# Inference for Regression with Variables Generated by AI or Machine Learning

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# Outline

# Motivation

Economists now routinely generate variables by AI/ML methods

- quantify unstructured data (text, images, ...)
- measure subtle concepts (uncertainty, sentiment, ...)
- generate variables previously too costly, labor-intensive, or infeasible to collect

The generated variables are inputs to downstream econometric models

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The generated variables are inputs to downstream econometric models

Almost all papers use a two-step strategy:

1. Generate estimate $\hat{\theta}_i$ of true variable $\theta_i$ using AI/ML algorithm
2. Plug estimates $(\hat{\theta}_i)$ into an econometric model, treating $\hat{\theta}_i$ as regular numeric data

# Examples

## Supervised Learning (Impute a Missing Label)

- Baker Bloom Davis (2016): economic policy uncertainty measured from newspaper text
- Gorodnichenko Pham Talavera (2023): tone-of-voice measured from FOMC press conferences
- Adukia et. al. (2023): race and gender of children book characters

## Unsupervised Learning (Learn Latent Representation)

- Hoberg Phillips (2016): latent industry type measured from corporate filings
- Hansen McMahon Prat (2018): policy deliberation measured from FOMC transcripts
- Magnolfi McClure Sorensen (2022): product differentiation measured from survey data
- Compiani Morozov Seiler (2023): substitutability measured from Amazon text + image data
- Gabaix Koijen Yogo (2023): firm characteristics measured from investor holdings

# This Paper

1. Two-step strategy leads to invalid inference: CIs have right width but wrong centering (bias)

   Bias depends on relative importance of

   (a) measurement error in $\hat{\theta}_i$
   (b) sampling error in downstream model

   Valid two-step inference requires (a) $\ll$ (b)

   This is not the case in most leading applications

2. Two solutions: bias correction + one-step strategy
   **NB**: Measurement error is AI/ML-generated variables in non-classical.

3. Shows empirical relevance in several empirical applications

# Related Work

Recent work (mainly stats/poli sci) has pointed out potential for generated variables to cause problems

- General ML-generated variables: Fong and Tyler (2021), Allon et al. (2023), Angelopoulos et al. (2023a, 2023b), Zhang et al. (2023), Zrnic and Candès (2024), and Miao and Lu (2024)
- Variables generated by LLMs: Egami et al. (2023, 2024), Ludwig et al. (2025), Carlson and Dell (2025).

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- Variables generated by LLMs: Egami et al. (2023, 2024), Ludwig et al. (2025), Carlson and Dell (2025).

These works propose bias corrections assuming a validation subsample in which $(Y_i, \boldsymbol{\theta}_i, \hat{\boldsymbol{\theta}}_i)$ are observed

- But one can estimate the model using $(Y_i, \boldsymbol{\theta}_i)$ in validation sample!

  AI/ML gen. vars only helpful for efficiency
- Not feasible in most economic use cases where $\boldsymbol{\theta}_i$ is truly latent (uncertainty, sentiment, ...)

# Outline

# Setup

**Want:** perform inference on $\gamma$ and/or $\alpha$ in the model

$$Y_i = \gamma^T \boldsymbol{\theta}_i + \boldsymbol{\alpha}^T \mathbf{q}_i + \varepsilon_i, \qquad \mathbb{E}\left[\varepsilon_i \middle| \boldsymbol{\theta}_i, \mathbf{q}_i\right] = 0,$$

- $\boldsymbol{\theta}_i$ is a latent variable of economic interest
- $\mathbf{q}_i$ are observed numeric covariates
- Unstructured dataset $\mathbf{x}_i$ available for estimating $\boldsymbol{\theta}_i$

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**Two-Step Strategy**:

1. Estimate $\hat{\boldsymbol{\theta}}_i$ of $\boldsymbol{\theta}_i$ obtained from unstructured data $\mathbf{x}_i$
2. Regress $Y_i$ on $\hat{\boldsymbol{\theta}}_i$ and $\mathbf{q}_i$. Perform inference treating $\hat{\boldsymbol{\theta}}_i$ as regular numeric data.

# Example 1: AI/ML-Generated Labels

- ML algorithms often deployed to impute missing observations from unstructured data.

