

Competitive Programming Reference

First, solve the problem. Then, write the code.

John Johnson

By

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Contents

Coding Resources	4
C++	4
Decimal Precision	4
Include All Libraries	4
Int To Binary	4
IO Optimization	4
Map Value To Int	4
Permutations	4
Print Vector	4
Priority Queue Of Object	4
Random	5
Read Line	5
Sort Pair	5
Sort Vector Of Object	5
Split String	5
Typedef	5
Python	5
Combinations	5
Fast IO	5
Permutations	5
Random	5
Sort List	6
Sort List Of Object	6
Data Structures	6
Geometry	6
K-D Tree	6
Graphs	6
UnionFind	6
Ranges	6
BIT	6
BIT Range Update	7
Segment Tree	7
Segment Tree Lazy Propagation	8
Sparse Table	8
Strings	9
Trie	9
Trees And Heaps	10
Red Black Tree	10
Treap	10
Geometry	10
Convex Hull	10
Max Interval Overlap	10
Graphs	11
Articulation Points And Bridges	11
Connected Components	11
Flood Fill	12
Heavy Light Decomposition	12
Is Bipartite	13
LCA	13
MST Kruskal	13
MST Prim	14
Strongly Connected Components	15
Topological Sort	15
Topological Sort (All possible sorts)	15

Cycles	15
Get All Simple Cycles	15
Get Some Cycles	15
Has Cycle	16
Flow	16
Max Flow Dinic	16
Maximum Bipartite Matching	17
ShortestPaths	17
Bellman Ford	17
Dijkstra	17
Shortest Path in Directed Acyclic Graph	18
Maths	18
Number Theory	18
Divisibility Criterion	18
Extended Euclidean	19
GCD	19
LCM	19
Prime Check Miller Rabin	19
Prime Sieve	20
Strings	20
KMP	20
Rabin Karp	20
Techniques	20
Binary Search	20
Multiple Queries	20
Mo	20
SQRT Decomposition	21

Coding Resources

C++

Decimal Precision

```
// rounds up the decimal number
cout << setprecision(N) << n << endl;
// specify N fixed number of decimals
cout << fixed << setprecision(N) << n << endl;
```

Include All Libraries

```
#include <bits/stdc++.h>
using namespace std;
```

Int To Binary

```
typedef long long int lli;

lli bitsInInt(lli n) { // clz = count leading zeroes
    return sizeof(n) * 8 - __builtin_clzll(n);
}

vector<bool> intToBitsArray(lli n) {
    n = abs(n);
    if (!n) return {};
    int length = bitsInInt(n), lastPos = length - 1;
    vector<bool> v(length);
    for (lli i = lastPos, j = 0; i > -1LL; i--, j++)
        v[j] = (n >> i) & 1LL;
    return v;
}
```

IO Optimization

```
int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}
```

Map Value To Int

```
// val = value
typedef string Val;
unordered_map<Val, int> intForVal;
unordered_map<int, Val> valForInt;
int mapId = 0;
```

```
int Map(Val val) {
    if (intForVal.count(val)) return intForVal[val];
    valForInt[mapId] = val;
    return intForVal[val] = mapId++;
}
```

```
Val IMap(int n) { return valForInt[n]; }
```

```
void initMapping() {
    mapId = 0;
    intForVal.clear();
    valForInt.clear();
}
```

Permutations

```
typedef vector<int> T; // typedef string T;

vector<T> permutations(T v) {
    vector<vector<int>>> ans;
    sort(v.begin(), v.end());
    do
        ans.push_back(v);
    while (next_permutation(v.begin(), v.end()));
    return ans;
}
```

Print Vector

```
void printv(vector<int> v) {
    if (v.size() == 0) {
        cout << "[]" << endl;
        return;
    }
    cout << "[" << v[0];
    for (int i = 1; i < v.size(); i++)
        cout << ", " << v[i];
    cout << "]" << endl;
}
```

Priority Queue Of Object

```
struct Object {
    char first;
    int second;
};

int main() {
    auto cmp = [](const Object& a, const Object& b) {
        return a.second > b.second;
    };
    priority_queue<Object, vector<Object>,
        decltype(cmp)> pq(cmp);
    vector<Object> v = {{'c', 3}, {'a', 1}, {'b', 2}};
    sort(v.begin(), v.end(), cmp);
    return 0;
}
```

Random

```
int random(int min, int max) {
    return min + rand() % (max - min + 1);
}

int main() {
    srand(time(0));
    // code ...
}
```

Read Line

```
// when reading lines, don't mix 'cin' with
// 'getline' just use getline and split
string input() {
    string ans;
    // cin >> ws; // eats all whitespaces.
    getline(cin, ans);
    return ans;
}
```

Sort Pair

```
pair<int, int> p;
// sorts array on the basis of the first element
sort(p.begin(), p.end());
```

Sort Vector Of Object

```
struct Object {
    char first;
    int second;
};

bool cmp(const Object& a, const Object& b) {
    return a.second > b.second;
}

int main() {
    vector<Object> v = {{'c', 3}, {'a', 1}, {'b', 2}};
    sort(v.begin(), v.end(), cmp);
    printv(v);
    return 0;
}
```

Split String

```
vector<string> split(string str, char token) {
    stringstream test(str);
    string seg;
    vector<string> seglist;
    while (getline(test, seg, token))
        seglist.push_back(seg);
    return seglist;
}
```

Typedef

```
typedef TYPE ALIAS;
// example:
typedef int T;
```

Python

Combinations

```
import itertools
# from arr choose k = > combinations(arr, k)
print(list(itertools.combinations([1, 2, 3], 3)))
```

