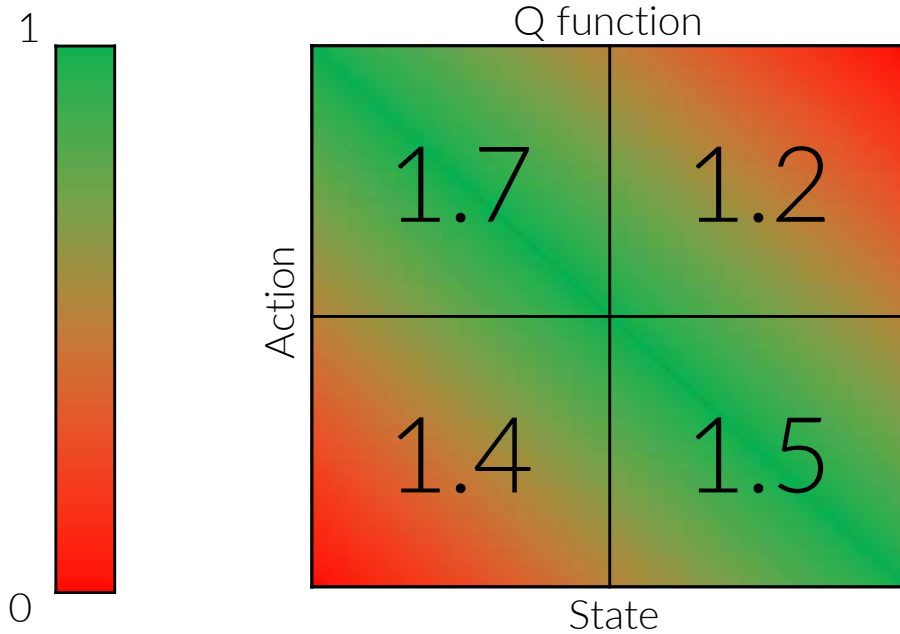


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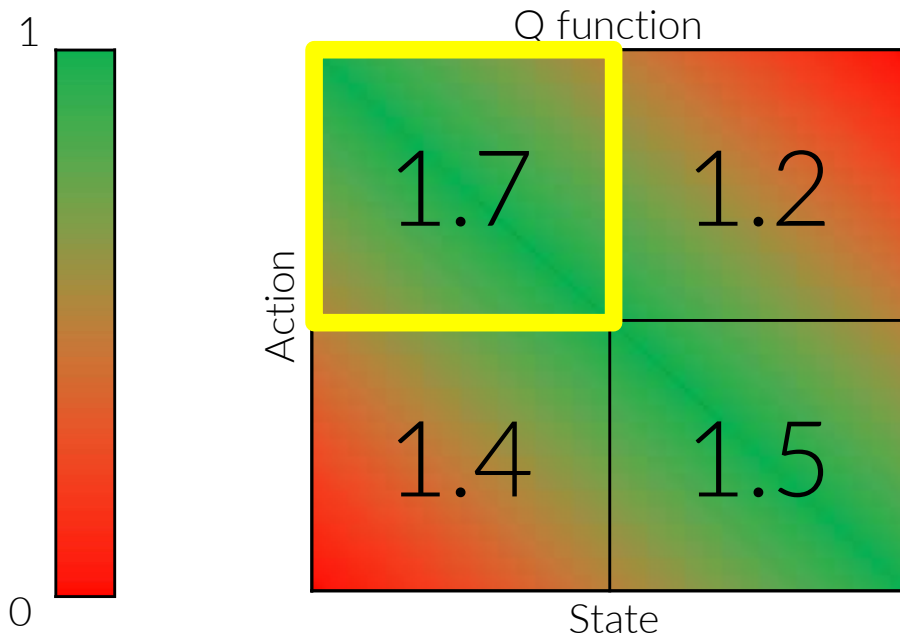