  Goldsmith-Pinkham and Shue (2023), Adams-Prassl et. al. (2023), Argyle et al. (2025), and Wu and Yang (2024)

- Leading use case: missing $\theta_i$ is binary (e.g., race indicator)

- Generate estimate $\hat{\theta}_i$ of $\theta_i$ using unstructured data $\mathbf{x}_i$ (e.g., voter registration data)

- Regress $Y_i$ on $\hat{\theta}_i$ and controls $\mathbf{q}_i$

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- Regress $Y_i$ on $\hat{\theta}_i$ and controls $\mathbf{q}_i$

- Measurement error due to misclassification error:

$$\Pr(\theta_i = 1 | \mathbf{x}_i, \mathbf{q}_i) \neq \Pr(\hat{\theta}_i = 1 | \mathbf{x}_i, \mathbf{q}_i)$$

## Example 2: Dimensionality Reduction

- Obtain low-dimensional representation of unstructured data which is plugged into regression:
  - Text data: Hansen McMahon Prat (2018); Mueller and Rauh (2018); Larsen and Thorsrud (2019); Thorsrud (2020); Bybee Kelly Manela Xiu (2024); Ash Morelli Vannoni (2025)
  - Survey data: Bandiera Prat Hansen Sadun (2020); Draca and Schwarz (2020)
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- Obs $i$ is a $V$-dim vector of feature counts $\mathbf{x}_i$

- Factor structure on multinomial probabilities (as in probabilistic latent semantic analysis/LDA):

$$\mathbf{x}_i | (C_i, \boldsymbol{\vartheta}_i) \sim \text{Multinomial}(C_i, \mathbf{B}^T \boldsymbol{\vartheta}_i)$$

  - $\mathbf{B}^T = [\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_K]$, each $\boldsymbol{\beta}_k \in \Delta^{V-1}$ is a topic
  - observation-specific topic weights $\boldsymbol{\vartheta}_i \in \Delta^{K-1}$
  - subset of interest: $\boldsymbol{\theta_i} = \boldsymbol{S}\boldsymbol{\vartheta}_i$

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- Measurement error due to upstream sampling error in $\hat{\boldsymbol{\theta}}_i$

# Example 3: Indices

- Several influential works generate indices by classifying documents + aggregating

  Baker Bloom Davis (2016), Caldara and Iacoviello (2022), Gorodnichenko Pham Talavera (2023)

- Each month observe $C_i$ documents (e.g., set of newspapers)
- Of these, $X_i$ are classified as pertaining to concept (e.g., policy uncertainty)
- Latent true uncertainty $\theta_i \in [0, 1]$
- Naive estimator: $\hat{\theta}_i = X_i / C_i$ (cf. BBD's EPU measure)

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- **Problem**: $\hat{\theta}_i$ is a signal of $\theta_i$
- e.g., could change set of newspapers and get a different (but related) measure

# Example 3: Indices

- Topic model representation:

$$\mathbf{x}_i | (C_i, \boldsymbol{\vartheta}_i) \sim \text{Multinomial}(C_i, \mathbf{B}^T \boldsymbol{\vartheta}_i),$$

for $\mathbf{x}_i = (X_i, C_i - X_i)^T$,

$$\mathbf{B}^T = \underbrace{\begin{bmatrix} \beta_1 & \beta_0 \\ (1-\beta_1) & (1-\beta_0) \end{bmatrix}}_{\text{misclass. rates}}, \qquad \boldsymbol{\vartheta}_i = \begin{bmatrix} \theta_i \\ 1-\theta_i \end{bmatrix}$$

- Measurement error due to misclassification error and upstream sampling error

# Outline

# Asymptotics: General Case

- Consider a sequence of DGPs for $(Y_i, \boldsymbol{\theta}_i, \hat{\boldsymbol{\theta}}_i, \mathbf{q}_i, \mathbf{x}_i)_{i=1}^n$ indexed by sample size $n$, in which

$$\sqrt{n}\left[\frac{1}{n}\sum_{i=1}^{n}\hat{\boldsymbol{\theta}}_i(\hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_i)^T\right] \rightarrow_p \kappa\,\boldsymbol{\Omega},$$