Fast IO

```
from sys import stdin, stdout

N = 10
# Reads N chars from stdin(it counts '\n' as char)
stdin.read(N)
# Reads until '\n' or EOF
line = stdin.readline()
# Reads all lines in stdin until EOF
lines = stdin.readlines()
# Writes a string to stdout, it doesn't add '\n'
stdout.write(line)
# Writes a list of strings to stdout
stdout.writelines(lines)
# Reads numbers separated by space in a line
numbers = list(map(int, stdin.readline().split()))
```

Permutations

```
import itertools
print(list(itertools.permutations([1, 2, 3])))
```

Random

```
import random
# Initialize the random number generator.
random.seed(None)
# Returns a random integer N such that a <= N <= b.
random.randint(a, b)
# Returns a random integer N such that 0 <= N < b
random.randrange(b)
# Returns a random integer N such that a <= N < b.
random.randrange(a, b)
# Returns and integer with k random bits.
random.getrandbits(k)
# shuffles a list
random.shuffle(li)
```

Sort List

```
li = ['a', 'c', 'b']
# sorts inplace in descending order
li.sort(reverse=True)
# returns sorted list ascending order
ol = sorted(li)
```

Sort List Of Object

```
class MyObject :
    def __init__(self, first, second, third):
        self.first = first
        self.second = second
        self.third = third

li = [MyObject('b', 3, 1), MyObject('a', 3, 2),
      → MyObject('b', 3, 3)]
# returns list sorted by first then by second then by
→ third in increasing order
ol = sorted(li, key = lambda x: (x.first, x.second,
→ x.third), reverse=False)
# sorts inplace by first then by second then by third
→ in increasing order
li.sort(key = lambda x: (x.first, x.second, x.third),
→ reverse=False)
```

Data Structures

Geometry

K-D Tree

Graphs

UnionFind

```
struct UnionFind {
    int n;
    vector<int> dad, size;

    UnionFind(int N) : n(N), dad(N), size(N, 1) {
        while (N--) dad[N] = N;
    }
}
```

```
int root(int u) {
    if (dad[u] == u) return u;
    return dad[u] = root(dad[u]);
}

bool areConnected(int u, int v) {
    return root(u) == root(v);
}

void join(int u, int v) {
    int Ru = root(u), Rv = root(v);
    if (Ru == Rv) return;
    --n, dad[Ru] = Rv;
    size[Rv] += size[Ru];
}

int getSize(int u) { return size[root(u)]; }

int numberOfSets() { return n; }
};
```

Ranges

BIT

```
typedef long long int T;
T neutro = 0;
vector<T> bit;

void initVars(int n) { bit.assign(++n, neutro); }

T F(T a, T b) {
    return a + b;
    // return a * b;
}

// Inverse of F
T I(T a, T b) {
    return a - b;
    // return a / b;
}

// O(N)
void build() {
    for (int i = 1; i < bit.size(); i++) {
        int j = i + (i & -i);
        if (j < bit.size()) bit[j] = F(bit[j], bit[i]);
    }
}

// O(lg(N))
void update(int i, T val) {
    for (i++; i < bit.size(); i += i & -i)
        bit[i] = F(bit[i], val);
}
```

```
// O(lg(N))
T query(int i) {
    T ans = neutro;
    for (i++; i; i -= i & -i) ans = F(ans, bit[i]);
    return ans;
}

// O(lg(N)), [l, r]
T query(int l, int r) {
    return I(query(r), query(--l));
}

void setValAt(T val, int i) { bit[++i] = val; }
```

BIT Range Update

```
typedef long long int T;
T neutro = 0;
vector<T> bit1, bit2;

void initVars(int n) {
    bit1.assign(++n, neutro);
    bit2 = bit1;
}

// O(lg(N))
void update(vector<T> &bit, int i, T val) {
    for (i++; i < bit.size(); i += i & -i)
        bit[i] += val;
}

// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    update(bit1, l, val);
    update(bit1, r + 1, -val);
    update(bit2, r + 1, val * r);
    update(bit2, l, -val * (l - 1));
}

// O(lg(N))
T query(vector<T> &bit, int i) {
    T ans = neutro;
    for (i++; i; i -= i & -i) ans += bit[i];
    return ans;
}

// O(lg(N))
T query(int i) {
    return query(bit1, i) * i + query(bit2, i);
}

// O(lg(N)), [l, r]
T query(int l, int r) {
    return query(r) - query(l - 1);
}
```

Segment Tree

```
// st = segment tree. st[1] = root;
// neutro = operation neutral value
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.

template <class T>
struct SegmentTree {
    T neutro = 0;
    int N;
    vector<T> st;

    SegmentTree(int n) : st(2 * n, neutro), N(n) {}

    T F(T a, T b) {
        return a + b;
        // return __gcd(a, b);
        // return a * b;
        // return min(a, b);
    }

    // O(2N)
    void build() {
        for (int i = N - 1; i > 0; i--)
            st[i] = F(st[i << 1], st[i << 1 | 1]);
    }

    // O(lg(2N))
    void update(int i, T val) {
        for (st[i += N] = val; i > 1; i >>= 1)
            st[i >> 1] = F(st[i], st[i ^ 1]);
    }

    // O(3N), [l, r]
    void update(int l, int r, T val) {
        if (l == r)
            update(l, val);
        else {
            for (l += N, r += N; l <= r; l++) st[l] = val;
            build();
        }
    }

    // O(lg(2N)), [l, r]
    T query(int l, int r) {
        T ans = neutro;
        for (l += N, r += N; l <= r; l >>= 1, r >>= 1) {
            if (l & 1) ans = F(ans, st[l++]);
            if (~r & 1) ans = F(ans, st[r--]);
        }
        return ans;
    }

    void setValAt(T val, int i) { st[i + N] = val; }
};
```

