

Competitive Programming Reference

First, solve the problem. Then, write the code.

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Coding Resources

C++

Competitive Programming Template

```

/*****
https://searleser97.gitlab.io/algorithms/template.cpp
*****/

#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define forr(_, x, n) for (int _ = x; _ > n; _--)
#define fos(_, x, n, s) for (int _ = x; _ < n; _ += s)
#define forn(_, x, n) fos(_, x, n, 1)
#define rep(_, n) forn(_, 0, n)
#define fi first
#define se second
#define pb push_back
#define pairii pair<int, int>
#define cerr(s) cerr << "\033[48;5;196m\033[38;5;15m"
↳ << s << "\033[0m"
// typedef __int128_t lli;
typedef long long int li;
typedef long double ld;

void _main(int tc) {

}

int main() {
    ios_base::sync_with_stdio(0), cin.tie(0);
    _main(0); return 0;
    int tc;
    cin >> tc;
    rep(i, tc) _main(i + 1);
}

```

Decimal Precision

```

// rounds up the decimal number
cout << setprecision(N) << n << endl;
// specify N fixed number of decimals
cout << fixed << setprecision(N) << n << endl;

```

Double Comparisons With Given Precision

```

typedef long double ld;
const ld eps = 1e-9;

bool eq(ld a, ld b) { return abs(a - b) <= eps; }
bool neq(ld a, ld b) { return abs(a - b) > eps; }
bool gr(ld a, ld b) { return a - b > eps; }
bool le(ld a, ld b) { return b - a > eps; }
bool geq(ld a, ld b) { return a - b >= -eps; }
bool leq(ld a, ld b) { return b - a >= -eps; }

```

Include All Libraries

```

#include <bits/stdc++.h>
using namespace std;

```

IO Optimization

```

int main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
}

```

Map Value To Int

```

// val = value
typedef string Val;
unordered_map<Val, int> intForVal;
unordered_map<int, Val> valForInt;
int mapId = 0;
int Map(Val val) {
    if (intForVal.count(val)) return intForVal[val];
    valForInt[mapId] = val;
    return intForVal[val] = mapId++;
}

Val IMap(int n) { return valForInt[n]; }
void initMapping() {
    mapId = 0;
    intForVal.clear();
    valForInt.clear();
}

```

Number To String

```

#include <bits/stdc++.h>
using namespace std;

int main() {
    // to_string method converts any type of number
    // (int, double, long long int, ...) to string
    string str = "str+" + to_string(123 + 1);
    cout << str << endl; // output: str+124
    return 0;
}

```

Permutations

```
typedef vector<int> T; // typedef string T;

vector<T> permutations(T v) {
    vector<vector<int>> ans;
    sort(v.begin(), v.end());
    do
        ans.push_back(v);
    while (next_permutation(v.begin(), v.end()));
    return ans;
}
```

Print Vector

```
void printv(vector<int> v) {
    if (v.size() == 0) {
        cout << "[]" << endl;
        return;
    }
    cout << "[" << v[0];
    for (int i = 1; i < v.size(); i++)
        cout << ", " << v[i];
    cout << "]" << endl;
}
```

Priority Queue Of Object

```
struct Object {
    char first;
    int second;
};

int main() {
    auto cmp = [](const Object& a, const Object& b) {
        return a.second > b.second;
    };
    priority_queue<Object, vector<Object>,
        decltype(cmp)>
        pq(cmp);
}
```

Random

```
mt19937_64 seed(chrono::steady_clock::now()
    .time_since_epoch()
    .count());

int random(int min, int max) { // [min, max]
    return uniform_int_distribution<int>(min,
        max)(seed);
}

double random(double min, double max) { // [min, max)
    return uniform_real_distribution<double>(min,
        max)(seed);
}
```

Read Line

```
// when reading lines, don't mix 'cin' with
// 'getline' just use getline and split
string input() {
    string ans;
    cin >> ws;
    cin.ignore(numeric_limits<streamsize>::max(), '\n');
    getline(cin, ans);
    return ans;
}
```

Set of Object

```
struct Object {
    char first;
    int second;
};

int main() {
    auto cmp = [](const Object& a, const Object& b) {
        return a.second > b.second;
    };
    set<Object, decltype(cmp)> pq(cmp);
}
```

Sort Vector Of Object

```
struct Object {
    char first;
    int second;
};

bool cmp(const Object& a, const Object& b) {
    return a.second > b.second;
}

int main() {
    vector<Object> v = {{'c', 3}, {'a', 1}, {'b', 2}};
    sort(v.begin(), v.end(), cmp);
}
```

Sort Vector of Pairs

```
vector<pair<int, int>> pairs;
// sorts array on the basis of the first element
sort(pairs.begin(), pairs.end());
```

Split String

```
vector<string> split(string str, char token) {
    stringstream ss(str);
    vector<string> v;
    while (getline(ss, str, token)) v.push_back(str);
    return v;
}
```

String To Int

```
#include <bits/stdc++.h>
using namespace std;

int main() {
    int n = stoi("123") + 1;
    cout << n << endl; // output: 124
    // stoll for long long int
    // stoull for unsigned long int
    // stod for double
    // stold for long double
}
```

Typedef

```
typedef TYPE ALIAS;
// example:
typedef int T;
```

Unordered Map with pair as key

```
typedef int T;

struct pairhash {
    template <class T1, class T2>
    size_t operator()(const pair<T1, T2> &p) const {
        return hash<T1>{}(p.first) ^
            (hash<T2>{}(p.second) << 32);
    }
};

int main() {
    unordered_map<pair<int, int>, T, hash_pair> um;
    um[{1, 2}] = 5;
    cout << um[{1, 2}] << endl;
}
```

Python

Combinations

```
import itertools
# from arr choose k => combinations(arr, k)
print(list(itertools.combinations([1, 2, 3], 3)))
```

Fast IO

```
from sys import stdin, stdout

N = 10
# Reads N chars from stdin(it counts '\n' as char)
stdin.read(N)
# Reads until '\n' or EOF
line = stdin.readline()
# Reads all lines in stdin until EOF
lines = stdin.readlines()
# Writes a string to stdout, it doesn't add '\n'
stdout.write(line)
# Writes a list of strings to stdout
stdout.writelines(lines)
# Reads numbers separated by space in a line
numbers = list(map(int, stdin.readline().split()))
```

Permutations

```
import itertools
print(list(itertools.permutations([1, 2, 3])))
```

Random

```
import random
# Initialize the random number generator.
random.seed(None)
# Returns a random integer N such that a <= N <= b.
random.randint(a, b)
# Returns a random integer N such that 0 <= N < b
random.randrange(b)
# Returns a random integer N such that a <= N < b.
random.randrange(a, b)
# Returns and integer with k random bits.
random.getrandbits(k)
# shuffles a list
random.shuffle(li)
```

Sort List

```
li = ['a', 'c', 'b']
# sorts inplace in descending order
li.sort(reverse=True)
# returns sorted list ascending order
ol = sorted(li)
```

Sort List Of Object

```
class MyObject :
    def __init__(self, first, second, third):
        self.first = first
        self.second = second
        self.third = third

li = [MyObject('b', 3, 1), MyObject('a', 3, 2),
      ↪ MyObject('b', 3, 3)]
# returns list sorted by first then by second then by
↪ third in increasing order
ol = sorted(li, key = lambda x: (x.first, x.second,
↪ x.third), reverse=False)
# sorts inplace by first then by second then by third
↪ in increasing order
li.sort(key = lambda x: (x.first, x.second, x.third),
↪ reverse=False)
```

BITS Manipulation

Bit Count

```
int bitCount(int n) {
    return sizeof(n) * 8 - __builtin_clz(n);
}
```

```
int bitCount(int n) {
    int c = 0;
    while (n) c++, n >>= 1;
    return c;
}
```

Bits To Int

```
typedef __int128_t lli

lli bitsToInt(string bits, bool isneg) {
    lli ans = 0;
    for (int i = bits.size() - 1, j = 0; ~i; i--, j++) {
        if (isneg) bits[i] = bits[i] == '0' ? '1' : '0';
        ans |= (lli)(bits[i] - '0') << j;
    }
    return isneg ? -(++ans) : ans;
}
```

Count Leading Zeroes

```
int clz(int n) {
    return __builtin_clz(n);
    // return __builtin_clzl(n); for long
    // return __builtin_clzll(n); for long long
}
```

```
int clz(int n) {
    // return sizeof(n) * 8 - bitCount(n);
    int c = 0;
    while (n) c++, n >>= 1;
    return sizeof(n) * 8 - c;
}
```

Count Set Bits

```
int popCount(int n) {
    return __builtin_popcount(n);
    // return __builtin_popcountl(n); for long
    // return __builtin_popcountll(n); for long long
}
```

```
int popCount(int n) {
    int c = 0;
    while (n) c++, n &= n - 1;
    return c;
}
```

Count Trailing Zeroes

```
int ctz(int n) {
    return __builtin_ctz(n);
    // return __builtin_ctzl(n); for long
    // return __builtin_ctzll(n); for long long
}
```

```
int ctz(int n) {
    int c = 0;
    n = ~n;
    while(n & 1) c++, n >>= 1;
    return c;
}
```

Divide By 2

```
int divideBy2(int n) { return n >> 1; }
```

Is Even

```
bool isEven(int n) { return ~n & 1; }
```

Is i-th Bit Set

```
bool isIthBitSet(int n, int i) {
    return n & (1 << i);
}
```

Is Odd

```
bool isOdd(int n) { return n & 1; }
```


Is Power Of 2

```
bool isPowerOf2(int n) { return n && !(n & (n - 1)); }
```

Least Significant Set Bit

```
int lsb(int n) { return n & -n; }
```

Log2

```
int Log2(int n) {
    return sizeof(n) * 8 - __builtin_clz(n) - 1;
}
```

```
int Log2(int n) {
    int lg2 = 0;
    while (n >= 1) lg2++;
    return lg2;
}
```

Modulo With Power Of 2

```
// b must be a power of 2
int mod(int a, int b) { return a & (b - 1); }
```

Most Significant Set Bit

```
int msb(int n) {
    return 1 << (sizeof(n) * 8 - __builtin_clz(n) - 1);
}
```

Multiply By 2

```
int multiplyBy2(int n) { return n << 1; }
```

One's Complement

```
int onesComplement(int n) { return ~n; }
```

Parity Check

```
#include "Count Set Bits.cpp"
#include "Is Even.cpp"
bool parityCheck(int n) {
    return !__builtin_parity(n);
    // return !__builtin_parityl(n); for long
    // return !__builtin_parityll(n); for long long
}

bool parityCheck(int n) {
    return isEven(popCount(n));
}
```

Print Bits

```
void printBits(int n) {
    for (int i = sizeof(n) * 8 - 1; ~i; i--)
        cout << ((n >> i) & 1);
    cout << endl;
}
```

Set i-th Bit

```
int setIthBit(int n, int i) { return n | (1 << i); }
```

Swap Integer Variables

```
void swap(int &a, int &b) {
    a ^= b;
    b ^= a;
    a ^= b;
}
```

To Lower Case

```
char lowerCase(char c) { return c | ' '; }
```

To Upper Case

```
char upperCase(char c) { return c & '_'; }
```

Toggle Case

```
char toggleCase(char c) { return c ^ ' '; }
```

Toggle i-th Bit

```
int toggleIthBit(int n, int i) {
    return n ^ (1 << i);
}
```

