

Competitive Programming Reference

First, solve the problem. Then, write the code.

John Johnson

By

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Coding Resources

C++

Decimal Precision

```
// rounds up the decimal number
cout << setprecision(N) << n << endl;
// specify N fixed number of decimals
cout << fixed << setprecision(N) << n << endl;
```

Include All Libraries

```
#include <bits/stdc++.h>
using namespace std;
```

Int To Binary

```
typedef long long int lli;

lli bitsInInt(lli n) { // clz = count leading zeroes
    return sizeof(n) * 8 - __builtin_clzll(n);
}

vector<bool> intToBitsArray(lli n) {
    n = abs(n);
    if (!n) return {};
    int length = bitsInInt(n), lastPos = length - 1;
    vector<bool> v(length);
    for (lli i = lastPos, j = 0; i > -1LL; i--, j++)
        v[j] = (n >> i) & 1LL;
    return v;
}
```

IO Optimization

```
int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}
```

Map Value To Int

```
// val = value
typedef string Val;
unordered_map<Val, int> intForVal;
unordered_map<int, Val> valForInt;
int mapId = 0;
```

```
int Map(Val val) {
    if (intForVal.count(val)) return intForVal[val];
    valForInt[mapId] = val;
    return intForVal[val] = mapId++;
}
```

```
Val IMap(int n) { return valForInt[n]; }
```

```
void initMapping() {
    mapId = 0;
    intForVal.clear();
    valForInt.clear();
}
```

Permutations

```
typedef vector<int> T; // typedef string T;

vector<T> permutations(T v) {
    vector<vector<int>>> ans;
    sort(v.begin(), v.end());
    do
        ans.push_back(v);
    while (next_permutation(v.begin(), v.end()));
    return ans;
}
```

Print Vector

```
void printv(vector<int> v) {
    if (v.size() == 0) {
        cout << "[]" << endl;
        return;
    }
    cout << "[" << v[0];
    for (int i = 1; i < v.size(); i++)
        cout << ", " << v[i];
    cout << "]" << endl;
}
```

Priority Queue Of Object

```
struct Object {
    char first;
    int second;
};

int main() {
    auto cmp = [](const Object& a, const Object& b) {
        return a.second > b.second;
    };
    priority_queue<Object, vector<Object>,
        decltype(cmp)> pq(cmp);
    vector<Object> v = {{'c', 3}, {'a', 1}, {'b', 2}};
    sort(v.begin(), v.end(), cmp);
    return 0;
}
```

Random

```
int random(int min, int max) {
    return min + rand() % (max - min + 1);
}

int main() {
    srand(time(0));
    // code ...
}
```

Read Line

```
// when reading lines, don't mix 'cin' with
// 'getline' just use getline and split
string input() {
    string ans;
    // cin >> ws; // eats all whitespaces.
    getline(cin, ans);
    return ans;
}
```

Sort Pair

```
pair<int, int> p;
// sorts array on the basis of the first element
sort(p.begin(), p.end());
```

Sort Vector Of Object

```
struct Object {
    char first;
    int second;
};

bool cmp(const Object& a, const Object& b) {
    return a.second > b.second;
}

int main() {
    vector<Object> v = {{'c', 3}, {'a', 1}, {'b', 2}};
    sort(v.begin(), v.end(), cmp);
    printv(v);
    return 0;
}
```

Split String

```
vector<string> split(string str, char token) {
    stringstream test(str);
    string seg;
    vector<string> seglist;
    while (getline(test, seg, token))
        seglist.push_back(seg);
    return seglist;
}
```

Typedef

```
typedef TYPE ALIAS;
// example:
typedef int T;
```

Python

Combinations

```
import itertools
# from arr choose k = > combinations(arr, k)
print(list(itertools.combinations([1, 2, 3], 3)))
```

Fast IO

```
from sys import stdin, stdout

N = 10
# Reads N chars from stdin(it counts '\n' as char)
stdin.read(N)
# Reads until '\n' or EOF
line = stdin.readline()
# Reads all lines in stdin until EOF
lines = stdin.readlines()
# Writes a string to stdout, it doesn't add '\n'
stdout.write(line)
# Writes a list of strings to stdout
stdout.writelines(lines)
# Reads numbers separated by space in a line
numbers = list(map(int, stdin.readline().split()))
```

Permutations

```
import itertools
print(list(itertools.permutations([1, 2, 3])))
```

Random

```
import random
# Initialize the random number generator.
random.seed(None)
# Returns a random integer N such that a <= N <= b.
random.randint(a, b)
# Returns a random integer N such that 0 <= N < b
random.randrange(b)
# Returns a random integer N such that a <= N < b.
random.randrange(a, b)
# Returns and integer with k random bits.
random.getrandbits(k)
# shuffles a list
random.shuffle(li)
```

Sort List

```
li = ['a', 'c', 'b']
# sorts inplace in descending order
li.sort(reverse=True)
# returns sorted list ascending order
ol = sorted(li)
```

Sort List Of Object

```
class MyObject :
    def __init__(self, first, second, third):
        self.first = first
        self.second = second
        self.third = third

li = [MyObject('b', 3, 1), MyObject('a', 3, 2),
      → MyObject('b', 3, 3)]
# returns list sorted by first then by second then by
→ third in increasing order
ol = sorted(li, key = lambda x: (x.first, x.second,
→ x.third), reverse=False)
# sorts inplace by first then by second then by third
→ in increasing order
li.sort(key = lambda x: (x.first, x.second, x.third),
→ reverse=False)
```