Segment Tree Lazy Propagation

```
// st = segment tree, st[1] = root, H = height of d
// u = updates, d = delayed updates
// neutro = operation neutral val
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.

template <class T>
struct SegmentTree {
    T neutro = 0;
    int N, H;
    vector<T> st, d;
    vector<bool> u;

    SegmentTree(int n)
        : st(2 * n, neutro), d(n), u(n, 0) {
        H = sizeof(int) * 8 - __builtin_clz(N = n);
    }

    T F(T a, T b) {
        return a + b;
        // return __gcd(a, b);
        // return a * b;
        // return min(a, b);
    }

    void apply(int i, T val, int k) {
        st[i] = val * k; // sum
        // st[i] = val; // min, max, gcd
        // st[i] = pow(a, k); // multiplication
        if (i < N) d[i] = val, u[i] = 1;
    }

    void calc(int i) {
        if (!u[i]) st[i] = F(st[i << 1], st[i << 1 | 1]);
    }

    // O(2N)
    void build() {
        for (int i = N - 1; i > 0; i--) calc(i);
    }

    // O(lg(N))
    void build(int p) {
        while (p > 1) p >>= 1, calc(p);
    }

    // O(lg(N))
    void push(int p) {
        for (int s = H, k = 1 << (H - 1); s > 0;
             s--, k >>= 1) {
            int i = p >> s;
            if (u[i]) {
                apply(i << 1, d[i], k);
                apply(i << 1 | 1, d[i], k);
                u[i] = 0, d[i] = neutro;
            }
        }
    }
}
```

```
// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    push(l += N);
    push(r += N);
    int ll = l, rr = r, k = 1;
    for (; l <= r; l >>= 1, r >>= 1, k <<= 1) {
        if (l & 1) apply(l++, val, k);
        if (~r & 1) apply(r--, val, k);
    }
    build(ll);
    build(rr);
}

// O(lg(2N)), [l, r]
T query(int l, int r) {
    push(l += N);
    push(r += N);
    T ans = neutro;
    for (; l <= r; l >>= 1, r >>= 1) {
        if (l & 1) ans = F(ans, st[l++]);
        if (~r & 1) ans = F(ans, st[r--]);
    }
    return ans;
}

void setValAt(T val, int i) { st[i + N] = val; }
};
```

Sparse Table

```
// st = sparse table, Arith = Arithmetic
typedef int T;
int neutro = 0;
vector<vector<T>> st;

T F(T a, T b) {
    // return min(a, b);
    return __gcd(a, b);
    // return a + b; // Arith
    // return a * b; // Arith
}

// O(N lg(N))
void build(vector<T> &arr) {
    st.assign(log2(arr.size()), vector<T>(arr.size()));
    st[0] = arr;
    for (int i = 1; (1 << i) <= arr.size(); i++)
        for (int j = 0; j + (1 << i) <= arr.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

// O(1), [l, r]
T query(int l, int r) {
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]);
}
```



```
// O(lg(N)), [l, r]
T queryArith(int l, int r) {
    T ans = neutro;
    while (true) {
        int k = log2(r - l + 1);
        ans = F(ans, st[k][l]);
        l += 1 << k;
        if (l > r) break;
    }
    return ans;
}
```

Strings

Trie

```
// wpt = number of words passing through
// w = number of words ending in the node
// c = character
```

```
struct Trie {

    struct Node {
        // for lexicographical order use 'map'
        // map<char, Node *> ch;
        unordered_map<char, Node *> ch;
        int w = 0, wpt = 0;
    };

    Node *root = new Node();

    // O(STR.SIZE)
    void insert(string str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c))
                curr->ch[c] = new Node();
            curr->wpt++, curr = curr->ch[c];
        }
        curr->wpt++, curr->w++;
    }

    // O(STR.SIZE)
    Node *find(string &str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c)) return nullptr;
            curr = curr->ch[c];
        }
        return curr;
    }

    // O(STR.SIZE) number of words with given prefix
    int prefixCount(string prefix) {
        Node *node = find(prefix);
        return node ? node->wpt : 0;
    }
}
```

```
// O(STR.SIZE) number of words matching str
int strCount(string str) {
    Node *node = find(str);
    return node ? node->w : 0;
}

// O(N)
void getWords(Node *curr, vector<string> &words,
              string &word) {
    if (!curr) return;
    if (curr->w) words.push_back(word);
    for (auto &c : curr->ch) {
        getWords(c.second, words, word += c.first);
        word.pop_back();
    }
}
```

```
// O(N)
vector<string> getWords() {
    vector<string> words;
    string word = "";
    getWords(root, words, word);
    return words;
}
```

```
// O(N)
vector<string> getWordsByPrefix(string prefix) {
    vector<string> words;
    getWords(find(prefix), words, prefix);
}
```

```
// O(STR.SIZE)
bool remove(Node *curr, string &str, int &i) {
    if (i == str.size()) {
        curr->wpt--;
        return curr->w ? !(curr->w == 0) : 0;
    }
    int c = str[i];
    if (!curr->ch.count(c)) return false;
    if (remove(curr->ch[c], str, ++i)) {
        if (!curr->ch[c]->wpt)
            curr->wpt--, curr->ch.erase(c);
        return true;
    }
    return false;
}
```

```
// O(STR.SIZE)
int remove(string str) {
    int i = 0;
    return remove(root, str, i);
}
};
```