  (expressions are DGP-specific)

- Scalar $\kappa \geq 0$ measures the importance of measurement error relative to sampling error
- Positive $\kappa$ allows both sampling error and measurement error to play a role
- Reflects prevailing trend: increasingly large data sets + increasingly accurate algorithms

# Asymptotics: $\kappa$ and $\Omega$

- ML-generated binary labels:

$$\sqrt{n} \times \underbrace{\mathbb{E}\left[\hat{\theta}_i(1-\theta_i)\right]}_{\text{false-positive rate}} \to \kappa, \qquad \boldsymbol{\Omega} = 1$$

- Topic models:

$$\sqrt{n} \times \mathbb{E}\left[\frac{1}{C_i}\right] \to \kappa, \qquad \boldsymbol{\Omega} = \boldsymbol{S}(\mathbf{B}\mathbf{B}^T)^{-1}\mathbf{B}\,\mathrm{diag}(\mathbf{B}^T\mathbb{E}[\boldsymbol{\vartheta}_i])\mathbf{B}^T(\mathbf{B}\mathbf{B}^T)^{-1}\boldsymbol{S}^T - \mathbb{E}\left[\boldsymbol{\theta}_i\boldsymbol{\theta}_i^T\right]$$

# Theorem on Two-Step Inference

**Theorem: Two-Step Inference is Invalid Unless $\kappa = 0$**

1. OLS estimator $\hat{\boldsymbol{\psi}} = (\hat{\gamma}, \hat{\boldsymbol{\alpha}})$ of $\boldsymbol{\psi} = (\gamma, \boldsymbol{\alpha})$ from regressing $Y_i$ on $\hat{\boldsymbol{\xi}}_i = (\hat{\boldsymbol{\theta}}_i, \mathbf{q}_i)$ has asy dist

$$\sqrt{n}\left(\hat{\boldsymbol{\psi}} - \boldsymbol{\psi}\right) \to_d N\left(-\kappa\mathbb{E}[\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T]^{-1}\begin{pmatrix} \boldsymbol{\Omega} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\boldsymbol{\psi}\,,\ \underbrace{\mathbb{E}[\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T]^{-1}\mathbb{E}[\varepsilon_i^2\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T]\mathbb{E}[\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T]^{-1}}_{=:\mathbf{V}}\right)$$

where $\boldsymbol{\xi}_i = (\boldsymbol{\theta}_i, \mathbf{q}_i)$ are the "true" covariates

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   where $\boldsymbol{\xi}_i = (\boldsymbol{\theta}_i, \mathbf{q}_i)$ are the "true" covariates

2. Eicker–Huber–White standard errors are consistent for all $\kappa \geq 0$:

$$\hat{\mathbf{V}} := \left(\frac{1}{n}\sum_{i=1}^{n} \hat{\boldsymbol{\xi}}_i \hat{\boldsymbol{\xi}}_i^T\right)^{-1} \left(\frac{1}{n}\sum_{i=1}^{n} \hat{\varepsilon}_i^2 \hat{\boldsymbol{\xi}}_i \hat{\boldsymbol{\xi}}_i^T\right) \left(\frac{1}{n}\sum_{i=1}^{n} \hat{\boldsymbol{\xi}}_i \hat{\boldsymbol{\xi}}_i^T\right)^{-1} \to_p \mathbf{V}$$

# Implications

- $\kappa \in (0, \infty)$: two-step inference is **biased**
  - degree of bias is increasing in $\kappa$ (relative importance of measurement vs sampling error)
  - no variance distortion, unlike generated regressors
- $\kappa = 0$: two-step inference is **valid**: treat $\hat{\boldsymbol{\theta}}_i$ as if they are the true latent $\boldsymbol{\theta}_i$
- Take-away: if $\kappa$ is large, consider using resources to improve precision of $\hat{\boldsymbol{\theta}}_i$

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- Take-away: if $\kappa$ is large, consider using resources to improve precision of $\hat{\boldsymbol{\theta}}_i$
- To the extent empirical papers flag concerns about 2-step inference, usually about std errors
- Common intuition is wrong: problem is measurement error not standard errors