Data Structures

Geometry

K-D Tree

Graphs

UnionFind

```
struct UnionFind {
    int n;
    vector<int> dad, size;

    UnionFind(int N) : n(N), dad(N), size(N, 1) {
        while (N--) dad[N] = N;
    }
}
```

```
int root(int u) {
    if (dad[u] == u) return u;
    return dad[u] = root(dad[u]);
}

bool areConnected(int u, int v) {
    return root(u) == root(v);
}

void join(int u, int v) {
    int Ru = root(u), Rv = root(v);
    if (Ru == Rv) return;
    --n, dad[Ru] = Rv;
    size[Rv] += size[Ru];
}

int getSize(int u) { return size[root(u)]; }

int numberOfSets() { return n; }
};
```

Ranges

BIT

```
typedef long long int T;
T neutro = 0;
vector<T> bit;

void initVars(int n) { bit.assign(++n, neutro); }

T F(T a, T b) {
    return a + b;
    // return a * b;
}

// Inverse of F
T I(T a, T b) {
    return a - b;
    // return a / b;
}

// O(N)
void build() {
    for (int i = 1; i < bit.size(); i++) {
        int j = i + (i & -i);
        if (j < bit.size()) bit[j] = F(bit[j], bit[i]);
    }
}

// O(lg(N))
void update(int i, T val) {
    for (i++; i < bit.size(); i += i & -i)
        bit[i] = F(bit[i], val);
}
```

```
// O(lg(N))
T query(int i) {
    T ans = neutro;
    for (i++; i; i -= i & -i) ans = F(ans, bit[i]);
    return ans;
}

// O(lg(N)), [l, r]
T query(int l, int r) {
    return I(query(r), query(--l));
}

void setValAt(T val, int i) { bit[++i] = val; }
```

BIT Range Update

```
typedef long long int T;
T neutro = 0;
vector<T> bit1, bit2;

void initVars(int n) {
    bit1.assign(++n, neutro);
    bit2 = bit1;
}

// O(lg(N))
void update(vector<T> &bit, int i, T val) {
    for (i++; i < bit.size(); i += i & -i)
        bit[i] += val;
}

// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    update(bit1, l, val);
    update(bit1, r + 1, -val);
    update(bit2, r + 1, val * r);
    update(bit2, l, -val * (l - 1));
}

// O(lg(N))
T query(vector<T> &bit, int i) {
    T ans = neutro;
    for (i++; i; i -= i & -i) ans += bit[i];
    return ans;
}

// O(lg(N))
T query(int i) {
    return query(bit1, i) * i + query(bit2, i);
}

// O(lg(N)), [l, r]
T query(int l, int r) {
    return query(r) - query(l - 1);
}
```

Segment Tree

```
// st = segment tree. st[1] = root;
// neutro = operation neutral value
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.

template <class T>
struct SegmentTree {
    T neutro = 0;
    int N;
    vector<T> st;

    SegmentTree(int n) : st(2 * n, neutro), N(n) {}

    T F(T a, T b) {
        return a + b;
        // return __gcd(a, b);
        // return a * b;
        // return min(a, b);
    }

    // O(2N)
    void build() {
        for (int i = N - 1; i > 0; i--)
            st[i] = F(st[i << 1], st[i << 1 | 1]);
    }

    // O(lg(2N))
    void update(int i, T val) {
        for (st[i += N] = val; i > 1; i >>= 1)
            st[i >> 1] = F(st[i], st[i ^ 1]);
    }

    // O(3N), [l, r]
    void update(int l, int r, T val) {
        if (l == r)
            update(l, val);
        else {
            for (l += N, r += N; l <= r; l++) st[l] = val;
            build();
        }
    }

    // O(lg(2N)), [l, r]
    T query(int l, int r) {
        T ans = neutro;
        for (l += N, r += N; l <= r; l >>= 1, r >>= 1) {
            if (l & 1) ans = F(ans, st[l++]);
            if (~r & 1) ans = F(ans, st[r--]);
        }
        return ans;
    }

    void setValAt(T val, int i) { st[i + N] = val; }
};
```

Segment Tree Lazy Propagation

```
// st = segment tree, st[1] = root, H = height of d
// u = updates, d = delayed updates
// neutro = operation neutral val
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.

template <class T>
struct SegmentTree {
    T neutro = 0;
    int N, H;
    vector<T> st, d;
    vector<bool> u;

    SegmentTree(int n)
        : st(2 * n, neutro), d(n, 0), u(n, 0) {
        H = sizeof(int) * 8 - __builtin_clz(N = n);
    }

    T F(T a, T b) {
        return a + b;
        // return __gcd(a, b);
        // return a * b;
        // return min(a, b);
    }

    void apply(int i, T val, int k) {
        st[i] = val * k; // sum
        // st[i] = val; // min, max, gcd
        // st[i] = pow(a, k); // multiplication
        if (i < N) d[i] = val, u[i] = 1;
    }

    void calc(int i) {
        if (!u[i]) st[i] = F(st[i << 1], st[i << 1 | 1]);
    }

    // O(2N)
    void build() {
        for (int i = N - 1; i > 0; i--) calc(i);
    }

    // O(lg(N))
    void build(int p) {
        while (p > 1) p >>= 1, calc(p);
    }

    // O(lg(N))
    void push(int p) {
        for (int s = H, k = 1 << (H - 1); s > 0;
             s--, k >>= 1) {
            int i = p >> s;
            if (u[i]) {
                apply(i << 1, d[i], k);
                apply(i << 1 | 1, d[i], k);
                u[i] = 0, d[i] = neutro;
            }
        }
    }
}
```

```
// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    push(l += N);
    push(r += N);
    int ll = l, rr = r, k = 1;
    for (; l <= r; l >>= 1, r >>= 1, k <<= 1) {
        if (l & 1) apply(l++, val, k);
        if (~r & 1) apply(r--, val, k);
    }
    build(ll);
    build(rr);
}

// O(lg(2N)), [l, r]
T query(int l, int r) {
    push(l += N);
    push(r += N);
    T ans = neutro;
    for (; l <= r; l >>= 1, r >>= 1) {
        if (l & 1) ans = F(ans, st[l++]);
        if (~r & 1) ans = F(ans, st[r--]);
    }
    return ans;
}

void setValAt(T val, int i) { st[i + N] = val; }
};
```