Two's Complement

```
int twosComplement(int n) { return ~n + 1; }
```

Unset i-th Bit

```
int unsetIthBit(int n, int i) {
    return n & ~(1 << i);
}
```

XOR From 1 To N

```
int xorToN(int n) {
    switch (n & 3) {
        case 0: return n;
        case 1: return 1;
        case 2: return n + 1;
        case 3: return 0;
    }
}
```

Geometry

Data Structures

Circle

```
// c = center, r = radius;
#include "Point.cpp"
```

```
struct Circle {
    Point c;
    ld r;
    Circle(Point c, ld r) : c(c), r(r) {}
};
```

Point

```
#include "../Coding Resources/C++/Double
↳ Comparisons With Given Precision.cpp"
```

```
const ld pi = acos(-1), inf = 1 << 30;
// (PointB - PointA) = vector from A to B.
struct Point {
    ld x, y;
    Point() : x(0), y(0) {}
    Point(ld x, ld y) : x(x), y(y) {}

    Point operator+(const Point &p) const {
        return Point(x + p.x, y + p.y);
    }

    Point operator-(const Point &p) const {
        return Point(x - p.x, y - p.y);
    }

    Point operator*(const ld &k) const {
        return Point(x * k, y * k);
    }

    Point operator/(const ld &k) const {
        return Point(x / k, y / k);
    }
}
```

```
bool operator==(const Point &p) const {
    return eq(x, p.x) && eq(y, p.y);
}

bool operator!=(const Point &p) const {
    return !(*this == p);
}

bool operator<(const Point &p) const {
    return eq(x, p.x) ? le(y, p.y) : le(x, p.x);
}

bool operator>(const Point &p) const {
    return eq(x, p.x) ? gr(y, p.y) : gr(x, p.x);
}

ld norm() const { return sqrt(x * x + y * y); }

ld dot(const Point &p) { return x * p.x + y * p.y; }

ld cross(const Point &p) {
    return x * p.y - y * p.x;
}

Point perpendicularLeft() { return Point(-y, x); }

Point perpendicularRight() { return Point(y, -x); }

Point rotate(ld deg) {
    ld rad = (deg * pi) / 180.0;
    return Point(x * cos(rad) - y * sin(rad),
                x * sin(rad) + y * cos(rad));
}

Point unit() const { return (*this) / norm(); }

Point projectOn(const Point &p) {
    return p.unit() * (dot(p) / p.norm());
}

ld angleWith(const Point &p) {
    ld x = dot(p) / norm() / p.norm();
    return acos(max(-1.0L, min(1.0L, x)));
}

bool isPerpendicularWith(const Point &p) {
    return dot(p);
}

// ans > 0 p is on the left
// ans < 0 p is on the right
// ans == 0 p has our same direction
ld positionOf(const Point &p) { return cross(p); }

istream &operator>>(istream &is, Point &p) {
    return is >> p.x >> p.y;
}

ostream &operator<<(ostream &os, const Point &p) {
    return os << "(" << p.x << ", " << p.y << ")";
}
```

Intersection Points Of Segments In A Plane

```
#include <bits/stdc++.h>
using namespace std;

#include "Data Structures/Point.cpp"

vector<Point> intersections(
    vector<pair<Point, Point>>& segs) {

    return {};
}
```

Is Point In Line

```
#include "Data Structures/Point.cpp"

bool isPointInLine(Point& a, Point& b, Point& p) {
    return !(b - a).cross(p - a);
}
```

Is Point In Segment

```
#include "Data Structures/Point.cpp"

bool isPointInSegment(Point& a, Point& b, Point& p) {
    return !(b - a).cross(p - a) &&
           (a - p).dot(b - p) <= 0;
}
```

Line-Line Intersection

```
#include "Data Structures/Point.cpp"

pair<int, Point> llintersection(Point& a, Point& b,
                               Point& u, Point& v) {
    if ((b - a).cross(v - u)) // single point
        return {1, a + (b - a) * ((u - a).cross(v - u) /
                                   (b - a).cross(v - u))};
    if ((b - a).cross(u - a))
        return {0, Point()}; // no points
    return {-1, Point()}; // infinity points
}
```

Line-Segment Intersection

```
#include "Data Structures/Point.cpp"

int sign(ld x) {
    if (x < 0) return -1;
    return x > 0;
}
```

```
// line: a-b, segment: u-v
// 0: no point, 1: single point, -1: infinity points
pair<int, Point> lsintersection(Point& a, Point& b,
                               Point& u, Point& v) {
    if ((b - a).cross(v - u))
        return {sign((b - a).cross(u - a)) !=
                 sign((b - a).cross(v - a)),
                a + (b - a) * ((u - a).cross(v - u) /
                               (b - a).cross(v - u))};
    if ((u - a).cross(b - a)) return {0, Point()};
    return {-1, Point()};
}
```

Max Interval Overlap

```
typedef long long int T;
typedef pair<T, T> Interval;
vector<Interval> maxIntervals;

// O(N * lg(N))
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    map<T, int> m;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (auto &i : arr) m[i.first]++, m[i.second + 1]--;
    for (auto &p : m) {
        curr += p.second;
        if (curr > maxI) maxI = curr, l = p.first;
        if (curr == maxI) r = p.first;
    }
    curr = 0;
    for (auto &p : m) {
        curr += p.second;
        if (curr == maxI && isFirst)
            l = p.first, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, p.first - 1}),
            isFirst = 1;
    }
    return maxI;
}
```

```
// O(MaxPoint) maxPoint < vector::max_size
int maxOverlap(vector<Interval> &arr) {
    maxIntervals.clear();
    T maxPoint = 0;
    for (auto &i : arr)
        if (i.second > maxPoint) maxPoint = i.second;
    vector<int> x(maxPoint + 2);
    for (auto &i : arr) x[i.first]++, x[i.second + 1]--;
    int maxI = 0, curr = 0, isFirst = 1;
    T l = -1LL, r = -1LL;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr > maxI) maxI = curr;
    }
    curr = 0;
    for (int i = 0; i < x.size(); i++) {
        curr += x[i];
        if (curr == maxI && isFirst) l = i, isFirst = 0;
        if (curr < maxI && !isFirst)
            maxIntervals.push_back({l, i - 1}), isFirst = 1;
    }
    return maxI;
}
```

Point Projection On Line

```
#include "Data Structures/Point.cpp"
```

```
Point projectionOnLine(Point& a, Point& b, Point& p) {
    Point pr = a + (p - a).projectOn(b - a);
    if (abs(pr.x) < eps) pr.x = 0;
    if (abs(pr.y) < eps) pr.y = 0;
    return pr;
}
```

Point Reflection On Line

```
#include "Data Structures/Point.cpp"
```

```
Point reflectionOnLine(Point& a, Point& b, Point& p) {
    Point r = a * 2 + (p - a).projectOn(b - a) * 2 - p;
    if (abs(r.x) < eps) r.x = 0;
    if (abs(r.y) < eps) r.y = 0;
    return r;
}
```

Point-Line Distance

```
#include "Data Structures/Point.cpp"
```

```
ld pointLineDist(Point& a, Point& b, Point& p) {
    return ((p - a).projectOn(b - a) - (p - a)).norm();
}
```

Point-Line Distance (Signed)

```
#include "Data Structures/Point.cpp"
// ans > 0 p is on the left of a-b
// ans < 0 p is on the right of a-b
ld pointLineDist(Point& a, Point& b, Point& p) {
    return (b - a).cross(p - a) / (b - a).norm();
}
```

Segment-Segment Intersection

```
#include "Data Structures/Point.cpp"
```

```
int sign(ld x) {
    if (x < 0) return -1;
    return x > 0;
}
```

```
void swap(Point& a, Point& b) {
    swap(a.x, b.x), swap(a.y, b.y);
}
```

```
// 0: no point, 1: single point, -1: infinity points
pair<int, Point> ssintersection(Point a, Point b,
                                Point u, Point v) {
    if ((b - a).cross(v - u))
        return {sign((b - a).cross(u - a)) !=
                sign((b - a).cross(v - a)) &&
                sign((v - u).cross(a - u)) !=
                sign((v - u).cross(b - u)),
                a + (b - a) * ((u - a).cross(v - u) /
                                (b - a).cross(v - u))};
    if ((b - a).cross(u - a)) return {0, Point()};
    if (a > b) swap(a, b);
    if (u > v) swap(u, v);
    if (a > u) swap(a, u), swap(b, v);
    if (u < b) return {-1, Point()};
    return {b == u, u};
}
```

Sort Points Along Line With Direction

```
#include "Data Structures/Point.cpp"
```

```
void sortAlongLine(Point& a, Point& b,
                   vector<Point>& ps) {
    sort(ps.begin(), ps.end(), [&](Point& u, Point& v) {
        return u.dot(b - a) < v.dot(b - a);
    });
}
```

Graphs

Data Structures

Union Find

```
struct UnionFind {
    int n;
    vector<int> dad, size;

    UnionFind(int N) : n(N), dad(N), size(N, 1) {
        while (N--) dad[N] = N;
    }

    // O(lg*(N))
    int root(int u) {
        if (dad[u] == u) return u;
        return dad[u] = root(dad[u]);
    }

    // O(1)
    void join(int u, int v) {
        int Ru = root(u), Rv = root(v);
        if (Ru == Rv) return;
        if (size[Ru] > size[Rv]) swap(Ru, Rv);
        --n, dad[Ru] = Rv;
        size[Rv] += size[Ru];
    }

    // O(lg*(N))
    bool areConnected(int u, int v) {
        return root(u) == root(v);
    }

    int getSize(int u) { return size[root(u)]; }

    int numberOfSets() { return n; }
};
```

Union Find (Partially Persistent)

```
// jTime = join time, t = time
struct UnionFind {
    int Time = 0;
    vector<int> dad, size, jTime;

    UnionFind(int N) : dad(N), size(N, 1), jTime(N) {
        while (N--) dad[N] = N;
    }

    // O(lg(N))
    int root(int u, int t) {
        while (jTime[u] <= t && u != dad[u]) u = dad[u];
        return u;
    }
};
```

```
// O(1)
void join(int u, int v, bool newTime = 1) {
    int Ru = root(u, Time), Rv = root(v, Time);
    if (newTime) Time++;
    if (Ru == Rv) return;
    if (size[Ru] > size[Rv]) swap(Ru, Rv);
    jTime[Ru] = Time;
    dad[Ru] = Rv;
    size[Rv] += size[Ru];
}

// O(lg(N))
bool areConnected(int u, int v, int t) {
    return root(u, t) == root(v, t);
}

// O(lg(N))
int getLastVersionSize(int u) {
    return size[root(u, Time)];
}

// O(lg(Time) * lg(N))
int joinTime(int u, int v) {
    int l = 0, r = Time, ans = -1;
    while (l <= r) {
        int mid = (l + r) >> 1;
        if (areConnected(u, v, mid))
            ans = mid, r = mid - 1;
        else
            l = mid + 1;
    }
    return ans;
}
};
```