# Analogy with Factor-Augmented Regressions

(i) Impute latent factor $\mathbf{F}_t$ from $N$-dim cross-section of predictors $\mathbf{x}_t \rightarrow \hat{\mathbf{F}}_t$

(ii) Regress $y_t$ on $\hat{\mathbf{F}}_t$ and covariates $\mathbf{q}_t$

(Stock and Watson, 2002; Boivin and Bernanke, 2003)

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- **Bai and Ng (2006)**: two-step inference valid if $\sqrt{T}/N \to 0$
  - analogous to $\kappa = 0$
  - $n$ analogous to $T$
  - for topic models, $\mathbb{E}[C_i^{-1}]$ analogous $1/N$

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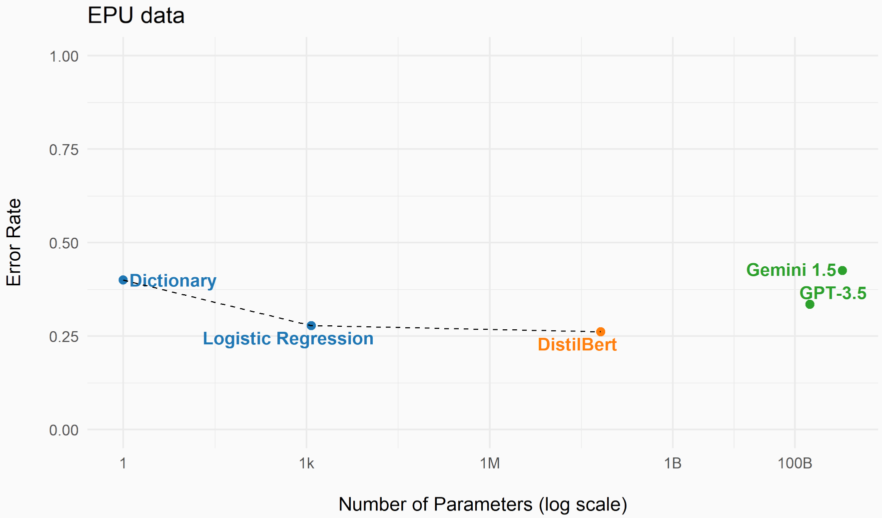
- **Gonçalves and Perron (2014)**: first-order bias when $\sqrt{T}/N \rightarrow c > 0$
  - analogous to $\kappa > 0$
  - validity of residual-based bootstraps

# Measurement Error in Baker, Bloom, and Davis (2016)

| | Classification Labels | |
|---|---|---|
| Human Labels | 0 | 1 |
| 0 | 1486 | 474 |
| 1 | 825 | 802 |

# Errors Remain with Modern Algorithms



## $\kappa$ for Dimensionality Reduction

- Minimum Data Set (MDS) for Nursing Homes
  - 24,000,000 patients
  - $\hat{\kappa} \approx 46$
- Lightcast (formerly Burning Glass) job postings data
  - 45,000,000 observations
  - $\hat{\kappa} \approx 20$
- Nielsen Homescan
  - 40,000 households

    $\hat{\kappa} \approx 3.8$
- US Patents in 2023
  - 315,000 filings
  - $\hat{\kappa} \approx 1$

# Outline

# How to Do Valid Inference

1. **Explicit Bias Correction**: use analytical expressions in Theorem to adjust two-step estimates/CIs

   Advantage: Simple and scalable

   Disadvantage: Not feasible in complex models; poor approximation with large $\kappa$

2. **One-Step Strategy**: MLE using joint likelihood for upstream IR model + regression model

   Advantage: General purpose and flexible

   Disadvantage: More computationally demanding

**NB**: Measurement error is AI/ML-generated variables in non-classical.

## Bias Correction

- First-order asymptotic bias of OLS estimator $\hat{\psi}$ is

$$-\kappa\,\mathbb{E}\left[\boldsymbol{\xi}_i\boldsymbol{\xi}_i^T\right]^{-1}\begin{pmatrix}\boldsymbol{\Omega} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}\psi$$

- Given estimators $\hat{\kappa}$ and $\hat{\boldsymbol{\Omega}}$ of $\kappa$ and $\boldsymbol{\Omega}$, can construct bias-corrected estimators:

Additive $$\hat{\psi}^{bca} = \left(\mathbf{I} + \frac{\hat{\kappa}}{\sqrt{n}}\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\boldsymbol{\xi}}_i\hat{\boldsymbol{\xi}}_i^T\right)^{-1}\begin{bmatrix}\hat{\boldsymbol{\Omega}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)\hat{\psi}$$

Multiplicative $$\hat{\psi}^{bcm} = \left(\mathbf{I} - \frac{\hat{\kappa}}{\sqrt{n}}\left(\frac{1}{n}\sum_{i=1}^{n}\hat{\boldsymbol{\xi}}_i\hat{\boldsymbol{\xi}}_i^T\right)^{-1}\begin{bmatrix}\hat{\boldsymbol{\Omega}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\right)^{-1}\hat{\psi}$$

- Bias-corrected CIs: center at $\hat{\psi}^{bca}$ or $\hat{\psi}^{bcm}$ and use 2-step std errors

# Theory for Bias-Correction

**Validity of Bias-Corrected Inference**

If $\hat{\kappa} \to_p \kappa$ and $\hat{\mathbf{\Omega}} \to_p \mathbf{\Omega}$, the under conditions of previous theorem, have

1. Bias-corrected estimators are asymptotically equivalent and correctly centered

$$\sqrt{n}\left(\hat{\psi}^{bcm} - \psi\right) = \sqrt{n}\left(\hat{\psi}^{bca} - \psi\right) + o_p(1) \to_d N\left(\mathbf{0}\,,\mathbf{V}\right)$$

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2. Bias-corrected CIs have correct coverage:

$$\lim_{n\to\infty} \Pr\left(\psi_i \in \hat{\psi}_i^{bc} \pm 1.96\sqrt{\frac{\hat{\mathbf{V}}_{ii}}{n}}\right) = 0.95.$$

# Bias Correction: Labels Example

- Here need to estimate $\kappa = \lim_{n\to\infty} \sqrt{n}\,\mathbb{E}\left[\hat{\theta}_i(1-\theta_i)\right]$
- Can use validation data, or assess validity externally (as in Bursztyn Chaney Hassan Rao (2024))

$$\hat{\kappa} = \sqrt{n}\widehat{FPR}, \qquad \widehat{FPR} = \frac{1}{m}\sum_{i=1}^{m}\hat{\theta}_i(1-\theta_i)$$

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- We show $\hat{\kappa} \to_p \kappa$ provided $n/m^2 \to 0$ (small subsample)
- Finite-sample correction to standard errors:

$$\tilde{\mathbf{V}}^{bca} = (\mathbf{I} + \widehat{FPR}\,\hat{\mathbf{\Gamma}})\hat{\mathbf{V}}(\mathbf{I} + \widehat{FPR}\,\hat{\mathbf{\Gamma}})' + \frac{1}{m}\widehat{FPR}(1-\widehat{FPR})\hat{\mathbf{\Gamma}}\hat{\mathbf{V}}\hat{\mathbf{\Gamma}}' + \frac{n}{m}\widehat{FPR}(1-\widehat{FPR})\hat{\mathbf{\Gamma}}\hat{\psi}\hat{\psi}'\hat{\mathbf{\Gamma}}',$$

$$\tilde{\mathbf{V}}^{bcm} = (\mathbf{I} - \widehat{FPR}\,\hat{\mathbf{\Gamma}})^{-1}\hat{\mathbf{V}}(\mathbf{I} - \widehat{FPR}\,\hat{\mathbf{\Gamma}})^{-1'} + \frac{1}{m}\widehat{FPR}(1-\widehat{FPR})\hat{\mathbf{\Gamma}}\hat{\mathbf{V}}\hat{\mathbf{\Gamma}}' + \frac{n}{m}\widehat{FPR}(1-\widehat{FPR})\hat{\mathbf{\Gamma}}\hat{\psi}\hat{\psi}'\hat{\mathbf{\Gamma}}',$$

$$\hat{\mathbf{\Gamma}} = \left(\frac{1}{n}\sum_{i=1}^{n}\hat{\boldsymbol{\xi}}_i\hat{\boldsymbol{\xi}}_i^T\right)^{-1}\begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0}\end{pmatrix}$$