Trees And Heaps

Red Black Tree

```
template <class K, class V>
struct RedBlackTree {

    struct Node {
        K key;
        V val;
        Node *l, *r; // left, right
        bool isRed;
        Node(K k, V v, bool isRed)
            : key(k), val(v), isRed(isRed) {}
    };

    Node *root = nullptr;

    int compare(K a, K b) {
        if (a < b) return -1;
        if (a > b) return 1;
        return 0;
    }

    // O(lg(N))
    V at(K key) {
        Node *x = root;
        while (x) {
            int cmp = compare(key, x->key);
            if (!cmp) return x->val;
            if (cmp < 0) x = x->l;
            if (cmp > 0) x = x->r;
        }
        throw runtime_error("Key doesn't exist");
    }

    Node *rotateLeft(Node *h) {
        Node *x = h->r;
        h->r = x->l;
        x->l = h;
        x->isRed = h->isRed;
        h->isRed = 1;
        return x;
    }

    Node *rotateRight(Node *h) {
        Node *x = h->l;
        h->l = x->r;
        x->r = h;
        x->isRed = h->isRed;
        h->isRed = 1;
        return x;
    }

    void flipColors(Node *h) {
        h->isRed = 1;
        h->l->isRed = 0;
        h->r->isRed = 0;
    }
};
```

```
// O(lg(N))
Node *insert(Node *h, K key, V val) {
    if (!h) return new Node(key, val, 1);
    int cmp = compare(key, h->key);
    if (!cmp) h->val = val;
    if (cmp < 0) h->l = insert(h->l, key, val);
    if (cmp > 0) h->r = insert(h->r, key, val);
    if (h->r && h->r->isRed && !(h->l && h->l->isRed))
        h = rotateLeft(h);
    if (h->l && h->l->isRed && h->l->l &&
        h->l->l->isRed)
        h = rotateRight(h);
    if (h->l && h->l->isRed && h->r && h->r->isRed)
        flipColors(h);
    return h;
}

// O(lg(N))
void insert(K key, V val) {
    root = insert(root, key, val);
}
};
```

Treap

Geometry

Convex Hull

Max Interval Overlap

```
typedef long long int T;
typedef pair<T, T> Interval;
vector<Interval> maxIntervals;
```

```
// O(N * lg(N))
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    map<T, int> m;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (auto &i : arr) m[i.first]++, m[i.second + 1]--;
    for (auto &p : m) {
        curr += p.second;
        if (curr > maxI) maxI = curr, l = p.first;
        if (curr == maxI) r = p.first;
    }
    curr = 0;
    for (auto &p : m) {
        curr += p.second;
        if (curr == maxI && isFirst)
            l = p.first, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, p.first - 1}),
            isFirst = 1;
    }
    return maxI;
}

// O(MaxPoint) maxPoint < vector::max_size
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    T maxPoint = 0;
    for (auto &i : arr)
        if (i.second > maxPoint) maxPoint = i.second;
    vector<int> x(maxPoint + 2);
    for (auto &i : arr) x[i.first]++, x[i.second + 1]--;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr > maxI) maxI = curr;
    }
    curr = 0;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr == maxI && isFirst) l = i, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, i - 1}), isFirst = 1;
    }
    return maxI;
}
```

Graphs

Articulation Points And Bridges

```
// APB = articulation points and bridges
// ap = Articulation Point
// br = bridges, p = parent
// disc = discovery time
// low = lowTime, ch = children

typedef pair<int, int> Edge;
int Time;
vector<vector<int>>> ady;
vector<int> disc, low, ap;
vector<Edge> br;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

int dfsAPB(int u, int p) {
    int ch = 0;
    low[u] = disc[u] = ++Time;
    for (int &v : ady[u]) {
        if (v == p) continue;
        if (!disc[v]) {
            ch++, dfsAPB(v, u);
            if (disc[u] <= low[v]) ap[u]++;
            if (disc[u] < low[v]) br.push_back({u, v});
            low[u] = min(low[u], low[v]);
        } else
            low[u] = min(low[u], disc[v]);
    }
    return ch;
}

// O(N)
void APB() {
    br.clear();
    ap = low = disc = vector<int>(ady.size());
    Time = 0;
    for (int u = 0; u < ady.size(); u++)
        if (!disc[u]) ap[u] = dfsAPB(u, u) > 1;
}
```

Connected Components

```
// comp = component
int compId;
vector<vector<int>>> ady;
vector<int> getComp;
```

```

void initVars(int N) {
    ady.assign(N, vector<int>());
    getComp.assign(N, -1);
    compId = 0;
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

void dfsCC(int u, vector<int> &comp) {
    if (getComp[u] > -1) return;
    getComp[u] = compId;
    comp.push_back(u);
    for (auto &v : ady[u]) dfsCC(v, comp);
}

// O(N)
vector<vector<int>> connectedComponents() {
    vector<vector<int>> comps;
    for (int u = 0; u < ady.size(); u++) {
        vector<int> comp;
        dfsCC(u, comp);
        if (!comp.empty())
            comps.push_back(comp), compId++;
    }
    return comps;
}

```

Flood Fill

```

int n, m, oldColor = 0, color = 1;
vector<vector<int>> mat;
vector<vector<int>> movs = {
    {1, 0}, {0, 1}, {-1, 0}, {0, -1}};

void floodFill(int i, int j) {
    if (i >= mat.size() || i < 0 ||
        j >= mat[i].size() || j < 0 ||
        mat[i][j] != oldColor)
        return;
    mat[i][j] = color;
    for (auto move : movs)
        floodFill(i + move[1], j + move[0]);
}

void floodFill() {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            if (mat[i][j] == oldColor) floodFill(i, j);
}

```

Heavy Light Decomposition

```

// p = parent;
#include "../Data Structures/Ranges/Segment Tree.cpp"
typedef int T;
vector<vector<int>> ady;
vector<int> p, heavy, depth, root, stPos, vals;
SegmentTree<T> st(0);