Sparse Table

```
// st = sparse table, Arith = Arithmetic
typedef int T;
int neutro = 0;
vector<vector<T>> st;

T F(T a, T b) {
    // return min(a, b);
    return __gcd(a, b);
    // return a + b; // Arith
    // return a * b; // Arith
}

// O(N lg(N))
void build(vector<T> &arr) {
    st.assign(log2(arr.size()), vector<T>(arr.size()));
    st[0] = arr;
    for (int i = 1; (1 << i) <= arr.size(); i++)
        for (int j = 0; j + (1 << i) <= arr.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

// O(1), [l, r]
T query(int l, int r) {
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]);
}
```



```
// O(lg(N)), [l, r]
T queryArith(int l, int r) {
    T ans = neutro;
    while (true) {
        int k = log2(r - l + 1);
        ans = F(ans, st[k][l]);
        l += 1 << k;
        if (l > r) break;
    }
    return ans;
}
```

Strings

Trie

// wpt = number of words passing through
// w = number of words ending in the node
// c = character

```
struct Trie {
    struct Node {
        // for lexicographical order use 'map'
        // map<char, Node *> ch;
        unordered_map<char, Node *> ch;
        int w = 0, wpt = 0;
    };

    Node *root = new Node();

    // O(STR.SIZE)
    void insert(string str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c))
                curr->ch[c] = new Node();
            curr->wpt++, curr = curr->ch[c];
        }
        curr->wpt++, curr->w++;
    }

    // O(STR.SIZE)
    Node *find(string &str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c)) return nullptr;
            curr = curr->ch[c];
        }
        return curr;
    }

    // O(STR.SIZE) number of words with given prefix
    int prefixCount(string prefix) {
        Node *node = find(prefix);
        return node ? node->wpt : 0;
    }
}
```

```
// O(STR.SIZE) number of words matching str
int strCount(string str) {
    Node *node = find(str);
    return node ? node->w : 0;
}

// O(N)
void getWords(Node *curr, vector<string> &words,
              string &word) {
    if (!curr) return;
    if (curr->w) words.push_back(word);
    for (auto &c : curr->ch) {
        getWords(c.second, words, word += c.first);
        word.pop_back();
    }
}

// O(N)
vector<string> getWords() {
    vector<string> words;
    string word = "";
    getWords(root, words, word);
    return words;
}

// O(N)
vector<string> getWordsByPrefix(string prefix) {
    vector<string> words;
    getWords(find(prefix), words, prefix);
}

// O(STR.SIZE)
bool remove(Node *curr, string &str, int &i) {
    if (i == str.size()) {
        curr->wpt--;
        return curr->w ? !(curr->w == 0) : 0;
    }
    int c = str[i];
    if (!curr->ch.count(c)) return false;
    if (remove(curr->ch[c], str, ++i)) {
        if (!curr->ch[c]->wpt)
            curr->wpt--, curr->ch.erase(c);
        return true;
    }
    return false;
}

// O(STR.SIZE)
int remove(string str) {
    int i = 0;
    return remove(root, str, i);
}
};
```

Trees And Heaps

Red Black Tree

```
template <class K, class V>
struct RedBlackTree {
```

```

struct Node {
    K key;
    V val;
    Node *l, *r; // left, right
    bool isRed;
    Node(K k, V v, bool isRed)
        : key(k), val(v), isRed(isRed) {}
};

Node *root = nullptr;

int compare(K a, K b) {
    if (a < b) return -1;
    if (a > b) return 1;
    return 0;
}

// O(lg(N))
V at(K key) {
    Node *x = root;
    while (x) {
        int cmp = compare(key, x->key);
        if (!cmp) return x->val;
        if (cmp < 0) x = x->l;
        if (cmp > 0) x = x->r;
    }
    throw runtime_error("Key doesn't exist");
}

Node *rotateLeft(Node *h) {
    Node *x = h->r;
    h->r = x->l;
    x->l = h;
    x->isRed = h->isRed;
    h->isRed = 1;
    return x;
}

Node *rotateRight(Node *h) {
    Node *x = h->l;
    h->l = x->r;
    x->r = h;
    x->isRed = h->isRed;
    h->isRed = 1;
    return x;
}

void flipColors(Node *h) {
    h->isRed = 1;
    h->l->isRed = 0;
    h->r->isRed = 0;
}