Articulation Points And Bridges

```
// APB = articulation points and bridges
// Ap = Articulation Point
// br = bridges, p = parent
// disc = discovery time
// low = lowTime, ch = children
// nup = number of edges from u to p

typedef pair<int, int> Edge;
int Time;
vector<vector<int>> adj;
vector<int> disc, low, isAp;
vector<Edge> br;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}
```

```

int dfsAPB(int u, int p) {
    int ch = 0, nup = 0;
    low[u] = disc[u] = ++Time;
    for (int &v : adj[u]) {
        if (v == p && !nup++) continue;
        if (!disc[v]) {
            ch++, dfsAPB(v, u);
            if (disc[u] <= low[v]) isAp[u]++;
            if (disc[u] < low[v]) br.push_back({u, v});
            low[u] = min(low[u], low[v]);
        } else
            low[u] = min(low[u], disc[v]);
    }
    return ch;
}

// O(N)
void APB() {
    br.clear();
    isAp = low = disc = vector<int>(adj.size());
    Time = 0;
    for (int u = 0; u < adj.size(); u++)
        if (!disc[u]) isAp[u] = dfsAPB(u, u) > 1;
}

```

Connected Components

```

// comp = component
int compId;
vector<vector<int>> adj;
vector<int> getComp;

void init(int N) {
    adj.assign(N, vector<int>());
    getComp.assign(N, -1);
    compId = 0;
}

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

void dfsCC(int u, vector<int> &comp) {
    if (getComp[u] > -1) return;
    getComp[u] = compId;
    comp.push_back(u);
    for (auto &v : adj[u]) dfsCC(v, comp);
}

// O(N)
vector<vector<int>> connectedComponents() {
    vector<vector<int>> comps;
    for (int u = 0; u < adj.size(); u++) {
        vector<int> comp;
        dfsCC(u, comp);
        if (!comp.empty())
            comps.push_back(comp), compId++;
    }
    return comps;
}

```

Flood Fill

```

int n, m, oldColor = 0, color = 1;
vector<vector<int>> mat;
vector<vector<int>> movs = {
    {1, 0}, {0, 1}, {-1, 0}, {0, -1}};

void floodFill(int i, int j) {
    if (i >= mat.size() || i < 0 ||
        j >= mat[i].size() || j < 0 ||
        mat[i][j] != oldColor)
        return;
    mat[i][j] = color;
    for (auto move : movs)
        floodFill(i + move[1], j + move[0]);
}

void floodFill() {
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            if (mat[i][j] == oldColor) floodFill(i, j);
}

```

Heavy Light Decomposition

```

// p = parent;
#include "../Data Structures/Ranges/Segment Tree.cpp"
typedef int T;
vector<vector<int>> adj;
vector<int> p, heavy, depth, root, stPos, vals;
SegmentTree<T> st(0);

void init(int n) {
    adj.assign(n, vector<int>());
    heavy.assign(n, -1);
    vals.assign(n, 0);
    p = root = stPos = depth = heavy;
    st = SegmentTree<T>(n);
}

void addEdge(int u, int v, T val) {
    adj[u].push_back(v);
    p[v] = u, vals[v] = val;
}

T F(T a, T b) { return a + b; }

// O(N)
int dfs(int u) {
    int size = 1, maxSubtree = 0;
    for (int &v : adj[u]) {
        depth[v] = depth[u] + 1;
        int subtree = dfs(v);
        if (subtree > maxSubtree)
            heavy[u] = v, maxSubtree = subtree;
        size += subtree;
    }
    return size;
}

```

```
// O(N)
void initHeavyLight() {
    for (int i = 0; i < adj.size(); i++)
        if (p[i] < 0) dfs(i);
    for (int i = 0, pos = 0; i < adj.size(); i++)
        if (p[i] < 0 || heavy[p[i]] != i)
            for (int j = i; ~j; j = heavy[j]) {
                st.setValAt(vals[j], stPos[j] = pos++);
                root[j] = i;
            }
    st.build();
}

// O(lg^2(N))
template <class Op>
void processPath(int u, int v, Op op) {
    for (; root[u] != root[v]; v = p[root[v]]) {
        if (depth[root[u]] > depth[root[v]]) swap(u, v);
        op(stPos[root[v]], stPos[v]);
    }
    if (depth[u] > depth[v]) swap(u, v);
    // for values on edges
    if (u != v) op(stPos[u] + 1, stPos[v]);
    // for values on nodes
    // op(stPos[u], stPos[v]);
}

// O(lg^2(N))
void update(int u, int v, T val) {
    processPath(u, v, [&val](int l, int r) {
        st.update(l, r, val);
    });
}

// O(lg^2(N))
T query(int u, int v) {
    T ans = T();
    processPath(u, v, [&ans](int l, int r) {
        ans = F(ans, st.query(l, r));
    });
    return ans;
}
```

Is Bipartite

```
vector<vector<int>>> adj;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}
```

```
// O(N)
bool isBipartite() {
    vector<int> color(adj.size(), -1);
    for (int s = 0; s < adj.size(); s++) {
        if (color[s] > -1) continue;
        color[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (int &v : adj[u]) {
                if (color[v] < 0)
                    q.push(v), color[v] = !color[u];
                if (color[v] == color[u]) return false;
            }
        }
    }
    return true;
}
```

LCA

```
// st = sparse table, p = parent
typedef pair<int, int> T;
int neutro = 0;
vector<vector<T>>> st;
vector<int> first;
vector<T> tour;
vector<vector<int>>> adj;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

T F(T a, T b) { return a.first < b.first ? a : b; }

void build() {
    st.assign(log2(tour.size()),
              vector<T>(tour.size()));
    st[0] = tour;
    for (int i = 1; (1 << i) <= tour.size(); i++)
        for (int j = 0; j + (1 << i) <= tour.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

void eulerTour(int u, int p, int h) {
    first[u] = tour.size();
    tour.push_back({h, u});
    for (int v : adj[u])
        if (v != p) {
            eulerTour(v, u, h + 1);
            tour.push_back({h, u});
        }
}
```



```
// O(N * lg(N))
void preprocess() {
    tour.clear();
    first.assign(adj.size(), -1);
    vector<int> roots = {0};
    for (auto &root : roots) eulerTour(root, -1, 0);
    build();
}

// O(1)
int lca(int u, int v) {
    int l = min(first[u], first[v]);
    int r = max(first[u], first[v]);
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]).second;
}
```

MST Kruskal

```
// N = number of nodes, Wedge = Weighted Edge
#include "../Data Structures/Graphs/UnionFind.cpp"
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
vector<Wedge> Wedges;
vector<Wedge> mst;
UnionFind uf(0);

void init(int N) {
    mst.clear();
    Wedges.clear();
    uf = UnionFind(N);
}

void addEdge(int u, int v, T w) {
    Wedges.push_back({w, {u, v}});
}

T kruskal() {
    T cost = 0;
    sort(Wedges.begin(), Wedges.end());
    // reverse(Wedges.begin(), Wedges.end());
    for (Wedge &wedge : Wedges) {
        int u = wedge.second.first,
            v = wedge.second.second;
        if (!uf.areConnected(u, v))
            uf.join(u, v), mst.push_back(wedge),
            cost += wedge.first;
    }
    return cost;
}
```

MST Prim

```
// st = spanning tree, p = parent
// vis = visited, dist = distance
typedef int T;
typedef pair<int, int> Edge;
typedef pair<T, Edge> Wedge;
typedef pair<T, int> DistNode;
long long int inf = (1ll << 62) - 1;
vector<vector<int>>> adj;
unordered_map<int, unordered_map<int, T>> weight;
vector<int> p, vis;
vector<T> dist;
vector<vector<Wedge>> msts;

void init(int N) {
    adj.assign(N, vector<int>());
    p.assign(N, 0);
    vis.assign(N, 0);
    dist.assign(N, inf);
    weight.clear();
    msts.clear();
}

void addEdge(int u, int v, T w) {
    adj[u].push_back(v);
    weight[u][v] = w;
    adj[v].push_back(u);
    weight[v][u] = w;
}

// ~ O(E * log(V))
T prim(int s) {
    vector<Wedge> mst;
    vector<T> dist(adj.size(), inf);
    priority_queue<DistNode> q;
    T cost = dist[s] = 0;
    q.push({0, s});
    while (q.size()) {
        pair<int, int> aux = q.top();
        int u = aux.second;
        q.pop();
        if (dist[u] < -aux.first) continue;
        vis[u] = 1, cost += dist[u];
        mst.push_back({dist[u], {p[u], u}});
        for (int &v : adj[u]) {
            T w = weight[u][v];
            if (!vis[v] && w < dist[v])
                q.push({-(dist[v] = w), v});
        }
    }
    msts.push_back(
        vector<Wedge>(mst.begin() + 1, mst.end()));
    return cost;
}
```



```
// O(V + E * log(V))
T prim() {
    T cost = 0;
    map<int, T> q;
    for (int i = 0; i < adj.size(); i++)
        if (!vis[i]) cost += prim(i);
    return cost;
}
```

Strongly Connected Components

```
// tv = top value from stack
// sccs = strongly connected components
// scc = strongly connected component
// disc = discovery time, low = low time
// s = stack, top = top index of the stack

int Time, top;
vector<vector<int>> adj, sccs;
vector<int> disc, low, s;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) { adj[u].push_back(v); }

void dfsSCCS(int u) {
    if (disc[u]) return;
    low[u] = disc[u] = ++Time;
    s[++top] = u;
    for (int &v : adj[u])
        dfsSCCS(v), low[u] = min(low[u], low[v]);
    if (disc[u] == low[u]) {
        vector<int> scc;
        while (true) {
            int tv = s[top--];
            scc.push_back(tv);
            low[tv] = adj.size();
            if (tv == u) break;
        }
        sccs.push_back(scc);
    }
}

// O(N)
void SCCS() {
    s = low = disc = vector<int>(adj.size());
    Time = 0, top = -1, sccs.clear();
    for (int u = 0; u < adj.size(); u++) dfsSCCS(u);
}
```

Topological Sort

```
// vis = visited
vector<vector<int>> adj;
vector<int> vis, toposorted;
```

```
void init(int N) {
    adj.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
}

void addEdge(int u, int v) { adj[u].push_back(v); }
// returns false if there is a cycle
// O(N)
bool toposort(int u) {
    vis[u] = 1;
    for (auto &v : adj[u])
        if (v != u && vis[v] != 2 &&
            (vis[v] || !toposort(v)))
            return false;
    vis[u] = 2;
    toposorted.push_back(u);
    return true;
}
// O(N)
bool toposort() {
    for (int u = 0; u < adj.size(); u++)
        if (!vis[u] && !toposort(u)) return false;
    return true;
}
```

Topological Sort (All possible sorts)

```
// indeg0 = indegree 0
vector<int> vis, indegree, path;
vector<vector<int>> adj, toposorts;
deque<int> indeg0;
void init(int n) {
    adj.assign(n, vector<int>());
    vis.assign(n, 0);
    indegree = vis;
}
void addEdge(int u, int v) {
    adj[u].push_back(v);
    indegree[v]++;
}
// O(V!)
void dfs() {
    for (int i = 0; i < indeg0.size(); i++) {
        int u = indeg0.front();
        indeg0.pop_front();
        path.push_back(u);
        for (auto &v : adj[u])
            if (!--indegree[v]) indeg0.push_back(v);
        if (!indeg0.size()) toposorts.push_back(path);
        dfs();
        for (int v = adj[u].size() - 1; ~v; v--) {
            indegree[adj[u][v]]++;
            if (indeg0.back() == adj[u][v])
                indeg0.pop_back();
        }
        indeg0.push_back(u);
        path.pop_back();
    }
}
```

```
// O(V + V!)
void allToposorts() {
    for (int u = 0; u < adj.size(); u++)
        if (!indegree[u]) indeg0.push_back(u);
    dfs();
}
```