```

```

void initVars(int n) {
    ady.assign(n, vector<int>());
    heavy.assign(n, -1);
    vals.assign(n, 0);
    p = root = stPos = depth = heavy;
    st = SegmentTree<T>(n);
}

void addEdge(int u, int v, T val) {
    ady[u].push_back(v);
    p[v] = u, vals[v] = val;
}

T F(T a, T b) { return a + b; }

// O(N)
int dfs(int u) {
    int size = 1, maxSubtree = 0;
    for (int &v : ady[u]) {
        depth[v] = depth[u] + 1;
        int subtree = dfs(v);
        if (subtree > maxSubtree)
            heavy[u] = v, maxSubtree = subtree;
        size += subtree;
    }
    return size;
}

// O(N)
void initHeavyLight() {
    for (int i = 0; i < ady.size(); i++)
        if (p[i] < 0) dfs(i);
    for (int i = 0, pos = 0; i < ady.size(); i++)
        if (p[i] < 0 || heavy[p[i]] != i)
            for (int j = i; ~j; j = heavy[j]) {
                st.setValAt(vals[j], stPos[j] = pos++);
                root[j] = i;
            }
    st.build();
}

// O(lg2 N)
template <class Op>
void processPath(int u, int v, Op op) {
    for (; root[u] != root[v]; v = p[root[v]]) {
        if (depth[root[u]] > depth[root[v]]) swap(u, v);
        op(stPos[root[v]], stPos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    // for values on edges
    if (u != v) op(stPos[u] + 1, stPos[v]);
    // for values on nodes
    // op(stPos[u], stPos[v]);
}

// O(lg2 N)
void update(int u, int v, T val) {
    processPath(u, v, [&val](int l, int r) {
        st.update(l, r, val);
    });
}

```

```
// O(lg2(N))
T query(int u, int v) {
    T ans = T();
    processPath(u, v, [&ans](int l, int r) {
        ans = F(ans, st.query(l, r));
    });
    return ans;
}
```

Is Bipartite

```
vector<vector<int>>> ady;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

// O(N)
bool isBipartite() {
    vector<int> color(ady.size(), -1);
    for (int s = 0; s < ady.size(); s++) {
        if (color[s] > -1) continue;
        color[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int &v : ady[u]) {
                if (color[v] < 0)
                    q.push(v), color[v] = !color[u];
                if (color[v] == color[u]) return false;
            }
        }
    }
    return true;
}
```

LCA

```
// st = sparse table
typedef pair<int, int> T;
int neutro = 0;
vector<vector<T>>> st;
vector<int> first;
vector<T> tour;
vector<vector<int>>> ady;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    ady[u].push_back(v);
    ady[v].push_back(u);
}

T F(T a, T b) { return a.first < b.first ? a : b; }
```

```
void build() {
    st.assign(log2(tour.size()),
              vector<T>(tour.size()));
    st[0] = tour;
    for (int i = 1; (1 << i) <= tour.size(); i++)
        for (int j = 0; j + (1 << i) <= tour.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

void eulerTour(int u, int p, int h) {
    first[u] = tour.size();
    tour.push_back({h, u});
    for (int v : ady[u])
        if (v != p) {
            eulerTour(v, u, h + 1);
            tour.push_back({h, u});
        }
}

// O(N * lg(N))
void preprocess() {
    tour.clear();
    first.assign(ady.size(), -1);
    eulerTour(0, 0, 0);
    build();
}

// O(1)
int lca(int u, int v) {
    int l = min(first[u], first[v]);
    int r = max(first[u], first[v]);
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]).second;
}
```

MST Kruskal

```
// N = number of nodes, Wedge = Weighted Edge
#include "../Data Structures/Graphs/UnionFind.cpp"
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
vector<Wedge> Wedges;
vector<Wedge> mst;
UnionFind uf(0);

void initVars(int N) {
    mst.clear();
    Wedges.clear();
    uf = UnionFind(N);
}

void addEdge(int u, int v, T w) {
    Wedges.push_back({w, {u, v}});
}
```

```

T kruskal() {
    T cost = 0;
    sort(Wedges.begin(), Wedges.end());
    // reverse(Wedges.begin(), Wedges.end());
    for (Wedge &wedge : Wedges) {
        int u = wedge.second.first,
            v = wedge.second.second;
        if (!uf.areConnected(u, v))
            uf.join(u, v), mst.push_back(wedge),
            cost += wedge.first;
    }
    return cost;
}

```

MST Prim

```

// st = spanning tree, p = parent
// vis = visited, dist = distance
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
typedef pair<T, int> DistNode;
int INF = 1 << 30;
vector<vector<int>> ady;
unordered_map<int, unordered_map<int, T>> weight;
vector<int> p, vis;
vector<T> dist;
vector<vector<Wedge>> msts;

void initVars(int N) {
    ady.assign(N, vector<int>());
    p.assign(N, 0);
    vis.assign(N, 0);
    dist.assign(N, INF);
    weight.clear();
    msts.clear();
}

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    ady[v].push_back(u);
    weight[v][u] = w;
}

```

```

// O(E * log(V))
T prim(int s) {
    vector<Wedge> mst;
    vector<set<Edge>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    set<Edge> q;
    T cost = dist[s] = 0;
    q.insert({0, s});
    while (q.size()) {
        int u = q.begin()->second;
        q.erase(q.begin());
        vis[u] = 1, cost += dist[u];
        mst.push_back({dist[u], {p[u], u}});
        for (int &v : ady[u]) {
            T w = weight[u][v];
            if (!vis[v] && w < dist[v]) {
                if (dist[v] != INF) q.erase(pos[v]);
                pos[v] = q.insert({dist[v] = w, v}).first;
            }
        }
    }
    msts.push_back(
        vector<Wedge>(mst.begin() + 1, mst.end()));
    return cost;
}

// ~ O(E * log(V))
T primLazy(int s) {
    vector<Wedge> mst;
    vector<set<Edge>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    priority_queue<DistNode> q;
    T cost = dist[s] = 0;
    q.push({0, s});
    while (q.size()) {
        pair<int, int> aux = q.top();
        int u = aux.second;
        q.pop();
        if (dist[u] < -aux.first) continue;
        vis[u] = 1, cost += dist[u];
        mst.push_back({dist[u], {p[u], u}});
        for (int &v : ady[u]) {
            T w = weight[u][v];
            if (!vis[v] && w < dist[v])
                q.push({-(dist[v] = w), v});
        }
    }
    msts.push_back(
        vector<Wedge>(mst.begin() + 1, mst.end()));
    return cost;
}

// O(V + E * log(V))
T prim() {
    T cost = 0;
    map<int, T> q;
    for (int i = 0; i < ady.size(); i++)
        if (!vis[i]) cost += prim(i);
    return cost;
}