```

```

// O(lg(N))
Node *insert(Node *h, K key, V val) {
    if (!h) return new Node(key, val, 1);
    int cmp = compare(key, h->key);
    if (!cmp) h->val = val;
    if (cmp < 0) h->l = insert(h->l, key, val);
    if (cmp > 0) h->r = insert(h->r, key, val);
    if (h->r && h->r->isRed && !(h->l && h->l->isRed))
        h = rotateLeft(h);
    if (h->l && h->l->isRed && h->l->l &&
        h->l->l->isRed)
        h = rotateRight(h);
    if (h->l && h->l->isRed && h->r && h->r->isRed)
        flipColors(h);
    return h;
}

// O(lg(N))
void insert(K key, V val) {
    root = insert(root, key, val);
}
};

```

Treap

Geometry

Convex Hull

Max Interval Overlap

```

typedef long long int T;
typedef pair<T, T> Interval;
vector<Interval> maxIntervals;

```

```
// O(N * lg(N))
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    map<T, int> m;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (auto &i : arr) m[i.first]++, m[i.second + 1]--;
    for (auto &p : m) {
        curr += p.second;
        if (curr > maxI) maxI = curr, l = p.first;
        if (curr == maxI) r = p.first;
    }
    curr = 0;
    for (auto &p : m) {
        curr += p.second;
        if (curr == maxI && isFirst)
            l = p.first, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, p.first - 1}),
            isFirst = 1;
    }
    return maxI;
}

// O(MaxPoint) maxPoint < vector::max_size
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    T maxPoint = 0;
    for (auto &i : arr)
        if (i.second > maxPoint) maxPoint = i.second;
    vector<int> x(maxPoint + 2);
    for (auto &i : arr) x[i.first]++, x[i.second + 1]--;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr > maxI) maxI = curr;
    }
    curr = 0;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr == maxI && isFirst) l = i, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, i - 1}), isFirst = 1;
    }
    return maxI;
}
```

Graphs

Articulation Points And Bridges

```
// APB = articulation points and bridges
// ap = Articulation Point
// br = bridges, p = parent
// disc = discovery time
// low = lowTime, ch = children

typedef pair<int, int> Edge;
int Time;
vector<vector<int>>> ady;
vector<int> disc, low, ap;
vector<Edge> br;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

int dfsAPB(int u, int p) {
    int ch = 0;
    low[u] = disc[u] = ++Time;
    for (int &v : ady[u]) {
        if (v == p) continue;
        if (!disc[v]) {
            ch++, dfsAPB(v, u);
            if (disc[u] <= low[v]) ap[u]++;
            if (disc[u] < low[v]) br.push_back({u, v});
            low[u] = min(low[u], low[v]);
        } else
            low[u] = min(low[u], disc[v]);
    }
    return ch;
}

// O(N)
void APB() {
    br.clear();
    ap = low = disc = vector<int>(ady.size());
    Time = 0;
    for (int u = 0; u < ady.size(); u++)
        if (!disc[u]) ap[u] = dfsAPB(u, u) > 1;
}
```

Connected Components

```
// comp = component
int compId;
vector<vector<int>>> ady;
vector<int> getComp;
```

```

void initVars(int N) {
    ady.assign(N, vector<int>());
    getComp.assign(N, -1);
    compId = 0;
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

void dfsCC(int u, vector<int> &comp) {
    if (getComp[u] > -1) return;
    getComp[u] = compId;
    comp.push_back(u);
    for (auto &v : ady[u]) dfsCC(v, comp);
}

// O(N)
vector<vector<int>>> connectedComponents() {
    vector<vector<int>>> comps;
    for (int u = 0; u < ady.size(); u++) {
        vector<int> comp;
        dfsCC(u, comp);
        if (!comp.empty())
            comps.push_back(comp), compId++;
    }
    return comps;
}

```

Flood Fill

```

int n, m, oldColor = 0, color = 1;
vector<vector<int>>> mat;
vector<vector<int>>> movs = {
    {1, 0}, {0, 1}, {-1, 0}, {0, -1}};

void floodFill(int i, int j) {
    if (i >= mat.size() || i < 0 ||
        j >= mat[i].size() || j < 0 ||
        mat[i][j] != oldColor)
        return;
    mat[i][j] = color;
    for (auto move : movs)
        floodFill(i + move[1], j + move[0]);
}

void floodFill() {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            if (mat[i][j] == oldColor) floodFill(i, j);
}

```

Heavy Light Decomposition

```

// p = parent;
#include "../Data Structures/Ranges/Segment Tree.cpp"
typedef int T;
vector<vector<int>>> ady;
vector<int> p, heavy, depth, root, stPos, vals;
SegmentTree<T> st(0);

```

```

void initVars(int n) {
    ady.assign(n, vector<int>());
    heavy.assign(n, -1);
    vals.assign(n, 0);
    p = root = stPos = depth = heavy;
    st = SegmentTree<T>(n);
}

void addEdge(int u, int v, T val) {
    ady[u].push_back(v);
    p[v] = u, vals[v] = val;
}

T F(T a, T b) { return a + b; }

// O(N)
int dfs(int u) {
    int size = 1, maxSubtree = 0;
    for (int &v : ady[u]) {
        depth[v] = depth[u] + 1;
        int subtree = dfs(v);
        if (subtree > maxSubtree)
            heavy[u] = v, maxSubtree = subtree;
        size += subtree;
    }
    return size;
}

// O(N)
void initHeavyLight() {
    for (int i = 0; i < ady.size(); i++)
        if (p[i] < 0) dfs(i);
    for (int i = 0, pos = 0; i < ady.size(); i++)
        if (p[i] < 0 || heavy[p[i]] != i)
            for (int j = i; ~j; j = heavy[j]) {
                st.setValAt(vals[j], stPos[j] = pos++);
                root[j] = i;
            }
    st.build();
}

// O(lg2 (N))
template <class Op>
void processPath(int u, int v, Op op) {
    for (; root[u] != root[v]; v = p[root[v]]) {
        if (depth[root[u]] > depth[root[v]]) swap(u, v);
        op(stPos[root[v]], stPos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    // for values on edges
    if (u != v) op(stPos[u] + 1, stPos[v]);
    // for values on nodes
    // op(stPos[u], stPos[v]);
}

// O(lg2 (N))
void update(int u, int v, T val) {
    processPath(u, v, [&val](int l, int r) {
        st.update(l, r, val);
    });
}