Topological Sort (lowest lexicographically)

Tree Diameter (Longest Path Between 2 Nodes)

```
int inf = (1 << 30) - 1;
vector<vector<int>> adj;

void init(int N) { adj.assign(N, vector<int>()); }

void addEdge(int u, int v) {
    adj[u].push_back(v);
    adj[v].push_back(u);
}

int bfs(int u) {
    vector<int> dist(adj.size(), inf);
    queue<int> q;
    q.push(u), dist[u] = 0;
    while (q.size()) {
        int u = q.front();
        q.pop();
        for (int& v : adj[u])
            if (dist[v] == inf)
                q.push(v), dist[v] = dist[u] + 1;
    }
    int node, maxx = -inf;
    for (int u = 0; u < adj.size(); u++)
        if (dist[u] > maxx) maxx = dist[u], node = u;
    return node;
}

vector<int> diameter() {
    int u = bfs(0), v = bfs(u);
    vector<int> path = {v}, vis(adj.size());
    function<bool(int)> dfs = [&](int a) {
        if (a == v) return true;
        vis[a] = 1;
        for (int& b : adj[a]) {
            if (vis[b] || !dfs(b)) continue;
            path.push_back(a);
            return true;
        }
        return false;
    };
    dfs(u);
    return path;
}
```

Cycles

Get Some Cycles

```
// at least detects one cycle per component
vector<vector<int>> adj, cycles;
vector<int> vis, cycle;
bool flag = false, isDirected = false;
int root = -1;

void init(int N) {
    adj.assign(N, vector<int>());
    vis.assign(N, 0);
    cycles.clear();
    root = -1, flag = false;
}

void addEdge(int u, int v) {
    adj[u].push_back(v);
    if (!isDirected) adj[v].push_back(u);
}

// O(N)
bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : adj[u]) {
        if (v == u || vis[v] == 2 ||
            (!isDirected && v == prev))
            continue;
        if (flag) {
            if (!vis[v]) hasCycle(v, u);
            continue;
        }
        if (vis[v] || hasCycle(v, u)) {
            if (root == -1) root = v, flag = true;
            cycle.push_back(u);
            if (root == u)
                flag = false, root = -1,
                cycles.push_back(cycle), cycle.clear();
        }
    }
    vis[u] = 2;
    return flag;
}

// O(N)
bool hasCycle() {
    for (int u = 0; u < adj.size(); u++)
        if (!vis[u]) cycle.clear(), hasCycle(u, -1);
    return cycles.size() > 0;
}

Has Cycle

vector<vector<int>> adj;
vector<int> vis;
bool isDirected = false;

void init(int N) {
    adj.assign(N, vector<int>());
    vis.assign(N, 0);
}
```

```

void addEdge(int u, int v) {
    adj[u].push_back(v);
    if (!isDirected) adj[v].push_back(u);
}

bool hasCycle(int u, int prev) {
    vis[u] = 1;
    for (auto &v : adj[u])
        if (v != u && vis[v] != 2 &&
            (isDirected || v != prev) &&
            (vis[v] || hasCycle(v, u)))
            return true;
    vis[u] = 2;
    return false;
}

// O(N)
bool hasCycle() {
    for (int u = 0; u < adj.size(); u++)
        if (!vis[u] && hasCycle(u, -1)) return true;
}

```

Flow

Max Flow Min Cut Dinic

```

// cap[a][b] = Capacity from a to b
// flow[a][b] = flow occupied from a to b
// level[a] = level in graph of node a
// iflow = initial flow, icap = initial capacity
// pathMinCap = capacity bottleneck for a path (s->t)

```

```

typedef int T;
vector<int> level;
vector<vector<int>> adj;
unordered_map<int, unordered_map<int, T>> cap, flow;
long long int inf = (1ll << 62) - 1;

```

```

void init(int N) {
    adj.assign(N, vector<int>());
    cap.clear();
    flow.clear();
}

```

```

void addEdge(int u, int v, T icap, T iflow = 0) {
    cap[u][v] = icap;
    flow[u][v] += iflow;
    flow[v][u] -= iflow;
    adj[u].push_back(v);
    adj[v].push_back(u);
}

```

```

bool levelGraph(int s, int t) {
    level.assign(adj.size(), 0);
    level[s] = 1;
    queue<int> q;
    q.push(s);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        for (int &v : adj[u]) {
            if (!level[v] && flow[u][v] < cap[u][v]) {
                q.push(v);
                level[v] = level[u] + 1;
            }
        }
    }
    return level[t];
}

T blockingFlow(int u, int t, T pathMinCap) {
    if (u == t) return pathMinCap;
    for (int v : adj[u]) {
        T capLeft = cap[u][v] - flow[u][v];
        if (level[v] == (level[u] + 1) && capLeft > 0)
            if (T pathMaxFlow = blockingFlow(
                v, t, min(pathMinCap, capLeft))) {
                flow[u][v] += pathMaxFlow;
                flow[v][u] -= pathMaxFlow;
                return pathMaxFlow;
            }
    }
    return 0;
}

```

```

// O(E * V^2)
T maxFlowMinCut(int s, int t) {
    if (s == t) return inf;
    T maxFlow = 0;
    while (levelGraph(s, t))
        while (T flow = blockingFlow(s, t, inf))
            maxFlow += flow;
    return maxFlow;
}

```

Maximum Bipartite Matching

```

// mbm = maximum bipartite matching
#include "Max Flow Min Cut Dinic.cpp"

```

```

void addEdgeMBM(int u, int v) {
    addEdge(u += 2, v += 2, 1);
    addEdge(0, u, 1);
    addEdge(v, 1, 1);
}

```

```

// O(E * V^2)
T mbm() { return maxFlowMinCut(0, 1); }

```

ShortestPaths

0 - 1 BFS

```
// s = source
typedef int T;
long long int inf = (1ll << 62) - 1;
vector<vector<int>> adj;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w, bool isDirected = 0) {
    adj[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    adj[v].push_back(u);
    weight[v][u] = w;
}

// O(E)
vector<T> bfs(int s) {
    vector<long long int> dist(adj.size(), inf);
    dist[s] = 0;
    deque<int> q;
    q.push_front(s);
    while (q.size()) {
        int u = q.front(); q.pop_front();
        for (auto& v : adj[u]) {
            T d = dist[u] + weight[u][v];
            if (d < dist[v])
                weight[u][v] ? q.push_back(v)
                             : q.push_front(v);
        }
    }
    return dist;
}
```

Bellman Ford

```
// s = source
// returns {} if there is a negative weight cycle
typedef int T;
long long int inf = (1ll << 62) - 1;
vector<vector<int>> adj;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    weight.clear();
}
```

```
void addEdge(int u, int v, T w, bool isDirected = 0) {
    adj[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    adj[v].push_back(u);
    weight[v][u] = w;
}

// O(V * E)
vector<T> bellmanFord(int s) {
    vector<long long int> dist(adj.size(), inf);
    dist[s] = 0;
    for (int i = 1; i <= adj.size(); i++)
        for (int u = 0; u < adj.size(); u++)
            for (auto &v : adj[u]) {
                T d = dist[u] + weight[u][v];
                if (dist[u] != inf && d < dist[v]) {
                    if (i == adj.size()) return {};
                    dist[v] = d;
                }
            }
    return dist;
}
```

Bellman Ford Fast

```
// s = source, its = iterations of node u
// returns {} if there is a negative weight cycle
typedef int T;
long long int inf = (1ll << 62) - 1;
vector<vector<int>> adj;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w, bool isDirected = 0) {
    adj[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    adj[v].push_back(u);
    weight[v][u] = w;
}
```

```
// O(V * E)
vector<T> bellmanFordFast(int s) {
    vector<long long int> dist(adj.size(), inf);
    vector<int> its(adj.size()), inqueue(adj.size());
    queue<int> q;
    q.push(s), dist[s] = 0, its[s] = 1;
    while (!q.empty()) {
        int u = q.front(); q.pop(), inqueue[u] = 0;
        for (auto &v : adj[u]) {
            T d = dist[u] + weight[u][v];
            if (d < dist[v]) {
                dist[v] = d;
                if (!inqueue[v]) q.push(v), its[v]++;
                if (its[v] == adj.size()) return {};
            }
        }
    }
    return dist;
}
```

Dijkstra

```
// s = source
typedef int T;
typedef pair<T, int> DistNode;
long long int inf = (1ll << 62) - 1;
vector<vector<int>>> adj;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    weight.clear();
}

void addEdge(int u, int v, T w, bool isDirected = 0) {
    adj[u].push_back(v);
    weight[u][v] = w;
    if (isDirected) return;
    adj[v].push_back(u);
    weight[v][u] = w;
}

// ~ O(E * lg(V))
vector<T> dijkstra(int s) {
    vector<long long int> dist(adj.size(), inf);
    priority_queue<DistNode> q;
    q.push({0, s}), dist[s] = 0;
    while (q.size()) {
        DistNode top = q.top();
        q.pop();
        int u = top.second;
        if (dist[u] < -top.first) continue;
        for (int &v : adj[u]) {
            T d = dist[u] + weight[u][v];
            if (d < dist[v]) q.push({-(dist[v] = d), v});
        }
    }
    return dist;
}
```

Floyd Warshall

```
// d = distance
typedef int T;
long long int inf = (1ll << 62) - 1;
vector<vector<long long int>>> d;

void init(int n) {
    d.assign(n, vector<long long int>(n, inf));
    for (int i = 0; i < n; i++) d[i][i] = 0;
}

void addEdge(int u, int v, T w, bool isDirected = 0) {
    d[u][v] = w;
    if (isDirected) return;
    d[v][u] = w;
}

// O(V^3)
void floydwarshall() {
    for (int k = 0; k < d.size(); k++)
        for (int u = 0; u < d.size(); u++)
            for (int v = 0; v < d.size(); v++)
                d[u][v] = min(d[u][v], d[u][k] + d[k][v]);
}
```

Shortest Path in Directed Acyclic Graph

```
// s = source
#include "../Topological Sort.cpp"
typedef int T;
long long int inf = (1ll << 62) - 1;
unordered_map<int, unordered_map<int, T>> weight;

void init(int N) {
    adj.assign(N, vector<int>());
    vis.assign(N, 0);
    toposorted.clear();
    weight.clear();
}

void addEdge(int u, int v, int w) {
    adj[u].push_back(v);
    weight[u][v] = w;
}

// O(N)
vector<T> dagsssp(int s) {
    vector<long long int> dist(adj.size(), inf);
    dist[s] = 0;
    toposort(s);
    while (toposorted.size()) {
        int u = toposorted.back();
        toposorted.pop_back();
        for (auto &v : adj[u]) {
            T d = dist[u] + weight[u][v];
            if (d < dist[v]) dist[v] = d;
        }
    }
    return dist;
}
```