## Adaptive Discretization



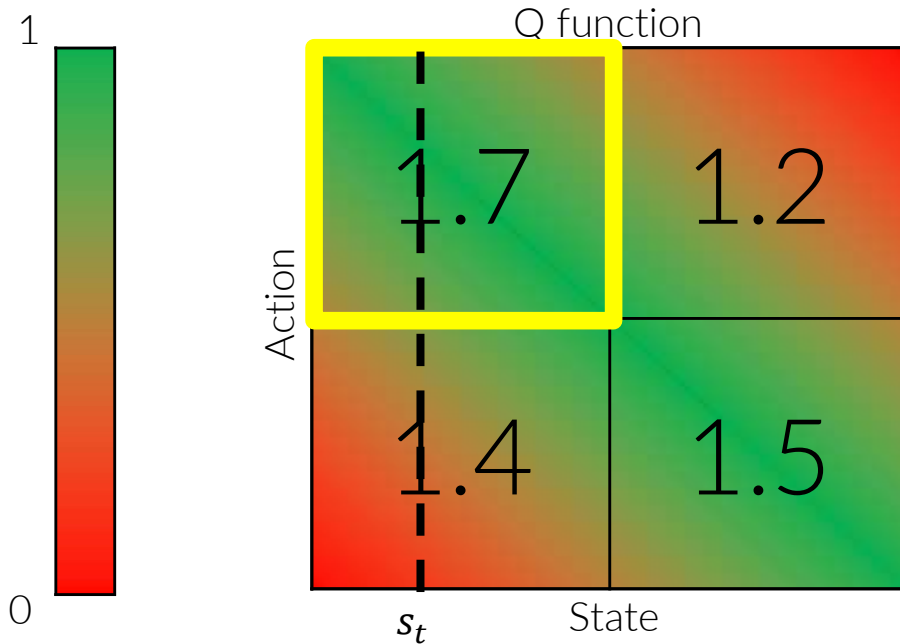
- (*Adaptive Partition*): Begin with coarse discretization of space
- (*Generate Estimates*): Maintain UCB estimates of Q function across regions
- (*Selection Rule*): Select “relevant” region that maximizes UCB for state

## Adaptive Discretization



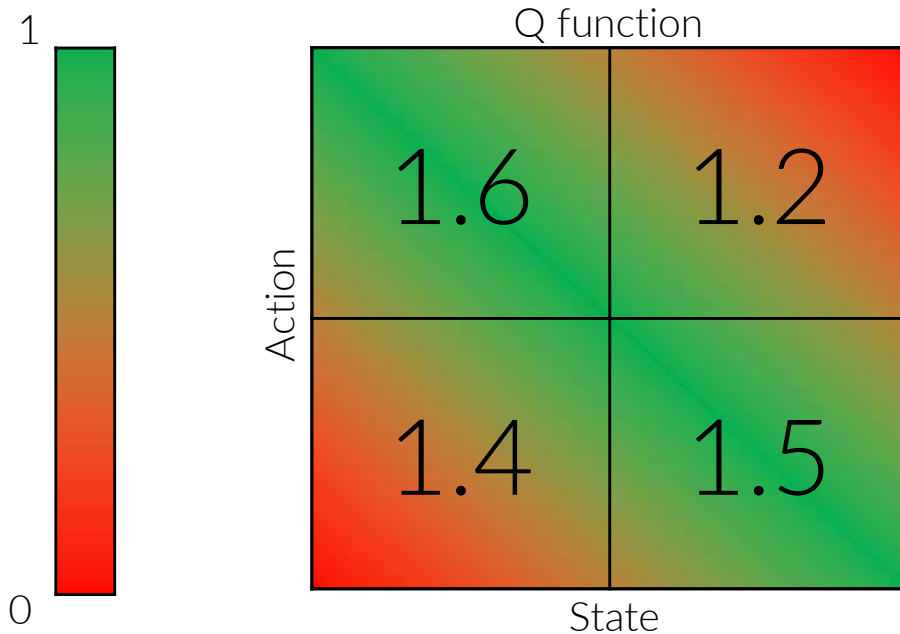
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## Adaptive Discretization



- (Update Estimates):* Update estimates with newly collected data:

$$(s_h, a_h, s_{h+1}, r_h)$$

## Model-Free:

$$\bar{Q}_h(B_{sel}) = (1 - \alpha_t) \bar{Q}_h(B_{sel}) + \alpha_t (r_h + \text{CONF}(B_{sel}) + \max_{x_{h+1} \in B} \bar{Q}_{h+1}(B))$$