```

```
// O(lg2(N))
T query(int u, int v) {
    T ans = T();
    processPath(u, v, [&ans](int l, int r) {
        ans = F(ans, st.query(l, r));
    });
    return ans;
}
```

Is Bipartite

```
vector<vector<int>>> ady;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

// O(N)
bool isBipartite() {
    vector<int> color(ady.size(), -1);
    for (int s = 0; s < ady.size(); s++) {
        if (color[s] > -1) continue;
        color[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int &v : ady[u]) {
                if (color[v] < 0)
                    q.push(v), color[v] = !color[u];
                if (color[v] == color[u]) return false;
            }
        }
    }
    return true;
}
```

LCA

```
// st = sparse table
typedef pair<int, int> T;
int neutro = 0;
vector<vector<T>>> st;
vector<int> first;
vector<T> tour;
vector<vector<int>>> ady;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

T F(T a, T b) { return a.first < b.first ? a : b; }
```

```
void build() {
    st.assign(log2(tour.size()),
              vector<T>(tour.size()));
    st[0] = tour;
    for (int i = 1; (1 << i) <= tour.size(); i++)
        for (int j = 0; j + (1 << i) <= tour.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

void eulerTour(int u, int p, int h) {
    first[u] = tour.size();
    tour.push_back({h, u});
    for (int v : ady[u])
        if (v != p) {
            eulerTour(v, u, h + 1);
            tour.push_back({h, u});
        }
}

// O(N * lg(N))
void preprocess() {
    tour.clear();
    first.assign(ady.size(), -1);
    eulerTour(0, 0, 0);
    build();
}

// O(1)
int lca(int u, int v) {
    int l = min(first[u], first[v]);
    int r = max(first[u], first[v]);
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]).second;
}
```

MST Kruskal

```
// N = number of nodes, Wedge = Weighted Edge
#include "../Data Structures/Graphs/UnionFind.cpp"
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
vector<Wedge> Wedges;
vector<Wedge> mst;
UnionFind uf(0);

void initVars(int N) {
    mst.clear();
    Wedges.clear();
    uf = UnionFind(N);
}

void addEdge(int u, int v, T w) {
    Wedges.push_back({w, {u, v}});
}
```

```

T kruskal() {
    T cost = 0;
    sort(Wedges.begin(), Wedges.end());
    // reverse(Wedges.begin(), Wedges.end());
    for (Wedge &wedge : Wedges) {
        int u = wedge.second.first,
            v = wedge.second.second;
        if (!uf.areConnected(u, v))
            uf.join(u, v), mst.push_back(wedge),
            cost += wedge.first;
    }
    return cost;
}

```

MST Prim

```

// st = spanning tree, p = parent
// vis = visited, dist = distance
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
typedef pair<T, int> DistNode;
int INF = 1 << 30;
vector<vector<int>> ady;
unordered_map<int, unordered_map<int, T>> weight;
vector<int> p, vis;
vector<T> dist;
vector<vector<Wedge>> msts;

void initVars(int N) {
    ady.assign(N, vector<int>());
    p.assign(N, 0);
    vis.assign(N, 0);
    dist.assign(N, INF);
    weight.clear();
    msts.clear();
}

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    ady[v].push_back(u);
    weight[v][u] = w;
}

```

```

// O(E * log(V))
T prim(int s) {
    vector<Wedge> mst;
    vector<set<Edge>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    set<Edge> q;
    T cost = dist[s] = 0;
    q.insert({0, s});
    while (q.size()) {
        int u = q.begin()->second;
        q.erase(q.begin());
        vis[u] = 1, cost += dist[u];
        mst.push_back({dist[u], {p[u], u}});
        for (int &v : ady[u]) {
            T w = weight[u][v];
            if (!vis[v] && w < dist[v]) {
                if (dist[v] != INF) q.erase(pos[v]);
                pos[v] = q.insert({dist[v] = w, v}).first;
            }
        }
    }
    msts.push_back(
        vector<Wedge>(mst.begin() + 1, mst.end()));
    return cost;
}

// ~ O(E * log(V))
T primLazy(int s) {
    vector<Wedge> mst;
    vector<set<Edge>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    priority_queue<DistNode> q;
    T cost = dist[s] = 0;
    q.push({0, s});
    while (q.size()) {
        pair<int, int> aux = q.top();
        int u = aux.second;
        q.pop();
        if (dist[u] < -aux.first) continue;
        vis[u] = 1, cost += dist[u];
        mst.push_back({dist[u], {p[u], u}});
        for (int &v : ady[u]) {
            T w = weight[u][v];
            if (!vis[v] && w < dist[v])
                q.push({-(dist[v] = w), v});
        }
    }
    msts.push_back(
        vector<Wedge>(mst.begin() + 1, mst.end()));
    return cost;
}

// O(V + E * log(V))
T prim() {
    T cost = 0;
    map<int, T> q;
    for (int i = 0; i < ady.size(); i++)
        if (!vis[i]) cost += prim(i);
    return cost;
}