Maths

Data Structures

Matrix

```
template <class T>
struct Matrix {
    int rows, cols;
    vector<vector<T>> m;

    Matrix(int r, int c) : rows(r), cols(c) {
        m.assign(r, vector<T>(c));
    }

    Matrix(const vector<vector<T>>& b)
        : rows(b.size()), cols(b[0].size()), m(b) {}

    Matrix(int n) {
        m.assign(n, vector<T>(n));
        while (n--) m[n][n] = 1;
    }

    vector<T>& operator[](int i) const {
        return const_cast<Matrix*>(this)->m[i];
    }

    // O(N * M)
    Matrix operator+(const Matrix& b) {
        Matrix ans(rows, cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < m[i].size(); j++)
                ans[i][j] = m[i][j] + b[i][j];
        return ans;
    }

    // O(N * M)
    Matrix operator-(const Matrix& b) {
        Matrix ans(rows, cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < m[i].size(); j++)
                ans[i][j] = m[i][j] - b[i][j];
        return ans;
    }

    // O(N^3)
    Matrix operator*(const Matrix& b) {
        if (cols != b.rows) return Matrix(0, 0);
        Matrix ans(rows, b.cols);
        for (int i = 0; i < rows; i++)
            for (int j = 0; j < b[i].size(); j++)
                for (int k = 0; k < b.rows; k++)
                    ans[i][j] += m[i][k] * b[k][j];
        return ans;
    }

    Matrix& operator+=(const Matrix& b) {
        return *this = *this + b;
    }
}
```

```
Matrix& operator--(const Matrix& b) {
    return *this = *this - b;
}

Matrix& operator*=(const Matrix& b) {
    return *this = *this * b;
}
};
```

Number Theory

Binary Exponentiation

```
typedef long long int li;

li binPow(li a, li p) {
    li ans = 1LL;
    while (p) {
        if (p & 1LL) ans *= a;
        a *= a, p >>= 1LL;
    }
    return ans;
}
```

Diophantine Equation (Base Solution)

```
#include "Extended Euclidean Algorithm.cpp"
typedef long long int li;

pair<li, li> diophantineBase(li a, li b, li c) {
    li d, x, y;
    tie(d, x, y) = extendedGCD(a, b);
    if (c % d) return {0, 0};
    return {c / d * x, c / d * y};
}
```

Diophantine Equation (Particular Solution)

```
#include "Extended Euclidean Algorithm.cpp"
typedef long long int li;

// returns the i-th solution or {0, 0} if no solutions
pair<li, li> diophantineSol(li a, li b, li c, li i) {
    li d, x, y;
    tie(d, x, y) = extendedGCD(a, b);
    if (c % d) return {0, 0};
    return {c / d * x + i / d * b,
            c / d * y - i / d * a};
}
```

Divisibility Criterion

```
def divisorCriteria(n, lim):
    results = []
    tenElevated = 1
    for i in range(lim):
        # remainder = pow(10, i, n)
        remainder = tenElevated % n
        negremainder = remainder - n
        if(remainder <= abs(negremainder)):
            results.append(remainder)
        else:
            results.append(negremainder)
        tenElevated *= 10
    return results

def testDivisibility(dividend, divisor,
    ↪ divisor_criteria):
    dividend = str(dividend)
    addition = 0
    dividendSize = len(dividend)
    i = dividendSize - 1
    j = 0
    while j < dividendSize:
        addition += int(dividend[i]) *
            ↪ divisor_criteria[j]
        i -= 1
        j += 1
    return addition % divisor == 0

if __name__ == '__main__':
    dividend, divisor = map(int, input().split())
    divisor_criteria = divisorCriteria(divisor,
    ↪ len(str(dividend)))
    print(divisor_criteria)
    print(testDivisibility(dividend, divisor,
    ↪ divisor_criteria))
```

Divisors

```
// divs = divisors
```

```
typedef long long int li;
typedef vector<li> V;
```

```
// O(sqrt(N))
```

```
V getDivisors(li n) {
    V divs;
    for (li i = 1; i * i <= n; i++)
        if (!(n % i)) {
            divs.push_back(i);
            if (i * i != n) divs.push_back(n / i);
        }
    return divs;
}
```

Divisors Pollard Rho

```
// pf = primeFactors, divs = divisors
```

```
#include "../Primes/Prime Factorization Pollard
    ↪ Rho.cpp"
```

```
typedef vector<li> V;
```

```
void divisors(Map &pf, V &divs, li ans, li p) {
    auto next = ++pf.find(p);
    int power = pf[p];
    for (li k = 0; k <= power; k++, ans *= p) {
        if (next == pf.end()) divs.push_back(ans);
        else divisors(pf, divs, ans, next->first);
    }
}
```

```
// O(number of pf)
```

```
V getDivisors(li n) {
    Map pf = getPrimeFactors(n);
    V divs;
    divisors(pf, divs, 1, pf.begin()->first);
    return divs;
}
```

Euler's Phi

```
#include "../Primes/Prime Factorization Pollard
    ↪ Rho.cpp"
// #include "../Primes/Prime Factorization.cpp"
```

```
// counts the number of integers (Xi) between 1 and n
// which are coprime (gcd(Xi, n) = 1) to n
```

```
li phi(li n) {
    Map pf = getPrimeFactors(n);
    if (pf.count(n)) return n - 1; // if n is prime
    for (auto &p : pf) n -= n / p.first;
    return n;
}
```

Extended Euclidean Algorithm

```
#include "Modulo with negative numbers.cpp"
typedef long long int li;
```

```
// gcd(a, b) = ax + by
```

```
tuple<li, li, li> extendedGCD(li a, li b) {
    if (!a) return {b, 0, 1};
    li d, x, y;
    tie(d, x, y) = extendedGCD(mod(b, a), a);
    return {d, y - floor((double) b / a) * x, x};
} // {gcd(a, b), x, y}
```


GCD

```
typedef long long int li;

li gcd(li a, li b) {
    return !b ? a : gcd(b, a % b);
}

// Iterative version
li gcdI(li a, li b) {
    while (b) a %= b, swap(a, b);
    return a;
}
```

LCM

```
#include "GCD.cpp"

int lcm(li a, li b) {
    int d = gcd(a, b);
    return d ? a / d * b : 0;
}
```

Modular Exponentiation

```
#include "../Modular Multiplication.cpp"

// O(lg(p))
li pow(li a, li p, li m) {
    a %= m;
    li ans = 1;
    while (p) {
        if (p & 1LL) ans = multiply(ans, a, m);
        a = multiply(a, a, m), p >>= 1;
    }
    return ans;
}
```

Modular Multiplication

```
typedef long long int li;

// O(lg(b))
li multiply(li a, li b, li m) {
    a %= m;
    li ans = 0;
    while (b) {
        if (b & 1)
            ans += a, ans = (ans >= m ? ans - m : ans);
        b >>= 1, a <<= 1;
        a = (a >= m ? a - m : a);
    }
    return ans;
}
```

Modulo with negative numbers

```
typedef long long int li;

// if b is positive the answer is positive
// if b is negative the answer is negative
li mod(li a, li b) { return (b + (a % b)) % b; }
```

Primes

Is Prime Miller Rabin

```
#include "../Number Theory/Modular Exponentiation.cpp"

bool isPrime(lli p, int k = 20) {
    if (p == 2 || p == 3) return 1;
    if ((~p & 1) || p == 1) return 0;
    lli d = p - 1, phi = d, r = 0;
    while (~d & 1) d >>= 1, r++;
    while (k--) {
        // set seed with: int main() { srand(time(0)); }
        lli a = 2 + rand() % (p - 3); // [2, p - 2]
        lli e = pow(a, d, p), r2 = r;
        if (e == 1 || e == phi) continue;
        bool flag = 1;
        while (--r2) {
            e = multiply(e, e, p);
            if (e == 1) return 0;
            if (e == phi) {
                flag = 0;
                break;
            }
        }
        if (flag) return 0;
    }
    return 1;
}
```

Prime Factorization

```
typedef long long int li;
// if li == __int128_t: use map<li, int> Map;
typedef unordered_map<li, int> Map;

// ~O(sqrt(N) * lg(N))
Map getPrimeFactors(li n) {
    Map pf;
    while (~n & 1) pf[2]++, n >>= 1;
    for (li i = 3; i * i <= n; i += 2)
        while (!(n % i)) pf[i]++, n /= i;
    if (n > 1) pf[n]++;
    return pf;
}
```


Prime Factorization (Sieve)

```
#include "Primes Sieve.cpp"

typedef unordered_map<int, int> Map;

// O(lg(N)) n <= sieve.size()
Map getPrimeFactors(int n) {
    Map pf;
    while (n > 1) {
        int p = n & 1 ? sieve[n] : 2, c;
        if (!p) p = n;
        while (n % p == 0) n /= p, c++;
        pf[p] = c;
    }
    return pf;
}
```

Prime Factorization (Sieve) 2

```
#include "Primes Sieve.cpp"

typedef long long int li;
// if li == __int128_t: use map<li, int> Map;
typedef unordered_map<li, int> Map;

// O(sqrt(N)) n <= sieve.size() ^ 2
Map getPrimeFactors(li n) {
    Map pf;
    for (int& p : primes) {
        if (p * p > n) break;
        int c;
        while (n % p == 0) n /= p, c++;
        if (c) pf[p] = c;
    }
    if (n > 1) pf[n] = 1;
    return pf;
}
```

Prime Factorization Pollard Rho

```
// pf = prime factors

#include "../Number Theory/GCD.cpp"
#include "../Is Prime Miller Rabin.cpp"

typedef long long int li;
// if li == __int128_t: use map<li, int> Map;
typedef unordered_map<li, int> Map;

li _abs(li a) { return a < 0 ? -a : a; }
```

```
li getRandomDivisor(li n) {
    if (~n & 1) return 2;
    li c = 1 + rand() % (n - 1), a = 2, b = 2, d = 1;
    auto f = [&](li x) {
        return (multiply(x, x, n) + c) % n;
    };
    while (d == 1)
        a = f(a), b = f(f(b)), d = gcd(_abs(a - b), n);
    return d;
}

void getpf(li n, Map &pf) {
    if (n == 1LL) return;
    if (isPrime(n)) {
        pf[n]++;
        return;
    }
    li divisor = getRandomDivisor(n);
    getpf(divisor, pf), getpf(n / divisor, pf);
}

// ~O(N^(1/4))
Map getPrimeFactors(li n) {
    Map pf;
    getpf(n, pf);
    return pf;
}
```

Primes Sieve

```
vector<int> sieve, primes;

// ~O(N * lg(lg(N)))
void primeSieve(int n) {
    sieve.assign(n + 1, 0);
    primes = {};
    for (int i = 3; i * i <= n; i += 2)
        if (!sieve[i])
            for (int j = i * i; j <= n; j += 2 * i)
                if (!sieve[j]) sieve[j] = i;
    primes.push_back(2);
    for (int i = 3; i < n; i++)
        if (!sieve[i] && (i & 1)) primes.push_back(i);
}
```

Ranges

Data Structures

BIT

```
template <class T>
struct BIT {
    T neutro = 0;
    vector<T> bit;

    BIT(int n) { bit.assign(++n, neutro); }

    inline T F(T a, T b) {
        return a + b;
        // return a * b;
    }

    // Inverse of F
    inline T I(T a, T b) {
        return a - b;
        // return a / b;
    }

    // O(N)
    void build() {
        for (int i = 1; i < bit.size(); i++) {
            int j = i + (i & -i);
            if (j < bit.size()) bit[j] = F(bit[j], bit[i]);
        }
    }

    // O(lg(N))
    void update(int i, T val) {
        for (i++; i < bit.size(); i += i & -i)
            bit[i] = F(bit[i], val);
    }

    // O(lg(N))
    T query(int i) {
        T ans = neutro;
        for (i++; i; i -= i & -i) ans = F(ans, bit[i]);
        return ans;
    }

    // O(lg(N)), [l, r]
    T query(int l, int r) {
        return I(query(r), query(--l));
    }

    T& operator[](int i) { return bit[++i]; }
};
```