```

Strongly Connected Components

```
// tv = top value from stack
// sccs = strongly connected components
// scc = strongly connected component
// disc = discovery time, low = low time
// s = stack, top = top index of the stack

int Time, top;
vector<vector<int>> ady, sccs;
vector<int> disc, low, s;

void initVars(int N) { ady.assign(N, vector<int>()); }

void addEdge(int u, int v) { ady[u].push_back(v); }

void dfsSCCS(int u) {
    if (disc[u]) return;
    low[u] = disc[u] = ++Time;
    s[++top] = u;
    for (int &v : ady[u])
        dfsSCCS(v), low[u] = min(low[u], low[v]);
    if (disc[u] == low[u]) {
        vector<int> scc;
        while (true) {
            int tv = s[top--];
            scc.push_back(tv);
            low[tv] = ady.size();
            if (tv == u) break;
        }
        sccs.push_back(scc);
    }
}

// O(N)
void SCCS() {
    s = low = disc = vector<int>(ady.size());
    Time = 0, top = -1, sccs.clear();
    for (int u = 0; u < ady.size(); u++) dfsSCCS(u);
}
```

Topological Sort

```
// vis = visited
vector<vector<int>> ady;
vector<int> vis, toposorted;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
}

void addEdge(int u, int v) { ady[u].push_back(v); }
```

```
// returns false if there is a cycle
// O(N)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}

// O(N)
bool toposort() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u] && !toposort(u)) return false;
    return true;
}
```

Topological Sort (All possible sorts)

Cycles

Get All Simple Cycles

Get Some Cycles

```
// at least detects one cycle per component
vector<vector<int>> ady, cycles;
vector<int> vis, cycle;
bool flag = false, isDirected = false;
int root = -1;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    cycles.clear();
    root = -1, flag = false;
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    if (!isDirected) ady[v].push_back(u);
}
```

```
// O(N)
bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : ady[u]) {
        if (v == u || vis[v] == 2 ||
            (!isDirected && v == prev))
            continue;
        if (flag) {
            if (!vis[v]) hasCycle(v, u);
            continue;
        }
        if (vis[v] || hasCycle(v, u)) {
            if (root == -1) root = v, flag = true;
            cycle.push_back(u);
            if (root == u)
                flag = false, root = -1,
                cycles.push_back(cycle), cycle.clear();
        }
    }
    vis[u] = 2;
    return flag;
}

// O(N)
bool hasCycle() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u]) cycle.clear(), hasCycle(u, -1);
    return cycles.size() > 0;
}
```

Has Cycle

```
vector<vector<int>>> ady;
vector<int> vis;
bool isDirected = false;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
}

void addEdge(int u, int v) {
    ady[u].push_back(v);
    if (!isDirected) ady[v].push_back(u);
}

bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (isDirected || v != prev) &&
            (vis[v] || hasCycle(v, u)))
            return true;
    vis[u] = 2;
    return false;
}
```

```
// O(N)
bool hasCycle() {
    for (int u = 0; u < ady.size(); u++)
        if (!vis[u] && hasCycle(u, -1)) return true;
}
```

Flow

Max Flow Dinic

```
// cap[a][b] = Capacity from a to b
// flow[a][b] = flow occupied from a to b
// level[a] = level in graph of node a

typedef int T;
vector<int> level;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>>> cap, flow;

void initVars(int N) {
    ady.assign(N, vector<int>());
    cap.clear();
    flow.clear();
}

void addEdge(int u, int v, T capacity) {
    cap[u][v] = capacity;
    ady[u].push_back(v);
}

bool levelGraph(int s, int t) {
    level = vector<int>(ady.size());
    level[s] = 1;
    queue<int> q;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int &v : ady[u]) {
            if (!level[v] && flow[u][v] < cap[u][v]) {
                q.push(v);
                level[v] = level[u] + 1;
            }
        }
    }
    return level[t];
}
```



```

T blockingFlow(int u, int t, T currPathMaxFlow) {
    if (u == t) return currPathMaxFlow;
    for (int v : ady[u]) {
        T capleft = cap[u][v] - flow[u][v];
        if ((level[v] == (level[u] + 1)) &&
            (capleft > 0)) {
            T pathMaxFlow = blockingFlow(
                v, t, min(currPathMaxFlow, capleft));
            if (pathMaxFlow > 0) {
                flow[u][v] += pathMaxFlow;
                flow[v][u] -= pathMaxFlow;
                return pathMaxFlow;
            }
        }
    }
    return 0;
}

// O(E * V^2)
T dinicMaxFlow(int s, int t) {
    if (s == t) return -1;
    T maxFlow = 0;
    while (levelGraph(s, t))
        while (T flow = blockingFlow(s, t, 1 << 30))
            maxFlow += flow;
    return maxFlow;
}

```

Maximum Bipartite Matching

```

// mbm = maximum bipartite matching
#include "Max Flow Dinic.cpp"

void addEdgeMBM(int u, int v) {
    addEdge(u += 2, v += 2, 1);
    addEdge(0, u, 1);
    addEdge(v, 1, 1);
}

// O(E * V^2)
T mbm() { return dinicMaxFlow(0, 1); }