```

Strongly Connected Components

```
// tv = top value from stack
// sccs = strongly connected components
// scc = strongly connected component
// disc = discovery time, low = low time
// s = stack, top = top index of the stack

int Time, top;
vector<vector<int>>> ady, sccs;
vector<int> disc, low, s;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) { ady[u].push_back(v); }

void dfsSCCS(int u) {
    if (disc[u]) return;
    low[u] = disc[u] = ++Time;
    s[++top] = u;
    for (int &v : ady[u])
        dfsSCCS(v), low[u] = min(low[u], low[v]);
    if (disc[u] == low[u]) {
        vector<int> scc;
        while (true) {
            int tv = s[top--];
            scc.push_back(tv);
            low[tv] = ady.size();
            if (tv == u) break;
        }
        sccs.push_back(scc);
    }
}

// O(N)
void SCCS() {
    s = low = disc = vector<int>(ady.size());
    Time = 0, top = -1, sccs.clear();
    for (int u = 0; u < ady.size(); u++) dfsSCCS(u);
}
```

Topological Sort

```
// vis = visited
vector<vector<int>>> ady;
vector<int> vis, toposorted;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
}

void addEdge(int u, int v) { ady[u].push_back(v); }
```

```
// returns false if there is a cycle
// O(N)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}

// O(N)
bool toposort() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u] && !toposort(u)) return false;
    return true;
}
```

Cycles

Get All Simple Cycles

Get Some Cycles

```
// at least detects one cycle per component
vector<vector<int>>> ady, cycles;
vector<int> vis, cycle;
bool flag = false, isDirected = false;
int root = -1;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    cycles.clear();
    root = -1, flag = false;
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    if (!isDirected) ady[v].push_back(u);
}
```

```
// O(N)
bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : ady[u]) {
        if (v == u || vis[v] == 2 ||
            (!isDirected && v == prev))
            continue;
        if (flag) {
            if (!vis[v]) hasCycle(v, u);
            continue;
        }
        if (vis[v] || hasCycle(v, u)) {
            if (root == -1) root = v, flag = true;
            cycle.push_back(u);
            if (root == u)
                flag = false, root = -1,
                cycles.push_back(cycle), cycle.clear();
        }
    }
    vis[u] = 2;
    return flag;
}

// O(N)
bool hasCycle() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u]) cycle.clear(), hasCycle(u, -1);
    return cycles.size() > 0;
}
```

Has Cycle

```
vector<vector<int>>> ady;
vector<int> vis;
bool isDirected = false;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    if (!isDirected) ady[v].push_back(u);
}

bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (isDirected || v != prev) &&
            (vis[v] || hasCycle(v, u)))
            return true;
    vis[u] = 2;
    return false;
}
```

```
// O(N)
bool hasCycle() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u] && hasCycle(u, -1)) return true;
}
```

Flow

Max Flow Dinic

```
// cap[a][b] = Capacity from a to b
// flow[a][b] = flow occupied from a to b
// level[a] = level in graph of node a

typedef int T;
vector<int> level;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>>> cap, flow;

void initVars(int N) {
    ady.assign(N, vector<int>());
    cap.clear();
    flow.clear();
}

void addEdge(int u, int v, T capacity) {
    cap[u][v] = capacity;
    ady[u].push_back(v);
}

bool levelGraph(int s, int t) {
    level = vector<int>(ady.size());
    level[s] = 1;
    queue<int> q;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int &v : ady[u]) {
            if (!level[v] && flow[u][v] < cap[u][v]) {
                q.push(v);
                level[v] = level[u] + 1;
            }
        }
    }
    return level[t];
}
```



```

T blockingFlow(int u, int t, T currPathMaxFlow) {
    if (u == t) return currPathMaxFlow;
    for (int v : ady[u]) {
        T capleft = cap[u][v] - flow[u][v];
        if ((level[v] == (level[u] + 1)) &&
            (capleft > 0)) {
            T pathMaxFlow = blockingFlow(
                v, t, min(currPathMaxFlow, capleft));
            if (pathMaxFlow > 0) {
                flow[u][v] += pathMaxFlow;
                flow[v][u] -= pathMaxFlow;
                return pathMaxFlow;
            }
        }
    }
    return 0;
}

// O(E * V^2)
T dinicMaxFlow(int s, int t) {
    if (s == t) return -1;
    T maxFlow = 0;
    while (levelGraph(s, t))
        while (T flow = blockingFlow(s, t, 1 << 30))
            maxFlow += flow;
    return maxFlow;
}

```

Maximum Bipartite Matching

```

// mbm = maximum bipartite matching
#include "Max Flow Dinic.cpp"

void addEdgeMBM(int u, int v) {
    addEdge(u += 2, v += 2, 1);
    addEdge(0, u, 1);
    addEdge(v, 1, 1);
}

// O(E * V^2)
T mbm() { return dinicMaxFlow(0, 1); }

```

ShortestPaths

Bellman Ford

```

// N = number of nodes
// returns {} if there is a negative weight cycle
typedef int T;
int MAXN = 20001, N, INF = 1 << 30, isDirected = true;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    weight.clear();
}