BIT Range Update

```
typedef long long int T;
T neutro = 0;
vector<T> bit1, bit2;
void initVars(int n) {
    bit1.assign(++n, neutro);
    bit2 = bit1;
}
// O(lg(N))
void update(vector<T> &bit, int i, T val) {
    for (i++; i < bit.size(); i += i & -i)
        bit[i] += val;
}
// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    update(bit1, l, val);
    update(bit1, r + 1, -val);
    update(bit2, r + 1, val * r);
    update(bit2, l, -val * (l - 1));
}
// O(lg(N))
T query(vector<T> &bit, int i) {
    T ans = neutro;
    for (i++; i; i -= i & -i) ans += bit[i];
    return ans;
}
// O(lg(N))
T query(int i) {
    return query(bit1, i) * i + query(bit2, i);
}
// O(lg(N)), [l, r]
T query(int l, int r) {
    return query(r) - query(l - 1);
}
```

Segment Tree

```
// st = segment tree. st[1] = root;
// neutro = operation neutral value
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.
template <class T>
struct SegmentTree {
    T neutro = 0;
    int N;
    vector<T> st;
    SegmentTree(int n) : st(2 * n, neutro), N(n) {}

    inline T F(T a, T b) {
        return a + b;
        // return __gcd(a, b);
        // return a * b;
        // return min(a, b);
    }

    // O(2N)
    void build() {
        for (int i = N - 1; i > 0; i--)
            st[i] = F(st[i << 1], st[i << 1 | 1]);
    }
};
```

```

// O(lg(2N)), works like replacing arr[i] with val
void update(int i, T val) {
    for (st[i += N] = val; i > 1; i >>= 1)
        st[i >> 1] = F(st[i], st[i ^ 1]);
}

// O(3N), [l, r]
void update(int l, int r, T val) {
    if (l == r)
        update(l, val);
    else {
        for (l += N, r += N; l <= r; l++) st[l] = val;
        build();
    }
}

// O(lg(2N)), [l, r]
T query(int l, int r) {
    T ans = neutro;
    for (l += N, r += N; l <= r; l >>= 1, r >>= 1) {
        if (l & 1) ans = F(ans, st[l++]);
        if (~r & 1) ans = F(ans, st[r--]);
    }
    return ans;
}

T& operator[](int i) { return st[i + N]; }
};

```

Segment Tree Lazy Propagation

// st = segment tree, st[1] = root, H = height of d
// u = updates, d = delayed updates
// neutro = operation neutral val
// e.g. for sum is 0, for multiplication
// is 1, for gcd is 0, for min is INF, etc.

```

template <class T>
struct SegmentTree {
    T neutro = 0;
    int N, H;
    vector<T> st, d;
    vector<bool> u;

    SegmentTree(int n, T val)
        : st(2 * n, val), d(n), u(n) {
        H = sizeof(int) * 8 - __builtin_clz(N = n);
    }

    inline T F(T a, T b) {
        return a + b;
        // return a * b;
        // return __gcd(a, b);
        // return min(a, b);
    }

    void apply(int i, T val, int k) {
        // operation to update st[i] in O(1) time
        st[i] += val * k; // updates the tree
        // operation to update d[i] in O(1) time
        // which updates values for future updates
        if (i < N) d[i] += val, u[i] = 1;
    }
}

```

```

void calc(int i) {
    if (!u[i]) st[i] = F(st[i << 1], st[i << 1 | 1]);
}

// O(2N)
void build() {
    for (int i = N - 1; i > 0; i--) calc(i);
}

// O(lg(N))
void build(int p) {
    while (p > 1) p >>= 1, calc(p);
}

// O(lg(N))
void push(int p) {
    for (int s = H, k = 1 << (H - 1); s > 0;
         s--, k >>= 1) {
        int i = p >> s;
        if (u[i]) {
            apply(i << 1, d[i], k);
            apply(i << 1 | 1, d[i], k);
            u[i] = 0, d[i] = 0;
        }
    }
}

// O(lg(N)), [l, r]
void update(int l, int r, T val) {
    push(l += N);
    push(r += N);
    int ll = l, rr = r, k = 1;
    for (; l <= r; l >>= 1, r >>= 1, k <== 1) {
        if (l & 1) apply(l++, val, k);
        if (~r & 1) apply(r--, val, k);
    }
    build(ll);
    build(rr);
}

// O(lg(2N)), [l, r]
T query(int l, int r) {
    push(l += N);
    push(r += N);
    T ans = neutro;
    for (; l <= r; l >>= 1, r >>= 1) {
        if (l & 1) ans = F(ans, st[l++]);
        if (~r & 1) ans = F(ans, st[r--]);
    }
    return ans;
}

T& operator[](int i) { return st[i + N]; }
};

```

Sparse Table

```
// st = sparse table, Arith = Arithmetic
typedef int T;
int neutro = 0;
vector<vector<T>> st;

T F(T a, T b) {
    // return min(a, b);
    return __gcd(a, b);
    // return a + b; // Arith
    // return a * b; // Arith
}

// O(N lg(N))
void build(vector<T> &arr) {
    st.assign(log2(arr.size()), vector<T>(arr.size()));
    st[0] = arr;
    for (int i = 1; (1 << i) <= arr.size(); i++)
        for (int j = 0; j + (1 << i) <= arr.size(); j++)
            st[i][j] = F(st[i - 1][j],
                          st[i - 1][j + (1 << (i - 1))]);
}

// O(1), [l, r]
T query(int l, int r) {
    int i = log2(r - l + 1);
    return F(st[i][l], st[i][r + 1 - (1 << i)]);
}

// O(lg(N)), [l, r]
T queryArith(int l, int r) {
    T ans = neutro;
    while (true) {
        int k = log2(r - l + 1);
        ans = F(ans, st[k][l]);
        l += 1 << k;
        if (l > r) break;
    }
    return ans;
}
```

Wavelet Tree

```
// pref = prefix sum
// lte = less than or equal, 1-indexed
// A = max_element(from, to)
// lcount = left children count
struct WaveletTree {
    WaveletTree *l, *r;
    int lo, hi;
    vector<int> lcount, pref;
```

```
// O(N lg(A))
WaveletTree(vector<int>::iterator from,
             vector<int>::iterator to, int lo,
             int hi) {
    this->lo = lo, this->hi = hi;
    if (lo == hi or from >= to) return;
    int mid = (lo + hi) >> 1;
    auto f = [mid](int x) { return x <= mid; };
    lcount.reserve(to - from + 1);
    pref.reserve(to - from + 1);
    lcount.push_back(0);
    pref.push_back(0);
    for (auto it = from; it != to; it++)
        lcount.push_back(lcount.back() + f(*it)),
        pref.push_back(pref.back() + *it);
    auto pivot = stable_partition(from, to, f);
    l = new WaveletTree(from, pivot, lo, mid);
    r = new WaveletTree(pivot, to, mid + 1, hi);
}
```

```
// O(lg(A)) frequency of k in [a, b]
int freq(int a, int b, int k) {
    if (a > b or k < lo or k > hi) return 0;
    if (lo == hi) return b - a + 1;
    int lc = lcount[a - 1], rc = lcount[b];
    if (k > ((lo + hi) >> 1))
        return r->freq(a - lc, b - rc, k);
    return l->freq(lc + 1, rc, k);
}
```

```
// O(lg(A)) kth-Smallest element in [a, b]
int kth(int a, int b, int k) {
    if (a > b) return 0;
    if (lo == hi) return lo;
    int lc = lcount[a - 1], rc = lcount[b],
        inleft = rc - lc;
    if (k > inleft)
        return r->kth(a - lc, b - rc, k - inleft);
    return l->kth(lc + 1, rc, k);
}
```

```
// O(lg(A)) count of elements <= to k in [a, b]
int lte(int a, int b, int k) {
    if (a > b or k < lo) return 0;
    if (hi <= k) return b - a + 1;
    int lc = lcount[a - 1], rc = lcount[b];
    return l->lte(lc + 1, rc, k) +
        r->lte(a - lc, b - rc, k);
}
```

```
// O(lg(A)) sum of numbers <= to k in [a, b]
int sumlte(int a, int b, int k) {
    if (a > b or k < lo) return 0;
    if (hi <= k) return pref[b] - pref[a - 1];
    int lc = lcount[a - 1], rc = lcount[b];
    return l->sumlte(lc + 1, rc, k) +
        r->sumlte(a - lc, b - rc, k);
}
};
```

Wavelet Tree Compressed

```
// lte = less than or equal, c = compressed
// o = original, 1-indexed
#include "../Techniques/Binary Search.cpp"
#include "Wavelet Tree.cpp"

template <class T>
struct WaveletTreeCompressed {
    unordered_map<int, T> imap;
    unordered_map<T, int> Map;
    WaveletTree* wt;
    vector<T> o;

    // O(N*lg(N))
    WaveletTreeCompressed(vector<T>& v) {
        o = v;
        int inf = 1 << 30, n = 0, lo = inf, hi = -inf;
        set<T> s(v.begin(), v.end());
        vector<int> c(v.size());
        for (auto& e : s) Map[e] = n++;
        for (int i = 0; i < v.size(); i++)
            c[i] = Map[v[i]], imap[Map[v[i]]] = v[i];
        for (auto& e : c)
            lo = min(lo, e), hi = max(hi, e);
        wt = new WaveletTree(c.begin(), c.end(), lo, hi);
    }

    // O(lg(N)) frequency of k in [a, b]
    int freq(int l, int r, T k) {
        return wt->freq(l, r, Map[k]);
    }

    // O(lg(N)) kth-Smallest element in [l, r]
    T kth(int l, int r, int k) {
        return imap[wt->kth(l, r, k)];
    }

    // O(lg(N)) count of numbers <= to k in [l, r]
    T lte(int l, int r, T k) {
        int kk = Map[bSearch<T>(o, k, l, r)[1]];
        return imap[wt->lte(l, r, kk)];
    }

    // O(lg(N)) sum of numbers <= to k in [l, r]
    T sumlte(int l, int r, T k) {
        int kk = Map[bSearch<T>(o, k, l, r)[1]];
        return imap[wt->sumlte(l, r, kk)];
    }
};
```