```

ShortestPaths

Bellman Ford

```

// N = number of nodes
// returns {} if there is a negative weight cycle
typedef int T;
int MAXN = 20001, N, INF = 1 << 30, isDirected = true;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    weight.clear();
}

```

```

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    ady[v].push_back(u);
    weight[v][u] = w;
}

// O(V * E)
vector<T> bellmanFord(int s) {
    vector<T> dist(ady.size(), INF);
    dist[s] = 0;
    for (int i = 1; i <= ady.size(); i++)
        for (int u = 0; u < ady.size(); u++)
            for (auto &v : ady[u]) {
                T w = weight[u][v], d = dist[u] + w;
                if (dist[u] != INF && d < dist[v]) {
                    if (i == ady.size()) return {};
                    dist[v] = d;
                }
            }
    return dist;
}

```

Dijkstra

```

#include <bits/stdc++.h>
using namespace std;

typedef int T;
typedef pair<T, int> DistNode;
int INF = 1 << 30, isDirected = false;
vector<vector<int>>> ady;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w) {
    ady[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    ady[v].push_back(u);
    weight[v][u] = w;
}

```

```
// O(E * lg(V))
vector<T> dijkstra(int s) {
    vector<set<DistNode>::iterator> pos(ady.size());
    vector<T> dist(ady.size(), INF);
    set<DistNode> q;
    q.insert({0, s}); dist[s] = 0;
    while (q.size()) {
        int u = q.begin()->second;
        q.erase(q.begin());
        for (int &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) {
                if (dist[v] != INF) q.erase(pos[v]);
                pos[v] = q.insert({dist[v] = d, v}).first;
            }
        }
    }
    return dist;
}

// ~ O(E * lg(V))
vector<T> dijkstraLazy(int s) {
    vector<T> dist(ady.size(), INF);
    priority_queue<DistNode> q;
    q.push({0, s}); dist[s] = 0;
    while (q.size()) {
        DistNode top = q.top();
        q.pop();
        int u = top.second;
        if (dist[u] < -top.first) continue;
        for (int &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) q.push({-(dist[v] = d), v});
        }
    }
    return dist;
}
```

Shortest Path in Directed Acyclic Graph

```
// vis = visited
typedef int T;
vector<vector<int>> ady;
vector<int> vis, toposorted;
int INF = 1 << 30;
unordered_map<int, unordered_map<int, T>> weight;

void initVars(int N) {
    ady.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
    weight.clear();
}

void addEdge(int u, int v, int w) {
    ady[u].push_back(v);
    weight[u][v] = w;
}
```

```
// returns false if there is a cycle
// O(N)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : ady[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}

// O(N)
vector<T> sssp(int s) {
    vector<T> dist(ady.size(), INF);
    dist[s] = 0;
    toposort(s);
    while (toposorted.size()) {
        int u = toposorted.back();
        toposorted.pop_back();
        for (auto &v : ady[u]) {
            T w = weight[u][v], d = dist[u] + w;
            if (d < dist[v]) dist[v] = d;
        }
    }
    return dist;
}
```

Maths

Number Theory

Divisibility Criterion

```
def divisorCriteria(n, lim):
    results = []
    tenElevated = 1
    for i in range(lim):
        # remainder = pow(10, i, n)
        remainder = tenElevated % n
        negremainder = remainder - n
        if (remainder <= abs(negremainder)):
            results.append(remainder)
        else:
            results.append(negremainder)
        tenElevated *= 10
    return results
```

```
def testDivisibility(dividend, divisor,
    ↪ divisor_criteria):
    dividend = str(dividend)
    addition = 0
    dividendSize = len(dividend)
    i = dividendSize - 1
    j = 0
    while j < dividendSize:
        addition += int(dividend[i]) *
            ↪ divisor_criteria[j]
        i -= 1
        j += 1
    return addition % divisor == 0

if __name__ == '__main__':
    dividend, divisor = map(int, input().split())
    divisor_criteria = divisorCriteria(divisor,
    ↪ len(str(dividend)))
    print(divisor_criteria)
    print(testDivisibility(dividend, divisor,
    ↪ divisor_criteria))
```

Extended Euclidean

```
// gcd(a, b) = ax + by
vector<long long int> extendedGCD(long long int a,
                                long long int b) {
    if (a > 0LL && b == 0LL) return {a, 1LL, 0LL};
    long long int x = 1LL, y = 0LL, prevx = 0LL,
        prevy = 1LL, q, remainder;
    while (true) {
        q = a / b;
        remainder = a - b * q;
        if (remainder == 0LL) break;
        a = b;
        b = remainder;
        x = x - prevx * q;
        swap(x, prevx);
        y = y - prevy * q;
        swap(y, prevy);
    }
    // gcd = b, x = prevx, y = prevy
    return {b, prevx, prevy};
}
```

GCD

```
// recursive
int gcd(int a, int b) {
    return !b ? a : gcd(b, a % b);
}
```

```
// iterative
int gcd(int a, int b) {
    while (b) {
        a %= b;
        swap(a, b);
    }
    return a;
}
```

LCM

```
int lcm(int a, int b) {
    int c = gcd(a, b);
    return c ? a / c * b : 0;
}
```

Prime Check Miller Rabin

```
from random import randrange
```

```
def is_prime(p):
    k = 100
    if p == 2 or p == 3:
        return True
    if (p & 1) == 0 or p == 1:
        return False
    phi = p - 1
    d = phi
    r = 0
    while (d & 1) == 0:
        d = int(d >> 1)
        r += 1
    for i in range(k):
        a = randrange(2, p - 2)
        exp = pow(a, d, p)
        if exp == 1 or exp == p - 1:
            continue
        flag = False
        for j in range(r - 1):
            exp = pow(exp, 2, p)
            if exp == 1:
                return False
            if exp == p - 1:
                flag = True
                break
        if flag:
            continue
        else:
            return False
    return True
```