```

```

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    ady[v].push_back(u);
    weight[v][u] = w;
}

// O(V * E)
vector<T> bellmanFord(int s) {
    vector<T> dist(ady.size(), INF);
    dist[s] = 0;
    for (int i = 1; i <= ady.size(); i++)
        for (int u = 0; u < ady.size(); u++)
            for (auto &v : ady[u]) {
                T w = weight[u][v], d = dist[u] + w;
                if (dist[u] != INF && d < dist[v]) {
                    if (i == ady.size()) return {};
                    dist[v] = d;
                }
            }
    return dist;
}

```

Dijkstra

```

#include <bits/stdc++.h>
using namespace std;

typedef int T;
typedef pair<T, int> DistNode;
int INF = 1 << 30, isDirected = false;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    ady[v].push_back(u);
    weight[v][u] = w;
}

```

```
// O(E * lg(V))
vector<T> dijkstra(int s) {
    vector<set<DistNode>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    set<DistNode> q;
    q.insert({0, s}); dist[s] = 0;
    while (q.size()) {
        int u = q.begin()->second;
        q.erase(q.begin());
        for (int &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) {
                if (dist[v] != INF) q.erase(pos[v]);
                pos[v] = q.insert({dist[v] = d, v}).first;
            }
        }
    }
    return dist;
}

// ~ O(E * lg(V))
vector<T> dijkstraLazy(int s) {
    vector<T> dist(ady.size(), INF);
    priority_queue<DistNode> q;
    q.push({0, s}); dist[s] = 0;
    while (q.size()) {
        DistNode top = q.top();
        q.pop();
        int u = top.second;
        if (dist[u] < -top.first) continue;
        for (int &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) q.push({-(dist[v] = d), v});
        }
    }
    return dist;
}
```

Directed Acyclic Graph

```
// vis = visited
typedef int T;
vector<vector<int>> ady;
vector<int> vis, toposorted;
int INF = 1 << 30;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
    weight.clear();
}

void addEdge(int u, int v, int w) {
    ady[u].push_back(v);
    weight[u][v] = w;
}
```

```
// returns false if there is a cycle
// O(N)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}

// O(N)
vector<T> sssp(int s) {
    vector<T> dist(ady.size(), INF);
    dist[s] = 0;
    toposort(s);
    while (toposorted.size()) {
        int u = toposorted.back();
        toposorted.pop_back();
        for (auto &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) dist[v] = d;
        }
    }
    return dist;
}
```

Maths

Number Theory

Divisibility Criterion

```
def divisorCriteria(n, lim):
    results = []
    tenElevated = 1
    for i in range(lim):
        # remainder = pow(10, i, n)
        remainder = tenElevated % n
        negremainder = remainder - n
        if (remainder <= abs(negremainder)):
            results.append(remainder)
        else:
            results.append(negremainder)
        tenElevated *= 10
    return results
```

```
def testDivisibility(dividend, divisor,
    ↪ divisor_criteria):
    dividend = str(dividend)
    addition = 0
    dividendSize = len(dividend)
    i = dividendSize - 1
    j = 0
    while j < dividendSize:
        addition += int(dividend[i]) *
            ↪ divisor_criteria[j]
        i -= 1
        j += 1
    return addition % divisor == 0

if __name__ == '__main__':
    dividend, divisor = map(int, input().split())
    divisor_criteria = divisorCriteria(divisor,
    ↪ len(str(dividend)))
    print(divisor_criteria)
    print(testDivisibility(dividend, divisor,
    ↪ divisor_criteria))
```

Extended Euclidean

```
// gcd(a, b) = ax + by
vector<long long int> extendedGCD(long long int a,
                                long long int b) {
    if (a > 0LL && b == 0LL) return {a, 1LL, 0LL};
    long long int x = 1LL, y = 0LL, prevx = 0LL,
        prevy = 1LL, q, remainder;
    while (true) {
        q = a / b;
        remainder = a - b * q;
        if (remainder == 0LL) break;
        a = b;
        b = remainder;
        x = x - prevx * q;
        swap(x, prevx);
        y = y - prevy * q;
        swap(y, prevy);
    }
    // gcd = b, x = prevx, y = prevy
    return {b, prevx, prevy};
}
```

GCD

```
// recursive
int gcd(int a, int b) {
    return !b ? a : gcd(b, a % b);
}
```

```
// iterative
int gcd(int a, int b) {
    while (b) {
        a %= b;
        swap(a, b);
    }
    return a;
}
```

LCM

```
int lcm(int a, int b) {
    int c = gcd(a, b);
    return c ? a / c * b : 0;
}
```

Prime Check Miller Rabin

```
from random import randrange
```

```
def is_prime(p):
    k = 100
    if p == 2 or p == 3:
        return True
    if (p & 1) == 0 or p == 1:
        return False
    phi = p - 1
    d = phi
    r = 0
    while (d & 1) == 0:
        d = int(d >> 1)
        r += 1
    for i in range(k):
        a = randrange(2, p - 2)
        exp = pow(a, d, p)
        if exp == 1 or exp == p - 1:
            continue
        flag = False
        for j in range(r - 1):
            exp = pow(exp, 2, p)
            if exp == 1:
                return False
            if exp == p - 1:
                flag = True
                break
        if flag:
            continue
        else:
            return False
    return True
```