Empirical Bellman Optimality Equation

## Adaptive Discretization

**Model-Based:** More complicated...as we require estimates of:

### Average Rewards

Easy – estimate as average reward of samples from region

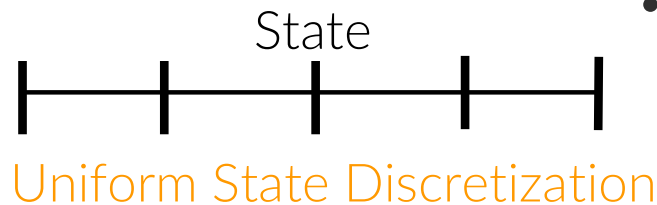
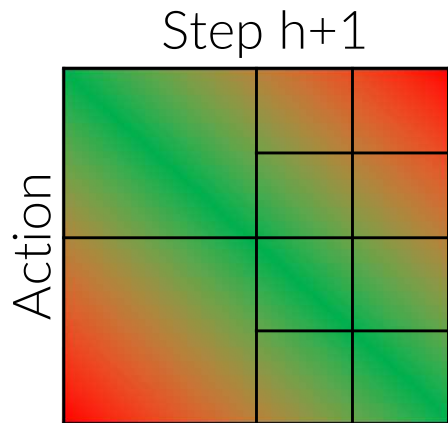
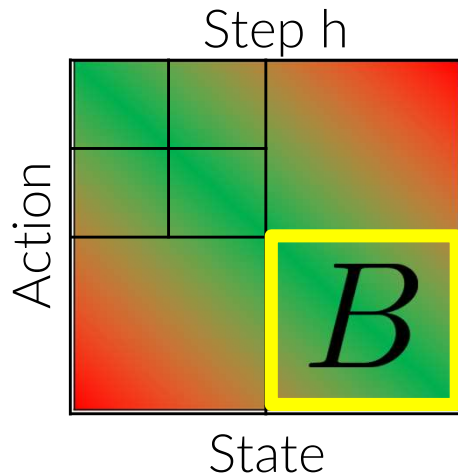
### Transition Distribution

Tough – domain is region, range is state space

$$\overline{T}_h(\cdot \mid B)$$

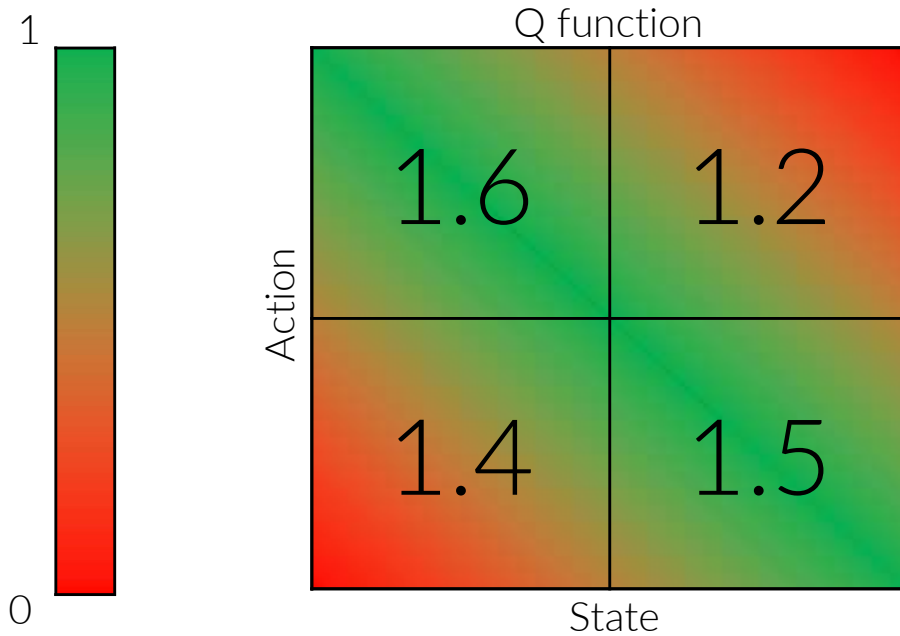


## Adaptive Discretization



- Estimate transition function over a uniform-discretization of the state space up to the precision of a ball
- Ensures accuracy of the estimate is proportional to the diameter
- Limits storage complexity for the estimates

## Adaptive Discretization



- *(Update Estimates)*: Update estimates with newly collected data:

$$(s_h, a_h, s_{h+1}, r_h)$$

## Model-Based:

$$\bar{Q}_h(B) = \bar{r}_h(B) + \mathbb{E}(\max_{X' \in B} \bar{Q}_{h+1}(B) \mid B) + \text{CONF}(B)$$

# Plan for Today

## Tree-Partitions

- Implement tree-based adaptive discretization from nonparametric RL algorithms.
  - Essentially a d-ary tree
  - Implement argmax over sub-tree
- Update Q estimates

## Run Experiments

- Use experiment instrumentation as part of ORSuite to run experiment of AdaMB, AdaQL on ambulance routing models under various arrival distributions:
  - beta distributed arrivals
  - uniform arrivals

## References

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<https://github.com/seanrsinclair/RLinOperations>