Prime Sieve

```
vector<int> primeSieve(int n) {
    vector<int> sieve(n + 1);
    for (int i = 4; i <= n; i += 2) sieve[i] = 2;
    for (int i = 3; i * i <= n; i += 2)
        if (!sieve[i])
            for (int j = i * i; j <= n; j += 2 * i)
                if (!sieve[j]) sieve[j] = i;
    return sieve;
}
```

Strings

KMP

```
// p = pattern, t = text
// f = error function, cf = create error function
// pos = positions where pattern is found in text
```

```
int MAXN = 1000000;
vector<int> f(MAXN + 1);

vector<int> kmp(string &p, string &t, int cf) {
    vector<int> pos;
    if (cf) f[0] = -1;
    for (int i = cf, j = 0; j < t.size(); i++) {
        while (i > -1 && p[i] != t[j]) i = f[i];
        i++; j++;
        if (cf) f[j] = i;
        if (!cf && i == p.size())
            pos.push_back(j - i), i = f[i];
    }
    return pos;
}
```

```
vector<int> search(string &p, string &t) {
    kmp(p, p, -1); // create error function
    return kmp(p, t, 0); // search in text
}
```

Rabin Karp

```
class RollingHash {
public:
    vector<unsigned long long int> pow;
    vector<unsigned long long int> hash;
    unsigned long long int B;
```

```
RollingHash(const string &text) : B(257) {
    int N = text.size();
    pow.resize(N + 1);
    hash.resize(N + 1);
    pow[0] = 1;
    hash[0] = 0;
    for (int i = 1; i <= N; ++i) {
        // in c++ an unsigned long long int is
        // automatically modulated by 2^64
        pow[i] = pow[i - 1] * B;
        hash[i] = hash[i - 1] * B + text[i - 1];
    }
}

unsigned long long int getWordHash() {
    return hash[hash.size() - 1];
}

unsigned long long int getSubstrHash(int begin,
                                     int end) {
    return hash[end] -
           hash[begin - 1] * pow[end - begin + 1];
}

int size() { return hash.size(); }
};

vector<int> rabinKarp(RollingHash &rhStr,
                     string &pattern) {
    vector<int> positions;
    RollingHash rhPattern(pattern);
    unsigned long long int patternHash =
        rhPattern.getWordHash();
    int windowSize = pattern.size(), end = windowSize;
    for (int i = 1; end < rhStr.size(); i++) {
        if (patternHash == rhStr.getSubstrHash(i, end))
            positions.push_back(i);
        end = i + windowSize;
    }
    return positions;
}
```

Techniques

Binary Search

Multiple Queries

Mo

```
// q = query
// qs = queries
```

```

struct Query {
    int l, r;
};

int blksize;
vector<Query> qs;
vector<int> arr;

void initVars(int N, int M) {
    arr = vector<int>(N);
    qs = vector<Query>(M);
}

bool cmp(Query &a, Query &b) {
    if (a.l == b.l) return a.r < b.r;
    return a.l / blksize < b.l / blksize;
}

void getResults() {
    blksize = (int)sqrt(arr.size());
    sort(qs.begin(), qs.end(), cmp);
    int prevL = 0, prevR = -1;
    int sum = 0;
    for (auto &q : qs) {
        int L = q.l, R = q.r;
        while (prevL < L) {
            sum -= arr[prevL]; // problem specific
            prevL++;
        }
        while (prevL > L) {
            prevL--;
            sum += arr[prevL]; // problem specific
        }
        while (prevR < R) {
            prevR++;
            sum += arr[prevR]; // problem specific
        }
        while (prevR > R) {
            sum -= arr[prevR]; // problem specific
            prevR--;
        }

        cout << "sum[" << L << ", " << R << "] = " << sum
              << endl;
    }
}

int main() {
    initVars(9, 2);
    arr = {1, 1, 2, 1, 3, 4, 5, 2, 8};
    qs = {{0, 8}, {3, 5}};
    getResults();
}

```

SQRT Decomposition

```

// sum of elements in range
int neutro = 0;
vector<int> arr;
vector<int> blks;

```

```

void initVars(int n) {
    arr.assign(n, neutro);
    blks.assign(sqrt(n), neutro);
}

void preprocess() {
    for (int i = 0, j = 0; i < arr.size(); i++) {
        if (i == blks.size() * j) j++;
        blks[j - 1] += arr[i]; // problem specific
    }
}

// problem specific
void update(int i, int val) {
    blks[i / blks.size()] += val - arr[i];
    arr[i] = val;
}

int query(int l, int r) {
    int sum = 0;
    int lblk = l / blks.size();
    if (l != blks.size() * lblk++) {
        while (l < r && l != lblk * blks.size()) {
            sum += arr[l]; // problem specific
            l++;
        }

        while (l + blks.size() <= r) {
            sum += blks[l / blks.size()]; // problem specific
            l += blks.size();
        }
        while (l <= r) {
            sum += arr[l]; // problem specific
            l++;
        }
        return sum;
    }
}

int main() {
    initVars(10);
    arr = {1, 5, 2, 4, 6, 1, 3, 5, 7, 10};
    preprocess();
    for (int i = 0; i < blks.size() + 1; i++)
        cout << blks[i] << " ";
    // output: 8 11 15 10
    cout << endl;
    cout << query(3, 8) << " ";
    cout << query(1, 6) << " ";
    update(8, 0);
    cout << query(8, 8) << endl;
    // output: 26 21 0
    return 0;
}

```