Strings

Data Structures

Suffix Automaton

```
// link[u]: links to the longest suffix which is
//           not in the same endpos-equivalence class
// len[u]: length of the longest suffix that
//           corresponds to u's endpos-equivalence class

struct SuffixAutomaton {
    vector<int> len, link, isClone, first;
    vector<map<char, int>> next;
    int size, last;

    void init(int n) {
        first = isClone = len = link = vector<int>(2 * n);
        next.resize(2 * n);
        len[0] = 0, link[0] = -1, size = 1, last = 0;
    }

    // O(N)
    SuffixAutomaton(const string& s) {
        init(s.size());
        for (const auto& c : s) add(c);
    }

    // O(1)
    void add(const char& c) {
        int p = last, u = size++;
        len[u] = len[p] + 1, first[u] = len[p];
        while (p != -1 && !next[p].count(c))
            next[p][c] = u, p = link[p];
        if (p == -1) link[u] = 0;
        else {
            int q = next[p][c];
            if (len[p] + 1 == len[q]) link[u] = q;
            else {
                int clone = size++;
                first[clone] = first[q];
                len[clone] = len[p] + 1, isClone[clone] = 1;
                link[clone] = link[q], next[clone] = next[q];
                while (p != -1 && next[p][c] == q)
                    next[p][c] = clone, p = link[p];
                link[q] = link[u] = clone;
            }
        }
        last = u;
    }
};
```

```
// O(N)
unordered_set<int> getTerminals() {
    unordered_set<int> terminals;
    for (int p = last; p; p = link[p])
        terminals.insert(p);
    return terminals;
}
};
```

Trie

// wpt = number of words passing through
// w = number of words ending in the node
// c = character

```
struct Trie {

    struct Node {
        // for lexicographical order use 'map'
        // map<char, Node *> ch;
        unordered_map<char, Node *> ch;
        int w = 0, wpt = 0;
    };

    Node *root = new Node();

    // O(STR.SIZE)
    void insert(string str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c))
                curr->ch[c] = new Node();
            curr->wpt++, curr = curr->ch[c];
        }
        curr->wpt++, curr->w++;
    }

    // O(STR.SIZE)
    Node *find(string &str) {
        Node *curr = root;
        for (auto &c : str) {
            if (!curr->ch.count(c)) return nullptr;
            curr = curr->ch[c];
        }
        return curr;
    }

    // O(STR.SIZE) number of words with given prefix
    int prefixCount(string prefix) {
        Node *node = find(prefix);
        return node ? node->wpt : 0;
    }

    // O(STR.SIZE) number of words matching str
    int strCount(string str) {
        Node *node = find(str);
        return node ? node->w : 0;
    }
};
```

```
// O(N)
void getWords(Node *curr, vector<string> &words,
              string &word) {
    if (!curr) return;
    if (curr->w) words.push_back(word);
    for (auto &c : curr->ch) {
        getWords(c.second, words, word += c.first);
        word.pop_back();
    }
}
```

```
// O(N)
vector<string> getWords() {
    vector<string> words;
    string word = "";
    getWords(root, words, word);
    return words;
}
```

```
// O(N)
vector<string> getWordsByPrefix(string prefix) {
    vector<string> words;
    getWords(find(prefix), words, prefix);
}
```

```
// O(STR.SIZE)
bool remove(Node *curr, string &str, int &i) {
    if (i == str.size()) {
        curr->wpt--;
        return curr->w ? !(curr->w = 0) : 0;
    }
    int c = str[i];
    if (!curr->ch.count(c)) return false;
    if (remove(curr->ch[c], str, ++i)) {
        if (!curr->ch[c]->wpt)
            curr->wpt--, curr->ch.erase(c);
        return true;
    }
    return false;
}
```

```
// O(STR.SIZE)
int remove(string str) {
    int i = 0;
    return remove(root, str, i);
}
};
```

Distinct Substring Count

```
#include "../Data Structures/Strings/Suffix
↪ Automaton.cpp"

// O(N)
int distinctSubstrCount(const string& s) {
    SuffixAutomaton sa(s);
    vector<int> dp(sa.size());
    function<int(int)> dfs = [&](int u) {
        if (dp[u]) return dp[u];
        for (auto& v : sa.next[u]) dp[u] += dfs(v.second);
        return ++dp[u];
    };
    return dfs(0) - 1;
}
```

KMP

```
// p = pattern, t = text
// f = error function, cf = create error function
// pos = positions where pattern is found in text

int MAXN = 1000000;
vector<int> f(MAXN + 1);

vector<int> kmp(string &p, string &t, int cf) {
    vector<int> pos;
    if (cf) f[0] = -1;
    for (int i = cf, j = 0; j < t.size(); ) {
        while (i > -1 && p[i] != t[j]) i = f[i];
        i++, j++;
        if (cf) f[j] = i;
        if (!cf && i == p.size())
            pos.push_back(j - i), i = f[i];
    }
    return pos;
}

vector<int> search(string &p, string &t) {
    kmp(p, p, -1); // create error function
    return kmp(p, t, 0); // search in text
}
```

Lexicographically K-th Substring

```
#include "../Data Structures/Strings/Suffix
↪ Automaton.cpp"

// O(N * ks.size)
vector<string> kthSubstr(string& s, vector<int>& ks) {
    SuffixAutomaton sa(s);
    vector<int> dp(sa.size());
    function<int(int)> dfs = [&](int u) {
        if (dp[u]) return dp[u];
        for (auto& v : sa.next[u]) dp[u] += dfs(v.second);
        return ++dp[u];
    };
}
```

```
dfs(0);
vector<string> ans;
for (auto k : ks) {
    int u = 0;
    string ss;
    while (k)
        for (auto& v : sa.next[u])
            if (k <= dp[v.second]) {
                ss += v.first, u = v.second, k--;
                break;
            } else
                k -= dp[v.second];
    ans.push_back(ss);
}
return ans;
}
```

Lexicographically Smallest Cyclic Shift

```
#include "../Data Structures/Strings/Suffix
↪ Automaton.cpp"

// O(N)
string smallestCyclicShift(const string& s) {
    SuffixAutomaton sa(s + s);
    int k = s.size(), u = 0;
    string ans;
    while (k)
        for (auto& v : sa.next[u]) {
            ans += v.first, u = v.second, k--;
            break;
        }
    return ans;
}
```

Longest Common Substring of 2 Strings

```
#include "../Data Structures/Strings/Suffix
↪ Automaton.cpp"

string lcs(string& a, string& b) {
    SuffixAutomaton sa(a);
    int bestLen = 0, bestPos = -1;
    for (int i = 0, u = 0, l = 0; i < b.size(); i++) {
        while (u && !sa.next[u].count(b[i]))
            u = sa.link[u], l = sa.len[u];
        if (sa.next[u].count(b[i]))
            u = sa.next[u][b[i]], l++;
        if (l > bestLen) bestLen = l, bestPos = i;
    }
    return b.substr(bestPos - bestLen + 1, bestLen);
}
```


Longest Common Substring of Multiple Strings

```
#include <bits/stdc++.h>
using namespace std;

#include "../Data Structures/Strings/Suffix
→ Automaton.cpp"

string lcs(vector<string>& ss) {

}

int main() {

}
```

Number of Occurrences

```
// t: text, ps: patterns
// cnt[u]: size of u's endpos set

#include "../Data Structures/Strings/Suffix
→ Automaton.cpp"

// O(T * Lg(T) + p.size() * ps.size())
vector<int> nOccurrences(string& t,
                        vector<string>& ps) {
    SuffixAutomaton sa(t);
    vector<int> cnt(sa.size(), aux(sa.size()), ans;
    for (int u = 0; u < sa.size; aux[u] = u, u++)
        if (!sa.isClone[u]) cnt[u] = 1;
    sort(aux.begin(), aux.end(), [&](int& a, int& b) {
        return sa.len[b] < sa.len[a];
    });
    for (auto& u : aux)
        if (u) cnt[sa.link[u]] += cnt[u];
    for (auto& p : ps)
        for (int u = 0, i = 0; i < p.size(); i++) {
            if (!sa.next[u].count(p[i])) {
                ans.push_back(0);
                break;
            }
            u = sa.next[u][p[i]];
            if (i + 1 == p.size()) ans.push_back(cnt[u]);
        }
    return ans;
}
```

Occurrences Positions

```
// invLink = inverse suffix-link

#include "../Data Structures/Strings/Suffix
→ Automaton.cpp"
```

```
// O(T + OCCURRENCES(ps[i]) * ps.size())
vector<vector<int>>> occurrencesPos(
    string& t, vector<string>& ps) {
    SuffixAutomaton sa(t);
    vector<vector<int>>> ans, invLink(sa.size);
    for (int u = 1; u < sa.size; u++)
        invLink[sa.link[u]].push_back(u);
    function<void(int, int, vector<int>&)> dfs =
        [&](int u, int pLen, vector<int>& oc) {
            if (!sa.isClone[u])
                oc.push_back(sa.first[u] - pLen + 1);
            for (auto& v : invLink[u]) dfs(v, pLen, oc);
        };

    for (auto& p : ps)
        for (int u = 0, i = 0; i < p.size(); i++) {
            if (!sa.next[u].count(p[i])) {
                ans.push_back({});
                break;
            }
            u = sa.next[u][p[i]];
            if (i + 1 == p.size()) {
                vector<int> oc;
                dfs(u, p.size(), oc), ans.push_back(oc);
            }
        }
    return ans;
}
```

Rabin Karp

```
class RollingHash {
public:
    vector<unsigned long long int> pow;
    vector<unsigned long long int> hash;
    unsigned long long int B;

    RollingHash(const string &text) : B(257) {
        int N = text.size();
        pow.resize(N + 1);
        hash.resize(N + 1);
        pow[0] = 1;
        hash[0] = 0;
        for (int i = 1; i <= N; ++i) {
            // in c++ an unsigned long long int is
            // automatically modulated by 2^64
            pow[i] = pow[i - 1] * B;
            hash[i] = hash[i - 1] * B + text[i - 1];
        }
    }

    unsigned long long int getWordHash() {
        return hash[hash.size() - 1];
    }
}
```



```

unsigned long long int getSubstrHash(int begin,
                                    int end) {
    return hash[end] -
           hash[begin - 1] * pow[end - begin + 1];
}

int size() { return hash.size(); }
};

vector<int> rabinKarp(RollingHash &rhStr,
                    string &pattern) {
    vector<int> positions;
    RollingHash rhPattern(pattern);
    unsigned long long int patternHash =
        rhPattern.getWordHash();
    int windowSize = pattern.size(), end = windowSize;
    for (int i = 1; end < rhStr.size(); i++) {
        if (patternHash == rhStr.getSubstrHash(i, end))
            positions.push_back(i);
        end = i + windowSize;
    }
    return positions;
}

```

Techniques

Binary Search

Binary Search

```

/* if e in v and lower = false (upper_bound):
   r = position of e in v
   l = r + 1
if e in v and lower = true (lower_bound):
   l = position of e in v
   r = l - 1
if e not in v and inv = false it means that:
   v[r] < e < v[l]
if e not in v and inv = true it means that:
   v[r] > e > v[l] */

```

```

// O(lg(r - l)) [l, r]
template <class T>
vector<T> bSearch(vector<T> &v, T e, int l, int r,
                 bool lower = 0, bool inv = 0) {
    int ll = l, rr = r;
    while (l <= r) {
        int mid = l + (r - l) >> 1;
        if (e < v[mid]) inv ? l = mid + 1 : r = mid - 1;
        else if (e > v[mid])
            inv ? r = mid - 1 : l = mid + 1;
        else
            lower ? r = mid - 1 : l = mid + 1;
    } // bSearch[0] tells if the element was found
    return {lower ?
           v[min(rr, l)] == e : v[max(ll, r)] == e, r, l};
}