# One-Step Strategy: Computation

- Joint likelihood: $f(Y_i, \mathbf{x}_i, \boldsymbol{\theta}_i | \mathbf{q}_i; \gamma, \boldsymbol{\alpha}, ...)$

- Integrated likelihood in terms of observables only:

$$f(Y_i, \mathbf{x}_i | \mathbf{q}_i; \gamma, \boldsymbol{\alpha}, ...) = \underbrace{\int f(Y_i, \mathbf{x}_i, \boldsymbol{\theta}_i | \mathbf{q}_i; \gamma, \boldsymbol{\alpha}, ...)\, d\boldsymbol{\theta}_i}_{\text{intractable}}$$

- Use Bayesian computation:
    - Integrates out $\boldsymbol{\theta}_i$ as part of the sampling algorithm
    - Resulting credible sets are valid frequentist confidence intervals for large $n$ by BvM theorem

- Sampling: Hamiltonian MC implemented in probabilistic programming language `NumPyro`
  $\Rightarrow$ allows for estimation of models on large scale

# Outline

# Hansen Lambert Bloom Davis Sadun Taska (WP, 2023)

- Consider $n = 16,315$ SD food+accom sector (NAICS code 72) job postings from January 2022
- Regress log wages $Y_i$ on ML-generated remote work indicator $\hat{\theta}_i$
- Estimate FPR from a subsample of size $m = 1,000$ postings, $\widehat{FPR} \approx 0.009$
- Fixed effects for SOC code (job type) and full/part time

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- Fixed effects for SOC code (job type) and full/part time
- For 1-step: use 3 component Gaussian mixture for errors $\varepsilon_i | \theta_i$

# Two-Step Estimates Smaller

| | No Fixed Effects | | | With Fixed Effects | | |
|---|---|---|---|---|---|---|
| | Est. | Std Err | 95% CI | Est. | Std Err | 95% CI |
| OLS | 0.648 | 0.024 | [0.599, 0.697] | 0.363 | 0.021 | [0.321, 0.406] |
| BCA | 0.897 | 0.119 | [0.663, 1.131] | 0.521 | 0.080 | [0.362, 0.680] |
| BCM | 1.052 | 0.140 | [0.777, 1.326] | 0.641 | 0.099 | [0.446, 0.836] |
| 1-Step | 0.563 | 0.016 | [0.532, 0.595] | 0.448 | 0.017 | [0.415, 0.480] |

# Bias-Corrected CIs to the right of Two-Step CIs

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| BCM | 1.052 | 0.140 | [0.777, 1.326] | 0.641 | 0.099 | [0.446, 0.836] |
| 1-Step | 0.563 | 0.016 | [0.532, 0.595] | 0.448 | 0.017 | [0.415, 0.480] |

# Outline

# Bandiera Hansen Prat Sadun (JPE, 2020)

- Time-use survey data for 916 CEOs
- 654 combinations of activities (e.g., meeting with suppliers) in 15min intervals
- LDA with $K = 2$: 2 types of CEO behaviors $\boldsymbol{\beta}_1$ (leaders) and $\boldsymbol{\beta}_2$ (managers).
- Two-step strategy: regress log sales $Y_i$ on leader weight $\hat{\theta}_{i,1}$ and firm characteristics $\mathbf{q}_i$.

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Original Paper: $\hat{\kappa} = 0.44$ (average $C_i = 88.4$).

Modified Sample: draw 10% of activities for each CEO (without replacement) $\longrightarrow \hat{\kappa} = 4.26$.

# Similar Estimates in Full Sample

**Table 1:** Estimates of Impact of CEO Behavior on Firm Performance

| Sample | Estimation Strategy | | |
|---|---|---|---|
| | Two-Step | Bias Correction | Joint |
| Full | 0.405 | 0.474 | 0.402 |
| | [0.224, 0.585] | [0.294, 0.655] | [0.240, 0.603] |
| 10% Subsample | 0.227 | 1.054 | 0.439 |
| | [-0.038, 0.492] | [0.789, 1.319] | [0.153, 0.711] |