Prime Sieve

```
vector<int> primeSieve(int n) {
    vector<int> sieve(n + 1);
    for (int i = 4; i <= n; i += 2) sieve[i] = 2;
    for (int i = 3; i * i <= n; i += 2)
        if (!sieve[i])
            for (int j = i * i; j <= n; j += 2 * i)
                if (!sieve[j]) sieve[j] = i;
    return sieve;
}
```

Strings

KMP

```
// p = pattern, t = text
// f = error function, cf = create error function
// pos = positions where pattern is found in text
```

```
int MAXN = 1000000;
vector<int> f(MAXN + 1);

vector<int> kmp(string &p, string &t, int cf) {
    vector<int> pos;
    if (cf) f[0] = -1;
    for (int i = cf, j = 0; j < t.size(); i++) {
        while (i > -1 && p[i] != t[j]) i = f[i];
        i++, j++;
        if (cf) f[j] = i;
        if (!cf && i == p.size())
            pos.push_back(j - i), i = f[i];
    }
    return pos;
}
```

```
vector<int> search(string &p, string &t) {
    kmp(p, p, -1); // create error function
    return kmp(p, t, 0); // search in text
}
```

Rabin Karp

```
class RollingHash {
public:
    vector<unsigned long long int> pow;
    vector<unsigned long long int> hash;
    unsigned long long int B;
```

```
RollingHash(const string &text) : B(257) {
    int N = text.size();
    pow.resize(N + 1);
    hash.resize(N + 1);
    pow[0] = 1;
    hash[0] = 0;
    for (int i = 1; i <= N; ++i) {
        // in c++ an unsigned long long int is
        // automatically modulated by 2^64
        pow[i] = pow[i - 1] * B;
        hash[i] = hash[i - 1] * B + text[i - 1];
    }
}

unsigned long long int getWordHash() {
    return hash[hash.size() - 1];
}

unsigned long long int getSubstrHash(int begin,
                                     int end) {
    return hash[end] -
           hash[begin - 1] * pow[end - begin + 1];
}

int size() { return hash.size(); }
};

vector<int> rabinKarp(RollingHash &rhStr,
                     string &pattern) {
    vector<int> positions;
    RollingHash rhPattern(pattern);
    unsigned long long int patternHash =
        rhPattern.getWordHash();
    int windowSize = pattern.size(), end = windowSize;
    for (int i = 1; end < rhStr.size(); i++) {
        if (patternHash == rhStr.getSubstrHash(i, end))
            positions.push_back(i);
        end = i + windowSize;
    }
    return positions;
}
```

Techniques

Binary Search

Multiple Queries

Mo

```
// q = query
// qs = queries
```

```

struct Query {
    int l, r;
};

int blksize;
vector<Query> qs;
vector<int> arr;

void initVars(int N, int M) {
    arr = vector<int>(N);
    qs = vector<Query>(M);
}

bool cmp(Query &a, Query &b) {
    if (a.l == b.l) return a.r < b.r;
    return a.l / blksize < b.l / blksize;
}

void getResults() {
    blksize = (int)sqrt(arr.size());
    sort(qs.begin(), qs.end(), cmp);
    int prevL = 0, prevR = -1;
    int sum = 0;
    for (auto &q : qs) {
        int L = q.l, R = q.r;
        while (prevL < L) {
            sum -= arr[prevL]; // problem specific
            prevL++;
        }
        while (prevL > L) {
            prevL--;
            sum += arr[prevL]; // problem specific
        }
        while (prevR < R) {
            prevR++;
            sum += arr[prevR]; // problem specific
        }
        while (prevR > R) {
            sum -= arr[prevR]; // problem specific
            prevR--;
        }

        cout << "sum[" << L << ", " << R << "] = " << sum
              << endl;
    }
}

int main() {
    initVars(9, 2);
    arr = {1, 1, 2, 1, 3, 4, 5, 2, 8};
    qs = {{0, 8}, {3, 5}};
    getResults();
}

```

SQRT Decomposition

```

// sum of elements in range
int neutro = 0;
vector<int> arr;
vector<int> blks;

```

```

void initVars(int n) {
    arr.assign(n, neutro);
    blks.assign(sqrt(n), neutro);
}

void preprocess() {
    for (int i = 0, j = 0; i < arr.size(); i++) {
        if (i == blks.size() * j) j++;
        blks[j - 1] += arr[i]; // problem specific
    }
}

// problem specific
void update(int i, int val) {
    blks[i / blks.size()] += val - arr[i];
    arr[i] = val;
}

int query(int l, int r) {
    int sum = 0;
    int lblk = l / blks.size();
    if (l != blks.size() * lblk++) {
        while (l < r && l != lblk * blks.size()) {
            sum += arr[l]; // problem specific
            l++;
        }

        while (l + blks.size() <= r) {
            sum += blks[l / blks.size()]; // problem specific
            l += blks.size();
        }
        while (l <= r) {
            sum += arr[l]; // problem specific
            l++;
        }
        return sum;
    }
}

int main() {
    initVars(10);
    arr = {1, 5, 2, 4, 6, 1, 3, 5, 7, 10};
    preprocess();
    for (int i = 0; i < blks.size() + 1; i++)
        cout << blks[i] << " ";
    // output: 8 11 15 10
    cout << endl;
    cout << query(3, 8) << " ";
    cout << query(1, 6) << " ";
    update(8, 0);
    cout << query(8, 8) << endl;
    // output: 26 21 0
    return 0;
}

```