```

Bitonic Search

```

/* assumes that the bitonic point is the greastest
 * value in v*/

#include "Binary Search.cpp"

template <class T>
vector<vector<int>>> bitonicSearch(vector<T> &v, T e) {
    int l = 0, r = v.size() - 1, mid;
    while (l <= r) {
        mid = l + (r - l) / 2;
        if (!mid || (mid >= v.size() - 1)) break;
        if (v[mid - 1] <= v[mid] && v[mid] > v[mid + 1])
            break;
        if (v[mid - 1] <= v[mid] && v[mid] <= v[mid + 1])
            l = mid + 1;
        if (v[mid - 1] > v[mid] && v[mid] > v[mid + 1])
            r = mid - 1;
    } // at the end of the loop mid = bitonic point
    return {
        bSearch<T>(v, e, 0, mid),
        bSearch<T>(v, e, mid, v.size() - 1, false,
            ↪ true)};
}

```

C++ lower bound

```

// ans[0] = true if e is in v else false
// ans[1] = index pointing to the first element in
// the range [l, r) which compares >= to e.
template <class T>
vector<int> lowerBound(vector<T>& v, T e, int l,
                     int r) {
    auto it = v.begin();
    int i = lower_bound(it + l, it + r, e) - it;
    return {v[i] == e, i};
}

```

C++ upper bound

```
// ans[0] = true if e is in v else false
// ans[1] = index pointing to the first element in
// the range [l, r) which compares > to e.
template <class T>
vector<int> upperBound(vector<T>& v, T e, int l,
                      int r) {
    auto it = v.begin();
    int i = upper_bound(it + l, it + r, e) - it;
    return {v[i - 1] == e, i};
}
```

Lower Bound

```
// ans[0] = true if e is in v else false
// ans[1] = index pointing to the first element in
// the range [l, r] which compares >= to e.

// O(lg(r - l)) [l, r]
template <class T>
vector<int> lowerBound(vector<T>& v, T e, int l,
                      int r) {
    int rr = r;
    while (l <= r) {
        int mid = l + (r - l) >> 1;
        e <= v[mid] ? r = mid - 1, l = mid + 1;
    }
    return {v[min(rr, l)] == e, l};
}
```

Upper Bound

```
// ans[0] = true if e is in v else false
// ans[1] = index pointing to the first element in
// the range [l, r] which compares > to e.

// O(lg(r - l)) [l, r]
template <class T>
vector<int> upperBound(vector<T>& v, T e, int l,
                      int r) {
    int ll = l;
    while (l <= r) {
        int mid = l + (r - l) >> 1;
        e < v[mid] ? r = mid - 1, l = mid + 1;
    }
    return {v[max(ll, r)] == e, l};
}
```

Multiple Queries

Mo

```
// q = query
// qs = queries
```

```
struct Query {
    int l, r;
};

int blksize;
vector<Query> qs;
vector<int> arr;

void initVars(int N, int M) {
    arr = vector<int>(N);
    qs = vector<Query>(M);
}

bool cmp(Query &a, Query &b) {
    if (a.l == b.l) return a.r < b.r;
    return a.l / blksize < b.l / blksize;
}

void getResults() {
    blksize = (int)sqrt(arr.size());
    sort(qs.begin(), qs.end(), cmp);
    int prevL = 0, prevR = -1;
    int sum = 0;
    for (auto &q : qs) {
        int L = q.l, R = q.r;
        while (prevL < L) {
            sum -= arr[prevL]; // problem specific
            prevL++;
        }
        while (prevL > L) {
            prevL--;
            sum += arr[prevL]; // problem specific
        }
        while (prevR < R) {
            prevR++;
            sum += arr[prevR]; // problem specific
        }
        while (prevR > R) {
            sum -= arr[prevR]; // problem specific
            prevR--;
        }

        cout << "sum[" << L << ", " << R << "] = " << sum
              << endl;
    }
}

int main() {
    initVars(9, 2);
    arr = {1, 1, 2, 1, 3, 4, 5, 2, 8};
    qs = {{0, 8}, {3, 5}};
    getResults();
}
```

SQRT Decomposition

```
// sum of elements in range
int neutro = 0;
vector<int> arr;
vector<int> blks;

void initVars(int n) {
    arr.assign(n, neutro);
    blks.assign(sqrt(n), neutro);
}

void preprocess() {
    for (int i = 0, j = 0; i < arr.size(); i++) {
        if (i == blks.size() * j) j++;
        blks[j - 1] += arr[i]; // problem specific
    }
}

// problem specific
void update(int i, int val) {
    blks[i / blks.size()] += val - arr[i];
    arr[i] = val;
}

int query(int l, int r) {
    int sum = 0;
    int lblk = l / blks.size();
    if (l != blks.size() * lblk++) {
        while (l < r && l != lblk * blks.size()) {
            sum += arr[l]; // problem specific
            l++;
        }

        while (l + blks.size() <= r) {
            sum += blks[l / blks.size()]; // problem specific
            l += blks.size();
        }

        while (l <= r) {
            sum += arr[l]; // problem specific
            l++;
        }
    }
    return sum;
}

int main() {
    initVars(10);
    arr = {1, 5, 2, 4, 6, 1, 3, 5, 7, 10};
    preprocess();
    for (int i = 0; i < blks.size() + 1; i++)
        cout << blks[i] << " ";
    // output: 8 11 15 10
    cout << endl;
    cout << query(3, 8) << " ";
    cout << query(1, 6) << " ";
    update(8, 0);
    cout << query(8, 8) << endl;
    // output: 26 21 0
    return 0;
}
```

Trees And Heaps

Red Black Tree

```
template <class K, class V>
struct RedBlackTree {

    struct Node {
        K key;
        V val;
        Node *l, *r; // left, right
        bool isRed;
        Node(K k, V v, bool isRed)
            : key(k), val(v), isRed(isRed) {}
    };

    Node *root = nullptr;

    int compare(K a, K b) {
        if (a < b) return -1;
        if (a > b) return 1;
        return 0;
    }

    // O(lg(N))
    V at(K key) {
        Node *x = root;
        while (x) {
            int cmp = compare(key, x->key);
            if (!cmp) return x->val;
            if (cmp < 0) x = x->l;
            if (cmp > 0) x = x->r;
        }
        throw runtime_error("Key doesn't exist");
    }

    Node *rotateLeft(Node *h) {
        Node *x = h->r;
        h->r = x->l;
        x->l = h;
        x->isRed = h->isRed;
        h->isRed = 1;
        return x;
    }

    Node *rotateRight(Node *h) {
        Node *x = h->l;
        h->l = x->r;
        x->r = h;
        x->isRed = h->isRed;
        h->isRed = 1;
        return x;
    }

    void flipColors(Node *h) {
        h->isRed = 1;
        h->l->isRed = 0;
        h->r->isRed = 0;
    }
}
```

```

// O(lg(N))
Node *insert(Node *h, K key, V val) {
    if (!h) return new Node(key, val, 1);
    int cmp = compare(key, h->key);
    if (!cmp) h->val = val;
    if (cmp < 0) h->l = insert(h->l, key, val);
    if (cmp > 0) h->r = insert(h->r, key, val);
    if (h->r && h->r->isRed && !(h->l && h->l->isRed))
        h = rotateLeft(h);
    if (h->l && h->l->isRed && h->l->l &&
        h->l->l->isRed)
        h = rotateRight(h);
    if (h->l && h->l->isRed && h->r && h->r->isRed)
        flipColors(h);
    return h;
}

// O(lg(N))
void insert(K key, V val) {
    root = insert(root, key, val);
}
};

```

Problems Solved

Maths

N as Sum of different numbers from 1 to M

```

int main() {
    int t;
    scanf("%d", &t);
    for (int j = 1; j <= t; j++) {
        lli x;
        scanf("%lld", &x);
        lli count = 0;
        V divisors = getDivisors(2LL * x);
        for (auto &div : divisors) {
            double d =
                ((double)x / div) + ((1.00 - div) / 2.00);
            if (d > 0 && div > 1 && d == floor(d) &&
                div < x &&
                (2 * d * div + (div - 1) * div == 2 * x)) {
                count++;
            }
        }
        printf("case %d: %lld\n", j, count);
    }
}

```

Extras

Data Structures

Strings

Suffix Automaton

- Each node u corresponds to an endpos-equivalence class for which its longest suffix corresponds to the path from root to u .

C++

Automatic Type Conversions

When performing operations with different primitive data types in C++, the operand with smaller rank gets converted to the data type of the operand with the greater rank.

Data Type Ranks	
Rank	Data Type
1	<code>bool</code>
2	<code>char</code> , <code>signed char</code> , <code>unsigned char</code>
3	<code>short int</code> , <code>unsigned short int</code>
4	<code>int</code> , <code>unsigned int</code>
5	<code>long int</code> , <code>unsigned long int</code>
6	<code>long long int</code> , <code>unsigned long long int</code>
7	<code>__int128_t</code>
8	<code>float</code>
9	<code>double</code>
10	<code>long double</code>

Source: <https://en.cppreference.com/w/c/language/conversion>

Integer Types Properties

Properties	
Width in bits	Data Type
8	<code>bool</code>
At least 8	<code>char</code> , <code>signed char</code> , <code>unsigned char</code>
At least 16	<code>short int</code> , <code>unsigned short int</code>
At least 16	<code>int</code> , <code>unsigned int</code>
At least 32	<code>long int</code> , <code>unsigned long int</code>
At least 64	<code>long long int</code> , <code>unsigned long long int</code>
128	<code>__int128_t</code>

Sources:

<https://en.cppreference.com/w/cpp/language/types>

<https://www.geeksforgeeks.org/c-data-types/>

<https://stackoverflow.com/questions/2064550/c-why-bool-is-8-bits-long>

Maths

Common Sums

$\sum_{k=0}^n k = \frac{n(n+1)}{2}$	$\sum_{k=0}^n k^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{k=0}^n k^3 = \frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^n k^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$		$\sum_{k=0}^n a^k = \frac{1-a^{n+1}}{1-a}$
$\sum_{k=0}^n ka^k = \frac{a[1-(n+1)a^n+na^{n+1}]}{(1-a)^2}$	$\sum_{k=0}^n k^2a^k = \frac{a[(1+a)-(n+1)^2a^n+(2n^2+2n-1)a^{n+1}-n^2a^{n+2}]}{(1-a)^3}$	
$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}, a < 1$	$\sum_{k=0}^{\infty} ka^k = \frac{a}{(1-a)^2}, a < 1$	$\sum_{k=0}^{\infty} k^2a^k = \frac{a^2+a}{(1-a)^3}, a < 1$
$\sum_{k=0}^{\infty} \frac{1}{a^k} = \frac{a}{a-1}, a > 1$	$\sum_{k=0}^{\infty} \frac{k}{a^k} = \frac{a}{(a-1)^2}, a > 1$	$\sum_{k=0}^{\infty} \frac{k^2}{a^k} = \frac{a^2+a}{(a-1)^3}, a > 1$
$\sum_{k=0}^{\infty} \frac{a^k}{k!} = e^a$	$\sum_{k=0}^n \binom{n}{k} = 2^n$	$\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$

Logarithm Rules

$\log_b(b^k) = k$	$\log_b(1) = 0$	$\log_b(X) = \frac{\log_c(X)}{\log_c(b)}$
$\log_b(X \cdot Y) = \log_b(X) + \log_b(Y)$	$\log_b\left(\frac{X}{Y}\right) = \log_b(X) - \log_b(Y)$	$\log_b(X^k) = k \cdot \log_b(X)$