# Difference in Subsample

**Table 2:** Estimates of Impact of CEO Behavior on Firm Performance

| Sample | Estimation Strategy | | |
|---|---|---|---|
| | Two-Step | Bias Correction | Joint |
| Full | 0.405 | 0.474 | 0.402 |
| | [0.224, 0.585] | [0.294, 0.655] | [0.240, 0.603] |
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# Outline

# Central Bank Communication

- Does written central bank communication drive long rates? Estimate

$$Y_i = \gamma\theta_i + \boldsymbol{\alpha}'\mathbf{q}_i + u_i$$

  - $Y_i$ is the path factor from Gürkaynak, Sack, and Swanson (2005) (mkt perceptions of future rates)
  - $\theta_i$ is a hawkish/dovish index (cf. Gorodnichenko, Pham, Talavera (2023))
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  - classify FOMC sentences as hawkish/dovish/neutral using fine-tuned BERT + aggregate
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$$\hat{\theta}_i = \frac{N_i^H - N_i^D}{N_i^H + N_i^D}$$

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- Compare 1 and 2 step methods over 02/1995-06/2023

# Central Bank Communication: One-Step Effect Size 3x Larger

| | Estimation Strategy | |
|---|---|---|
| | Two-Step | One-Step |
| Sentiment ($\theta_i$) | 0.039 | 0.114 |
| | [0.012, 0.066] | [0.027, 0.198] |
| Policy Rate ($q_i$) | -0.004 | -0.003 |
| | [-0.011, 0.003] | [-0.011, 0.004] |
| $\beta_0$ | | 0.009 |
| | | [0.001, 0.026] |
| $\beta_1$ | | 0.676 |
| | | [0.585, 0.768] |
| Observations | 200 | 200 |
| $R^2$ | 0.0425 | 0.1429 |

# Central Bank Communication: Material Misclassification Error

| | Estimation Strategy | |
|---|---|---|
| | Two-Step | One-Step |
| Sentiment ($\theta_i$) | 0.039 | 0.114 |
| | [0.012, 0.066] | [0.027, 0.198] |
| Policy Rate ($q_i$) | -0.004 | -0.003 |
| | [-0.011, 0.003] | [-0.011, 0.004] |
| $\beta_0$ | | 0.009 |
| | | [0.001, 0.026] |
| $\beta_1$ | | 0.676 |
| | | [0.585, 0.768] |
| Observations | 200 | 200 |
| $R^2$ | 0.0425 | 0.1429 |

# Central Bank Communication: Simulation

| | Bias | | | RMdSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|
| $\sqrt{n}\,\mathbb{E}[C_i^{-1}]$ | 4.57 | 2.28 | 1.14 | 4.57 | 2.28 | 1.14 | 4.57 | 2.28 | 1.14 |
| | | | | $n = 200$ | | | | | |
| 2-Step | -0.433 | -0.218 | -0.037 | 0.048 | 0.025 | 0.018 | 0.378 | 0.824 | 0.931 |
| Joint | -0.003 | 0.007 | 0.004 | 0.024 | 0.020 | 0.018 | 0.945 | 0.948 | 0.938 |
| | | | | $n = 800$ | | | | | |
| 2-Step | -0.215 | -0.041 | 0.084 | 0.024 | 0.010 | 0.012 | 0.507 | 0.942 | 0.894 |
| Joint | 0.004 | -0.006 | -0.006 | 0.011 | 0.010 | 0.010 | 0.956 | 0.950 | 0.950 |
| | | | | $n = 3200$ | | | | | |
| 2-Step | -0.042 | 0.085 | 0.158 | 0.006 | 0.009 | 0.017 | 0.887 | 0.739 | 0.353 |
| Joint | -0.005 | -0.002 | -0.003 | 0.005 | 0.005 | 0.005 | 0.942 | 0.941 | 0.943 |

*Note:* distribution of $C_i$ varying with $n$. Misclassification rates held fixed.

# Outline

# Conclusion

- Empirical work routinely uses AI/ML algorithms to generate new variables
- Common empirical practice leads to invalid inference
- We propose two solutions: bias correction + one-step strategy
- Illustrate important differences in simulations + applications
- Works in progress: specific methods tailored to important use cases
    - VARs and impulse response analysis w/ Hansen and Shin